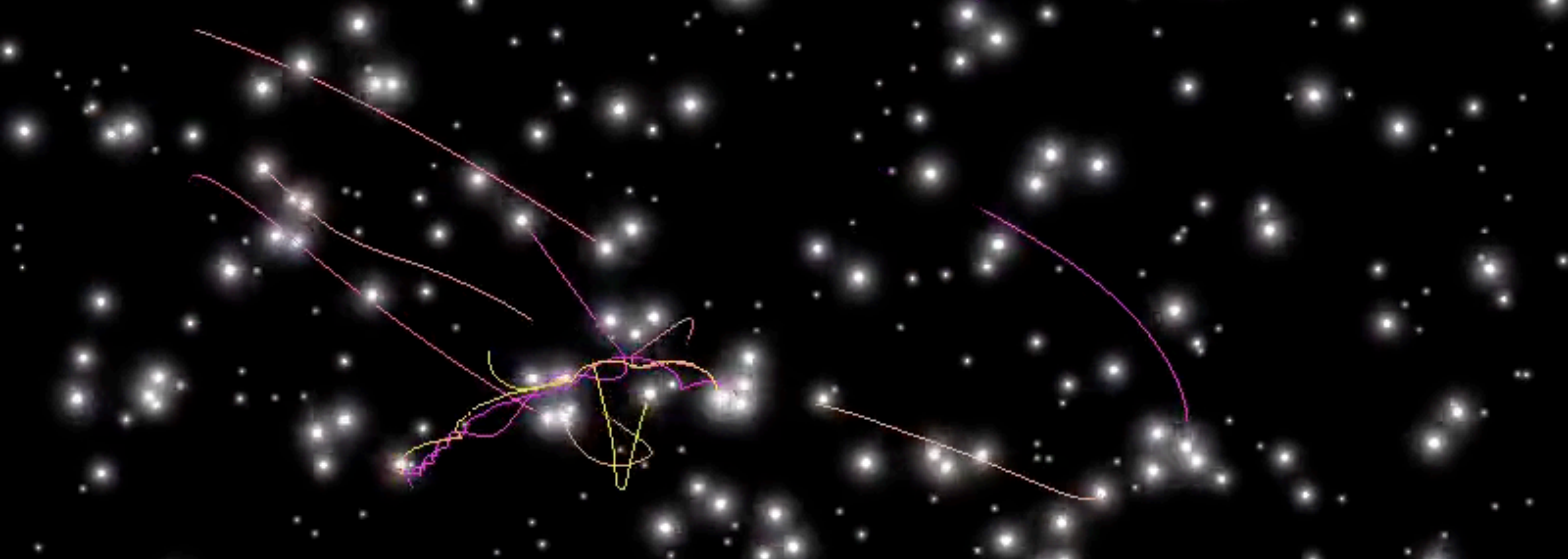
Rates and populations
of dynamically formed binary black holes

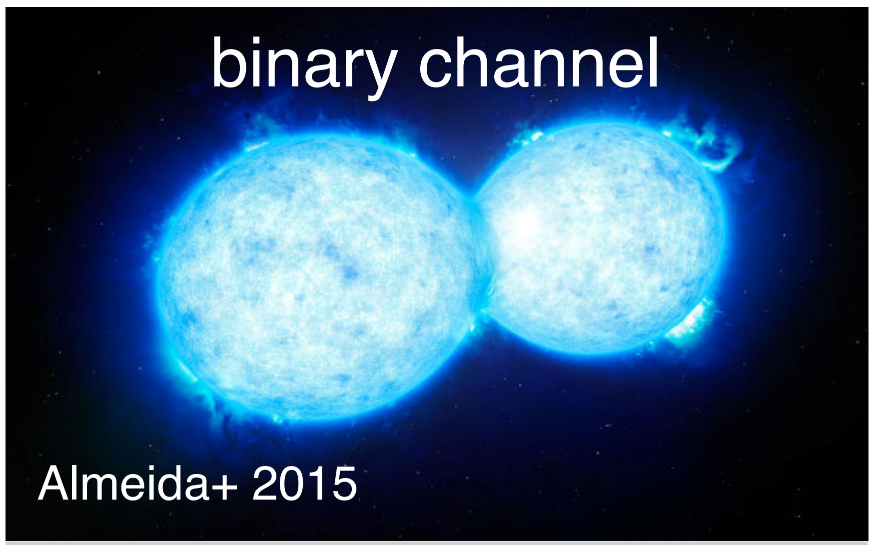


Mark Gieles

Fabio Antonini (Cardiff)
Daniel Marín Pina (ICCUB)

GW modelling workshop
ICCUB October 10, 2022

How do binary black hole binaries (BBHs) form?



Belczynski+ 2002; de Mink & Mandel 2016; Mandel & de Mink 2016; Marchant+ 2016; Farr+ 2017; Mapelli+ 2017; Schneider+ 2017; Gerosa+ 2018

Portegies & Zwart & McMillan 2000; Samsing+2014; Rodriguez+ 2015; Farr+ 2017; Silsbee & Tremaine 2017; Antonini+ 2018; Hong+ 2018; Rodriguez & Loeb 2018; Antonini & Gieles 2020a,b

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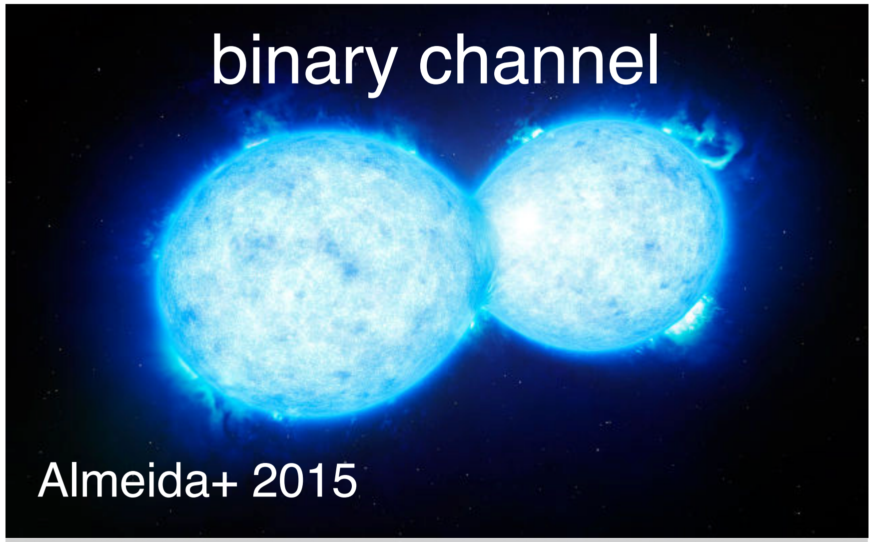
How do binary black hole binaries (BBHs) form?

Eccentric BBHs (eBBHs)



