Lepton Flavour Violation in the SMEFT

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Leptons:

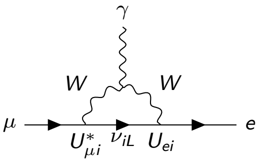
$$\ell_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}, \alpha_R \qquad \text{with } \alpha = e, \mu, \tau$$

- The Standard Model with $m_{\nu_\alpha} = 0$ has exact $U(1)_{B/3-L_\alpha}$
- Neutrino oscillations break all three symmetries. We can observe a μ turn into an e via neutrino oscillations (with two interaction vertices)

$$\mu \to e\bar{\nu}_e\nu_\mu \longrightarrow (\nu_\mu \to \nu_e) + (L_e = 0) \to e + (L_e = 0)$$

- but we have never observed a contact interaction between the charged leptons that change lepton flavour ≡ LFV.

Why LFV?



In **SM+ν_R** LFV is small:

$Br(\mu \to e\gamma) \simeq G_F^2(\Delta m_\nu^2)^2 \sim 10^{-50}$

- Smoking gun signal of New Physics
- LFV can give insight on neutrino masses? (also on BAU, if is via leptogenesis?)
- Many models predict potentially observable LFV rates

For some LFV reviews see Kuno+Okada hep-ph/9909265, Calibbi+Signorelli 1709.00294

Experimental searches

Process	Current bound on BR	Future Sensitivity
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	10^{-14} MEGII
$\mu \to \bar{e}ee$	$< 1.0 \times 10^{-12}$ SINDRUM	10^{-16} Mu3e
$\mu A \to eA$	$< 7 \times 10^{-13}$ SINDRUMII	$10^{-16} \to 10^{-18}$ COMET, Mu2e
$\tau \to l\gamma$	$< 3.3 \times 10^{-8}$	$3 \times 10^{-9}(e), 10^{-9}(\mu)$
$\tau \to e\bar{e}e$	$< 2.7 \times 10^{-8}$	5×10^{-9}
$\tau \to \mu\bar{\mu}\mu$	$< 2.1 \times 10^{-8}$	4×10^{-9}
$\tau \to \mu\bar{e}e, e\bar{\mu}\mu$	$< 1.8, 2.7 \times 10^{-8}$ Belle	$3, 5 \times 10^{-9}$ BelleII
...	...	...
$\tau \to l\pi^0$	$< 8.0 \times 10^{-8}$	4×10^{-9}
$\tau \to l\eta$	$< 6.5 \times 10^{-8}$	7×10^{-9}
$\tau \to l\rho$	$< 1.2 \times 10^{-8}$ Belle	10^{-9} BelleII
$K^0 \to \mu^\pm e^\mp$	$< 4.7 \times 10^{-12}$	
$B_d^0 \to \tau^\pm \mu^\mp$	$< 1.2 \times 10^{-5}$ LHCb	$\sim 10^{-6}$?
...	...	...
$h \to e^\pm \mu^\mp$	$< 6.1 \times 10^{-5}$ Atlas	2.1×10^{-5}
$h \to e^\pm \tau^\mp$	$< 2.2 \times 10^{-3}$ CMS	2.4×10^{-4}
$h \to \tau^\pm \mu^\mp$	$< 1.5 \times 10^{-3}$ CMS	2.3×10^{-4} ILC
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ Atlas	
$Z \to l^\pm \tau^\mp$	$< 10^{-7}$ Atlas	

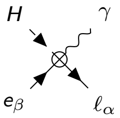
- Muon searches lead in sensitivity
- Tau searches have a large number of possible decay channels

Outline

Outline

Assuming that New Physics responsible for LFV is heavy $\Lambda \gtrsim 4$ TeV, we can parametrise it with SMEFT operators

Dipoles



$$2\sqrt{2}G_F C_{e\gamma}^{\alpha\beta} y_\mu (\bar{\ell}_\alpha H\sigma \cdot F e_\beta)$$

