Cosmic Axion Background: the QCD axion as a hot relic

Alessio Notari [1]

Universitat de Barcelona

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[1] In collaboration with R.Z. Ferreira, F. Rompineve, F. D'Eramo, F. Arias-Aragon, J.L. Bernal, L.Merlo.

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- Why so small?

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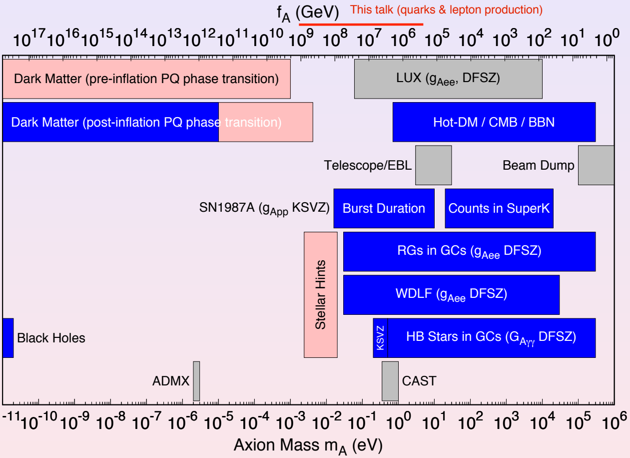
- *f (Axion "decay constant")* $\Leftrightarrow m_a$

Axion: constraints



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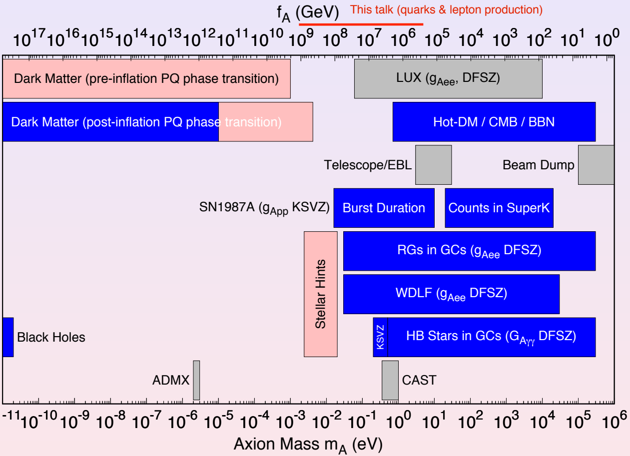

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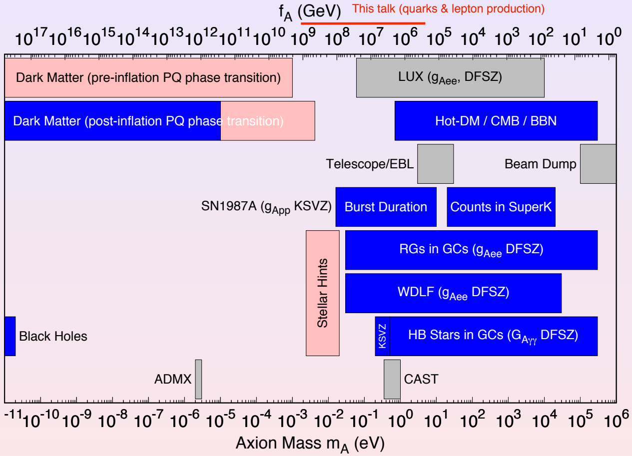

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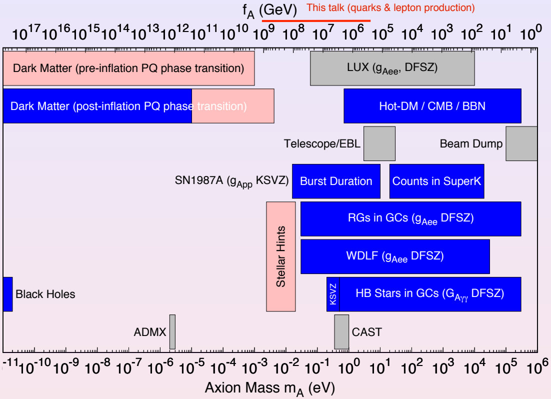

