Positivity bounds in the Standard Model EFT

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2110.01624 and *2205.03301* and ongoing work

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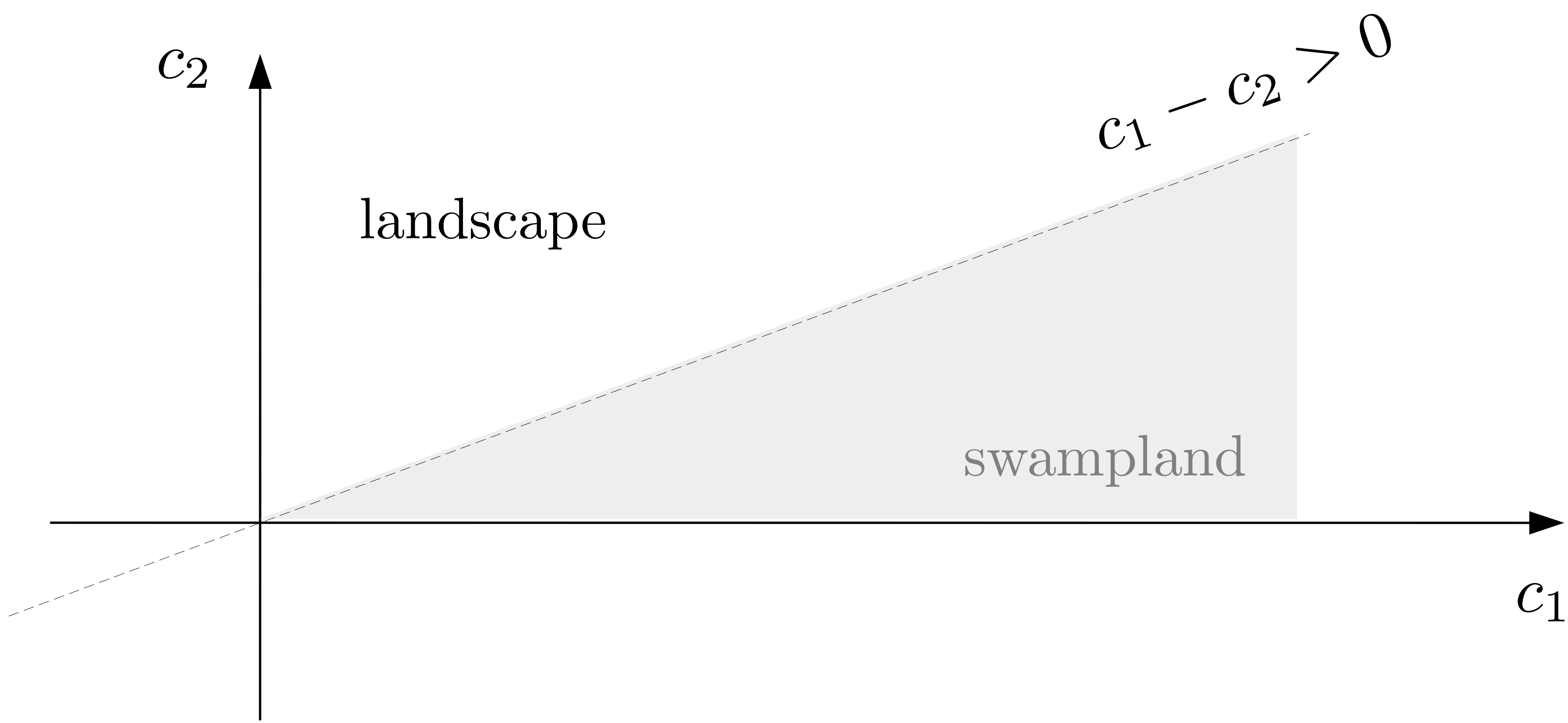
Positivity bounds are restrictions on the Wilson coefficients of EFT operators implied by the unitarity and locality of the S-matrix



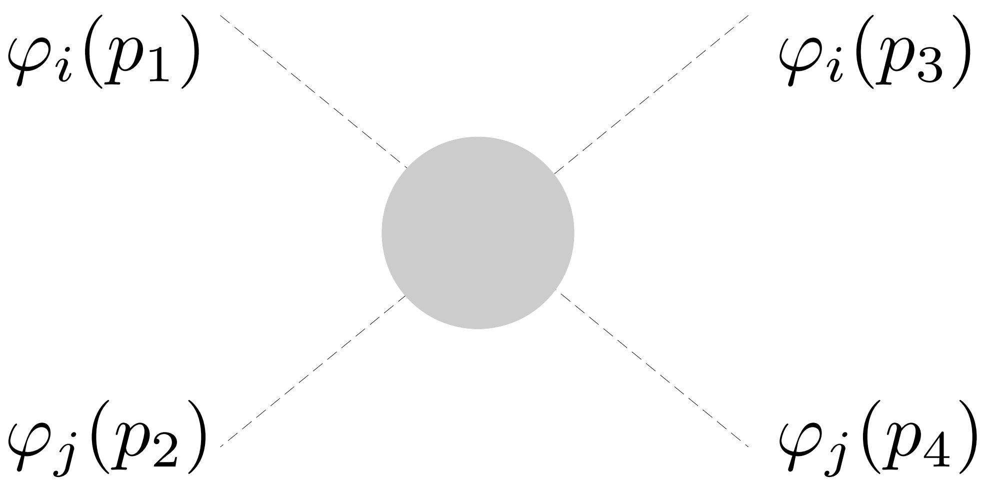
If positivity is violated

Positivity bounds are restrictions on the Wilson coefficients of EFT operators implied by the unitarity and locality of the S-matrix

Violation of positivity would imply the invalidity of the EFT or the breakdown of some of the fundamental principles of modern physics

More realistically, positivity bounds can be enforced as Bayesian priors in fits aiming at constraining the Wilson coefficients of the EFT

Deriving positivity



$$s = (p_1 + p_2)^2$$

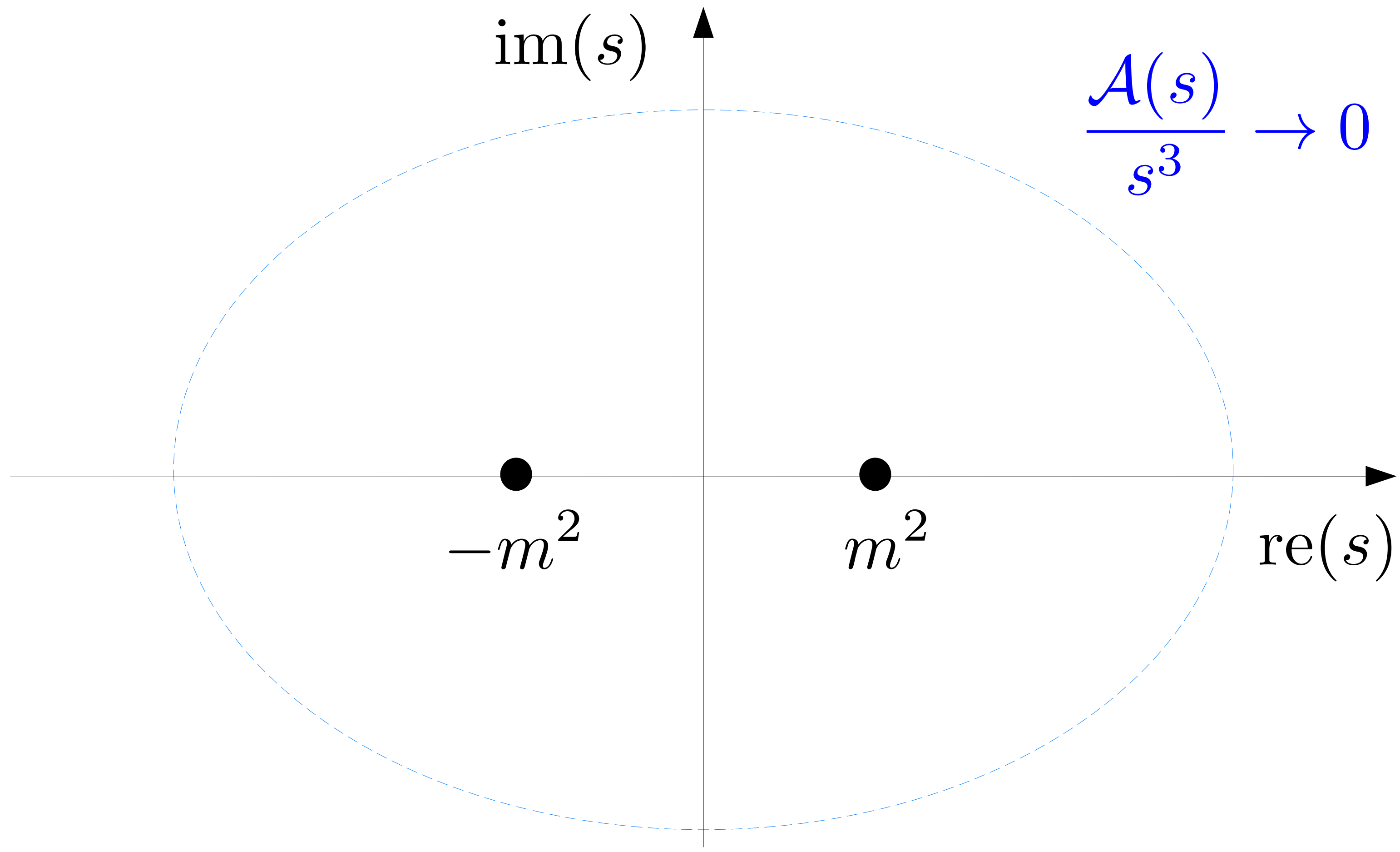
$$t = (p_1 + p_3)^2$$

$$u = (p_2 + p_3)^2$$

$$s + t + u = 0$$

$$\mathcal{A}(s) = \mathcal{A}(-s)$$

$$\mathcal{A}(s) = a_0 + {\color{red} a_1} s + a_2 s^2 + \cdots$$



$$0 = \mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = 0] + 2\,\mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = m^2]$$