At tree-level, the rate of $\mu \to e\gamma$ is ($\langle H \rangle = v$ and broken $SU(2) \otimes U(1)_Y$)

$$Br(\mu \to e\gamma) = 384\pi^2 \left(\left| C_{e\gamma}^{\mu e} \right|^2 + \left| C_{e\gamma}^{e\mu} \right|^2 \right) < 4.3 \times 10^{-13}$$

$$\to \left| C_{e\gamma}^{e\mu} \right| \lesssim 10^{-8}$$

Assuming

$$m_\mu 2\sqrt{2}G_F C_{e\gamma} \sim \frac{e m_\mu}{16\pi^2\Lambda^2}$$

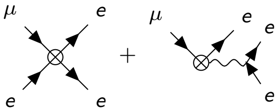
MEG can probe up to

$$\Lambda \sim 100 \text{ TeV}$$

The rate is at tree-level here. Low energy observables are calculated in broken SU(2) and integrating out heavy fields: SMEFT $\leftrightarrow$ WET.

$$\text{WET convention : } \delta\mathcal{L}^{d>4} = \sum_d \frac{C^{flavour}_{Lorentz,Chirality}\mathcal{O}^{flavour}_{Lorentz,Chirality}}{v^{d-4}} \qquad \text{with } 2\sqrt{2}G_F = \frac{1}{v^2}$$

Four lepton operators



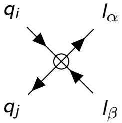
$$Br(\mu \to eee) = 2\left|C^{e\mu ee}_{V,LL} + 4eC^{e\mu}_{D,R}\right|^2 + \left|C^{e\mu ee}_{V,LR} + 4eC^{\mu e}_{D,R}\right|^2 + \frac{\left|C^{e\mu ee}_{S,LL}\right|^2}{8}$$
$$+ \left(64 \log\frac{m_\mu}{m_e} - 136\right)\left|eC^{e\mu}_{D,R}\right|^2 + \{L \leftrightarrow R\}$$

Okada+Okumura+Shimizu, hep-ph/9906446

Current (future) upper limit $Br(\mu \to 3e) < 10^{-12} \to 10^{-16}$ implies

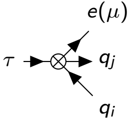
$$\left|C^{e\mu ee}_{V,XX}\right| \leq 7 \times 10^{-7(\to -9)}, \qquad \left|C^{e\mu ee}_{V,XY}\right| \leq 10^{-6(\to -8)}$$
$$\left|C^{e\mu ee}_{S,XX}\right| \leq 2.8 \times 10^{-6(\to -8)}$$

Two-lepton two-quarks operators

- 

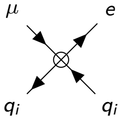
: LFV hadron decays ($K^0 \to \mu e, B^0 \to \tau\mu, \ldots$)

Davidson+Saporta 1807.10288

- 

: LFV τ hadron decays ($\tau \to \pi e(\mu), \tau \to \eta e(\mu), \ldots$)

Black et al. hep-ph/0206056; Husek et al. 2009.10428; Banerjee et al. 2203.14919

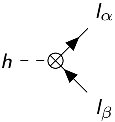
- 

: $\mu \to e$ conversion in nuclei (muon in a $1s$ state converts into an electron while in orbit)

Kitano+Koike+Okada hep-ph/0203110; Cirigliano et al. 0904.0957

Yukawas and Penguins

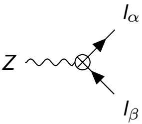
- $C_{eH}^{\alpha\beta}(\bar{\ell}_\alpha H e_\beta)(H^\dagger H)$ contributes to the lepton mass matrix $\longrightarrow$ SM Higgs h acquire LFV couplings in the mass basis



$$\propto C_{eH}^{\alpha\beta}\frac{v^2}{\Lambda^2}$$

Giudice+Lebedev 0804.1753

- $iC_{He}^{\alpha\beta}(\bar{e}_\alpha\gamma e_\beta)(H^\dagger \overleftrightarrow{D} H) + \mathcal{O}_{H\ell}^{(1),(3)}$ give LFV Z interactions



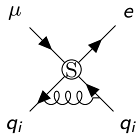
$$\propto (C_{He}^{\alpha\beta}\gamma P_R + C_{H\ell(1)+(3)}^{\alpha\beta}\gamma P_L)\frac{v^2}{\Lambda^2}$$