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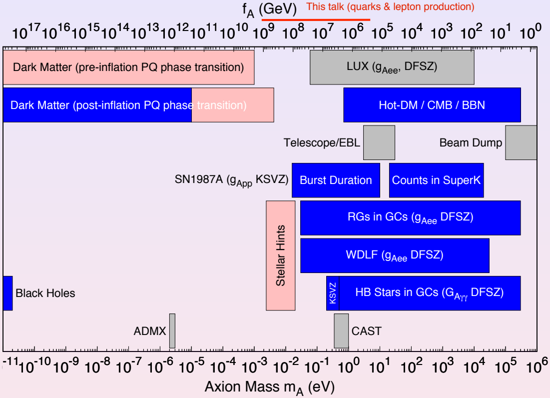

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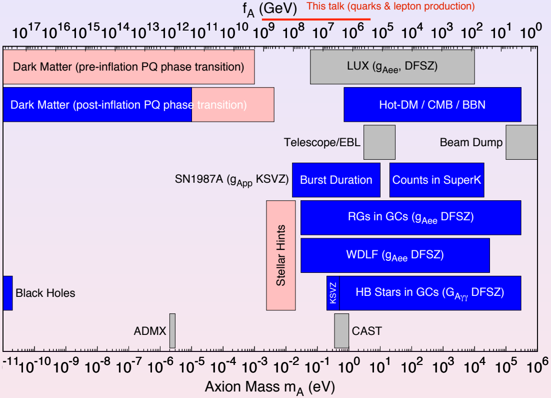

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- Caveat: Constraints based on individual couplings with e, γ, nucleons... Expected $\mathcal{O}(1/f)$, but model dependent.

QCD Axion

- Axion effective lagrangian:
  1. May couple with continuous shift symmetry with all SM
  2. Only breaking: Instanton-induced (tiny) mass

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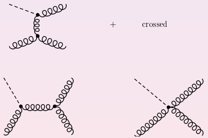

Figure: (Massò et al. Phys.Rev. D66 (2002).).

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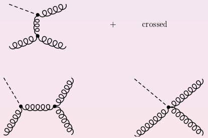

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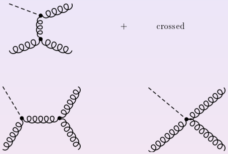

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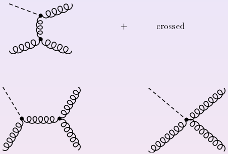

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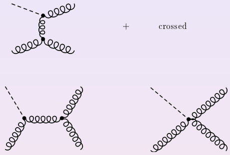

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- Example:
  1. $f = 10^8 GeV \implies T_{DEC} \approx TeV$
  2. $f = 10^9 GeV \implies T_{DEC} \approx 100 TeV$

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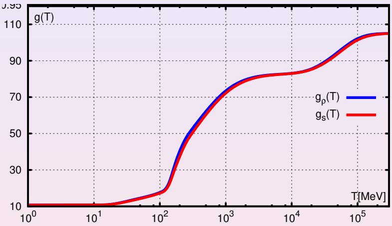

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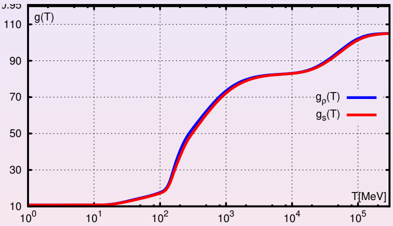

- $$\Delta N_{eff} \approx \frac{13.6}{g_{*,DEC}^{4/3}}$$
- If $T_{DEC} \gg 100$ GeV, $\Longrightarrow$ $g_{*,DEC} \geq 106.75$
- $\Longrightarrow$ $\Delta N_{eff} \lesssim 0.027$ (only upper bound!) (marginally detectable, 1σ, by CMB-Stage 4 experiments)

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- If $f \lesssim 10^9$-10^{10} GeV dominant channels can be via quarks & leptons [2] with $T_{DEC} \leq$ Electroweak scale

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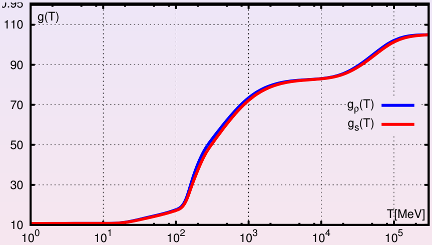

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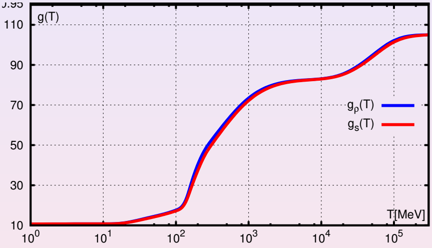

