γ_5 in Dimensional Regularization: The BMHV Scheme at Two-Loop

Paul Kühler, TU Dresden, Institut für Kern- und Teilchenphysik

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In collaboration with
Hermès Bélusca-Maïto, University of Zagreb
Marija Mađor-Božinović, University of Zagreb
Amon Ilakovac, University of Zagreb
Dominik Stöckinger, TU Dresden

γ_5 in DREG

Divergent (multi-)loop Feynman integrals require regularization

$$\int \mathrm{d}^4\mathrm{k}\,\frac{1}{k^2(k-p)^2} \to \infty.$$

The go-to method is **Dimensional Regularization** which preserves **BRST symmetry** for vector-like gauge theories needed to e.g. ensure unitarity.

DREG: Promote integral measure, momenta and algebra to $D = 4 - 2\epsilon$ dimensions $\int \mathrm{d}^4\mathrm{k} \to \mu^{4-D} \int \mathrm{d}^\mathrm{D}\mathrm{x}\,.$

How do we treat intrinsically 4-dimensional objects such as $\epsilon^{\mu\nu\rho\sigma}$ and γ_5?

γ_5 in DREG

Some 4-dimensional properties of γ_5

(i) $\{\gamma_\mu, \gamma_5\} = 0$

(ii) $\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 4i\varepsilon_{\mu\nu\rho\sigma}$

(iii) $\mathrm{Tr}(\gamma_\mu\gamma_\nu) = \mathrm{Tr}(\gamma_\nu\gamma_\mu)$

From which one can deduce $2(D-4)\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 0$, leading to an inconsistency for $D \neq 4$.

So we have to give up some of the above properties. The **BMHV** (G. 't Hooft, M. Veltman, P.Breitenlohner, D. Maison) scheme amounts to giving up the anticommutativity of γ_5.

γ_5 in DREG

- The BMHV scheme is proven to be consistent to all orders but explicitly breaks gauge invariance.
- There are several other schemes: NDR, Larin scheme, Reading Point prescription…
  But their range of applicability is not always known.
- However we need a reliable method for higher loop electroweak calculations necessitated by increasing needs for precision.

Goal: Apply the scheme to the **SM** and provide the necessary (symmetry-restoring) counterterms.
For now: study toy models like 1-loop generic Yang-Mills theory [arXiv:2004.14398], 2-loop chiral QED [arXiv:2109.11042], 2-loop YM/3-loop chiral QED [work in progress]

γ_5 in DREG

The space of DREG decomposes as $\mathbb{M}_D = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}$. We define the symbols

$$\bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4, \qquad \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = -2\epsilon, \qquad \bar{g}^{\mu\rho}\hat{g}_{\rho\nu} = 0.$$

For the 4-dimensional quantities we have e.g.

$g_{\mu\mu_1}\varepsilon_{\mu_1\mu_2\mu_3\mu_4} = \varepsilon_{\mu\mu_2\mu_3\mu_4}$ and
$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\nu_1\nu_2\nu_3\nu_4} = \sum_{\pi\in S_4} sgn(\pi)\prod_{i=1}^4 \bar{g}_{\mu_i\nu_{\pi(i)}}$,
and the γ_5-algebra becomes

$$\{\bar{\gamma}^\mu, \gamma_5\} = 0, \qquad [\hat{\gamma}^\mu, \gamma_5] = 0, \qquad \{\hat{\gamma}^\mu, \gamma_5\} = 2\hat{\gamma}^\mu\gamma_5.$$

The trace is cyclic and we have $\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 4i\varepsilon_{\mu\nu\rho\sigma}$.

Slavnov-Taylor Identities

At tree-level the BRST symmetry of the action can be expressed by the Slavnov-Taylor identity

$$\mathcal{S}(S_0) = \int \mathrm{d}^4\mathrm{x}\, \frac{\delta S_0}{\delta \phi_i(x)} \frac{\delta S_0}{\delta K_{\phi_i}(x)} = 0,$$

where $K_{\phi_i}(x)$ denotes a source coupling to the BRST transformation of the quantum field ϕ.

