

# Consistent automatic calculations to $1/\Lambda^4$ in SMEFT

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# Outline

Introduction - higher order SMEFT calculations

SmeftFR Mathematica package

Code structure

Operator basis and input parameter choice

Parametrizations of SMEFT Lagrangian and interaction vertices

Interfaces and options

Conclusions

Based on work with A. Dedes, M. Ryzkowski, K. Suxho, L. Trifyllis, to be published soon

EFT approach to New Physics searches since a decade is one of the most commonly used techniques.

$$L_{EFT} = L^{(4)} + \sum_{k=1}^N \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}$$

Thousands of analyses published, both theoretical and experimental.  
Many calculational tools developed and publicly available.

With time and progress, increasingly difficult issues attacked and corresponding progress in relevant computational techniques required – SMEFT is a very complicate model!

**Current attempts: “going beyond the leading order”**

NLO could mean including loop corrections - see Ken Mimasu talk.

But “beyond leading order” could also mean **going beyond the dimension-6 order in SMEFT expansion.**

This talk: how to consistently perform and automatise calculations including terms above dim-6.

No UV matching here, purely low-energy SMEFT parametrization considered.

And very “tool like-talk”, project not completely finished, no phenomenology considered yet.

Terms of higher order in  $1/\Lambda$  could be potentially important, both from theoretical and experimental point of view:

- theoretical issues - positivity problems for SMEFT when limited to dim-6 terms
- $\sim 2500$  Wilson coefficients (“WCs”) in the dim-6 SMEFT Lagrangian - obviously many of them still weakly constrained, dim-6<sup>2</sup> effects not necessarily small
- scattering amplitudes are growing with collision energy, amplifying higher dimension operator effects.

Example: unitarity violation of the  $S$  matrix in the Vector Boson Scattering. Existing analyses for LHC explicitly show importance of dim-6<sup>2</sup> and dim-8 terms.

Calculations above the leading dim-6 order technically a complicated problem - for consistency both dim-6<sup>2</sup> and dim-8 contributions should be simultaneously included.

In principle, all steps are known:

- choose SMEFT operator basis to the required  $1/\Lambda$  order. Usually defined in terms of fields before Spontaneous Symmetry Breaking
- choose SMEFT parametrization - both for Wilson coefficients and SM parameters
- perform the SSB and Higgs mechanism; find physical (mass eigenstates) fields and their interactions for easier direct comparison with experiments
- derive the Feynman rules
- calculate amplitudes and observables to the required order in terms of chosen SMEFT parameters
- find relations between parameters and set of relevant input observables
- re-insert such relations into the (squared-)amplitudes and for consistency re-expand to the required order again

For loop calculation additional steps necessary: quantization of the theory (selecting gauge fixing) and choice of the renormalization scheme.

## Usual (or “default”) SMEFT parametrization - set of “standard” SM parameters + WCs:

- gauge and Higgs couplings  $g, g', g_s, \lambda, v$  (eventually rescaled to absorb some oblique corrections)
- fermion masses (quark, lepton and neutrino) masses
- CKM and PMNS matrix elements
- Wilson coefficients of higher order operators in a chosen basis

Standard approach to calculations within SMEFT - find an expression for an observable

$$\mathcal{O} = \mathcal{O}(g, g', \dots, \text{WCs})$$

But determination of SM parameters is affected by presence of new operators!

They need to be related to some other measured quantities  $\mathcal{O}_1, \mathcal{O}_2, \dots$

$$\begin{aligned}g &= g(\mathcal{O}_1, \mathcal{O}_2, \dots) \\g' &= g'(\mathcal{O}_1, \mathcal{O}_2, \dots) \\&\dots\end{aligned}$$

Relations of SM parameters to input parameters needs to be re-inserted to calculated observable

$$\mathcal{O} = \mathcal{O}(g(\mathcal{O}_1, \mathcal{O}_2, \dots), g'(\mathcal{O}_1, \mathcal{O}_2, \dots), \dots, \text{WCs})$$

and re-expanded in  $1/\Lambda$  powers to required order.

Consistent calculations to given  $1/\Lambda$  order require proper (and technically feasible!) choice of “input parameters”, usually some well-measured experimental quantities.

Typical choices in EW sector: muon lifetime or  $\alpha_{em}$  and physical masses of gauge and Higgs bosons.

