Shaken, not stirred: test particles in binary black hole mergers.

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Abstract

In 2015 the advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) detected the first ever gravitational event, gravitational wave event GW150914, with multiple new gravitational wave events, originating from both binary neutron stars and binary black hole (BBH) mergers, detected in subsequent years. In light of these detections, we simulate the dynamics of ambient test particles in the gravitational potential well of a BBH system close to its inspiral phase with the goal of simulating the associated electromagnetic radiation and resulting spectral energy density distribution of such a BBH system. This could shed light on possible detection ranges of electromagnetic counterparts to BBH mergers. The potentials are numerically calculated using finite difference methods, under the assumption of non-rotating black holes with the post-Newtonian Paczynski-Wiita potential approximation in tandem with retarded time concepts analogous to electrodynamics. We find that the frequencies of potential electromagnetic radiation produced by these systems are produced by frequencies far too low to be detectible on earth.

Keywords: Binary black hole merger, binary black hole, binary black hole merger simulation, particle acceleration, gravity.

1 The Paczynski-Wiita Potential

The Paczynski-Wiita potential is an important pseudo-Newtonian approximation to the general relativistic Schwarzschild geometry developed by B. Paczynski and P. Wiita while studying thick accretion disks and supercritical luminosities [1]:

$$\Phi_{PW}(r) = -\frac{GM}{r - r_s}, \quad r_s = \frac{2GM}{c^2}. \tag{1}$$

The Paczynski-Wiita potential is a Newton-like potential that (for the case of non-rotating BHs) exactly reproduces the marginally bound ($r_{mb} = 2r_s$) and marginally stable circular ($r_{ms} = 3r_s$) orbits of the Schwarzschild geometry, as well as the form of the Keplerian angular momentum.

2 Solving the effective one-body problem

The effective one-body (EOB) problem entails treating two bodies, with masses M_1 and M_2, with a separation $\vec{r} = \vec{r}_2 - \vec{r}_1$, orbiting a common center of mass as a single body with mass $\mu = \frac{M_1 M_2}{M_1 + M_2}$ under the influence of an external potential due to mass $M = M_1 + M_2$. To first post newtonian order, the analytical solution of the circularized, inspiral BBH orbit can be derived as

$$R(t) = R_0 \left(\frac{t_c - t}{t_c}\right)^{-0.25} \frac{M_c}{M}, \tag{2}$$

$$\Phi(t) = -2\,(5M_c)^{-0.625}\,(t_c - t)^{-0.25}, \tag{3}$$

with t_c the time to coalescense of the BHs, R_0 the initial seperation of the BHs, and $M_c \equiv \frac{(M_1 M_2)^{0.6}}{(M_1+M_2)^{0.2}}$ the chirp mass of the BBHs. From this the position vectors $\vec{r}_{M_1}$ and $\vec{r}_{M_2}$ can be found similarly to the classical EOB problem.

3 Calculating the acceleration of a charged particle in a near-inspiral phase BBH merger

Consider a charged particle, with mass m_e, position $\vec{r}_{m_e}$, and velocity $\vec{v}_{m_e}$ in the lab frame. If the particle is under the influence of a force $\vec{F}$ due to the Paczynski-Wiita potentials Φ_{PW} resulting from the BBHs, calculated at retarded position $\vec{r}_k$ (that is calculated numerically from the definition of retarded time, $t_r = t - \frac{1}{c}|\vec{r} - \vec{r}_k|$, and equation 2 using Newton's method), then we have a set of discritized equations

$$\vec{p}_{i+1} = \vec{F}_i dt + \vec{p}_i, \qquad \vec{v}_i = \frac{\vec{p}_i}{m_e}\sqrt{1 - \frac{p_i^2}{m_e^2 c^2}\left(1 + \frac{p_i^2}{m_e^2 c^2}\right)^{-1}},$$

$$\vec{r}_{i+1} = \vec{v}_i dt + \vec{r}_i, \tag{4}$$

$$\gamma_i = \left(1 - \frac{p_i^2}{m_e^2 c^2}\left(1 + \frac{p_i^2}{m_e^2 c^2}\right)^{-1}\right)^{-0.5}, \quad \text{and} \quad \vec{a}_i = \frac{1}{m_e \gamma}\left[\vec{F}_i - \frac{\left(\vec{v}_i \cdot \vec{F}_i\right)}{c^2}\vec{v}_i\right],$$

that describes the evolution of the particle in a near-inspiral BBH system.