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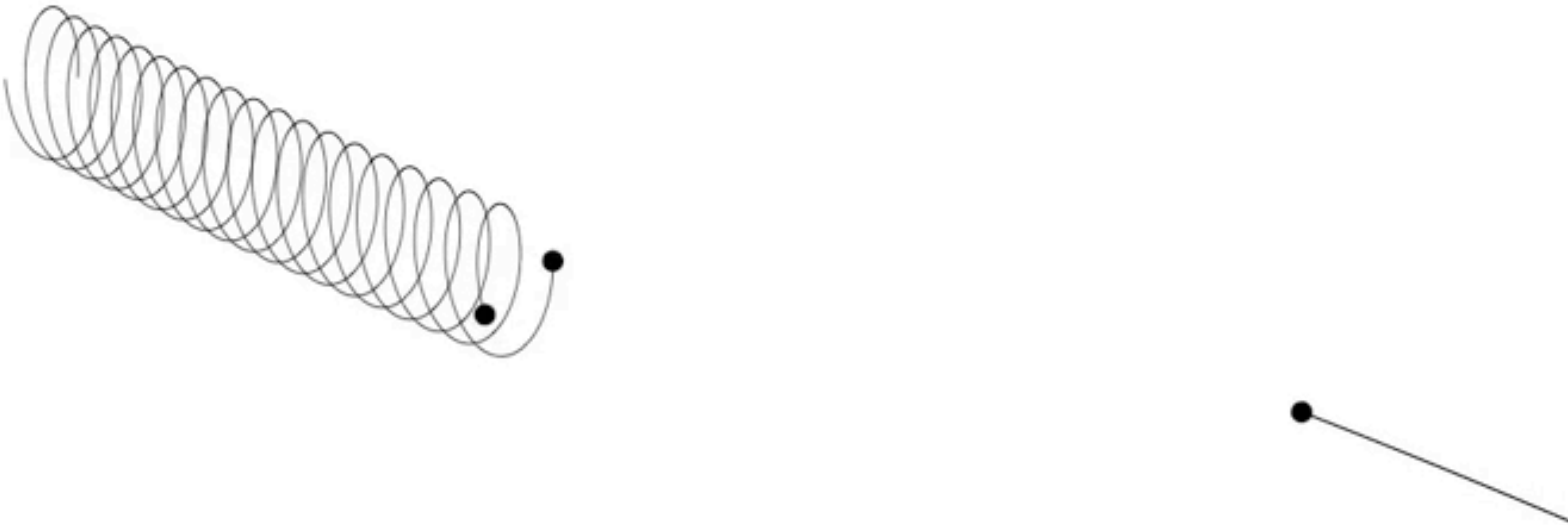
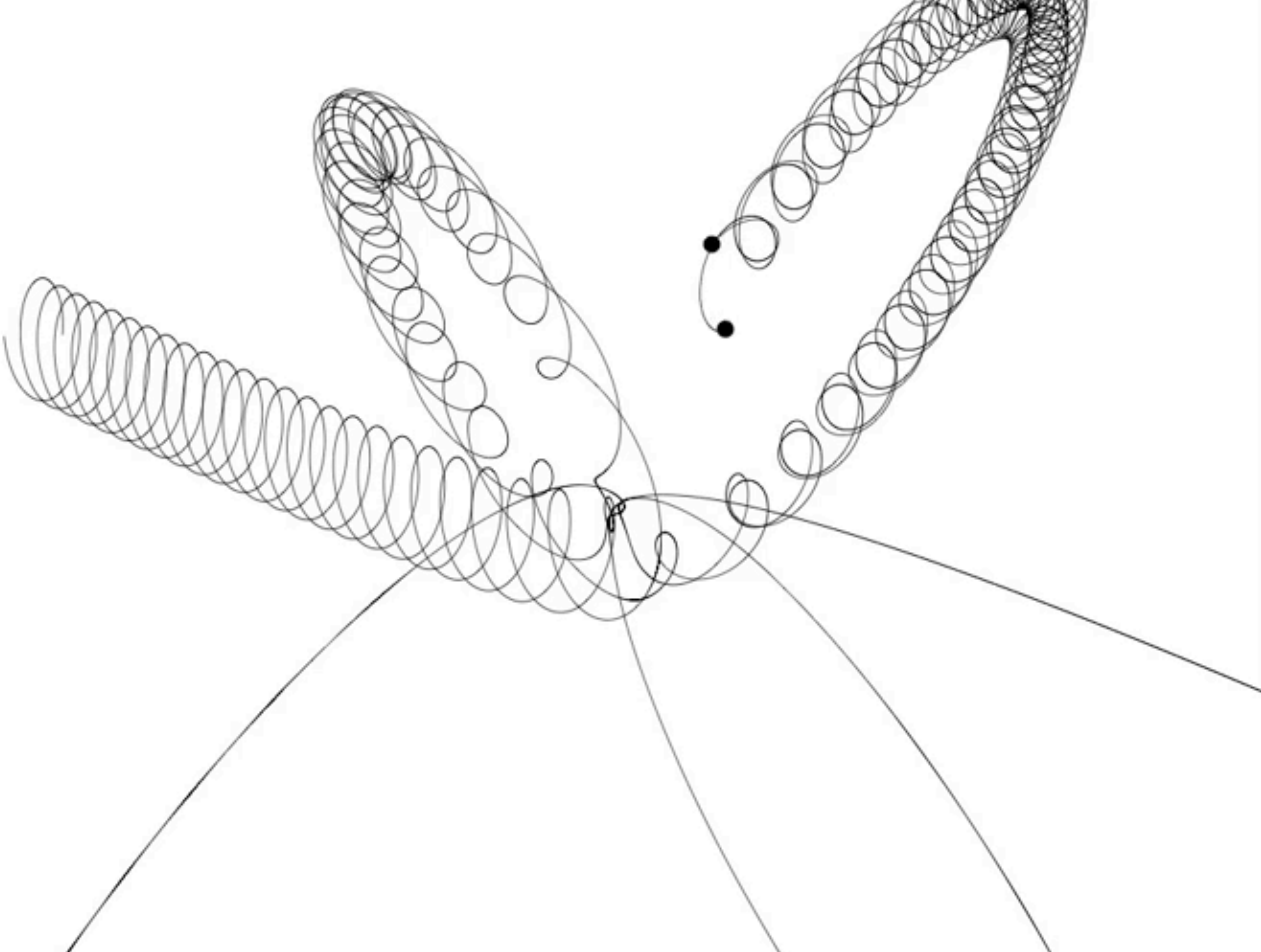
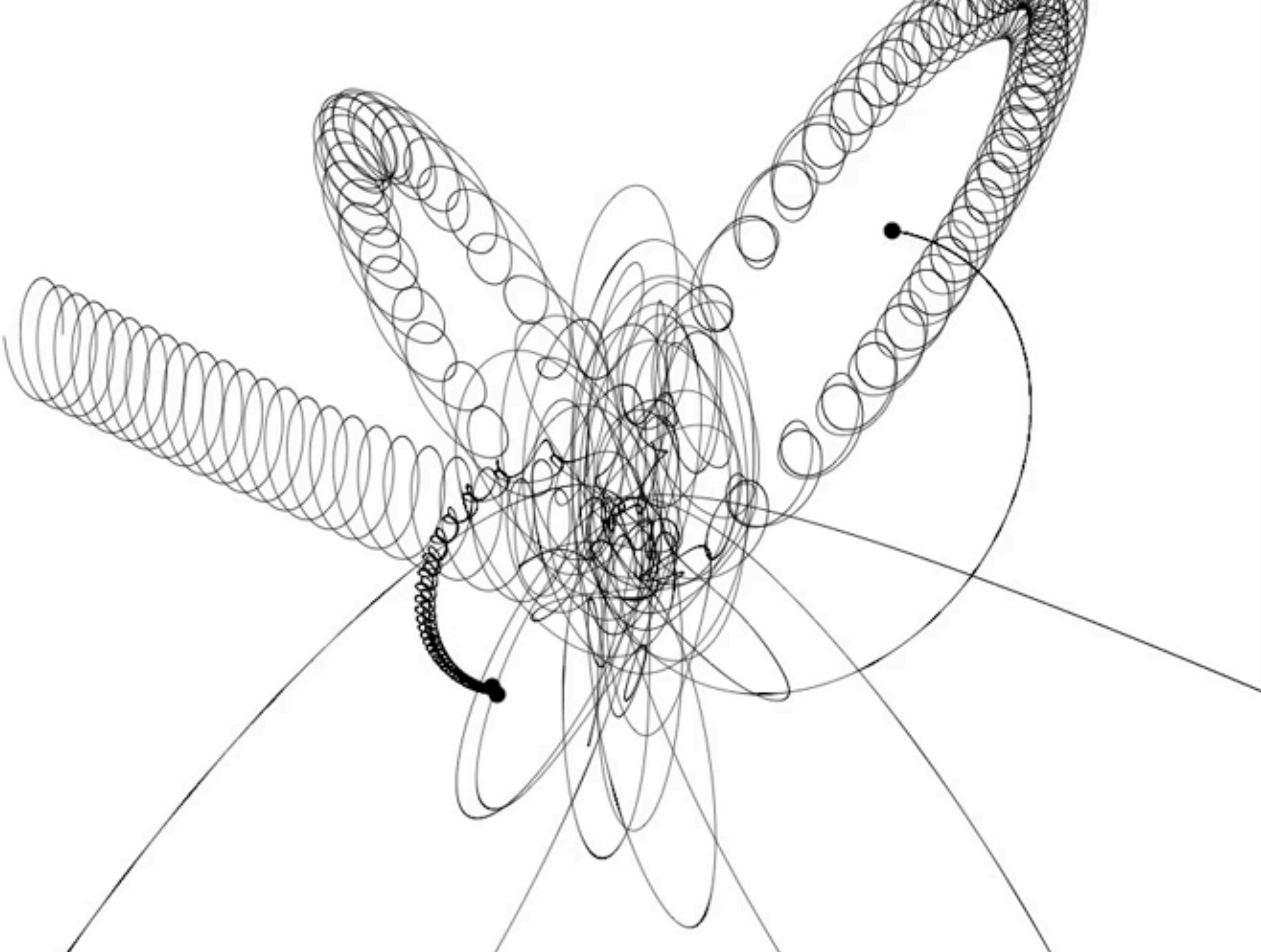
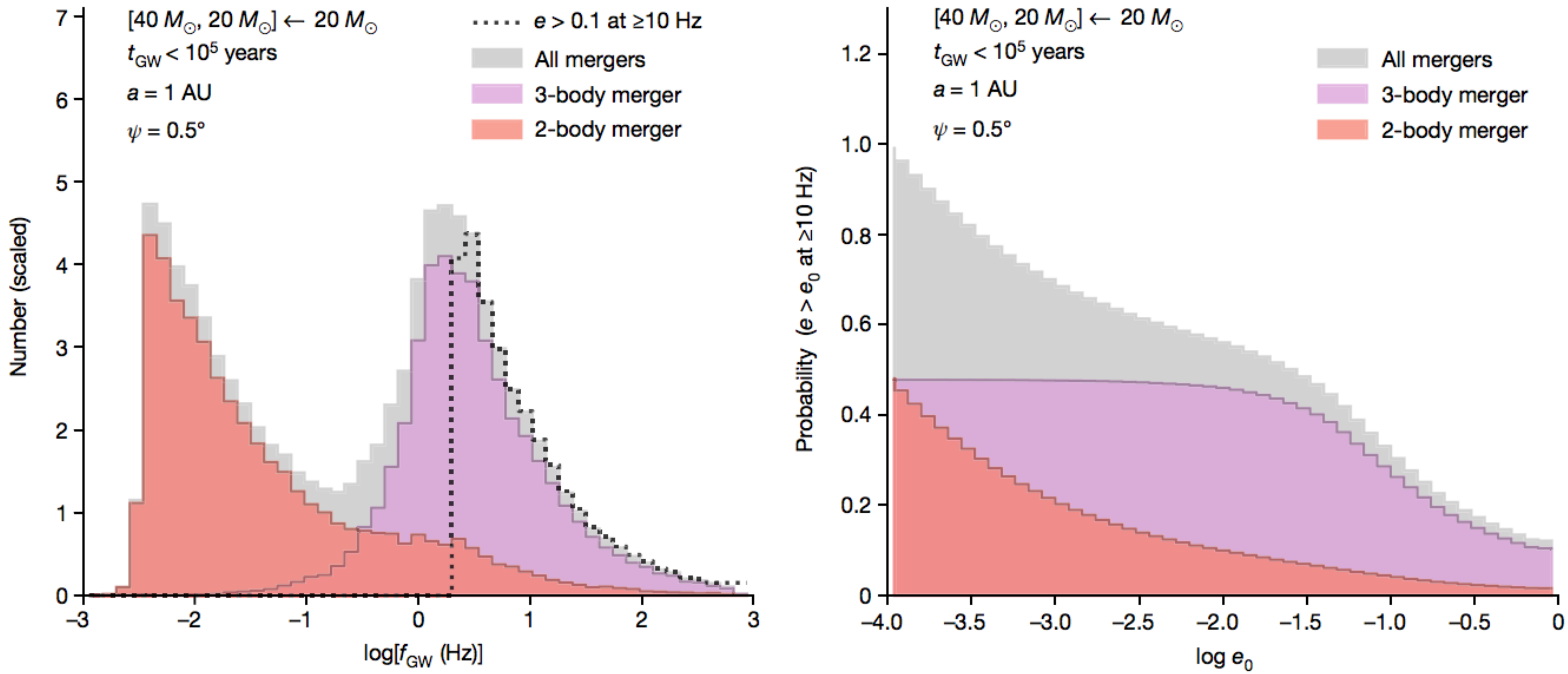