Other ideas in the literature: determination of CKM elements within SMEFT, Descotes-Genon et al, 1812.08163

Procedure straightforward but technically complicated and error-prone (especially dim-6<sup>2</sup> terms).



Idea for simplification: instead of using “default” set of SM parameters for the intermediate step calculations, express Lagrangian and interaction vertices **directly** in terms of predefined set of input observables.

Examples:

- EW sector:  $(g, g', \lambda, v) \longrightarrow (\tau_\mu, M_h, M_W, M_Z)$
- quark sector: masses + CKM elements  $\longrightarrow$   
 masses +  $Br(B \rightarrow \tau\nu_\tau), \Gamma(K \rightarrow \mu\nu_\mu)/\Gamma(\pi \rightarrow \mu\nu_\mu), \Delta m_{B_d}, \Delta m_{B_s}$   
 (1812.08163)
- ...

Transition needs to be known to the required order, see for example vev-muon lifetime relation to  $1/\Lambda^4$ :

$$\begin{aligned} \tau_\mu &= \frac{384\pi^3 v^4}{m_\mu^5} \left( 1 - Av^2 + (A^2 - B)v^4 \right) \\ A &= -\frac{2}{\Lambda^2} (C_{2112}^{ll} - C_{11}^{\varphi l3} - C_{22}^{\varphi l3}) \\ B &= \frac{1}{\Lambda^4} \left( -2C_{2112}^{ll} C_{11}^{\varphi l3} - 2C_{2112}^{ll} C_{22}^{\varphi l3} + C_{2112}^{ll^2} + C_{21}^{\varphi l1} C_{12}^{\varphi l3} - C_{12}^{\varphi l1} C_{21}^{\varphi l3} \right. \\ &\quad \left. + C_{12}^{\varphi l1} C_{21}^{\varphi l1} + C_{11}^{\varphi l3^2} + C_{22}^{\varphi l3^2} - C_{12}^{\varphi l3} C_{21}^{\varphi l3} + 4C_{11}^{\varphi l3} C_{22}^{\varphi l3} \right) \end{aligned}$$

Advantages of such approach:

- simple truncating of calculated transition amplitudes at required  $1/\Lambda$  order is sufficient - no need for re-expressing the final results in terms of measurable input quantities.
- in principle approach extendable to any order in  $1/\Lambda$ , providing that corresponding SMEFT expressions for the input quantities are also known accurately enough
- flexible - if properly implemented, allow users to choose their own preferred set of physical input

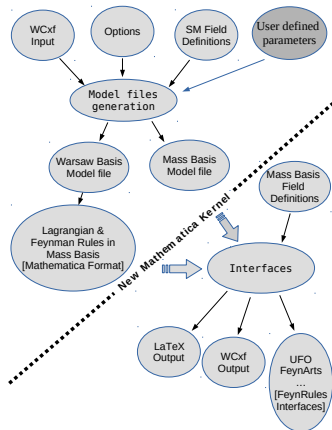
Implementation: last point requires defining some format and its “mini-compiler” **to let users define their own parameter sets** with relations to “default” parameters and transfer them automatically to [FeynRules](#) and later to [UFO/MadGraph](#), [FeynArts](#) or other codes.

## Practical realisation - SmeftFR v3 (Dedes, JR, Ryzkowski, Suxho, Trifyllis).

Main steps:

- define user-chosen input parameters and their relations to “default” set of SMEFT parameters (up to maximum  $1/\Lambda^4$ )
- import numerical values of WCs from file in WCxf format
- generate FeynRules model files containing newly defined parameters and chosen subset of higher dimension WCs
- find physical (mass) basis for all fields. Express field normalisation constants and shifts of SM couplings in terms of both “default” and user-defined variables
- calculate interaction vertices in terms of SM normalisation constants/coupling shifts
- for loop calculation define  $R_\xi$  gauge fixing terms and ghost interactions valid to any SMEFT order (1812.11513)
- expand interaction vertices in terms of preferred parameter set
- pass the Lagrangian to interfaces (FeynRules UFO or FeynArts generators, WCxf output, dedicated Latex generator)

## SmeftFR v3 code structure:



Still work in progress - in principle all points above are implemented and working, but thorough checks and testing necessary before making the code public.