4 Calculating the SED of charged particles in a near-inspiral BBH merger

The spectral energy distribution (SED) of a single accelerated charged particle is given by

$$I(\nu) = \frac{\mu_0 q^2}{3\pi c}\left[\left|\gamma^2(\nu)\,\vec{a}_{\parallel}(\nu)\right|^2 + \left|\gamma^3(\nu)\,\vec{a}_{\perp}(\nu)\right|^2\right]. \tag{5}$$

From this, it is evident that $I_{\parallel,k} = \left|\gamma^2(\nu_k)\,\vec{a}_{\parallel}(\nu_k)\right|^2$, and $I_{\perp,k} = \left|\gamma^3(\nu_k)\,\vec{a}_{\perp}(\nu_k)\right|^2$ needs to be determined in frequency space. This is done by simply taking the nonuniform discrete Fourier transform of the relevant products

$$I_{\parallel,k} = \sum_{j=0}^{N-1}\left|\gamma_i^2 \vec{a}_{\parallel,j}\right|^2 e^{-i\nu_k t_j}, \quad I_{\perp,k} = \sum_{j=0}^{N-1}\left|\gamma_j^3 \vec{a}_{\perp,j}\right|^2 e^{-i\nu_k t_j}, \tag{6}$$

with $i = \sqrt{-1}$ and N is the number of data points. From this

$$I(\nu_k) = \frac{\mu_0 q^2}{3\pi c}\left[I_{\parallel,k} + I_{\perp,k}\right], \tag{7}$$

where ν_k is the set of N preditermined frequencies in the range $[-\frac{1}{2t_N}, \frac{1}{2t_N}]$. The total SED from the system is found by summing over all of the particles evolving through the system.

5 Implementing the model in code

The model described in the earlier sections of this chapter is implemented in two separate programs. The first program is written in C++, with the time evolution of each of the position, velocity and acceleration vectors, as well as that of the Lorentz factor, saved as separate output datasets calculated using a embedded Runge Kutta 7(8) adaptive timestep method. The second program is a *Python* program that takes the acceleration and Lorentz factor datasets of the previous program as input, and calculates the Fourier transform of each particle (as described in Section 4). This is done by using the fiNUFFT non-uniform FFT package [2, 3].

6 Results

The model, briefly described above, is applied to a BBH system with masses $M_1 = 30M_\odot$ and $M_2 = 35M_\odot$. The initial separation of the BHs are taken to be $r_0 = 1.0 \times 10^8$cm. A set of 25 particles are randomly distributed in the system, with a uniform probability distribution.

Figure 1 shows the time evolution of the speed of three different particles evolved in the system, with 2 of the particles falling into one of the BHs and a third leaving the system. Figure 2 shows the components of the acceleration parallel and perpendicular to the velocity of 3 typical particles evolving through the system. Figure 3 shows the total SED of the system calculated by summing over the SEDs of all 25 particles.

7 Discussion and Conclusion

From the results given in the previous section, it becomes evident that the bulk of possible EM radiation that originates from charged particles, accelerated in inspiral BBH merger systems (determined from our model), is distributed at frequencies well below the operating ranges of current radio telescopes. The current operating range of the Low-Frequency Array (LOFAR) is $10 - 240MHz$ [4]. Figure 3 illustrates how the total SED calculated from the model drops off at 10^3Hz, which lies below the frequency range of LOFAR. If we now consider that the inter stellar medium (ISM) plasma frequency is $\backsim 2kHZ$ (for an assumed electron density of $0.03cm^{-3}$), we know that radiation emitted at frequencies below this threshold will be absorbed by the ISM, and, therefore, the bulk of radiation emitted by the system will be absorbed by the ISM.

8 Future Work

For this model we have also that neither of the BHs have a Magnetosphere, with only gravity acting in on the particles distributed within the system. If at least one of the BHs are significantly magnetized, particle dynamics within the system will likely be dominated by the resulting Lorentz forces, rather than the gravitational forces.