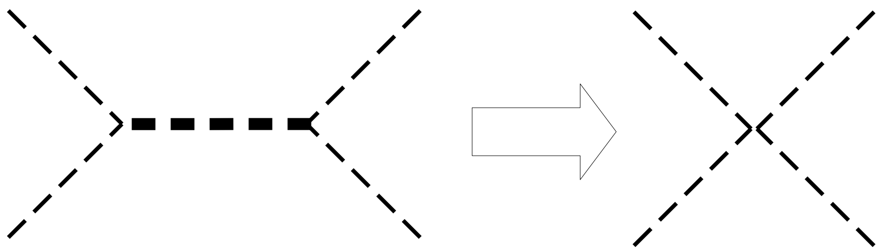
$$= a_2 - \frac{1}{\pi} \int s \frac{\sigma(s)}{(m^2)^3} \Rightarrow a_2 > 0$$

Adams et al '06

Application to the SMEFT

Murphy '20

$$\mathcal{O}^{(1)}_{H^4D^4} \quad (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$$
$$\mathcal{O}^{(2)}_{H^4D^4} \quad (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$
$$\mathcal{O}^{(3)}_{H^4D^4} \quad (D_\mu H^\dagger D^\mu H)(D^\nu H^\dagger D_\nu H)$$



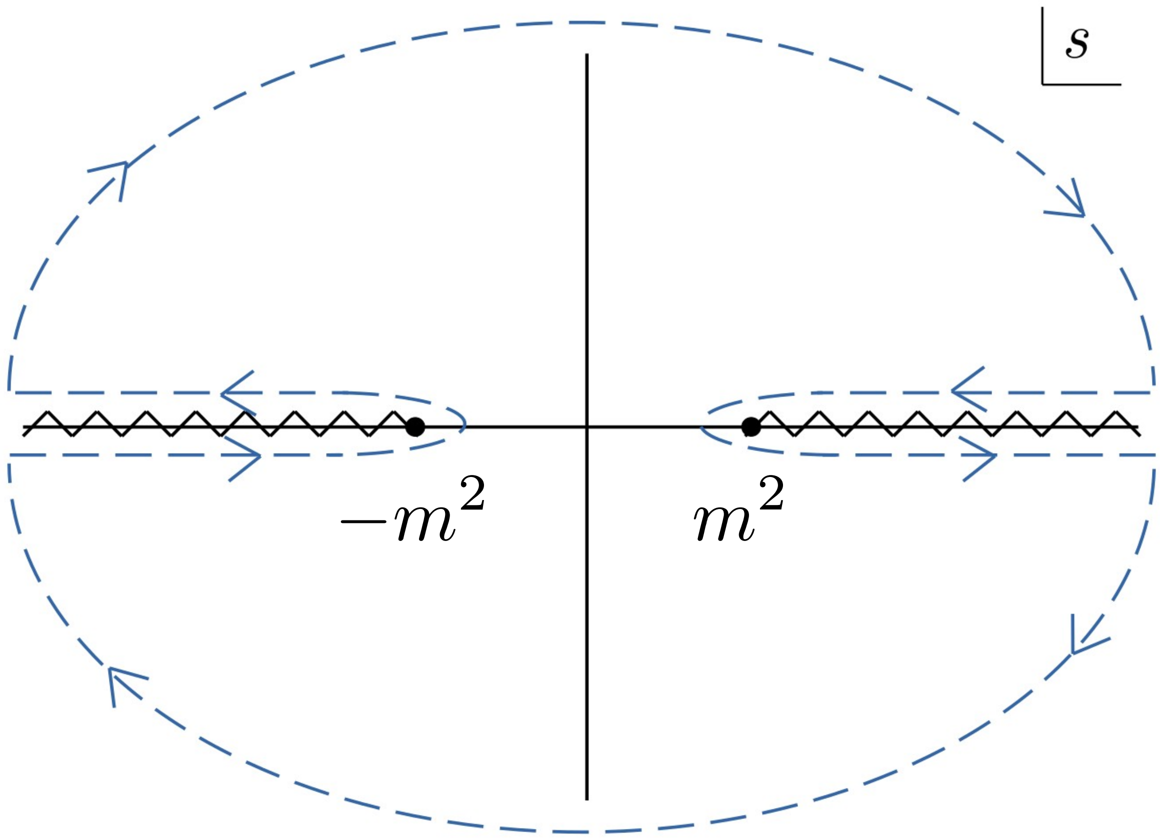
Remmen, Rodd '19

Positivity constraints

$$c_2 \geq 0$$
$$c_1 + c_2 \geq 0$$
$$c_1 + c_2 + c_3 \geq 0$$

$$\mathcal{S} \sim (1,1)_0 \longmapsto c^{(1,2,3)}_{H^4D^4} \sim (0,0,1)\,,$$
$$\Xi \sim (1,3)_0 \longmapsto c^{(1,2,3)}_{H^4D^4} \sim (2,0,-1)\,,$$
$$\mathcal{B} \sim (1,1)_0 \longmapsto c^{(1,2,3)}_{H^4D^4} \sim (-1,1,0)\,,$$
$$\mathcal{B}_1 \sim (1,1)_1 \longmapsto c^{(1,2,3)}_{H^4D^4} \sim (1,0,-1)\,,$$
$$\mathcal{W} \sim (1,3)_0 \longmapsto c^{(1,2,3)}_{H^4D^4} \sim (1,1,-2)\,.$$

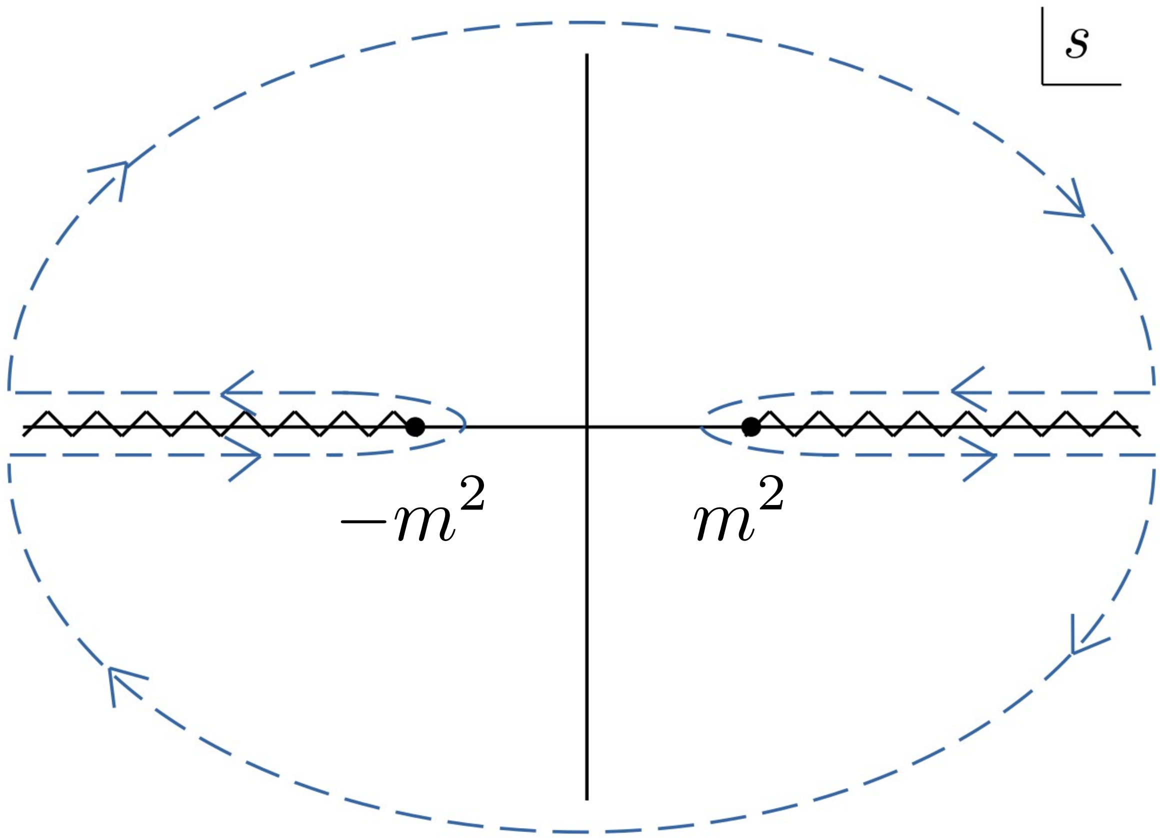
$$\frac{\mathcal{A}(s)}{s^3} \to 0$$



$$\int \frac{\mathcal{A}(s)}{s^3} = 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)]$$

$$= 2\int_{m^2}^{\infty} \frac{1}{s^3} \lim_{\epsilon\to 0}[\mathcal{A}(s+i\epsilon) - \mathcal{A}(s+i\epsilon)^*] = 2i\int_{m^2}^{\infty} \frac{\sigma(s)}{s^2}$$

$$\frac{\mathcal{A}(s)}{s^3} \to 0$$

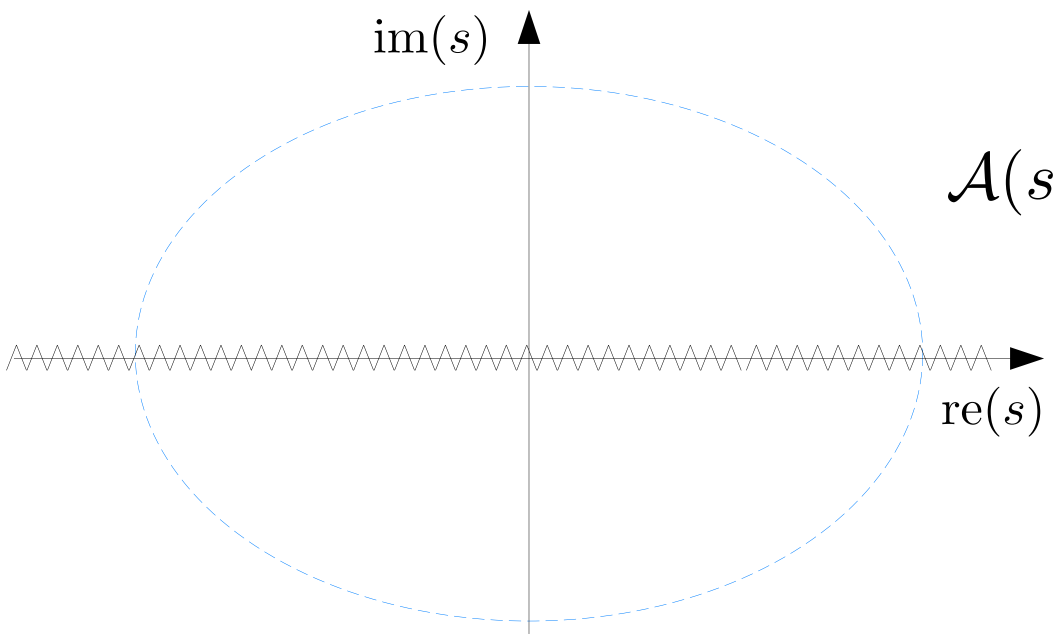


$$2i \int_{m^2}^{\infty} \frac{\sigma(s)}{s^2} = 2\pi i \,\mathrm{Res}[\frac{\mathcal{A}(s)}{s^3}, s = 0] = 2\pi i a_2$$

