More on the Geometric SMEFT...

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Key collaborators and developments in geoSMEFT:

A. Helset

T. Corbett

A. Martin

C. Hays

J. Talbert

1803.08001 Helset, Paraskevas, Trott.
2001.01453 Helset, Martin, Trott.
2010.08451 Corbett, Trott
2010.15852 Corbett,
2107.03951 Talbert, Trott.
2106.10284 Corbett,
2110.03694 Corbett, Rasmussen

1909.08470 Corbett, Helset, Trott
2007.00565 Hays, Helset, Martin, Trott
2102.02819 Helset, Corbett, Martin, Trott
2106.13794 Trott
2107.07470 Corbett, Martin, Trott
2109.05595 Martin, Trott
2208.11139. Talbert

Other groups joining in:

2202.06965 Cohen, Craig, Lu, Sutherland
2202.06972 Cheung, Helset, Parra-Martinez
2111.03045 Cheung, Helset, Parra-Martinez

Confession: I really hate to code.

So SMEFT-Tools 2022 PAULI CENTER Zürich ?

$\mathcal{L}_6$ Was hard. We also need one loop $\mathcal{L}_6$ and $(\mathcal{L}_6)^2, \mathcal{L}_8$

- Is pure automation the only way forward? It has been claimed a few times.
- Maybe not. Can the SMEFT for precise experiment be practically simpler?
- Physical results are field redefinition invariant. Operator bases are the result of using field redefinitions to fix variables for calculations. Any non redundant operator basis is not field redefinition invariant. And this is a pain in the…

  Your development of tools in a fixed operator basis is guaranteed to have many technical hurdles. So, let us minimise them.

> Main message: If you start to think field space geometrically, and make sure your tools are consistent with conventions of the geoSMEFT, long term you (and everyone using your tools) will benefit.

GeoSMEFT basics.

Field coord. invariance leads to field space geometry

geoSMEFT

- Field coordinate choice invariance in amplitudes

- Mathematical quantities: metrics, Curvature, tensors of INTERACTION TERMS (field spaces)

$$\mathcal{L}_{SMEFT} = \frac{1}{2} h_{IJ}(\phi)(D_\mu \phi)^I (D_\mu \phi)^J - \frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} W^{B,\mu\nu} + \cdots$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix} \qquad \mathcal{W}^A = (W^1, W^2, W^3, B)$$

General Relativity.

- Space-time coordinate general covariance

- Mathematical quantities: metrics, Curvature, tensors for SPACE TIME

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \kappa T_{\mu\nu}$$

Field coord. invariance leads to field space geometry

$$\mathcal{L}_{SMEFT} = \frac{1}{2} h_{IJ}(\phi)(D_\mu \phi)^I (D_\mu \phi)^J - \frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} W^{B,\mu\nu} + \cdots$$

- Dimensionless expansion into operator bases $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\Box} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1 + \tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1 + \tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1 + \tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1 + \tilde{C}_{HB} \end{bmatrix}$$

(Small perturbations so positive semi-definite matrix and unique square root)

- Geometric field space quantities are useful (True independent of mass dimension of ops) Amp. perturb. are:

Fun. of 4 vectors (kinematics)

$$\mathcal{A} \simeq \mathcal{A}_{SM} + \langle \mathcal{O} \rangle_1 N_1 + \langle \mathcal{O} \rangle_2 N_2 + \cdots$$

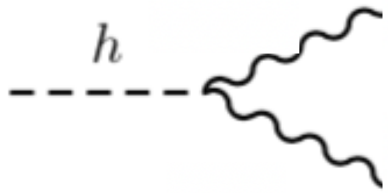
Defined by field space geometries

The intermediate language between
Pure amplitudes, and pure operators,
is field space geometry. You can use it.

What is better about it?

More basis independent results are possible!

All orders expression for Higgs to gamma gamma can be defined in closed form as:



Yes! Its a Vielbein of a Higgs space defining asy. particle states in SMEFT

$$\langle h|\mathcal{A}(p_1)\mathcal{A}(p_2)\rangle = -\langle h A^{\mu\nu} A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right],$$

Kinematic structure

Geometric Dressings

No explicit SMEFT expansion op forms. *But all orders in vev expansion*

How do we get to such results?

Curved SMEFT spaces: scalar fields

- Curved SMEFT field space manifest in background field formulation

In general terms: G. A. Vilkovisky, Nucl. Phys. B234 (1984) 125.

Metric on Higgs field space, SM a **FLAT** field space

$$\mathcal{L}_{\rm scalar,kin} = \frac{1}{2} h_{IJ}(\phi) \left(D_\mu \phi\right)^I \left(D^\mu \phi\right)^J ,$$

Where $H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\Box} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

Small perturbations so positive semi-definite
Matrix and unique square root

(sqrt) Metric in SMEFT, a *curved* field space

$$R^I_{JKL} \neq 0$$

1002.2730 Burgess, Lee, Trott
1511.00724 Alonso, Jenkins, Manohar
1605.03602 Alonso, Jenkins, Manohar

Curved SMEFT space: gauge fields

- Similarly in the gauge coupling space a curved field space

Metric on gauge field space, SM a **FLAT** field space

$$\mathcal{L}_{\text{gauge,kin}} = -\frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu},$$

Where $\mathcal{W}^A = (W^1, W^2, W^3, B)$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1+\tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1+\tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1+\tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1+\tilde{C}_{HB} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

(sqrt) Metric in SMEFT, a *curved* field space

1803.08001 Helset, Paraskevas, Trott.
1909.08470 Corbett, Helset, Trott

All orders SM Lagrangian parameters

- Low n-point interactions of fields are parameterised in terms of couplings,

1909.08470 Corbett, Helset, Trott

$$\bar{g}_2 = g_2\sqrt{g}^{11} = g_2\sqrt{g}^{22},$$
$$\bar{g}_Z = \frac{g_2}{c^2_{\bar\theta_Z}}\left(c_{\bar\theta}\sqrt{g}^{33} - s_{\bar\theta}\sqrt{g}^{34}\right) = \frac{g_1}{s^2_{\bar\theta_Z}}\left(s_{\bar\theta}\sqrt{g}^{44} - c_{\bar\theta}\sqrt{g}^{34}\right),$$
$$\bar{e} = g_2\left(s_{\bar\theta}\sqrt{g}^{33} + c_{\bar\theta}\sqrt{g}^{34}\right) = g_1\left(c_{\bar\theta}\sqrt{g}^{44} + s_{\bar\theta}\sqrt{g}^{34}\right),$$

- Masses

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4}\sqrt{h_{11}}^2\bar{v}_T^2, \qquad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4}\sqrt{h_{33}}^2\bar{v}_T^2 \qquad \bar{m}_A^2 = 0.$$

- Mixing angles:

$$s^2_{\bar\theta_Z} = \frac{g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})}{g_2(\sqrt{g}^{33}c_{\bar\theta} - \sqrt{g}^{34}s_{\bar\theta}) + g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})},$$
$$s^2_{\bar\theta} = \frac{(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1g_2\sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

(Interesting way to think of the Weinberg angle)

All orders expressions are known now

- All orders scalar metric -leading to gauge boson masses in SMEFT

$$h_{IJ} = \left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+2}\left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right]\delta_{IJ} + \frac{\Gamma^I_{A,J}\phi_K\Gamma^K_{A,L}\phi^L}{2}\left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right).$$

- All orders gauge metric - gives mass eigenstate couplings in SMEFT

$$\begin{aligned} g_{AB}(\phi_I) = & \left[1 - 4\sum_{n=0}^{\infty}\left(C^{(6+2n)}_{HW}(1-\delta_{A4}) + C^{(6+2n)}_{HB}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ & - \sum_{n=0}^{\infty} C^{(8+2n)}_{HW,2}\left(\frac{\phi^2}{2}\right)^{n}\left(\phi_I\Gamma^I_{A,J}\phi^J\right)\left(\phi_L\Gamma^L_{B,K}\phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\ & + \left[\sum_{n=0}^{\infty} C^{(6+2n)}_{HWB}\left(\frac{\phi^2}{2}\right)^{n}\right]\left[(\phi_I\Gamma^I_{A,J}\phi^J)(1-\delta_{A4})\delta_{B4} + (A \leftrightarrow B)\right], \end{aligned}$$

- Number of operator forms saturate in geosmeft.
  This is due to reducing possible generator insertions on the Higgs manifold

$$T^a_{ij}T^a_{k\ell} = \frac{1}{2}\left(\delta_{i\ell}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{k\ell}\right)$$

SM weak-mass eigenstate relations

- Weak eigenstates Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \delta^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \delta^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \delta^{JK} V_{KL} \hat{\Phi}^{L},$$

Rotations

Flat field space's.
Due to $D \leq 4$

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix} \qquad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^J = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^K = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^A = \{g_2\, g_2, g_2, g_1\}, \qquad \mathcal{W}^A = \{W_1, W_2, W_3, B\},$$

$$\beta^C = \left\{ \frac{g_2\,(1-i)}{\sqrt{2}}, \frac{g_2\,(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c_{\overline{\theta}}^2 - s_{\overline{\theta}}^2), \frac{2\, g_1\, g_2}{\sqrt{g_1^2+g_2^2}} \right\}, \qquad \mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A})\,.$$

What else could you write?

