Automatic generation of EFT operators

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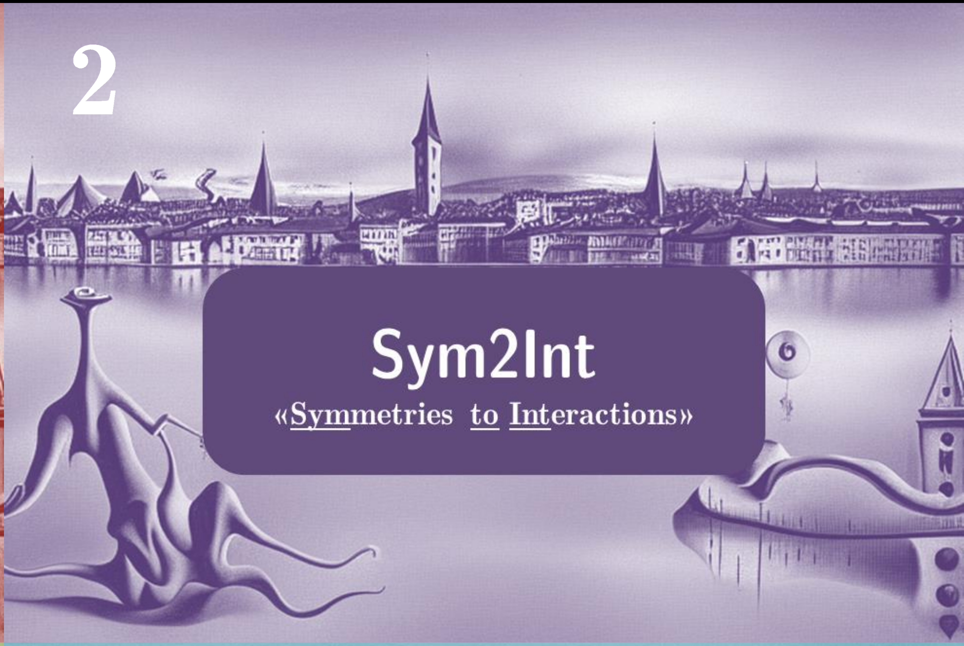
"*Zurich skyline, by Salvador Dalí*", Image generated with the machine learning model Stable Diffusion

SMEFT-Tools 2022, Zürich, 14 September 2022

1

Counting operators



2

Sym2Int

«Symmetries to Interactions»

3

Extending Sym2Int

Building operators explicitly

4

Flavor &

Application

A basis of operators for a general EFT

Counting operators

Rapid progress in recent years

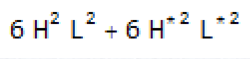
Using the Hilbert series, it became possible to count all SMEFT operators up to very high dimensions

!

Benvenuti, Feng, Hanany, He hep-th/0608050
Feng, Hanany, He hep-th/0701063
Hanany, Jenkins, Manohar, Torri 1010.3161
Lehman, Martin 1503.07537, 1510.00372
Henning, Lu, Melia, Murayama 1512.03433
...

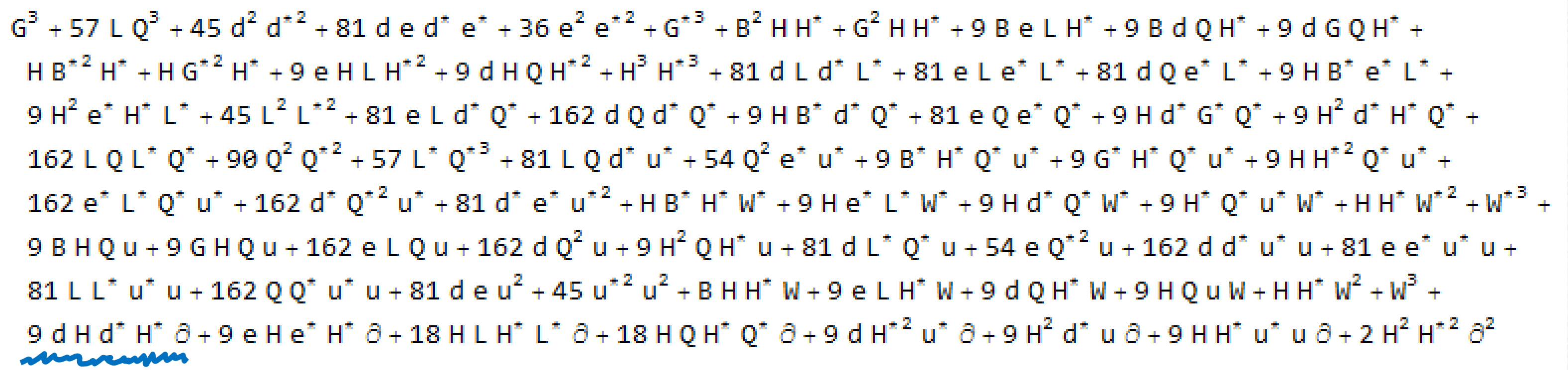
Sample

Dim 5



$$6\,H^2\,L^2 + 6\,H^{*2}\,L^{*2}$$

Dim 6

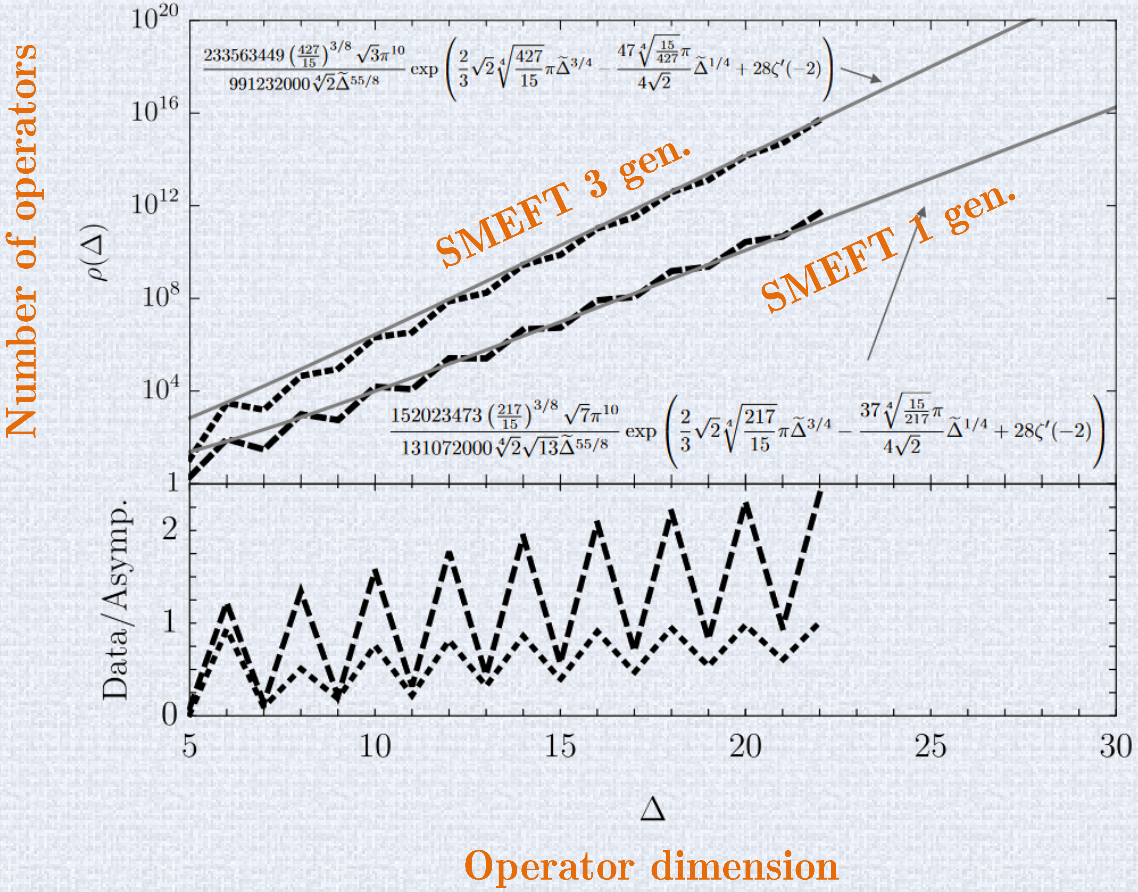


$$\begin{aligned}
&G^3 + 57\,L\,Q^3 + 45\,d^2\,d^{*2} + 81\,d\,e\,d^*\,e^* + 36\,e^2\,e^{*2} + G^{*3} + B^2\,H\,H^* + G^2\,H\,H^* + 9\,B\,e\,L\,H^* + 9\,B\,d\,Q\,H^* + 9\,d\,G\,Q\,H^* +\\
&H\,B^{*2}\,H^* + H\,G^{*2}\,H^* + 9\,e\,H\,L\,H^{*2} + 9\,d\,H\,Q\,H^{*2} + H^3\,H^{*3} + 81\,d\,L\,d^*\,L^* + 81\,e\,L\,e^*\,L^* + 81\,d\,Q\,e^*\,L^* + 9\,H\,B^*\,e^*\,L^* +\\
&9\,H^2\,e^*\,H^*\,L^* + 45\,L^2\,L^{*2} + 81\,e\,L\,d^*\,Q^* + 162\,d\,Q\,d^*\,Q^* + 9\,H\,B^*\,d^*\,Q^* + 81\,e\,Q\,e^*\,Q^* + 9\,H\,d^*\,G^*\,Q^* + 9\,H^2\,d^*\,H^*\,Q^* +\\
&162\,L\,Q\,L^*\,Q^* + 90\,Q^2\,Q^{*2} + 57\,L^*\,Q^{*3} + 81\,L\,Q\,d^*\,u^* + 54\,Q^2\,e^*\,u^* + 9\,B^*\,H^*\,Q^*\,u^* + 9\,G^*\,H^*\,Q^*\,u^* + 9\,H\,H^{*2}\,Q^*\,u^* +\\
&162\,e^*\,L^*\,Q^*\,u^* + 162\,d^*\,Q^{*2}\,u^* + 81\,d^*\,e^*\,u^{*2} + H\,B^*\,H^*\,W^* + 9\,H\,e^*\,L^*\,W^* + 9\,H\,d^*\,Q^*\,W^* + 9\,H^*\,Q^*\,u^*\,W^* + H\,H^*\,W^{*2} + W^{*3} +\\
&9\,B\,H\,Q\,u + 9\,G\,H\,Q\,u + 162\,e\,L\,Q\,u + 162\,d\,Q^2\,u + 9\,H^2\,Q\,H^*\,u + 81\,d\,L^*\,Q^*\,u + 54\,e\,Q^{*2}\,u + 162\,d\,d^*\,u^*\,u + 81\,e\,e^*\,u^*\,u +\\
&81\,L\,L^*\,u^*\,u + 162\,Q\,Q^*\,u^*\,u + 81\,d\,e\,u^2 + 45\,u^{*2}\,u^2 + B\,H\,H^*\,W + 9\,e\,L\,H^*\,W + 9\,d\,Q\,H^*\,W + 9\,H\,Q\,u\,W + H\,H^*\,W^2 + W^3 +\\
&\underbrace{9\,d\,H\,d^*\,H^*\,\partial}_{} + 9\,e\,H\,e^*\,H^*\,\partial + 18\,H\,L\,H^*\,L^*\,\partial + 18\,H\,Q\,H^*\,Q^*\,\partial + 9\,d\,H^{*2}\,u^*\,\partial + 9\,H^2\,d^*\,u\,\partial + 9\,H\,H^*\,u^*\,u\,\partial + 2\,H^2\,H^{*2}\,\partial^2
\end{aligned}$$

Format of each term: (#operators) x (field combinations)

- The **Hilbert series method counts operators**
  It does not build them explicitly
- This method also does not indicate where to apply the **derivatives**

Rapid progress in recent years



Melia, Pal 2010.08560

Eco

Marinissen, Rahn, Waalewijn 2004.09521

The traditional way

The Hilbert series (HS) gained prominence only in recent years

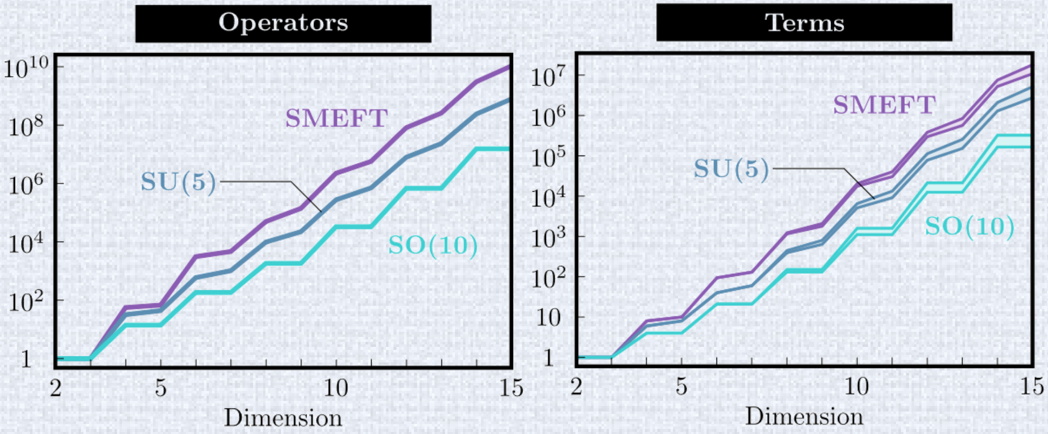
For decades, physicists have been building models and listing operators taking all combinations of fields, and picking out the ones which are gauge and Lorentz invariant (the *traditional method*)

Can it be used to reproduce the Hilbert series counting?
Yes. There are programs doing that.

BasisGen Criado 1901.03501

Sym2Int RF 1703.05221, 1907.12584

more on it later

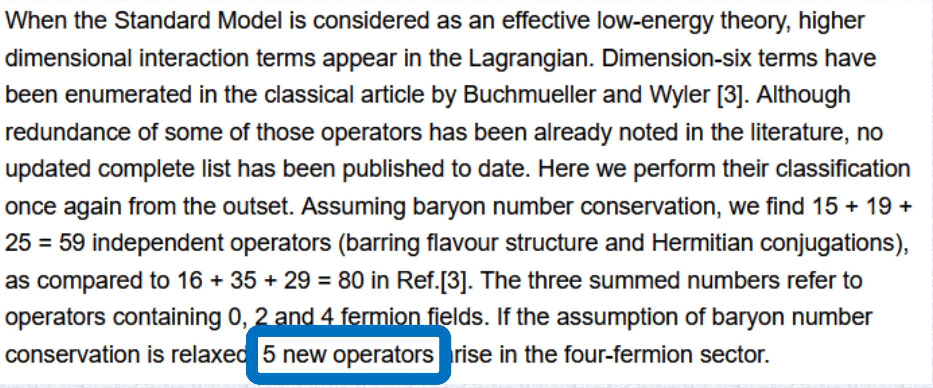


RF 1907.12584

- Viable to high dimensions
- Works out of the box with any group, representations
- Yields more information than just the number of operators, namely permutation symmetries of flavor indices
- Can't tell where to apply derivatives (same as HS method)

QQQL in SMEFT

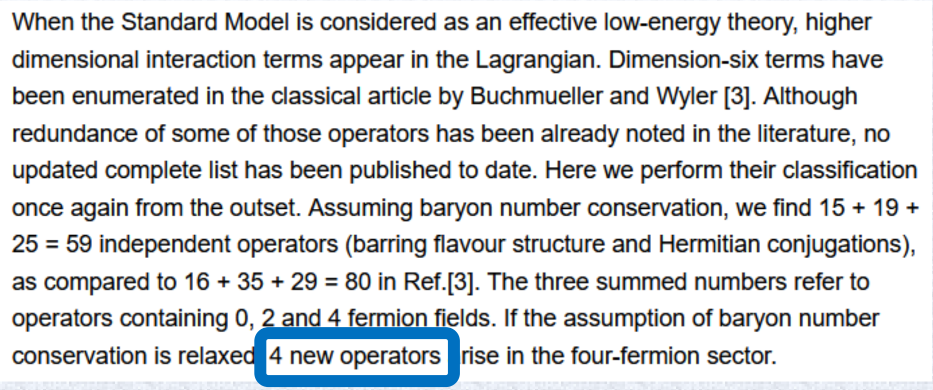
Counting *Lagrangian terms* is not always simple



When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed 5 new operators arise in the four-fermion sector.

Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884
("*Warsaw paper*")

v3 in arXiv



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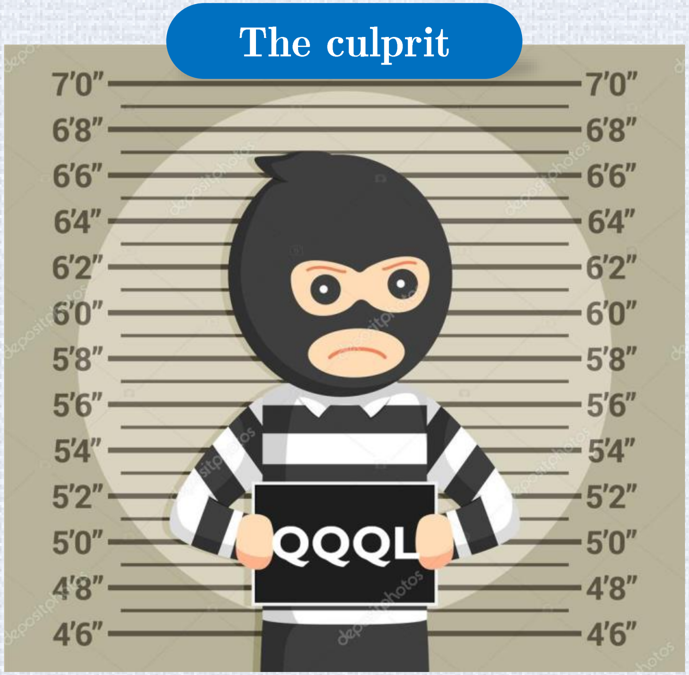
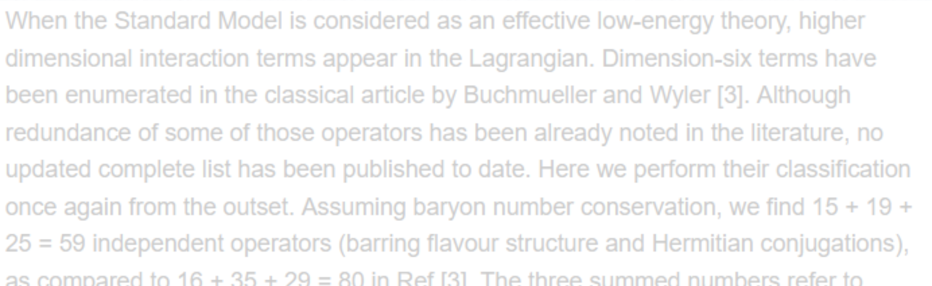


However, one can tackle
this kind of problem systematically
(but not with the HS, as far as I know)

$QQQL$ in SMEFT



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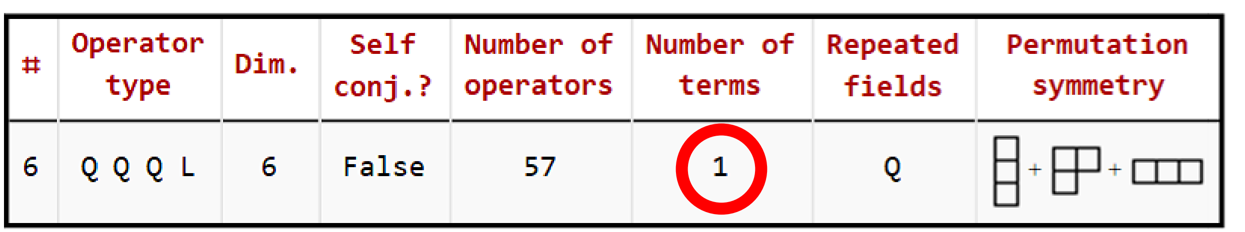
conservation is r

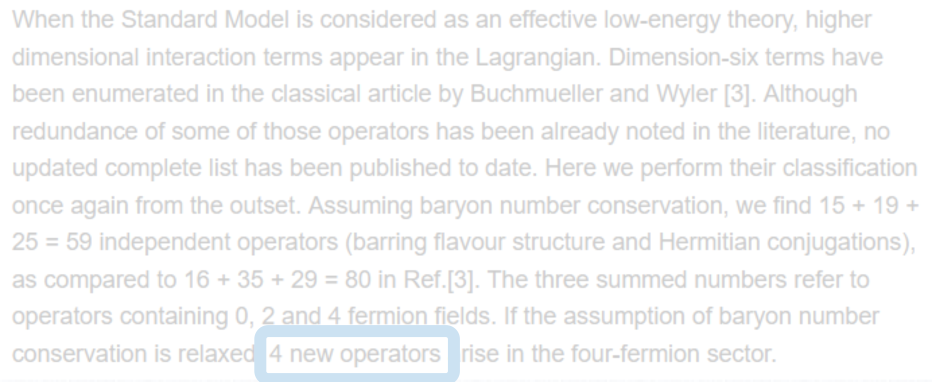
Grzad



#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
6	Q Q Q L	6	False	57	1	Q	$\yng(1,1,1) + \yng(2,1) + \yng(3)$

The culprit

7 years later (20‥)
v3 in arXiv of the same work



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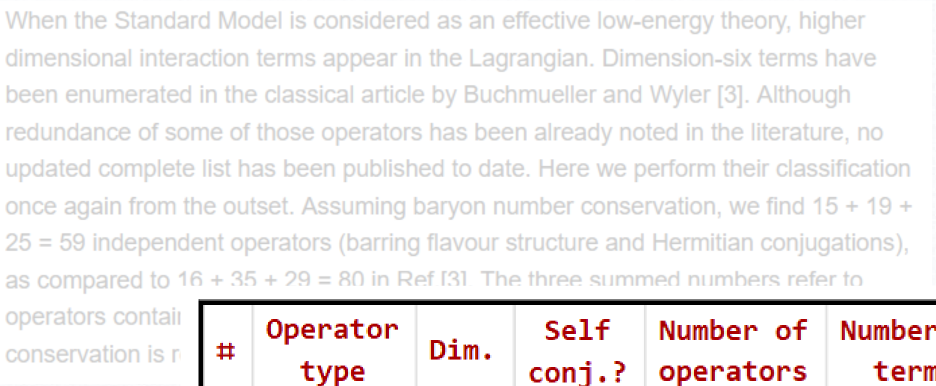
Easy to tackle this kind of problem systematically (see extra slides)

QQQL in SMEFT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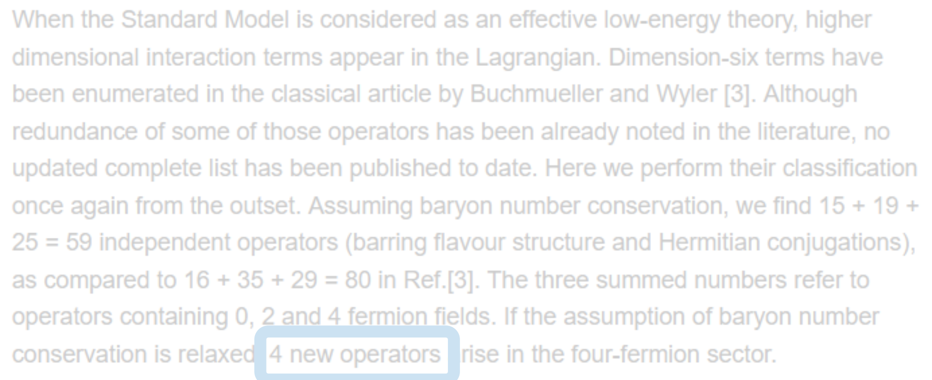
7 years later (20

v3 in arXiv of the same

I'll say more on this topic later (when discussing flavor)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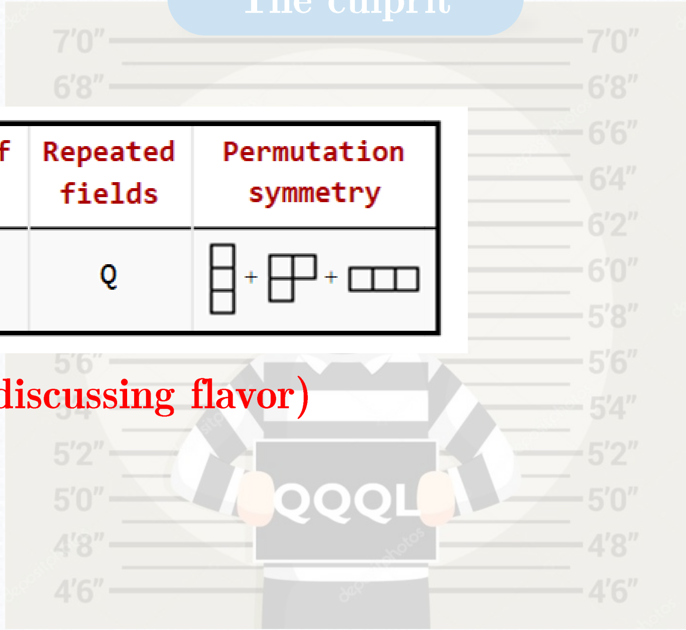
The culprit



Easy to tackle this kind of problem systematically (see extra slides)

Sym2Int

«Symmetries to Interactions»

GroupMath

A Mathematica package for the
group theory computations

RF 2011.01764

Basis-independent functions

Adjoint | Casimir | ConjugateIrrep | DynkinIndex | DimR | PermutationSymmetryOfInvariants | ReduceRepProduct | RepName | RepsUpToDimN | Weights | TriangularAnomalyValue | ...

Basis-dependent functions

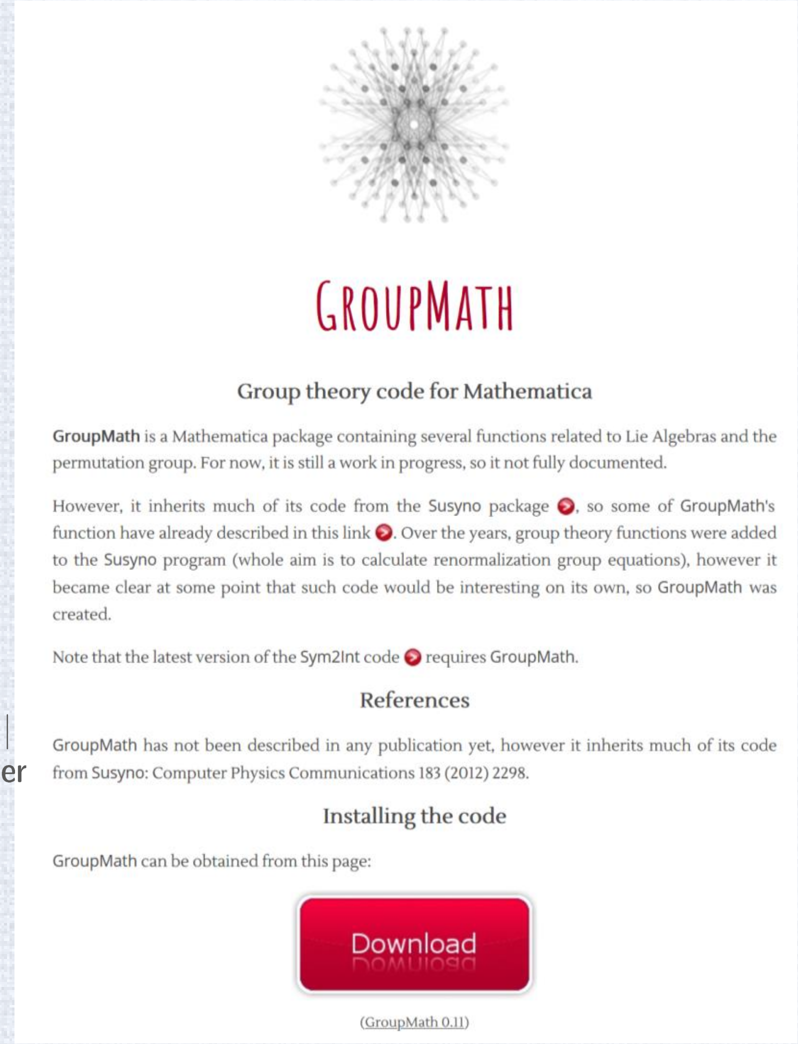
IrrepInProduct | RepMatrices | Invariants

Permutation group functions

DecomposeSnProduct | DrawYoungDiagram | GenerateStandardTableaux | HookContentFormula | LittlewoodRichardsonCoefficients | SnClassCharacter | SnClassOrder | SnIrrepDim | SnIrrepGenerators | ...

Symmetry breaking functions

DecomposeRep | FindAllEmbeddings | MaximalSubgroups | RegularSubgroupProjectionMatrix | SubgroupEmbeddingCoefficients



GROUPMATH

Group theory code for Mathematica

GroupMath is a Mathematica package containing several functions related to Lie Algebras and the permutation group. For now, it is still a work in progress, so it not fully documented.

However, it inherits much of its code from the Susyno package, so some of GroupMath's function have already described in this link. Over the years, group theory functions were added to the Susyno program (whole aim is to calculate renormalization group equations), however it became clear at some point that such code would be interesting on its own, so GroupMath was created.

Note that the latest version of the Sym2Int code requires GroupMath.

References

GroupMath has not been described in any publication yet, however it inherits much of its code from Susyno: Computer Physics Communications 183 (2012) 2298.

Installing the code

GroupMath can be obtained from this page:

Download

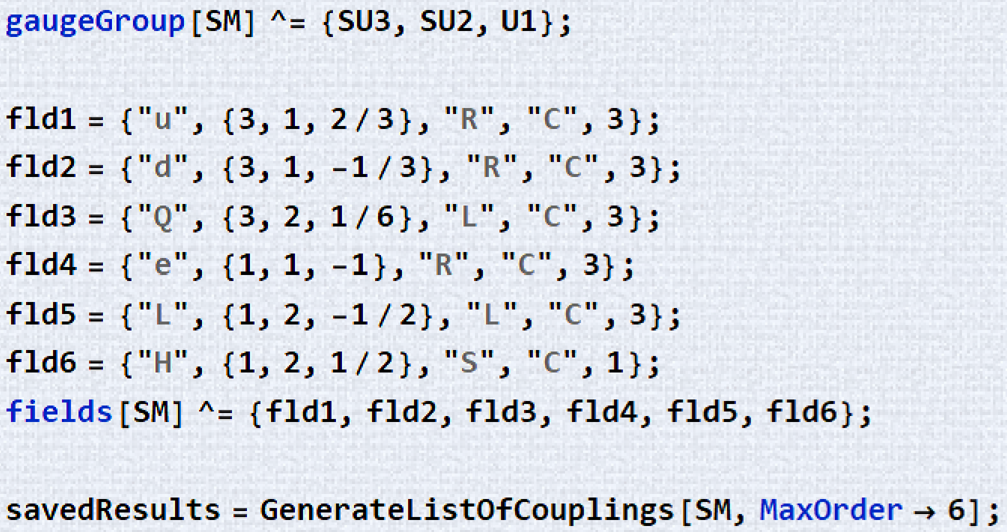
(GroupMath 0.11)

Sym2Int

«Symmetries to Interactions»

A Mathematica package to list the operators in a model
Works out of the box for any gauge group and representations