$$\Rightarrow a_2 \geq 0$$

Adams et al '06

Positivity with massless loops



$$\mathcal{A}(s) \sim (a_0 + a_2 s^2 + \cdots) \log \frac{\Lambda^2}{s}$$

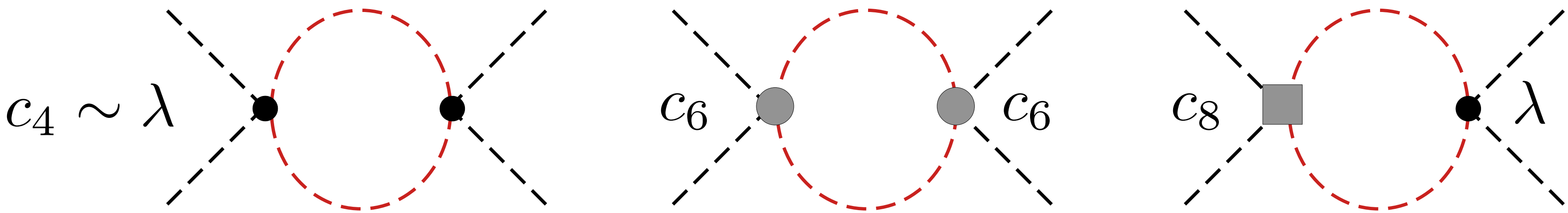
Bellazzini '16

We can deform it by adding one mass m to regulate the IR. Caveats with spin 1, 2: new degrees of freedom, forward-limit singularities...

It can be tricky even for spin-0: $$\log \frac{\Lambda^2}{s} \to \log \frac{\Lambda^2}{s + m^2}$$

Arkani-Hamed et al '16

Consider an UV completion that, at tree level, generates only **black-red** operators, so **black-black** scattering occurs first at one loop

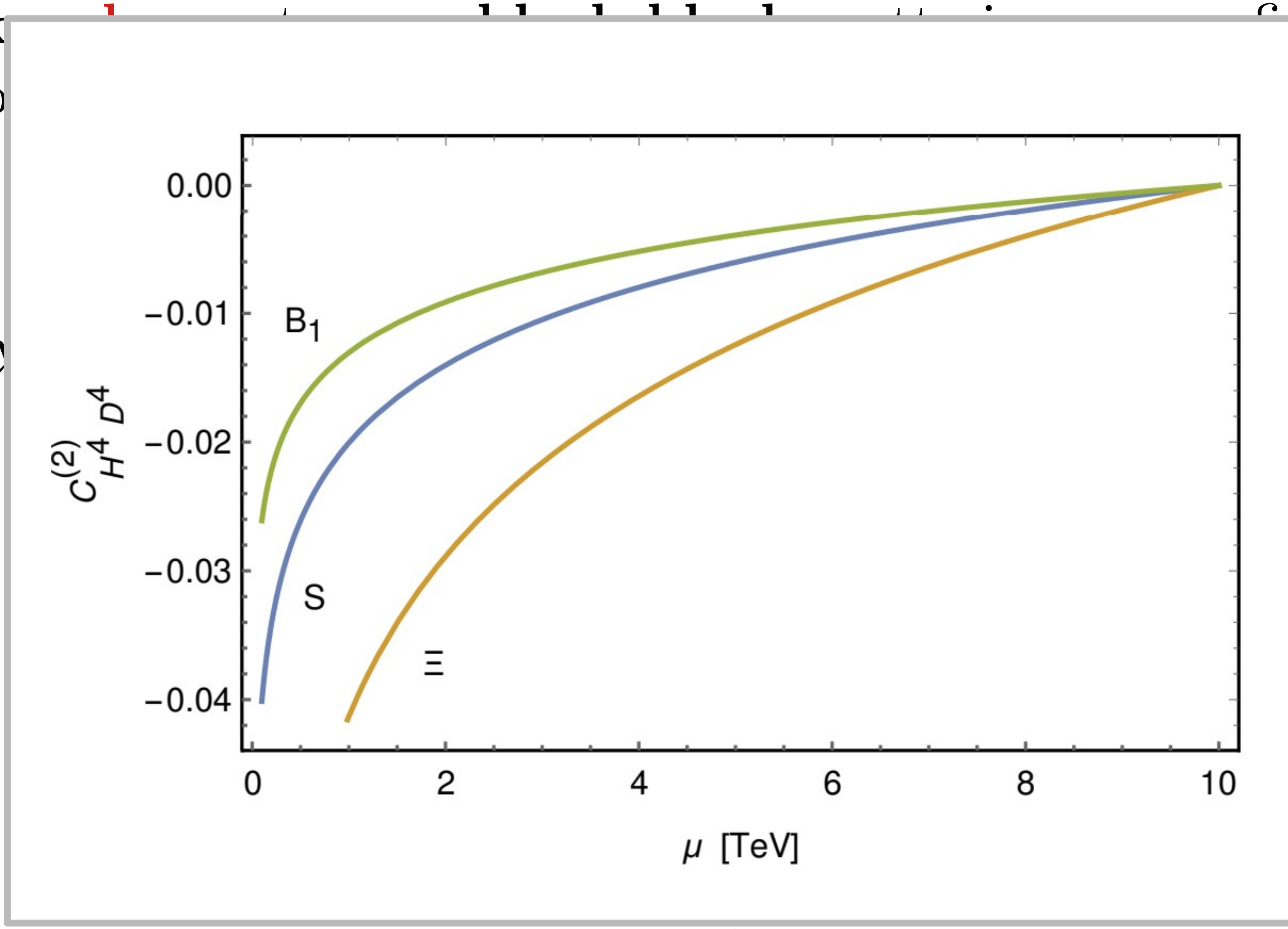


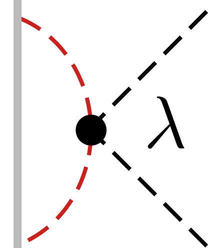
$$\mathcal{A}(s) \sim \left[\frac{\lambda^2}{2m^4} + \underbrace{(c_6^2 + \lambda c_8)}\log\frac{\Lambda^2}{m^2}\right]\frac{s^2}{\Lambda^4}$$

running of dimension-8
black-black operator

Consider an UV completion that, at tree level, generates only
black ... st at
one lo ...

$c_4 \sim \lambda$ ...





$\mathcal{A}($ ... $\dfrac{s^2}{\Lambda^4}$

running of dimension-8
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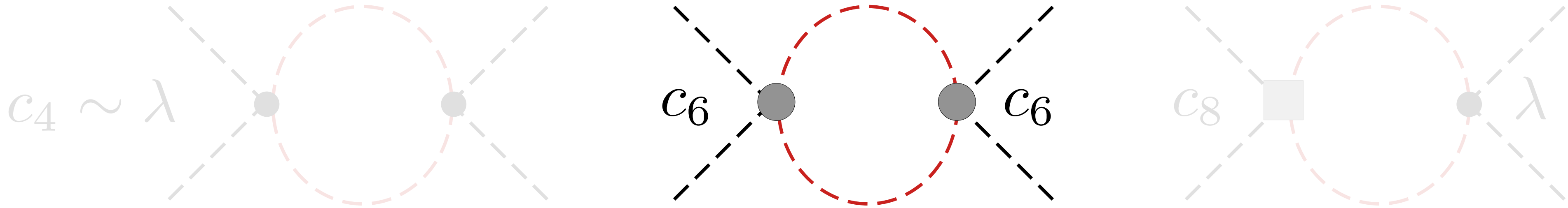
Consider an UV completion that, at tree level, generates only **black-red** operators, so **black-black** scattering occurs first at one loop



$$\mathcal{A}(s) \sim \left[\frac{\lambda^2}{2m^4} + \underbrace{\left(c_6^2 + \lambda c_8\right)}_{\text{running of dimension-8 black-black operator}} \log\frac{\Lambda^2}{m^2}\right]\frac{s^2}{\Lambda^4}$$

running of dimension-8
black-black operator

Consider an UV completion that, at tree level, generates only **black**-**red** operators, so **black-black** scattering occurs first at

$$16\pi^2\beta^{(1)}_{H^4D^4} = \frac{8}{3}\Big[-2(c^{(1)}_{H^4D^2})^2 - \frac{11}{8}(c^{(2)}_{H^4D^2})^2 + 4c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2}$$

$$\underline{+3c^2_{Hd}}\ \underline{\underline{+c^2_{He}}}\ \underline{\underline{\underline{+2(c^{(1)}_{Hl})^2}}} - 2(c^{(3)}_{Hl})^2 \underline{\underline{\underline{\underline{+6(c^{(1)}_{Hq})^2}}}} - 6(c^{(3)}_{Hq})^2 + \underline{\underline{\underline{\underline{3c^2_{Hu}}}}} - 3c^2_{Hud}\Big],$$

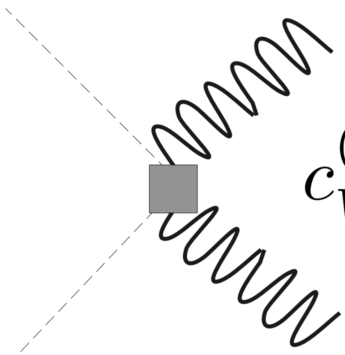
$$16\pi^2\beta^{(2)}_{H^4D^4} = \frac{8}{3}\Big[-2(c^{(1)}_{H^4D^2})^2 - \frac{5}{8}(c^{(2)}_{H^4D^2})^2 - 2c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2}$$

$$\underline{-3c^2_{Hd}}\ \underline{\underline{-c^2_{He}}}\ \underline{\underline{\underline{-2(c^{(1)}_{Hl})^2}}} - 2(c^{(3)}_{Hl})^2 \underline{\underline{\underline{\underline{-6(c^{(1)}_{Hq})^2}}}} - 6(c^{(3)}_{Hq})^2 \underline{\underline{\underline{\underline{-3c^2_{Hu}}}}}\Big],$$

$$16\pi^2\beta^{(3)}_{H^4D^4} = \frac{8}{3}\Big[-5(c^{(1)}_{H^4D^2})^2 + \frac{7}{8}(c^{(2)}_{H^4D^2})^2 - 2c^{(1)}_{H^4D^2}c^{(2)}_{H^4D^2} + 4(c^{(3)}_{Hl})^2 + 12(c^{(3)}_{Hq})^2 + 3c^2_{Hud}\Big]$$