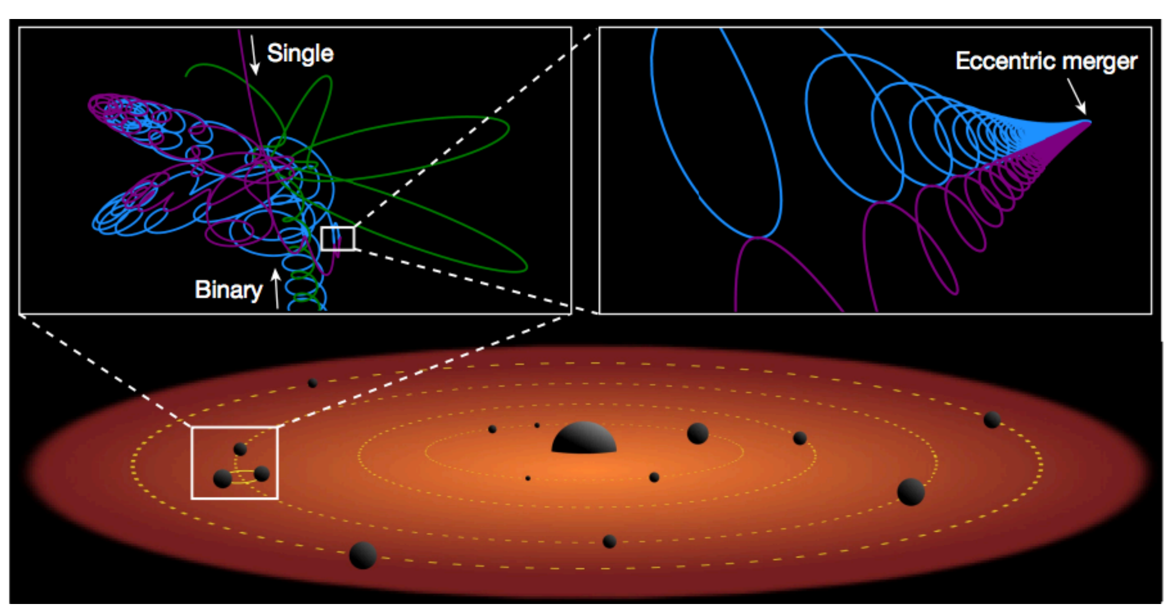
Credit: Johan Samsing 2018





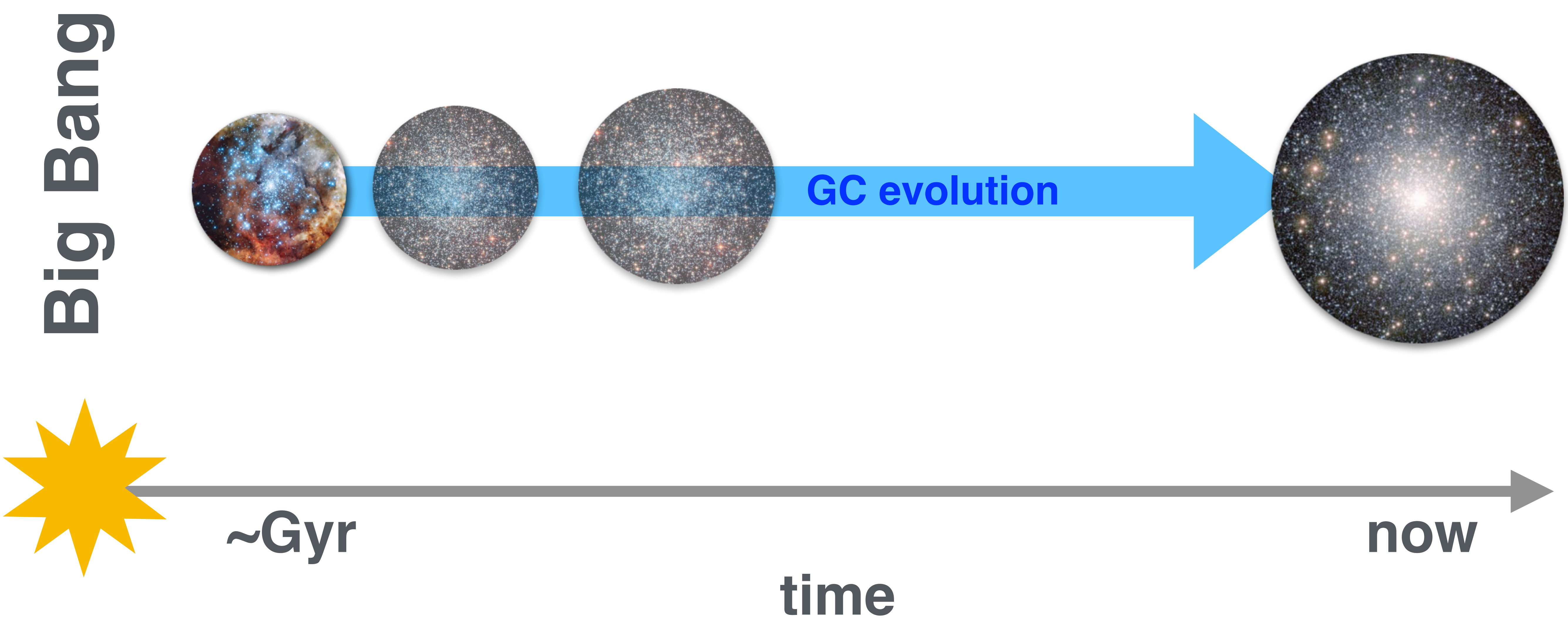
Formation of eBBHs in AGN discs





Samsing+ 2022

Dynamical formation of (e)BBHs in star clusters



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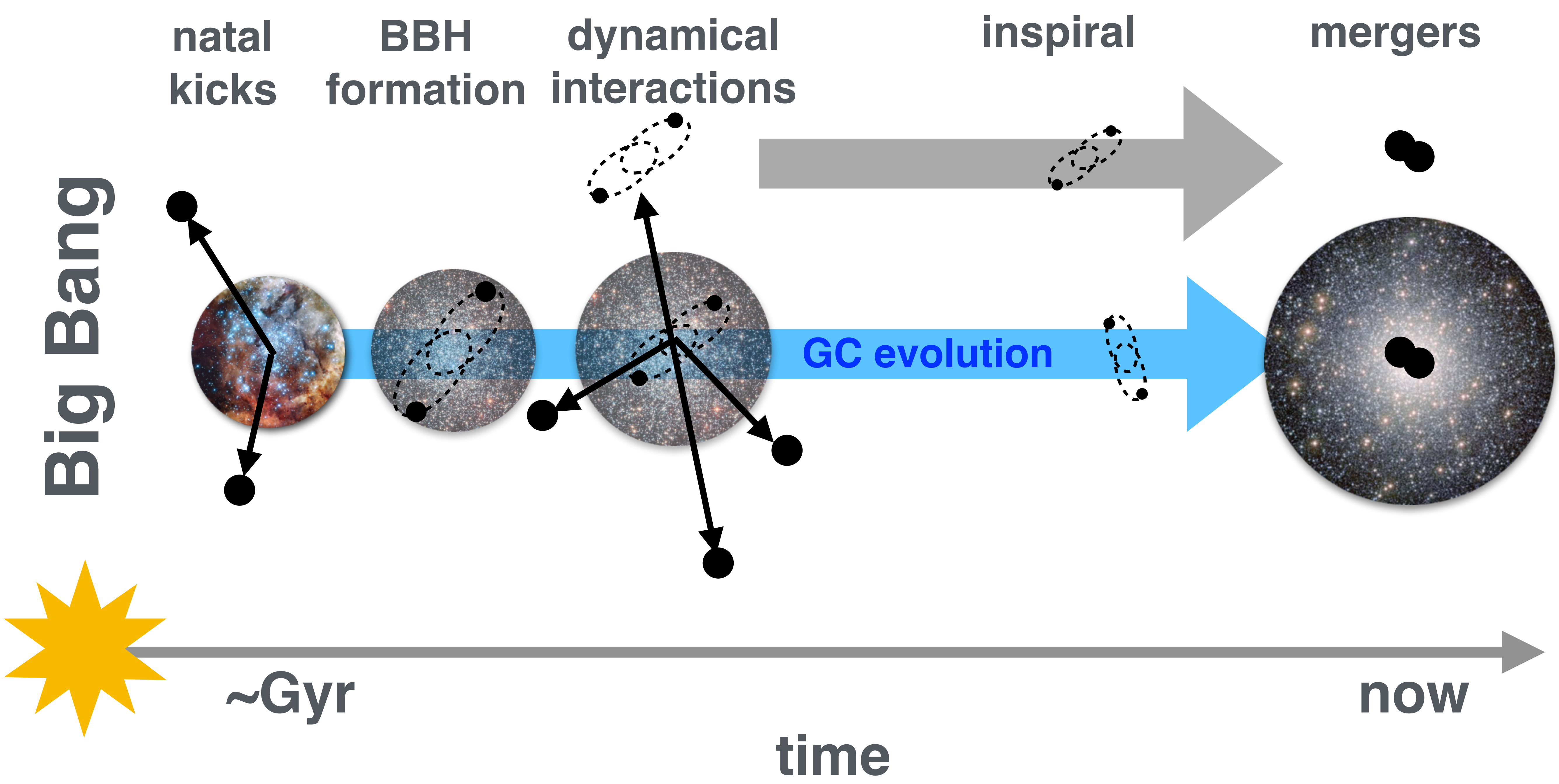

I must admit …

Globular cluster formation?

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EVOLVE!

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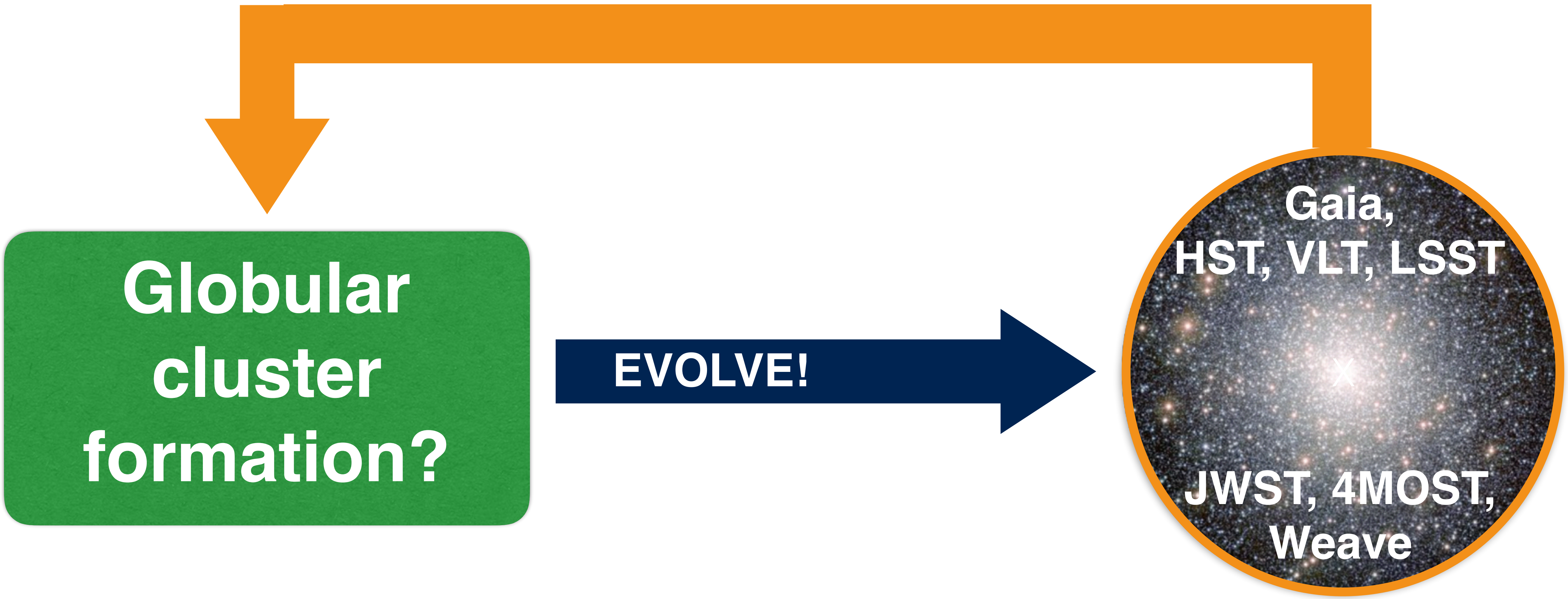

