Gauge Invariance and Finite Counterterms in Chiral Gauge Theories

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Outline

- Intro: Dimensional Regularization and the BMHV scheme
- Regularization of chiral gauge theories
- Symmetries: spurious anomalies and counterterms
- Background field method
- Result: 1-loop gauge-restoring counterterm for chiral theories in Dim-Reg.
- Motivation: useful for checks and automation

INTRO:
Dimensional Regularization
and the BMHV scheme

't Hooft-Veltman (1972)
Breitenlohner-Maison (1977)

Dimensional Regularization

4-dimensional (d-4)-dimensional

Recipe:

— Space-time dimension continued to (complex) d. Coordinates split $\mu = \bar{\mu} \oplus \hat{\mu}$

— Kinetic terms (propagators) promoted to d-dimensions

— Interactions (vertices) are scheme-dependent: just need to reduce to the familiar 4-dim theory

— Regularized bosonic part can respect all 4-dim symmetries (we make natural choice).

— **Regularized fermionic action cannot respect the 4-dim chiral symmetries…**

Chirality?

There is no notion of chirality in arbitrary d-dimensions!

➔ Chirality-projectors must be trivial

➔ The usual 4-dim relations must become inconsistent

$$\left.\begin{array}{r}\{\gamma^\mu, \gamma_5\} = 0 \\ \text{Cyclicity of the trace}\end{array}\right\} \Rightarrow$$

All traces with one ɣ5 vanish: cannot have

$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 4i\epsilon^{\mu\nu\rho\sigma}$$

’t Hooft-Veltman:

— Cyclicity of trace is sacred (no anti-commuting ɣ5)

— Chirality (and hence Levi-Civita) is a purely 4-dimensional concept

$$\gamma_5 = -\frac{i}{4!}\epsilon_{\bar{\mu}\bar{\nu}\bar{\alpha}\bar{\beta}}\gamma^{\bar{\mu}}\gamma^{\bar{\nu}}\gamma^{\bar{\alpha}}\gamma^{\bar{\beta}} \implies \begin{cases} \{\gamma^{\bar{\mu}}, \gamma_5\} = 0 \\ [\gamma^{\hat{\mu}}, \gamma_5] = 0 \end{cases}$$

Breitenlohner-Maison:

— Algebraically consistent definitions

— Allows to identify an unambiguous scheme at all orders

Note: no alternative prescription has been proven consistent (ambiguities).

Chiral Theories
in Dim-Reg with BMHV

Implications: Chiral gauge theories

Consider the following 4-dimensional theory

$$S_{\text{Fermions}} = \int d^4x\, \overline{\Psi} i\gamma^\mu [\partial_\mu - iA_\mu^a T^a]\Psi$$

Arbitrary compact gauge theory: product of U(1)'s and simple factors.
Arbitrary fermion content: LH and RH charged under different (reducible) representations

$$T^a = T_L^a P_L + T_R^a P_R, \qquad [T^a, T^b] = if^{abc}T^c \qquad P_L = \frac{1}{2}(1-\gamma_5) \qquad P_R = \frac{1}{2}(1+\gamma_5)$$

Same for LH and RH generators

Chiral projectors

Regularized version in Dim-Reg with BMHV:

— Kinetic term must be promoted to d-dimensions.

Denominator of propagator falls off as p^2 in any d-direction

$$S = \int d^4x\, \overline{\Psi} i \gamma^{\bar{\mu}} \partial_{\bar{\mu}} \Psi + \cdots \to S^{(d)} = \int d^dx\, \overline{\Psi} i \gamma^{\mu} \partial_{\mu} \Psi + \cdots$$

— Interaction: a lot of freedom, just needs to recover the familiar 4-dim limit

$$J^\mu_L = \overline{\Psi}\gamma^\mu P_L \Psi \quad \text{or} \quad \overline{\Psi} P_R \gamma^\mu \Psi \quad \text{or} \quad \overline{\Psi} P_R \gamma^\mu P_L \Psi \ ???$$

Chiral projectors defined as in d=4

$$\boxed{J^\mu_L \equiv \overline{\Psi} P_R \gamma^\mu P_L \Psi = \overline{\Psi}\gamma^{\bar{\mu}} P_L \Psi = [J^\mu_L]^\dagger}$$

Our choice:
It is hermitian (unitarity retained by regulator)
It minimizes the spurious anomaly.

