The Higgs: a promising portal to New Physics

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Based on: ***I. Asiáin, D .Espriu and F .Mescia. Phys. Rev. D 105, 015009***

ICCUB Winter Meeting 2022

Motivation

SM issues:

From observations:

- Only $\sim 5\,\%$ is explained by **ordinary matter**

  $\sim 27\,\%$ of **dark matter** which we do not know

  massive neutrinos are not predicted

Internal:

- **Flavour puzzle:** 22 free parameters

  CP violation in the strong sector: $\theta_{QCD} \approx 0$

  EW **Hierarchy problem**: $m_h \lll M_{pl}$

  Gravity?

Motivation

The SM **Higgs boson**:

- **predicted** in 1964
- Gives masses to EW gauge bosons (W, Z) **EWSBS**
- Where does it **come from**?

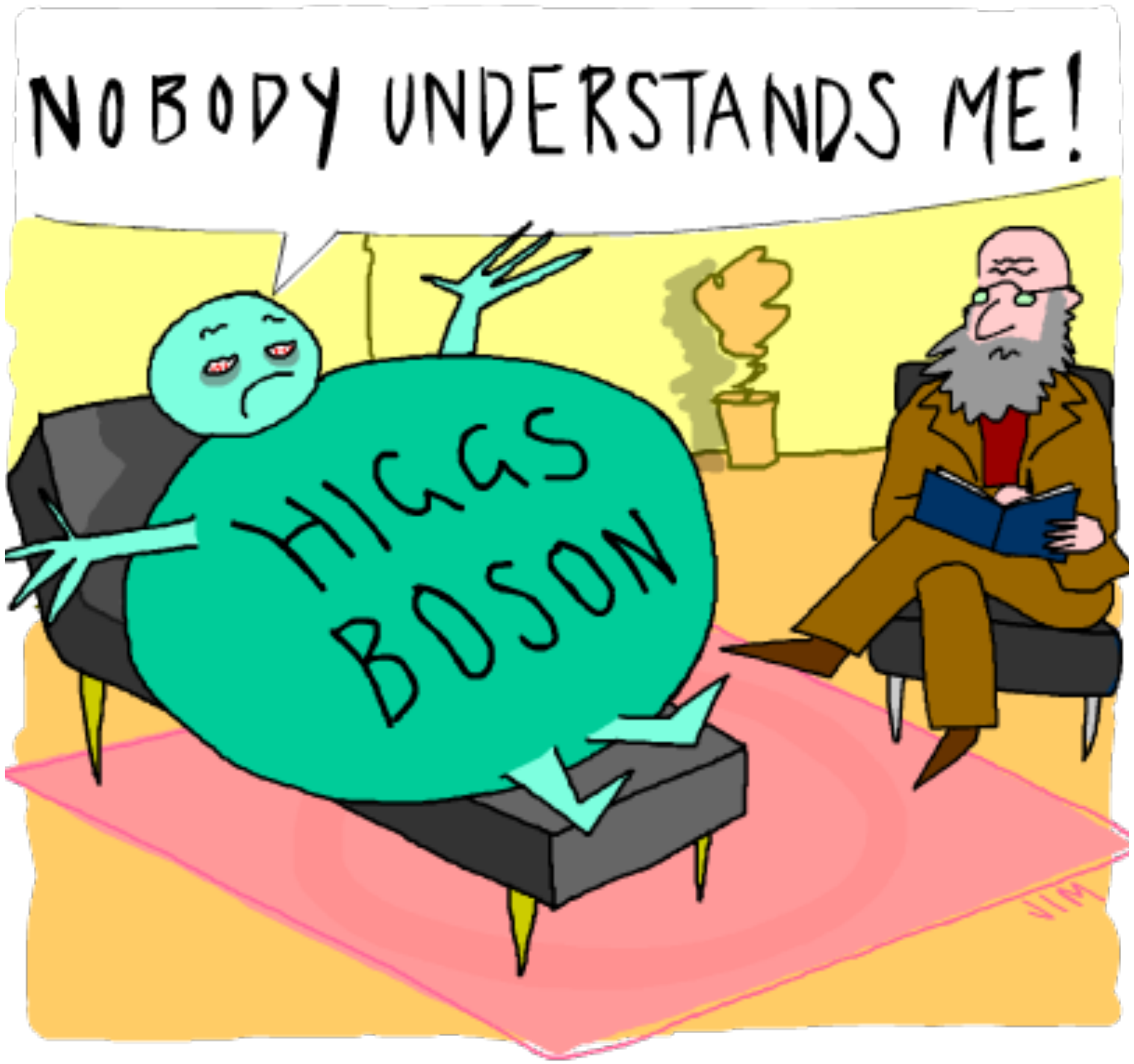


Fig I: The poor misunderstood Higgs boson

Motivation

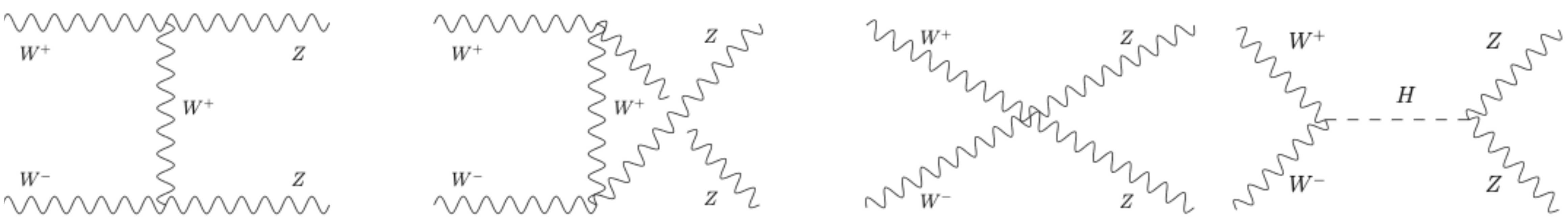
What is the origin of the Electroweak Symmetry Breaking Sector (EWSBS)?

BSM theories

- **Dynamically** generated EWSBS
  - **Strong dynamics with SB at** $f \gg v = 246\,GeV$
    - New **resonances** at new scales!