For the full quantum theory we require that the STI be satisfied for every loop order

$$\mathcal{S}(\Gamma_{\mathrm{ren}}) = \int \mathrm{d}^4\mathrm{x}\, \frac{\delta \Gamma_{\mathrm{ren}}}{\delta \phi_i(x)} \frac{\delta \Gamma_{\mathrm{ren}}}{K_{\phi_i}(x)} = 0.$$

Slavnov-Taylor Identities

The linearized ST-operator, defined by
$\mathcal{S}_D(S_0 + \hbar\mathcal{F}) = S_D(S_0) + \hbar b_D \mathcal{F} + \mathcal{O}(\hbar^2)$, corresponds to

$$b_D = \int \mathrm{d}^{\mathrm{D}}\mathrm{x}\; \underbrace{\frac{\delta S_0}{\delta K_{\phi_i}} \frac{\delta}{\delta \phi}}_{s_D} + \frac{\delta S_0}{\delta \phi_i} \frac{\delta}{\delta K_{\phi_i}},$$

and we have $b^2 = 0$ ($S_0 \equiv S_0^{D=4}$) expressing BRST nilpotency (but $b_D^2 \neq 0$).

Central to our formalism is the **Quantum Action Principle** which connects the breaking of the STI to the insertion of a local operator

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma,$$

which for our purposes is given by $\Delta_* = b\, S_*$.

Slavnov-Taylor Identities

Instead of working directly with

$$\mathcal{S}(\Gamma) \stackrel{!}{=} 0,$$

the **QAP** allows us to systematically compute a certain list of Green's functions with operator insertion Δ.

This will generate both **essential anomalies** as well as **spurious anomalies**.

The latter can be written as total b-variations

$$bX = \Delta_{\mathrm{fct}}.$$

This determines the renormalized action

$$\Gamma_{\mathsf{ren}} = \mathsf{LIM}_{D\to 4}(\Gamma + S_{\mathsf{sct}} + S_{\mathsf{fct}}),$$

satisfying $\mathcal{S}(\Gamma_{\mathsf{ren}}) = 0$.

Chiral QED

The 4-dimensional Lagrangian of χQED

$$\mathcal{L}^{4-dim}_{\chi QED} = i\overline{\psi_{Ri}}\not{D}\psi_{Rj} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \textit{ghost terms} + \textit{gauge fixing},$$

with covariant derivative $D^{\mu}_{ij} = \partial^{\mu}\delta_{ij} - ieA^{\mu}T_{Rij}$
and BRST transformations (here $s \equiv b$)

$$sA_{\mu} = \partial_{\mu}c \quad s\psi_i \equiv \psi_{Ri} = iecT_{Rij}\psi_{Rj} \quad s\overline{\psi_i} = ie\overline{\psi_{Rj}}cT_{Rji},$$

is gauge invariant, hence $s\mathcal{L}^{4-dim}_{\chi QED} = 0$.

Chiral QED

Now **generalize to D dimensions** $\mathcal{L}^{4-dim}_{\chi QED} \to \mathcal{L}^{D}_{\chi QED} \equiv \mathcal{L}^{(0)}$.

The kinetic term $i\overline{\Psi_i}\partial\!\!\!/\Psi_i$ must be D-dimensional to ensure regularization.

The interaction term is more ambiguous since in 4 dimensions, we have

$$\overline{\Psi_R}\gamma^\mu\Psi_R A_\mu = \overline{\Psi}P_L\gamma^\mu P_R\Psi A_\mu = \overline{\Psi}P_L\gamma^\mu\Psi A_\mu = \overline{\Psi}\gamma^\mu P_R\Psi A_\mu,$$

but the latter equals are violated by our γ_5-treatment!