Our regularized fermion action finally reads (= choice of scheme):

d-dimensional

$$S^{(d)}_{\text{Fermions}} \equiv \int d^dx \, [\overline{\Psi} i\gamma^{\mu}\partial_{\mu}\Psi + \overline{\Psi}\gamma^{\bar{\mu}} A^a_{\bar{\mu}} T^a \Psi]$$

4-dimensional

$$= \int d^dx \, [\overline{\Psi} i\gamma^{\bar{\mu}}(\partial_{\bar{\mu}} - iA^a_{\bar{\mu}}T^a)\Psi + \overline{\Psi} i\gamma^{\hat{\mu}}\partial_{\hat{\mu}}\Psi]$$

Conserved global symmetries:

— **SO(1,3)xSO(d-4)** → no need of Lorentz-restoring counterterms
— **CP**
— **Spurious P** (under which generators transform)
— **Vector-like rotations**
— **Chiral rotations (even non-abelian) are classically anomalous!**

Our regularized fermion action finally reads (choice of scheme):

$$S^{(d)}_{\text{Fermions}} \equiv \int d^d x \left[\overline{\Psi} i\gamma^{\mu}\partial_{\mu}\Psi + \overline{\Psi}\gamma^{\bar{\mu}}A^a_{\bar{\mu}}T^a\Psi\right]$$

$$= \int d^d x \left[\overline{\Psi} i\gamma^{\bar{\mu}}\left(\partial_{\bar{\mu}} - iA^a_{\bar{\mu}}T^a\right)\Psi + \overline{\Psi} i\gamma^{\hat{\mu}}\partial_{\hat{\mu}}\Psi\right]$$

Local symmetries?

They must be defined in d-dimensions…

We declare they are purely 4-dimensional.

— This way vector-like symmetries are preserved.

— Axial symmetries are broken….

$$U = e^{i\alpha^a(\bar{x})T^a} \quad \rightarrow \quad \begin{cases} \Psi \to U\Psi \\ \overline{\Psi} \to \overline{\Psi}\gamma_0 U^\dagger \gamma_0 \\ A_{\bar{\mu}} \to U A_{\bar{\mu}} U^\dagger - iU\partial_{\bar{\mu}}U^\dagger \\ A_{\hat{\mu}} \to U A_{\hat{\mu}} U^\dagger \end{cases}$$

Classical anomaly

$$\delta_\alpha S^{(d)} \equiv \int d^d x \ \alpha_a(\bar{x}) L_a S^{(d)} = - \int d^d x \ \alpha_a(\bar{x}) \boxed{\overline{\Psi}(T_R^a - T_L^a)\gamma_5 \gamma^{\hat{\mu}} \partial_{\hat{\mu}} \Psi}$$

Small parameter

Generator of infinitesimal gauge transformations of fields

Unavoidable: d-dim kinetic term mixes L with R → explicit breaking of chiral symmetry.
Evanescent: the anomaly must vanish as d → 4.
Minimality: our assumptions lead to minimal, irreducible anomaly (practical utility).
Generality: same anomaly found including Yukawas (result is general at ren. level).