References

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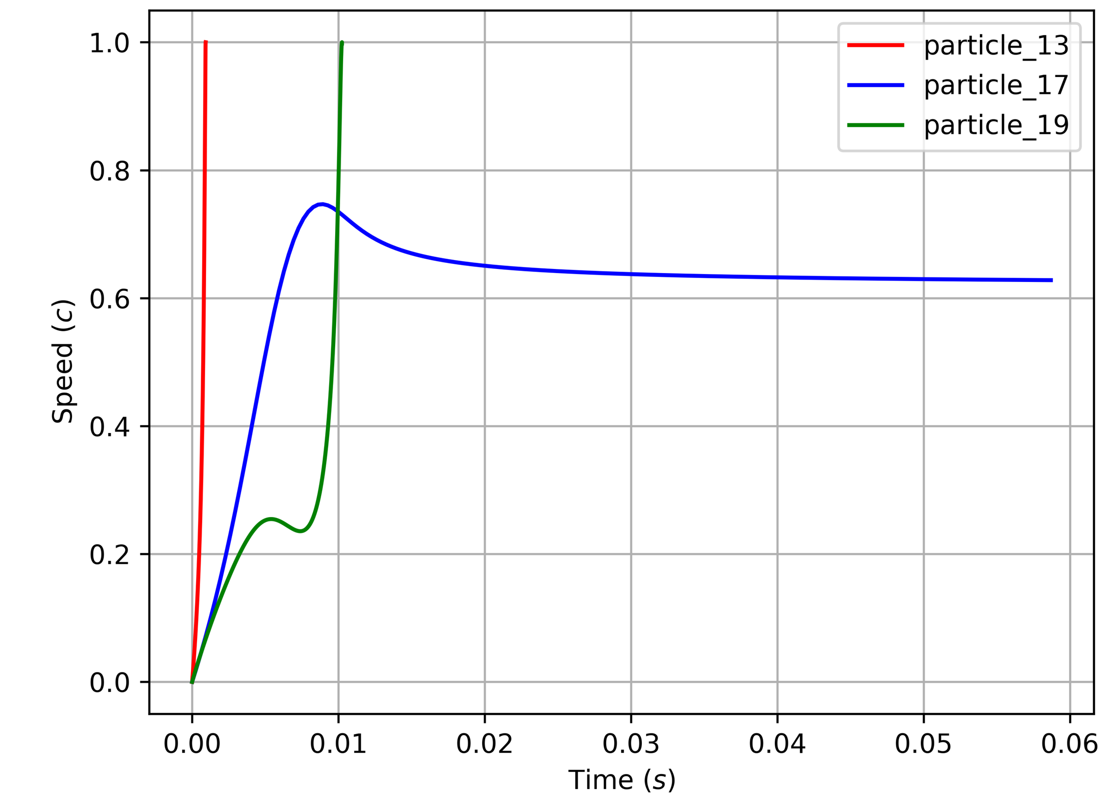

Figure 1: Plot of the speed as a function of time of three different particles evolved using the model.