SMEFT weak-mass eigenstate relations

- Weak eigenstates

Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \sqrt{g}^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

Generator transform

$$\gamma^{I}_{C,J} = \frac{1}{2}\tilde{\gamma}^{I}_{A,J}\sqrt{g}^{AB} U_{BC}.$$

1909.08470 Corbett, Helset, Trott

SMEFT Field space metrics
(Now known to all orders)

Rotations

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix} \qquad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^{J} = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^{K} = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^{A} = \{g_2\, g_2, g_2, g_1\}, \qquad \mathcal{W}^{A} = \{W_1, W_2, W_3, B\},$$

$$\beta^{C} = \left\{ \frac{g_2\,(1-i)}{\sqrt{2}}, \frac{g_2\,(1+i)}{\sqrt{2}}, \sqrt{g_1^2+g_2^2}(c^2_{\overline{\theta}} - s^2_{\overline{\theta}}), \frac{2\, g_1\, g_2}{\sqrt{g_1^2+g_2^2}} \right\}, \qquad \mathcal{A}^{C} = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A}).$$

What else could you write? Nothing that generalises to all orders.

Dim 6 SMEFT EW Lagrangian terms

- EW sector parameters redefined in the SMEFT (already in SMEFTsim)

$$\begin{bmatrix} \mathcal{W}^3_\mu \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} v_T^2 C_{HWB} \\ -\frac{1}{2} v_T^2 C_{HWB} & 1 \end{bmatrix} \begin{bmatrix} \cos\overline{\theta} & \sin\overline{\theta} \\ -\sin\overline{\theta} & \cos\overline{\theta} \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix},$$

Mass redefinitions

$$M_W^2 = \frac{\overline{g}_2{}^2 v_T^2}{4},$$
$$M_Z^2 = \frac{v_T^2}{4}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{8} v_T^4 C_{HD} (\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{2} v_T^4 \overline{g}_1 \overline{g}_2 C_{HWB}.$$

Mixing angle redefinitions

$$\sin\overline{\theta} = \frac{\overline{g}_1}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 + \frac{v_T^2}{2} \frac{\overline{g}_2}{\overline{g}_1} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$
$$\cos\overline{\theta} = \frac{\overline{g}_2}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 - \frac{v_T^2}{2} \frac{\overline{g}_1}{\overline{g}_2} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$

Interactions to remaining SM fields via:

$$D_\mu = \partial_\mu + i\frac{\overline{g}_2}{\sqrt{2}}\left[\mathcal{W}^+_\mu T^+ + \mathcal{W}^-_\mu T^-\right] + i\,\overline{g}_Z\left[T_3 - \overline{s}^2 Q\right]\mathcal{Z}_\mu + i\,\overline{e}\,Q\,\mathcal{A}_\mu,$$

$$\overline{e} = \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}}\left[1 - \frac{\overline{g}_1 \overline{g}_2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} v_T^2 C_{HWB}\right]$$

$$\overline{g}_Z = \sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} v_T^2 C_{HWB}$$

$$\overline{s}^2 = \sin^2\overline{\theta} = \frac{\overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2 (\overline{g}_2{}^2 - \overline{g}_1{}^2)}{(\overline{g}_1{}^2 + \overline{g}_2{}^2)^2} v_T^2 C_{HWB}.$$

- Dim 8,10 etc solved in closed form. Just expand.

Generalisation for composite ops

$$\mathcal{A} \simeq \mathcal{A}_{SM} + \langle \mathcal{O} \rangle_1 N_1 + \langle \mathcal{O} \rangle_2 N_2 + \cdots$$

Fun. of 4 vectors (kinematics)

Defined by field space geometries

Making this decomposition more manifest from the start

The usual operator focused language is not particularly helpful.

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots, \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

Generalisation for composite ops

$$\mathcal{A} \simeq \mathcal{A}_{SM} + \langle \mathcal{O} \rangle_1 N_1 + \langle \mathcal{O} \rangle_2 N_2 + \cdots$$

Reformulate!
Reformulate!

$$\mathcal{L}_{\rm SMEFT} = \sum_i f_i(\alpha \cdots) G_i(I, A \cdots),$$

Derivative expansion

Composite operator form
With minimal scalar field
coordinate dependence

Vev expansion

Scalar field coordinate dependence
And insertions of symmetry generators

$D^\mu \phi$

Mixes expansions, but grouped with derivative forms.

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu \phi)^I \, \delta(D_\nu \phi)^J}\Bigg|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu \Phi)^K, \bar\psi \sigma^\mu \psi, \bar\psi \psi \cdots\}$

- Limited number of such connections for up to three point functions

$V(\phi)$ $\quad h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}, \quad k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A,$
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$

With fermions $\quad Y(\phi)\bar\psi_1\psi_2, \quad L_{I,A}(\phi)\bar\psi_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar\psi_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$

Gluon fields $\quad k_{\mathcal{AB}}(\phi)G^{\mathcal{A}}_{\mu\nu}G^{\mathcal{B},\mu\nu}, \quad k_{\mathcal{ABC}}(\phi)G^{\mathcal{A}}_{\nu\mu}G^{\mathcal{B},\rho\nu}G^{\mathcal{C},\mu\rho}, \quad c(\phi)\bar\psi_1\sigma^{\mu\nu}T_{\mathcal{A}}\psi_2 G^{\mathcal{A}}_{\mu\nu}.$

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu\phi)^I \, \delta(D_\nu\phi)^J}\Big|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu\Phi)^K, \bar{\psi}\sigma^\mu\psi, \bar{\psi}\psi\cdots\}$

- Limited number of such connections for up to three point functions

This is a non trivial fact proven in 2001.01453 Helset, Martin, Trott

There is a theory choice here - its REMOVE DERIVATIVE OPS, USE EOM.

Same reasoning built into, and led to the "Warsaw basis".
Also why we were able to renormalise the Warsaw basis completely in 2013.

EFT Industry standard in flavour physics, chiral pert theory etc.

GeoSMEFT Pushing to higher n points

- Limited number of such connections for up to three point functions

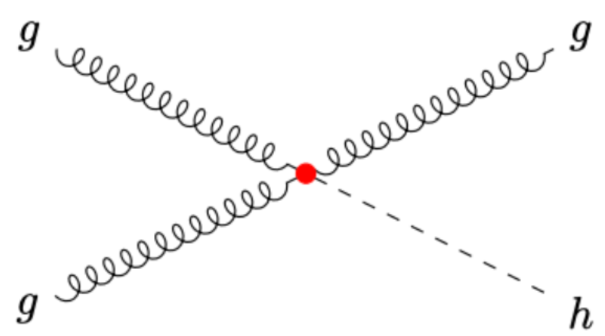
  This is a non trivial fact proven for: $F = \{H, \psi, \mathcal{W}^{\mu\nu}\}$ via the following:

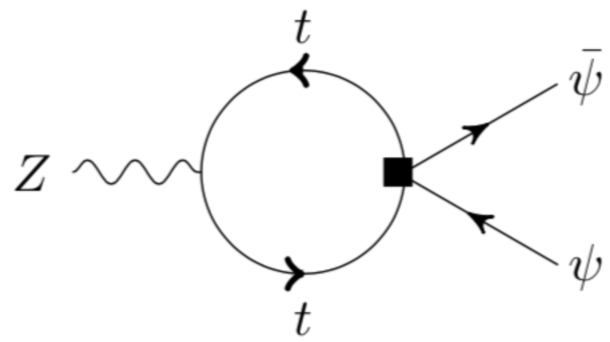
$$D^2 F \;\Rightarrow\; \boxed{\text{EOM}} \text{ and higher-points,}$$

2001.01453 Helset, Martin, Trott

$$f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \;\Rightarrow\; \boxed{\text{EOM}} \text{ and higher-points.}$$

$$f(\phi)\,F_1\,(D_\mu F_2)\,(D_\mu F_3) \Rightarrow (D_\mu f(\phi))\,(D_\mu F_1)\,F_2\,F_3 + \frac{1}{2}(D^2 f(\phi))\,F_1\,F_2\,F_3 + \boxed{\text{EOM}},$$





- How to incorporate such higher n-point effects is the key challenge.
- Pert corrections advancing fast- higher n points also moving.

GeoSMEFT Pushing to higher n points

- Note these integration by parts steps were used

$$
\begin{aligned}
& f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \\
&= - f(H)\left[(D^2 F_1)(D_\nu F_2) + (D_\mu F_1)(D_\mu D_\nu F_2) + (D_\mu D_\nu F_1)(D_\mu F_2) + (D_\nu F_1)(D^2 F_2)\right](D_\nu F_3) \\
&\quad - (D_\mu f(H))\left[(D_\mu F_1)(D_\nu F_2) + (D_\nu F_1)(D_\mu F_2)\right](D_\nu F_3)
\end{aligned}
$$

$$
f(\phi)\, F_1\, (D_\mu F_2)\, (D_\mu F_3) \Rightarrow (D_\mu f(\phi))\, (D_\mu F_1)\, F_2\, F_3 + \frac{1}{2}(D^2 f(\phi))\, F_1\, F_2\, F_3 + \boxed{\mathrm{EOM}},
$$

These steps were critical to reducing the number of connections for two and three point functions. This just fails for four points and higher.