Quantum Symmetries:
Spurious anomalies and Counterterms

Quantum Symmetries in Dim-Reg

$$e^{i\Gamma[\phi_c]} = \int_{1\mathrm{PI}} \mathcal{D}\phi \; e^{iS[\phi+\phi_c]}$$

1) In Dim-Reg the measure is invariant under local transformations of fields since δ(0)=0:

$$\mathcal{D}\phi' = e^{i\delta^{(d)}(0)\int d^dx\, f(x)}\mathcal{D}\phi = \mathcal{D}\phi$$

2) The transformation of the background is given by:

$$e^{i\Gamma[\phi_c']} = \int_{1\mathrm{PI}} \mathcal{D}\phi \; e^{iS[\phi+\phi_c']} = \int_{1\mathrm{PI}} \mathcal{D}\phi' \; e^{iS[\phi'+\phi_c']} = \int_{1\mathrm{PI}} \mathcal{D}\phi \; e^{iS[\phi'+\phi_c']}$$

At infinitesimal level the variation of the 1PI effective action is given by the matrix elements of the classical anomaly (Quantum Action Principle)

$$L_a^{\mathrm{bckgrd}}\Gamma[\phi_c] = \frac{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS[\phi+\phi_c]} L_a S[\phi+\phi_c]}{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS[\phi+\phi_c]}}$$

Infinitesimal transformation of background fields

Infinitesimal transformation of quantum+classical fields

— Symmetries of the classical action hold at all orders (4-dim Lorentz, vector-like, CP, P).
— What happens to anomalous symmetries?
Spurious (gauge, non-abelian axial) or Physical (abelian axial, scale invariance)

Theorem (anomaly of non-abelian global symmetries):

If the **renormalized** 1PI effective action is symmetric up to order (n-1) in $\hbar$

then the anomaly is the variation of a local functional ➔ it is spurious.

Proof:

Casimir (invertible because anomaly is non-trivial)

$$\left.\begin{aligned} L_a\Gamma^{(n)} &= \mathcal{A}_a^{(n)} \\ [L_a, L_b] &= i f_{abc} L_c \end{aligned}\right\} L_a\mathcal{A}_b^{(n)} - L_b\mathcal{A}_a^{(n)} = i f_{abc}\mathcal{A}_c^{(n)} \quad \Rightarrow \quad L^2\mathcal{A}_a^{(n)} = L_a(L_b\mathcal{A}_b^{(n)})$$

$$\Rightarrow \quad \mathcal{A}_a^{(n)} = L_a\Big[\underbrace{L^{-2}L_b\mathcal{A}_b^{(n)}}_{-S_{\rm ct}^{(n)}}\Big]$$

It is local at each order!

Result:
At order n the spurious anomaly can be removed by an appropriate counterterm.

$$\Gamma_{\rm inv}^{(n)} \equiv \Gamma^{(n)} + S_{\rm ct}^{(n)}$$

Gauge theories are self-consistent as long as

$$D^{abc} = \mathrm{tr}(T^a_L\{T^b_L, T^c_L\}) - \mathrm{tr}(T^a_R\{T^b_R, T^c_R\}) = 0\,.$$

Georgi-Glashow (1972)

No new anomalies emerge in perturbation theory (even beyond renormalizable).

See, e.g., Gomis-Weinberg (1995)
Luscher (1999)

Breaking due to Dim-Reg is artificial ⇛ the anomaly can be removed via counterterms.

Tonin et al. (1977)

Explicit form of the Counterterm?
Background Field Method:
1-loop results

Gauge theories: Background Field Method

See, e.g. Abbott

$$e^{i\Gamma[\phi_c]} = \int_{\rm 1PI} \mathcal{D}\phi \; e^{iS[\phi+\phi_c]}$$

— Split physical fields in bckgrnd + quantum fluctuations.
— Gauge-fixing can be chosen to <u>preserve bckgrnd gauge-invariance:</u>
— Then: the symmetry <u>acts linearly</u> on the 1PI effective action ➔ easier.

$$\mathcal{L}_{\rm g.f.} = -\frac{1}{2\xi}(D_c^\mu A_\mu)_a (D_c^\mu A_\mu)_a$$

Alternatively:
Gauge-fixing leaves BRST ➔ (non-linear) Slavnov-Taylor Identities.

