

# Automated One-Loop Matching with MATCHETE

#### Matthias König

Technische Universität München

w/ J. Fuentes, J. Pages, A. E. Thomsen, F. Wilsch

"*SMEFT Tools*" Universität Zürich September 14, 2022 Match any high-scale New Physics to an agreed-upon parameterization\*.

Low-energy phenomenology can simply be **extracted from reference results** derived using said parameterization!

As an added benefit of the EFT framework, **logarithmic corrections** are easily included.

Still, this is a **lenghty task**, heavy on book-keeping, easy to make mistakes, miss contributions, ...

Find an automated solution, in order to

# MATCH Effective Theories Efficiently

## Outline



#### 1 Functional Methods

- Background Field Expansion
- Method of Regions
- Covariant Derivative Expansion

#### 2 A Guided Tour of MATCHETE

Demonstration

#### 3 Outlook and Conclusions



# **Functional Methods**



**Scenario**: Two-scales, separated by a cutoff  $\Lambda$ . Want to compute scattering at energies  $E < \Lambda$ .

Split field content  $\phi$  into **modes**  $\omega > \Lambda$  ("hard") and  $\omega < \Lambda$  ("soft"):

$$\phi \to \phi_H + \phi_S$$

Matrix elements at low energies derived from the generating functional:

$$Z[J_S] = \int \mathcal{D}\phi_S \mathcal{D}\phi_H \exp\left\{iS(\phi_S, \phi_H) + i\int d^4x \, J_S(x)\phi_S(x)\right\}$$

From this, one defines the Wilsonian effective action:

$$\begin{split} \int \mathcal{D}\phi_H \exp\left\{iS(\phi_S,\phi_H)\right\} &\equiv \exp\left\{iS_{\Lambda}(\phi_S)\right\}\\ \text{and also} \quad S_{\Lambda} &= \int d^4x \,\mathcal{L}_{\text{eff}}(x) \end{split}$$

Can compute  $S_{\Lambda}$  directly from this definition!

# The Background Field Expansion



Want to compute the path integral perturbatively, split

classical field, satisfies eom quantum fluctuation

Expand in the quantum field  $\eta$ :

$$\mathcal{L} = \frac{\mathcal{L}(\varphi)}{\bigwedge} + \eta_i \cdot \left[\frac{\delta \mathcal{L}}{\delta \phi_i}\right](\varphi) + \frac{1}{2}\bar{\eta}_i\eta_j \left[\frac{\delta^2 \mathcal{L}}{\delta \bar{\phi}_i \delta \phi_j}\right](\varphi) + \mathcal{O}(\eta^3)$$
tree-level eom  $\to 0$  fluctuation operator  $\mathcal{Q}_{ij}(\varphi)$ 
(indices on fields here denote hard or soft)

 $\phi = \frac{\varphi}{\checkmark} + \frac{\eta}{\checkmark}$ 

The path-integral of the one-loop piece is Gaussian:

$$\exp\left\{iS_{\text{eff}}^{(1)}\right\} = \int \mathcal{D}\eta \exp\left\{\frac{1}{2}\bar{\eta}\mathcal{Q}\eta\right\}$$

#### **Supertraces**



From this, one finds the effective action given by

$$S_{\text{eff}}^{(1)} = \frac{i}{2} \text{STr}(\log \mathcal{Q}) = \pm \frac{i}{2} \int \frac{d^d k}{(2\pi)^d} \langle k | \text{tr} \log \mathcal{Q} | k \rangle$$

This **supertrace** generalizes the usual trace to operators with fermionic and bosonic fields with appropriate signs.

[Cohen et al (2020), 2011.02484]

One now **splits** the fluctuation operator into:

$$\begin{aligned} \mathcal{Q}_{ij} &= \delta_{ij} \Delta_i^{-1} - X_{ij} = \Delta_i^{-1} \left( \delta_{ij} - \Delta_i X_{ij} \right) \\ \Delta_i^{-1} &= \begin{cases} -(D^2 + M_i^2) & \text{scalar} \\ i \not D - M_i & \text{fermion} \\ g^{\mu\nu} (D^2 + M_i^2) & \text{vector} \end{cases} X_{ij} = (\text{interactions}) \end{aligned}$$

and expands the log in the interactions:

$$S_{\text{eff}}^{(1)} = \frac{i}{2} \text{STr}(\log \Delta^{-1}) - \frac{i}{2} \sum_{n=1}^{\infty} \text{STr}[(\Delta X)^n]$$

This is just the usual **one-loop effective action**  $S_{\text{eff}}$ . How to get to the **Wilsonian** effective action  $S_{\Lambda}$  from here?