Vector Boson Scattering (VBS)

- Within SM, Higgs **unitarizes** $V_L V_L \to V_L V_L,\ V = \{W, Z\}$



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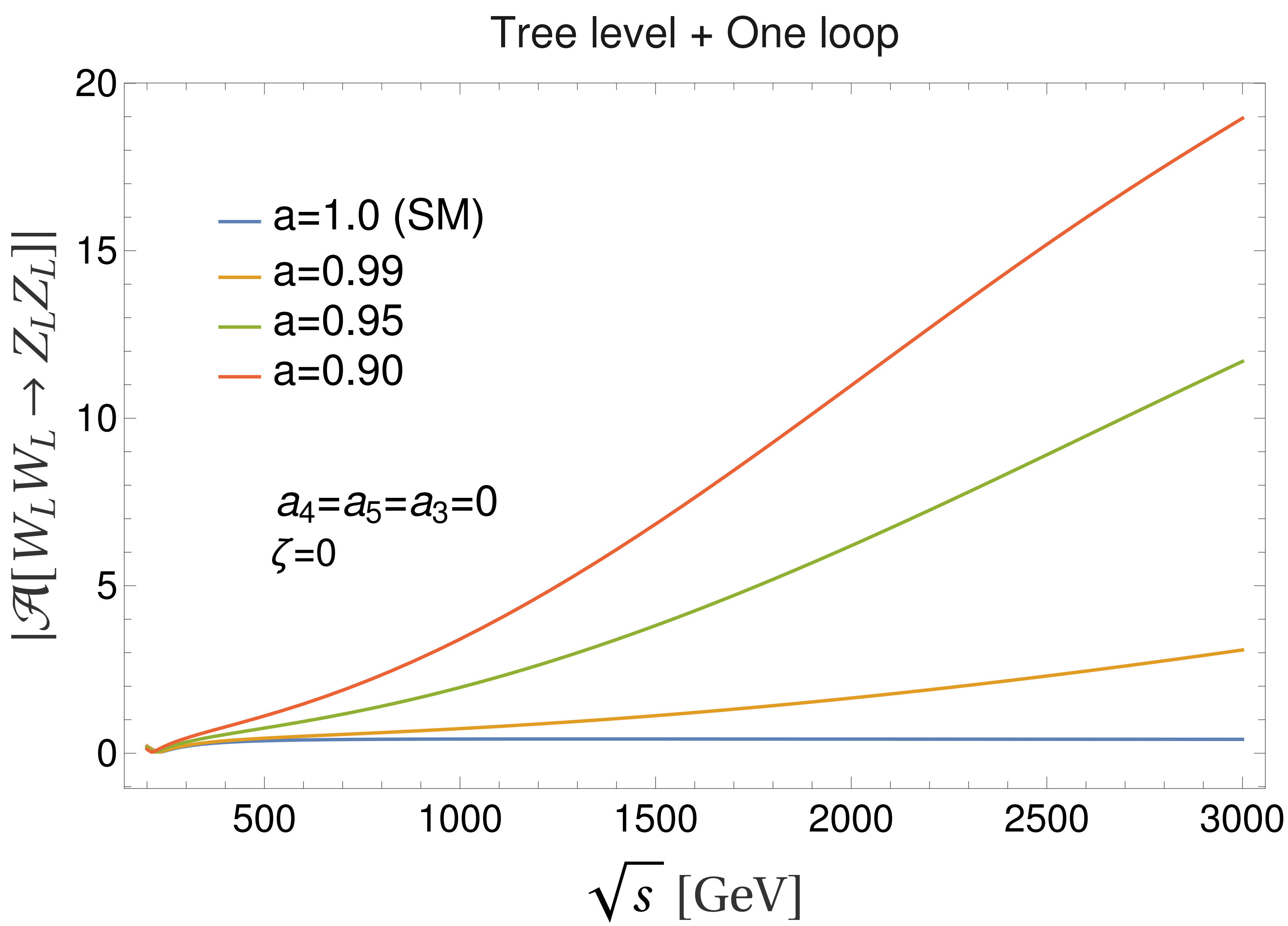
- New Physics **modifies** SM HVV interactions and breaks unitarity

Anomalous couplings in EW sector → Spoiled unitarity in longitudinally polarized scattering → Restoration by the appearance of resonances



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Effective Framework

Effective Field Theory (EFT)

- **Bottom-up EFT:** Symmetries + Low Energy degrees of freedom
  - **No UV completion** needed
  - **Breaks down** at some energy scale. Unitarity no longer fulfilled
- **Assumptions:**
  - **Strongly coupled dynamics** with resonances at $\sqrt{S} \lesssim \Lambda \sim 4\pi v \sim 3\,TeV$
  - **Minimal EWSB pattern:**
    - **Chiral EW symmetry:** $SU(2)_L \times SU(2)_R \to SU(2)_V$ (L = R) Custodial $\{g, g'\} \to \{g, 0\}$
    - **Gauge EW symmetry:** $SU(2)_L \times U(1)_Y$
  - Goldstones in **non-linear** realization $U(x) \to LU(x)R^{\dagger}$
  - **Light** degrees of freedom: $\omega^{\pm}, z, W^{\pm}, Z, h$

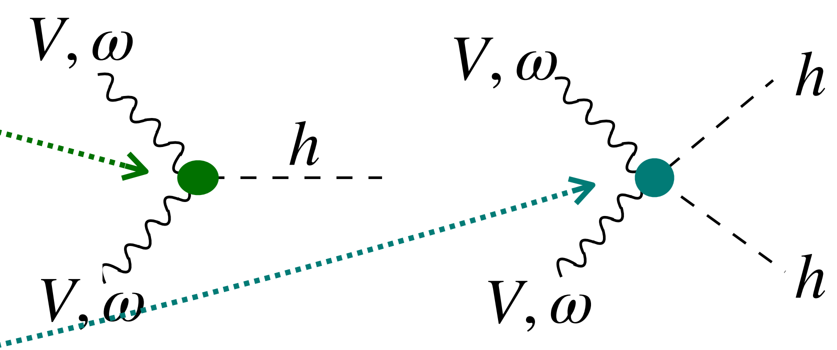
Effective Framework

Electroweak Chiral Lagrangian (EChL)

- Extension of **ChPT** to EW sector
- Expansion in powers of the **momentum (derivatives):** $\mathscr{L}_{EChL} = \mathscr{L}_2 + \mathscr{L}_4 + \ldots$
- **Building blocks**

$$U = e^{\frac{i\omega^a \tau^a}{v}} \qquad \mathscr{F}\left(\frac{h}{v}\right) = 1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2 + \ldots \qquad \mathscr{V}_\mu = \left(D_\mu U\right) U^\dagger \qquad \hat{W}_{\mu\nu}, \hat{B}_{\mu\nu}$$

$$\mathscr{L}_2 \supset \frac{v^2}{4}\left[1 + 2a\left(\frac{h}{v}\right) + b\left(\frac{h}{v}\right)^2\right] Tr\left[D^\mu U^\dagger D_\mu U\right]$$