In principle we could have several parametrizations

$$\overline{\Psi_R}\bar{\gamma}^\mu\Psi_R A_\mu, \quad \overline{\Psi_L}\bar{\gamma}^\mu\Psi_L A_\mu, \quad \overline{\Psi_R}\hat{\gamma}^\mu\Psi_L A_\mu, \quad \overline{\Psi_L}\hat{\gamma}^\mu\Psi_R A_\mu.$$

Chiral QED

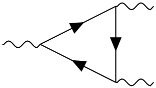
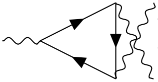
Choosing the purely 4-dimensional vertex we find

$$\mathcal{L}^{(0)} = i\overline{\Psi_i}\partial\!\!\!/\Psi_i + \underbrace{e\overline{\Psi_i}\bar{\gamma}^{\mu}\Psi_j\mathcal{Y}_{Rij}A_{\mu}}_{\mathcal{L}_{\overline{\psi}_R A\psi_R}} + \mathcal{L}^{(0)}_{\mathrm{rest}},$$

where

$$\mathcal{L}^{(0)}_{\mathrm{rest}} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\mathcal{L}_{AA}} - \frac{1}{2\xi}(\partial A)^2 - \overline{c}\,\Box\,c + K_{\Psi_i}s\Psi_i + \ldots$$

For our Abelian model we have $\mathcal{Y}_{Rij} = (\mathrm{diag}(T^1_R, \ldots, \mathcal{Y}^{N_f}_R))_{ij}$ and we have to require $\mathrm{Tr}(\mathcal{Y}^3_R) = 0$ to ensure cancellation of the chiral anomaly

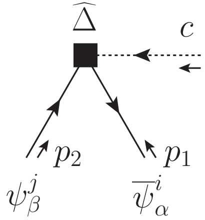
$p_\mu V^{\mu\nu\rho} = p\cdot($  $+$  $) \propto \varepsilon^{\cdot\nu\cdot\rho}q_1q_2 + \mathcal{O}(\epsilon)$

χQED: BRST Symmetry Breaking

In D dimensions the BRST symmetry is broken

$$s_D\, S_0 \equiv \hat{\Delta} = \int \mathrm{d^D x}\, \hat{\Delta}(x) = \int \mathrm{d^D x}\, e\mathcal{Y}_{Rij}\, c(\overline{\psi}_i(\overleftarrow{\hat{\not\partial}}\, P_R + \overrightarrow{\hat{\not\partial}}\, P_L)\psi_j)$$

by a local, dimension 4 field operator corresponding to the Feynman rule



$$= \frac{e}{2}\,\mathcal{Y}_{R\,ij}\Big((\widehat{\not p_1} + \widehat{\not p_2}) + (\widehat{\not p_1} - \widehat{\not p_2})\gamma_5\Big)_{\alpha\beta}$$

$$= e\,\mathcal{Y}_{R\,ij}\Big(\widehat{\not p_1} P_R + \widehat{\not p_2} P_L\Big)_{\alpha\beta}\,.$$

χQED One-Loop Renormalization

At one-loop the breaking of the STI is given by

$$\mathcal{S}(\Gamma)^{(1)} = (\Delta \cdot \Gamma)^{(1)} = \underbrace{\Delta^{(0)} \cdot \Gamma^{(1)}}_{(2)} + \underbrace{\Delta^{(1)}_{\mathrm{sct}} \cdot \Gamma^{(0)}}_{(3)} + \underbrace{\Delta^{(1)}_{\mathrm{fct}} \cdot \Gamma^{(0)}}_{(4)},$$

where $b_D\, \mathcal{L}^{(1)}_{\mathrm{ct}} = \Delta^{(1)}_{\mathrm{ct}}$ and $\Delta^{(1)}_{\mathrm{ct}} \cdot \Gamma^{(0)} = \Delta^{(1)}_{\mathrm{ct}}$.