Remember, in  $S_{\Lambda}$  only the integral over **hard modes** was carried out!  $\rightarrow$  Need to "select" only those pieces of the integrals

$$\int \frac{d^d k}{(2\pi)^d} \langle k | \operatorname{tr} \log \mathcal{Q} | k \rangle$$

An **intuitive** way to realize this, would be to evaluate the integral with a lower momentum **cutoff** in place.

In **dimensional regularization** however, we can simply expand the **integrands** around this limit, by power-counting the loop momentum k like the heavy masses M.

[Beneke, Smirnov (1997), hep-ph/9711391] [Jantzen (2011), 1111.2589]

Each propagator now has virtuality  $k^2 \sim M^2$ .

## The Method of Regions - Example



# Take this simple scalar loop graph with $p_1^2 \sim p_2^2 \sim (p_1 \cdot p_2) \ll M^2$ :

$$\int \frac{d^{d}l}{(2\pi)^{d}} \frac{1}{l^{2}} \frac{1}{(l-p_{1})^{2}} \frac{1}{(l+p_{2})^{2} - M^{2}}$$

$$\int \frac{d^{d}l}{(2\pi)^{d}} \frac{1}{l^{2}} \frac{1}{l^{2} - M^{2}}$$

$$\int \frac{d^{d}l}{(2\pi)^{d}} \frac{1}{l^{2}} \frac{1}{l^{2} - M^{2}}$$

$$\int \frac{d^{d}l}{(2\pi)^{d}} \frac{1}{l^{2}} \frac{1}{l^{2} - M^{2}}$$

$$\int \frac{d^{d}l}{(2\pi)^{d}} \frac{1}{l^{2}} \frac{1}{(l-p_{1})^{2}} \frac{1}{(-M^{2})}$$

$$(\text{"bard region"} (\text{"soft region"})$$

The hard region corresponds to the one-loop matching correction to an operator  $\phi^3$ , the soft region to a matrix element of an operator  $\phi^4$ 

$$\left\langle T\{\phi^4, i\int d^4x \mathcal{L}(x)\} \right\rangle$$

ТЛТ

Further features:

- Power-corrections: The method of regions allows to extract matching coefficients to any order in power-counting!
- **Overcounting?** Said that "hard region":  $l^2 \sim M^2$ , but we still integrate over all  $d^d l$ .
  - $\rightarrow$  Not in **dimensional regularization** and as long as we identify  $\epsilon_{IR} = \epsilon_{UV}$ . Then the overlaps are all scaleless integrals.
- Matching: Along those lines, graphs with purely soft modes are automatically zero and thus the method automatically "selects" the correct amplitudes and directly yields matching coefficients.

From this it follows that evaluating the **effective action** in the **hard region**, directly yields the **Wilsonian effective action**!

[Fuentes et al (2016), 1607.02142]

$$S_{\text{eff}}^{(1)}\Big|_{\text{hard}} = \pm \frac{i}{2} \int_{h} \frac{d^{d}k}{(2\pi)^{d}} \langle k | \text{tr} \log \mathcal{Q} | k \rangle = S_{\Lambda}^{(1)} \,.$$



We can now evaluate the supetraces in the hard region:

$$S_{\Lambda}^{(1)} = \frac{i}{2} \operatorname{STr}_{h}(\log \Delta^{-1}) - \frac{i}{2} \sum_{n=1}^{\infty} \operatorname{STr}_{h}[(\Delta X)^{n}]$$

The fluctuation operator and thus  $\Delta$  and X depend on the classical fields  $\varphi$  and derivatives.

$$\Rightarrow S_{\Lambda}$$
 is of the form  $i \int d^4x \sum_i C_i(\mu) \cdot \mathcal{O}_i(\varphi, iD_{\mu})$   
with loop functions  $C_i(\mu)$  and composite operators of **classical** fields and

covariant derivatives.

This is almost an effective Lagrangian!

The expression is not manifestly **gauge-invariant** and contains **redundant operators**.



The supertraces contain open covariant derivatives:

$$\int \frac{d^d k}{(2\pi)^d} \mathrm{tr}\left(\mathcal{Q}(\varphi, iD_\mu)\right) \cdot \mathbf{1} \supset \dots D_\mu \mathbf{1}$$

that **cannot** be set to zero, as is clear from e.g  $F_{\mu\nu} = i[D_{\mu}, D_{\nu}]/g$ .