RF 1703.05221, 1907.12584



```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", 3};
fld2 = {"d", {3, 1, -1/3}, "R", "C", 3};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", 3};
fld4 = {"e", {1, 1, -1}, "R", "C", 3};
fld5 = {"L", {1, 2, -1/2}, "L", "C", 3};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};

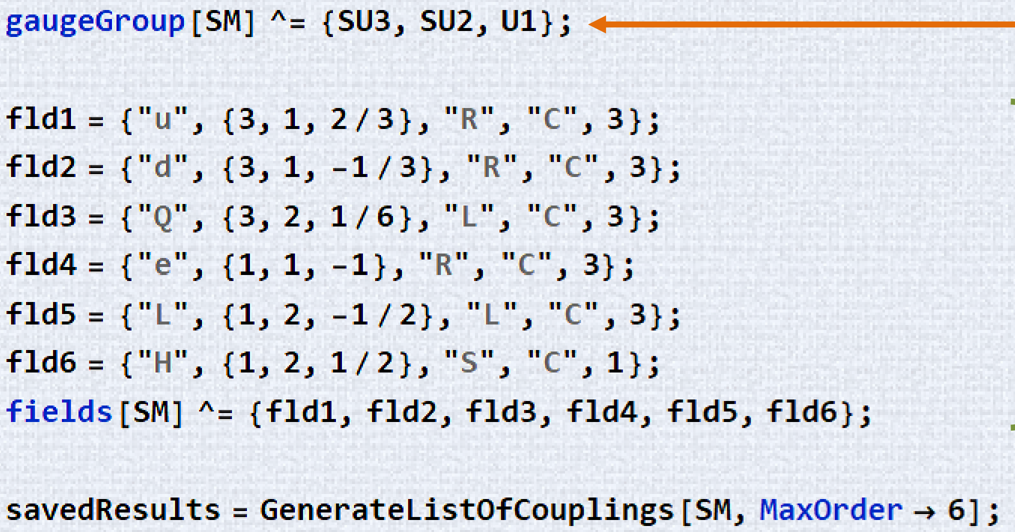
savedResults = GenerateListOfCouplings[SM, MaxOrder → 6];
```

Sym2Int

«Symmetries to Interactions»

A Mathematica package to list the operators in a model
Works out of the box for any gauge group and representations

RF 1703.05221, 1907.12584



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fld1 = {"u", {3, 1, 2 / 3}, "R", "C", 3};
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```

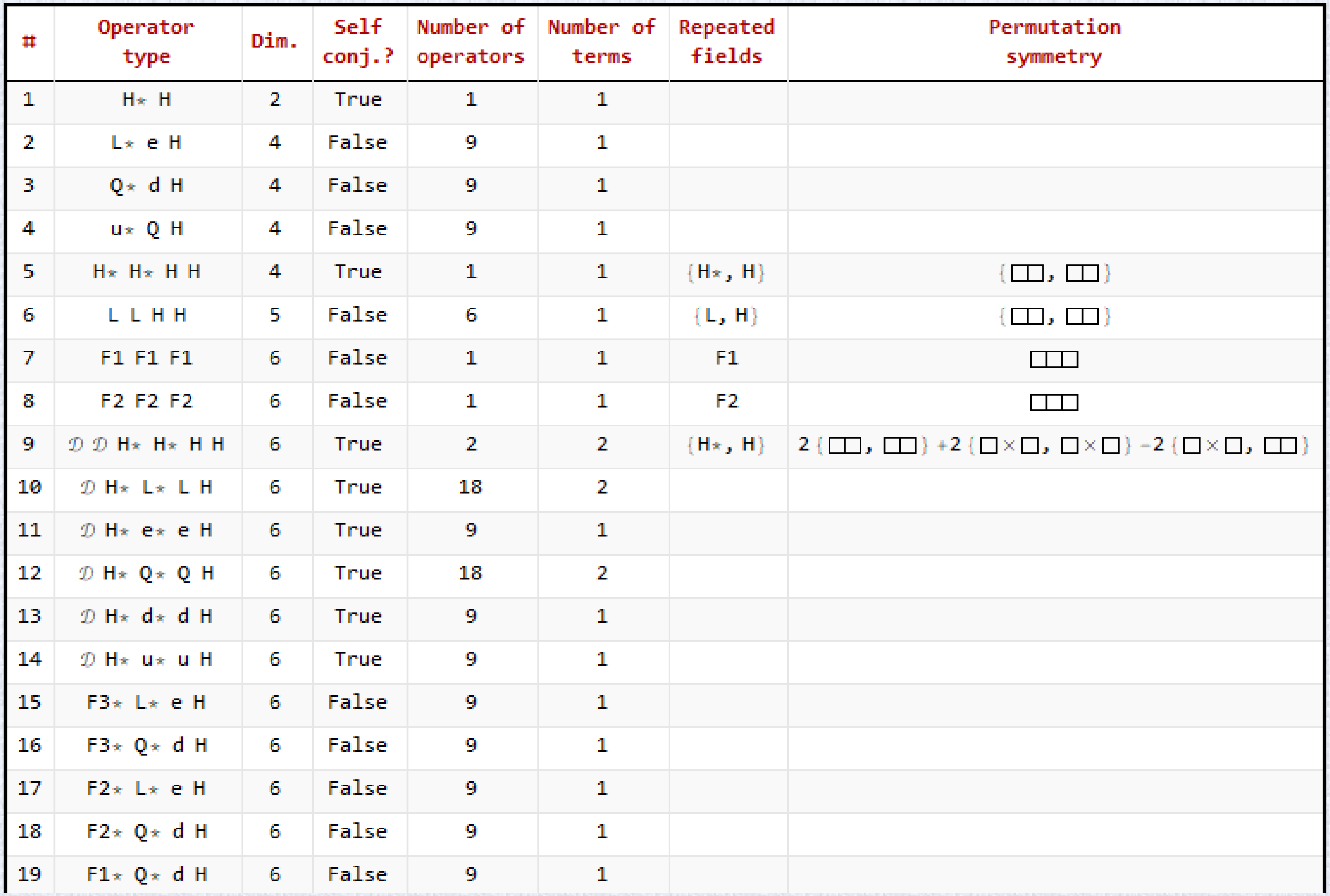
A name to the model
(e.g. SM)

The gauge group
(e.g. SU(3) x SU(2) x U(1))

The fields, i.e. the irreps under the gauge and Lorentz groups, including #flavors

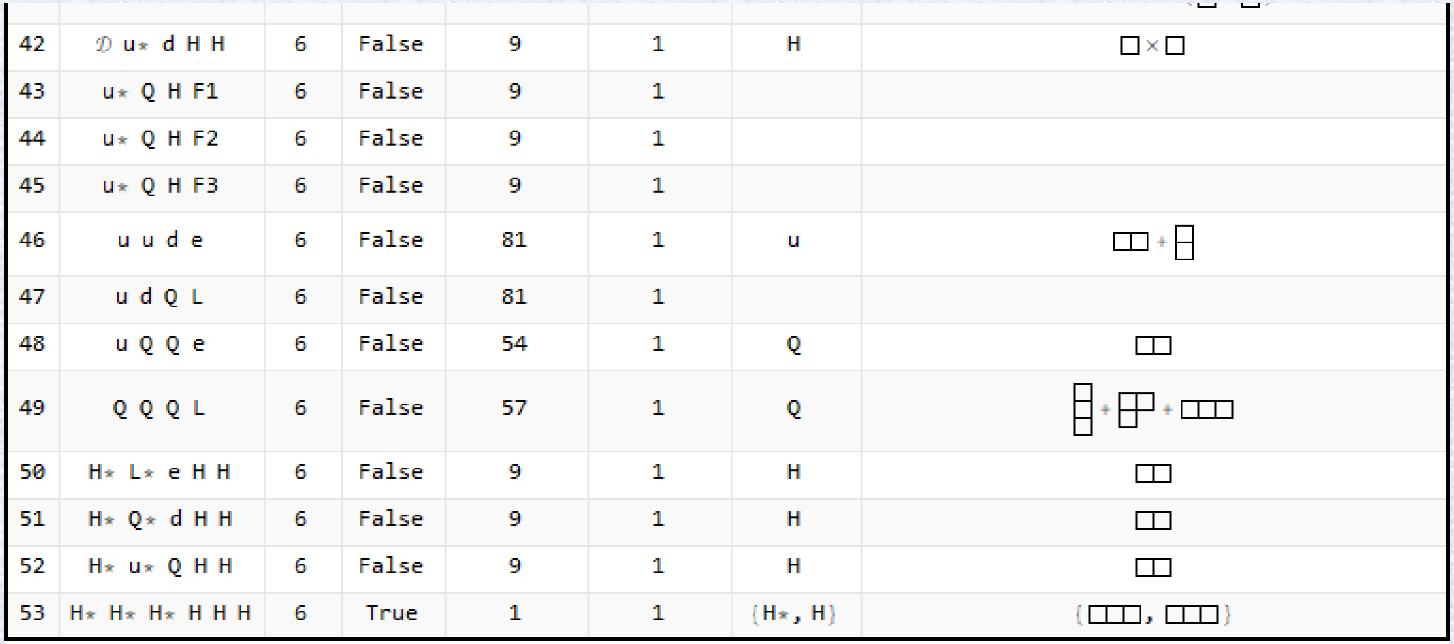
Max dimension of interactions
(e.g.: 6)

Example: SMEFT up to dim 6

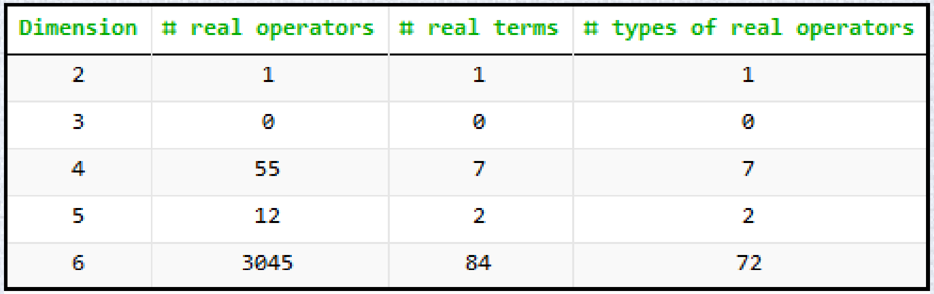


#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	H* H	2	True	1	1		
2	L* e H	4	False	9	1		
3	Q* d H	4	False	9	1		
4	u* Q H	4	False	9	1		
5	H* H* H H	4	True	1	1	{H*, H}	{☐☐, ☐☐}
6	L L H H	5	False	6	1	{L, H}	{☐☐, ☐☐}
7	F1 F1 F1	6	False	1	1	F1	☐☐☐
8	F2 F2 F2	6	False	1	1	F2	☐☐☐
9	𝒟 𝒟 H* H* H H	6	True	2	2	{H*, H}	2 {☐☐, ☐☐} +2 {☐×☐, ☐×☐} -2 {☐×☐, ☐☐}
10	𝒟 H* L* L H	6	True	18	2		
11	𝒟 H* e* e H	6	True	9	1		
12	𝒟 H* Q* Q H	6	True	18	2		
13	𝒟 H* d* d H	6	True	9	1		
14	𝒟 H* u* u H	6	True	9	1		
15	F3* L* e H	6	False	9	1		
16	F3* Q* d H	6	False	9	1		
17	F2* L* e H	6	False	9	1		
18	F2* Q* d H	6	False	9	1		
19	F1* Q* d H	6	False	9	1		

Example: SMEFT up to dim 6



42	𝒟 u* d H H	6	False	9	1	H	□ × □
43	u* Q H F1	6	False	9	1		
44	u* Q H F2	6	False	9	1		
45	u* Q H F3	6	False	9	1		
46	u u d e	6	False	81	1	u	□□ + $\substack{\square\\\square}$
47	u d Q L	6	False	81	1		
48	u Q Q e	6	False	54	1	Q	□□
49	Q Q Q L	6	False	57	1	Q	$\substack{\square\\\square\\\square}$ + $\substack{\square\square\\\square}$ + □□□
50	H* L* e H H	6	False	9	1	H	□□
51	H* Q* d H H	6	False	9	1	H	□□
52	H* u* Q H H	6	False	9	1	H	□□
53	H* H* H* H H H	6	True	1	1	{H*, H}	{□□□, □□□}



Dimension	# real operators	# real terms	# types of real operators
2	1	1	1
3	0	0	0
4	55	7	7
5	12	2	2
6	3045	84	72

Extending Sym2Int

Building operators explicitly

Known results for SMEFT

SMEFT dim 6

Buchmüller, Wyler NPB 268 (1986) 621
Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884

1986-2017

SMEFT dim 7

Lehman 1410.4193

2014

SMEFT dim 8

Murphy 2005.00059
Li, Ren, Shu, Xiao, Yu, Zheng, 2005.00008

2020

SMEFT dim 9

Li, Ren, Xiao, Yu, Zheng, 2007.07899

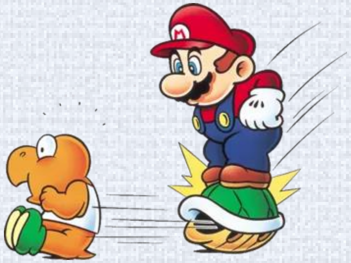
2020

DEFT Gripaios, Sutherland 1807.07546

ABC4EFT Li, Ren, Xiao, Yu, Zheng 2201.04639

Off shell

EOMs are not used
(Green basis)



SMEFT dim 6

Gherardi, Marzocca, Venturini, 2003.12525

SMEFT dim 8 (bosons)

Chala, Díaz-Carmona, Guedes 2112.12724

Operators = polynomials in many variables

Operators are just **homogenous polynomials in many variables**

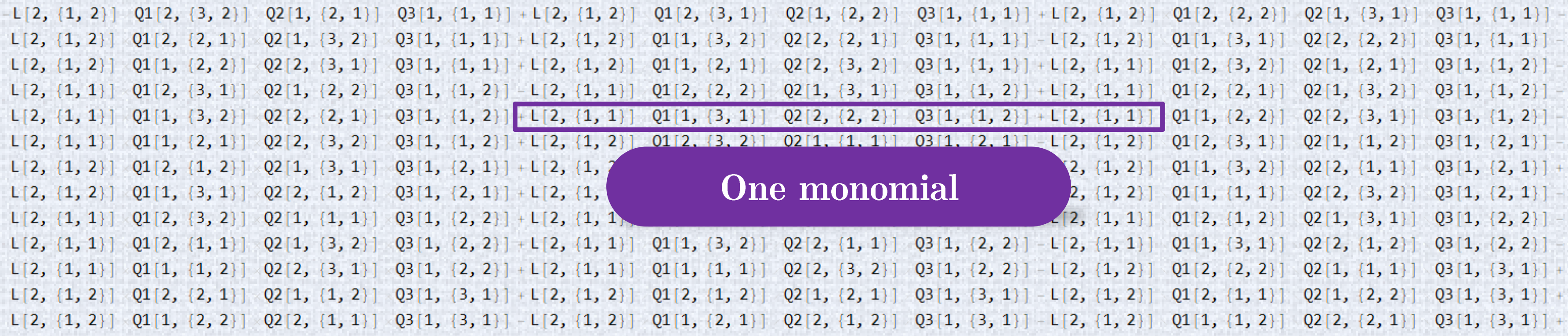
The **variables are field components**

Once we have a (potential over-complete) basis of operators of some kind, we can take each monomial to be a basis of a vector space and **covert operators into vectors**

At this stage we have a **Linear Algebra problem**

EOMs and IBPs are linear relations among the operators; they define **directions (vectors) in this vector space**