Renormalization Procedure

1. Renormalize the divergent Green's functions to obtain the divergent counter terms $\mathcal{L}^{(1)}_{\mathrm{sct}}$

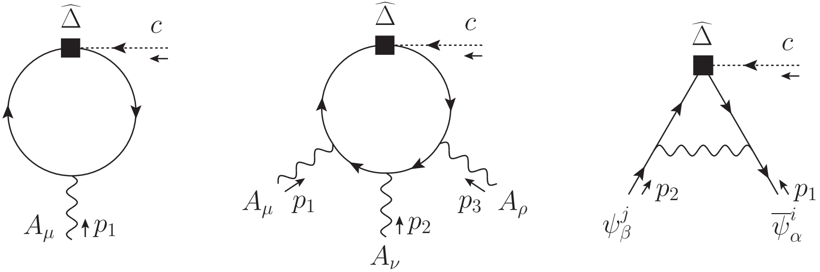
2. Compute the insertion of tree-level $\Delta^{(0)}$ into one-loop diagrams to obtain $\Delta^{(0)} \cdot \Gamma^{(1)}_{\mathrm{div}} + \Delta^{(0)} \cdot \Gamma^{(1)}_{\mathrm{fin}}$

3. Check that $\Delta^{(1)}_{\mathrm{sct}}$ cancels $\Delta^{(0)} \cdot \Gamma^{(1)}_{\mathrm{div}}$

4. Determine $\mathcal{L}^{(1)}_{\mathrm{fct}}$ such that $\Delta^{(1)}_{\mathrm{fct}} = -\mathrm{LIM}_{D\to 4}\Delta^{(0)} \cdot \Gamma^{(1)}_{\mathrm{fin}}$

χQED One-Loop Renormalization

Only power-counting divergent diagrams are needed for the finite symmetry breaking since $\epsilon_{\text{evanescent}} \times (\frac{1}{\epsilon_{\text{UV}}} + \text{finite}) = \text{finite} + \mathcal{O}(\epsilon)$.



$$\widehat{\Delta} \cdot \Gamma^{(1)} = \frac{\hbar}{16\pi^2} \int \mathrm{d^D x}\, \frac{e^2\,\mathrm{Tr}(\mathcal{Y}_R^2)}{3} \left(\frac{1}{\epsilon} (\overline{\partial}_\mu c)\, (\widehat{\partial}^2 \bar{A}^\mu) + (\overline{\partial}_\mu c)(\overline{\partial}^2 \bar{A}^\mu) \right) + \frac{e^4\,\mathrm{Tr}(\mathcal{Y}_R^4)}{3}\, c\, \overline{\partial}_\mu (\bar{A}^\mu \bar{A}^2) - \frac{(\xi+5) e^3 (\mathcal{Y}_R^3)_{jk}}{6}\, c\, \overline{\partial}^\mu (\overline{\psi}_j \overline{\gamma}_\mu P_R \psi_k)$$

χQED One-Loop Renormalization

$$S^{(1)}_{\rm sct} = \frac{-\hbar\, e^2}{16\pi^2\epsilon}\left(\frac{2\,\mathrm{Tr}(\mathcal{Y}_R^2)}{3}\overline{S_{AA}} + \xi\sum_j (\mathcal{Y}_R^j)^2\left(\overline{S^j_{\bar\psi\psi_R}} + \overline{S^j_{\overline{\psi_R}A\psi_R}}\right)\right.$$
$$\left. + \frac{\mathrm{Tr}(\mathcal{Y}_R^2)}{3}\int \mathrm{d^Dx}\,\frac{1}{2}\bar{A}_\mu\widehat{\partial}^2\bar{A}^\mu\right)$$

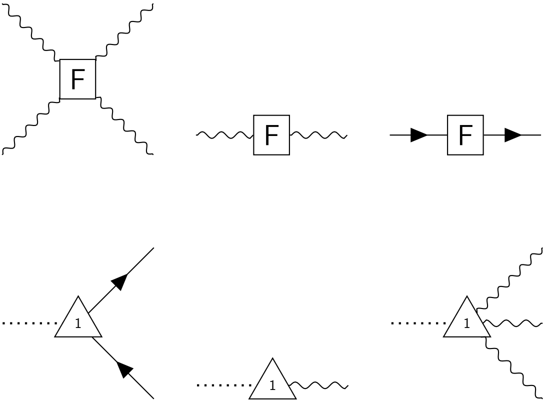
4-dimensional gauge invariant terms and evanescent $\times\frac{1}{\epsilon}$