One knows that there are an infinite set of higher derivative terms lurking In higher n points, dependent on $\{D_\mu\phi^I, D_{\{\mu,\nu\}}\phi^I, D_{\{\mu,\nu,\rho\}}\phi^I, \cdots\},$

This is a problem for measurements away from SM resonances.

An instant pay off of this approach

- Growth in operator forms in connections
  Always saturate to fixed number, this is just the simplest organization exploiting this

Field space connection	6	8	10	12	14
	Mass Dimension				
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

- Once we have things to dim eight it is sufficient in many observables

Mases
Couplings and mixing angles
TGC, Higgs to ZZ,WW
QGC,TGC + Higgs

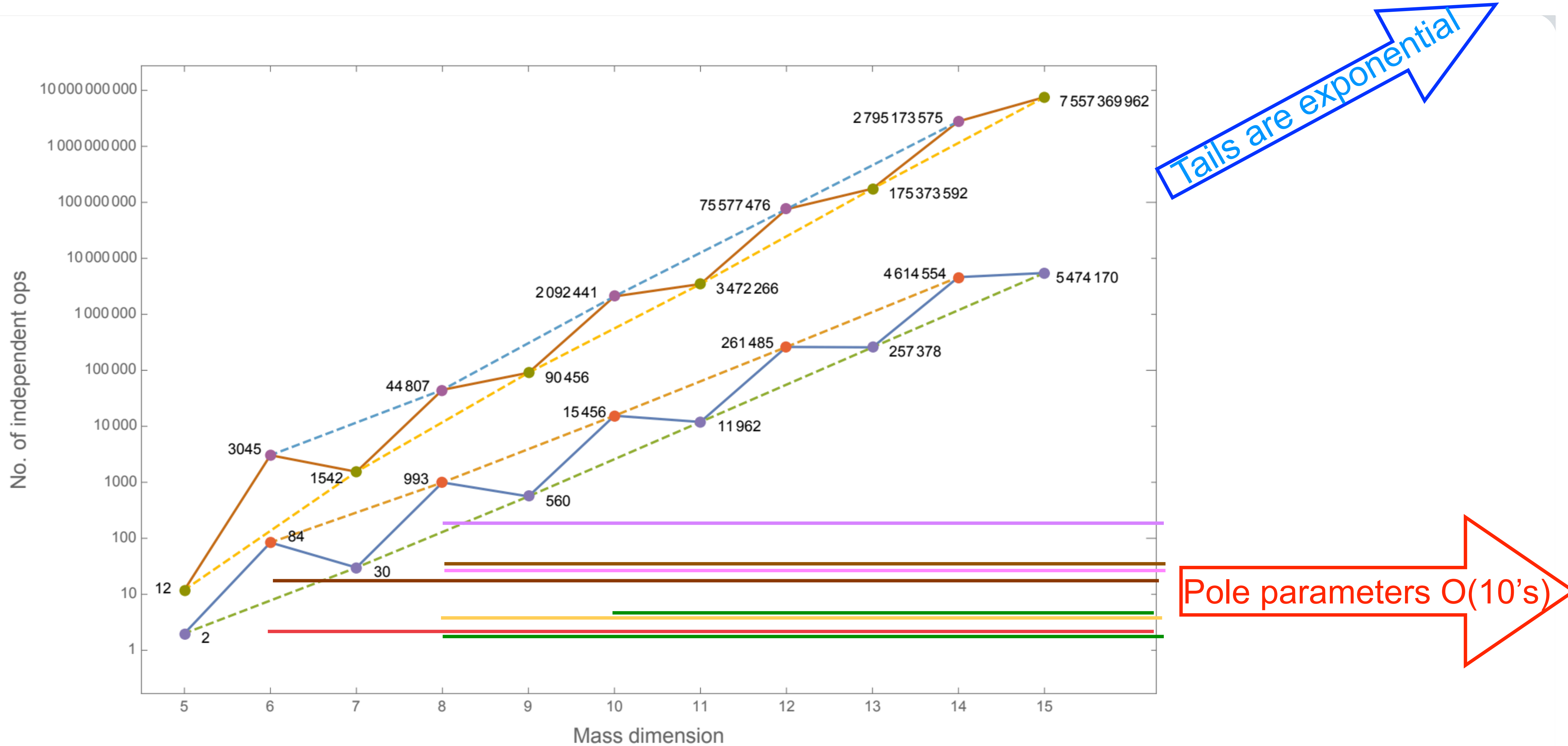
Yukawas

Dipoles

W,Z couplings to fermions +higgs

2001.01453 Helset, Martin, Trott

Application to theory error

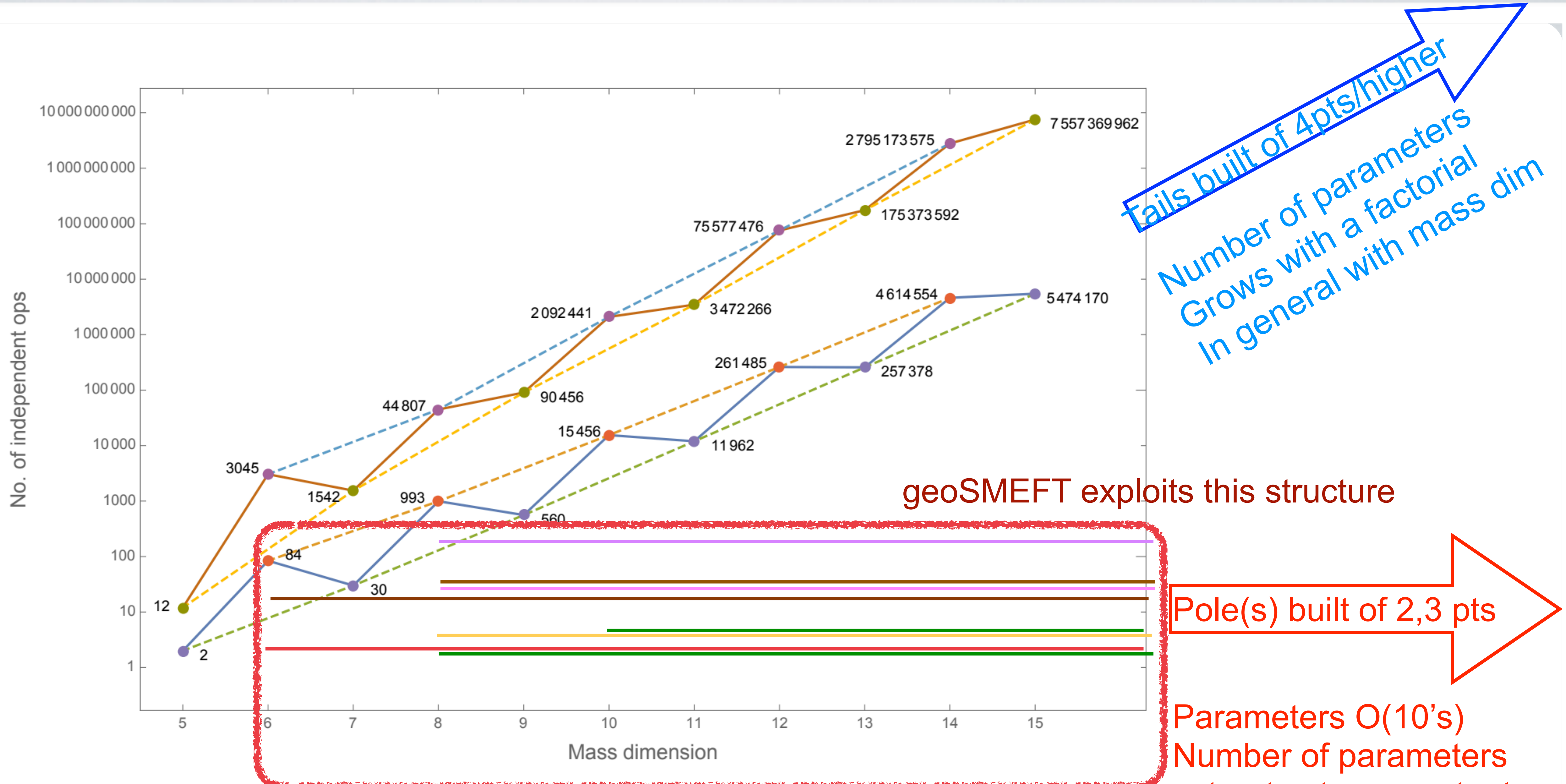


- General growth in operator forms from Hilbert series

  1503.07537 Lehman, Martin. 1512.03433 Henning, Lu, Melia, Murayama

  1510.00372 Lehman, Martin. 1706.08520 Henning, Lu, Melia, Murayama

Application to theory error



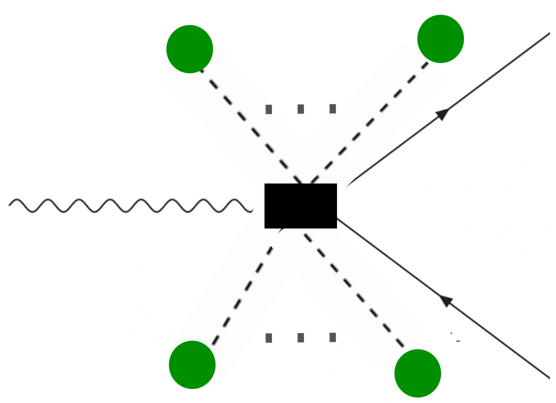
- General growth in operator forms from Hilbert series

1503.07537 Lehman, Martin. 1512.03433 Henning, Lu, Melia, Murayama

1510.00372 Lehman, Martin. 1706.08520 Henning, Lu, Melia, Murayama

GeoSMEFT example

- What does this allow one to do? 2001.01453 Helset, Martin, Trott



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

The all orders coupling in the SMEFT is a sum of two field space connections.