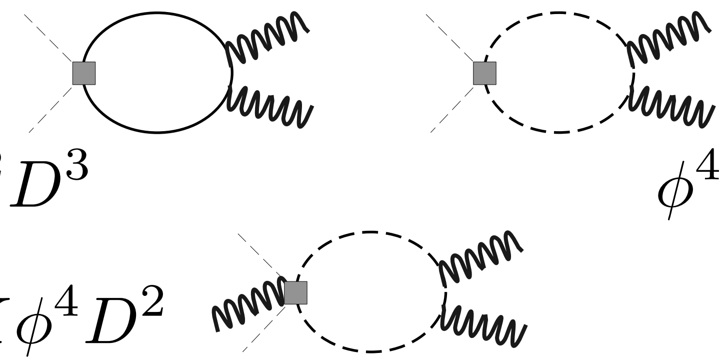
running of dimension-8
black-black operator

Example of RGE preserving positivity



$c^{(1)}_{W^2\phi^2 D^2} < 0$

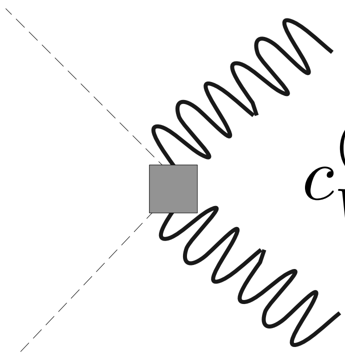
a priori, it can be renormalised
by several classes of operators



$\psi^2\phi^2 D^3$ $\phi^4 D^4$ $X\phi^4 D^2$

$$c^{(1)}_{W^2\phi^2 D^2}(\tilde{\mu}) = c^{(1)}_{W^2\phi^2 D^2}(\Lambda) - \frac{1}{16\pi^2}\dot{c}^{(1)}_{W^2\phi^2 D^2}(\Lambda)\log\frac{\Lambda}{\tilde{\mu}} < 0$$

Example of RGE preserving positivity



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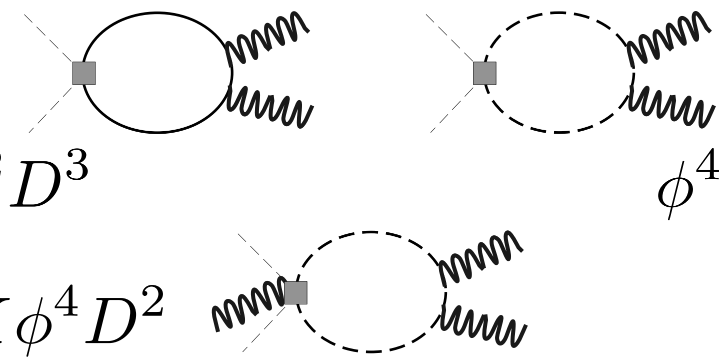
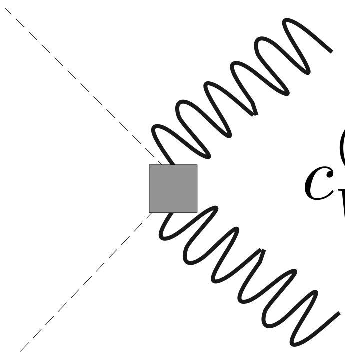


$\psi^2\phi^2D^3$ ϕ^4D^4 $X\phi^4D^2$

$$c^{(1)}_{W^2\phi^2D^2}(\tilde{\mu}) = c^{(1)}_{W^2\phi^2D^2}(\Lambda) - \frac{1}{16\pi^2}\dot{c}^{(1)}_{W^2\phi^2D^2}(\Lambda)\log\frac{\Lambda}{\tilde{\mu}} < 0$$

$$\dot{c}^{(1)}_{W^2\phi^2D^2} \left\{ + \frac{1}{6}g_2^2\underbrace{(2c^{(1)}_{\phi^4} + 3c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4})}_{\underbrace{c^{(2)}_{\phi^4}}_{\geq 0} + \underbrace{[c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4}]}_{\geq 0} + \underbrace{[c^{(1)}_{\phi^4} + c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}]}_{\geq 0}} \right.$$

Example of RGE preserving positivity



$c^{(1)}_{W^2\phi^2D^2} < 0$

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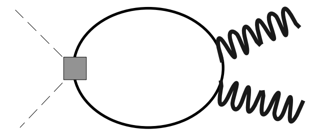
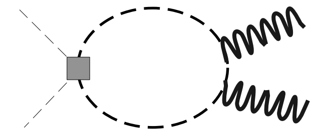
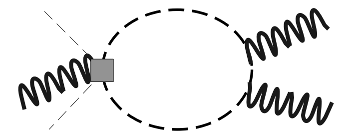


$\psi^2\phi^2D^3$



ϕ^4D^4

$X\phi^4D^2$



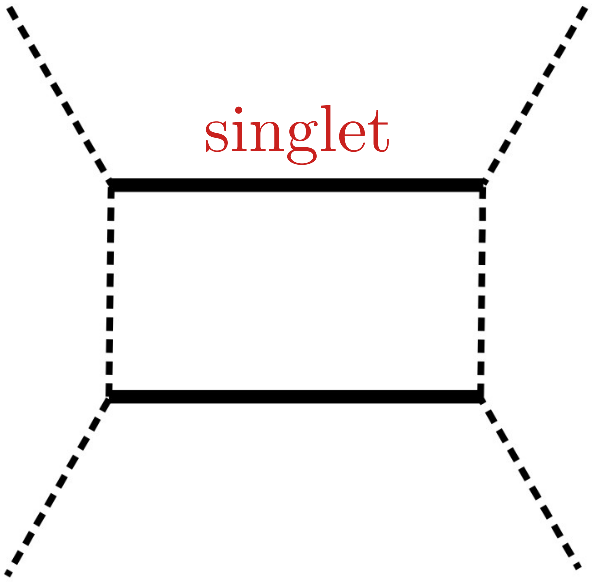
$$c^{(1)}_{W^2\phi^2D^2}(\tilde{\mu}) = \underline{c^{(1)}_{W^2\phi^2D^2}(\Lambda)} - \frac{1}{16\pi^2}\dot{c}^{(1)}_{W^2\phi^2D^2}(\Lambda)\log\frac{\Lambda}{\tilde{\mu}} < 0$$

$$\dot{c}^{(1)}_{W^2\phi^2D^2} \left\{ \begin{array}{l} +\frac{1}{6}g_2^2(2c^{(1)}_{\phi^4} + 3c^{(2)}_{\phi^4} + c^{(3)}_{\phi^4}) \\ -\frac{8}{3}g_2^2\left[c^{(1)}_{l^2\phi^2D^3} + c^{(2)}_{l^2\phi^2D^3} + 3(c^{(1)}_{q^2\phi^2D^3} + c^{(2)}_{q^2\phi^2D^3})\right] \end{array} \right.$$

Positivity breaking in matching

$$\mathcal{L}_{\mathcal{S}} = \kappa_{\mathcal{S}} \mathcal{S} H^\dagger H$$

$$c_{H^4 D^4}^{(1)\,\mathrm{tree}} = c_{H^4 D^4}^{(2)\,\mathrm{tree}} = 0\,, \qquad c_{H^4 D^4}^{(3)\,\mathrm{tree}} = 2\,\frac{\kappa_{\mathcal{S}}^2}{M^2}$$



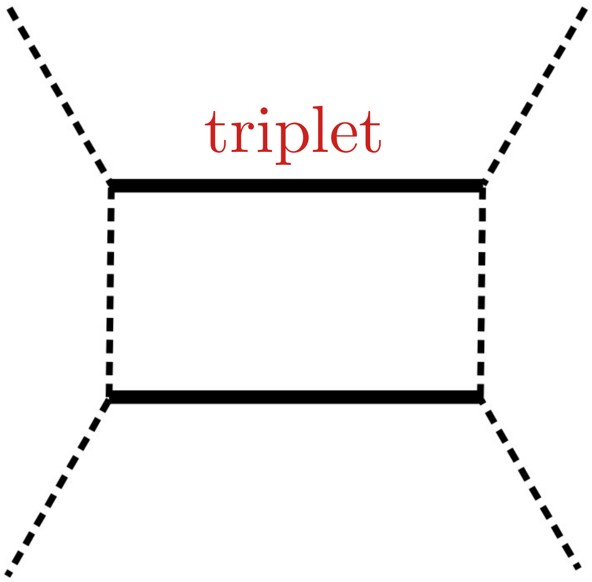
$$c_{H^4 D^4}^{(1)\,\mathrm{loop}} = -\frac{39}{144\pi^2}\frac{\kappa_{\mathcal{S}}^4}{M^4}\,,$$

$$c_{H^4 D^4}^{(2)\,\mathrm{loop}} = -\frac{39}{144\pi^2}\frac{\kappa_{\mathcal{S}}^4}{M^4}\,,$$

$$c_{H^4 D^4}^{(3)\,\mathrm{loop}} = -\frac{187}{720\pi^2}\frac{\kappa_{\mathcal{S}}^4}{M^4}\,.$$