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Gaia,
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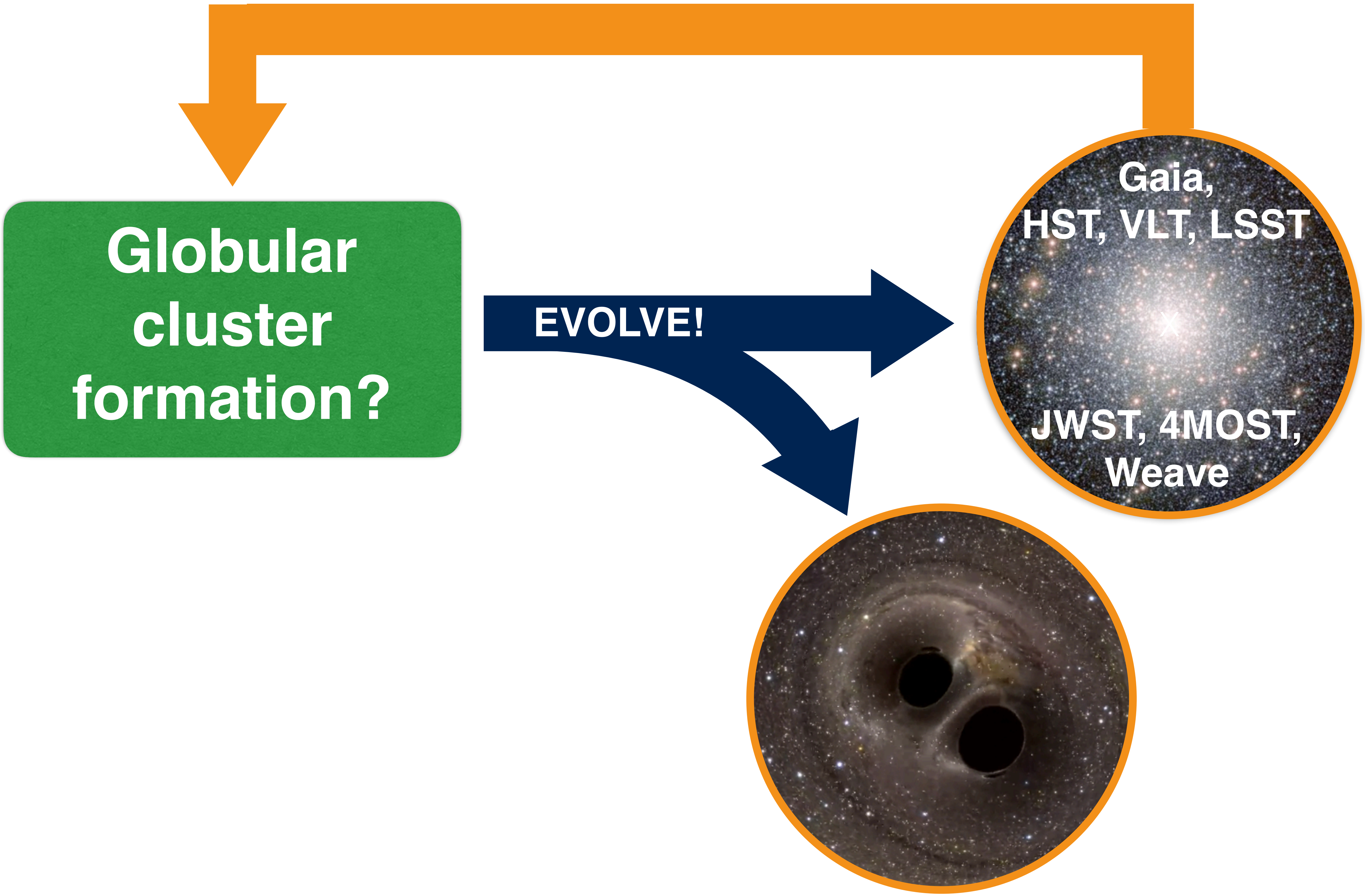
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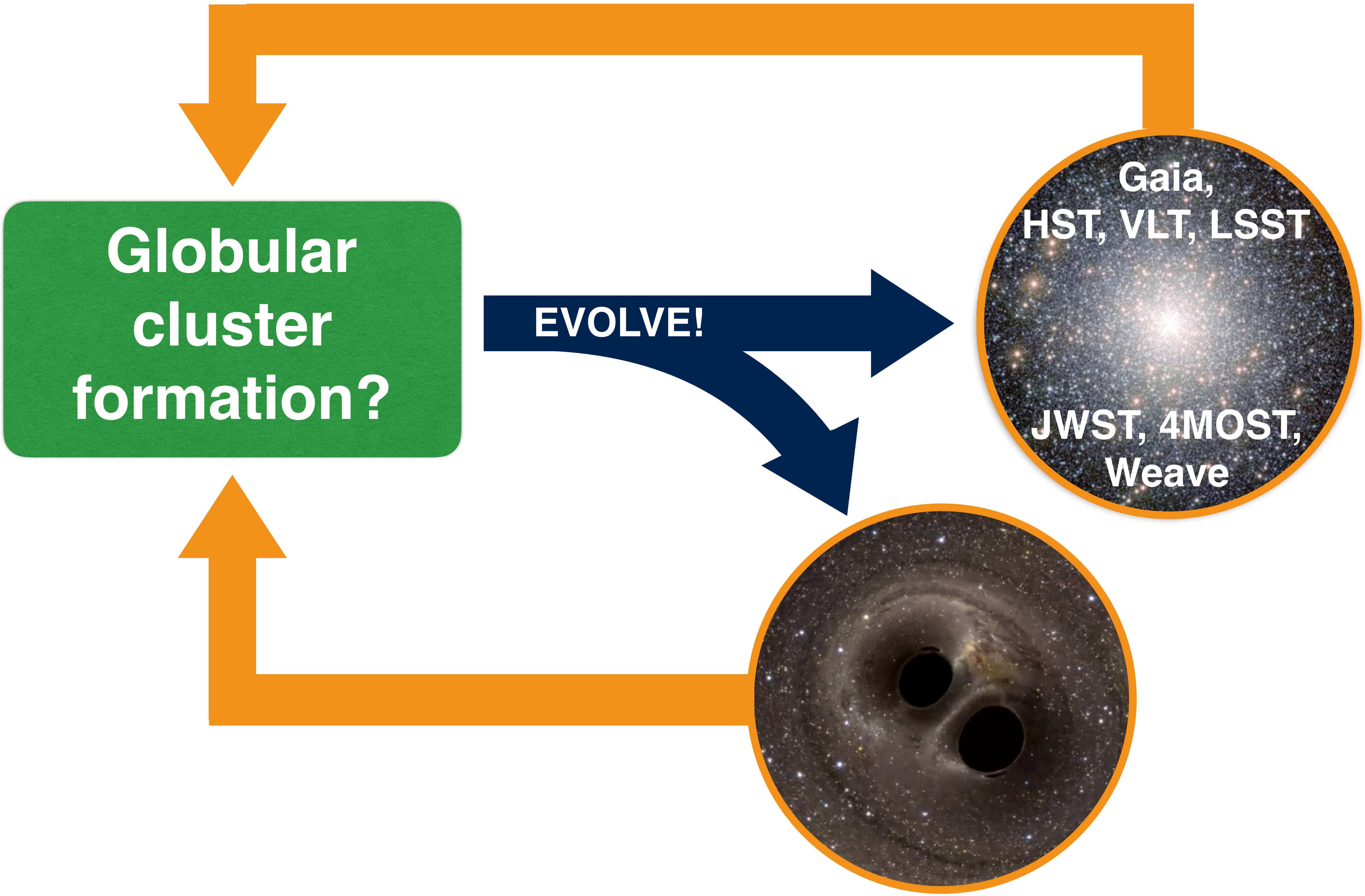

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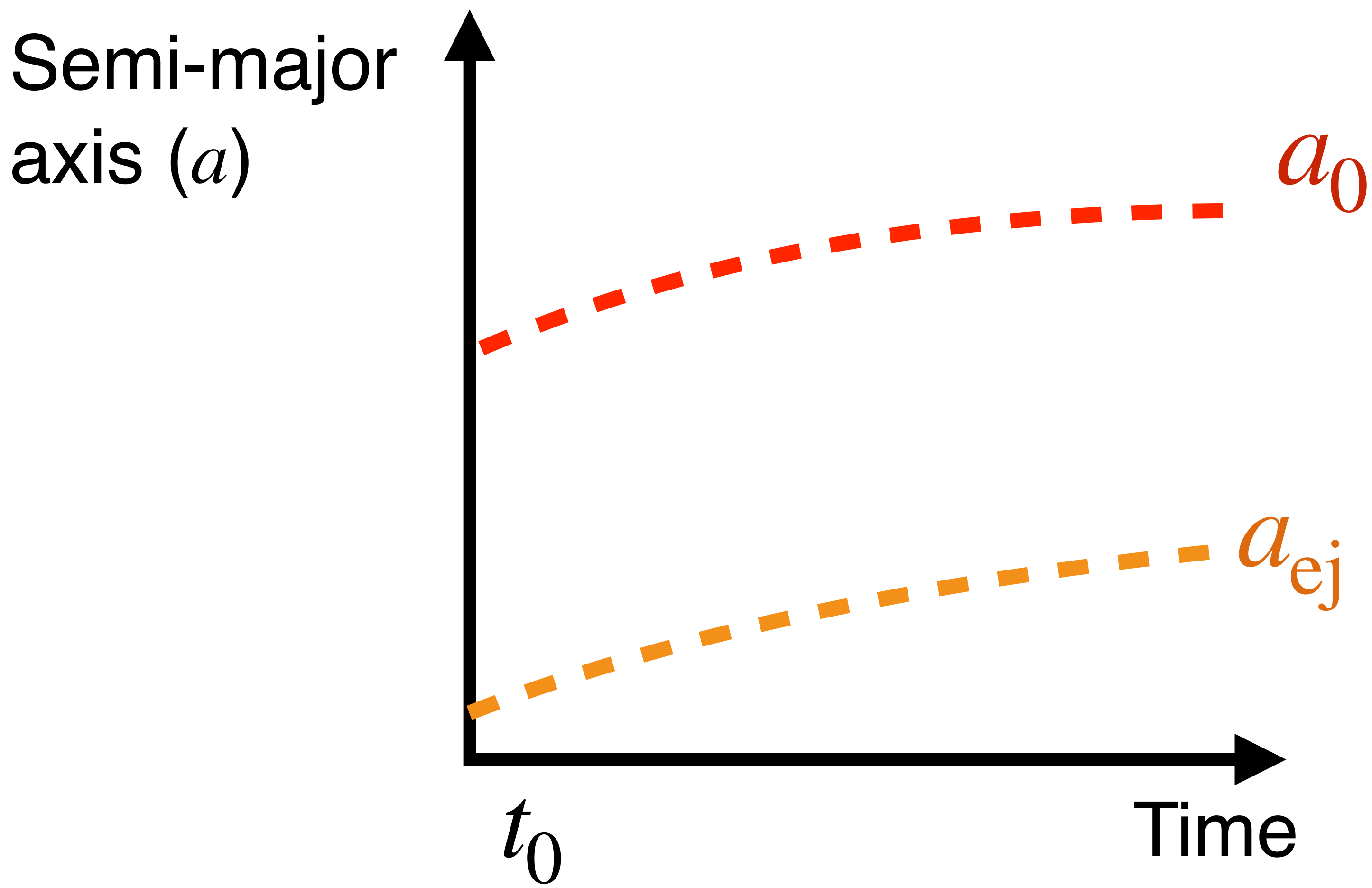
Dynamical BBHs, the idea