ADVANTAGES:

1. g_*^{SM} is smaller $\Longrightarrow$ larger N_{eff}
2. Here we are confident on g_*^{SM} $\Longrightarrow$ Precise predictions
3. Lower f $\Longrightarrow$ more accessible by direct searches (CAST, IAXO)

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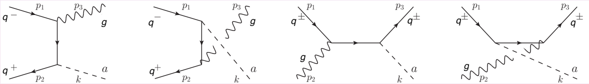

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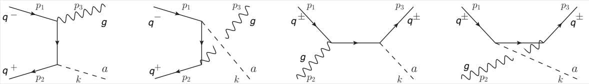

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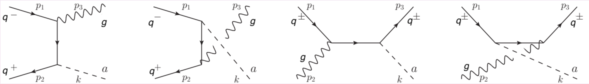

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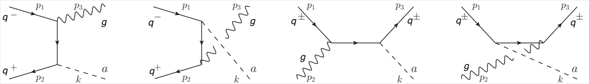

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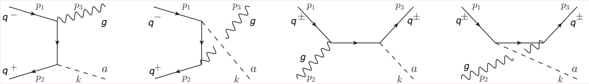

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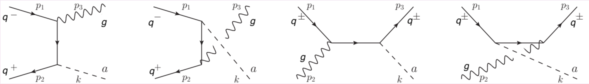

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Axions as Hot Relics

The QCD Axion

Solution: The QCD Axion

Axions via Gluons

Axion via Quarks

Axion via Leptons

Axions via Pions

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- Ratio peaks at $\boxed{T \approx m_q}$

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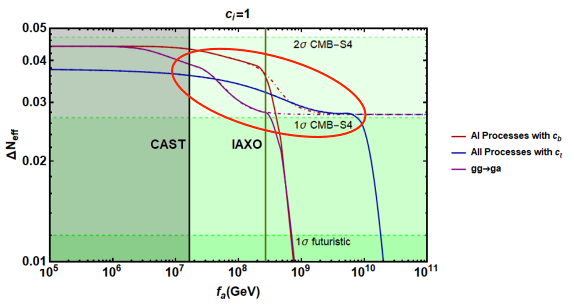
- Solving Boltzmann equations for n_a:

(R.Ferreira & A.N., PRL 2018; F.Arias-Aragon et al. JCAP, 2021)



$$10^9\mathrm{GeV} \gtrsim f/c_i \gtrsim 10^7\mathrm{GeV} \quad , 5\times10^{-3}\mathrm{eV} \lesssim m_a \lesssim 0.5\mathrm{eV}$$
$(c_i = 1$, for QCD Axion)

QCD Axion through N_{eff}

- Potentially larger for c-quark: $N_{\text{eff}} \lesssim 0.05 - 0.06$ (but uncertain)

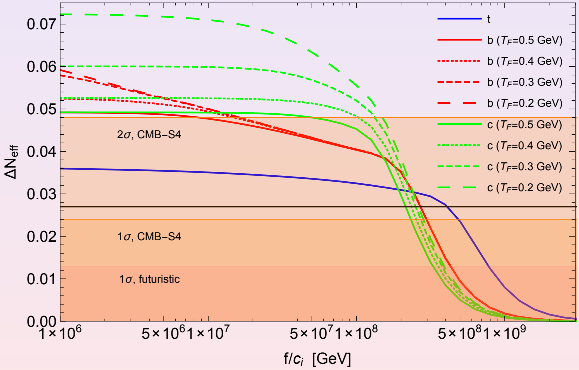


Figure: R.Ferreira & A.N., PRL 2018.

Hot Axions via Quark Decays

- $a - q$ interaction can be flavor non-diagonal

$$\mathcal{L}_{a-q} = \partial_\mu a \sum_{q \neq q'} \bar{q}' \gamma^\mu \left(\mathcal{V}_{q'q} + \mathcal{A}_{q'q} \gamma^5 \right) q + \text{h.c.} \ ,$$

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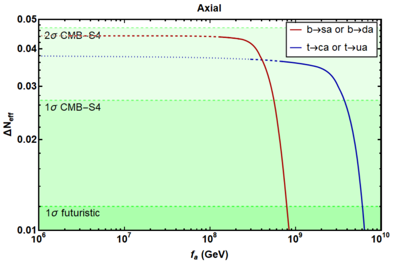

Figure: F.Arias-Aragon, F.D'Eramo, R.Z.Ferreira, A. N , L.Merlo, JCAP 2021.