$$S^{(1)}_{\rm fct} = \frac{\hbar}{16\pi^2}\int \mathrm{d^Dx}\left\{\frac{-e^2\,\mathrm{Tr}(\mathcal{Y}_R^2)}{6}\bar{A}\cdot(\overline{\partial}^2\bar{A}) + \frac{e^4\,\mathrm{Tr}(\mathcal{Y}_R^4)}{12}(\bar{A}^2)^2\right.$$
$$\left. + \left(\frac{5+\xi}{6}\right)e^2\sum_j(\mathcal{Y}_R^j)^2\, i\overline{\psi}_j\overline{\gamma}^\mu\overline{\partial}_\mu P_R\psi_j\right\}$$

purely 4-dimensional non-gauge-invariant terms

χQED One-Loop Renormalization

New finite counter term vertices and breaking corrections



Here for two-loop and the Abelian model we only require the structures present at tree-level

χQED Two-Loop Renormalization

STI breaking at two-loop

$$\mathcal{S}(\Gamma)^{(2)} = (\Delta \cdot \Gamma)^{(2)} = \Delta^{(0)} \cdot \Gamma^{(2)} + \Delta_{\mathrm{sct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathrm{fct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathrm{sct}}^{(2)} \cdot \Gamma^{(0)} + \Delta_{\mathrm{fct}}^{(2)} \cdot \Gamma^{(0)}$$

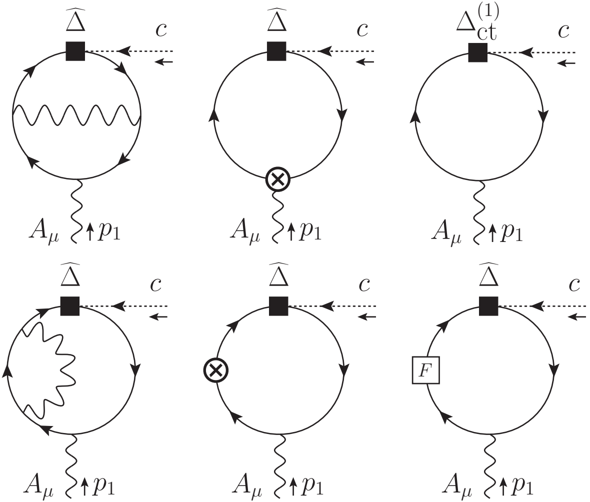
Renormalization at two-loop

$$S_{\mathsf{sct}}^{(2)} + \Gamma_{\mathsf{div}}^{(2)} = 0$$

$$\left(\widehat{\Delta} \cdot \Gamma^{(2)} + \Delta_{\mathsf{sct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathsf{fct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathsf{sct}}^{(2)}\right)_{\mathsf{div}} = 0$$

$$\mathrm{LIM}_{d\to 4}\left(\widehat{\Delta} \cdot \Gamma^{(2)} + \Delta_{\mathsf{sct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathsf{fct}}^{(1)} \cdot \Gamma^{(1)} + \Delta_{\mathsf{fct}}^{(2)}\right)_{\mathsf{fin}} = 0$$

χQED Two-Loop Renormalization



χQED Two-Loop Renormalization

Structure of two-loop counter term Lagrangians

Same structure for $S^{(2)}_{\mathrm{sct}}$ as at one-loop except for

$$-\left(\frac{\hbar\, e^2}{16\pi^2}\right)^2 \sum_j \frac{(\mathcal{Y}_R^j)^2}{3\epsilon} \left(\frac{5}{2}(\mathcal{Y}_R^j)^2 - \frac{2}{3}\,\mathrm{Tr}(\mathcal{Y}_R^2)\right) \overline{S^j_{\overline{\psi}\psi_R}} \;.$$

violating the Ward identity for electron self energy and vertex correction in $\frac{1}{\epsilon}$.