Definiton: Binding energy BBH: $E_{\rm b} \equiv Gm_{\bullet}^2/(2a)$



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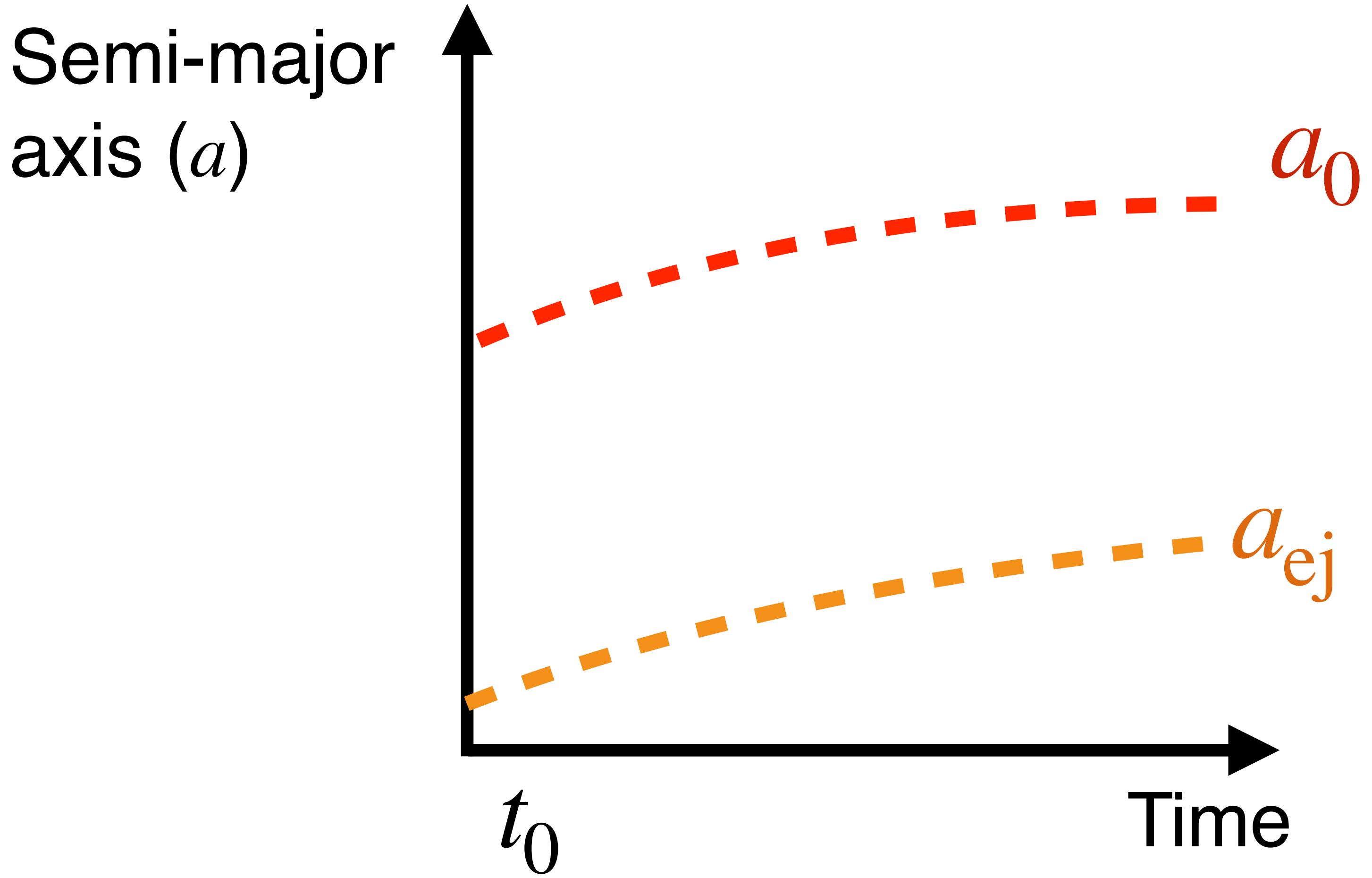

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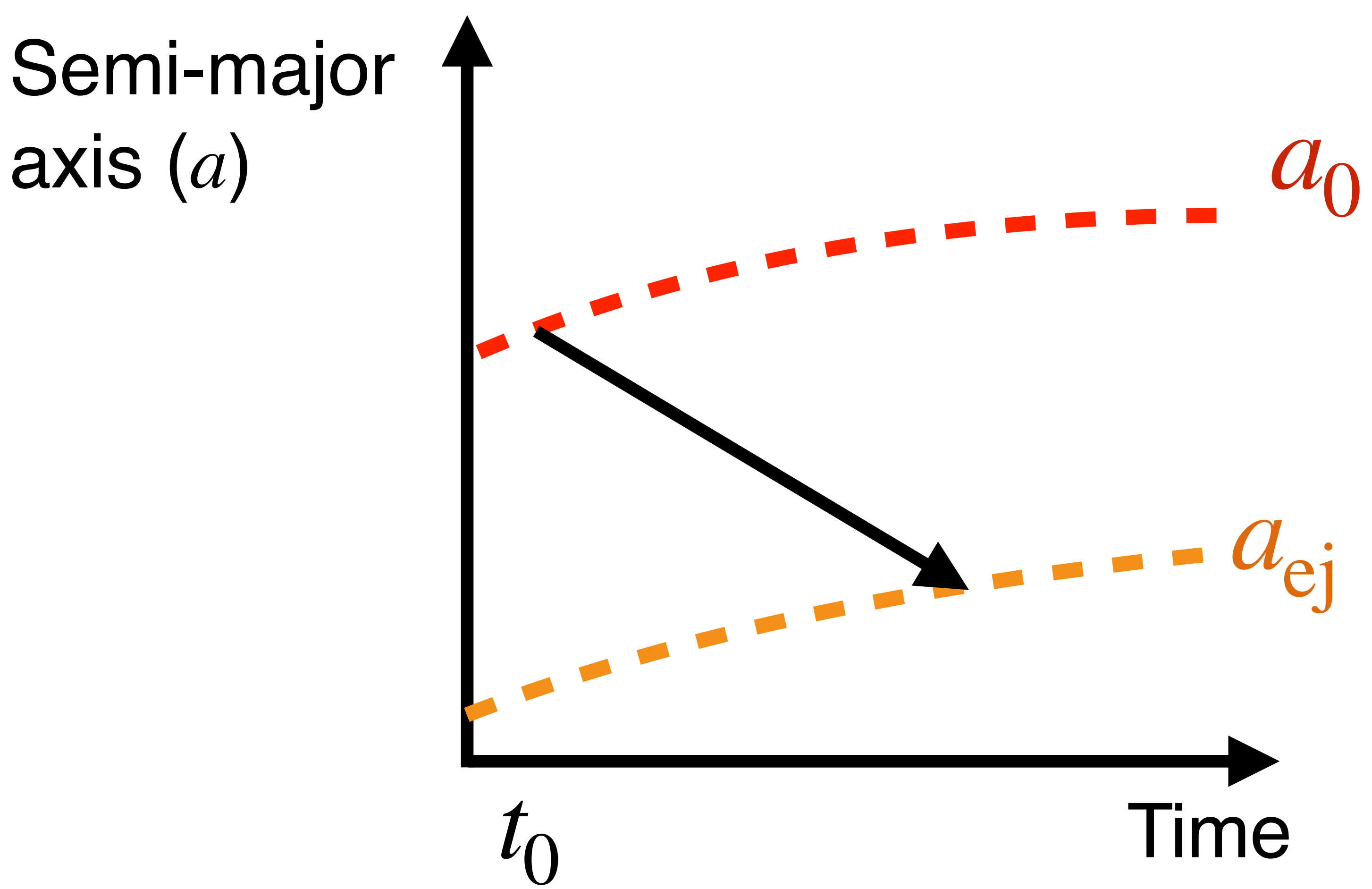

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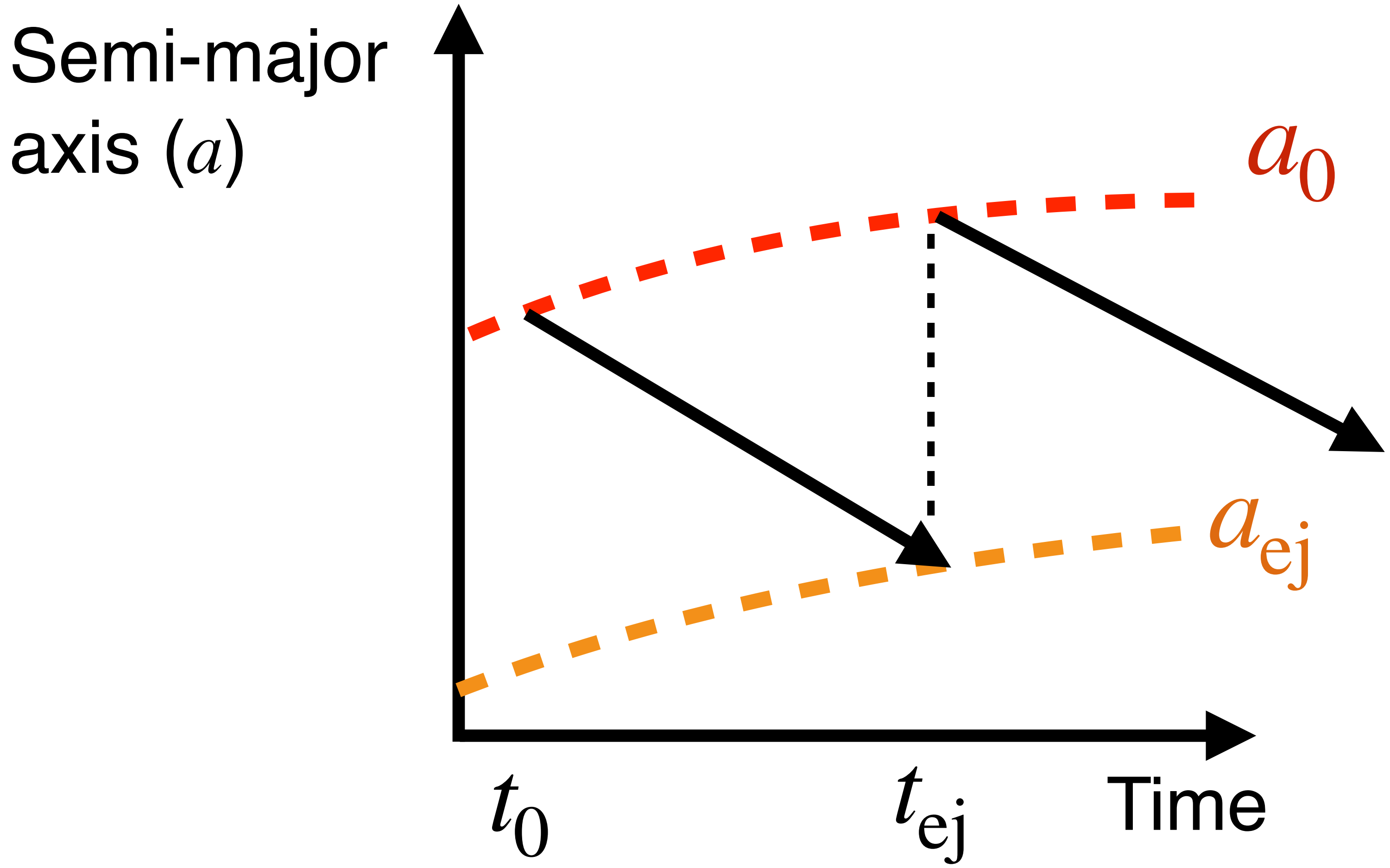