Implemented steps:

## 1. SMEFT basis choice

- for dim-5 and -6 operators we use the “Warsaw basis”.
- all dim-7 operators are lepton and/or baryon number violating - we do not include them (treatment of charge conjugated fields in FeynRules is currently broken anyway)
- for dim-8 operators we implemented bosonic terms from basis defined in [Murphy 2005.00059](#). No fermionic dim-8 operators are included.

Including all dim-8 fermionic operators currently does not seem neither plausible ( $\sim 44000$  operators before flavor expansion, CPU time will exceed thermal death of Universe) nor necessary - fermionic parameters are typically known with lower accuracy the quantities in the EW sector, no need (yet?) for higher order fermionic operators.

## 2. User-defined parameters

In addition to values of WCs the SmeftFR v3 code uses 2 sets of parameters for the SM Lagrangian.

“Internal” or “default” parameters based on natural SM parametrization:  $g, g', g_s, \lambda, \text{vev}$ , fermion masses, CKM and PMNS elements.

“User-defined” parameters - can be any set of quantities sufficient to uniquely express “default” parameters in terms of them and the WCs.

Assumption is that user-defined parameters are chosen as experimental observables so independent of values of WCs.

NOTE: for practical purposes physical SM masses are almost obligatory members of both “default” and “user” input sets - if masses are calculated from other input quantities, WCs appear in the denominators of propagators!

User-defined parameters should be collected in file `smeft_input.m`.

File header should contain definitions of preferred parameters in FeynRules-like format. Example:

```
SM$GaugeParameters = {
  alphas == { ParameterType    -> External,
              Value           -> 0.1176,
              InteractionOrder -> {QCD,2},
              Description      -> "alpha_s at MZ scale"},
  taumu == { ParameterType    -> External,
             Value           -> 3.337862504 1018,
             Description      -> "muon lifetime in GeV-1"},
  Zmass == { ParameterType    -> External,
            Value           -> 91.1876,
            Description      -> "Pole Z boson mass" },
  ...
  mBs == { ParameterType    -> External,
          Value           -> 5.36689,
          Description      -> "B_s meson mass" },
  ...
}
```

All user-defined parameters should be real scalars, no indices!

`smeft_input.m` content: must define relations between **user-defined** parameters and “**default**” SM parametrization.

One has to define routines expressing variables named

```
UserInput$vev  
UserInput$GW  
UserInput$G1  
UserInput$GS  
UserInput$hlambda  
UserInput$MZ  
...  
UserInput$CKM  
UserInput$PMNS
```

i.e. “**default**” SM parameters, in terms of **user-defined** set of observables and WCs to the required  $1/\Lambda$  order.



## Pre-defined input schemes:

```
SMEFTInputScheme = Function[{}],  
(* routine initialising of parameters in various sectors -  
gauge, quark and lepton. *)  
SMEFTGaugeInputsGF[];  
(* alternative input scheme: SMEFTGaugeInputsAEM[]; *)  
SMEFTQuarkInputs[];  
SMEFTLeptonInputs[];  
SMEFT$UserInputInitialization = True;  
(* end of SMEFTInputScheme *)  
]
```

Users who prefer after other set of input quantities should edit the the file [smft\\_input.m](#) replacing routines above by their own set of relations to “default” parameters.

Predefined input schemes included in SmeftFR v3 distribution:

- **EW sector**, input scheme  $G_F, M_W, M_Z, M_h$ :  $G_F$  and  $v_{ev}$  calculated from muon lifetime up to  $1/\Lambda^4$
- **EW sector**, alternative input scheme  $\alpha_{em}, M_W, M_Z, M_h$ , again related to other SM parameters up to  $1/\Lambda^4$
- **QCD sector** -  $\alpha_s(M_Z)$  used as input, no  $1/\Lambda$  corrections included yet!
- **quark sector** - input scheme based on physical quark masses and implementation of **1812.08163**, CKM Wolfenstein parametrization related to measurements of  $Br(B \rightarrow \tau\nu_\tau)$ ,  $\Gamma(K \rightarrow \mu\nu_\mu)/\Gamma(\pi \rightarrow \mu\nu_\mu)$ ,  $\Delta m_{B_d}$ ,  $\Delta m_{B_s}$ . Expansion up to  $1/\Lambda^2$ , more accurate formulae not available in the literature.
- **lepton sector** - input scheme based on masses and PMNS elements, no  $1/\Lambda$  corrections to PMNS included

Various levels of input scheme accuracy currently implemented:

- EW sector most precisely measured -  $1/\Lambda^4$  accuracy most important here and already available.
- QCD sector is tricky - published value of  $\alpha_s(M_Z)$  is average from various measurements, SMEFT corrections should be calculated to each measurement separately and combined afterwards - no such analysis yet?
- fermion sector:  $1/\Lambda^2$  accuracy probably sufficient at present (but - 1812.08163 gives sometimes large corrections to CKM for some operators, issue related to constraints on flavor structure of WCs).