E.g.: Q1 Q2 Q3 L



```
- L[2, {1, 2}]  Q1[2, {3, 2}]  Q2[1, {2, 1}]  Q3[1, {1, 1}] + L[2, {1, 2}]  Q1[2, {3, 1}]  Q2[1, {2, 2}]  Q3[1, {1, 1}] + L[2, {1, 2}]  Q1[2, {2, 2}]  Q2[1, {3, 1}]  Q3[1, {1, 1}] -
  L[2, {1, 2}]  Q1[2, {2, 1}]  Q2[1, {3, 2}]  Q3[1, {1, 1}] + L[2, {1, 2}]  Q1[1, {3, 2}]  Q2[2, {2, 1}]  Q3[1, {1, 1}] - L[2, {1, 2}]  Q1[1, {3, 1}]  Q2[2, {2, 2}]  Q3[1, {1, 1}] -
  L[2, {1, 2}]  Q1[1, {2, 2}]  Q2[2, {3, 1}]  Q3[1, {1, 1}] + L[2, {1, 2}]  Q1[1, {2, 1}]  Q2[2, {3, 2}]  Q3[1, {1, 1}] + L[2, {1, 1}]  Q1[2, {3, 2}]  Q2[1, {2, 1}]  Q3[1, {1, 2}] -
  L[2, {1, 1}]  Q1[2, {3, 1}]  Q2[1, {2, 2}]  Q3[1, {1, 2}] - L[2, {1, 1}]  Q1[2, {2, 2}]  Q2[1, {3, 1}]  Q3[1, {1, 2}] + L[2, {1, 1}]  Q1[2, {2, 1}]  Q2[1, {3, 2}]  Q3[1, {1, 2}] -
  L[2, {1, 1}]  Q1[1, {3, 2}]  Q2[2, {2, 1}]  Q3[1, {1, 2}] + L[2, {1, 1}]  Q1[1, {3, 1}]  Q2[2, {2, 2}]  Q3[1, {1, 2}] + L[2, {1, 1}]  Q1[1, {2, 2}]  Q2[2, {3, 1}]  Q3[1, {1, 2}] -
  L[2, {1, 1}]  Q1[1, {2, 1}]  Q2[2, {3, 2}]  Q3[1, {1, 2}] + L[2, {1, 2}]  Q1[2, {3, 2}]  Q2[1, {1, 1}]  Q3[1, {2, 1}] - L[2, {1, 2}]  Q1[2, {3, 1}]  Q2[1, {1, 2}]  Q3[1, {2, 1}] -
  L[2, {1, 2}]  Q1[2, {1, 2}]  Q2[1, {3, 1}]  Q3[1, {2, 1}] + L[2, {1,                                                      2, {1, 2}]  Q1[1, {3, 2}]  Q2[2, {1, 1}]  Q3[1, {2, 1}] +
  L[2, {1, 2}]  Q1[1, {3, 1}]  Q2[2, {1, 2}]  Q3[1, {2, 1}] + L[2, {1,                                                      2, {1, 2}]  Q1[1, {1, 1}]  Q2[2, {3, 2}]  Q3[1, {2, 1}] -
  L[2, {1, 1}]  Q1[2, {3, 2}]  Q2[1, {1, 1}]  Q3[1, {2, 2}] + L[2, {1, 1                                                    L[2, {1, 1}]  Q1[2, {1, 2}]  Q2[1, {3, 1}]  Q3[1, {2, 2}] -
  L[2, {1, 1}]  Q1[2, {1, 1}]  Q2[1, {3, 2}]  Q3[1, {2, 2}] + L[2, {1, 1}]  Q1[1, {3, 2}]  Q2[2, {1, 1}]  Q3[1, {2, 2}] - L[2, {1, 1}]  Q1[1, {3, 1}]  Q2[2, {1, 2}]  Q3[1, {2, 2}] -
  L[2, {1, 1}]  Q1[1, {1, 2}]  Q2[2, {3, 1}]  Q3[1, {2, 2}] + L[2, {1, 1}]  Q1[1, {1, 1}]  Q2[2, {3, 2}]  Q3[1, {2, 2}] - L[2, {1, 2}]  Q1[2, {2, 2}]  Q2[1, {1, 1}]  Q3[1, {3, 1}] +
  L[2, {1, 2}]  Q1[2, {2, 1}]  Q2[1, {1, 2}]  Q3[1, {3, 1}] + L[2, {1, 2}]  Q1[2, {1, 2}]  Q2[1, {2, 1}]  Q3[1, {3, 1}] - L[2, {1, 2}]  Q1[2, {1, 1}]  Q2[1, {2, 2}]  Q3[1, {3, 1}] +
  L[2, {1, 2}]  Q1[1, {2, 2}]  Q2[2, {1, 1}]  Q3[1, {3, 1}] - L[2, {1, 2}]  Q1[1, {2, 1}]  Q2[2, {1, 2}]  Q3[1, {3, 1}] - L[2, {1, 2}]  Q1[1, {1, 2}]  Q2[2, {2, 1}]  Q3[1, {3, 1}] +
```

One monomial

+ ...

Lorentz contractions

Distribute the derivatives by the fields in all possible ways

Bosons

Vector indices: contract them in all possible ways with g's and ε's

Explicitly build the expressions and check for redundancies

Fermions

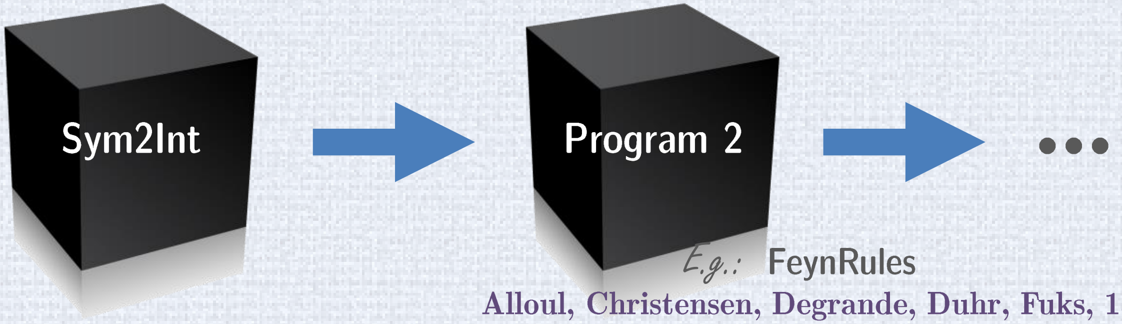
- Place Weyl spinors in 4-D Dirac spinors
- Form fermion bilinears
- Use Dirac gamma matrices and C to convert spinor indices into vector indices

$$\left(\begin{array}{c:c} \begin{gathered}\boxed{L^*L}\\ \gamma^0\gamma^\mu\end{gathered} & \begin{gathered}\boxed{L^*R}\\ \gamma^0\\ \gamma^0\left[\gamma^\mu,\gamma^\nu\right]\end{gathered} \\ \hdashline \begin{gathered}\boxed{R^*L}\\ \gamma^0\\ \gamma^0\left[\gamma^\mu,\gamma^\nu\right]\end{gathered} & \begin{gathered}\boxed{R^*R}\\ \gamma^0\gamma^\mu\end{gathered} \end{array}\right) \qquad \left(\begin{array}{c:c} \begin{gathered}\boxed{LL}\\ C\\ C\left[\gamma^\mu,\gamma^\nu\right]\end{gathered} & \begin{gathered}\boxed{LR}\\ C\gamma^\mu\end{gathered} \\ \hdashline \begin{gathered}\boxed{RL}\\ C\gamma^\mu\end{gathered} & \begin{gathered}\boxed{RR}\\ C\\ C\left[\gamma^\mu,\gamma^\nu\right]\end{gathered} \end{array}\right)$$

Gauge contractions (#1)

GroupMath can find the explicit gauge invariant contractions of a set of representations of arbitrary Lie algebras

It works fine. However ... it might not be ideal.



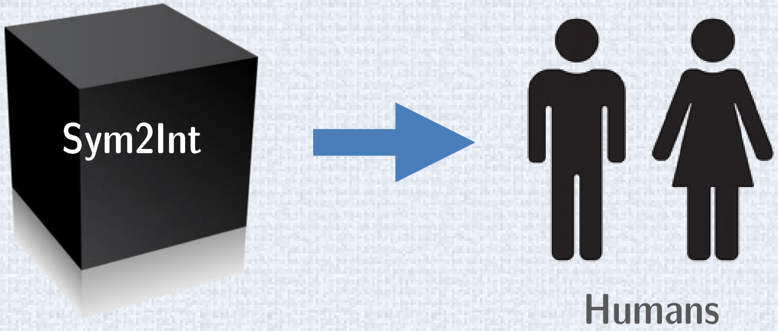
Large tensors are passed to other programs. Output of Sym2Int does not need to be easy to read

E.g.: FeynRules

Alloul, Christensen, Degrande, Duhr, Fuks, 1310.1921

MatchMakereft

Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787



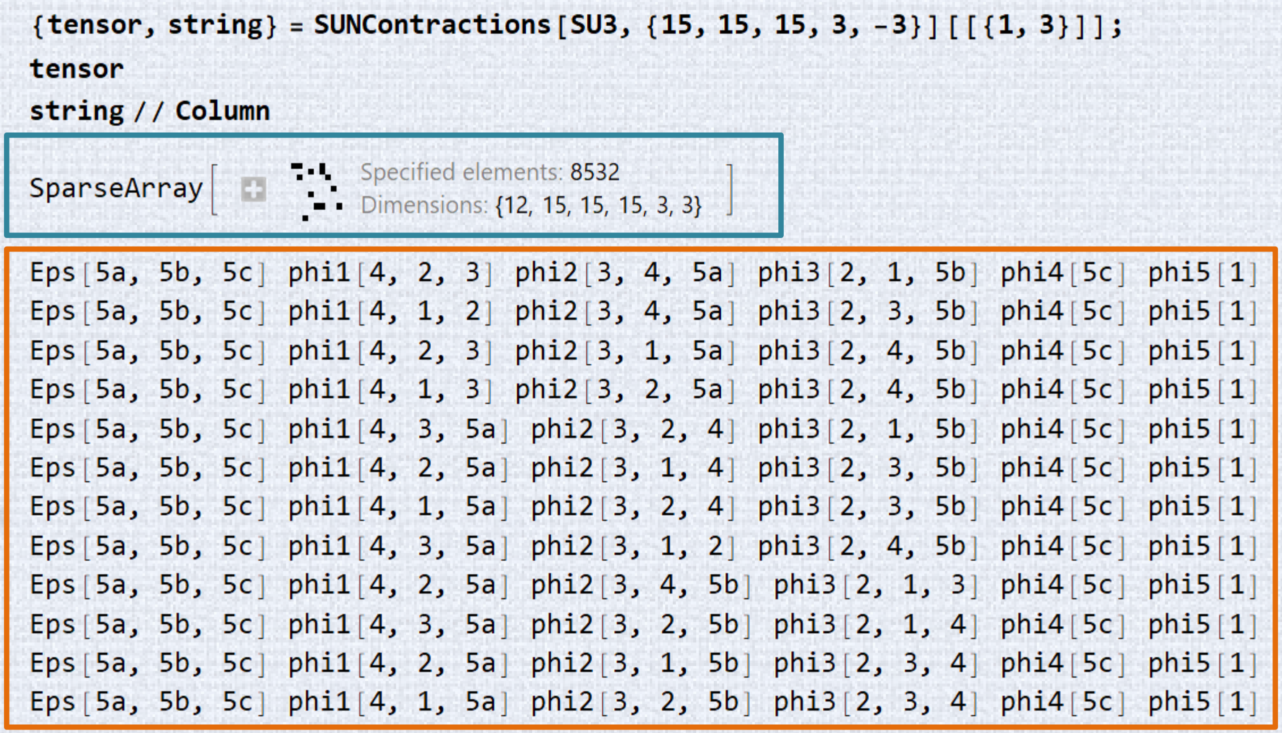
Output of Sym2Int should be **human readable**

No right/wrong answers here. But in the end, in both cases it is convenient that the gauge contractions used are similar to what a human would write

Gauge contractions (#2)

To this of end, I've been extending GroupMath so that in the case of *SU(n)* groups contractions are done via the tensor method.

The program outputs a tensor with the result, but also a string identifying which type of contraction was made



```
{tensor, string} = SUNContractions[SU3, {15, 15, 15, 3, -3}][[{1, 3}]];
tensor
string // Column
```

The tensor

```
SparseArray[ Specified elements: 8532
             Dimensions: {12, 15, 15, 15, 3, 3} ]
```

The contractions

```
Eps[5a, 5b, 5c] phi1[4, 2, 3] phi2[3, 4, 5a] phi3[2, 1, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 2] phi2[3, 4, 5a] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 3] phi2[3, 1, 5a] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 3] phi2[3, 2, 5a] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 2, 4] phi3[2, 1, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 1, 4] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 5a] phi2[3, 2, 4] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 1, 2] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 4, 5b] phi3[2, 1, 3] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 2, 5b] phi3[2, 1, 4] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 1, 5b] phi3[2, 3, 4] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 5a] phi2[3, 2, 5b] phi3[2, 3, 4] phi4[5c] phi5[1]
```

Assumed indices:

$15^{*}{}_{**}$

3_{*}

$\overline{3}^{*}$

EOM/ IBP relations summary

Long story short:

- EOM/IBP relations are calculated explicitly.
- The program computes that some polynomials of the field components are redundant, i.e. null for the present purposes.
- These formally null polynomials must be a linear combinations of the basis of operators previously computed. Therefore,

IOMs/EOMs
=
vectors (linear relations among operators)

Example

Consider 4 operators: $\mathcal{O}_{1,2,3,4}$ (calculated explicitly by the code)

Suppose there are two EOM/IBP relations:

$$\mathcal{O}_1 + 7\mathcal{O}_3 \sim 0$$
$$\mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4 \sim 0$$

(their existence and form computed by explicitly calculating the redundant operators)

We associate these relations to two vectors, which we place as rows in a matrix

$$\begin{pmatrix} 1 & 0 & 7 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Valid basis of non-redundant operators: Any two vectors which, when added to this matrix, make it full rank.
Non-unique/a matter of choice.

A major problem ... and its solution

The problem: Repeated fields

Operators with repeated fields (such as *LLHH*) are much harder to handle.

Even ignoring derivatives, just consider that
(# **contractions**) ≠ (# **gauge contr.**) × (# **Lorentz contr.**)

The solution: Differentiate fields

$$LLHH \rightarrow L_1 L_2 H_1 H_2$$

Side comment: manipulation of Grassmann variables alone recommends this

1. Obtain a "*super basis*" of operators

   Contains extra operators which are null when fields are made equal

2. Permutations of equal fields = redundancies of the "super basis"

   We add these to IBPs and EOMs relations

Grid of *super basis* of operators

I think it is very useful to picture all operators in a grid

		Lorentz contractions								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**...**
Gauge contractions	**1**	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Operators are being displayed here as a grid, to emphasize the different gauge and Lorentz representations. But in computer calculation, it is better to "flatten" this grid

IBPs, EOMs and equal-field relations are vectors in a vector space whose basis are all the operators in the grid.

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		Lorentz contractions								
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Operators are being displayed here as a grid, to emphasize the different gauge and Lorentz representations. But in computer calculation, it is better to "flatten" this grid

IBPs, EOMs and equal-field relations are vectors in a vector space whose **basis are all the operators** in the grid.

Example

Consider 6 operators:

$$\begin{matrix} \mathcal{O}_{1,1} & \mathcal{O}_{1,2} & \mathcal{O}_{1,3} \\ \mathcal{O}_{2,1} & \mathcal{O}_{2,2} & \mathcal{O}_{2,3} \end{matrix}$$

If for some reason $\mathcal{O}_{1,1} + 7\mathcal{O}_{2,2} \sim 0$, then we should consider the vector

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 7 & 0 \end{matrix}$$

In this example, any 5 vectors which together with (1,0,0,0,7,0) form a full-rank matrix, would represent valid non-redundant operators

Grid of *super basis* of operators

I think it is very useful to picture all operators in a grid

		Lorentz contractions								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**...**
Gauge contractions	**1**	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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		Lorentz contractions								
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
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	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

EOM's

Horizontal relations; the **same for all rows** (i.e. all gauge contractions)

Grid of *super basis* of operators

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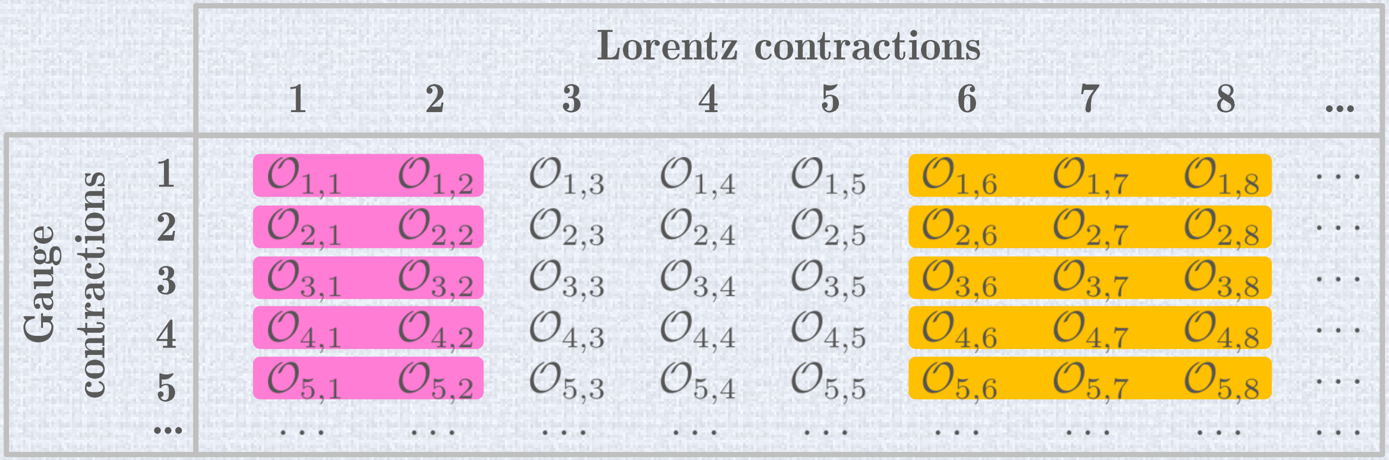