SM Loops in the EFT

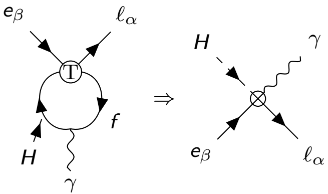
- SM loops decorate the contact interactions, causing the Wilson coefficients to run

$$\vec{C}(\mu_f) = \vec{C}(\mu_i)U(\mu_i, \mu_f)$$

- Solving the RGEs at n-loop resum $\alpha^{m+n-1} log^{m+n}(\mu_f/\mu_i)$ contributions
- One-loop QCD running of scalar and tensor quark operators can be numerically relevant ($\sim$ few $\times$ 10%)



- Electroweak loops can mix operators with different Lorentz structure and external legs



Probe a difficult-to-detect operator via its mixing into a tightly constrained one

Top-down vs bottom-up?

Loops are interesting for LFV

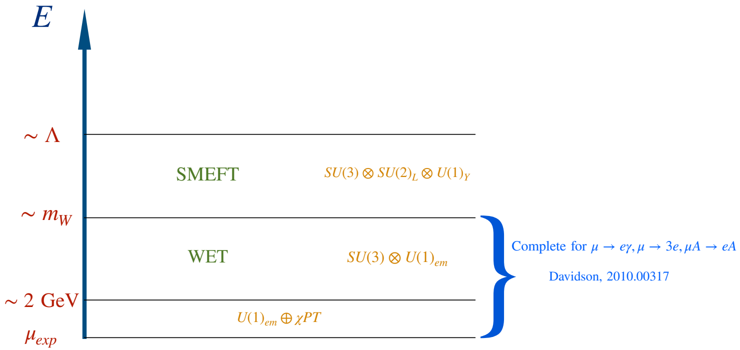
- Top-down: model-build, match onto EFT, solve the RGEs down to the experimental scale and check with experiments.
- Bottom-up: calculate observables in the appropriate EFT at the experimental scale, run to the high scale and identify the region in coefficient space accessible to experiment

Bottom-up:

- repeat only when new data are available (not so frequently for LFV...);
- no model-building needed: works for any model at $\sim \Lambda$;
- maybe one can learn something about BSM?

Outline

Running from data to the high scale



- Focus on $\mu \to e\gamma, \mu \to 3e, \mu A \to eA$ because they are the most sensitive and expect the best improvement
- Complete in bottom-up = every contribution that could be detected in the experiments

WET Running $m_\mu \to m_W$

Davidson, 2010.00317

$\mu \to e\gamma$ probe dipoles at m_μ: $C^{e\mu}_{D,X}(m_\mu)$

$$
\begin{aligned}
C_{D,X}(m_\mu) \;=\;& C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\
&-\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\
&+16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\
&-8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\ \mathrm{GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right) f_{TD} \\
&+16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\ \mathrm{GeV}}\left(\sum_{u,c} 4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)
\end{aligned}
$$

- Solve the RGEs to express in terms of coefficient at m_W
- Resummed one-loop QCD running: $\lambda = \alpha_S(2\ \mathrm{GeV})/\alpha_S(m_W)$, $f_{TD} \simeq .862$, $a_S = 12/23$, $a_T = -4/23$
- Include QED one-loop at leading log, some $\alpha_e^2 log^2$ effect and two-loop vector to dipole mixing

WET Running $m_\mu \to m_W$

Davidson, 2010.00317

MEG is sensitive to $\sim$ every operator in the basis (at m_W)

coefficient	$\mu \to e\gamma$
$\lvert C_{D,X}\rvert$	1.12×10^{-8}
$\lvert C_{GG,X}\rvert$	
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.84×10^{-4}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.45×10^{-4}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	3.62×10^{-5}
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.07×10^{-8}

$+\ldots$

These are sensitivities $\neq$ constraints

Apply the same procedure for $\mu \to 3e$, $\mu A \to eA$

Flat directions in bottom-up

- If we combine $\mu \to e\gamma$, $\mu \to 3e$, $\mu A \to eA(\times 2$ for Al, $Ti)$ we can constrain 12 (with chiral leptons) directions in coefficient space at low-energy
- But the RGEs mix the low energy operator with all the ~ 90 operators in the basis at higher energies $\Rightarrow$ many "flat" (unconstrainable) directions

What to do?