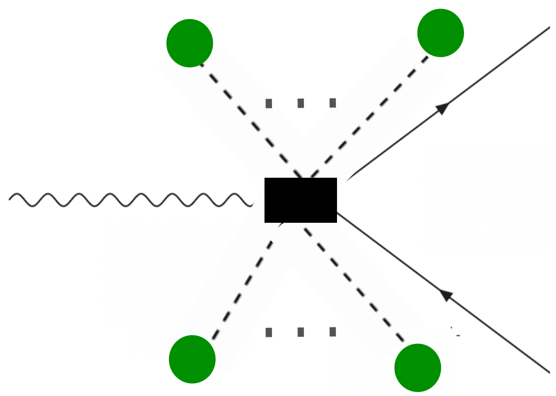
$\bar{\psi} i \not{D} \psi$:with a consistent change weak to mass eigenstates in SMEFT

Added to this is the scalar, fermion connection (with a background field expectation)

$$L^{\psi_R}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,R} \gamma_\mu \sigma_A \psi_{r,R})$$
$$L^{\psi_L}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,L} \gamma_\mu \sigma_A \psi_{r,L})$$

GeoSMEFT example

- What does this allow one to do? 2001.01453 Helset, Martin, Trott



$\mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A})$

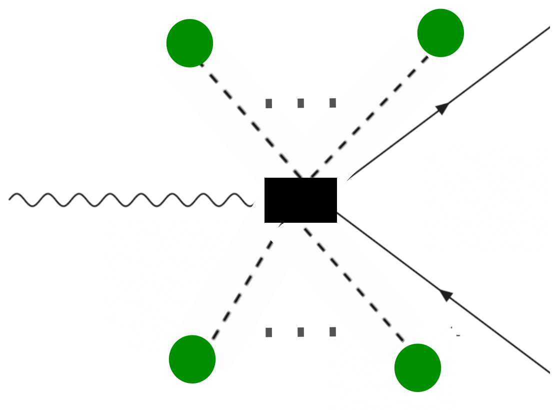
Consider a $W^\pm, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^I_{C,4})\bar{v}_T,$$

Compact all $\bar{v}_T/\Lambda$ orders answer!

GeoSMEFT example

- What does this allow one to do? 2001.01453 Helset, Martin, Trott



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

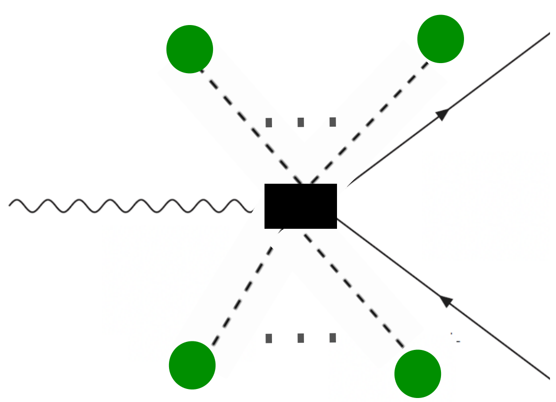
$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\gamma_{C,4}^{I})\bar{v}_T,$$

The coupling of the canonically normalised mass eigenstate fields is then

$$\begin{aligned}
\langle \mathcal{Z}|\bar{\psi}_p \psi_r\rangle &= \frac{\bar{g}_Z}{2}\,\bar{\psi}_p\,\not{\epsilon}_{\mathcal{Z}}\left[(2 s_{\theta_Z}^2 Q_\psi - \sigma_3)\delta_{pr} + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi,pr}\rangle + \bar{v}_T \langle L_{3,4}^{\psi,pr}\rangle\right]\psi_r,\\
\langle \mathcal{A}|\bar{\psi}_p \psi_r\rangle &= -\bar{e}\,\bar{\psi}_p\,\not{\epsilon}_{\mathcal{A}}\,Q_\psi\,\delta_{pr}\,\psi_r,\\
\langle \mathcal{W}_{\pm}|\bar{\psi}_p \psi_r\rangle &= -\frac{\bar{g}_2}{\sqrt{2}}\bar{\psi}_p(\not{\epsilon}_{\mathcal{W}^{\pm}})T^{\pm}\left[\delta_{pr} - \bar{v}_T\langle L_{1,1}^{\psi,pr}\rangle \pm i\bar{v}_T\langle L_{1,2}^{\psi,pr}\rangle\right]\psi_r.
\end{aligned}$$

GeoSMEFT example

- Can build up observable quantities, such as a decay width.



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p\gamma_\mu\bar{\tau}_A\psi_r)\delta_{pr}+\mathcal{A}^{C,\mu}(\bar{\psi}_p\gamma_\mu\sigma_A\psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\gamma_{C,4}^{I})\bar{v}_T,$$

- Two body decay widths:

$$\bar{\Gamma}_{Z\to\bar{\psi}\psi}=\sum_\psi\frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi}=\frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}^2\,Q_\psi-\sigma_3)\delta_{pr}+\bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle+\sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right]$$

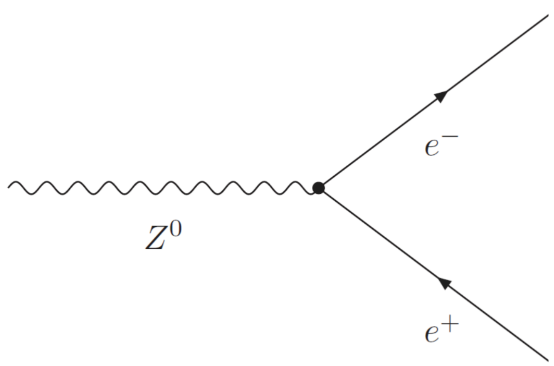
$$\bar{\Gamma}_{W\to\bar{\psi}\psi}=\sum_\psi\frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_W^2}|g_{\rm eff}^{W,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_W^2}\right)^{3/2}$$

$$g_{\rm eff}^{W,q_L}=-\frac{\bar{g}_2}{\sqrt{2}}\left[V_{\rm CKM}^{pr}-\bar{v}_T\langle L_{1,1}^{q_L,pr}\rangle\pm i\bar{v}_T\langle L_{1,2}^{q_L,pr}\rangle\right],$$

$$g_{\rm eff}^{W,\ell_L}=-\frac{\bar{g}_2}{\sqrt{2}}\left[U_{\rm PMNS}^{pr,\dagger}-\bar{v}_T\langle L_{1,1}^{\ell_L,pr}\rangle\pm i\bar{v}_T\langle L_{1,2}^{\ell_L,pr}\rangle\right],$$

SMEFT expansion correction error estimate by process.

- Process by process many of the dimension 8 corrections are also fully known now. We can examine process by process to be informed in defining the error. Ex. 2102.02819 Helset, Corbett, Martin, Trott



$$= \langle g_{\rm SM,pr}^{\mathcal{Z},\psi}\rangle + \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^2/\Lambda^2)} + \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^4/\Lambda^4)} + \cdots .$$

- LO SMEFT in SMEFTsim

SMEFT corrections in the $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme							
$\mathcal{O}(\frac{v^2}{\Lambda^2})$	$\langle g_{\rm eff,pp}^{\mathcal{Z},u_R}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},d_R}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},\ell_R}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},u_L}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},d_L}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},\ell_L}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},\nu_L}\rangle$
$\delta G_F^{(6)}$	-0.08/0.15	0.04/-0.07	0.12/-0.22	0.18/0.41	-0.22/-0.34	-0.15/-0.49	0.26/0.26
$\tilde{C}_{HD}^{(6)}$	-0.22/0.05	0.11/-0.03	0.33/-0.08	-0.13/0.15	0.02/-0.12	0.24/-0.17	0.09/0.09
$\tilde{C}_{HWB}^{(6)}$	-0.21/0.39	0.10/-0.19	0.31/-0.58	-0.21/0.39	0.10/-0.19	0.31/-0.58	
$\tilde{C}_{H\psi}^{(6)}$	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37	0.37/0.37
$\tilde{C}_{H\psi}^{3,(6)}$				-0.37/-0.37	0.37/0.37	0.37/0.37	-0.37/-0.37

SMEFT expansion correction error estimate by process.