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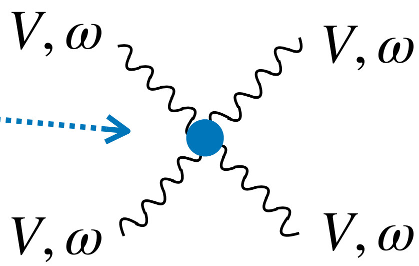
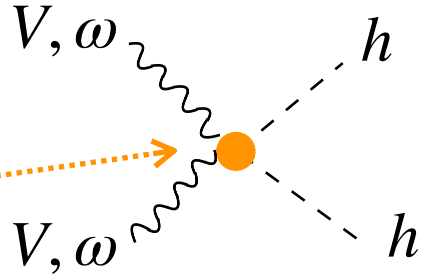
$$\mathscr{L}_4 \supset a_4 \left(Tr\left[\mathscr{V}_\mu \mathscr{V}_\nu\right] Tr\left[\mathscr{V}^\mu \mathscr{V}^\nu\right]\right) + a_5 \left(Tr\left[\mathscr{V}_\mu \mathscr{V}^\mu\right] Tr\left[\mathscr{V}_\nu \mathscr{V}^\nu\right]\right)$$

$$+ \frac{\delta}{v^2}\partial_\mu h \partial h^\mu \left(Tr\left[D_\nu U^\dagger D^\nu U\right]\right) + \frac{\eta}{v^2}\partial_\mu h \partial h_\nu \left(Tr\left[D^\mu U^\dagger D^\nu U\right]\right)$$

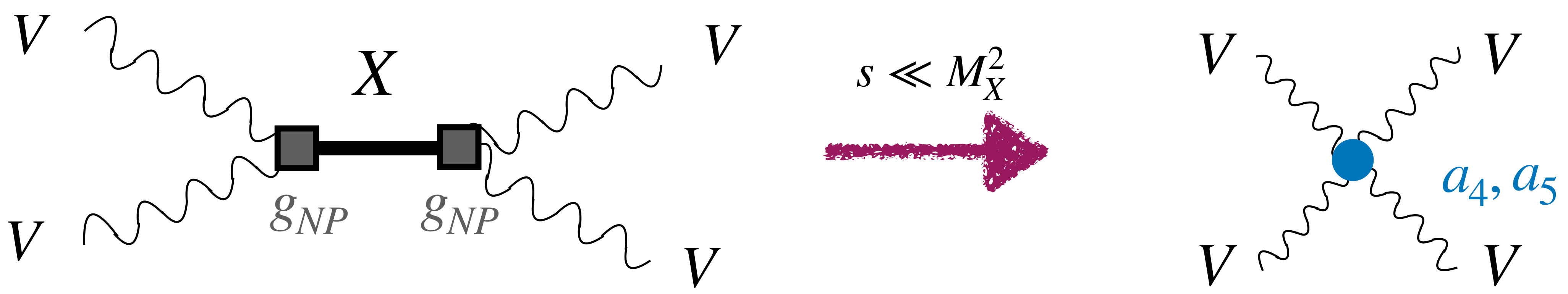




Effective Framework

Electroweak Chiral Lagrangian (EChL)

- Extension of **ChPT** to EW sector
- Expansion in powers of the **momentum (derivatives):** $\mathscr{L}_{EChL} = \mathscr{L}_2 + \mathscr{L}_4 + \ldots$
- **Integrate out** new content at high energies



Unitarization

- Expansion in powers of the **momentum (derivatives)**
- Realistic predictions only with **unitary amplitudes:** $|t_{IJ}| < 1$
- Unitarization methods **required**: IAM, K matrix, N/D,…
- Based on **partial wave** analysis

$$t_{IJ}(s) = \frac{1}{64\pi}\int_{-1}^{1} d\,(cos\theta)\,P_J T_I(s, cos\theta) \approx t_{IJ}^{(2)} + t_{IJ}^{(4)} + \ldots$$

- T_I **fixed isospin** amplitudes. In the **charged basis** have the form

$$T_0 = 3\mathscr{A}^{+-00} + \mathscr{A}^{++++}$$
$$T_1 = 2\mathscr{A}^{+-+-} - 2\mathscr{A}^{+-00} - \mathscr{A}^{++++}$$
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Violation of unitarity!!

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The other ones are obtained via **crossing symmetry**!

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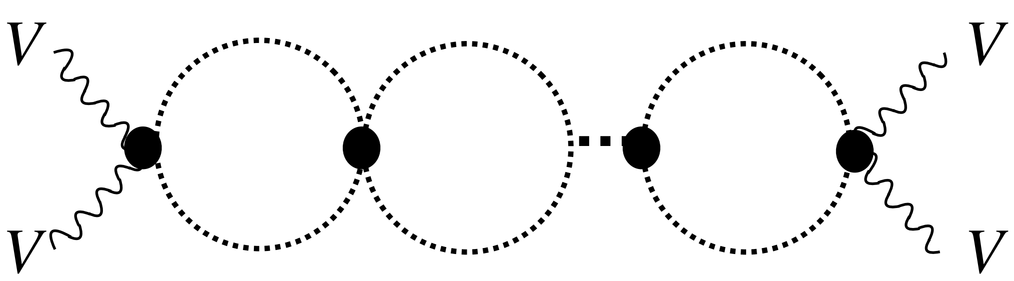
re-summation of infinite chain of bubble diagrams

$$t_{IJ}^{IAM} = \frac{\left(t_{IJ}^{(2)}\right)^2}{t_{IJ}^{(2)} - t_{IJ}^{(4)}}$$



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re-summation of infinite chain of bubble diagrams

$$t_{IJ}^{IAM} = \frac{\left(t_{IJ}^{(2)}\right)^2}{t_{IJ}^{(2)} - t_{IJ}^{(4)}}$$

Dynamically generated **resonances** with mass $M\left(a, b, \{a_i\}\right)$, width $\Gamma\left(a, b, \{a_i\}\right)$ and quantum numbers IJ

$$M = \sqrt{Re\, s_R} \qquad \Gamma = -\frac{1}{m} Im\, s_R$$

NLO calculation

- a **full NLO** $V_L V_L \to V_L V_L$ is available in the literature Too complicated for for our purposes.

In[20]:=

```
topologies = CreateTopologies[1, 2 → 2];
amp = InsertFields[topologies, {V[3], -V[3]} → {V[2], V[2]}, (*WW→ZZ*)
    InsertionLevel → {Particles}, Model → "EChL_custodial_p2",
    GenericModel → "EChL_custodial_p2"];
Paint[amp]
in total: 474 Particles insertions
```

diagrams in **custodial limit (no e.m.)!**

- **Shortcut:** $t_{IJ}^{(4)} = Re\, t_{IJ}^{(4)} + iIm\, t_{IJ}^{(4)}$

Maria J. Herrero and Roberto A. Morales
Phys. Rev. D104, 075013
Published 12 October 2021

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- **Shortcut:** $t_{IJ}^{(4)} = Re\, t_{IJ}^{(4)} + i Im\, t_{IJ}^{(4)}$

- $Re\, t_{IJ}^{(4)}: \{a_i\} - terms + NLO - ET\ amplitude$

  ET: Equivalence Theorem

  $$\mathscr{A}\left(V_L V_L \to V_L V_L\right) \approx \mathscr{A}\left(\omega\omega \to \omega\omega\right) + o\left(\frac{M_V}{\sqrt{S}}\right)$$

  ω is the **Goldstone Boson** associated to V

reduced to 94 scalar diagrams!!