1. Sensitivities are useful = smaller values are okay, larger values require cancellations
2. Define a (scale-dependent) basis that coincides with the directions probed by experiments
   Davidson+Echenard 2204.00564 - Davidson 2010.00317

$$\Gamma(X) \propto \left| \vec{C}(\Lambda) \cdot \vec{v}_X(\Lambda) \right|$$

In this basis one can identify a 12-dimensional ellipse where models at Λ should sit

Outline

WET to SMEFT matching

We want to continue running up towards Λ in SMEFT

Match WET onto SMEFT at m_W: what basis for leptons? Dimension six Yukawas contribute to the mass matrix

$$[m_e]_{\alpha\beta} = v\left([Y_e]_{\alpha\beta} + C_{eH}^{\alpha\beta}\frac{v^2}{\Lambda^2}\right)$$

We stay in the mass-basis for the leptons $\Rightarrow$ Y_e is not diagonal

Upper limits on $h \to \bar{l}_\alpha l_\beta$ suggest that the off-diagonal Yukawas can be neglected in the RGEs (within upcoming sensitivities) A+Davidson 2103.07212

SMEFT running $m_W \to \Lambda$

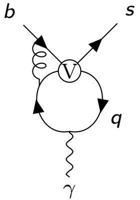
- State-of-the-art: one-loop running of dimension six operators Alonso+Jenkins+Manohar+Trott 1308.2627 - 1310.4838 - 1312.2014
- some one-loop anomalous dimension of dimension eight Davidson+Gorbahn 1909.07406, Chala+Guedes+... 2205.03301 2106.05291
- +many tools represented at this workshop

Is it enough for LFV?

Two loops can be interesting

Vectors-to-dipoles mixing is two-loop at leading order in WET.

Ex: In QCD the leading corrections to the $b \to s\gamma$ dipole due to four-fermion operators are from the 2-loop RGEs

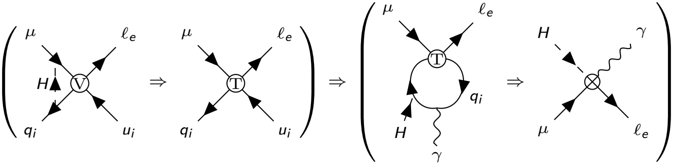


Ciuchini+Franco+Reina+Silvestrini hep-ph/9311357

SMEFT running: two-loops for LFV?

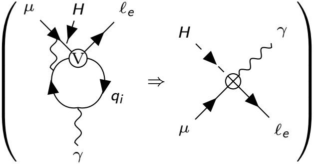
A+Davidson 2103.07212

In SMEFT **vector-to-dipole mixing** (ex: $\mathcal{O}_{\ell q} = (\bar{\ell}\gamma\ell)(\overline{q}\gamma q)$) can be a second-order one-loop RGE effect

(1) 

$$\Rightarrow C^{e\mu}_{e\gamma}(m_W) \sim \frac{eC^{e\mu ii}_{\ell q}(\Lambda)}{(16\pi^2)^2} y^2_{u_i} \log^2\left(\frac{\Lambda}{m_W}\right)$$

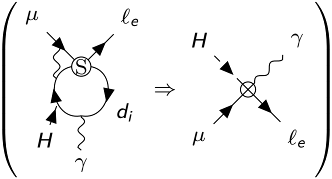
or at two-loop anomalous dimension directly Some calculated Miro+Fernandez. . . 2112.12131

(2) 

$$\Rightarrow C^{e\mu}_{e\gamma}(m_W) \sim \frac{e^3 C^{e\mu ii}_{\ell q}(\Lambda)}{(16\pi^2)^2} \log\left(\frac{\Lambda}{m_W}\right)$$

If $u_i = c$, $\frac{(1)}{(2)} \sim 10^{-3}$. Only for $u_i = t$ the one-loop RGEs dominate

Scalar operators $\mathcal{O}^{\alpha\beta ii}_{\ell edq} = (\bar{\ell}_\alpha e_\beta)(\bar{d}_i q_i)$ can mix with the dipole at two-loop? (leading order because there are no tensors with down quarks at dimension six)



MEGII ($\mu \to e\gamma$) could probe scalar coefficients with $i = b$ up to $\Lambda \sim 100$ TeV

SMEFT running: dimension eight?