- EWPD to dim 8 now known in both input parameter schemes.

2102.02819 Helset, Corbett, Martin, Trott

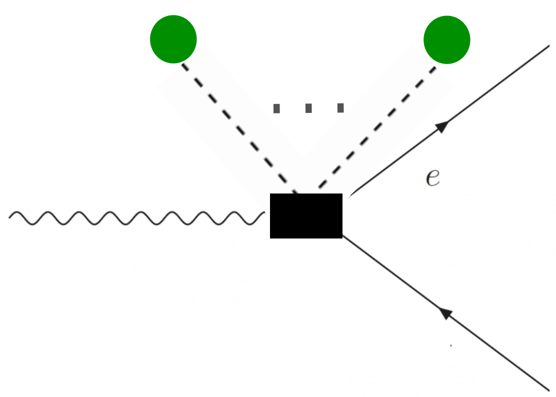
Many cross terms *NOT in quadratics* but known for error Estimates

Few new dimension Eight parameters

SMEFT corrections in $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme			
$\mathcal{O}(\frac{v^4}{\Lambda^4})$	$\langle g^{\mathcal{Z},u_R}_{\rm eff,pp}\rangle$	$\langle g^{\mathcal{Z},d_R}_{\rm eff,pp}\rangle$	$\langle g^{\mathcal{Z},\ell_R}_{\rm eff,pp}\rangle$
$\langle g^{\mathcal{Z},\psi}_{\rm eff}\rangle^2$	14/5.5	-27/-11	-9.1/-3.6
$\tilde{C}_{HB}\tilde{C}_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^2_{HD}$	0.28/-0.026	-0.14/0.013	-0.42/0.040
$\tilde{C}_{HD}\tilde{C}^{(6)}_{H\psi}$	-0.83/-0.19	-0.83/-0.19	-0.83/-0.19
$\tilde{C}_{HD}\tilde{C}_{HWB}$	0.59/-0.19	-0.29/0.097	-0.88/0.29
$\tilde{C}_{HD}\langle g^{\mathcal{Z},\psi}_{\rm eff}\rangle$	4.0/0.50	4.0/0.50	4.0/0.50
$(\tilde{C}^{(6)}_{H\psi})^2$	0.62/1.4	-1.2/-2.8	-0.42/-0.93
$\tilde{C}_{HWB}\tilde{C}^{(6)}_{H\psi}$	-0.69/0.58	-0.69/0.58	-0.69/0.58
$\tilde{C}^{(6)}_{H\psi}\langle g^{\mathcal{Z},\psi}_{\rm eff}\rangle$	-6.7/-5.8	13/12	4.5/3.9
$\tilde{C}_{HWB}\langle g^{\mathcal{Z},\psi}_{\rm eff}\rangle$	3.7/0.26	3.7/0.26	3.7/0.26
$\tilde{C}_{HW}\tilde{C}_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^{(8)}_{HD}$	-0.014/0.026	0.0069/-0.013	0.021/-0.040
$\tilde{C}^{(8)}_{HD,2}$	-0.21/0.026	0.10/-0.013	0.31/-0.040
$\tilde{C}^{(8)}_{H\psi}$	0.19/0.19	0.19/0.19	0.19/0.19
$\tilde{C}^{(8)}_{HW,2}$	-0.38/0	0.19/0	0.58/0
$\tilde{C}^{(8)}_{HWB}$	-0.10/0.19	0.051/-0.097	0.15/-0.29
$\delta G^{(8)}_F$	-0.078/0.15	0.039/-0.075	0.12/-0.22
$(\tilde{C}^{(6)}_{HWB})^2$	0.19/-0.35	-0.096/0.18	-0.29/0.53

GeoSMEFT directly informs theory error

- Can build up observable quantities, such as a decay width.



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p\gamma_\mu\bar{\tau}_A\psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p\gamma_\mu\sigma_A\psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\gamma_{C,4}^{I})\bar{v}_T,$$

- Dim 8 effects on Z decay now known

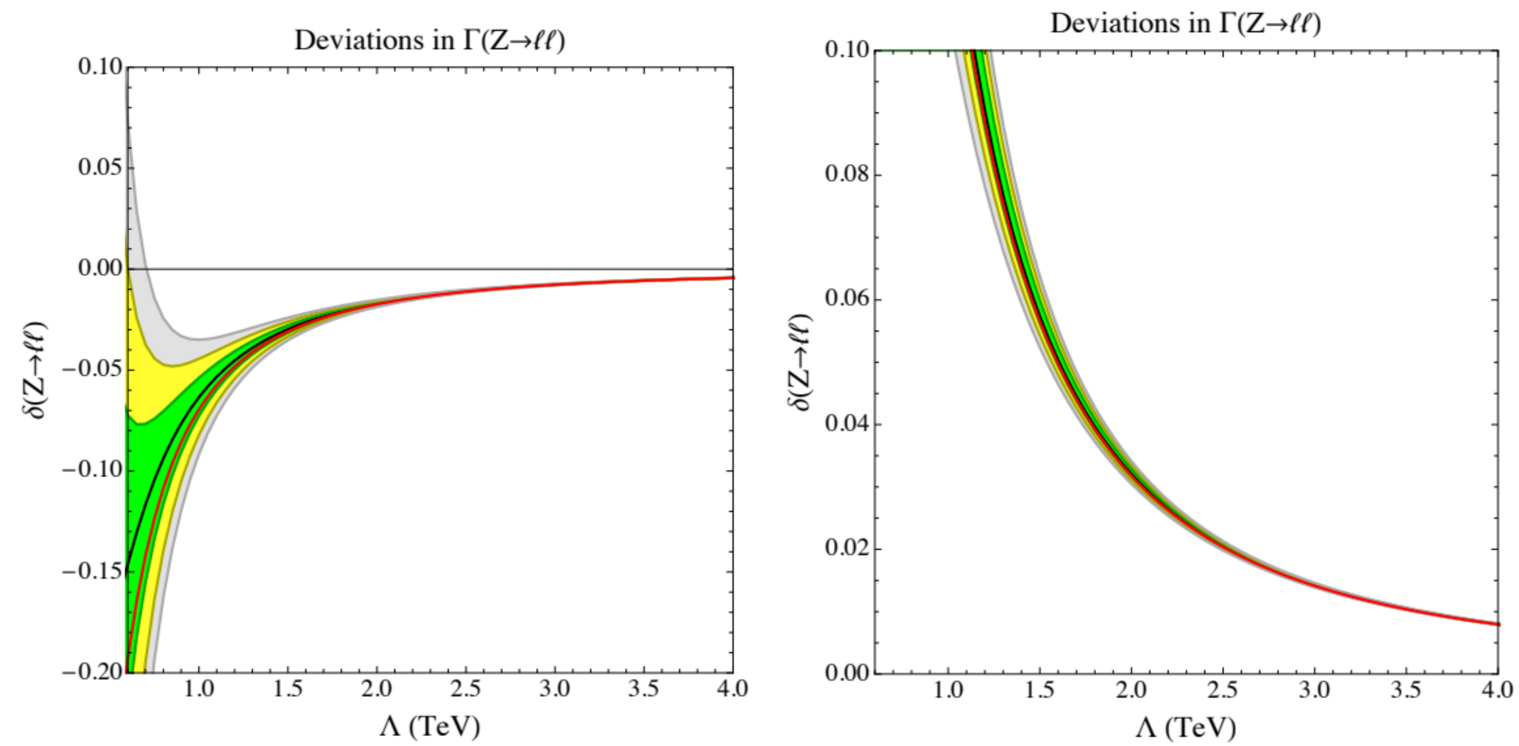
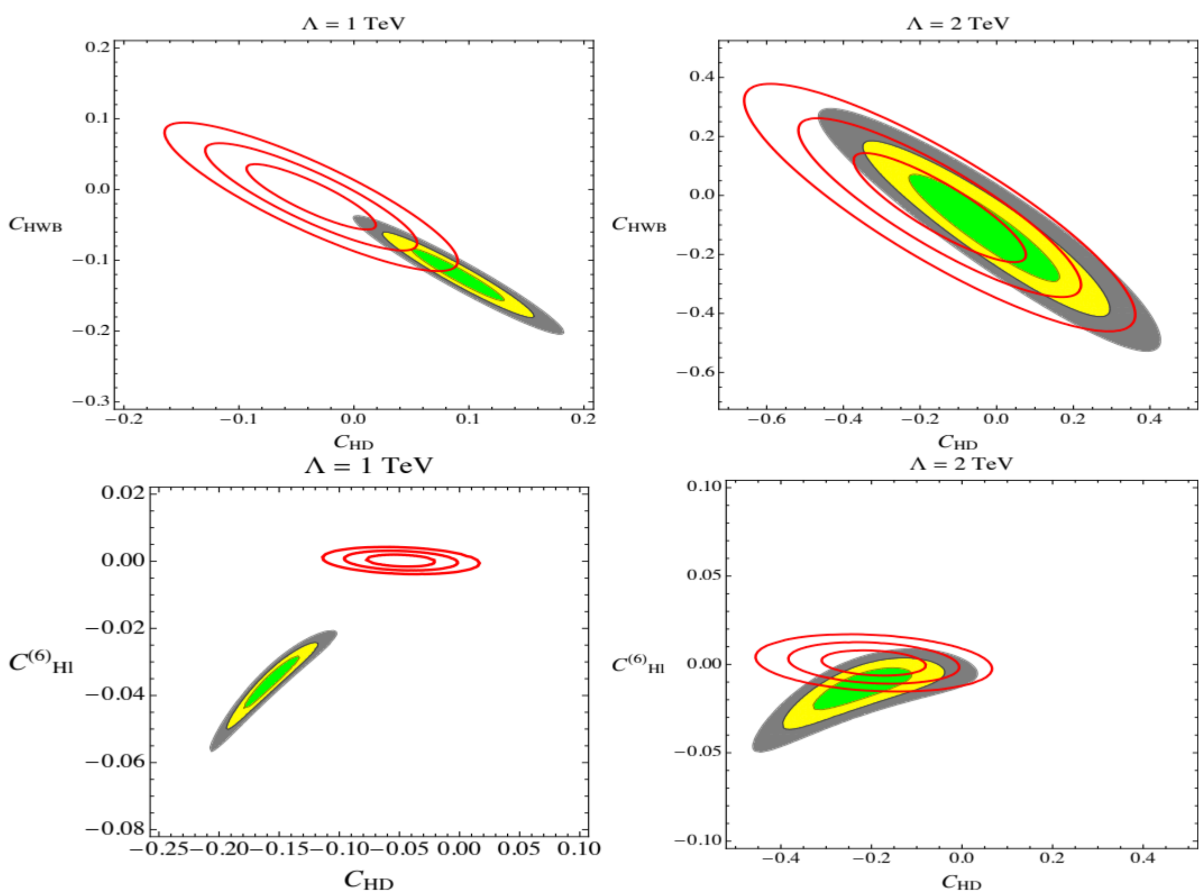