The finite counter terms are also of the same kind

$$S^{(2)}_{\mathrm{fct}} = \left(\frac{\hbar}{16\pi^2}\right)^2 \int \mathrm{d^D x}\, e^4 \left\{ \mathrm{Tr}(\mathcal{Y}_R^4)\frac{11}{48}\bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2 \frac{\mathrm{Tr}(\mathcal{Y}_R^6)}{8}\, \bar{A}_\mu \bar{A}^\mu \bar{A}_\nu \bar{A}^\nu \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36}(\mathcal{Y}_R^j)^2 - \frac{1}{27}\,\mathrm{Tr}(\mathcal{Y}_R^2)\right) \left(\bar{\psi}_j i\bar{\partial\!\!\!/}\, P_R\, \psi_j\right) \right\} \;.$$

χQED Ward Identities

Since in $U(1)$ χQED ghosts do not propagate, we always have

$$\frac{\delta \Gamma_{\rm ren}}{\delta c} = \text{linear expression, and we can require } \frac{\delta \Gamma_{\rm ren}}{\delta c} = \frac{\delta S^{(0)}}{\delta c}.$$

Thus from the STI $\mathcal{S}(\Gamma_{\rm ren}) = 0$ we can derive the **Ward Identities**

$$\left(\partial^\mu \frac{\delta}{\delta A^\mu(x)} - ie\mathcal{Y}_R^j \sum_\phi (\pm)\phi(x) \frac{\delta}{\delta \phi(x)}\right)\Gamma_{\rm ren} = -\partial^2 B(x)\,,$$

implying well-known QED Ward identities

$$ip_\nu \frac{\delta^2 \widetilde{\Gamma}_{\rm ren}}{\delta A_\mu(p) \delta A_\nu(-p)} = 0,$$

$$-ie\mathcal{Y}_R \frac{\partial}{\partial p_\mu} \frac{\delta^2 \widetilde{\Gamma}_{\rm ren}}{\delta \overline{\psi}(-p) \delta \psi(p)} + i\, \frac{\delta^3 \widetilde{\Gamma}_{\rm ren}}{\delta A_\mu(0) \delta \overline{\psi}(-p) \delta \psi(p)} = 0\,.$$

χQED Ward Identities

The finite part of photon self energy at the two-loop level is given by

$$
\begin{aligned}
ip_\nu \widetilde{\Gamma}^{\mu\nu}_{AA}(p)\Big|^2_{\text{fin}} &= \frac{ie^4}{256\pi^4}\frac{\mathrm{Tr}(\mathcal{Y}_R^4)}{3}\Big[\Big(\frac{673}{23} - 6\log(-\overline{p}^2) - 24\zeta(3)\Big) \\
&\quad \times \underbrace{p_\nu(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu})}_{=0} + \frac{11}{8}p_\nu\overline{p}^\mu\overline{p}^\nu\Big] \\
&= -\Big([\widehat{\Delta} + \Delta^1_{\text{ct}}]\cdot\widetilde{\Gamma}\Big)^2_{\text{fin},\,A_\mu(-p)c(p)}\,,
\end{aligned}
$$

and for the electron self energy and vertex correction

$$
\begin{aligned}
& - i\,e\mathcal{Y}_R\frac{\partial}{\partial p_\mu}\widetilde{\Gamma}_{\psi(p)\overline{\psi}(-p)}\Big|^2_{\text{fin}} + i\,\widetilde{\Gamma}^\mu_{\psi(p)\overline{\psi}(-p)A(0)}\Big|^2_{\text{fin}} \\
&= i\mathcal{Y}_R\frac{e^5}{256\pi^4}\overline{\gamma}^\mu\,P_R\Big(\frac{\mathcal{Y}_R^2\ \mathrm{Tr}(\mathcal{Y}_R^2)}{27} - \frac{127\mathcal{Y}_R^4}{36}\Big) \\
&= -\frac{\partial}{\partial q_\mu}\Big([\widehat{\Delta} + \Delta^1_{\text{ct}}]\cdot\widetilde{\Gamma}\Big)^2_{\text{fin},\,\psi(p-q)\overline{\psi}(-p)c(q)}(q=0)\,.
\end{aligned}
$$