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- **Shortcut:** $t_{IJ}^{(4)} = Re\, t_{IJ}^{(4)} + iIm\, t_{IJ}^{(4)}$

- $Im\, t_{IJ}^{(4)}$: **exact** calculation through perturbative **Optical Theorem**

$$Im\, t_{IJ}^{(4)}(s) = \sigma(s)\,|\,t_{IJ}^{(2)}\,|^2 + \sigma_h(s)\,|\,t_{h,I}^{(2)}\,|^2\,\delta_{I0}$$

$VV \to VV$ $\qquad$ $VV \to hh$

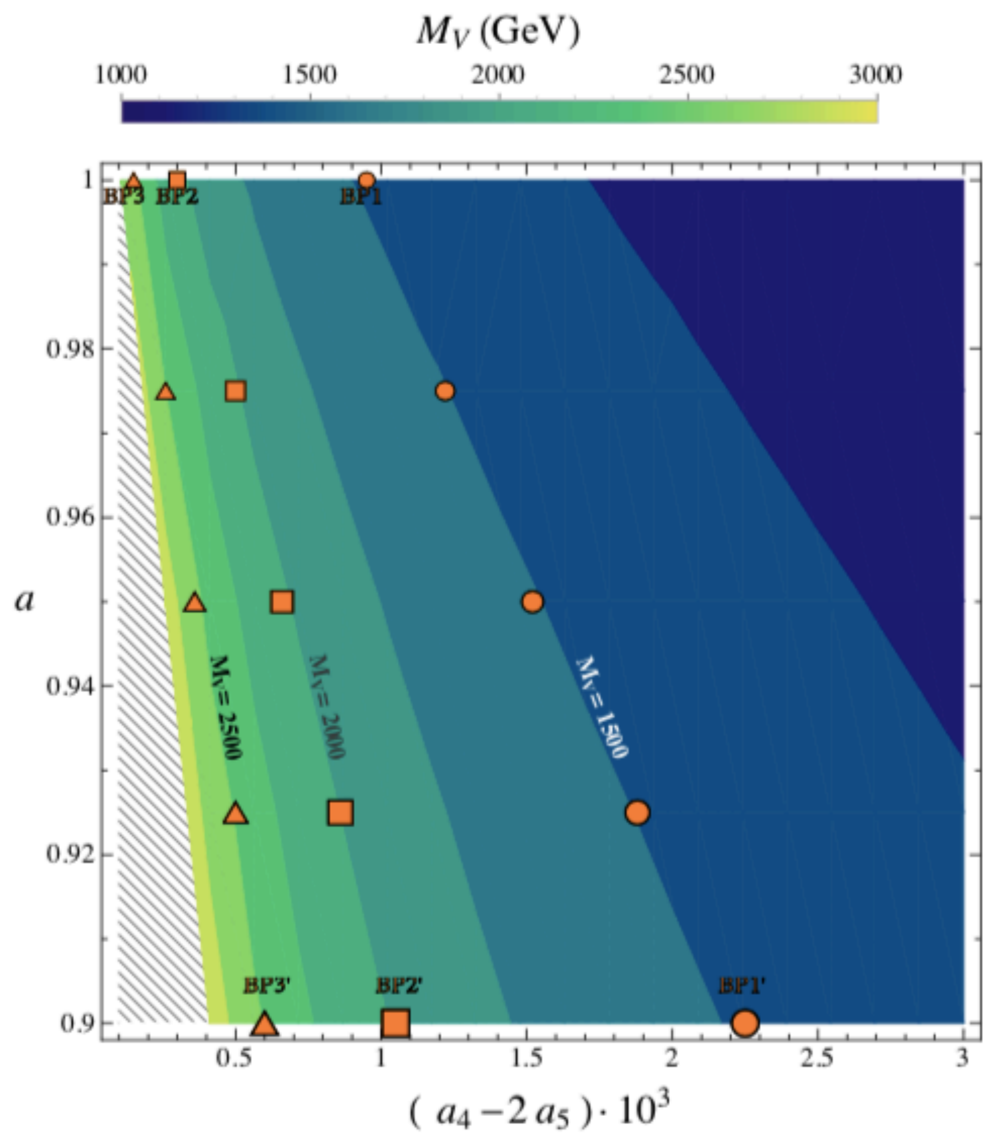
Tree level calculation!!