Predefined routines could be easily adapted to include higher order corrections or completely replaced by users who prefer other choice of input observables - as long as standard SM variables are properly defined.

To save CPU time, actual expansion order is controlled by `SMEFT$ExpansionOrder` variable - could be  $1/\Lambda^2$  or  $1/\Lambda^4$ .

### 3. Next steps:

- creating FeynRules model files, including user-chosen set of WCs and definitions of BOTH user-defined and default parameters, the latter with numerical values updated with WC corrections
- WC values read from the file in WCxf format ([1712.05298](#))
- starting FeynRules engine and calculating bilinear terms and physical fields normalisation/SM parameter shifts:

$$\begin{aligned} G &= Z_g g \\ G' &= Z_{g'} g' \\ \dots & \\ \Phi &= \begin{pmatrix} Z_{G^+}^{-1} G^+ \\ \frac{1}{\sqrt{2}} (v + Z_h^{-1} h + i Z_{G^0}^{-1} G^0) \\ \dots \end{pmatrix} \end{aligned}$$

$Z_X$  shifts are evaluated to required  $1/\Lambda$  order both in terms of default and user-defined parameters and stored for future use.

Examples of SM normalisation constants/shifts at higher orders:

$$\begin{aligned}Z_{G^\pm}^2 &= 1, \\Z_{G^0}^2 &= 1 + \sum_{n=0} \left(\frac{v^2}{2}\right)^{n+1} C^{\varphi D,n}, \\Z_h^2 &= 1 + \sum_{n=0} \left(\frac{v^2}{2}\right)^{n+1} \left[ C^{\varphi D,n} - 4C^{\varphi \square,n} \right], \\Z_{g'}^2 &= 1 - 4 \sum_{n=0} \left(\frac{v^2}{2}\right)^{n+1} C^{\varphi B,n} \\Z_g^2 &= 1 - 4 \sum_{n=0} \left(\frac{v^2}{2}\right)^{n+1} C^{\varphi W,n} \\&\dots\end{aligned}$$

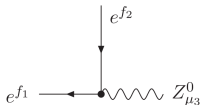
In the code expanded to maximum  $1/\Lambda^4$ .

**Lagrangian and vertex calculation** - done at first without replacing  $Z_X$  constants by their WC expansions.

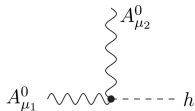
- significantly speeds up the calculations, unexpanded Lagrangian is simpler
- formulae for interaction vertices more compact
- approach can easily accommodate including higher order corrections, if necessary above  $1/\Lambda^4$
- $R_\xi$  gauge fixing terms and ghost terms also calculated at this stage (unless unitary gauge is requested)

Results stored in variables with self explanatory names  
(`LeptonGaugeLagrangian`, `LeptonGaugeVertices`, ...).

Examples:



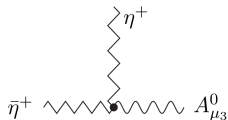
$$\begin{aligned}
 & + \frac{i}{2} \delta_{f_1 f_2} (\bar{g}' Z_{g'} Z_{\gamma Z}^{21} (\gamma^{\mu_3} P_L + 2\gamma^{\mu_3} P_R) + \bar{g} Z_g Z_{\gamma Z}^{11} \gamma^{\mu_3} P_L) \\
 & + \frac{iv^2}{2} C_{f_1 f_2}^{\varphi l 1} (\bar{g} Z_g Z_{\gamma Z}^{11} - \bar{g}' Z_{g'} Z_{\gamma Z}^{21}) \gamma^{\mu_3} P_L \\
 & + \frac{iv^2}{2} C_{f_1 f_2}^{\varphi l 3} (\bar{g} Z_g Z_{\gamma Z}^{11} - \bar{g}' Z_{g'} Z_{\gamma Z}^{21}) \gamma^{\mu_3} P_L
 \end{aligned}$$