χQED at Two-Loop Summary

- We were able to obtain the divergent and finite counter term structure up to two-loop needed to render the theory finite and restore the symmetry
- The counter terms are rather compact and did not change appreciably
- Ward identities are checked to be satisfied after renormalization
- The necessary diagrams could be systematically identified

  Outlook:

  Continue at three-loop [work in progress];

  Alternative couplings for FFV to come closer to the SM;

  Non-Abelian theory at two-loop [work in progress]

  Explore relation to other schemes

From χQED to $SU(N)$

Now external fields appear in loop calculations and certain simplifications are no longer possible due to ccG-interaction

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{g-fix}} + \mathcal{L}_{\text{ext}}$$

$$\mathcal{L}_{\text{fermion}} = i\overline{\psi_i}\partial\!\!\!/\delta_{ij}\psi_j + g\overline{\psi_{Ri}}\overline{G\!\!\!/}^a T^a_{ij}\psi_{Rj}$$

$$\mathcal{L}_{\text{FP}} = \partial^\mu \bar{c}^a(\partial_\mu \delta_{ac} + g f_{abc} G^b_\mu)c^c$$

$$\mathcal{L}_{\text{ext}} = \rho^\mu_a s G^a_\mu + \zeta_a s c^a + \bar{R}^i s\psi_{Ri} + R^i s\overline{\psi_{Ri}}$$

$$s(G^a_\mu) = \partial_\mu c^a + g f_{abc} G^b_\mu c^c$$
$$s(c^a) = -\frac{1}{2} g f_{abc} c^b c^c$$

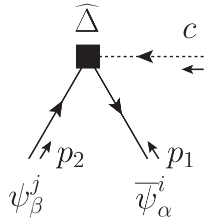
Also, STI now more involved [C.P.Martin]

$$\mathcal{S}(\Gamma_{\text{DR}}) = \Delta \cdot \Gamma_{\text{DR}} + \Delta_{\text{ct}} \cdot \Gamma_{\text{DR}} + \int \mathrm{d^D x}[\frac{\delta S^{(n)}_{\text{ct}}}{\delta K_\phi} \cdot \Gamma_{\text{DR}}]\frac{\delta \Gamma_{\text{DR}}}{\phi(x)}$$

From χQED to $SU(N)$

The model has been studied at one-loop [C.P. Martin, Sanchez-Ruiz '99], [Bélusca-Maïto, Ilakovac, Mađor-Božinović, Stöckinger, 2020] (with scalars)

The tree-level breaking is essentially the same



$$= \frac{g}{2}\, T^a_{Rij} \left((\widehat{\not p_1} + \widehat{\not p_2}) + (\widehat{\not p_1} - \widehat{\not p_2})\gamma_5 \right)_{\alpha\beta}$$

$$= g\, T^a_{Rij} \left(\widehat{\not p_1} P_R + \widehat{\not p_2} P_L \right)_{\alpha\beta} .$$

$$S^{(1)}_{\text{fct}} = \frac{\hbar}{16\pi^2}\Big(g^2 \frac{S_2(R)}{6}\Big(5 S_{GG} + S_{GGG} - \int \mathrm{d}^4\mathrm{x}\, G^{a\mu}\partial^2 G^a_\mu\Big)$$

$$+ g^2 \frac{(T_R)^{abcd}}{3} \int \mathrm{d}^4\mathrm{x}\, \frac{g^2}{4} G^a_\mu G^{b\mu} G^c_\nu G^{d\nu}$$

$$+ g^2 \Big(1 + \frac{\xi - 1}{6}\Big) C_2(R) S_{\overline{\psi}\psi}\Big) + \textit{anomalies} + \textit{external fields}$$