Martin-SanchezRuiz (2000)
SanchezRuiz (2003)
BeluscaMaito et al. (2020-2021)

Chiral gauge theories at 1-loop

Use MSbar: introduce divergent counterterms (symmetric & evanescent non-symmetric!) to make the regularized 1PI effective action finite. At 1-loop:

$$e^{i\Gamma^{(d)}_{\mathrm{fin},1}[\phi_c]} = \int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS^{(d)}[\phi+\phi_c]+iS^{(d)}_{\mathrm{div},1}[\phi+\phi_c]}$$

$$L_a^{\mathrm{bckgrd}}\Gamma^{(d)}_{\mathrm{fin},1} = \frac{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS^{(d)}+iS^{(d)}_{\mathrm{div},1}} L_a \left[S^{(d)} + S^{(d)}_{\mathrm{div},1}\right]}{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS^{(d)}+iS^{(d)}_{\mathrm{div},1}}}$$

$$= \langle L_a S^{(d)} \rangle_1 + L_a S^{(d)}_{\mathrm{div},1}$$

Cancels the divergent part of the 1-loop matrix element of classical anomaly.

$$= \langle L_a S^{(d)} \rangle_1 \Big|_{\mathrm{finite\ part}}$$

Combining evanescent & divergent (local) we get a local finite quantum anomaly.

In summary, at 1-loop:

— the quantum anomaly is given by the finite part of the matrix element of the classical anomaly

— it is local because LS is evanescent: to survive in 4-dim it must be multiplied by a divergence.

$$L_a^{\mathrm{bckgrd}} \Gamma^{(d)}_{\mathrm{fin},1} = \left. \frac{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS^{(d)}} L_a S^{(d)}}{\int_{\mathrm{1PI}} \mathcal{D}\phi \; e^{iS^{(d)}}} \right|_{\mathrm{finite}}$$

Note it is trivial to automatize:

Introduce η Anomaly=η LS as a new vertex and evaluate finite part of diagrams with 1 external η.

$$\lim_{d\to4} L_a \Gamma^{(d)}_{\text{Gauge}}\Big|_{(1)} = -\text{tr}\left[T^a_A\left(a_2^{\epsilon}(x)+a_2^{\not\epsilon}(x)\right)\right]$$

$$a_2^{\epsilon} = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\left[\mathcal{V}_{\mu\nu}\mathcal{V}_{\alpha\beta}+\frac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\alpha\beta}-\frac{8}{3}i\left(\mathcal{A}_\alpha\mathcal{A}_\beta\mathcal{V}_{\mu\nu}+\mathcal{A}_\alpha\mathcal{V}_{\mu\nu}\mathcal{A}_\beta+\mathcal{V}_{\mu\nu}\mathcal{A}_\alpha\mathcal{A}_\beta\right)-\frac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right]$$

$$a_2^{\not\epsilon} = \frac{1}{16\pi^2}\left[\frac{4}{3}D^{\mathcal{V}}_\nu D^{\mathcal{V}}_\nu D^{\mathcal{V}}_\mu\mathcal{A}_\mu+\frac{8}{3}i[\mathcal{A}_\mu, D^{\mathcal{V}}_\nu\mathcal{V}_{\mu\nu}]-\frac{2}{3}i[\mathcal{A}_{\mu\nu},\mathcal{V}_{\mu\nu}]\right]$$
$$+\frac{1}{16\pi^2}\left[-8\mathcal{A}_\mu(D^{\mathcal{V}}_\nu\mathcal{A}^\nu)\mathcal{A}_\mu-\frac{8}{3}\left\{D^{\mathcal{V}}_\mu\mathcal{A}_\nu+D^{\mathcal{V}}_\nu\mathcal{A}_\mu,\mathcal{A}_\mu\mathcal{A}_\nu\right\}+\frac{4}{3}\left\{D^{\mathcal{V}}_\mu\mathcal{A}^\mu,\mathcal{A}_\nu\mathcal{A}_\nu\right\}\right]$$

$$\lim_{d\to4} L_c \Gamma^{(d)}_{\text{Fermions}}\Big|_{(1)} = -\frac{2G_{aa}}{16\pi^2}\left(1+\frac{\xi-1}{6}\right)\bar{f}\gamma_5\left(\overrightarrow{\not\partial}+\overleftarrow{\not\partial}\right)T^aT^c_AT^af$$
$$+\frac{2G_{aa}}{16\pi^2}\left(1+\frac{\xi-1}{6}\right)\bar{f}\gamma_5\gamma^\mu\left\{[T^c,T^aT^m_AT^a]-if^{cmn}T^aT^n_AT^a\right\}f\,A^m_\mu$$