Figure 3. The deviations in $\mathcal{Z} \to \ell\ell$ from the $\mathcal{O}(v^2/\Lambda^2)$ (red line) and partial-square (black line) results, and the full $\mathcal{O}(v^4/\Lambda^4)$ results (green $\pm 1\,\sigma_\delta$, yellow $\pm 2\,\sigma_\delta$, and grey $\pm 3\,\sigma_\delta$ regions). In the left panel the coefficients determining the $\mathcal{O}(v^2/\Lambda^2)$ and partial-square results are $C_{H\ell}^{1,(6)} = -0.46, C_{H\ell}^{3,(6)} = 1.24, C_{He}^{(6)} = 1.53, C_{HD}^{(6)} = -0.79, C_{HWB}^{(6)} = 0.007$, and $\delta G_F^{(6)} = 0.16$. In the right panel they are $C_{H\ell}^{1,(6)} = 1.55, C_{H\ell}^{3,(6)} = -0.71, C_{He}^{(6)} = 0.23, C_{HD}^{(6)} = -0.51, C_{HWB}^{(6)} = -0.008$, and $\delta G_F^{(6)} = -0.44$.

Yes people do over interpret EWPD.

- (Heretical) Doubts were raised years ago that dim 8 neglect and loop neglect In EWPD can significantly impact naive bounds with significant implications
1502.02570, 1508.05060, Berthier, MT ,1606.06693 Berthier, Bjorn, MT , arXiv:1701.06424 Brivio, MT

2102.02819 Helset, Corbett, Martin, Trott



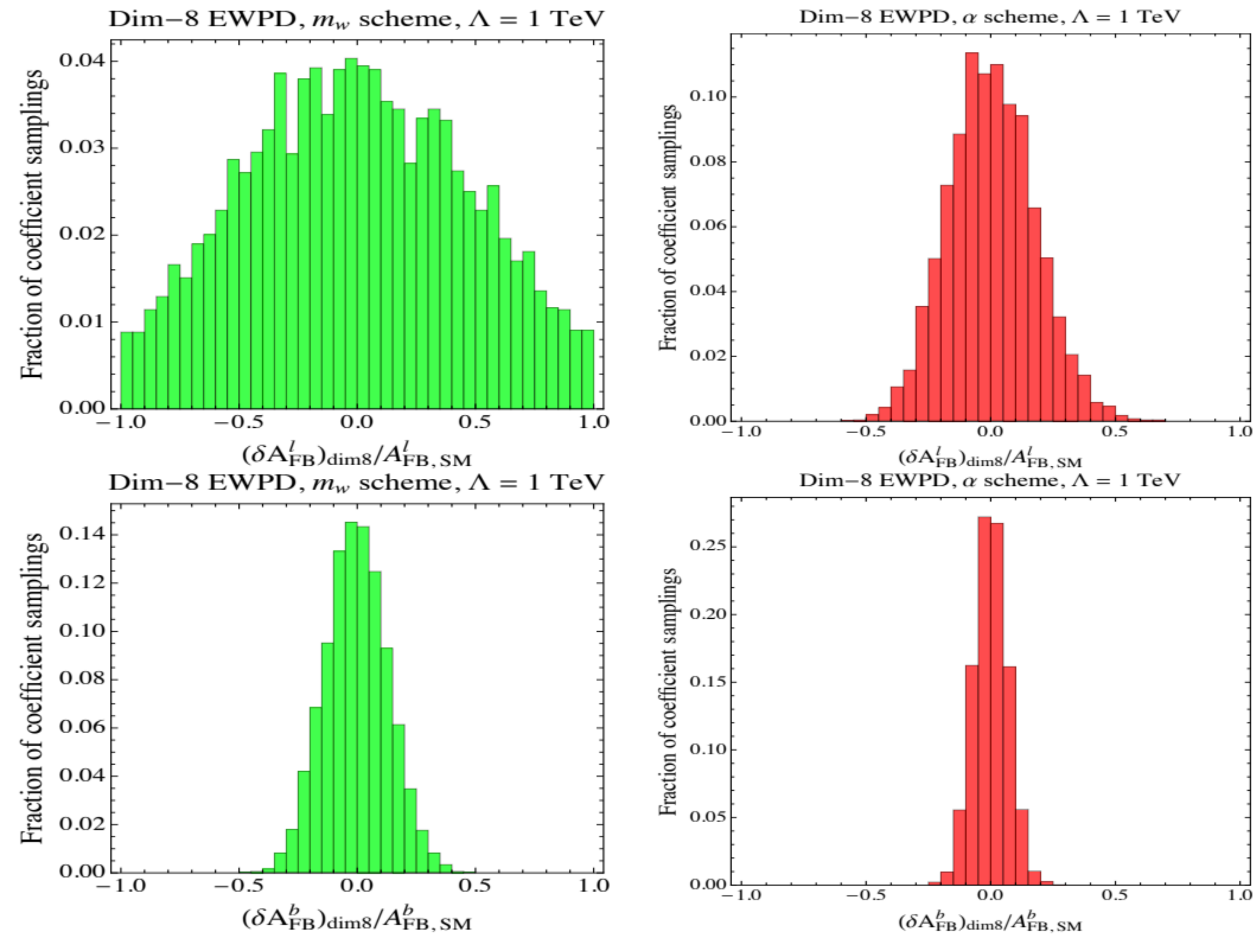
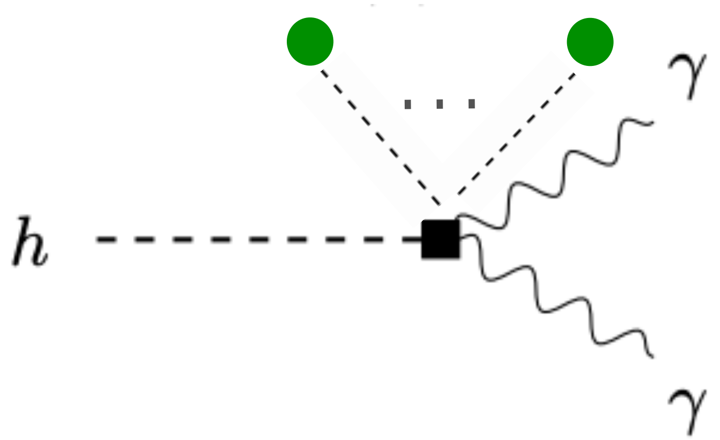


Figure 3. The green/yellow/gray contours correspond to the 68%/95%/99.9% CL two par... ter fit determined by $\Delta\chi^2_{\mathcal{O}(v^4/\Lambda^4)}$, while the red rings correspond to the same CL determined using $\Delta\chi^2_{\mathcal{O}(v^2/\Lambda^2)}$. In the top panels the free parameters are C_{HD} and C_{HWB}, while in the bottom panels the free parameters are C_{HD} and $C^{(6)}_{H\ell}$. Note that the axes ranges vary from panel to panel. In the left panels, we have taken the scale $\Lambda = 1$ TeV, while in the right panels $\Lambda = 2$ TeV. All calculations use the $\hat{m}_W$ scheme.