		Lorentz contractions								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**...**
Gauge contractions	**1**	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
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	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
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	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

EOM's **Horizontal relations**; the **same for all rows** (i.e. all gauge contractions)

IBP's **Horizontal relations**; the **same for all rows** (i.e. all gauge contractions)

Grid of *super basis* of operators

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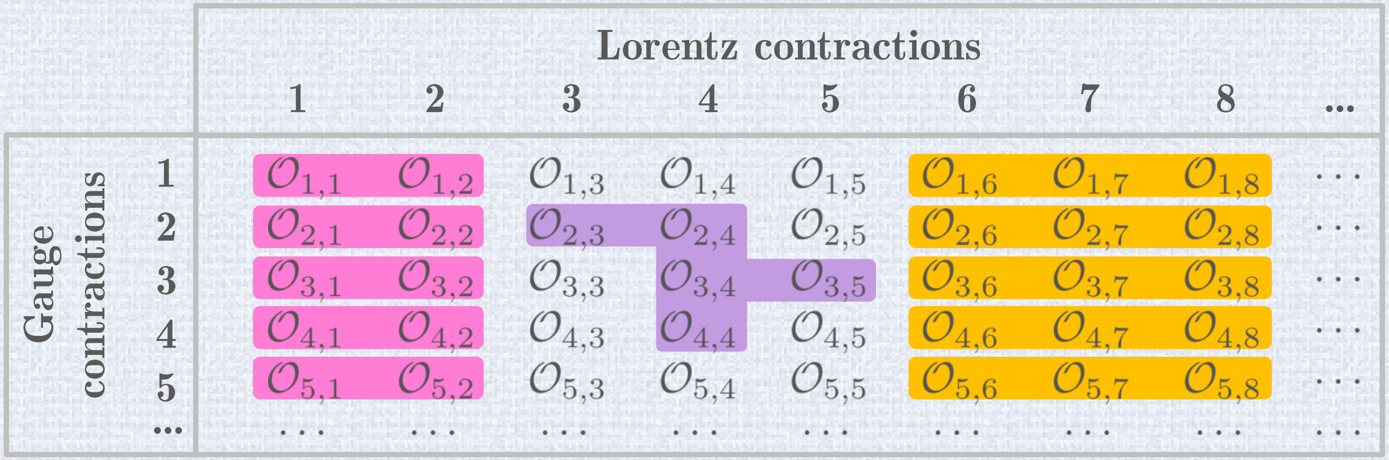

		Lorentz contractions								
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
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	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

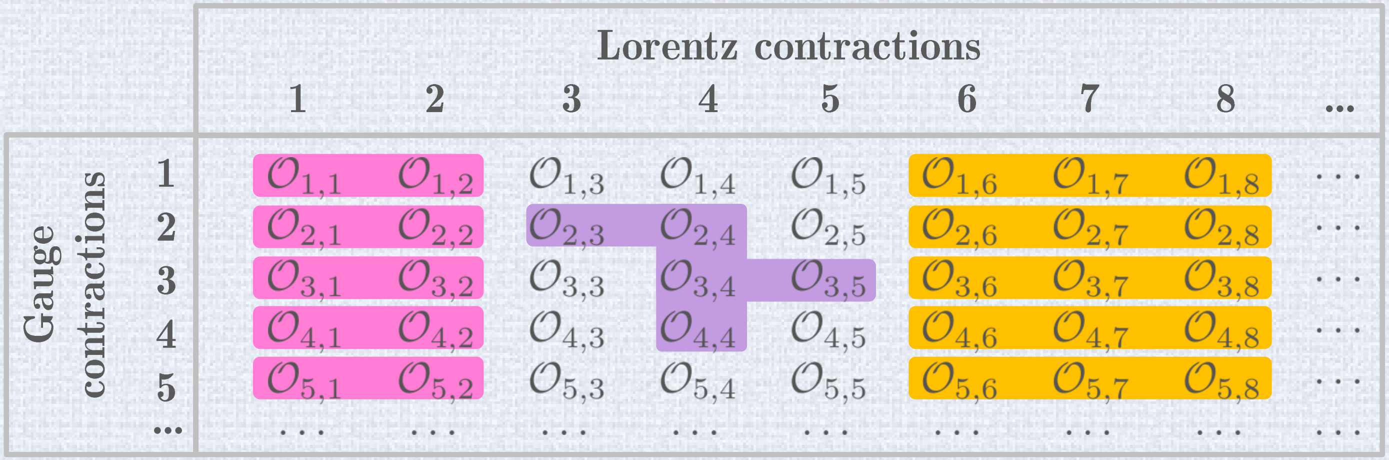
EOM's Horizontal relations; the same for all rows (i.e. all gauge contractions)

IBP's Horizontal relations; the same for all rows (i.e. all gauge contractions)

Repeated fields Oblique relations in general! Not the same for each row

Grid of *super basis* of operators

I think it is very useful to picture all operators in a grid



		Lorentz contractions							
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

EOM's Horizontal relations; the same for all rows (i.e. all gauge contractions)

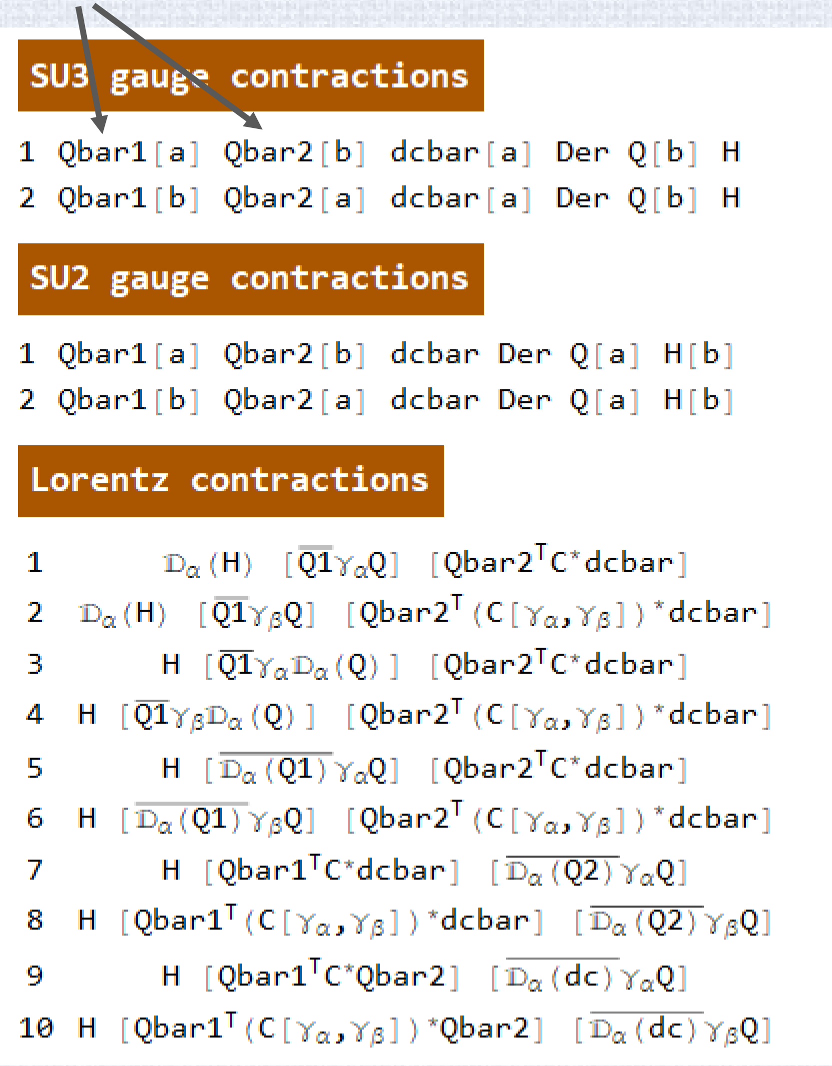
IBP's Horizontal relations; the same for all rows (i.e. all gauge contractions)

Repeated fields Oblique relations in general! Not the same for each row

A nice fact: in order to know the "repeated field redundancies" it is not necessary to know the details of the gauge contractions – only how permutation symmetries act on them (elegant; one can change the group/reps and still reuse results)

Example: $D_\mu\overline{Q}\,\overline{Q}Q\overline{d^c}H$

Discriminate
the $\overline{Q}$’s



SU3 gauge contractions

```
1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H
```

SU2 gauge contractions

```
1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]
```

Lorentz contractions

1 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\alpha\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

2 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\beta\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

3 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\alpha\mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

4 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\beta\mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

5 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\alpha\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

6 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\beta\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

7 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\alpha\mathrm{Q}]$

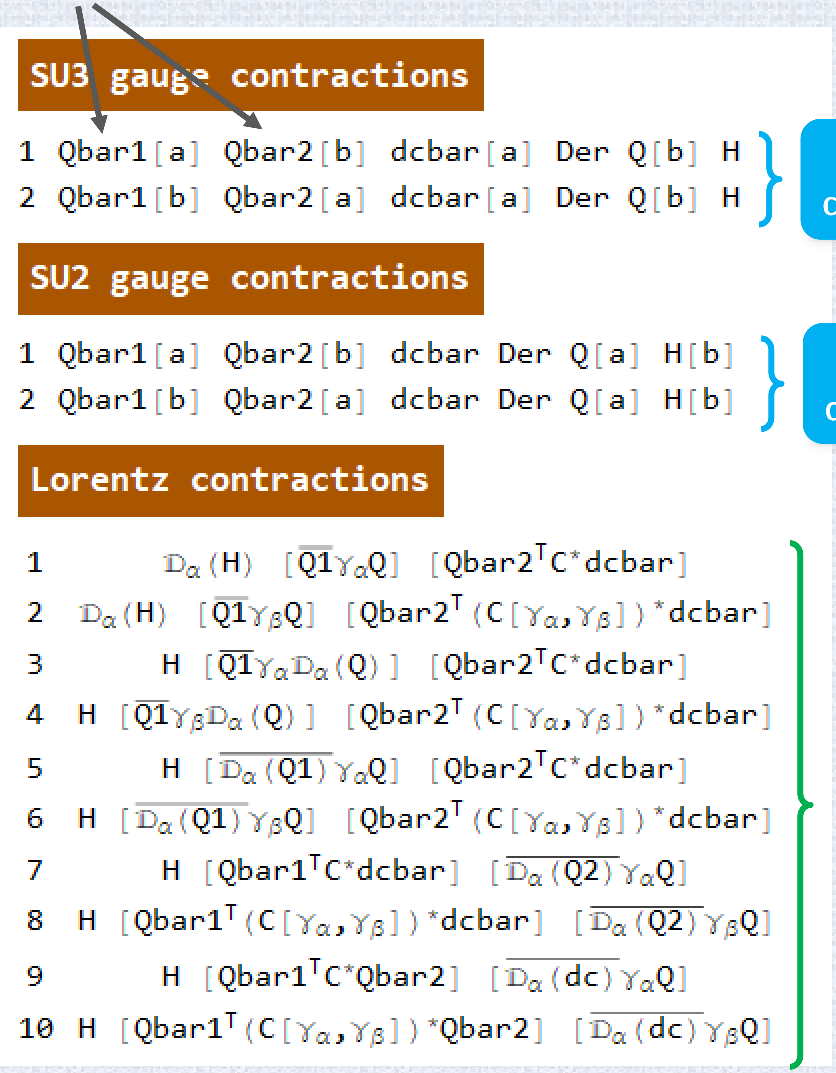
8 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\beta\mathrm{Q}]$

9 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}\mathrm{C}^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\alpha\mathrm{Q}]$

10 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\beta\mathrm{Q}]$

Example: $D_\mu \overline{Q}\overline{Q}Q\overline{d^c}H$

Discriminate the $\overline{Q}$'s



SU3 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H

SU2 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]

Lorentz contractions

1 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\alpha \mathrm{Q}]\ [\mathrm{Qbar2}^{\mathrm{T}}\mathrm{C}^*\mathrm{dcbar}]$
2 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\beta \mathrm{Q}]\ [\mathrm{Qbar2}^{\mathrm{T}}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$
3 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\alpha \mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^{\mathrm{T}}\mathrm{C}^*\mathrm{dcbar}]$
4 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\beta \mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^{\mathrm{T}}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$
5 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\alpha \mathrm{Q}]\ [\mathrm{Qbar2}^{\mathrm{T}}\mathrm{C}^*\mathrm{dcbar}]$
6 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\beta \mathrm{Q}]\ [\mathrm{Qbar2}^{\mathrm{T}}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$
7 $\mathrm{H}\ [\mathrm{Qbar1}^{\mathrm{T}}\mathrm{C}^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\alpha \mathrm{Q}]$
8 $\mathrm{H}\ [\mathrm{Qbar1}^{\mathrm{T}}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\beta \mathrm{Q}]$
9 $\mathrm{H}\ [\mathrm{Qbar1}^{\mathrm{T}}\mathrm{C}^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\alpha \mathrm{Q}]$
10 $\mathrm{H}\ [\mathrm{Qbar1}^{\mathrm{T}}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\beta \mathrm{Q}]$

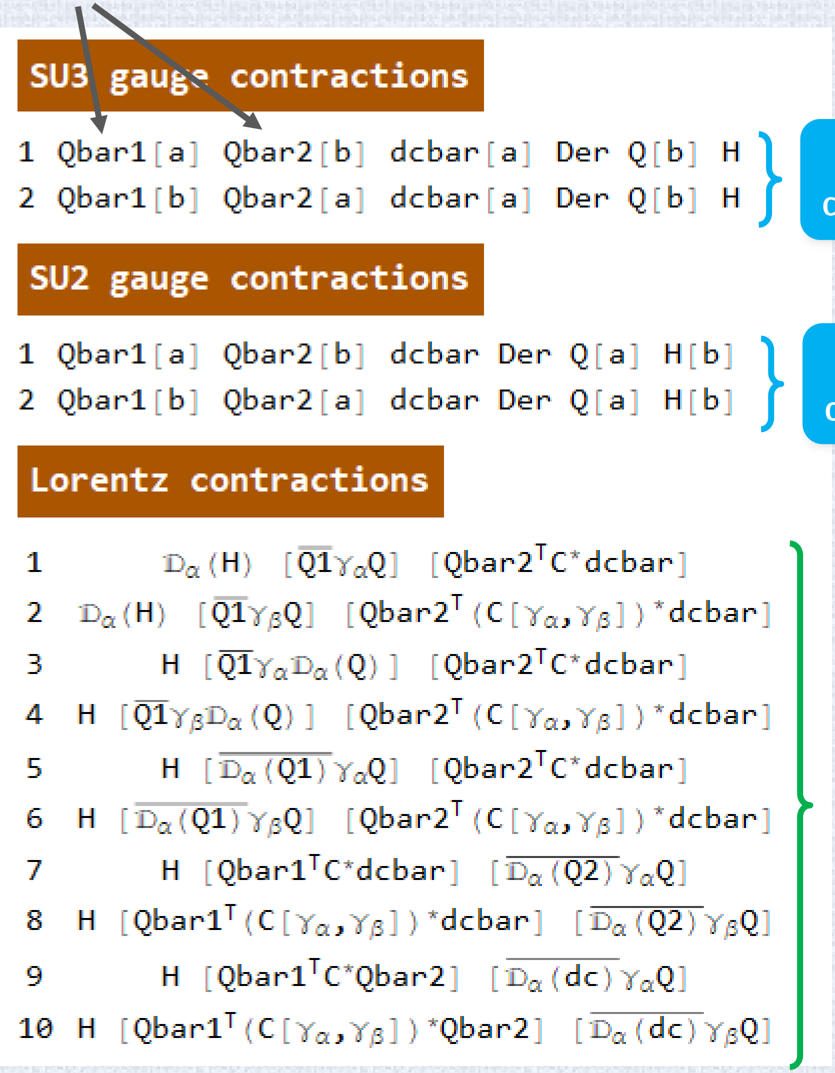
2 SU(3) contractions

2 SU(2) contractions

10 Lorentz contractions

Example: $D_\mu \overline{Q}\,\overline{Q} Q \overline{d^c} H$

Discriminate
the $\overline{Q}$'s



SU3 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H

SU2 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]