To this end, one uses the **Covariant-Derivative Expansion (CDE)** and introduces the unitary operator  $U_D$  satisfying

$$U_D = \exp(iD \cdot \partial_k), \qquad U_D^{\dagger} U_D = \mathbf{1}, \qquad U_D \cdot \mathbf{1} = \mathbf{1}.$$

Now insert  $Q \cdot \mathbf{1} = U_D^{\dagger} Q U_D \cdot \mathbf{1}$  and commute  $U_D$  to the left.

The resulting expression will have all all **open derivatives in commutators** and thus be expressed through covariant derivatives and Field-strength tensors. The hard-region expansion produces terms with derivatives in  $\mathcal{L}_{\rm eff}$  , containing terms like

 $D^2\phi, \quad i \not\!\!D\psi, \quad D_\mu F^{\mu\nu}, \quad \dots$ 

Operators containing such expressions are deemed **redundant** and can be removed.

Using equations of motion here is **subtle**, because the duality

field redefinitions  $\longleftrightarrow$  equations of motion is broken in the presence of **power-corrections**.

Not relevant in the SMEFT at dim 6, but in a general framework, anything can happen.



The functional matching procedure with CDE yields the **complete effective action**. Just extract the fluctuation operator from the Lagrangian, compute

$$S_{\Lambda}^{(1)} = \frac{i}{2} \operatorname{STr}_{h}(\log \Delta^{-1}) - \frac{i}{2} \sum_{n=1}^{\infty} \operatorname{STr}_{h}[(\Delta X)^{n}]$$

and truncate the sum in the last term.

No need to enumerate diagram topologies, compute symmetry factors.

The result is **manifestly gauge-invariant**, operators with  $F_{\mu\nu}$  and  $iD_{\mu}$  emerge in a **transparent** way.

Lends itself extremely well to automation.

$$\begin{array}{l} [{\rm Das \ Bakshi \ et \ al \ (2018), \ 1808.04403]} \\ [{\rm Cohen \ et \ al \ (2020), \ 2012.07851]} \\ [{\rm Fuentes \ et \ al \ (2020), \ 2012.08506]} \\ [{\rm Carmona \ et \ al \ (2021), \ 2112.10787]} \\ \end{array}$$

Straightforward algorithm:

- $\blacksquare$  Define your **UV Lagrangian**  $\mathcal{L}_{\rm UV}$  and identify heavy fields.
- Compute the equations of motions for the heavy fields and solve for them to express them through soft (classical) modes:

$$\frac{\delta \mathcal{L}_{\rm UV}}{\delta \phi_H} = 0 \qquad \Rightarrow \qquad \phi_H = \varphi_H(\varphi_S)$$

Split φ = φ + η Compute the fluctuation operator with at least one of the η a hard mode:

$$\mathcal{Q}_{ij} = \frac{\delta^2 \mathcal{L}_{\rm UV}}{\delta \bar{\eta}_i \delta \eta_j}$$

- Compute the hard supertraces, with power-type traces up to the desired order, replace classical hard modes with their eom.
- Perform the **CDE expansion**.

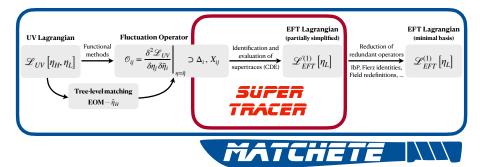


# A Guided Tour of MATCHETE



#### **MATCHETE** is a Mathematica package, that:

- Takes as an input the field content, power-counting and interaction Lagrangian in an intuitive form.
- Computes the effective Lagrangian up to one-loop order using functional methods, extending TRACER.
- Provides automatic reduction of redundant operators (IBP, Field Redefinition, Fierz transforms).
- Handles evanescent contributions.
- Provides the output in an **easy-to-interpret** form.
- Goes up to any order in **power-counting** (given enough computer time).
- Can easily be used to implement multi-step matching.
- Is fully open-source.



The user defines the **UV theory**, by **field** content, **symmetries**, mass **hierarchies** and interaction **Lagrangian**.

MATCHETE computes from that the **tree-level matching** and **fluctuation operator**, passing this to **SUPER**.

After hard STraces are computed, MATCHETE simplifies the output.

ТЛП

There is a lot to do:

- ✓ Defining and implementing an **input standard** for Lagrangians.
- Modular handling for group theory, allowing arbitrary symmetry groups and representations.
- Implementation of functional methods to derive EOMs and fluctuation operators.
- V Evaluation of hard supertraces  $\rightarrow$
- ✓ **IBP identities** for derivative operators.
- ... Kinetic **diagonalization**, **field-redefinitions** for redundant operators.
- ... Fierz transformations, handling of evanescent operators.
- ... Targeting a **specific basis**.