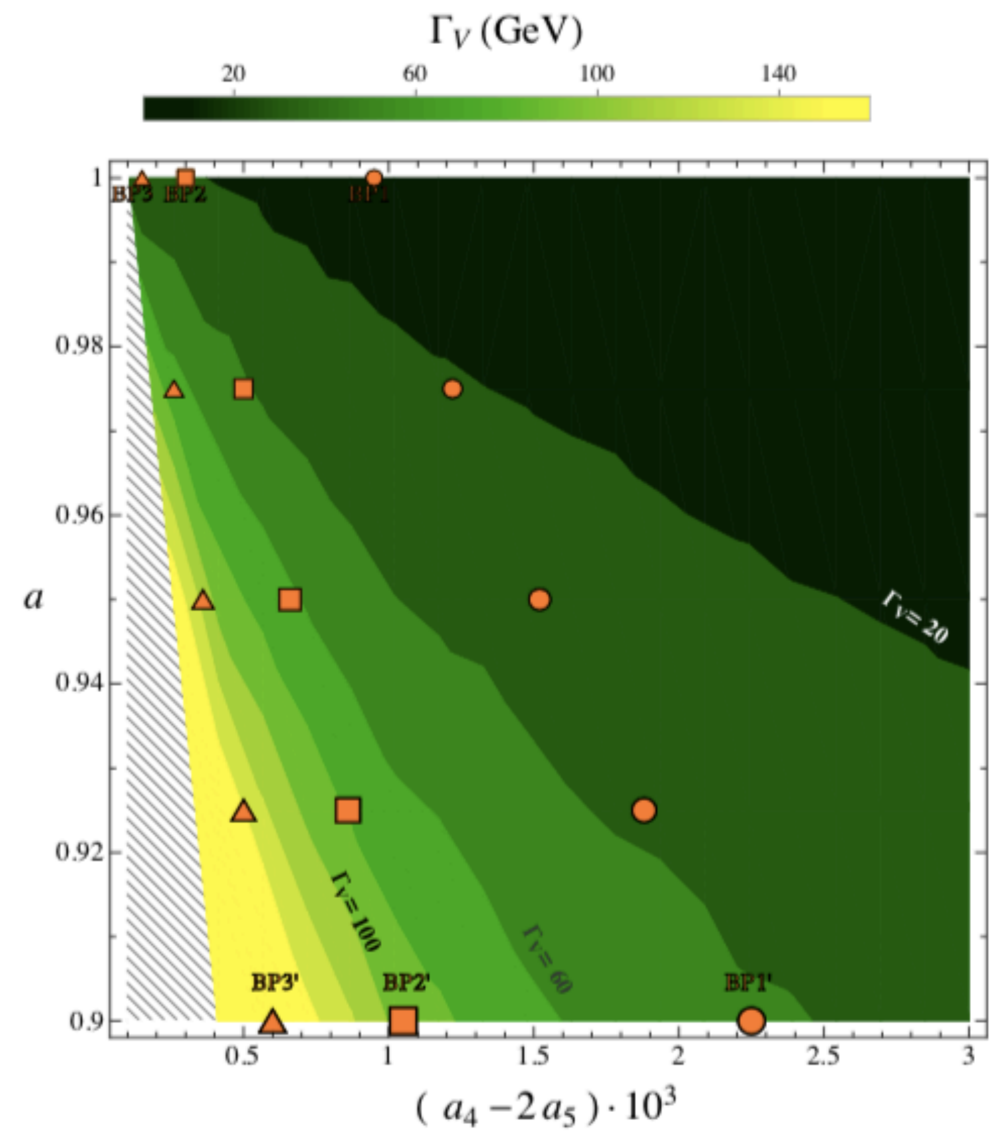
Previous work

- **Extreme version of ET**. No $W^{\pm}, Z$ allowed at all

Espriu, Mescia, Yencho
Phys. Rev. D 88, 055002

- t_{11} :Scenarios where **no scalar-isoscalar nor scalar-isotensor** poles appear





arXiv 1707.04580v2 [hep-ph]

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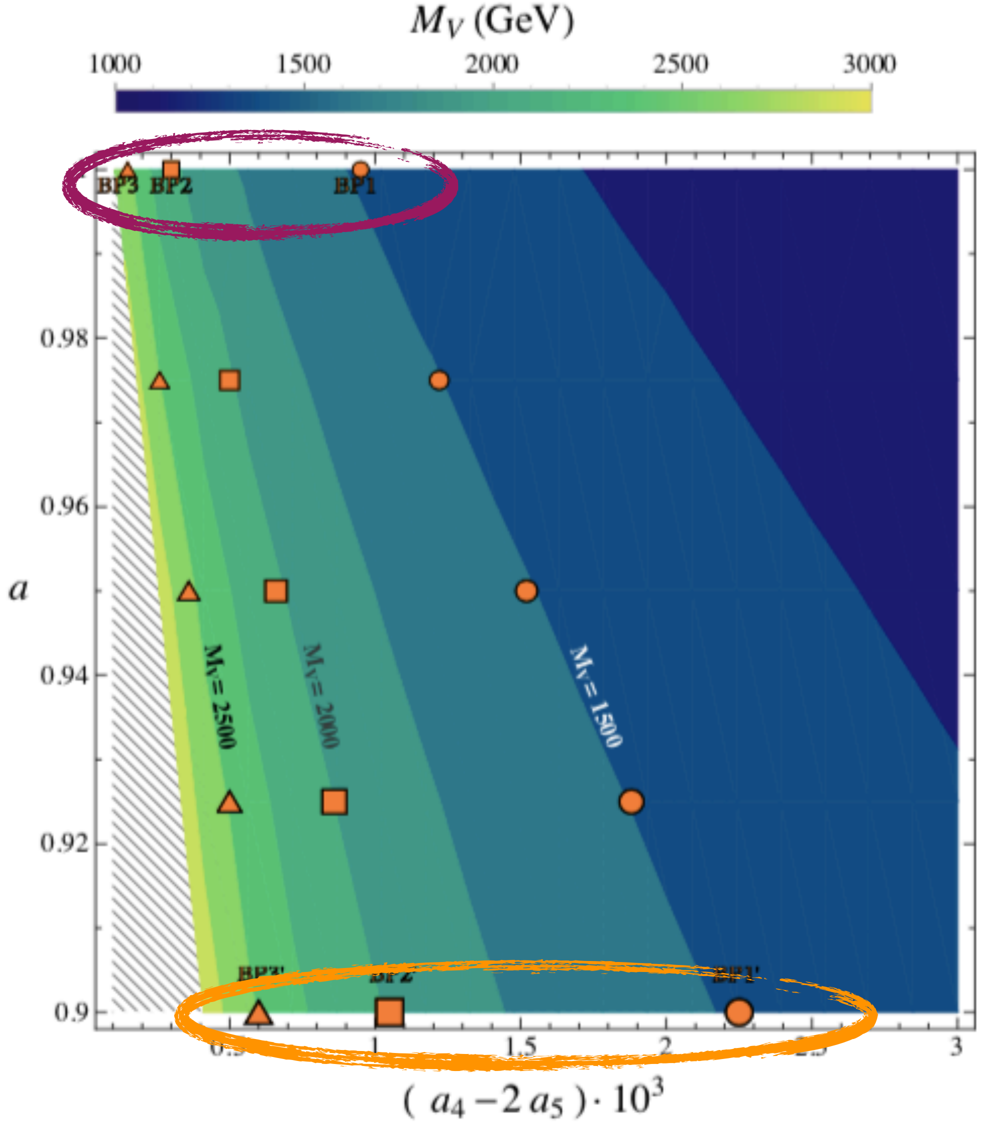


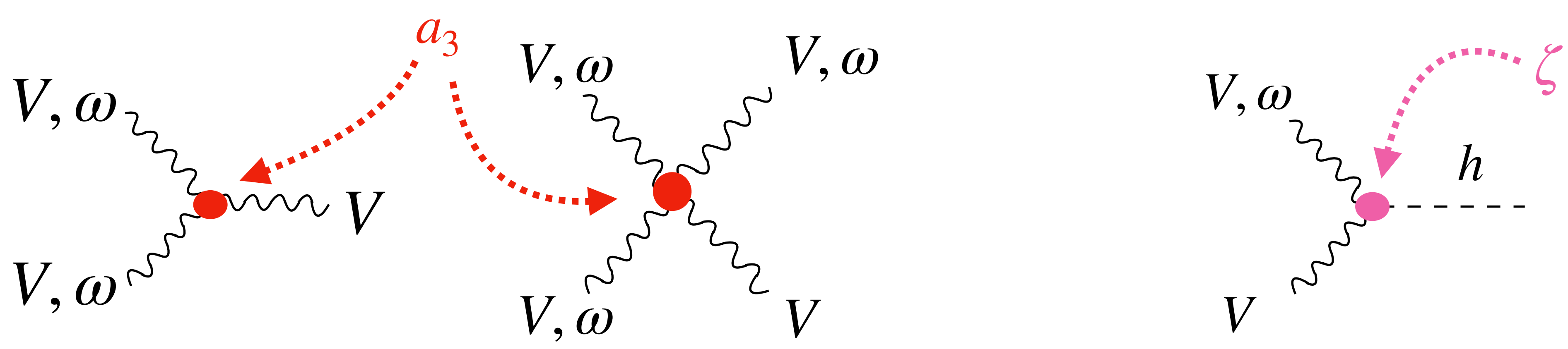
arXiv 1707.04580v2 [hep-ph]

	a	$a_4 \cdot 10^4$	$a_5 \cdot 10^4$	
BP1	1	3.5	-3	1.5 TeV
BP2	1	1	-1	2.0 TeV
BP3	1	0.5	-0.5	2.5 TeV
BP1’	0.9	9.5	-6.5	1.5 TeV
BP2’	0.9	5.5	-2.5	2.0 TeV
BP3’	0.9	4	-1	2.5 TeV