- ***Significant implications for future collider studies.***

GeoSMEFT for the Higgs



- How much does dim 8 effect things? Alot.
  2007.00565 Hays, Helset, Martin, Trott

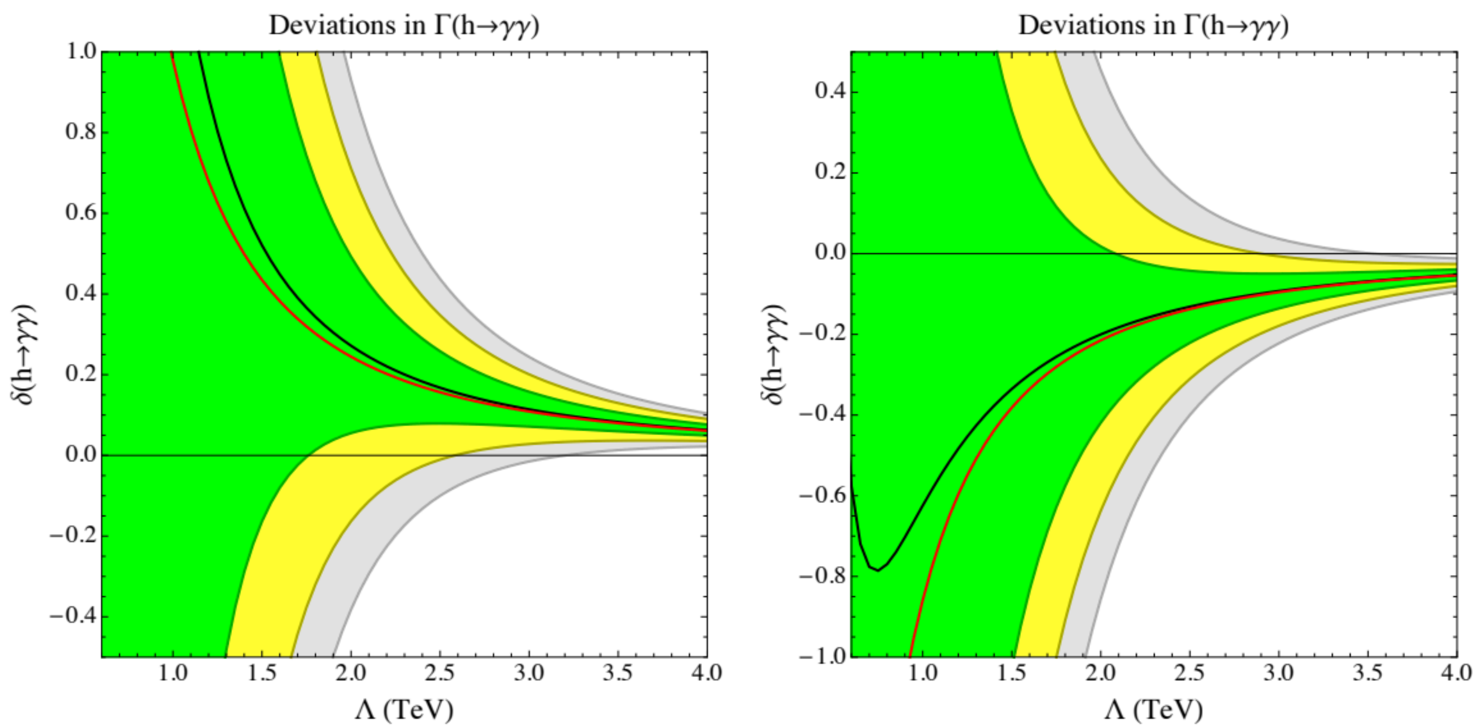


Figure 1. The deviations in $h \to \gamma\gamma$ from the $\mathcal{O}(v^2/\Lambda^2)$ (red line) and partial-square (black line) results, and the full $\mathcal{O}(v^4/\Lambda^4)$ results (green $\pm 1\,\sigma_\delta$, yellow $\pm 2\,\sigma_\delta$, and grey $\pm 3\,\sigma_\delta$ regions). In the left panel the coefficients determining the $\mathcal{O}(v^2/\Lambda^2)$ and partial-square results are $C^{(6)}_{HB} = -0.01$, $C^{(6)}_{HW} = 0.004$, $C^{(6)}_{HWB} = 0.007$, $C^{(6)}_{HD} = -0.74$, and $\delta G^{(6)}_F = -1.6$. In the right panel they are $C^{(6)}_{HB} = 0.007$, $C^{(6)}_{HW} = 0.007$, $C^{(6)}_{HWB} = -0.015$, $C^{(6)}_{HD} = 0.50$, and $\delta G^{(6)}_F = 1.26$.

- EFT studies that ignore this geoSMEFT enabled information can be misleading.

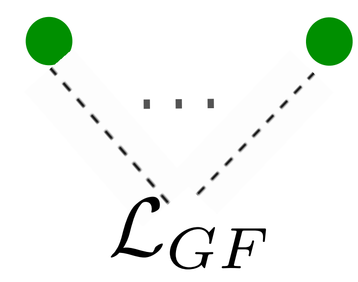
- Loop processes are more subject to SMEFT theoretical errors in the cases we looked at.

GeoSMEFT based loop corrections?

- The simplicity of the results for two and three point functions points to radiative corrections being more elegant than expected.

  The renormalisation follows the dependence on the Wilson coefficients.

- Do we have hints of this yet? Yes.



- Background field gauge fixing with preserved background Gauge invariance 1803.08001 Helset, Paraskevas, Trott.

$$\mathcal{L}_{\rm GF} = -\frac{\hat{g}_{AB}}{2\,\xi}\mathcal{G}^A\,\mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{\ CD}\hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\,\hat{h}_{IK}\,\tilde{\gamma}^K_{C,J}\hat{\phi}^J.$$

- Gauge fixing confusion directly solved generalising to GeoSMEFT

Consistency checks at one loop/dim8 for tools

- The operator and loop expansion are not independent.

$$\mathcal{A} = \mathcal{A}_{SM} + \tilde{C}_i^{(6)} \, a_i + \cdots$$

If you choose to rescale (or not) the Wilson coefficient at L6 it changes the one loop result and the dimension 8 result in a correlated way.

Only game in town for full dimension 8 with all input redefinitions etc is geoSMEFT. Loops should be done in BFM for consistency with background field independent formulation defining geoSMEFT.

Consistency checks at one loop/dim8

Benefits of the Background Field method one loop approach in SMEFT.

- Many cross checks afforded (Ward identities and more).
- Clean understanding of ward identities.
- One loop redefinition of input parameters INDIVIDUALLY gauge independent.
- Cross checks of

$$\Delta Z_e = -\tfrac{1}{2}\Delta Z_{\hat{\mathcal{A}}},$$
$$\Delta R_e = -\tfrac{1}{2}\Delta R_{\hat{\mathcal{A}}}.$$

Our calc in 2107.07470
Stoffer/Denkens in 1908.05295

$$\Delta R_{\hat{\mathcal{A}}} = \frac{\bar{g}_1^2 \bar{g}_2^2}{(\bar{g}_1^2 + \bar{g}_2^2)} \left[-\frac{7}{16\pi^2} \log\left(\frac{\mu^2}{\bar{m}_W^2}\right) + \sum_\psi \frac{N_c^\psi Q_\psi^2}{12\pi^2} \log\left(\frac{\mu^2}{\bar{m}_\psi^2}\right) - \frac{1}{24\pi^2} \right].$$

Consistency checks at one loop/dim8

Cancelation of large mt dependent logs in relations between observables:
Expected and anticipated in Hartmann/Trott. 1505.02646

- Expected cancelation confirmed in 2107.07470 and 1908.05295

$$\bar{v}_T = \hat{v}_T \left[1 + \frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2}\left[1 + \log\left(\frac{\mu^2}{m_f^2}\right)\right] + \cdots\right].$$

$$\frac{\Delta v}{\bar{v}_T} \propto -\frac{2y_t^2}{16\pi^2} N_C \frac{m_f^2}{\bar{m}_h^2}\left[1 + \log\left(\frac{\mu^2}{m_f^2}\right)\right].$$

- Cancelation in single Higgs, single dev observables with tadpole term and GF extraction.