— Calculation performed via heat kernel
— Multiple checks using Feynman diagrams
— Result satisfies Wess-Zumino consistency conditions
— Respects spurious P and CP and vector gauge symmetries

Do not commute

$$\mathcal{V}_\mu = \frac{1}{2}(T^a_R+T^a_L)A^a_\mu \qquad T^a = T^a_RP_R+T^a_LP_L$$
$$\mathcal{A}_\mu = \frac{1}{2}(T^a_R-T^a_L)A^a_\mu \qquad G_{aa} = \delta^G_{ab}g^2_G$$

The explicit form of the gauge-restoring counterterm (up to gauge-invariant terms) is:

$$\mathcal{L}_{\rm ct}\big|_{(1)} = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\mathrm{Tr}\left\{\frac{8}{3}\partial_\mu \mathcal{V}_\nu \left\{\mathcal{V}_\alpha, \mathcal{A}_\beta\right\} + 4i\mathcal{V}_\mu\mathcal{V}_\nu\mathcal{V}_\alpha\mathcal{A}_\beta + \frac{4}{3}i\mathcal{V}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right\}$$

$$+\frac{1}{16\pi^2}\mathrm{Tr}\left\{-\frac{4}{3}(D^{\mathcal{V}}_\mu \mathcal{A}_\nu)^2 + 2(D^{\mathcal{V}}_\mu \mathcal{A}^\mu)^2 - \frac{4}{3}[\mathcal{A}_\mu, \mathcal{A}_\nu]^2 + \frac{4}{3}(\mathcal{A}_\mu\mathcal{A}_\nu)^2 + \mathcal{A}^2_{\mu\nu}\right\}$$

$$-\frac{2}{16\pi^2}\left(1+\frac{\xi-1}{6}\right)G_{aa}\bar{f}\gamma_5\gamma^\mu T^a \mathcal{A}_\mu T^a f$$

Holds for most general fermion + gauge theory

Very compact: CP and (spurious) P are manifest.

Cornella-Feruglio-LV (2022)

Finally…

Previous slide

$$L_a^{\text{bckgrd}} \Gamma^{(d)}_{\text{fin},1} = -L_a^{\text{bckgrd}} S^{(d)}_{\text{ct},1} + \text{finite evanescent}$$

$$e^{i\Gamma^{(d)}_{\text{fin,invariant},1}[\phi_c]} = \int_{1\text{PI}} \mathcal{D}\phi \; e^{iS^{(d)}[\phi+\phi_c]+iS^{(d)}_{\text{div},1}[\phi+\phi_c]+iS^{(d)}_{\text{ct},1}[\phi+\phi_c]}$$

At 1-loop
(local and global symmetries)

Renormalized

$$L_a^{\text{bckgrd}} \Gamma^{(4)}_{\text{fin,invariant},1} = +\frac{1}{48\pi^2} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\alpha\beta} \; \text{tr}\left(\left[T^a_L \left\{T^b_L, T^c_L\right\}\right] - \left[T^a_R \left\{T^b_R, T^c_R\right\}\right] \right)$$

Standard result (Bardeen)

In the Standard Model:

— QCD & QED are vector-like and manifest

— no terms with Levi-Civita, peculiarity of SU(2)xU(1)

— Contains all interactions that respect QCD & QED but violate SU(2)xU(1)

VVDD: $D_\mu W^-_\nu D^\mu W^{+\nu}$ $\quad \partial_\mu Z_\nu \partial^\mu Z^\nu$

VVVD: $iF^{\mu\nu} W^+_\mu W^-_\nu$ $\quad iD^\mu W^-_\mu W^+_\nu Z^\nu$ $\quad iD^\nu W^-_\mu W^+_\nu Z^\mu$ $\quad iD_\nu W^-_\mu W^{+\mu} Z^\nu$ $\quad$ +hc