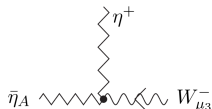
$$\begin{aligned}
 & + \frac{4iv}{Z_h} (Z_{\gamma Z}^{22})^2 C^{\varphi B} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2}) \\
 & + \frac{4i\bar{g}'^2 Z_{g'}^2 v}{\bar{g}^2 Z_g^2 Z_h} (Z_{\gamma Z}^{22})^2 C^{\varphi W} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2}) \\
 & - \frac{4i\bar{g}' Z_{g'} v}{\bar{g} Z_g Z_h} (Z_{\gamma Z}^{22})^2 C^{\varphi WB} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2})
 \end{aligned}$$

(direct dim-8 operator contributions not included here)

Examples continued, ghost vertices:



$$-i\bar{g}' Z_{g'} Z_{\gamma Z}^{22} p_1^{\mu_3}$$



$$-\frac{i\bar{g}^2}{Z_{\gamma Z}^{22} (\bar{g} Z_g Z_{\gamma Z}^{11} - g' Z_{g'} Z_{\gamma Z}^{21})} Z_{\gamma Z}^{21} p_1^{\mu_3}$$

Worth noting: all ghost interactions dependent on WCs only through  $Z_X$ ,  
expressions valid to any SMEFT order!



4. Final step of vertex generation: expanding  $Z_X$  shifts and truncating higher order in  $1/\Lambda$

```
SMEFTExpandVertices[ Input -> "smeft",  
                    ExpOrder-> SMEFT$ExpansionOrder,  
                    ExpName->"Exp" ]
```

Options:

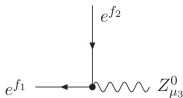
ExpOrder=0,1,2 - expressions are expanded to  $1/\Lambda^{2\text{ExpOrder}}$

ExpName - suffix of the variable names storing expanded expressions (here `LeptonGaugeLagrangianExp`, `LeptonGaugeVerticesExp` etc.)

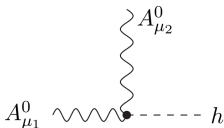
Input =

- “none” no expansion of  $Z_X$  shifts
- “smeft” vertices expressed in terms of “default” parameters
- “user” vertices expressed in terms of “user-defined” parameters

## Examples cd: vertices expanded in “default” parametrization



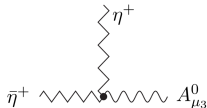
$$\begin{aligned}
 & -\frac{i}{2\sqrt{\bar{g}'^2 + \bar{g}^2}} \delta_{f_1 f_2} ((\bar{g}'^2 - \bar{g}^2) \gamma^{\mu_3} P_L + 2\bar{g}'^2 \gamma^{\mu_3} P_R) \\
 & + \frac{i\bar{g}'\bar{g}v^2}{2(\bar{g}'^2 + \bar{g}^2)^{3/2}} \delta_{f_1 f_2} C^{\varphi WB} ((\bar{g}'^2 - \bar{g}^2) \gamma^{\mu_3} P_L - 2\bar{g}^2 \gamma^{\mu_3} P_R) \\
 & + \frac{1}{2} i v^2 \sqrt{\bar{g}'^2 + \bar{g}^2} C_{f_1 f_2}^{\varphi l1} \gamma^{\mu_3} P_L + \frac{1}{2} i v^2 \sqrt{\bar{g}'^2 + \bar{g}^2} C_{f_1 f_2}^{\varphi l3} \gamma^{\mu_3} P_L
 \end{aligned}$$



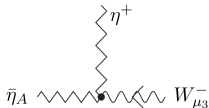
$$\begin{aligned}
 & + \frac{4i\bar{g}^2 v}{\bar{g}'^2 + \bar{g}^2} C^{\varphi B} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2}) \\
 & + \frac{4i\bar{g}'^2 v}{\bar{g}'^2 + \bar{g}^2} C^{\varphi W} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2}) \\
 & - \frac{4i\bar{g}'\bar{g}v}{\bar{g}'^2 + \bar{g}^2} C^{\varphi WB} (p_1^{\mu_2} p_2^{\mu_1} - p_1 \cdot p_2 \eta_{\mu_1 \mu_2})
 \end{aligned}$$

(dim-8 operator contributions not included here)