Consistency checks at one loop/dim8

Gauge independence of a common partial matrix element in single Higgs processes in BFM:

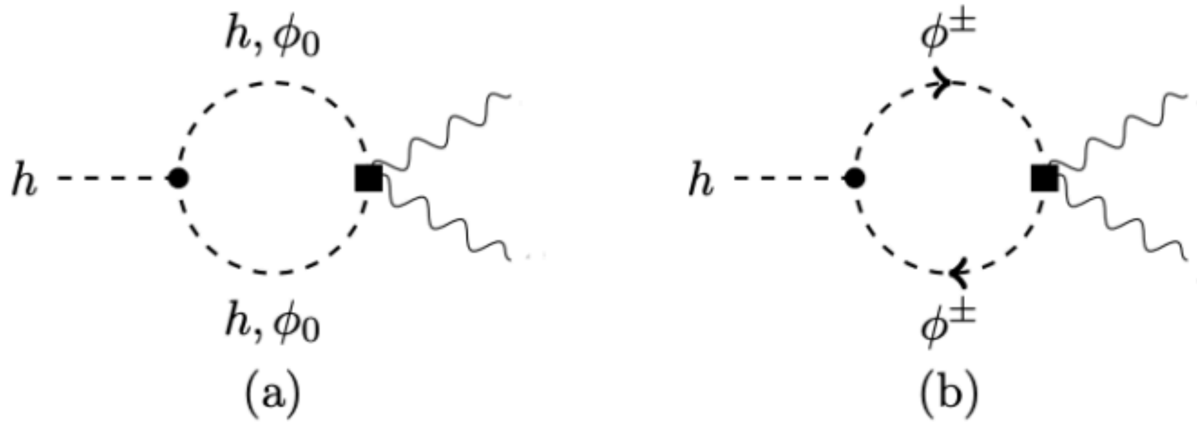


Figure 2. One loop contributions to $\langle \phi_4 | F\, F \rangle \langle \frac{\delta M_{AB}}{\delta \phi_4} \rangle$.

$$\frac{\langle \phi_4 F(p_1) F(p_2) \rangle^1}{\langle \phi_4 F^{\mu\nu} F_{\mu\nu} \rangle^0 \langle \frac{\delta M_{AB}(\phi)}{\delta \phi_4} \rangle^0} \propto M_1$$

$$M_1 \equiv \left(\frac{\Delta R_h}{2} + \frac{\Delta v}{v} + \frac{(\sqrt{3}\pi - 6)\lambda}{16\pi^2} + \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} + \frac{3\bar{g}_2^2}{4} + 6\lambda \right) \log\left[\frac{\bar{m}_h^2}{\mu^2} \right] \right),$$
$$+ \frac{1}{16\pi^2} \left(\frac{\bar{g}_1^2}{4} \mathcal{I}[\bar{m}_Z] + (\frac{\bar{g}_2^2}{4} + \lambda)(\mathcal{I}[\bar{m}_Z] + 2\mathcal{I}[\bar{m}_W]) \right).$$

GeoSMEFT based loop corrections?

- Will this simplify the NLO SMEFT radiative correction program?(Yes)

  Immediate BFM Ward Identities have already been derived:

$$\frac{\delta\Gamma[\hat{F},0]}{\delta\hat{\alpha}^B} = 0.$$

Background field gauge transformation

1909.08470 Corbett, Helset, Trott

$$0 = \left(\partial^\mu \delta^A_B - \tilde{\epsilon}^A_{\ BC}\, \hat{\mathcal{W}}^{C,\mu}\right)\frac{\delta\Gamma}{\delta\hat{\mathcal{W}}^\mu_A} - \frac{\tilde{\gamma}^I_{B,J}}{2}\hat{\phi}^J\frac{\delta\Gamma}{\delta\hat{\phi}^I} + \sum_j \left(\bar{f}_j\bar{\Lambda}^j_{B,i}\frac{\delta\Gamma}{\delta\bar{f}_i} - \frac{\delta\Gamma}{\delta f_i}\Lambda^i_{B,j}f_j\right).$$

Photon identities:

$$0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\mathcal{A}}^{Y\nu}}, \qquad 0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{4\mu}\delta\hat{\Phi}^I}.$$

$$\Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{L,\mathrm{SMEFT}}(k^2) = 0, \qquad \Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{T,\mathrm{SMEFT}}(0) = 0.$$

One loop behaviour works!

Z identities:

Geometric mass

$$0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\mathcal{A}}^{Y\nu}} - \bar{M}_Z \frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\mathcal{A}}^{Y\nu}},$$

$$0 = \partial^\mu \frac{\delta^2\Gamma}{\delta\hat{\mathcal{A}}^{3\mu}\delta\hat{\Phi}^I} - \bar{M}_Z \frac{\delta^2\Gamma}{\delta\hat{\Phi}^3\delta\hat{\Phi}^I} + \frac{\bar{g}_Z}{2}\frac{\delta\Gamma}{\delta\hat{\Phi}^4}\left(\sqrt{h}_{[4,4]}\sqrt{h}^{[3,3]} - \sqrt{h}_{[4,3]}\sqrt{h}^{[4,3]}\right)\delta^3_I - \frac{\bar{g}_Z}{2}\frac{\delta\Gamma}{\delta\hat{\Phi}^4}\left(\sqrt{h}_{[4,4]}\sqrt{h}^{[3,4]} - \sqrt{h}_{[4,3]}\sqrt{h}^{[4,4]}\right)\delta^4_I,$$

GeoSMEFT based loop corrections?

- Expanding out the Ward ID you get expressions like this:

$$0 = \Sigma_L^{\hat{\mathcal{Z}}\hat{\mathcal{Z}}}(k^2) - i\bar{M}_{\mathcal{Z}}\Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2),$$

Geometric mass

Consider the operator C_{HWB}

$$-i\bar{M}_{\mathcal{Z}}\Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2) = -i\,\frac{\sqrt{g_1^2+g_2^2}\,\bar{v}_T}{2}\left[\Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2)\right]^{div}_{\tilde{C}_{HWB}} - i\frac{g_1\,g_2\,\tilde{C}_{HWB}\,\bar{v}_T}{2\sqrt{g_1^2+g_2^2}}\left[\Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2)\right]^{div}_{SM},$$

$$= -\tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\left[\frac{g_1\,g_2\,(3\,g_1^2+5\,g_2^2)}{256\,\pi^2\,\epsilon} + \frac{g_1\,g_2\,(g_1^2+3g_2^2)}{256\pi^2\epsilon}\right],$$

$$= -\tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\,\frac{g_1^2(g_1^2+2g_2^2)}{64\,\pi^2\,\epsilon}.$$

2010.08451 Corbett, Trott

2010.15852 Corbett

Which exactly cancels: $\left[\Sigma_L^{\hat{\mathcal{Z}}\hat{\mathcal{Z}}}(k^2)\right]^{div}_{\tilde{C}_{HWB}} = \tilde{C}_{HWB}\,\bar{v}_T^2\,(\xi+3)\,\frac{g_1^2(g_1^2+2g_2^2)}{64\,\pi^2\,\epsilon}.$

All one and two point ward ID working out at one loop. Powerful new NLO code tool is being developed using this as a theory cross check by Tyler Corbett.

GeoSMEFT based loop corrections?

- There was some debate about something called the "higgs basis" Years ago. Here is an explicit example of the problem using such A construction, you need to track the modifications of the tadpoles Somehow to not violate gauge invariance:

2010.08451 Corbett, Trott

$$0 = \Sigma_L^{\hat{\mathcal{Z}}\hat{\mathcal{Z}}}(k^2) - i\bar{M}_Z \Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2), \quad (37)$$

$$0 = k^2 \Sigma^{\hat{\mathcal{Z}}\hat{\chi}}(k^2) - i\bar{M}_Z\, \Sigma^{\hat{\chi}\hat{\chi}}(k^2) + i\,\frac{\bar{g}_Z}{2} T^H \left(1 - \tilde{C}_{H\Box}\right), \quad (38)$$

and

$$0 = \Sigma_L^{\hat{\mathcal{W}}^{\pm}\hat{\mathcal{W}}^{\mp}}(k^2) \pm \bar{M}_W \Sigma^{\hat{\Phi}^{\pm}\hat{\mathcal{W}}^{\mp}}(k^2), \quad (39)$$

$$0 = k^2 \Sigma^{\hat{\mathcal{W}}^{\pm}\hat{\Phi}^{\mp}}(k^2) \pm \bar{M}_W\, \Sigma^{\hat{\Phi}^{\pm}\hat{\Phi}^{\mp}}(k^2) \mp \frac{\bar{g}_2}{2} T^H \left(1 - \tilde{C}_{H\Box} + \frac{\tilde{C}_{HD}}{4}\right). \quad (40)$$

- If you violate gauge invariance, you can write down many other gauge dependent interactions. The resulting construction is difficult to map To a consistent SMEFT result

GeoSMEFT based Theory error/higher order.

- GeoSMEFT efficiently organised the physics for dimension 8 calc that follow from rescaling dimension 6 results with the same kinematics.

- An algorithm to develop calc to dimension 8 efficiently to use, and also inform theory errors 2106.13794 Trott "Methodology…"

  SMEFT RGE is transparent to simulation chain. Thus infer logs of missing pert corrections for theory error.

  Use geoSMEFT to rescale dim 6 results with common kinematics to post-facto (avoid redundant Monte Carlo) get dim 8 from SMEFTsim

  Extra dim 8 bits with novel kinematics define in geoSMEFT consistent basis. Put in code. (A SMEFTsim mod by T Corbett exits for a few terms)

- Being applied to associated production, already done for $\Gamma(h \to \gamma\gamma)$
  $\sigma(\mathcal{G}\,\mathcal{G} \to h)$, $\Gamma(h \to \mathcal{G}\,\mathcal{G})$

Conclusions.

Tools designed consistent with geoSMEFT allow you to leverage some dim 6 results with analytic rescaling, to obtain higher order results.

Higgs/SMEFT physics is the physics of curved field space(s).

Still true (and cool!)