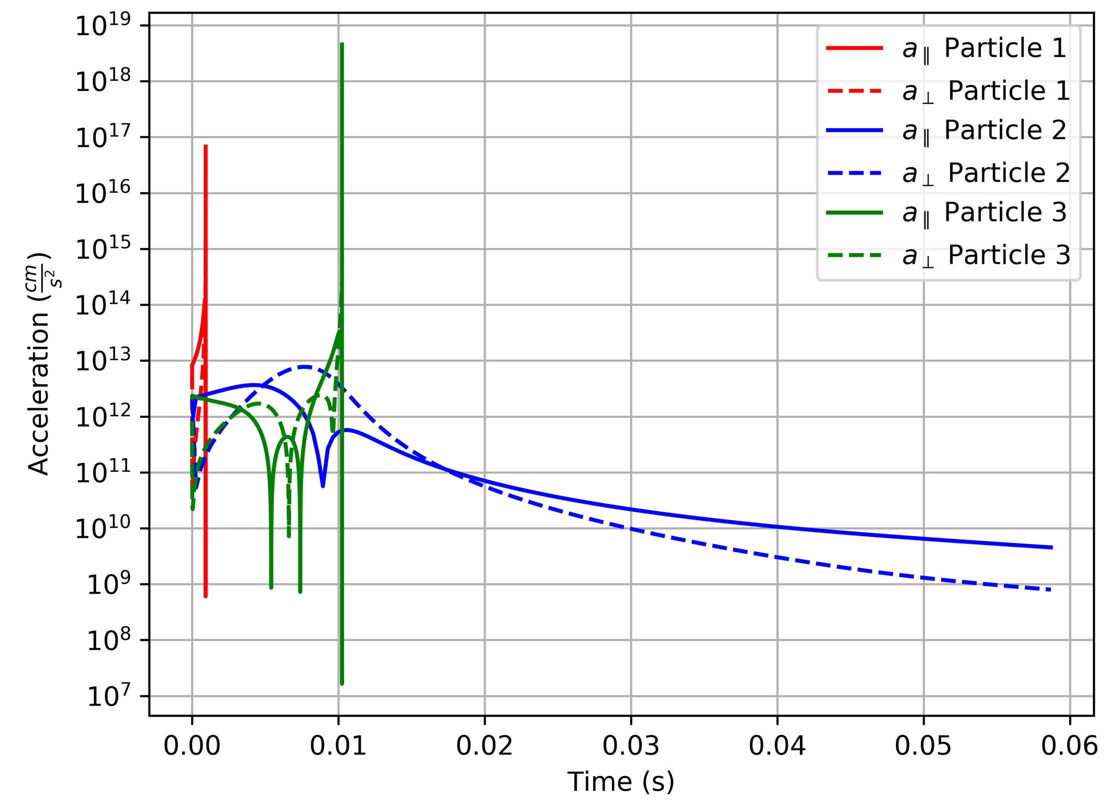

Figure 2: Acceleration components as a function of time for three seperate particles evolved using the model.

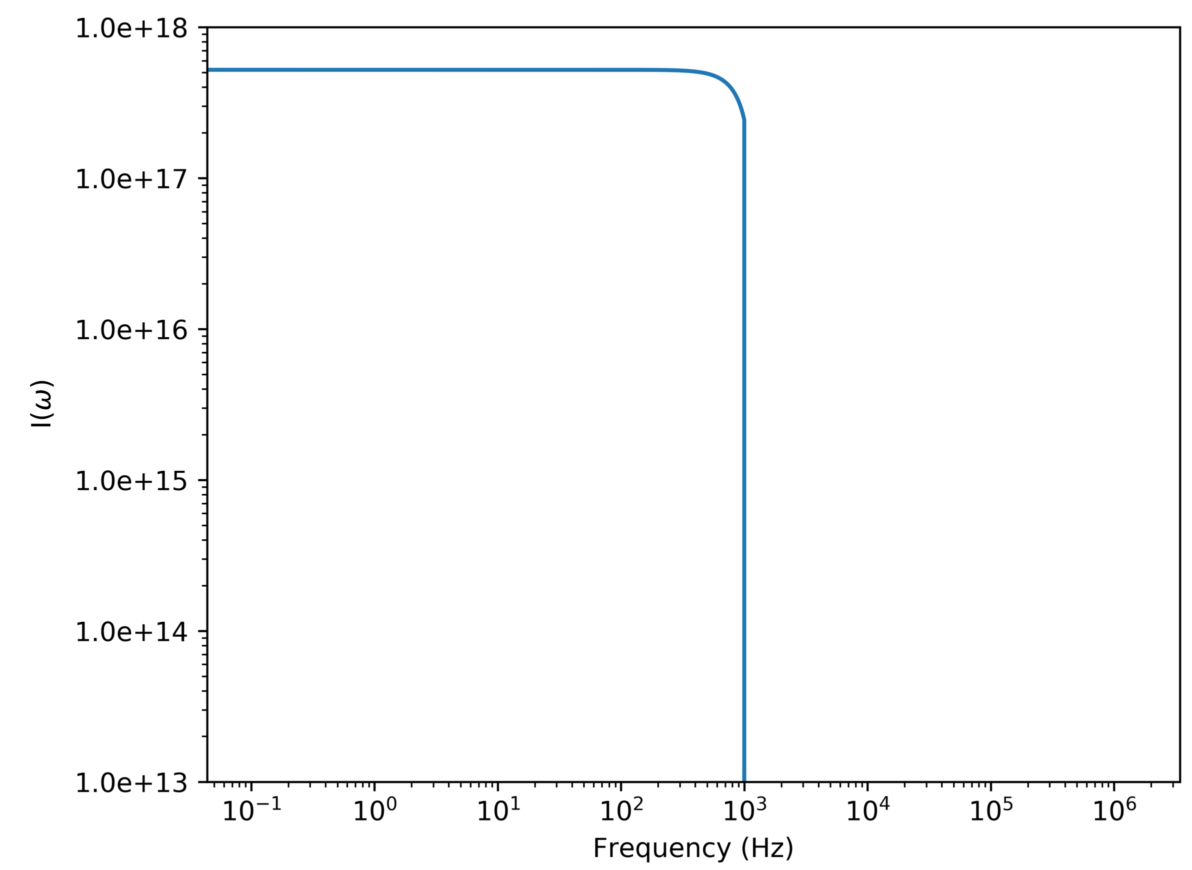

Figure 3: Total SED, determined by integrating over all 50 particles distributed in the system. There is a steep drop off that occurs at $\sim 10^5$Hz. In addition to the drop off, there is also a cut of at $\sim 10^6$Hz that occurs due to limitations on the numerical resolution.