Examples cd, ghost vertices expanded in “default” parametrization:



$$-\frac{i\bar{g}'\bar{g}}{\sqrt{\bar{g}'^2 + \bar{g}^2}}p_1^{\mu_3} + \frac{i\bar{g}'^2\bar{g}^2v^2}{(\bar{g}'^2 + \bar{g}^2)^{3/2}}C^{\varphi WB}p_1^{\mu_3}$$



$$+\frac{i\bar{g}'\bar{g}}{\sqrt{\bar{g}'^2 + \bar{g}^2}}p_1^{\mu_3} + \frac{i\bar{g}^4v^2}{(\bar{g}'^2 + \bar{g}^2)^{3/2}}C^{\varphi WB}p_1^{\mu_3}$$

(dim-8 operator contributions not included here)

Examples cd: vertices expanded in “user-defined” parametrization, here  $(G_F, M_W, M_Z, M_h)$  input scheme

In[15]= **LeptonGaugeVerticesExp[[4]]**

Out[15]=  $\{\{\{l, 1\}, \{l, 2\}, \{Z, 3\}\},$

$$\frac{i 2^{1/4} \sqrt{G_F} \delta_{f_1, f_2} \left( (2 M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{-s_1, s_2} + 2 (M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{+s_1, s_2} \right)}{M_Z} + \frac{1}{\Lambda^2} \left( \frac{i 2^{3/4} C^{0WB} M_W \sqrt{\frac{-M_W^2 + M_Z^2}{G_F}} \gamma_{s_1, s_2}^{\mu 3} \delta_{f_1, f_2}}{M_Z} + \frac{i M_Z C^{011}{}_{f_1, f_2} \gamma^{\mu 3} \cdot P_{-s_1, s_2}}{2^{1/4} \sqrt{G_F}} + \frac{i C^{11}{}_{2, 1, 1, 2} \delta_{f_1, f_2} \left( (2 M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{-s_1, s_2} + 2 (M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{+s_1, s_2} \right)}{2 \times 2^{1/4} \sqrt{G_F} M_Z} + \frac{i C^{00} \delta_{f_1, f_2} \left( (2 M_W^2 + M_Z^2) \gamma^{\mu 3} \cdot P_{-s_1, s_2} + 2 (M_W^2 + M_Z^2) \gamma^{\mu 3} \cdot P_{+s_1, s_2} \right)}{4 \times 2^{1/4} \sqrt{G_F} M_Z} + \frac{i \left( 2 M_Z^2 C^{013}{}_{f_1, f_2} \gamma^{\mu 3} \cdot P_{-s_1, s_2} - (C^{013}{}_{1, 1} + C^{013}{}_{2, 2}) \delta_{f_1, f_2} \left( (2 M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{-s_1, s_2} + 2 (M_W^2 - M_Z^2) \gamma^{\mu 3} \cdot P_{+s_1, s_2} \right) \right)}{2 \cdot 2^{1/4} \sqrt{G_F} M_Z} \right)$$

In[21]= **GaugeHiggsVerticesExp[[17]]**

Out[21]=  $\{\{\{A, 1\}, \{A, 2\}, \{H, 3\}\},$

$$\frac{1}{\Lambda^2} \left( \frac{2 i 2^{3/4} C^{0B} M_W^2 \left( p_1^{\mu 2} p_2^{\mu 1} - \eta_{\mu 1, \mu 2} p_1 \cdot p_2 \right)}{\sqrt{G_F} M_Z^2} + \frac{2 i 2^{3/4} C^{0W} \left( -M_W^2 + M_Z^2 \right) \left( p_1^{\mu 2} p_2^{\mu 1} - \eta_{\mu 1, \mu 2} p_1 \cdot p_2 \right)}{\sqrt{G_F} M_Z^2} - \frac{2 i 2^{3/4} C^{0WB} M_W \sqrt{\frac{-M_W^2 + M_Z^2}{G_F}} \left( p_1^{\mu 2} p_2^{\mu 1} - \eta_{\mu 1, \mu 2} p_1 \cdot p_2 \right)}{M_Z^2} \right)$$

(dim-8 operator contributions not included here)

Examples cd, ghost vertices expanded in “user-defined” parametrization, here  $(G_F, M_W, M_Z, M_h)$  input scheme