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$M_{\rm GC} = 5\times10^5\,{\rm M_\odot}$
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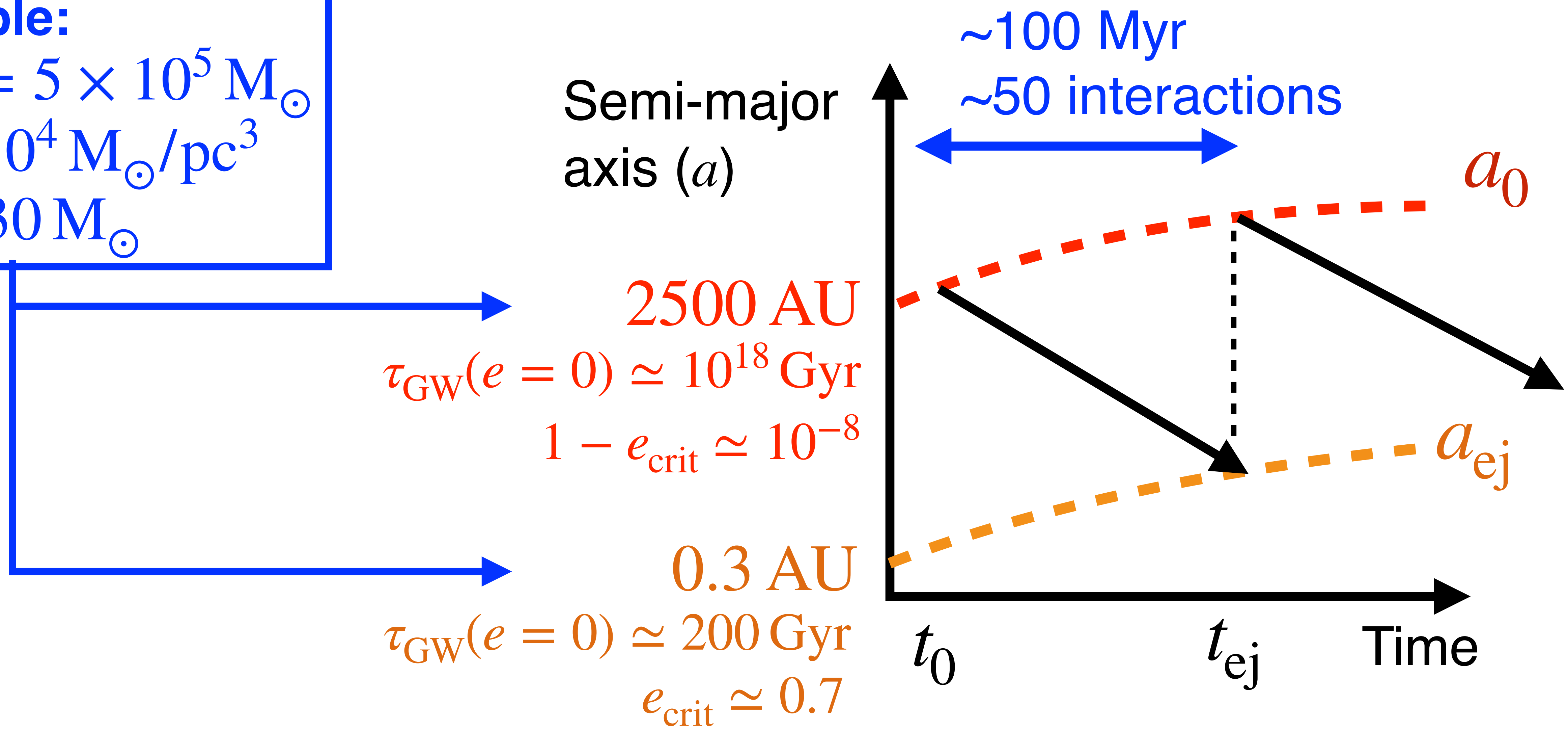

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depend on cluster!

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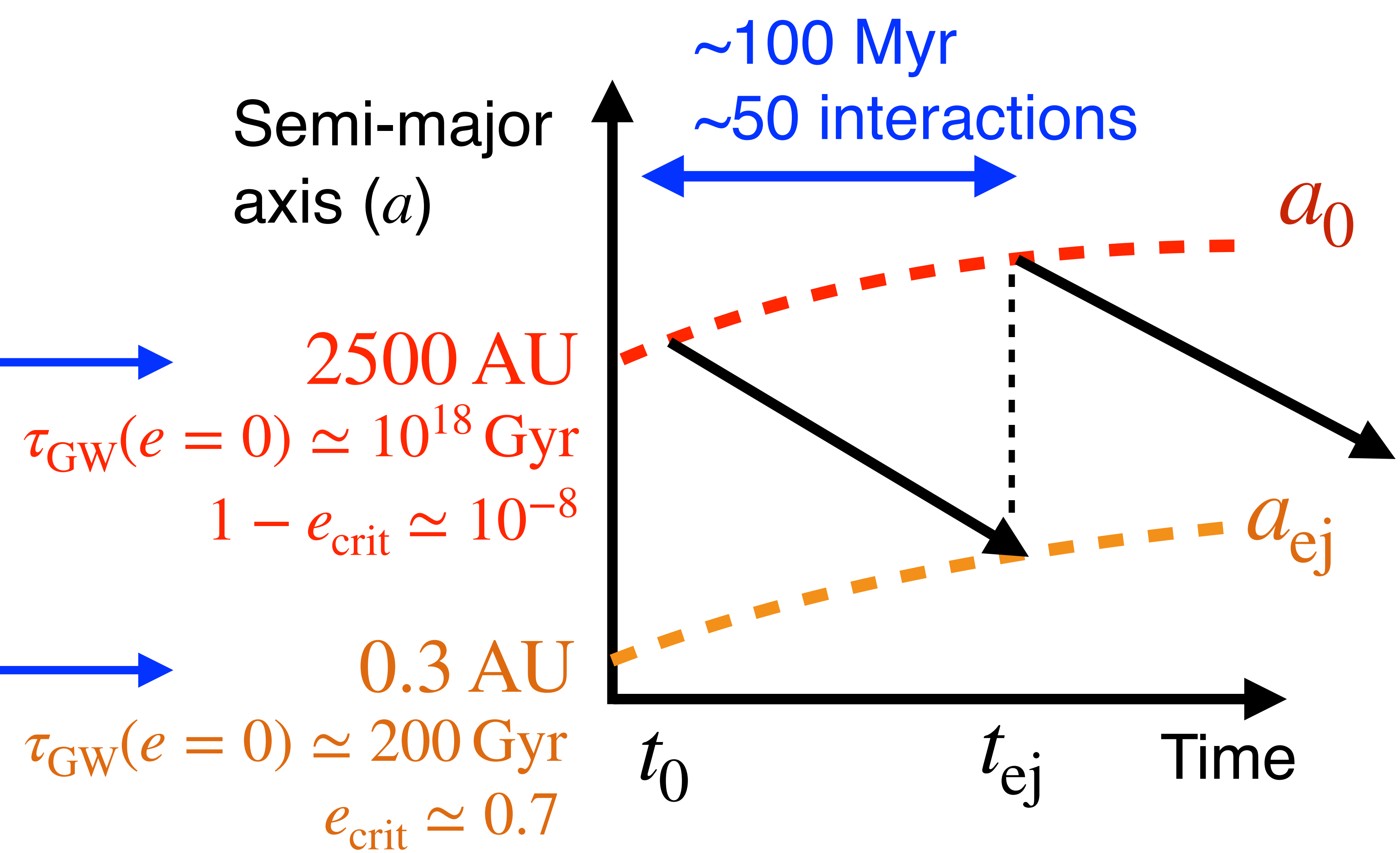
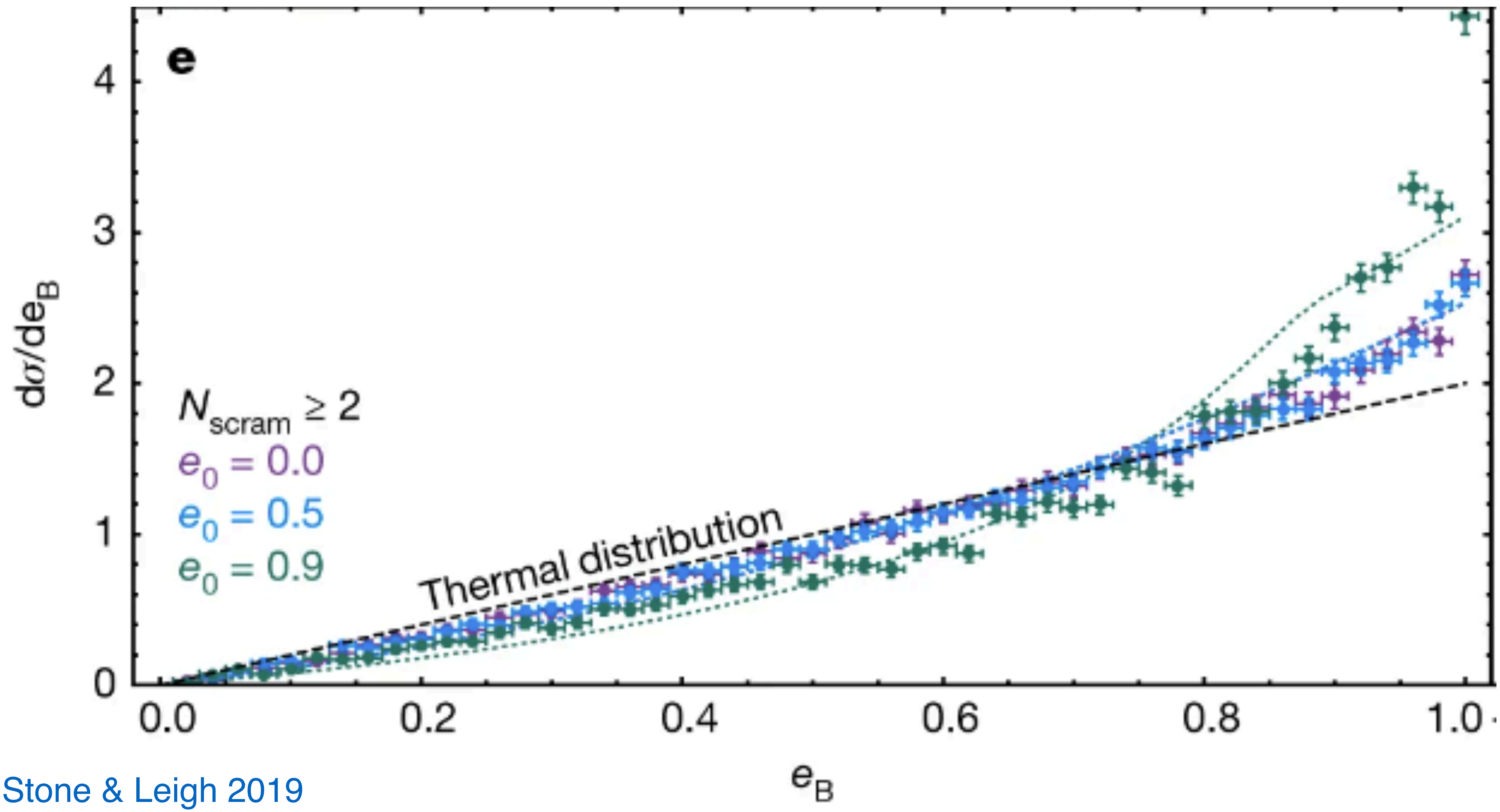