A+Davidson 2103.07212

LFV observables can be sensitive to dimension eight amplitudes.
With $Br(\mu \to e\gamma) \sim 10^{-14}$, experiments could probe

$$C_T^{e\mu\tau\tau} \sim 10^{-9}$$

Tensors with leptons are at dimension eight in SMEFT

$$(2\sqrt{2}G_F)^2 C_T^{e\mu\tau\tau} \sim \frac{1}{\Lambda^4} \quad \Rightarrow \quad \Lambda \sim 30 \text{ TeV}$$

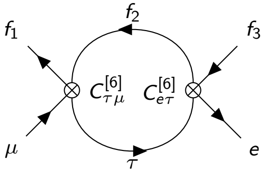
A subset of dimension eight operator are interesting (especially those that match onto WET interactions only at dimension eight)

- Include in the SMEFT$\leftrightarrow$WET matching these dimension eight operators
- but still misses the SMEFT running...

$\mu \to e = \mu \to \tau \times \tau \to e$

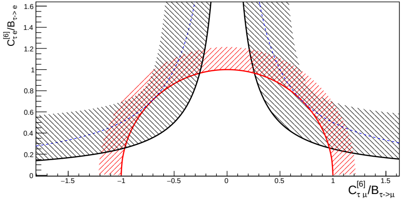
A+Davidson+Gorbahn 2202.09246

- $\mu \to e$ observables can be sensitive to dimension eight operators
- make a $\mu \to e$ dimension eight with two insertion of dimension six $\tau \leftrightarrow \mu$ and $\tau \leftrightarrow e$



$$\frac{\Delta C^{[8]}_{e\mu}}{\Lambda^4} \sim \frac{C^{[6]}_{\tau\mu}}{\Lambda^2}\frac{C^{[6]}_{e\tau}}{\Lambda^2} \times \frac{\log}{16\pi^2}$$

- with the exceptional upcoming sensitivity of $\mu \to e$, can probe parameter space beyond the reach of direct $\tau \to l$ $(l = e, \mu)$ searches

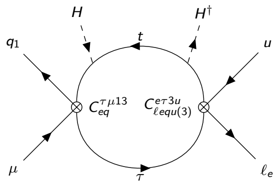


- $\mu \to e : \left| C^{[6]\tau\mu} C^{[6]e\tau} \right| \lesssim B_{\mu\leftrightarrow e}$
- $\tau \to l : \frac{\left| C^{[6]\tau\mu} \right|^2}{B^2_{\tau\leftrightarrow\mu}} + \frac{\left| C^{[6]e\tau} \right|^2}{B^2_{\tau\leftrightarrow e}} \lesssim 1$

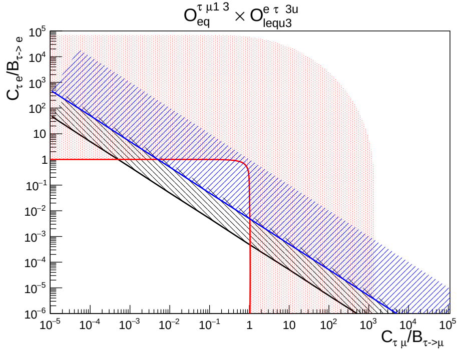
Hyperbola enters the ellipse if

$$B_{\mu\leftrightarrow e} < 1/2 B_{\tau\leftrightarrow e} B_{\tau\leftrightarrow\mu}$$

$\mu A \to eA$ vs $B \to \tau l$



Comparison:



- The pair $\mathcal{O}_{eq}^{\tau\mu 13} = (\overline{\tau}\gamma\mu)(\overline{q}_1\gamma q_3)$, $\mathcal{O}_{\ell equ(3)} = (\overline{\ell}_e\sigma\tau)(\overline{q}_3\sigma u)$ mix with a dimension eight scalar with up quarks that contributes to the rate of $\mu A \to eA$

$$\frac{\Delta C_S^{e\mu uu}}{\Lambda^4} \sim y_t^2 \frac{C_{eq}^{\tau\mu 13} C_{\ell equ(3)}^{e\tau 3u}}{16\pi^2\Lambda^4} \log$$