In[24]:= GhostVerticesExp[[12]]

$$\text{Out[24]} = \left\{ \left\{ \{ghWm^1, 1\}, \{ghWm, 2\}, \{A, 3\} \right\}, \right. \\ \left. \frac{2 i 2^{1/4} M_W \sqrt{G_F (-M_W^2 + M_Z^2)} p_1^{\mu 3}}{M_Z} + \frac{1}{\Lambda^2} \left( -\frac{i 2^{3/4} C^{0WB} M_W^2 p_1^{\mu 3}}{\sqrt{G_F} M_Z} - \frac{i C^{0D} M_W^3 p_1^{\mu 3}}{2 \times 2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} + \frac{i (-M_W^3 + M_W M_Z^2) C^{11}_{2,1,1,2} p_1^{\mu 3}}{2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} + \frac{i M_W (M_W^2 - M_Z^2) (C^{013}_{1,1} + C^{013}_{2,2}) p_1^{\mu 3}}{2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} \right) \right\}$$

In[26]:= GhostVerticesExp[[19]]

$$\text{Out[26]} = \left\{ \left\{ \{ghA^1, 1\}, \{ghWp, 2\}, \{W^1, 3\} \right\}, \right. \\ \left. \frac{2 i 2^{1/4} M_W \sqrt{G_F (-M_W^2 + M_Z^2)} p_1^{\mu 3}}{M_Z} + \frac{1}{\Lambda^2} \left( -\frac{i C^{0D} M_W^3 p_1^{\mu 3}}{2 \times 2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} + \frac{i (-M_W^3 + M_W M_Z^2) C^{11}_{2,1,1,2} p_1^{\mu 3}}{2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} + \frac{i M_W (M_W^2 - M_Z^2) (C^{013}_{1,1} + C^{013}_{2,2}) p_1^{\mu 3}}{2^{1/4} M_Z \sqrt{G_F (-M_W^2 + M_Z^2)}} \right) \right\}$$

(dim-8 operator contributions not included here)

Examples cd,  $HZZ$  vertex including dim-8 terms for chosen operator subset, two parametrizations:

$$Out[*]= \left\{ \left\{ \{H, 1\}, \{Z, 2\}, \{Z, 3\} \right\}, \frac{i \text{vev} \left( G_W \text{gwnorm} \text{AZnorm}_{1,1} - G_1 \text{g1norm} \text{AZnorm}_{2,1} \right)^2 \eta_{\mu_2, \mu_3}}{2 \text{Hnorm}} + \frac{i C^{\phi D} \frac{1}{\Lambda^2} \text{vev}^3 \left( G_W \text{gwnorm} \text{AZnorm}_{1,1} - G_1 \text{g1norm} \text{AZnorm}_{2,1} \right)^2 \eta_{\mu_2, \mu_3}}{2 \text{Hnorm}} + \frac{3}{8} i C^{\phi 6D2} \left( \frac{1}{\Lambda^2} \right)^2 \text{vev}^5 \left( G_W \text{gwnorm} \text{AZnorm}_{1,1} - G_1 \text{g1norm} \text{AZnorm}_{2,1} \right)^2 \eta_{\mu_2, \mu_3} \right\}$$

$$Out[*]= \left\{ \left\{ \{H, 1\}, \{Z, 2\}, \{Z, 3\} \right\}, 2 i 2^{1/4} \sqrt{G_F} M_Z^2 \eta_{\mu_2, \mu_3} + \frac{i \left( 32 C^{\phi 6Box} + 12 C^{\phi 6D2} + 48 (C^{\phi Box})^2 - 8 C^{\phi Box} C^{\phi D} - 9 (C^{\phi D})^2 \right) \left( \frac{1}{\Lambda^2} \right)^2 M_Z^2 \eta_{\mu_2, \mu_3}}{16 \times 2^{3/4} G_F^{3/2}} + \frac{1}{\Lambda^2} \left( \frac{i 2^{3/4} C^{\phi Box} M_Z^2 \eta_{\mu_2, \mu_3}}{\sqrt{G_F}} + \frac{i C^{\phi D} M_Z^2 \eta_{\mu_2, \mu_3}}{2 \times 2^{1/4} \sqrt{G_F}} \right) \right\}$$

Notation  $\text{gwnorm} = Z_g$ ,  $\text{g1norm} = Z'_g$ ,  $\text{AZnorm}_{2,1} = Z_{\gamma Z}^{12}$ ,  $\text{Hnorm} = Z_H$  etc.

The sequence of steps described above starts from weak basis Lagrangian (“Warsaw” basis for dim-6, bosonic subset of Murphy basis for dim-8).