Our work: improves

- One step further than extreme version of ET: **allow physical $W^{\pm}, Z$ inside loops (94 → 294 one loop diagrams)**

- **new** chiral parameters entering t_{11} (renormalization and evaluation)



- How the **previously calculated vector resonances** (BPs) are affected?

- Not only VBS but also $IJ = 00$ final states $WW \to hh$ and $hh \to hh$
  ~ 1500 diagrams already renormalized (my particular hell)

Our work: results

- How the **previously calculated resonances** (BPs) are affected?
- $a_3, \zeta = 0$

$\sqrt{s_V}$ (GeV)	$g = 0$	$g \neq 0$	a	$a_4 \cdot 10^4$	$a_5 \cdot 10^4$
BP1	$1476 - \frac{i}{2}14$	$1503 - \frac{i}{2}13$	1	3.5	-3
BP2	$2039 - \frac{i}{2}21$	$2087 - \frac{i}{2}20$	1	1	-1
BP3	$2473 - \frac{i}{2}27$	$2540 - \frac{i}{2}27$	1	0.5	-0.5
BP1′	$1479 - \frac{i}{2}42$	$1505 - \frac{i}{2}44$	0.9	9.5	-6.5
BP2′	$1981 - \frac{i}{2}97$	$2025 - \frac{i}{2}98$	0.9	5.5	-2.5
BP3′	$2481 - \frac{i}{2}183$	$2547 - \frac{i}{2}183$	0.9	4	-1

Asiáin, Espriu, Mescia. Phys. Rev. D 105, 015009

- Variations around $+(2\% - 3\,\%)$ when physical gauge bosons are allowed

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$\sqrt{s_V}$ (GeV)
BP1
BP2
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BP1′
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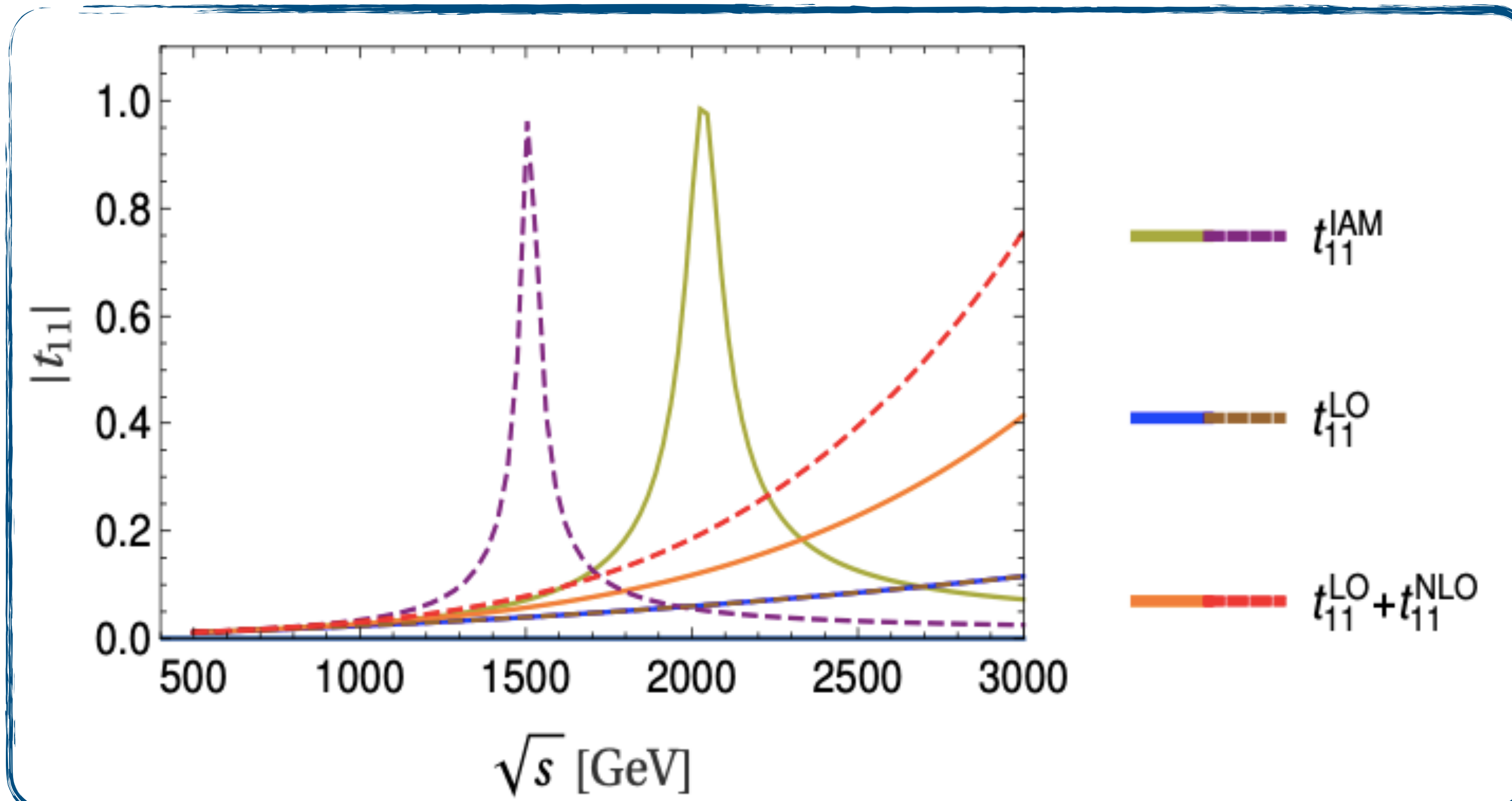
BP3′



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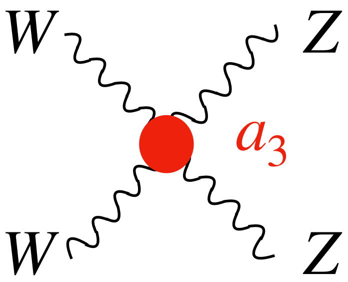