- The contact interactions $C_{eq}^{\tau\mu 13}$, $C_{\ell equ(3)}^{e\tau 3u}$ contribute to $B_d^0 \to \mu^\pm\tau^\mp$, $B^+ \to \overline{\tau}\nu$

- current $\mu A \to eA$
- future $\mu A \to eA$
- B decays

Summary

- LFV is New Physics that is expected to occur and the next generation of muon experiments are expected to deliver impressive sensitivities
- If LFV is heavy, EFT parametrisation is a natural choice
- Including loops in the EFT calculations is interesting because of operator mixing (if $\sim$ any $\mu \to e$ interactions is mediated by heavy states with masses $\Lambda \lesssim 100$ TeV, we should see it)
- Running from data to a high scale Λ we can identify a region in coefficient space (12$-$dimensional for $\mu \to e$) where BSM models must sit
- In SMEFT, the known RGEs for LFV are missing contributions within future sensitivity (selected two-loop anomalous dimension and some dimension eight running)
- The sensitivity of $\mu \to e$ to the product of $\mu \to \tau \times \tau \to e$ interactions can compete with direct $\tau \to e(\mu)$ searches, probing parameter space beyond their reach.

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THANK YOU!

WET Basis

$$\mathcal{O}^{ll}_{V,YY} = (\bar{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma_\alpha P_Y l), \quad \mathcal{O}^{ll}_{V,YX} = (\bar{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma_\alpha P_X l)$$

$$\mathcal{O}^{ll}_{S,YY} = (\bar{e}P_Y \mu)(\bar{l}P_Y l) \qquad \mathcal{O}^{\tau\tau}_{S,YX} = (\bar{e}P_Y \mu)(\bar{\tau}P_X \tau)$$

$$\mathcal{O}^{\tau\tau}_{T,YY} = (\bar{e}\sigma^{\alpha\beta} P_Y \mu)(\bar{\tau}\sigma_{\alpha\beta} P_Y \tau)$$

$$\mathcal{O}^{qq}_{V,YY} = (\bar{e}\gamma^\alpha P_Y \mu)(\bar{q}\gamma_\alpha P_Y q) \quad , \quad \mathcal{O}^{qq}_{V,YX} = (\bar{e}\gamma^\alpha P_Y \mu)(\bar{q}\gamma_\alpha P_X q)$$

$$\mathcal{O}^{qq}_{S,YY} = (\bar{e}P_Y \mu)(\bar{q}P_Y q) \quad , \quad \mathcal{O}^{qq}_{S,YX} = (\bar{e}P_Y \mu)(\bar{q}P_X q)$$

$$\mathcal{O}^{qq}_{T,YY} = (\bar{e}\sigma^{\alpha\beta} P_Y \mu)(\bar{q}\sigma_{\alpha\beta} P_Y q)$$

$$\mathcal{O}_{D,L} = m_\mu \overline{e_R}\sigma^{\alpha\beta}\mu_L F_{\alpha\beta} \qquad m_\mu \overline{e_L}\sigma^{\alpha\beta}\mu_R F_{\alpha\beta}$$

$$\mathcal{O}_{GG,Y} = \frac{1}{v}(\bar{e}P_Y \mu)G_{\alpha\beta}G^{\alpha\beta} \quad , \quad \mathcal{O}_{G\tilde{G},Y} = \frac{1}{v}(\bar{e}P_Y \mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta}$$

$$\mathcal{O}_{GGV,Y} = \frac{1}{v^2}(\bar{e}\gamma_\sigma P_Y \mu)G_{\alpha\beta}\partial_\beta G^{\alpha\sigma} \quad , \quad \mathcal{O}_{G\tilde{G}V,Y} = \frac{1}{v^2}(\bar{e}\gamma_\sigma P_Y \mu)G_{\alpha\beta}\partial_\beta \widetilde{G}^{\alpha\sigma}$$

$$\mathcal{O}_{FF,Y} = \frac{1}{v}(\bar{e}P_Y \mu)F_{\alpha\beta}F^{\alpha\beta} \quad , \quad \mathcal{O}_{F\tilde{F},Y} = \frac{1}{v}(\bar{e}P_Y \mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