Positivity breaking in matching

$$\mathcal{L}_{\Xi} = \kappa_{\Xi} H^{\dagger} \Xi^{I} \sigma^{I} H$$



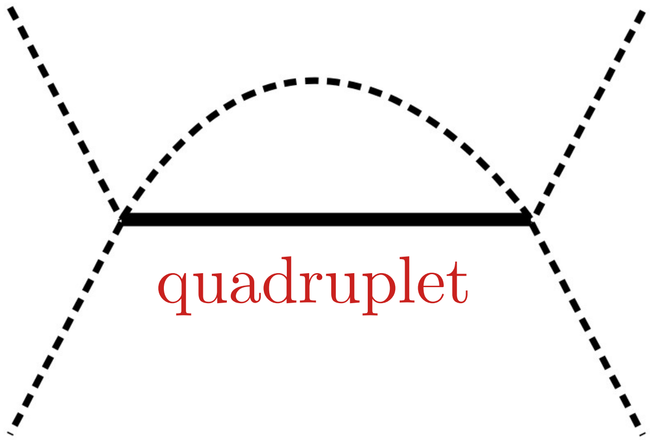
$$c^{(1)}_{H^4 D^4} = 4\frac{\kappa_{\Xi}^2}{M^2} - \frac{107}{144\pi^2}\frac{\kappa_{\Xi}^4}{M^4}$$

$$c^{(2)}_{H^4 D^4} = -\frac{61}{144\pi^2}\frac{\kappa_{\Xi}^4}{M^4},$$

$$c^{(3)}_{H^4 D^4} = -2\frac{\kappa_{\Xi}^2}{M^2} - \frac{271}{720\pi^2}\frac{\kappa_{\Xi}^4}{M^4}$$

Positivity breaking in matching

Scalar quadruplets



$Y = \frac{1}{2}$

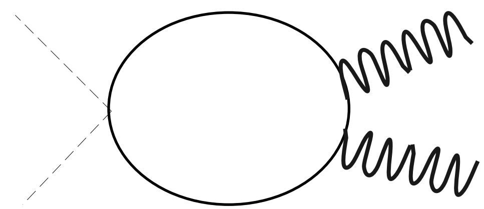
$$c^{(1)}_{H^4D^4} = \frac{|\lambda_{\Theta_1}|^2}{9\pi^2}\,,$$
$$c^{(2)}_{H^4D^4} = \frac{|\lambda_{\Theta_1}|^2}{36\pi^2}\,,$$
$$c^{(3)}_{H^4D^4} = -\frac{|\lambda_{\Theta_1}|^2}{18\pi^2}\,;$$

$Y = \frac{3}{2}$

$$c^{(1)}_{H^4D^4} = 0\,,$$
$$c^{(2)}_{H^4D^4} = \frac{|\lambda_{\Theta_3}|^2}{4\pi^2}$$
$$c^{(3)}_{H^4D^4} = 0\,;$$

First reflection: phenomenological relevance

Not clear to me that we are more sensitive to the tree-level operators, e.g.:



Zhang '21

In this section we consider a concrete example. We consider a vector-like SU(2) singlet fermion F with hypercharge $-\frac{1}{3}$, which interacts with the SM left-handed quark doublet q and the Higgs boson H:

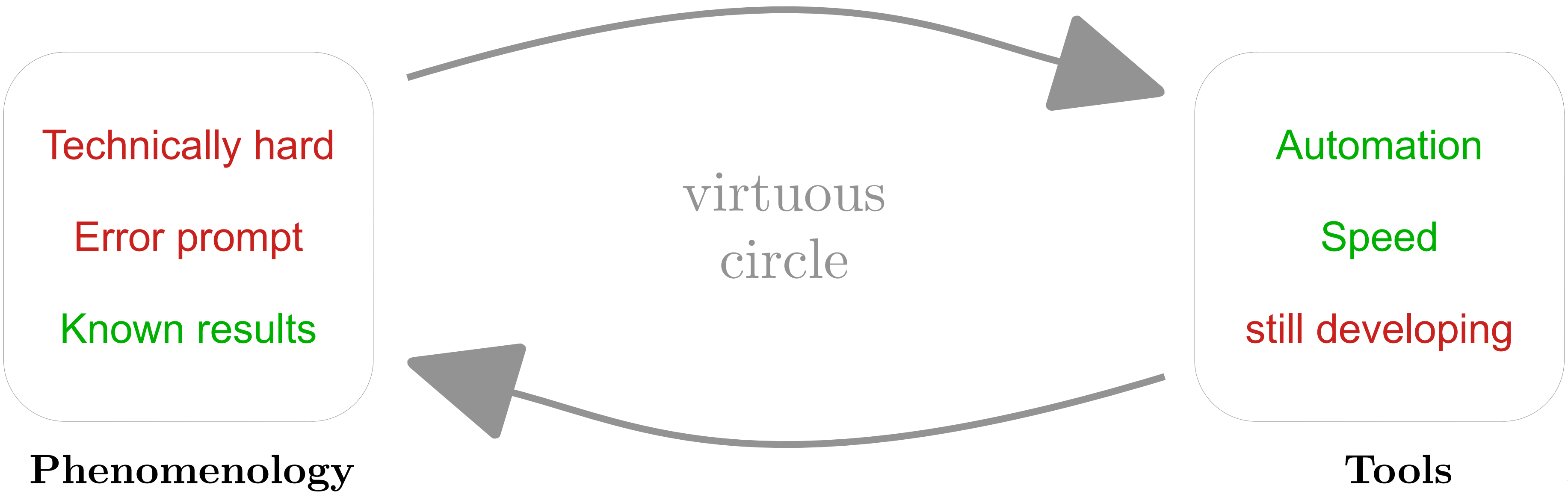
$$\mathcal{L} = y(\bar{q}H)F + h.c. \tag{6.29}$$

We see that the heavy F exchange generates an $HH^\dagger q\bar{q}$ amplitude already at the tree level, and therefore in practice its more realistic to study the $HH^\dagger q\bar{q}$ operators rather than the 4-Higgs operators that are loop-induced. This is in general true when mixed loops are present. The discussion in this section is therefore mostly for the completeness of the picture.

Second reflection: the role of tools

Not an easy task: one-loop matching/running to dimension 8

Fantastic phenomenological/theoretical scenario for **application** and **testing** of tools



Lagrangian in the UV

FeynRules
QGRAF
FORM

user input ☞ $$L_{\text{Green}} = \sum_i c_i^G \mathcal{O}_i^G$$

see Renato's talk

user input

see Matthias's talk

user input ☞ $$L_{\text{Physical}} = \sum_j c_j^P \mathcal{O}_j^P$$

	Operator	Notation	Operator	Notation
ϕ^8	$(\phi^\dagger\phi)^4$	$\mathcal{O}_{\phi^8}$		
$\phi^6 D^2$	$(\phi^\dagger\phi)^2(D_\mu\phi^\dagger D^\mu\phi)$	$\mathcal{O}_{\phi^6}^{(1)}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)(D_\mu\phi^\dagger\sigma^I D^\mu\phi)$	$\mathcal{O}_{\phi^6}^{(2)}$
	$(\phi^\dagger\phi)^2(\phi^\dagger D^2\phi+\text{h.c.})$	$\mathcal{O}_{\phi^6}^{(3)}$	$(\phi^\dagger\phi)^2 D_\mu(\phi^\dagger \mathrm{i}\overleftrightarrow{D}^\mu\phi)$	$\mathcal{O}_{\phi^6}^{(4)}$
$\phi^4 D^4$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi)$	$\mathcal{O}_{\phi^4}^{(1)}$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi)$	$\mathcal{O}_{\phi^4}^{(2)}$
	$(D^\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D_\nu\phi)$	$\mathcal{O}_{\phi^4}^{(3)}$	$D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger D^2\phi+\text{h.c.})$	$\mathcal{O}_{\phi^4}^{(4)}$
	$D_\mu\phi^\dagger D^\mu\phi(\phi^\dagger \mathrm{i}D^2\phi+\text{h.c.})$	$\mathcal{O}_{\phi^4}^{(5)}$	$(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger D_\mu\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(6)}$
	$(D_\mu\phi^\dagger\phi)(D^2\phi^\dagger \mathrm{i}D_\mu\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(7)}$	$(D^2\phi^\dagger\phi)(D^2\phi^\dagger\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(8)}$
	$(D^2\phi^\dagger\phi)(\mathrm{i}D^2\phi^\dagger\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(9)}$	$(D^2\phi^\dagger D^2\phi)(\phi^\dagger\phi)$	$\mathcal{O}_{\phi^4}^{(10)}$
	$(\phi^\dagger D^2\phi)(D^2\phi^\dagger\phi)$	$\mathcal{O}_{\phi^4}^{(11)}$	$(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger D^2\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(12)}$
	$(D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger \mathrm{i}D^2\phi)+\text{h.c.}$	$\mathcal{O}_{\phi^4}^{(13)}$		
$X^3\phi^2$	$f^{ABC}(\phi^\dagger\phi)G_\mu^{A,\nu}G_\nu^{B,\rho}G_\rho^{C,\mu}$	$\mathcal{O}_{G^3\phi^2}^{(1)}$	$f^{ABC}(\phi^\dagger\phi)G_\mu^{A,\nu}G_\nu^{B,\rho}\tilde{G}_\rho^{C,\mu}$	$\mathcal{O}_{G^3\phi^2}^{(1)}$
	$\epsilon^{IJK}(\phi^\dagger\phi)W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{W^3\phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger\phi)W_\mu^{I\nu}W_\nu^{J\rho}\widetilde{W}_\rho^{K\mu}$	$\mathcal{O}_{W^3\phi^2}^{(2)}$
	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)B_\mu^{\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{W^2B\phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(\widetilde{B}^{\mu\nu}W_{\nu\rho}^{J}W_\mu^{K\rho}+B^{\mu\nu}W_{\nu\rho}^{J}\widetilde{W}_\mu^{K\rho})$	$\mathcal{O}_{W^2B\phi^2}^{(2)}$
$X^2\phi^4$	$(\phi^\dagger\phi)^2 G_{\mu\nu}^A G^{A\mu\nu}$	$O_{G^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$O_{G^2\phi^4}^{(2)}$
	$(\phi^\dagger\phi)^2 W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(2)}$
	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)W_{\mu\nu}^I W^{J\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(3)}$	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)\widetilde{W}_{\mu\nu}^I W^{J\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(4)}$
	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{WB\phi^4}^{(1)}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{WB\phi^4}^{(2)}$
	$(\phi^\dagger\phi)^2 B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{B^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2\widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{B^2\phi^4}^{(2)}$