$\sqrt{s_V}\,(GeV)$	$a_3 = 0$	$a_3 = 0.1$	$a_3 = -0.1$	$a_3 = 0.01$	$a_3 = -0.01$
BP1	$1503 - \frac{i}{2}13$	$1795 - \frac{i}{2}11$	$1215 - \frac{i}{2}15$	$1532 - \frac{i}{2}13$	$1474 - \frac{i}{2}13$
BP2	$2087 - \frac{i}{2}20$	$2721 - \frac{i}{2}15$	$1505 - \frac{i}{2}23$	$2150 - \frac{i}{2}19$	$2025 - \frac{i}{2}21$
BP1′	$1505 - \frac{i}{2}44$	$1663 - \frac{i}{2}46$	$1335 - \frac{i}{2}43$	$1520 - \frac{i}{2}44$	$1488 - \frac{i}{2}44$
BP2′	$2025 - \frac{i}{2}98$	$2278 - \frac{i}{2}104$	$1752 - \frac{i}{2}89$	$2052 - \frac{i}{2}98$	$1999 - \frac{i}{2}97$

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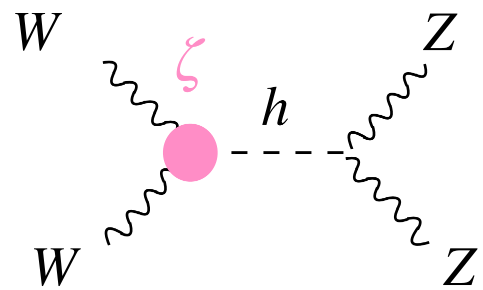
- For reasonable values of a_3 **are not significatively modified**
- **Inverse effect** for positive and negative values

Our work: results

- How the **previously calculated resonances** (BPs) are affected?
- $\zeta \neq 0,\ a_3 = 0$



$\sqrt{s_V}\ (GeV)$	$\zeta = 0$	$\zeta = 0.1$	$\zeta = -0.1$	$\zeta = 0.01$	$\zeta = -0.01$
BP1	$1503 - \frac{i}{2}13$	$1637 - \frac{i}{2}13$	$1377 - \frac{i}{2}14$	$1516 - \frac{i}{2}13$	$1489 - \frac{i}{2}13$
BP2	$2087 - \frac{i}{2}20$	$2393 - \frac{i}{2}18$	$1809 - \frac{i}{2}22$	$2117 - \frac{i}{2}20$	$2058 - \frac{i}{2}21$
BP1′	$1505 - \frac{i}{2}44$	$1570 - \frac{i}{2}46$	$1439 - \frac{i}{2}43$	$1510 - \frac{i}{2}45$	$1497 - \frac{i}{2}45$
BP2′	$2025 - \frac{i}{2}98$	$2136 - \frac{i}{2}100$	$1915 - \frac{i}{2}94$	$2036 - \frac{i}{2}98$	$2014 - \frac{i}{2}97$

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- For reasonable values of ζ **are not significatively modified**

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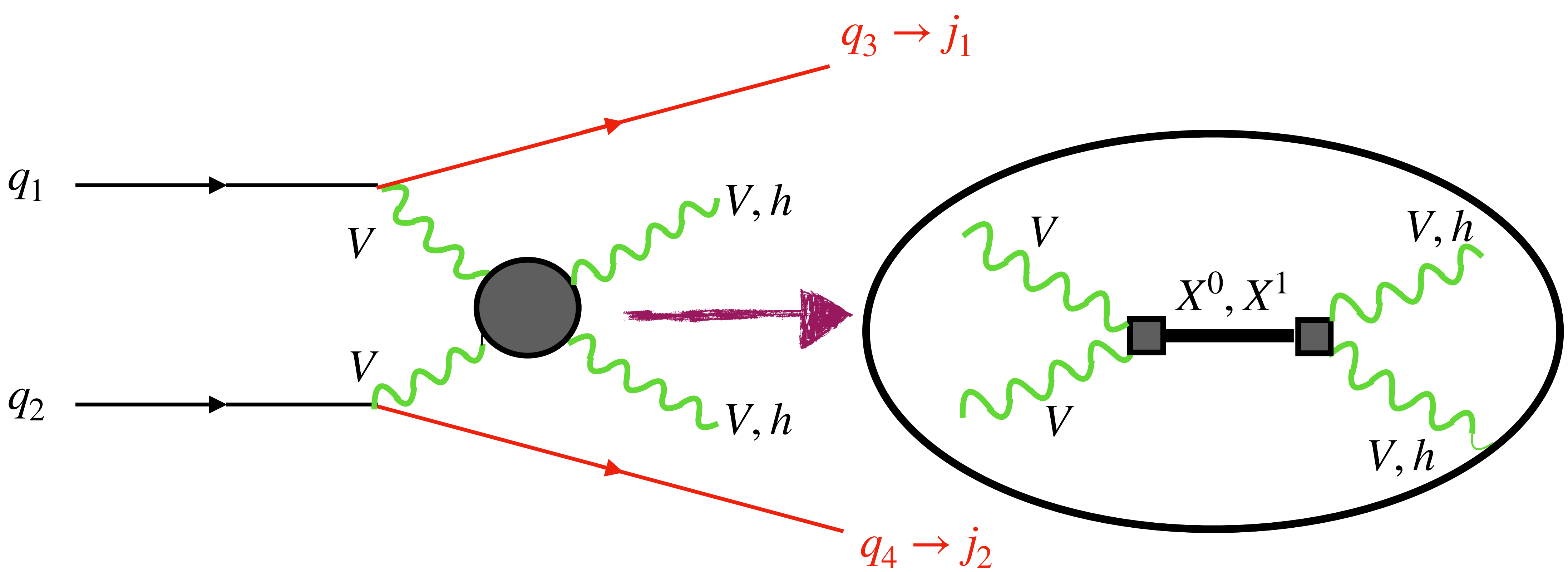
Our work: objectives

- **Monte Carlo @LHC** to make predictions for vector/scalar BSM resonances
- **Subprocess:** $pp \to VVjj$ / $hhjj$



Conclusions

- Effective field theories are **powerful tools** to explore High Energy Physics in a model-indepent way

- Unitary amplitudes can help to **constrain anomalous couplings** by studying the predicted resonances

- An extended EWSBS typically **have such resonances**

- VBS promising channel to to look for these **BSM resonances**

- LHC **has not seen this resonances yet**. Diboson excess at CMS with 1.9σ two years ago

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THANK YOU!! :)