The (super) thermal distribution of eccentricities

$$f(e)de = 2e\mathrm{d}e$$



Stone & Leigh 2019
Jeans 1919; Heggie 1975
https://joe-antognini.github.io/astronomy/thermal-eccentricities

Fast model for dynamical BBH mergers

Population modelling and compare to GWTC

Ongoing work

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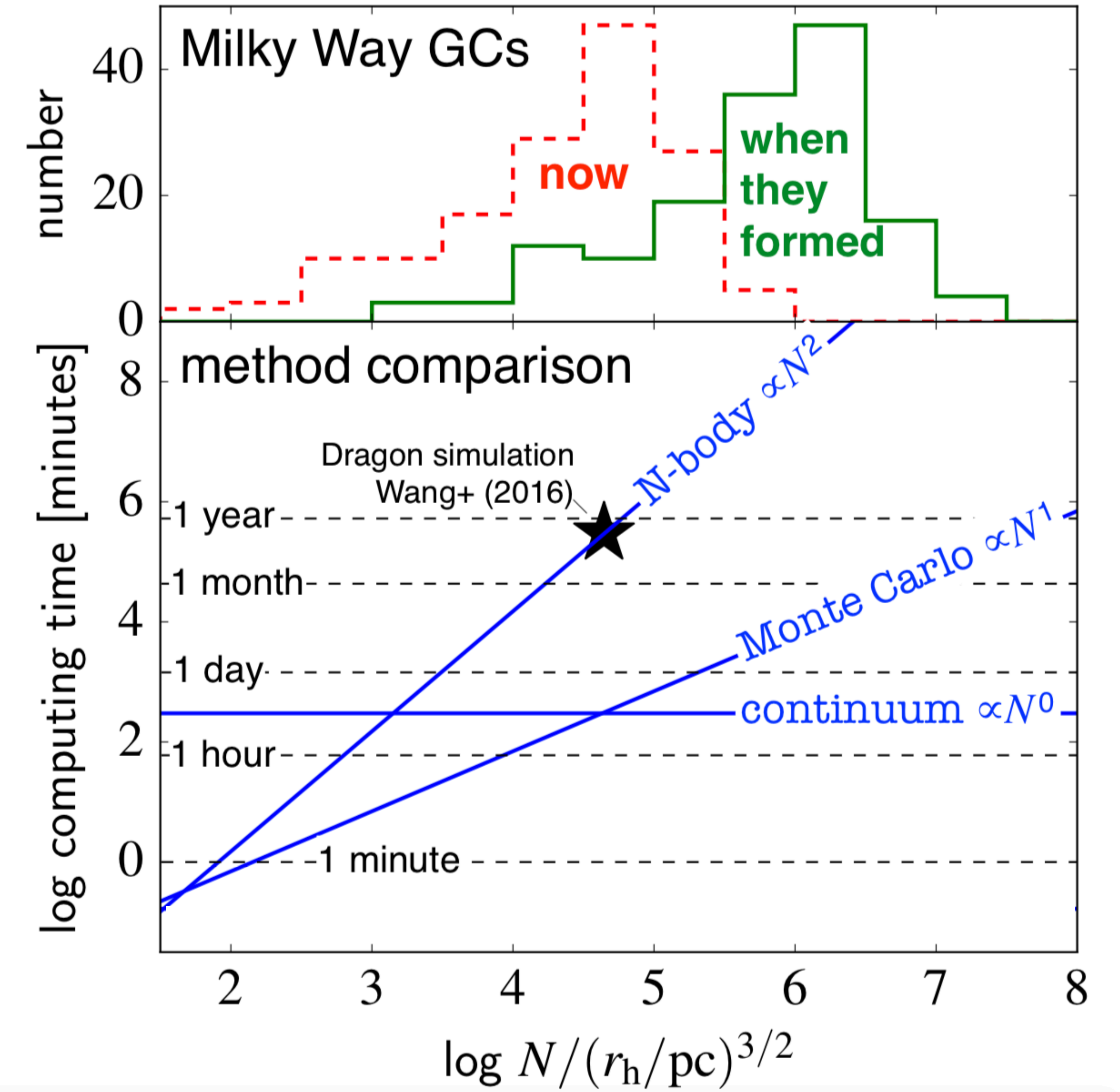
Computational effort of GC evolution



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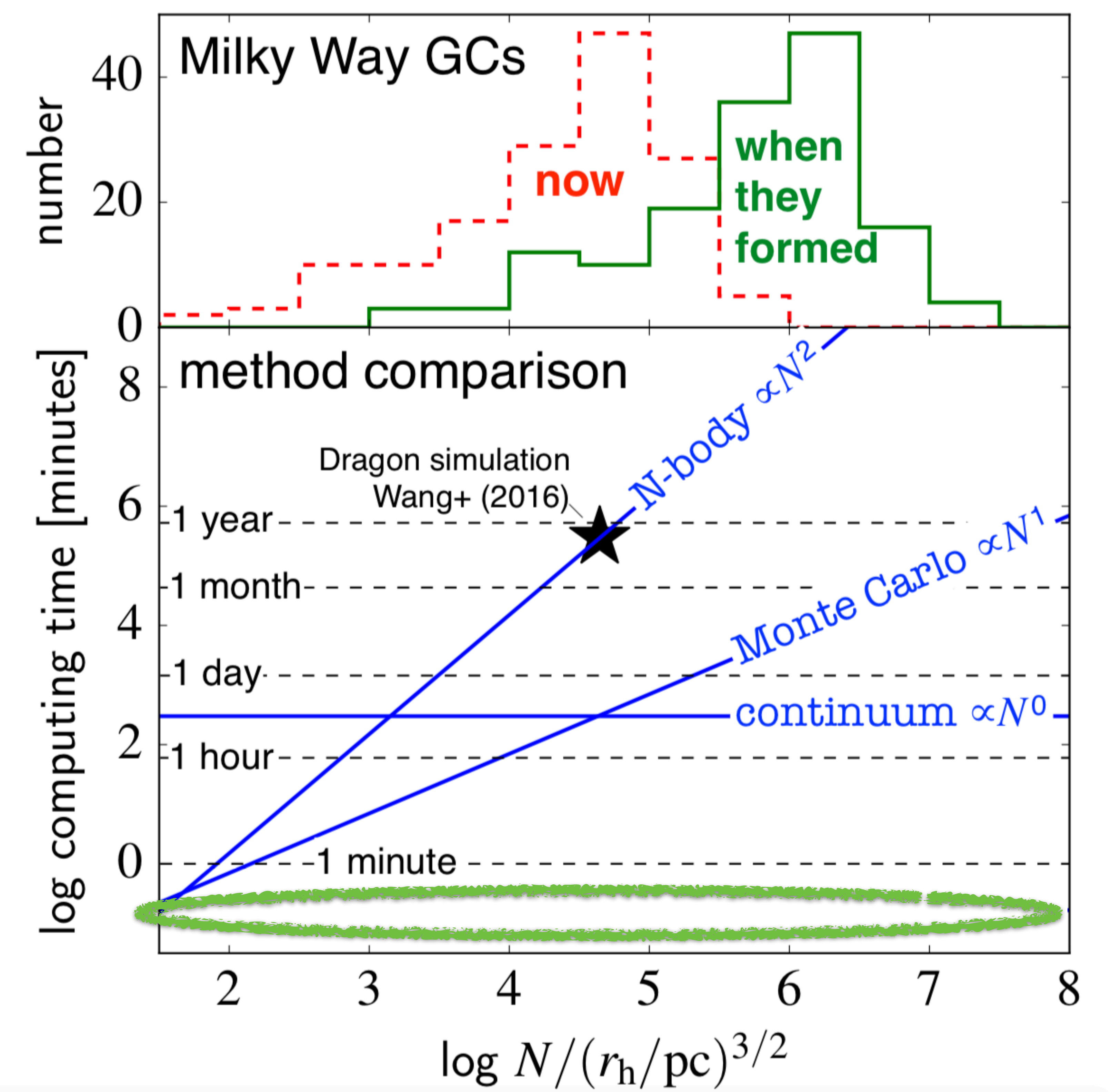
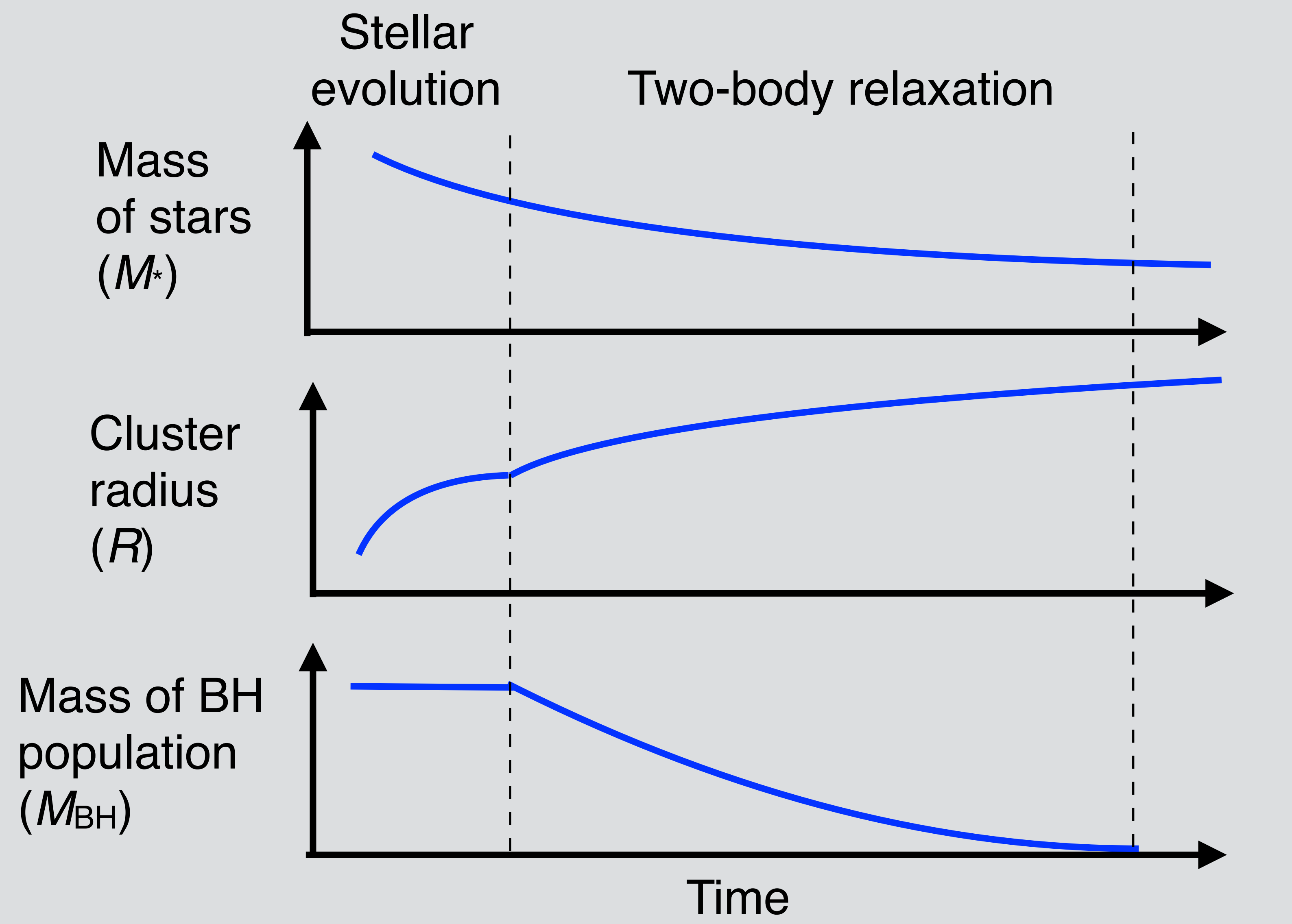