VVVV: $(W^-_\mu W^{+\mu})^2$ $\quad (W^-_\mu W^{-\mu})(W^+_\nu W^{+\nu})$ $\quad (Z_\mu Z^\mu)^2$ $\quad (W^+_\mu Z^\mu)(W^-_\nu Z^\nu)$ $\quad (W^+_\mu W^{-\mu})(Z_\nu Z^\nu)$

ffW: $W^+_\mu \overline{f_u} \gamma^\mu P_L f_d$ $\quad W^+_\mu \overline{f_u} \gamma^\mu P_R f_d$ $\quad$ +hc

ffZ: $Z_\mu \overline{f} \gamma^\mu P_L f$ $\quad Z_\mu \overline{f} \gamma^\mu P_R f$ $\quad$ +hc

Our calculation was performed in a specific scheme.

Identified scheme-independent (1-loop) mapping Spurious Anomaly $\Longrightarrow$ Counterterm

I_a^0	$\Box\partial^\mu A_{a\mu}$
I_{ab}^1	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$
I_{ab}^2	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$
I_{ab}^3	$A_{a\mu}\Box A_b^\mu$
I_{ab}^4	$(\partial_\nu A_{a\mu})(\partial^\nu A_b^\mu)$
I_{ab}^5	$(\partial_\nu A_{a\mu})(\partial^\mu A_b^\nu)$
I_{ab}^6	$(\partial^\mu A_{a\mu})(\partial^\nu A_{b\nu})$
I_{abd}^7	$(\partial_\mu A_a^\mu)A_{b\nu}A_d^\nu$
I_{abd}^8	$(\partial_\mu A_a^\nu)A_{b\mu}A_d^\nu$
I_{abd}^9	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$
I_{abde}^{10}	$A_{a\mu}A_b^\mu A_{d\nu}A_e^\nu$
I_{abde}^{11}	$\epsilon^{\mu\nu\rho\sigma}A_{a\mu}A_{b\nu}A_{d\rho}A_{e\sigma}$
I_{Xij}^{12}	$\bar f_{Xi}\overrightarrow{\not\partial} f_{Xj}$
I_{Xij}^{13}	$\bar f_{Xi}\overleftarrow{\not\partial} f_{Xj}$
I_{Xaij}^{14}	$\bar f_{Xi}\not A_a f_{Xj}$

$\Longrightarrow$

$\mathcal{I}_{ghl}^1$	$(\partial^\nu A_g^\mu)A_{h\nu}A_{l\mu}$
$\mathcal{I}_{gh}^2$	$A_{g\mu}\Box A_h^\mu$
$\mathcal{I}_{gh}^3$	$A_{g\mu}\partial^\mu\partial^\nu A_{h\nu}$
$\mathcal{I}_{ghl}^4$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}(\partial_\rho A_{l\sigma})$
$\mathcal{I}_{ghlm}^5$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}\ A_{l\rho}A_{m\sigma}$
$\mathcal{I}_{ghlm}^6$	$A_{g\mu}A_h^\mu\ A_{l\nu}A_m^\nu$
$\mathcal{I}_{Xij}^7$	$\bar f_{Xi}\overrightarrow{\not\partial} f_{Xj}$
$\mathcal{I}_{Xaij}^8$	$\bar f_{Xi}\not A_a f_{Xj}$

Assume 4-dim. Lorentz, vectorial gauge, spurious P&CP
Impose Wess-Zumino consistency conditions
Then
Given the “spurious anomaly”, the Counterterm is fixed up to gauge invariant terms.

Conclusions

- ☑ In concrete calculations, counterterms needed to restore chiral invariance
- ☑ **1-loop counterterm in dim-reg & BMHV for general fermionic reps**
  - (i) Non-trivial check of explicit calculations
  - (ii) Useful for automation
- ☑ **General map anomaly → counterterm, valid for general regularizations**
- ☐ **Extendable to Yukawa sector and SM-EFT**