$$\mathcal{O}_{FFV,Y} = \frac{1}{v}(\bar{e}\gamma^\sigma P_Y \mu)F^{\alpha\beta}\partial_\beta F_{\alpha\sigma} \quad , \quad \mathcal{O}_{F\tilde{F}V,Y} = \frac{1}{v}(\bar{e}\gamma^\sigma P_Y \mu)F^{\alpha\beta}\partial_\beta \widetilde{F}_{\alpha\sigma}$$

where $l \in \{e, \mu\}$, $q \in \{u, d, s, c, b\}$.
The operators are added to the Lagrangian as $2\sqrt{2}G_F C_\Box \mathcal{O}_\Box$

SMEFT dimension six

$1: X^3$		$2: H^6$		$3: H^4D^2$		$5: \psi^2H^3 + \text{h.c.}$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$			Q_{HD}	$\left(H^\dagger D^\mu H\right)^* \left(H^\dagger D_\mu H\right)$	Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$						

$4: X^2H^2$		$6: \psi^2 XH + \text{h.c.}$		$7: \psi^2H^2 D$	
Q_{HG}	$H^\dagger H\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p \gamma^\mu l_r)$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_p \tau^I \gamma^\mu l_r)$
Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde H\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p \gamma^\mu e_r)$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde H\, W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p \gamma^\mu q_r)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde H\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p \gamma^\mu u_r)$
Q_{HWB}	$H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p \gamma^\mu d_r)$
$Q_{H\widetilde WB}$	$H^\dagger \tau^I H\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$	Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p \gamma^\mu d_r)$

$8: (\bar LL)(\bar LL)$		$8: (\bar RR)(\bar RR)$		$8: (\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$8: (\bar LR)(\bar RL) + \text{h.c.}$		$8: (\bar LR)(\bar LR) + \text{h.c.}$		$8: (\not B) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r) \epsilon_{jk} (\bar q_s^k d_t)$	Q_{duql}	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} (d^{\alpha T}_p C u^\beta_r)(q^{j\gamma T}_s C l^k_t)$
		$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r) \epsilon_{jk} (\bar q_s^k T^A d_t)$	Q_{qque}	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} (q^{j\alpha T}_p C q^{k\beta}_r)(u^{\gamma T}_s C e_t)$
		$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r) \epsilon_{jk} (\bar q_s^k u_t)$	Q_{qqql}	$\epsilon_{\alpha\beta\gamma} \epsilon_{mn} \epsilon_{jk} (q^{m\alpha T}_p C q^{j\beta}_r)(q^{k\gamma T}_s C l^n_t)$
		$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar q_s^k \sigma^{\mu\nu} u_t)$	Q_{duue}	$\epsilon_{\alpha\beta\gamma} (d^{\alpha T}_p C u^\beta_r)(u^{\gamma T}_s C e_t)$

Dimension eight but not dimension six?

The tensors with down-type quarks and charged leptons match at dimension eight at leading order in the SMEFT:

$$C^{e\mu\tau\tau}_{T,RR} = \frac{v^2}{\Lambda^2_{\rm NP}} C^{e\mu\tau\tau}_{L^2E^2H^2(4)}$$
$$C^{e\mu\tau\tau}_{T,LL} = \frac{v^2}{\Lambda^2_{\rm NP}} C^{\mu e\tau\tau *}_{L^2E^2H^2(4)}$$
$$C^{e\mu d_n d_n}_{T,RR} = \frac{v^2}{\Lambda^2_{\rm NP}} \sum_j V_{jn} C^{e\mu jn}_{LEQDH^2(5)}$$
$$C^{e\mu d_n d_n}_{T,LL} = \frac{v^2}{\Lambda^2_{\rm NP}} \sum_j V^*_{jn} C^{\mu e jn *}_{LEQDH^2(5)}$$

where $n = d, s, b$ and

$$\mathcal{O}_{L^2E^2H^2(4)} = (\bar{\ell}H\sigma e)(\bar{\ell}H\sigma e)$$
$$\mathcal{O}_{LEQDH^2(5)} = (\bar{\ell}H\sigma e)(\overline{q}H\sigma d)$$

Otherwise, the dimension eight operators are $\sim dim6 \times (H^\dagger H)$