$X\phi^2D^4$	$i(D_\nu\phi^\dagger\sigma^I D^2\phi - D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(1)}_{W\phi^2D^4}$	$(D_\nu\phi^\dagger\sigma^I D^2\phi + D^2\phi^\dagger\sigma^I D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(2)}_{W\phi^2D^4}$
	$i(D_\rho D_\nu\phi^\dagger\sigma^I D^\rho\phi - D^\rho\phi^\dagger\sigma^I D_\rho D_\nu\phi)D_\mu W^{I\mu\nu}$	$\mathcal{O}^{(3)}_{W\phi^2D^4}$		
	$i(D_\nu\phi^\dagger D^2\phi - D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(1)}_{B\phi^2D^4}$	$(D_\nu\phi^\dagger D^2\phi + D^2\phi^\dagger D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(2)}_{B\phi^2D^4}$
	$i(D_\rho D_\nu\phi^\dagger D^\rho\phi - D^\rho\phi^\dagger D_\rho D_\nu\phi)D_\mu B^{\mu\nu}$	$\mathcal{O}^{(3)}_{B\phi^2D^4}$		
$X\phi^4D^2$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{W\phi^4D^2}$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)\widetilde{W}^I_{\mu\nu}$	$\mathcal{O}^{(2)}_{W\phi^4D^2}$
	$\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)W^K_{\mu\nu}$	$\mathcal{O}^{(3)}_{W\phi^4D^2}$	$\mathrm{i}\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)\widetilde{W}^K_{\mu\nu}$	$\mathcal{O}^{(4)}_{W\phi^4D^2}$
	$(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger\sigma^I\phi + \mathrm{h.c.})$	$\mathcal{O}^{(5)}_{W\phi^4D^2}$	$(\phi^\dagger\phi)D_\nu W^{I\mu\nu}(D_\mu\phi^\dagger\mathrm{i}\sigma^I\phi + \mathrm{h.c.})$	$\mathcal{O}^{(6)}_{W\phi^4D^2}$
	$\epsilon^{IJK}(D_\mu\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J D_\nu\phi)W^{K\mu\nu}$	$\mathcal{O}^{(7)}_{W\phi^4D^2}$	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\nu}$	$\mathcal{O}^{(1)}_{B\phi^4D^2}$
	$\mathrm{i}(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)\widetilde{B}_{\mu\nu}$	$\mathcal{O}^{(2)}_{B\phi^4D^2}$	$(\phi^\dagger\phi)D_\nu B^{\mu\nu}(D_\mu\phi^\dagger\mathrm{i}\phi + \mathrm{h.c.})$	$\mathcal{O}^{(3)}_{B\phi^4D^2}$
ϕ^2D^6	$D^2\phi^\dagger D_\mu D_\nu D^\mu D^\nu\phi$	$\mathcal{O}_{\phi^2}$		

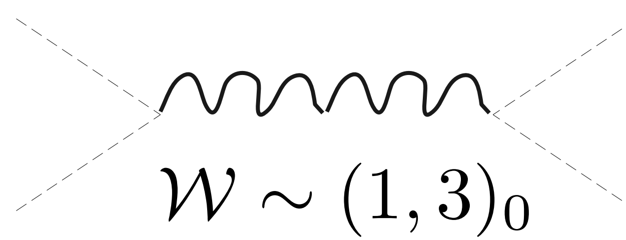
$$c^{(1)}_{\phi^4} \to c^{(1)}_{\phi^4} + c_{B^2D^4}g_1^2 - c^{(3)}_{B\phi^2D^4}g_1 - c_{W^2D^4}g_2^2 + c^{(3)}_{W\phi^2D^4}g_2\,,$$
$$c^{(2)}_{\phi^4} \to c^{(2)}_{\phi^4} - c_{B^2D^4}g_1^2 + c^{(3)}_{B\phi^2D^4}g_1 - c_{W^2D^4}g_2^2 + c^{(3)}_{W\phi^2D^4}g_2\,,$$
$$c^{(3)}_{\phi^4} \to c^{(3)}_{\phi^4} + 2c_{W^2D^4}g_2^2 - 2c^{(3)}_{W\phi^2D^4}g_2\,,$$

Cross-checks with other tools

Criado '17

Tree-level results with MatchingTools

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EFT}}^{(8)} = \frac{(g_{\mathcal{W}}^{\phi})^2}{m_{\mathcal{W}}^4}\Big[& 2(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + 4(D_\nu\phi^\dagger D^\nu D^\mu\phi)(D_\mu\phi^\dagger\phi) - 2(D_\mu\phi^\dagger D_\nu\phi)(\phi^\dagger D^\mu D^\nu\phi) \\
& - 4(D_\mu\phi^\dagger\phi)(D^\mu D_\nu\phi^\dagger D^\nu\phi) + 2(D_\mu\phi^\dagger D_\nu\phi)(D^\mu D^\nu\phi^\dagger\phi) - 4(D_\mu\phi^\dagger D^\mu\phi)(D_\nu\phi^\dagger D^\nu\phi) \\
& + 2(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + \frac{1}{2}(\phi^\dagger D_\mu D_\nu\phi)(\phi^\dagger D^\mu D^\nu\phi) - 2(D_\nu D_\rho\phi^\dagger D^\nu D^\rho\phi)(\phi^\dagger\phi) \\
& + (D_\mu D_\nu\phi^\dagger\phi)(\phi^\dagger D^\mu D^\nu\phi) - 4(\phi^\dagger D_\rho\phi)(D_\nu\phi^\dagger D^\rho D^\nu\phi) + 2(\phi^\dagger D_\nu D_\mu\phi)(D^\mu\phi^\dagger D^\nu\phi) \\
& + \frac{1}{2}(D_\mu D_\nu\phi^\dagger\phi)(D^\mu D^\nu\phi^\dagger\phi) + 4(D_\rho D_\nu\phi^\dagger D^\rho\phi)(D^\nu\phi^\dagger\phi) - 2(D_\nu D_\mu\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi) \\
& - \frac{1}{2}(\phi^\dagger D_\nu D_\mu\phi)(\phi^\dagger D^\mu D^\nu\phi) + 2(D_\rho D_\nu\phi^\dagger D^\nu D^\rho\phi)(\phi^\dagger\phi) - (D^\nu D^\mu\phi^\dagger\phi)(\phi^\dagger D_\mu D_\nu\phi) \\
& - \frac{1}{2}(D_\nu D_\mu\phi^\dagger\phi)(D^\mu D^\nu\phi^\dagger\phi)\Big] .
\end{aligned}
$$



Cross-checks with other tools

Criado '17

Tree-level results with MatchingTools

- Export the Lagrangian to FeynRules
- Import in FeynArts + FormCalc
- Compute amplitudes off-shell to reduce to Green's basis, from where previous results can be used

$$\mathcal{L}_{\text{EFT}}^{(8)} = \frac{(g_{\mathcal{W}}^{\phi})^2}{m_{\mathcal{W}}^4}\Big[2\mathcal{O}_{\phi^4}^{(1)} + 2\mathcal{O}_{\phi^4}^{(2)} - 4\mathcal{O}_{\phi^4}^{(3)} - \frac{1}{4}g_2^2\mathcal{O}_{W^2\phi^4}^{(1)} + \frac{1}{2}g_1 g_2\mathcal{O}_{WB\phi^4}^{(1)}$$
$$+ \frac{3}{4}g_1^2\mathcal{O}_{B^2\phi^4}^{(1)} - 2g_2\mathcal{O}_{W\phi^4D^2}^{(1)} + 6g_1\mathcal{O}_{B\phi^4D^2}^{(1)} + 2g_1\mathcal{O}_{B\phi^4D^2}^{(3)}\Big]$$

Cross-checks with other tools

Fuentes-Martin et al '20

Loop-level results with SuperTracer

After (huge) simplification:

```
rita = SuperSimplify[(rete /. CovD[a_, G[b___], c___] → 0) // Tr] /. Plus → List
```

$$\Big\{-\frac{13}{3}\,\text{alpha}^4\bar{H}^a H^b D_\mu D_\nu H^a D_\nu D_\mu \bar{H}^b,\ -\frac{97}{45}\,\text{alpha}^4\bar{H}^a H^a D_\mu D_\nu H^b D_\nu D_\mu \bar{H}^b,\ -\frac{13}{3}\,\text{alpha}^4\bar{H}^a \bar{H}^b D_\mu D_\nu H^b D_\nu D_\mu H^a,\ \frac{7}{4}\,\text{alpha}^4 H^a H^b D^2\bar{H}^b D^2\bar{H}^a,\ \frac{4}{3}\,\text{alpha}^4\bar{H}^a H^b D^2 H^a D^2\bar{H}^b,$$
$$-\frac{221}{45}\,\text{alpha}^4\bar{H}^a H^a D^2 H^b D^2\bar{H}^b,\ -\frac{5}{12}\,\text{alpha}^4\bar{H}^a \bar{H}^b D^2 H^b D^2 H^a,\ -\frac{13}{3}\,\text{alpha}^4\bar{H}^a H^b D_\mu H^a D_\mu D^2\bar{H}^b,\ -\frac{233}{30}\,\text{alpha}^4\bar{H}^a H^a D_\mu H^b D_\mu D^2\bar{H}^b,\ -\frac{23}{12}\,\text{alpha}^4\bar{H}^a \bar{H}^b D_\mu H^b D_\mu D^2 H^a,$$
$$-\frac{103}{30}\,\text{alpha}^4\bar{H}^a H^a D_\mu\bar{H}^b D_\mu D^2 H^b,\ -\frac{29}{12}\,\text{alpha}^4\bar{H}^a \bar{H}^b D_\mu H^a D_\mu D^2 H^b,\ -\frac{319}{60}\,\text{alpha}^4\bar{H}^a H^a H^b D^2 D^2\bar{H}^b,\ -\frac{37}{30}\,\text{alpha}^4\bar{H}^a \bar{H}^b H^b D^2 D^2 H^a,\ -\frac{23}{12}\,\text{alpha}^4\bar{H}^a \bar{H}^b H^a D^2 D^2 H^b\Big\}$$

$$c^{(1)}_{H^4D^4} = c^{(2)}_{H^4D^4} = -\frac{13}{48\pi^2}\alpha^4$$

It matches matchmakereft!

Automatising operator reduction?

Current results must be cross-checked.