Lorentz contractions

1 $\mathbb{D}_\alpha(H)\ [\overline{Q1}\gamma_\alpha Q]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
2 $\mathbb{D}_\alpha(H)\ [\overline{Q1}\gamma_\beta Q]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
3 $H\ [\overline{Q1}\gamma_\alpha \mathbb{D}_\alpha(Q)]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
4 $H\ [\overline{Q1}\gamma_\beta \mathbb{D}_\alpha(Q)]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
5 $H\ [\overline{\mathbb{D}_\alpha(Q1)}\gamma_\alpha Q]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
6 $H\ [\overline{\mathbb{D}_\alpha(Q1)}\gamma_\beta Q]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
7 $H\ [\mathrm{Qbar1}^T C^* \mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(Q2)}\gamma_\alpha Q]$
8 $H\ [\mathrm{Qbar1}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(Q2)}\gamma_\beta Q]$
9 $H\ [\mathrm{Qbar1}^T C^* \mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(dc)}\gamma_\alpha Q]$
10 $H\ [\mathrm{Qbar1}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(dc)}\gamma_\beta Q]$

2 SU(3) contractions

2 SU(2) contractions

10 Lorentz contractions

Same-field redundancies

$$
\begin{array}{c|cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
(1,1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(1,2) & \frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(2,1) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(2,2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

+19 others

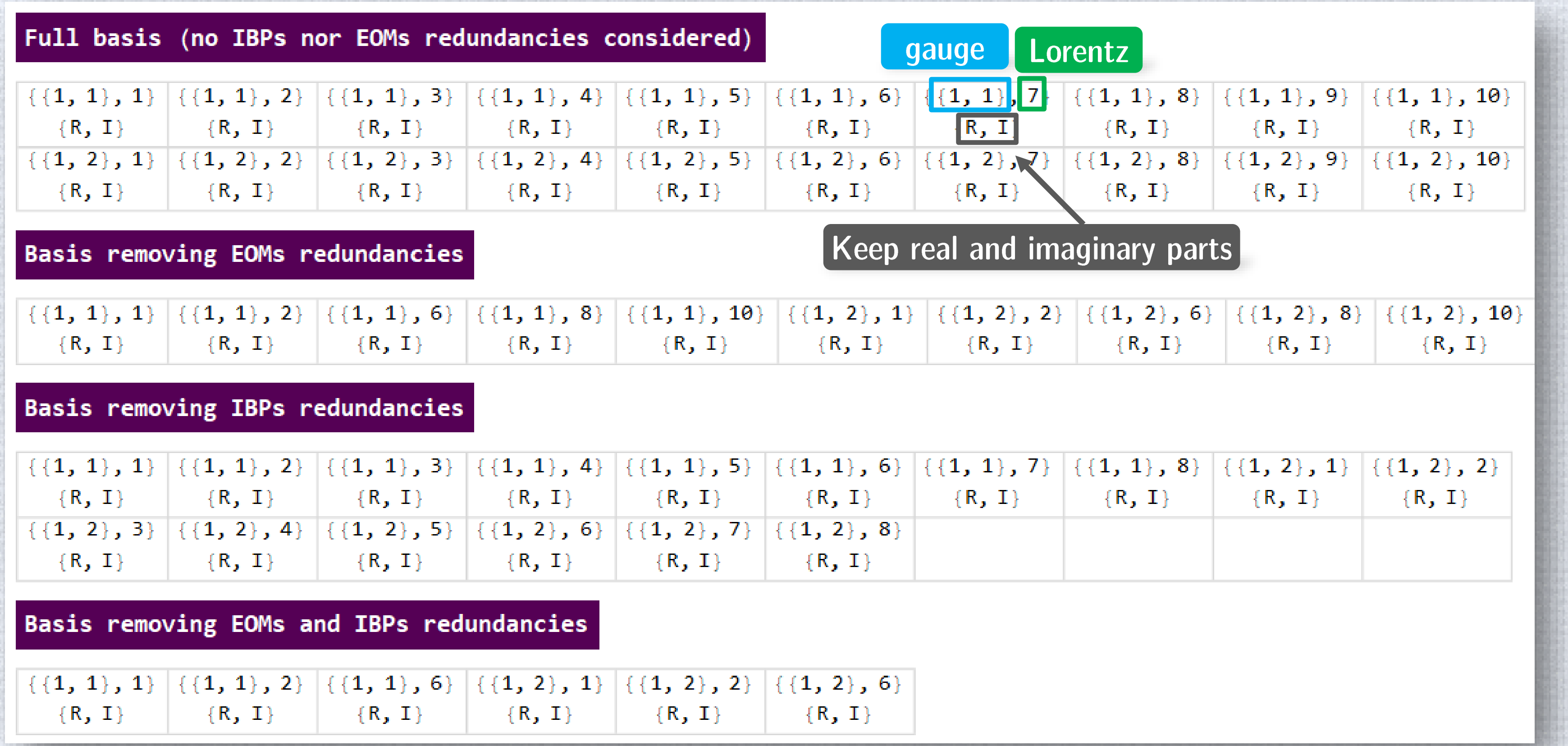
IBP redundancies

$$
\begin{array}{r}
\text{for each (i,j)} \\
\text{for each (i,j)}
\end{array}
\begin{pmatrix}
1 & 0 & 1 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\
0 & 1 & 0 & 1 & 0 & 1 & -3 & -\frac{1}{2} & -3 & -\frac{1}{2}
\end{pmatrix}
$$

EOM redundancies

$$
\text{for each (i,j)} \left\{
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\right.
$$

Example: $D_\mu \overline{QQ} Q \overline{d^c} H$



Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R, I}	{{1, 1}, 5} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 7} {R, I}	{{1, 1}, 8} {R, I}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}
{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 3} {R, I}	{{1, 2}, 4} {R, I}	{{1, 2}, 5} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 7} {R, I}	{{1, 2}, 8} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 10} {R, I}

Basis removing EOMs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 8} {R, I}	{{1, 1}, 10} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 8} {R, I}	{{1, 2}, 10} {R, I}

Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R, I}	{{1, 1}, 5} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 7} {R, I}	{{1, 1}, 8} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}
{{1, 2}, 3} {R, I}	{{1, 2}, 4} {R, I}	{{1, 2}, 5} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 7} {R, I}	{{1, 2}, 8} {R, I}				

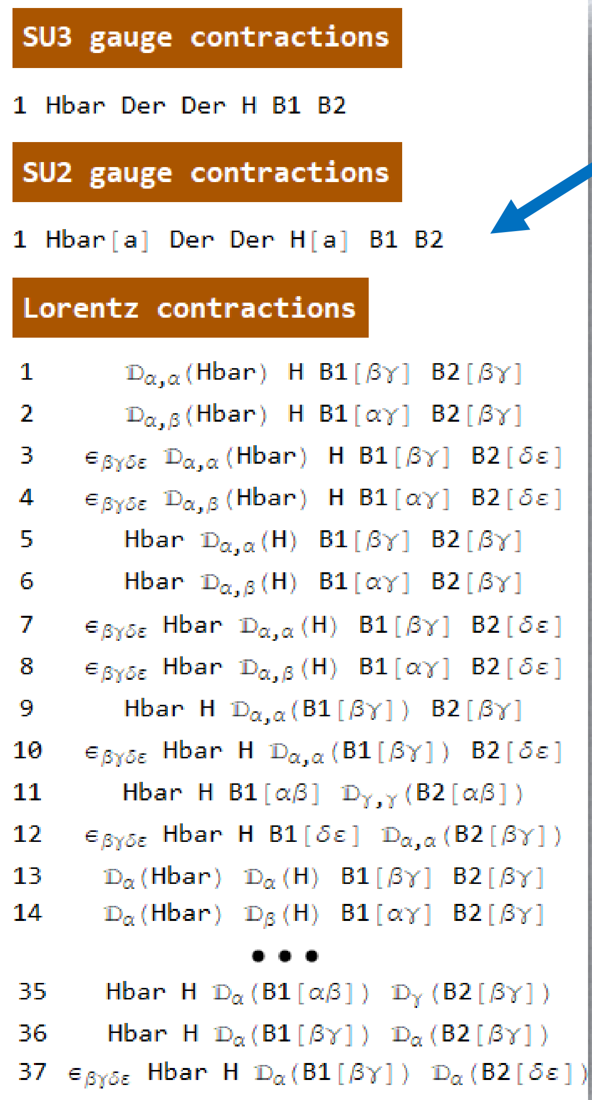
Basis removing EOMs and IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 6} {R, I}

This is one possibility: sets of operators that work are picked automatically. With the redundancies calculated, another conceivable scenario is to allow the user to ask the code “Do the operators A,B,C form a basis?”.

Interface & output format require thinking (work in progress)

Example: $D_\mu D_\nu B B \overline{H} H$



SU3 gauge contractions

1 Hbar Der Der H B1 B2

SU2 gauge contractions

1 Hbar[a] Der Der H[a] B1 B2

Lorentz contractions

1 $\mathbb{D}_{\alpha,\alpha}$(Hbar) H B1[$\beta\gamma$] B2[$\beta\gamma$]
2 $\mathbb{D}_{\alpha,\beta}$(Hbar) H B1[$\alpha\gamma$] B2[$\beta\gamma$]
3 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\alpha}$(Hbar) H B1[$\beta\gamma$] B2[$\delta\varepsilon$]
4 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\beta}$(Hbar) H B1[$\alpha\gamma$] B2[$\delta\varepsilon$]
5 Hbar $\mathbb{D}_{\alpha,\alpha}$(H) B1[$\beta\gamma$] B2[$\beta\gamma$]
6 Hbar $\mathbb{D}_{\alpha,\beta}$(H) B1[$\alpha\gamma$] B2[$\beta\gamma$]
7 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\alpha}$(H) B1[$\beta\gamma$] B2[$\delta\varepsilon$]
8 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\beta}$(H) B1[$\alpha\gamma$] B2[$\delta\varepsilon$]
9 Hbar H $\mathbb{D}_{\alpha,\alpha}$(B1[$\beta\gamma$]) B2[$\beta\gamma$]
10 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha,\alpha}$(B1[$\beta\gamma$]) B2[$\delta\varepsilon$]
11 Hbar H B1[$\alpha\beta$] $\mathbb{D}_{\gamma,\gamma}$(B2[$\alpha\beta$])
12 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H B1[$\delta\varepsilon$] $\mathbb{D}_{\alpha,\alpha}$(B2[$\beta\gamma$])
13 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\alpha}$(H) B1[$\beta\gamma$] B2[$\beta\gamma$]
14 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\beta}$(H) B1[$\alpha\gamma$] B2[$\beta\gamma$]

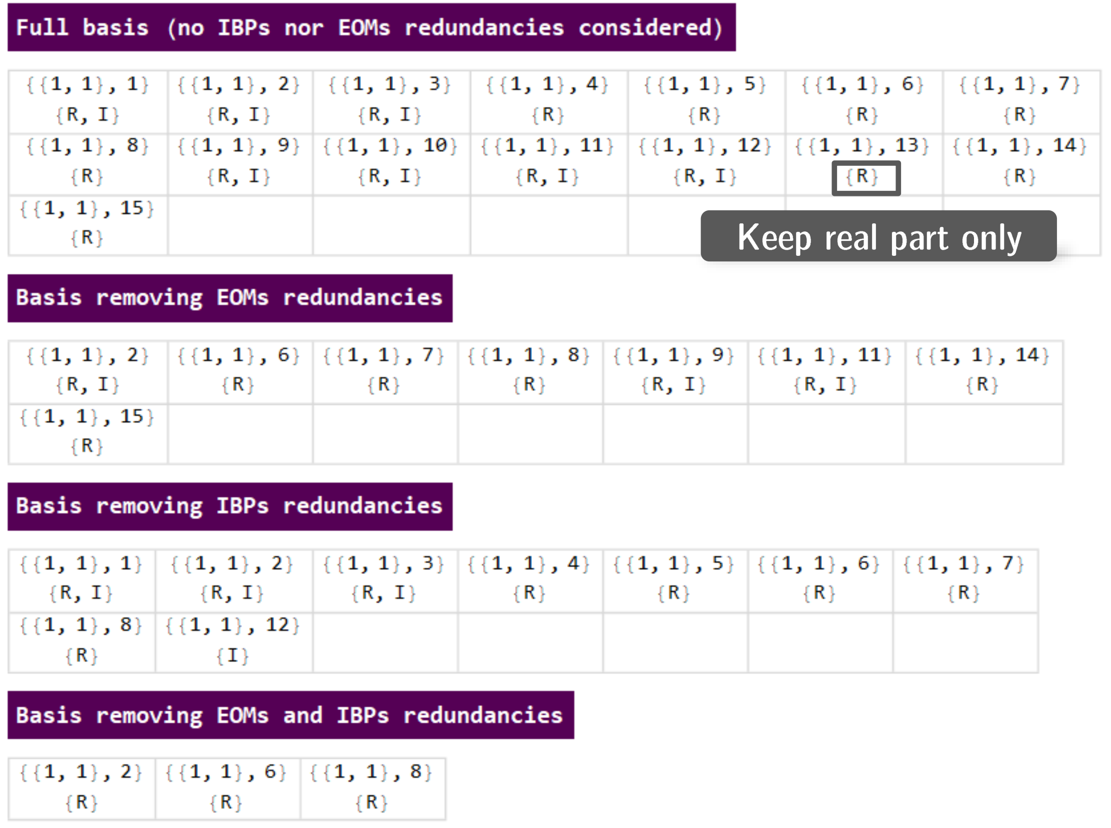
• • •

35 Hbar H $\mathbb{D}_{\alpha}$(B1[$\alpha\beta$]) $\mathbb{D}_{\gamma}$(B2[$\beta\gamma$])
36 Hbar H $\mathbb{D}_{\alpha}$(B1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(B2[$\beta\gamma$])
37 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha}$(B1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(B2[$\delta\varepsilon$])

Symmetric under exchange of B1 and B2

This is all that matters

H, B could transform differently (even under some different group), but the results would be the same as long as B1 and B2 are symmetrically contracted



Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}
{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 12} {R, I}	{{1, 1}, 13} {R}	{{1, 1}, 14} {R}
{{1, 1}, 15} {R}						

Keep real part only

Basis removing EOMs redundancies

{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 14} {R}
{{1, 1}, 15} {R}						

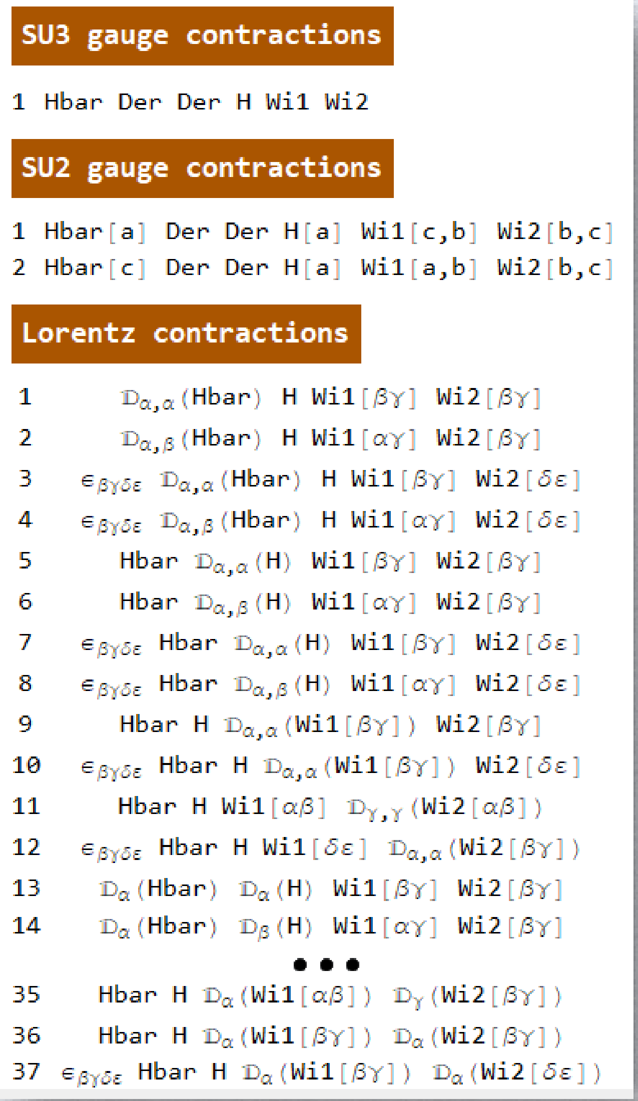
Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}
{{1, 1}, 8} {R}	{{1, 1}, 12} {I}					