As previously mentioned, the SuperTracer output contains **derivative operators** that can be reduced with IBP:

$$\delta \mathcal{L} = c_1 \phi^3 (D_\mu D^\mu \phi) + c_2 \phi^2 (D_\mu \phi)^2 = \left(c_1 - \frac{c_2}{3}\right) \phi^3 (D^2 \phi)$$

Particularly nasty with charged fields:

$$\delta \mathcal{L} = c_1 \phi \, \bar{\psi} \gamma^{\mu} \gamma^{\nu} (D_{\mu} D_{\nu} + D_{\nu} D_{\mu}) \psi$$
  
=  $-2c_1 (D_{\mu} \phi) (\bar{\psi} \gamma_{\mu} \not{D} \psi) - 2c_1 \phi (\bar{\psi} \overleftarrow{D} \not{D} \psi) + c_1 \phi F^{\mu\nu} (\bar{\psi} i \Gamma_{\mu\nu} \psi)$ 

Not unique  $\rightarrow$  Choice: Prefer  $derivative \ operators$  proportional to EoMs!

This removes as many terms as possible from the Lagrangian, when **field redefinitions** are applied.

Four-fermion operators can be reduced by Fierz transforms:

$$\begin{split} \delta \mathcal{L} &= c_1^{prst}(\bar{\ell}^p \gamma_\mu q^t)(\bar{d}^s \gamma^\mu e^r) + c_2^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t) \\ &\to (c_2 - 2c_1)(\bar{\ell}^p e^r)(\bar{d}^s q^t) \end{split}$$

But this identity holds **only** in d = 4. Beyond that, **evanescent operators** need to be included.

Compute hard IR-regularized supertraces of the **difference Lagrangian** between original and 4d-Fierz'ed basis.

 $\Rightarrow$  Yields finite corrections to the physical operators.



These corrections are **finite one-loop effects** and are especially important for "loop-level operators".

Integrate out a vector leptoquark, yields at tree-level:

$$\delta \mathcal{L} = c_1^{prst} (\bar{\ell}^p \gamma_\mu q^t) (\bar{d}^s \gamma^\mu e^r) = c_1^{prst} O_1^{prst}$$

We want the SMEFT basis! So Fierz:  $O_1 = -2Q_{ledq}$ . This generates the evanescent operator

$$E_{lqde} = O_1 + 2Q_{ledq} \,.$$

This evanescent gives an important contribution, for example, to the dipole operators:

$$S_{\Lambda}^{(E)} = \int d^4x \, c_1^{pstr} \frac{Y_d^{st}}{64\pi^2} \left( g_Y Q_{eB}^{pr} - 3g_2 Q_{eW}^{pr} \right) + \text{many more!}$$

The full result is **much** more complicated, and you will be glad not having to derive it by hand:

$$\begin{split} \frac{\overline{c}_{lpde}^{rest}}{\overline{\mathcal{R}}_{lpde}^{pred}} + c_{lpde}^{rest}\overline{\mathcal{R}}_{lpde}^{rest} - 2c_{lpde}^{rest}Q_{lpde}^{lest} + \frac{1}{(58\pi^2)} \left( 8c_{lpde}^{rest}\overline{Y}_{des}^{rest} Y_{des}^{lest} Y_{des}^{le$$



#### A Guided Tour of MATCHETE Demonstration

Consider a simple toy theory with U(1) symmetry and following field content:

Field	Туре	Charge	Mass
$\phi$	Scalar	0	0
$\psi$	Fermion	+1	0
$\Psi$	Fermion	+1	$M_\psi$ (heavy)

The interaction Lagrangian is:

$$\mathcal{L} = \mathcal{L}_0 - y\bar{\psi}\phi P_R\Psi + \text{h.c.}$$

Want to compute the **complete effective Lagrangian** after integrating out  $\Psi$ , at one-loop order.