Current results only for dimension-8 **bosonic** operators

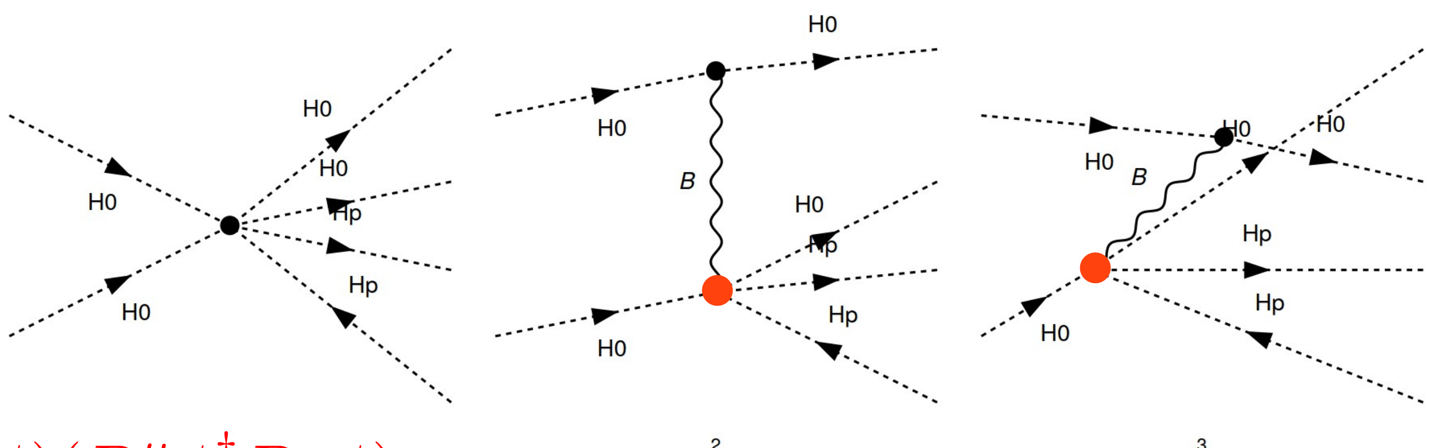
Eventually, going beyond the SMEFT

Eventually, going beyond dimension-8 (maybe for formal aspects)

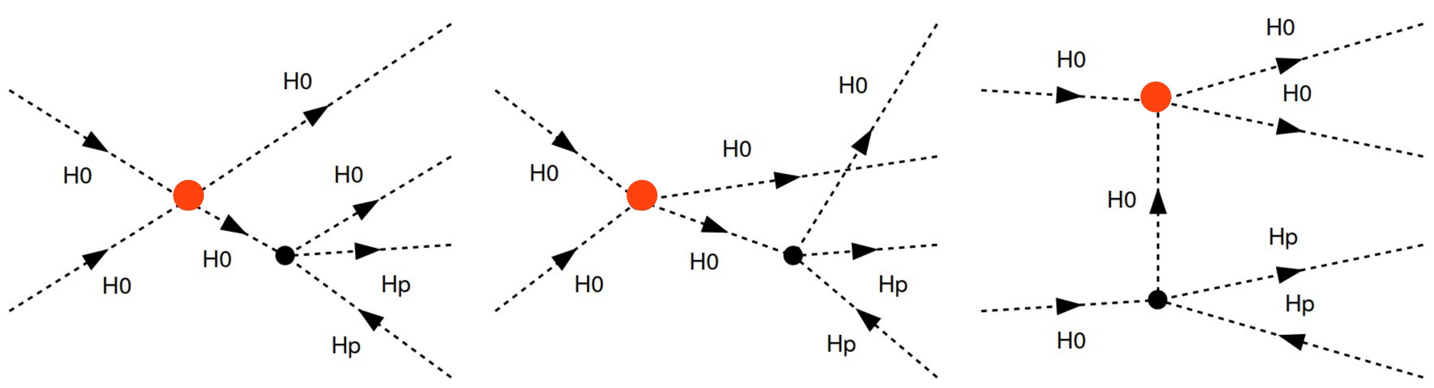
...

Strategy

Diagrammatically: Match redundant Lagrangian onto physical Lagrangian at **tree level** and **on-shell**



e.g. $(\phi^\dagger \phi)(D^\mu \phi^\dagger D_\mu \phi)$



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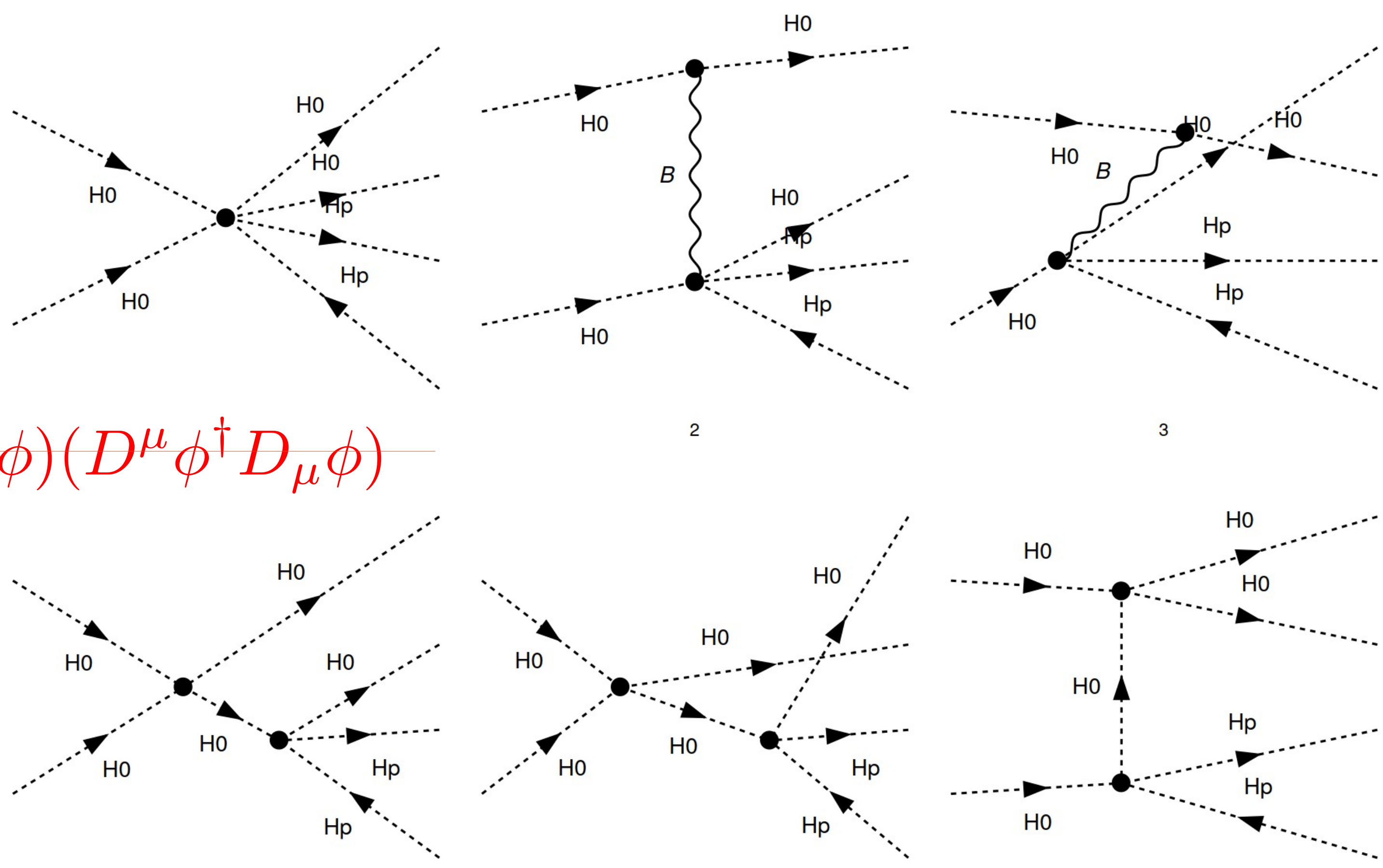

e.g. $(\phi^\dagger\phi)(D^\mu\phi^\dagger D_\mu\phi)$

$$\mathcal{M}_{\text{phys+red}} = \mathcal{M}_{\text{phys'}}$$

Addressing the matching symbolically is hopeless...

```
final /. {Den[x_, y_] → 1 / (x - y)} // Expand
```

```
12 i aHdim6 - (i g1^2 rHDpdim6 Pair[k[1],k[2]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[2]])/(8 Pair[k[1],k[4]]) - (i g1^2 rHDpdim6 Pair[k[1],k[3]])/(8 Pair[k[1],k[4]]) -
 (i g1^2 rHDpdim6 Pair[k[1],k[4]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[5]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[5]])/(8 Pair[k[1],k[4]]) +
 (i g1^2 rHDpdim6 Pair[k[1],k[6]])/(8 Pair[k[1],k[3]]) + ··· 1142 ··· + (i g1^2 rHDpdim6 Pair[k[3],k[4]]×Pair[k[4],k[6]])/(2 Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^4 Pair[k[3],k[4]]×Pair[k[4],k[6]])/(8 Pair[k[2],k[3]]×Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^4 Pair[k[3],k[4]]×Pair[k[4],k[6]])/(8 Pair[k[2],k[4]]×Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (4 i lmbd rHDpdim6 Pair[k[5],k[6]])/(-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]]) - (i g1^2 rHDpdim6 Pair[k[2],k[3]]×Pair[k[5],k[6]])/(2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[2],k[4]]×Pair[k[5],k[6]])/(2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[3],k[4]]×Pair[k[5],k[6]])/(2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[3],k[4]]×Pair[k[5],k[6]])/(2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]]))
```

large output | show less | show more | show all | set size limit...