Basis removing EOMs and IBPs redundancies

{{1, 1}, 2} {R}	{{1, 1}, 6} {R}	{{1, 1}, 8} {R}

Example: $D_\mu D_\nu W W \overline{H} H$



```
SU3 gauge contractions

1 Hbar Der Der H Wi1 Wi2

SU2 gauge contractions

1 Hbar[a] Der Der H[a] Wi1[c,b] Wi2[b,c]
2 Hbar[c] Der Der H[a] Wi1[a,b] Wi2[b,c]

Lorentz contractions

1        𝔻α,α(Hbar) H Wi1[βγ] Wi2[βγ]
2        𝔻α,β(Hbar) H Wi1[αγ] Wi2[βγ]
3    ϵβγδε 𝔻α,α(Hbar) H Wi1[βγ] Wi2[δε]
4    ϵβγδε 𝔻α,β(Hbar) H Wi1[αγ] Wi2[δε]
5        Hbar 𝔻α,α(H) Wi1[βγ] Wi2[βγ]
6        Hbar 𝔻α,β(H) Wi1[αγ] Wi2[βγ]
7    ϵβγδε Hbar 𝔻α,α(H) Wi1[βγ] Wi2[δε]
8    ϵβγδε Hbar 𝔻α,β(H) Wi1[αγ] Wi2[δε]
9        Hbar H 𝔻α,α(Wi1[βγ]) Wi2[βγ]
10   ϵβγδε Hbar H 𝔻α,α(Wi1[βγ]) Wi2[δε]
11       Hbar H Wi1[αβ] 𝔻γ,γ(Wi2[αβ])
12   ϵβγδε Hbar H Wi1[δε] 𝔻α,α(Wi2[βγ])
13     𝔻α(Hbar) 𝔻α(H) Wi1[βγ] Wi2[βγ]
14     𝔻α(Hbar) 𝔻β(H) Wi1[αγ] Wi2[βγ]
                    ●●●
35     Hbar H 𝔻α(Wi1[αβ]) 𝔻γ(Wi2[βγ])
36     Hbar H 𝔻α(Wi1[βγ]) 𝔻α(Wi2[βγ])
37 ϵβγδε Hbar H 𝔻α(Wi1[βγ]) 𝔻α(Wi2[δε])
```

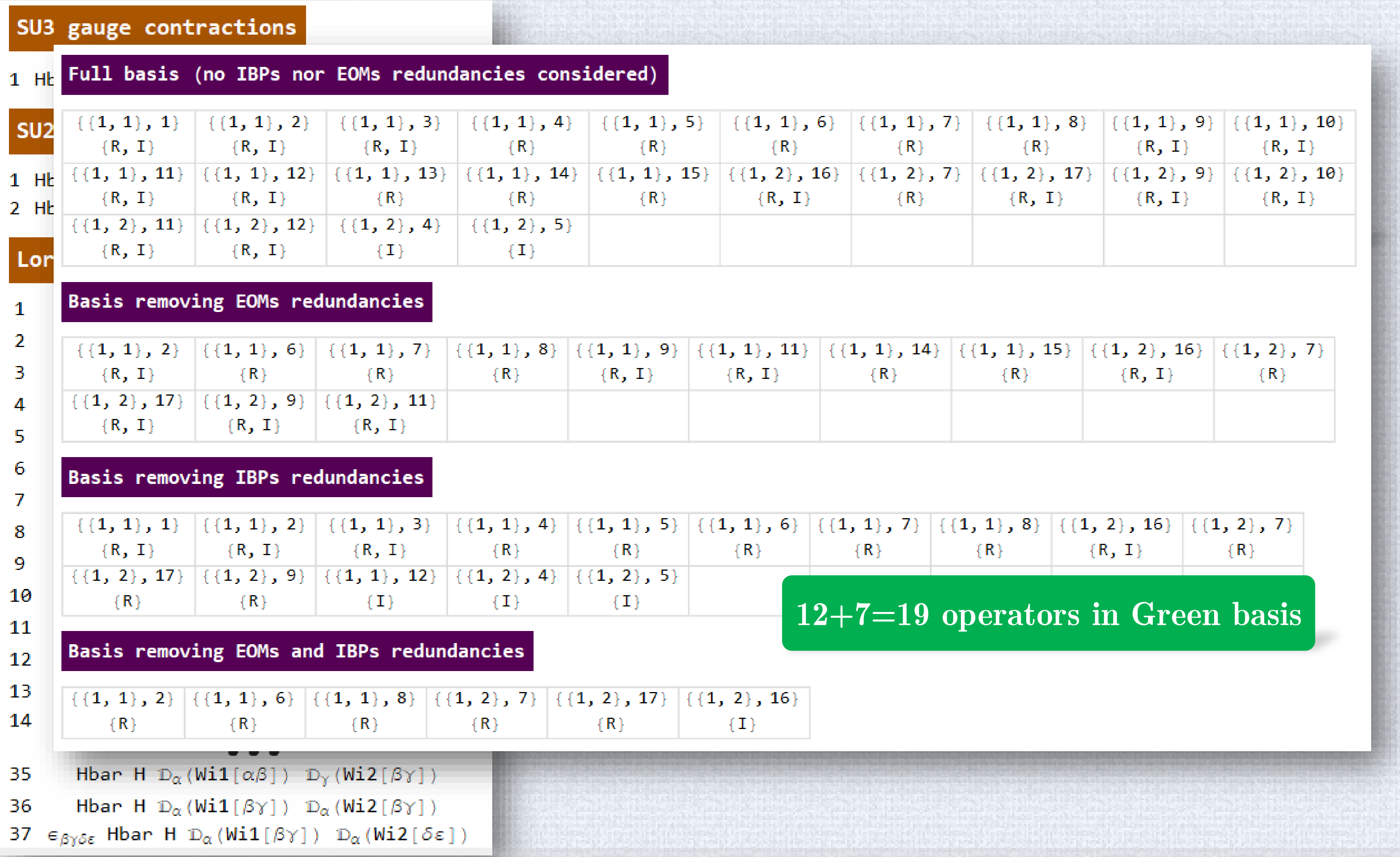
One contraction is symmetric (S) under exchange of W1 and W2, and the other is anti-symmetric (A)

Written in this form, the S and A are mixed (they are not cleanly separated)

For the symmetric (S) contraction the results on the previous slide apply! For example, there are 12 operators after application of IBPs

For the anti-symmetric (A) gauge contraction, there are an addition 7 operators in the Green basis. Total: 12+7=19

Example: $D_\mu D_\nu W W \overline{H} H$



SU3 gauge contractions

1 Hb

SU2

1 Hb
2 Hb

Lor

1
2
3
4
5
6
7
8
9
10
11
12
13
14

...

35 Hbar H $\mathbb{D}_\alpha$(Wi1[$\alpha\beta$]) $\mathbb{D}_\gamma$(Wi2[$\beta\gamma$])
36 Hbar H $\mathbb{D}_\alpha$(Wi1[$\beta\gamma$]) $\mathbb{D}_\alpha$(Wi2[$\beta\gamma$])
37 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_\alpha$(Wi1[$\beta\gamma$]) $\mathbb{D}_\alpha$(Wi2[$\delta\varepsilon$])

Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}
{{1, 1}, 11} {R, I}	{{1, 1}, 12} {R, I}	{{1, 1}, 13} {R}	{{1, 1}, 14} {R}	{{1, 1}, 15} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}	{{1, 2}, 17} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 10} {R, I}
{{1, 2}, 11} {R, I}	{{1, 2}, 12} {R, I}	{{1, 2}, 4} {I}	{{1, 2}, 5} {I}						

Basis removing EOMs redundancies

{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 14} {R}	{{1, 1}, 15} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}
{{1, 2}, 17} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 11} {R, I}							

Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}
{{1, 2}, 17} {R}	{{1, 2}, 9} {R}	{{1, 1}, 12} {I}	{{1, 2}, 4} {I}	{{1, 2}, 5} {I}					

Basis removing EOMs and IBPs redundancies

{{1, 1}, 2} {R}	{{1, 1}, 6} {R}	{{1, 1}, 8} {R}	{{1, 2}, 7} {R}	{{1, 2}, 17} {R}	{{1, 2}, 16} {I}

12+7=19 operators in Green basis

Status of the code

🙂 Seems to work well

🙂 All SMEFT operators, with 3 generations, can be computed up to dimension 10 in a couple of hours

😐 Decide how to interact with users

- How to present/export results?
- Select automatically non-redundant operators vs provide EOM/IBPs and let the user pick
- Input: (1) compute gauge contractions or (2) user does it?
  A viable and very interesting third alternative: user only needs to say permutation symmetry of the gauge contractions [*no need to know the group/representations/Clebsch Gordons!*]

Dealing (elegantly and automatically) with flavor: ongoing

Flavor &

A basis of operators for a general EFT

No repeated field = trivial flavor

For an operator type with no repeated fields, such as $Q_i^* Q_j L_k^* L_l$, whatever is happening for a particular set of flavor indices (*i,j,k,l*) is independent of what is happening for other values.

So...

(1) Run the code once for a particular (*i,j,k,l*)
(2) Analyze what non-redundant operators to keep
(3) Slap generic Wilson coefficients in front of them, with flavor indices.

At most, one might have to care about hermiticity of the WCs (real vs complex operators)


AND THEY LIVED Happily EVER AFTER

... if it were not for repeated fields

Repeated fields

(villain which is still alive)

Idea: remove the gauge structure

The problem of flavor is more acute if there are
few distinctions (other than flavor) among the fields

Best model to study flavor
(most stringent test):
A model with no gauge symmetry

Model contains arbitrary number
of copies/flavors of a left-handed
Weyl spinors, real scalars, $F_{\mu\nu}$'s

This also describes the most general EFT one can have

SMEFT and other EFTs can be obtained from it by imposing gauge invariant on the various Wilson coefficients

Renormalizable terms

$$\mathscr{L}_{d\leq 4} = -\frac{1}{4}F^A_{\mu\nu}F^{B\,\mu\nu} + \frac{1}{2}D_\mu\phi_a D^\mu\phi_b + \bar{\psi}_i \mathrm{i}\not{D}\psi_j - \frac{1}{2}\left[(m_f)_{ij}\psi_i^T C\psi_j + \text{h.c.}\right]$$

$$-\frac{1}{2}(m_\phi^2)_{ab}\phi_a\phi_b - \frac{1}{2}\left[Y_{ija}\psi_i^T C\psi_j\phi_a + \text{h.c.}\right] - \frac{\kappa_{abc}}{3!}\phi_a\phi_b\phi_c - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

Coefficient × **Operator**

$$D_\mu\psi_i = \partial_\mu\psi_i - \mathrm{i}g t^A_{ij} V^A_\mu \psi_j$$

$$D_\mu\phi_a = \partial_\mu\phi_a - \mathrm{i}g\theta^A_{ab} V^A_\mu \phi_b$$

t^A and θ^A are Hermitian matrices
(θ^A are also anti-symmetric)

In a particular model one has to specify the shape of generic tensor coefficients shown here

In practice, this usually involves simply enforcing gauge invariance on these tensor coefficients

E.g.: in SMEFT one has 45 Weyl fermions and 4 real scalars: the t^A are 45-dim; the θ^A are 4-dim. The Yukawa couplings are given by the most general Y tensor obeying

$$t^A_{ii'}Y_{i'ja} + t^A_{jj'}Y_{i'ja} + \theta^A_{aa'}Y_{ija'} = 0$$

In the SM, Y has 27 complex degrees of freedom

RGEs for a general model

In a series of classical papers, the 2-loop RGEs of a general model have been derived

Jack, Osborn (1982,1983,1985)
Machacek, Vaughn (1983,1984,1985)
Luo, Wang, Xiao, hep-ph/0211440 (2003)

Martin, Vaughn, hep-ph/9311340 (1994)
Yamada, hep-ph/9401241 (1994)
(SUSY)

The job of getting the RGEs of a specific model is not over, but a significant amount of work was done once and for all.

There is no need to go back and calculate divergences from diagrams for every new model. One only needs to compute the specific gauge invariant Lagrangian of a model, and apply the general RGEs [i.e. the work is no longer about the RG but rather about gauge invariance = group theory/linear algebra essentially.]

The model-specific work is still non-trivial and there are programs to help

SARAH
Staub (2010, 2012,...)

ARGES
Litim, Steudtner (2020)

Susyno
R.F. (2011)

RGBeta
Thomsen (2021)

PyR@TE
Lyonnet, Schienbein, Staub, Wingerter (2014) | Lyonnet, Schienbein (2017) | Sartore, Schienbein (2021)

Why not do the same for EFTs?

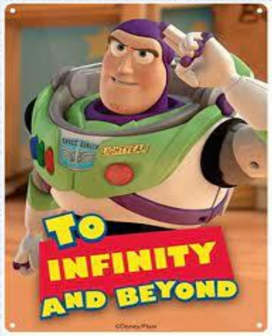
With José Santiago and using
[see his talk]

Matchmakereft
Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787

we are in the process of computing the general 1-loop RGEs up to dimension 6 EFT

[See also the talk by Mikolaj Misiak and Nalecz Ignacy tomorrow on this topic]

But one can go beyond RGEs with this approach



Matching

In the same spirit, why not calculate the matching for a general light+heavy set of fields? (Diagrammatic *vs* functional *vs* 'do the matching once and for all' method?)

Generate operators

Maybe one do the same with SymInt to generate operators (main topic of this talk): run it once to get the results for a general EFT, and from there just deal with gauge invariance on a model-by model basis

Dimension 5 Green basis

$$\begin{aligned}\mathscr{L}_5^{\text{phys}} &= \left[\frac{1}{2}\left(a_{\psi F}^{(5)}\right)_{Aij}\psi_i^T C\sigma^{\mu\nu}\psi_j F_{\mu\nu}^A + \frac{1}{4}\left(a_{\psi\phi^2}^{(5)}\right)_{ijab}\psi_i^T C\psi_j\phi_a\phi_b + \text{h.c.}\right] \\ &+ \frac{1}{2}\left(a_{\phi F}^{(5)}\right)_{ABa}F^{A\,\mu\nu}F_{\mu\nu}^B\phi_a + \frac{1}{2}\left(a_{\phi\widetilde{F}}^{(5)}\right)_{ABa}F^{A\,\mu\nu}\widetilde{F}_{\mu\nu}^B\phi_a + \frac{1}{5!}\left(a_{\phi}^{(5)}\right)_{abcde}\phi_a\phi_b\phi_c\phi_d\phi_e \\ \mathscr{L}_5^{\text{red}} &= \frac{1}{2}\left(r_{\phi\Box}^{(5)}\right)_{abc}(D_\mu D^\mu\phi_a)\phi_b\phi_c + \left[\frac{1}{2}\left(r_{\psi}^{(5)}\right)_{ij}(D_\mu\psi_i)^T C D^\mu\psi_j + \left(r_{\psi\phi}^{(5)}\right)_{ija}\bar{\psi}_i \text{i}\not{D}\psi_j\phi_a + \text{h.c.}\right]\end{aligned}$$

The Wilson coefficients have important symmetries (in some cases non-trivial)

$$\left(a_{\psi F}^{(5)}\right)_{ij} = -\left(a_{\psi F}^{(5)}\right)_{ji} \qquad \left(a_{\psi\phi^2}^{(5)}\right)_{ijab} = \left(a_{\psi\phi^2}^{(5)}\right)_{jiab} = \left(a_{\psi\phi^2}^{(5)}\right)_{ijba}$$

$$\left(a_{\phi F}^{(5)}\right)_{ABa} = \left(a_{\phi F}^{(5)}\right)_{BAa} \qquad \left(a_{\phi\widetilde{F}}^{(5)}\right)_{ABa} = \left(a_{\phi\widetilde{F}}^{(5)}\right)_{BAa} \qquad \left(a_{\phi}^{(5)}\right)_{abcde} = \text{fully symmetric}$$

$$\left(r_{\psi}^{(5)}\right)_{ij} = \left(r_{\psi}^{(5)}\right)_{ji} \qquad \left(r_{\phi\Box}^{(5)}\right)_{abc} = \left(r_{\phi\Box}^{(5)}\right)_{acb}$$