BACK UP SLIDES

The Lagrangian

- The **complete Lagrangian**

$$\mathcal{L}_2 = -\frac{1}{2g^2}\mathrm{Tr}\left(\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right) - \frac{1}{2g'^2}\mathrm{Tr}\left(\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right) + \frac{v^2}{4}\mathcal{F}(h)\mathrm{Tr}\left(D^\mu U^\dagger D_\mu U\right) + \frac{1}{2}\partial_\mu h\partial^\mu h$$
$$- V(h)$$

$$\mathcal{L}_4 = -ia_3\mathrm{Tr}\left(\hat{W}_{\mu\nu}\left[V^\mu, V^\nu\right]\right) + a_4\left(\mathrm{Tr}\left(V_\mu V_\nu\right)\right)^2 + a_5\left(\mathrm{Tr}\left(V_\mu V^\mu\right)\right)^2 + \frac{\gamma}{v^4}\left(\partial_\mu h\partial^\mu h\right)^2$$
$$+ \frac{\delta}{v^2}\left(\partial_\mu h\partial^\mu h\right)\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) + \frac{\eta}{v^2}\left(\partial_\mu h\partial_\nu h\right)\mathrm{Tr}\left(D^\mu U^\dagger D^\nu U\right)$$
$$+ i\chi\,\mathrm{Tr}\left(\hat{W}_{\mu\nu}V^\mu\right)\partial^\nu\mathcal{G}(h)$$

- **Building blocks**

$$U = \exp\left(\frac{i\omega^a\sigma^a}{v}\right) \in SU(2)_V, \quad V_\mu = D_\mu U^\dagger U, \quad \mathcal{F}(h) = 1 + 2a\left(\frac{h}{v}\right) + b\left(\frac{h}{v}\right)^2 + \ldots,$$

$$D_\mu U = \partial_\mu U + i\hat{W}_\mu U, \quad \hat{W}_\mu = g\frac{\vec{W}_\mu\cdot\vec{\sigma}}{2}, \quad \hat{W}_{\mu\nu} = \partial_\mu\hat{W}_\nu - \partial_\nu\hat{W}_\mu + i\left[\hat{W}_\mu, \hat{W}_\nu\right],$$

$$V(h) = \frac{1}{2}M_h^2h^2 + \lambda_3 vh^3 + \frac{\lambda_4}{4}h^4 + \ldots, \quad \mathcal{G}(h) = 1 + b_1\left(\frac{h}{v}\right) + b_2\left(\frac{h}{v}\right)^2 + \ldots$$

Experimental bounds on chiral couplings

Couplings	Ref.	Experiments
$0.89 < a < 1.13$	[47]	LHC
$-0.76 < b < 2.56$	[48]	ATLAS
$-3.3\lambda < \lambda_3 < 8.5\lambda$	[49]	CMS
$\|a_1\| < 0.004$	[50]	LEP (S-parameter)
$-0.06 < a_2 - a_3 < 0.20$	[51]	LEP & LHC
$-0.0061 < a_4 < 0.0063$	[52]	CMS (from $WZ \to 4l$)
$\|a_5\| < 0.0008$	[53]	CMS (from $WZ/WW \to 2l2j$)

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The couterterms

- **The counterterms for:** $\omega\omega \to \omega\omega,\ \omega\omega \to hh\ and\ hh \to hh$

$$\delta v^2_{div} = \frac{\Delta}{16\pi^2}\left((b-a^2)M_h^2 + 3(a^2+2)M_W^2\right), \quad \delta T_{div} = -\frac{\Delta}{32\pi^2 v}3\left(d_3 M_h^4 + 6aM_W^4\right),$$

$$\delta a = \frac{\Delta}{32\pi^2 v^2}\left(6\,a\left(-2a^2+b+1\right)M_W^2 + (5a^3 - a(2+3b) - 3d_3(a^2-b))M_h^2\right),$$

$$\delta b = \frac{\Delta}{32\pi^2 v^2}\left(6\left(3a^4 - 6a^2 b + b(b+2)\right)M_W^2\right.$$
$$\left. - \left(21a^4 - a^2(8+19b) + b(4+2b) + 6ad_3(1+2b-3a^2) - 3d_4(b-a^2)\right)M_h^2\right),$$

$$\delta\lambda_{div} = \frac{\Delta}{64\pi^2 v^4}\left(\left(5a^2 - 2b + 3\left(d_3(3d_3-1)+d_4\right)\right)M_h^4 - 12\left(2a^2+1\right)M_W^2 M_h^2\right.$$
$$\left. + 18(a(2a-1)+b)M_W^4\right),$$

$$\delta\lambda_3 = \frac{\Delta}{64\pi^2 v^4}\left(36abM_W^4 + 6(3a^3 - 3ab - d_3(5a^2+1))M_W^2 M_h^2\right.$$
$$\left. + (-9a^3 + 3ab + d_3(10a^2 - b) + 9d_3 d_4)M_h^4\right),$$

$$\delta\lambda_4 = \frac{\Delta}{64\pi^2 v^4}\left(36b^2 M_W^4 - 12(a^2-b)(8a^2 - 2b - 9ad_3)M_W^2 M_h^2\right.$$
$$\left. + (96a^4 + 4b^2 - d_3(114a^3 - 42ab) + 9d_4^2 + a^2(-64b + 27d_3^2 + 12d_4))M_h^4\right),$$

$$\delta a_3 = -\frac{\Delta}{384\pi^2}\left(1-a^2\right), \quad \delta a_4 = -\frac{\Delta}{192\pi^2}\left(1-a^2\right)^2,$$

$$\delta a_5 = -\frac{\Delta}{768\pi^2}\left(5a^4 - 2a^2(3b+2) + 3b^2 + 2\right),$$

$$\delta\gamma = -\frac{\Delta}{64\pi^2}3(b-a^2)^2, \quad \delta\delta = -\frac{\Delta}{192\pi^2}(b-a^2)(7a^2-b-6), \quad \delta\eta = -\frac{\Delta}{48\pi^2}(b-a^2)^2,$$

$$\delta\zeta = \frac{\Delta}{96\pi^2}a(b-a^2).$$

The couterterms

- **The** **counterterms** **for:** $\omega\omega \to \omega\omega,\ \omega\omega \to hh\ and\ hh \to hh$

$$\delta M^2_{h,div} = \frac{\Delta}{32\pi^2 v^2}\left(3\left[6\left(2a^2+b\right)M_W^4 - 6a^2 M_W^2 M_h^2 + \left(3d_3^2+d_4+a^2\right)M_h^4\right]\right),$$
$$\delta M^2_{W,div} = \frac{\Delta}{48\pi^2 v^2}\left(M_W^2\left[3\left(b-a^2\right)M_h^2 + \left(-69+10a^2\right)M_W^2\right]\right),$$
$$\delta Z_{h,div} = \frac{\Delta}{16\pi^2 v^2}\left(3a^2\left(3M_W^2 - M_h^2\right)\right),$$
$$\delta Z_{\omega,div} = \frac{\Delta}{16\pi^2 v^2}\left(\left(b-a^2\right)M_h^2 + 3\left(a^2+2\right)M_W^2\right)$$

- In total: **17 counterterms + 1 tadpole**