#### Backup: Code



```
In[34]:= DefineGaugeGroup[U1e, U[1], e, A]
```

```
DefineCoupling[y, EFTorder \rightarrow 0]
```

```
DefineField[⊈, Fermion, Charges → {Ule@1}, Mass → Heavy]
DefineField[⊭, Fermion, Charges → {Ule@1}, Mass → 0]
DefineField[¢, Scalar, Mass → 0, SelfConjugate → True]
```

```
LInt = -y[]×φ[]×Bar[ψ[]] ** PR ** Ψ[] // PlusHc;
LUV = LInt + FreeLag[];
L1 = EchoTiming[Match(LUV, LoopOrder → 1] // IBPSimplify] // NiceForm
```

```
0 23.2519
```

Out[41]//NiceForm=

$$\begin{split} & -\frac{2}{15} \ \hbar \frac{1}{Mu^2} \ D_{\nu} A^{\mu\nu} \ D_{\mu} A^{\mu\nu} + i \ \left( \overline{\psi} \cdot \gamma_{\mu} \cdot D_{\mu} \psi \right) + \frac{i}{6} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D_{\mu} \overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D^2 \psi \right) - \frac{i}{6} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D^2 \overline{\psi} \cdot \gamma_{\nu} \ P_{L} \cdot D_{\nu} \psi \right) + \frac{i}{3} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D_{\mu} \overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D^2 \psi \right) - \frac{i}{6} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D^2 \overline{\psi} \cdot \gamma_{\nu} \ P_{L} \cdot D_{\nu} \psi \right) + \frac{i}{3} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D_{\mu} \overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D^2 \psi \right) - \frac{i}{6} \ \hbar \ \overline{y} \ y \ \frac{1}{Mu^2} \ \left( D^2 \overline{\psi} \cdot \gamma_{\nu} \ P_{L} \cdot D_{\nu} \psi \right) + \frac{i}{3} \ \hbar \ \overline{y} \ dD^2 \ D^2 \phi \ D^2 \phi + \left( -\frac{1}{4} \ \frac{1}{4^2} - \frac{1}{3} \ \hbar \ \log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \right] \right) \ A^{\mu\nu2} - \hbar \ \overline{y}^2 \ y^2 \ \phi^4 \ \log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \ \right] + y \ M\overline{u}^2 \ \left( -2 \ \hbar \ \overline{y} - 2 \ \hbar \ \overline{y} \ Log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \right] \right) \ \phi^2 + \frac{i}{4} \ \hbar \ \overline{y} \ Log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \ \right] \right) \ (\overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D_{\mu} \psi) + \frac{1}{3} \ \hbar \ \overline{y}^3 \ \frac{1}{Mu^2} \ \phi^6 + \frac{13}{18} \ \hbar \ \overline{y}^2 \ y^2 \ y^2 \ d^4 \ \log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \ \right] \right) \ (\overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D_{\mu} \psi) + \frac{1}{3} \ \hbar \ \overline{y}^3 \ \frac{1}{Mu^2} \ \phi^6 + \frac{13}{18} \ \hbar \ \overline{y}^2 \ y^2 \ d^4 \ \log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \ \right] \right) \ (\overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D_{\mu} \psi) + \frac{1}{3} \ \hbar \ \overline{y}^3 \ \frac{1}{Mu^2} \ \phi^6 + \frac{13}{18} \ \hbar \ \overline{y}^2 \ y^2 \ d^4 \ \log \left[ \overline{\mu}^2 \ \frac{1}{Mu^2} \ \right] \ \left( \overline{\psi} \cdot \gamma_{\mu} \ P_{L} \cdot D_{\mu} \psi) + \frac{1}{3} \ \hbar \ \overline{y}^3 \ \frac{1}{Mu^2} \ \phi^6 + \frac{13}{18} \ \hbar \ \overline{y}^2 \ y^2 \ \frac{1}{Mu^2} \ D^2 \ \phi^6 + \frac{1}{3} \ \hbar \ \overline{y}^2 \ y^2 \ \frac{1}{Mu^2} \ D^2 \ \psi^6 \ \psi^6 \ \gamma^6 \ \gamma^6 \ \gamma^6 \ \psi^6 \ \gamma^6 \$$



# **Outlook and Conclusions**

- One-loop matching is a repetitive, mechanical, but crucial task in BSM physics.
- Functional matching with the CDE and the Method of Regions are an economical way to compute effective Lagrangians.
- Well-suited for automation, requires minimal user input.
- Most-importantly, the target basis needs not to be known.
- MATCHETE is a Mathematica package providing an easy-to-use framework for SMEFT-like EFT matching.
- Still lots to do: Many steps trivial to humans are complicated to teach a machine in full generality.
- Derivation of the EFT beta functions possible by methods already implemented.
- Future: Interface to exisiting pheno codes (smelly, flavio, ...), export of UFO files to interface with Monte Carlo generators like MADGraph, ...



# **Bonus slides**



There are no bonus slides.