IBPs/EOMs may act only on some subspaces (e.g. they can remove the symmetric part of some WC and leave untouched the anti-symmetric)

To understand in a systematic way what is going on we need to discuss the permutation group, its representations, and how it acts on tensors

Dimension 6 Green basis

$$\begin{aligned}
\mathscr{L}_6^{\text{phys}} = {} & \frac{1}{3!}\left(a_{3F}^{(6)}\right)_{ABC}(F^A)_\mu^{\ \nu}(F^B)_\nu^{\ \rho}(F^C)_\rho^{\ \mu} + \frac{1}{3!}\left(a_{3\widetilde{F}}^{(6)}\right)_{ABC}(F^A)_\mu^{\ \nu}(F^B)_\nu^{\ \rho}(\widetilde{F}^C)_\rho^{\ \mu} \\
& + \frac{1}{4}\left(a_{\phi F}^{(6)}\right)_{ABab}F_{\mu\nu}^A F^{B\,\mu\nu}\phi_a\phi_b + \frac{1}{4}\left(a_{\phi\widetilde{F}}^{(6)}\right)_{ABab}F_{\mu\nu}^A \widetilde{F}^{B\,\mu\nu}\phi_a\phi_b \\
& + \frac{1}{4}\left(a_{\phi D}^{(6)}\right)_{abcd}(D_\mu\phi_a)(D^\mu\phi_b)\phi_c\phi_d + \frac{1}{6!}\left(a_{\phi}^{(6)}\right)_{abcdef}\phi_a\phi_b\phi_c\phi_d\phi_e\phi_f \\
& + \frac{1}{2}\left(a_{\phi\psi}^{(6)}\right)_{ijab}\bar{\psi}_i\gamma^\mu\psi_j[\phi_a D_\mu\phi_b - \phi_b D_\mu\phi_a] + \frac{1}{4}\left(a_{\bar{\psi}\psi}^{(6)}\right)_{ijkl}(\bar{\psi}_i\gamma^\mu\psi_j)(\bar{\psi}_k\gamma_\mu\psi_l) \\
& + \left[\frac{1}{2}\left(a_{\psi F}^{(6)}\right)_{Aija}F_{\mu\nu}^A\psi_i^T C\sigma^{\mu\nu}\psi_j\phi_a + \frac{1}{2!3!}\left(a_{\psi\phi}^{(6)}\right)_{ijabc}\psi_i^T C\psi_j\phi_a\phi_b\phi_c\right. \\
& \left. + \frac{1}{4!}\left(a_{\psi\psi}^{(6)}\right)_{ijkl}(\psi_i^T C\psi_j)(\psi_k^T C\psi_l) + \text{h.c.}\right]
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}_6^{\text{red}} = {} & \frac{1}{2!}\left(r_{2F}^{(6)}\right)_{AB}(D_\mu F^{A\,\mu\nu})(D^\rho F_{\rho\nu}^B) + \frac{1}{2!}\left(r_{FD\phi}^{(6)}\right)_{Aab}(D_\nu F^{A,\mu\nu})\left[(D_\mu\phi_a)\,\phi_b - (a \leftrightarrow b)\right] \\
& + \frac{1}{2!}\left(r_{D\phi}^{(6)}\right)_{ab}(D_\mu D^\mu\phi_a)\,(D_\nu D^\nu\phi_b) + \frac{1}{3!}\left(r_{\phi D}^{(6)}\right)_{abcd}(D_\mu D^\mu\phi_a)\,\phi_b\phi_c\phi_d \\
& + \ldots
\end{aligned}$$

So let us talk about flavor and
permutation symmetries

Flavor tensors with symmetries

1. The flavor symmetry of WC is dictated by the symmetry of the Lorentz and gauge contractions under permutations of same fields

2. From these symmetries one can compute everything (number of operators = free parameters in the WC; number of terms, ...)

Derivatives can be problematic

3. However, these symmetries might be complicated (mixed symmetries = irreps of the permutation group with dimension >1)

Flavor tensors with symmetries

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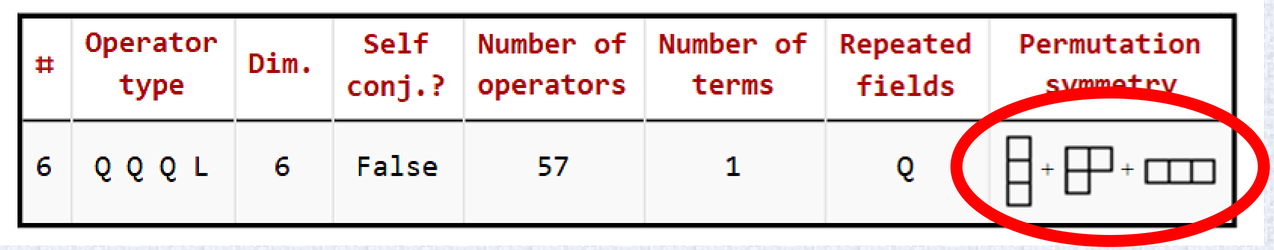

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
6	Q Q Q L	6	False	57	1	Q	+ +

RF 1907.12584

4 contractions: 1 totally sym; 1 totally anti-sym; two of them form a 2-dimensional irrep of the permutation group S3

From this data: $\frac{n_L n_Q\left(2n_Q^2+1\right)}{3}$ **operators; 1 term.**

An example: $D^2\phi^4$

Consider 4 distinct scalars: phi[1], phi[2], phi[3], phi[4] (don't think about flavor for now)

10 distinct operators

1 $\mathbb{D}_{\alpha,\alpha}$(phi[1]) phi[2] phi[3] phi[4]
2 phi[1] $\mathbb{D}_{\alpha,\alpha}$(phi[2]) phi[3] phi[4]
3 phi[1] phi[2] $\mathbb{D}_{\alpha,\alpha}$(phi[3]) phi[4]
4 phi[1] phi[2] phi[3] $\mathbb{D}_{\alpha,\alpha}$(phi[4])
5 $\mathbb{D}_{\alpha}$(phi[1]) $\mathbb{D}_{\alpha}$(phi[2]) phi[3] phi[4]
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8 phi[1] $\mathbb{D}_{\alpha}$(phi[2]) $\mathbb{D}_{\alpha}$(phi[3]) phi[4]
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10 phi[1] phi[2] $\mathbb{D}_{\alpha}$(phi[3]) $\mathbb{D}_{\alpha}$(phi[4])

IBPs

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

EOMs

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So for D^2 phi[1] phi[2] phi[3] phi[4]:
6 Green operators; 2 Physical. ✓

Now add flavor. The phi[i] are flavors of a common scalar, and we add WCs with flavor indices (summed over)

$$\text{wc}^{(1)}_{ijkl}\mathcal{O}^{(1)}_{ijkl} = \text{wc}^{2)}_{ijkl}\mathcal{O}^{(2)}_{ijkl} = \text{wc}^{(3)}_{ijkl}\mathcal{O}^{(3)}_{ijkl} = \text{wc}^{(4)}_{ijkl}\mathcal{O}^{(4)}_{ijkl}$$

(same for operators 5 to 10)

<u>What were 10 different operators, seem to reduce to only 2</u>
(e.g. we can keep #1 and #5).

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4 phi[1] phi[2] phi[3] $\mathbb{D}_{\alpha,\alpha}$(phi[4])
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IBPs

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

EOMs

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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(same for operators 5 to 10)

So what do the IBP’s/EOM’s remove then?

An example: $D^2\phi^4$

Study the effect of permuting the 4 scalar

phi[1] <-> phi[2]
induces this linear transformation

$$\begin{pmatrix}
0&1&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

All 4! permutations can be obtained from these two

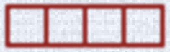
phi[1] -> phi[2] -> phi[3] -> phi[4] -> phi[1]
induces this linear transformation

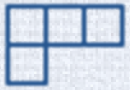
$$\begin{pmatrix}
0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0
\end{pmatrix}$$

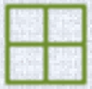
With a change of operator basis these become:

$$\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0\\
0&0&1&0&-1&0&0&0&0&0\\
0&0&0&1&-1&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&1&0&-1&0&0\\
0&0&0&0&0&0&1&-1&0&0\\
0&0&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&1&-1\\
0&0&0&0&0&0&0&0&0&-1
\end{pmatrix}
\qquad
\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0\\
0&0&-1&1&0&0&0&0&0&0\\
0&0&-1&0&1&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&1&0&0&0\\
0&0&0&0&0&-1&0&1&0&0\\
0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&-1&1
\end{pmatrix}$$

We've got:

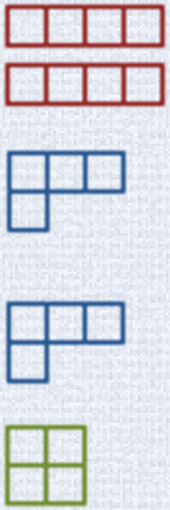
2 

2 

1 

The new basis

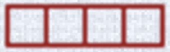
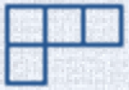
$$\begin{pmatrix} \mathcal{O}^1+\mathcal{O}^2+\mathcal{O}^3+\mathcal{O}^4 \\ \mathcal{O}^5+\mathcal{O}^6+\mathcal{O}^7+\mathcal{O}^8+\mathcal{O}^9+\mathcal{O}^{10} \\ \mathcal{O}^3-\mathcal{O}^2 \\ \mathcal{O}^4-\mathcal{O}^2 \\ \mathcal{O}^1-\mathcal{O}^2 \\ \mathcal{O}^5-\mathcal{O}^6+\mathcal{O}^9-\mathcal{O}^{10} \\ \mathcal{O}^5-\mathcal{O}^7+\mathcal{O}^8-\mathcal{O}^{10} \\ -\mathcal{O}^6-\mathcal{O}^7+\mathcal{O}^8+\mathcal{O}^9 \\ \mathcal{O}^5-\mathcal{O}^7-\mathcal{O}^8+\mathcal{O}^{10} \\ \mathcal{O}^6-\mathcal{O}^7-\mathcal{O}^8+\mathcal{O}^9 \end{pmatrix}$$



In the new basis:

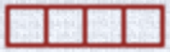
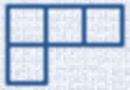
IBPs

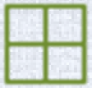
$$\begin{pmatrix} 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

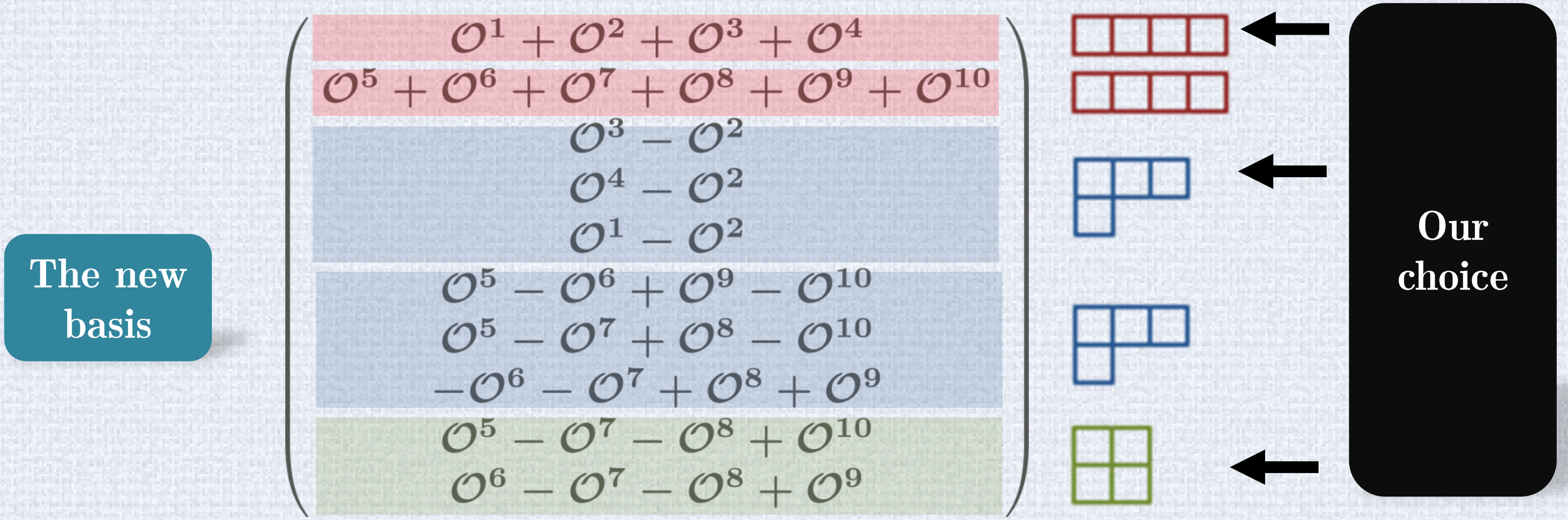
Remove one of the  and one of the 3-plet 

EOMs

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Remove the remaining  and also the remaining 3-plet 

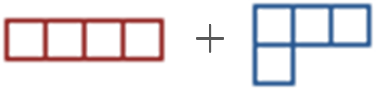
IBPs and EOMs remove all but the  **= physical basis**



The new basis

$$\begin{pmatrix} \mathcal{O}^1+\mathcal{O}^2+\mathcal{O}^3+\mathcal{O}^4 \\ \mathcal{O}^5+\mathcal{O}^6+\mathcal{O}^7+\mathcal{O}^8+\mathcal{O}^9+\mathcal{O}^{10} \\ \mathcal{O}^3-\mathcal{O}^2 \\ \mathcal{O}^4-\mathcal{O}^2 \\ \mathcal{O}^1-\mathcal{O}^2 \\ \mathcal{O}^5-\mathcal{O}^6+\mathcal{O}^9-\mathcal{O}^{10} \\ \mathcal{O}^5-\mathcal{O}^7+\mathcal{O}^8-\mathcal{O}^{10} \\ -\mathcal{O}^6-\mathcal{O}^7+\mathcal{O}^8+\mathcal{O}^9 \\ \mathcal{O}^5-\mathcal{O}^7-\mathcal{O}^8+\mathcal{O}^{10} \\ \mathcal{O}^6-\mathcal{O}^7-\mathcal{O}^8+\mathcal{O}^9 \end{pmatrix}$$

Our choice



$$\left(r_{\phi D}^{(6)}\right)_{abcd}\left(D_\mu D^\mu \phi_a\right)\phi_b\phi_c\phi_d$$

$$\left(r_{\phi D}^{(6)}\right)_{abcd} = \text{fully symmetric in } (b,c,d) \in \mathbb{R}$$

$$+$$

$$\left(a_{\phi D}^{(6)}\right)_{abcd}\left[(D_\mu\phi_a)(D^\mu\phi_b)\phi_c\phi_d + (ab \leftrightarrow cd) - (b \leftrightarrow d) - (a \leftrightarrow c)\right]$$

$$\left(a_{\phi D}^{(6)}\right)_{abcd} = -\left(a_{\phi D}^{(6)}\right)_{cbad} = -\left(a_{\phi D}^{(6)}\right)_{adcb} = \left(a_{\phi D}^{(6)}\right)_{cdab} \text{ and}$$
$$\left(a_{\phi D}^{(6)}\right)_{abcd} + \left(a_{\phi D}^{(6)}\right)_{adbc} + \left(a_{\phi D}^{(6)}\right)_{acdb} = 0,\ \left(a_{\phi D}^{(6)}\right)_{abcd} \in \mathbb{R}$$

Summary

Summary

From a list of fields and some symmetries, we want to get a basis of EFT operators. Maybe also tweak them (change basis)

I've described the possibility of making **GroupMath** + **Sym2Int** not just list, but also **build explicitly EFT operators**. Flavor & interface: ongoing work.

Interesting application: **generate the most general EFT** (up to some mass dimension) **and study, for example, its RGEs**. This can be done once and for all, so that obtaining the RGEs of specific models would only require group theory/algebra.

Thank you