$\mathcal{M}_{\text{phys+red}} = \mathcal{M}_{\text{phys'}}$

Let's simply give numbers to the kinematic invariants

```
final /. {Den[x_, y_] → 1/(x - y)} // Expand
```

```
12 i aHdim6 - (i g1^2 rHDpdim6 Pair[k[1],k[2]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[2]])/(8 Pair[k[1],k[4]]) - (i g1^2 rHDpdim6 Pair[k[1],k[3]])/(8 Pair[k[1],k[4]]) -
 (i g1^2 rHDpdim6 Pair[k[1],k[4]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[5]])/(8 Pair[k[1],k[3]]) - (i g1^2 rHDpdim6 Pair[k[1],k[5]])/(8 Pair[k[1],k[4]]) +
 (i g1^2 rHDpdim6 Pair[k[1],k[6]])/(8 Pair[k[1],k[3]]) + … 1142 … + (i g1^2 rHDpdim6 Pair[k[3],k[4]] × Pair[k[4],k[6]])/(2 Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^4 Pair[k[3],k[4]] × Pair[k[4],k[6]])/(8 Pair[k[2],k[3]] × Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^4 Pair[k[3],k[4]] × Pair[k[4],k[6]])/(8 Pair[k[2],k[4]] × Pair[k[5],k[6]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (4 i lmbd rHDpdim6 Pair[k[5],k[6]])/(-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]]) - (i g1^2 rHDpdim6 Pair[k[2],k[3]] × Pair[k[5],k[6]])/(2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[2],k[4]] × Pair[k[5],k[6]])/(2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[3],k[4]] × Pair[k[5],k[6]])/(2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])) -
 (i g1^2 rHDpdim6 Pair[k[3],k[4]] × Pair[k[5],k[6]])/(2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]]))
```

large output | show less | show more | show all | set size limit...

$$\mathcal{M}_{\text{phys+red}} = \mathcal{M}_{\text{phys'}}$$

Let's simply give numbers to the kinematic invariants

fi

Warning! The assignment must be compatible with on-shellness: **physical configuration** of momenta

$$p_1 \to -(p_2 + \cdots + p_N)$$

$$p_1^2 = 0 \Rightarrow p_2 p_3 \to -(p_2 p_4 + p_2 p_5 + \cdots + \cdots p_{N-1} p_N)$$

```
2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])

            i g1^2 rHDpdim6 Pair[k[3],k[4]] Pair[k[5],k[6]]
  ─────────────────────────────────────────────────────────────────── -
2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])

            i g1^2 rHDpdim6 Pair[k[3],k[4]] Pair[k[5],k[6]]
  ───────────────────────────────────────────────────────────────────
2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])
```

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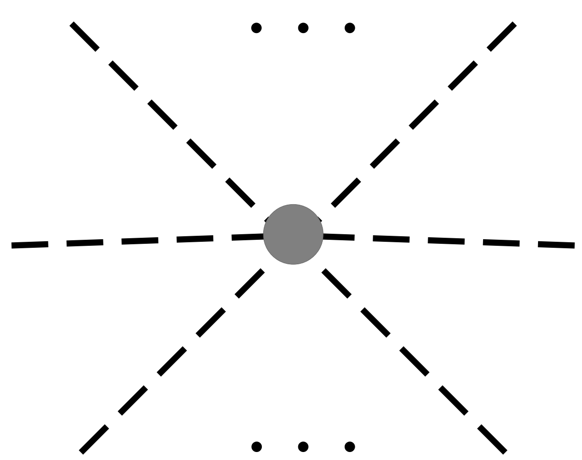
$$\mathcal{M}_{\text{phys+red}} = \mathcal{M}_{\text{phys'}}$$

Let's simply give numbers to the kinematic invariants

$n \to n$



	$6n-10$	previous $n(2n-3)$
$n=2$	2	2
$n=3$	8	9
$n=4$	14	20

Chiachian and Fischer '88

```
fi
2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])
              i g1^2 rHDpdim6 Pair[k[3],k[4]] × Pair[k[5],k[6]]
------------------------------------------------------------------------- -
2 Pair[k[2],k[3]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])
              i g1^2 rHDpdim6 Pair[k[3],k[4]] × Pair[k[5],k[6]]
-------------------------------------------------------------------------
2 Pair[k[2],k[4]] (-2 (Pair[k[1],k[5]]+Pair[k[1],k[6]])+2 Pair[k[5],k[6]])
```

large output | show less | show more | show all | set size limit...

A NEW MONTE CARLO TREATMENT OF MULTIPARTICLE PHASE SPACE AT HIGH ENERGIES

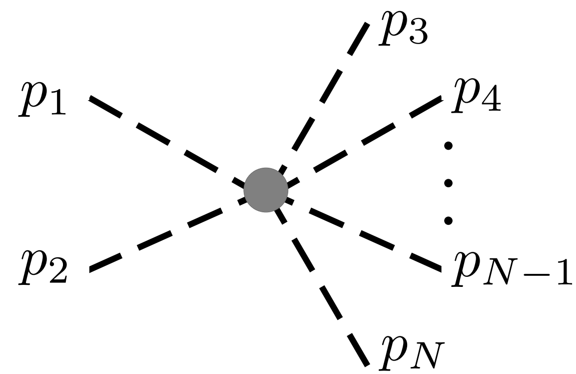
R. KLEISS, W.J. STIRLING

CERN, Geneva, Switzerland

and

S.D. ELLIS *

Dept. of Physics, University of Washington, Seattle, WA 98195, USA




```
  For[yy = 1, yy ≤ numP + 2, yy++,
   polarizations = AppendTo[polarizations, givePol[momenta[[yy]]]];
  ];
  rule = Table[Pair[k[i], k[j]] → sprod4D[momenta[[i]], momenta[[j]]] // Simplify, {i, 1, 2 + numP}, {j, 1, 2 + numP}];
  AppendTo[Rules, Flatten[rule]];
 ];
 Return[Rules];)
```

In[10]:= `momenta[[6]] // FullSimplify`

Out[10]=

$$\left\{\frac{1843}{4683}, -\frac{5014\sqrt{2}+\frac{358355}{\sqrt{9366}}}{9366+265\sqrt{4683}}, \frac{7150\sqrt{\frac{6}{1561}}+80\sqrt{2}}{9366+265\sqrt{4683}}, \frac{54833}{4683\sqrt{2}\left(265+2\sqrt{4683}\right)}\right\}$$

In[11]:= `createRules[4, 5]`

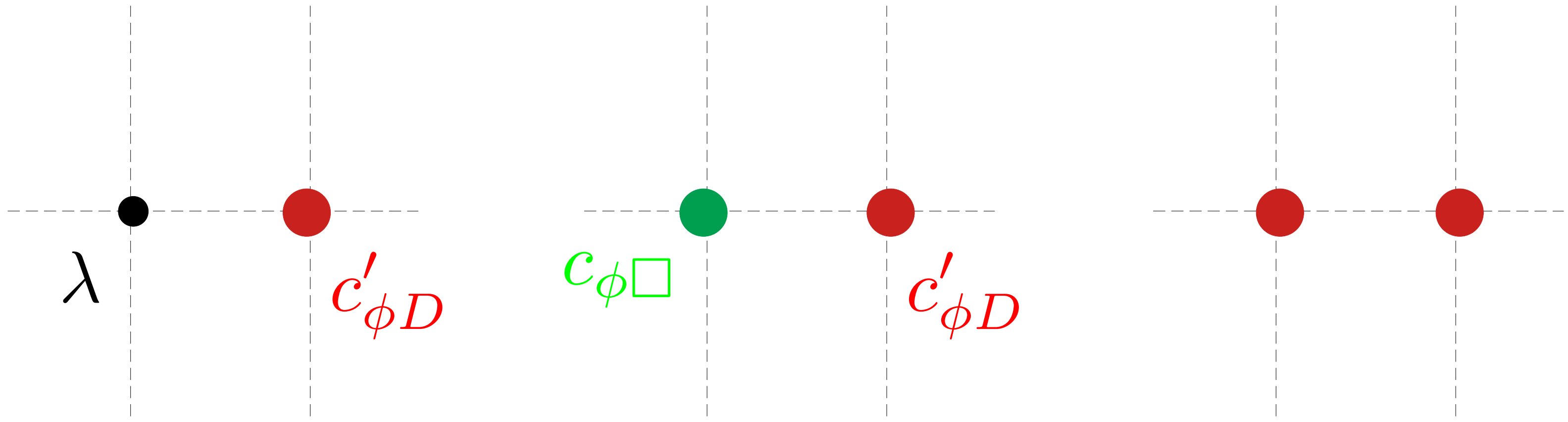
Out[11]=

$$\Big\{\Big\{\text{Pair}[k[1],k[1]]\to 0,\ \text{Pair}[k[1],k[2]]\to\frac{1}{2},\ \text{Pair}[k[1],k[3]]\to\frac{2168652+463898\sqrt{127}-106379\sqrt{254}}{36576\times\left(762+163\sqrt{127}\right)},$$

$$\text{Pair}[k[1],k[4]]\to\frac{6877812+1471238\sqrt{127}+346999\sqrt{254}}{36576\times\left(762+163\sqrt{127}\right)},\ \text{Pair}[k[1],k[5]]\to\frac{4463796+954854\sqrt{127}-203801\sqrt{254}}{36576\times\left(762+163\sqrt{127}\right)},$$

$$\text{Pair}[k[1],k[6]]\to\frac{47244+10106\sqrt{127}-4091\sqrt{254}}{4064\times\left(762+163\sqrt{127}\right)},\ \text{Pair}[k[2],k[1]]\to\frac{1}{2},\ \text{Pair}[k[2],k[2]]\to 0,$$

Example of application



Out[∘]=

$$\{\{\mathtt{lmbd} \to \mathtt{lmbdUV},\ \mathtt{aHDdim6} \to \mathtt{aHDdim6UV},\ \mathtt{aHDDdim6} \to \mathtt{aHDDdim6UV} + \frac{\mathtt{rHDpdim6}}{2},$$

$$\mathtt{aHdim6} \to \mathtt{aHdim6UV} + \mathtt{lmbdUV}\ \mathtt{rHDpdim6},\ \mathtt{aH41} \to \mathtt{aH41UV},\ \mathtt{aH42} \to \mathtt{aH42UV},$$

$$\mathtt{aH43} \to \mathtt{aH43UV},\ \mathtt{aH61} \to \mathtt{aH61UV} + \mathtt{lmbdUV}\ (2\ \mathtt{rH412} - 2\ \mathtt{rH44} - \mathtt{rH46}) -$$

$$4\ \mathtt{aHDDdim6UV}\ \mathtt{rHDpdim6} - \frac{\mathtt{aHDdim6UV}\ \mathtt{rHDpdim6}}{2} - \frac{7\ \mathtt{rHDpdim6}^2}{4} + \mathtt{rHDppdim6}^2,$$

$$\mathtt{aH62} \to \mathtt{aH62UV} + \mathtt{lmbdUV}\ (\mathtt{rH412} - \mathtt{rH46}) - \mathtt{aHDdim6UV}\ \mathtt{rHDpdim6}\}\}$$

Current status

Almost everything working in matchmakereft (including masses), modulo some issues with fermions; important cross-checks from FeynArts+FormCalc

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Almost everything working in matchmakereft (including masses), modulo some issues with fermions; important cross-checks from FeynArts+FormCalc

Conclusions

Positivity bounds are powerful constraints in the search for new physics, but they must be considered with care for loop operators.

Positivity studies benefit strongly from tools, which in turn are tested in a highly non-trivial context.

Two main obstacles for fully automatised matching/running: operators bases and redundancies.

We are addressing redundancies via tree-level on-shell matching within matchmakereft.

Thank you!