Output:

- 2 sets of model files for FeynRules - one for weak basis, another for physical (mass eigenstates) basis
- interaction Lagrangian and vertices for fields in physical basis, in terms of  $Z_X$  shifts and in terms of chosen (user-defined) set of input parameters, expanded to maximum  $1/\Lambda^4$
- result stored in disk file

To run interfaces to other codes, one has to quit Mathematica kernel and re-initialise FeynRules using the mass-basis parameter files.

## 5. Interfacing to other codes

Available format of output:

- *Mathematica/FeynRules* syntax
- WC numerical values may be dumped to file in the *WCxf* format
- *Latex* (with some limitations)
- *UFO* format (→ interface to MC generators)
- *FeynArts* format

FeynRules package offers other types of interface routines and formats – good chance that they work, too, but have not been tested by us.



## Comments on interfaces:

- *WCxf* output file allows to transfer numerical values of WCs to other compatible SMEFT public packages, *Flavio*, *FlavorKit*, *Spheno*, *DSixTools*, *wilson*, *FormFlavor*, *SMEFTSim*, ...
- *Latex interface*: SmeftFR includes dedicated Latex generator optimised to make output more readable, draw diagrams etc.  
Note: generated Latex file shows only terms up to dim-6, in “unexpanded” version or expressed in terms of “default” parameter set (difficult to beautify output for freely defined “user” parameters...)
- *FeynArts interface* - seems to work properly without problems (up to issues with signs in 4-fermion vertices and charge conjugated fields).

- *UFO interface* - most tricky and still some problems remain.

Solved: UFO syntax sensitive to missing/double defined/complex parameters values - requires care when transferring user-defined input parameter sets and updating values of “default” parameters.

Not fully solved: UFO requires defining “interaction order” (IO) for couplings, to truncate higher order diagrams.

SM assignments of IOs straightforward:

QCD=1 for  $g_s$ ,

QED=1 for  $g, g'$ ,

QED=2 for quartic Higgs  $\lambda$  etc.

Additional common assignment: QED=-1 for  $vev$ . Works only if all couplings does not explicitly contain momenta (like in the SM).

SMEFT: many vertices directly momentum-dependent leading to “negative IO” warnings and problems with diagram generation in MadGraph.

In our approach problem even more difficult: SMEFT can be parametrized by any set of variables, not necessarily including *vev*, no universal IO assignments.

I have an idea for solution, but not fully implemented/tested yet. To be done after SMEFT Tools ...

Various additional options available (included in the code since SmeftFR v2.0, [1704.03888](#) and [1904.03204](#), download at [www.fuw.edu.pl/smeft](http://www.fuw.edu.pl/smeft))

- unitary or  $R_\xi$  gauge can be selected for tree/loop calculations
- neutrino fields treated as massless Weyl or massive Majorana (in the presence of dim-5 Weinberg operator) spinors
- correction of FeynRules 4-fermion sign issues
- corrected BL violating 4-fermion vertices and  $4-\nu$  vertex (treatment of charge conjugated fields in FeynRules still broken)
- ...

Options yet to be tested after modifications from v2 to v3.

Before conclusions: current status of the project

- all main ideas implemented and working. Few final problems remain (IO assignment for UFO etc.)
- significant code modifications from v2 - extensive testing and bug fixing still necessary
- testing on real calculations of physical processes? Sakis Dedes proved with this code Goldstone Boson Equivalence Theorem in SMEFT to  $1/\Lambda^4$  for several exclusive processes, highly non-trivial and requires many cancellations.
- manual to be updated from v2
- include more (or more accurate) input schemes? User input warmly welcomed!

# Conclusions

1. Higher order SMEFT calculations of physical observables technically quite involved - automatising highly advised.
2. Consistent expansion in  $1/\Lambda$  orders much simpler if SMEFT interactions are expressed in terms of WC-invariant quantities, like experimental observables.
3. Realisation: *Mathematica* SmeftFR v3 package:
  - ▶ Feynman rules calculated in physical field basis and in terms of **user-defined input parameters**
  - ▶ accuracy at present up to  $1/\Lambda^4$
  - ▶ unitary or  $R_\xi$  gauge selectable
  - ▶ interfaces to many other commonly used codes and formats

SmeftFR v3 to be published soon (SmeftFR v2 available on [www.fuw.edu.pl/smeft](http://www.fuw.edu.pl/smeft)).