- More efficient than scatterings (larger $f/c \lesssim 10^{10}$ GeV)

Hot Axions via Leptons

- The same can be done with leptons (μ and τ) [4]
- *a*-electron uninteresting (strongly constrained)

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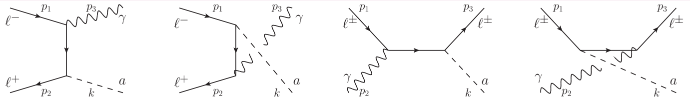

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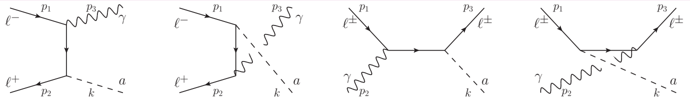

- Slightly smaller f/c_ℓ
- Ratio peaks at $\boxed{T \approx m_\ell}$ $\Longrightarrow$ Larger N_{eff}

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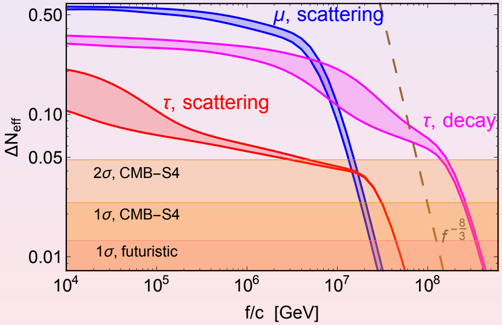
Hot Axions via Leptons

- Smaller $f/c_i \lesssim \text{few} \cdot 10^7$ GeV
- Ratio peaks at $\boxed{T \approx m_\ell}$ $\Longrightarrow$ Larger N_{eff}



- Caveat: μ scattering constrained by SN cooling at $f/c_\mu \gtrsim 10^8 GeV$ (Bolling et al. PRL 2020, Croon et al. JHEP 2021)

Axion-Pion coupling

- DFSZ model: a couples to u-type and d-type quarks,
- KSVZ model: no coupling to SM fermions

$$\textbf{DFSZ}: \quad c_u^0 = \frac{1}{3}\cos^2(\beta)\,, \quad c_d^0 = \frac{1}{3}\sin^2(\beta)\,,$$
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CMB Bounds on DFSZ

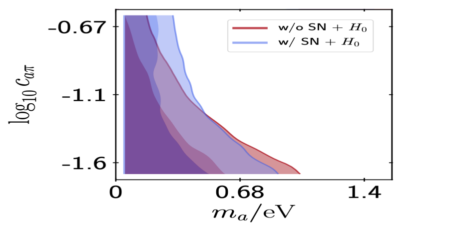

Figure: Constraints due to pion production Planck 18 + BAO (+ Pantheon + SH0ES H_0)

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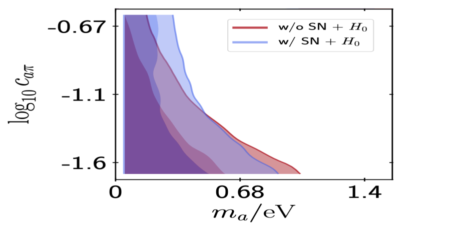

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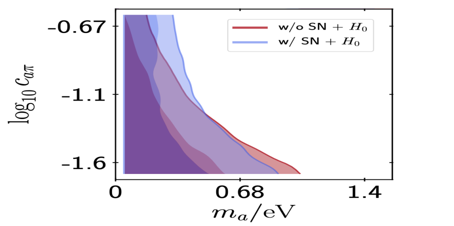

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Caveat! Perturbative pion cross-section calculation in Chiral PT breaks down at $T \gtrsim 60$ MeV (Di Luzio et al. 2021, arXiv 2101.10330.)

Conclusions

1. If $f \lesssim \mathcal{O}(10^9)$ GeV, coupling with quarks and leptons (with $c_i = \mathcal{O}(1)$) dominates over $\frac{\alpha_s}{8\pi}\frac{a}{f}G\tilde{G}$
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8. Future CMB experiments will tell in a few years, plus direct detection (*e.g.* IAXO)