A fast model for dynamical BBH mergers

Antonini & Gieles 2020a

clusterBH:



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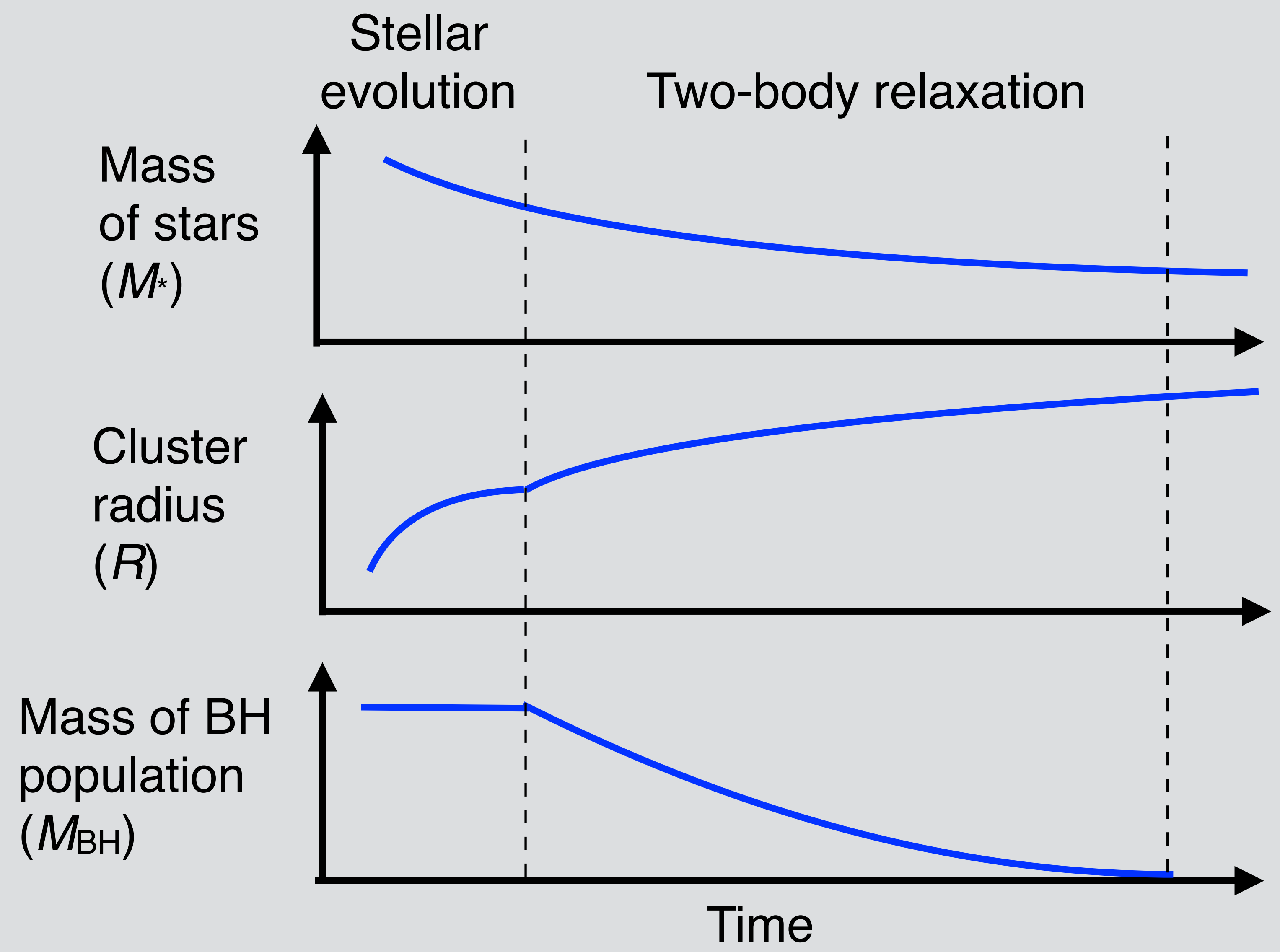
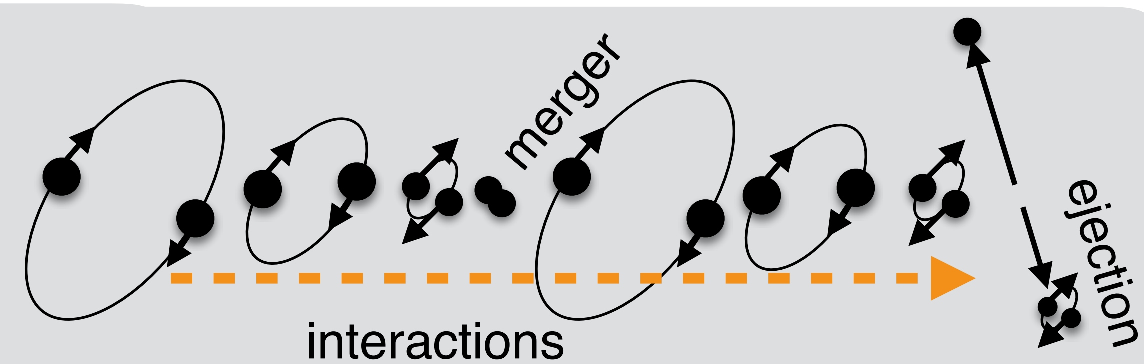


BHBdynamics:



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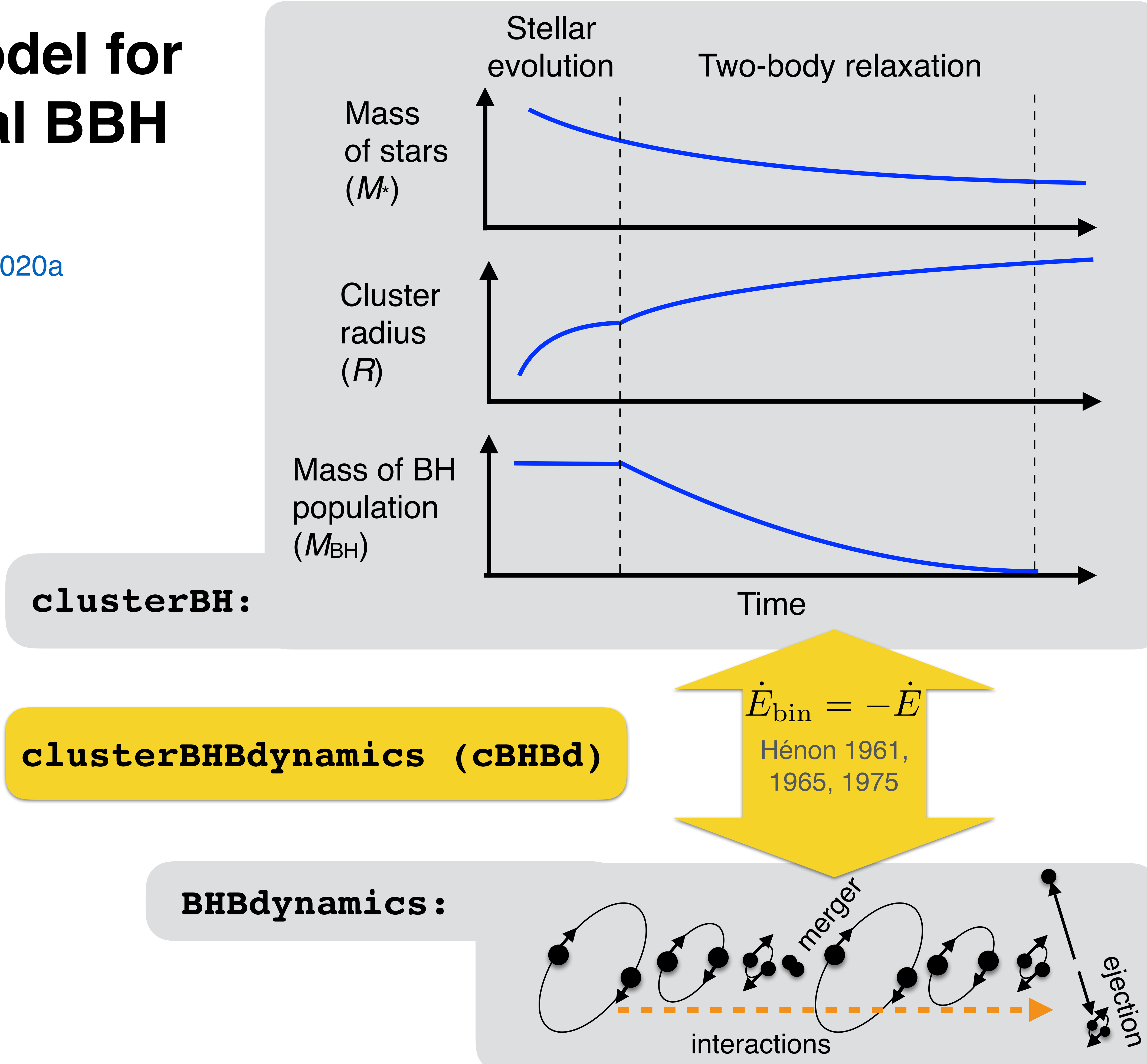

Cluster evolution: `clusterBH`

Assumptions:

1. No Galactic tides *(for now!)*
2. Two component clusters: stars + BHs
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Stellar evolution:

$$M_* \propto t^{-\nu}, \quad \nu \simeq 0.07$$

$$R \propto M^{-1}$$ (Hills 1980)

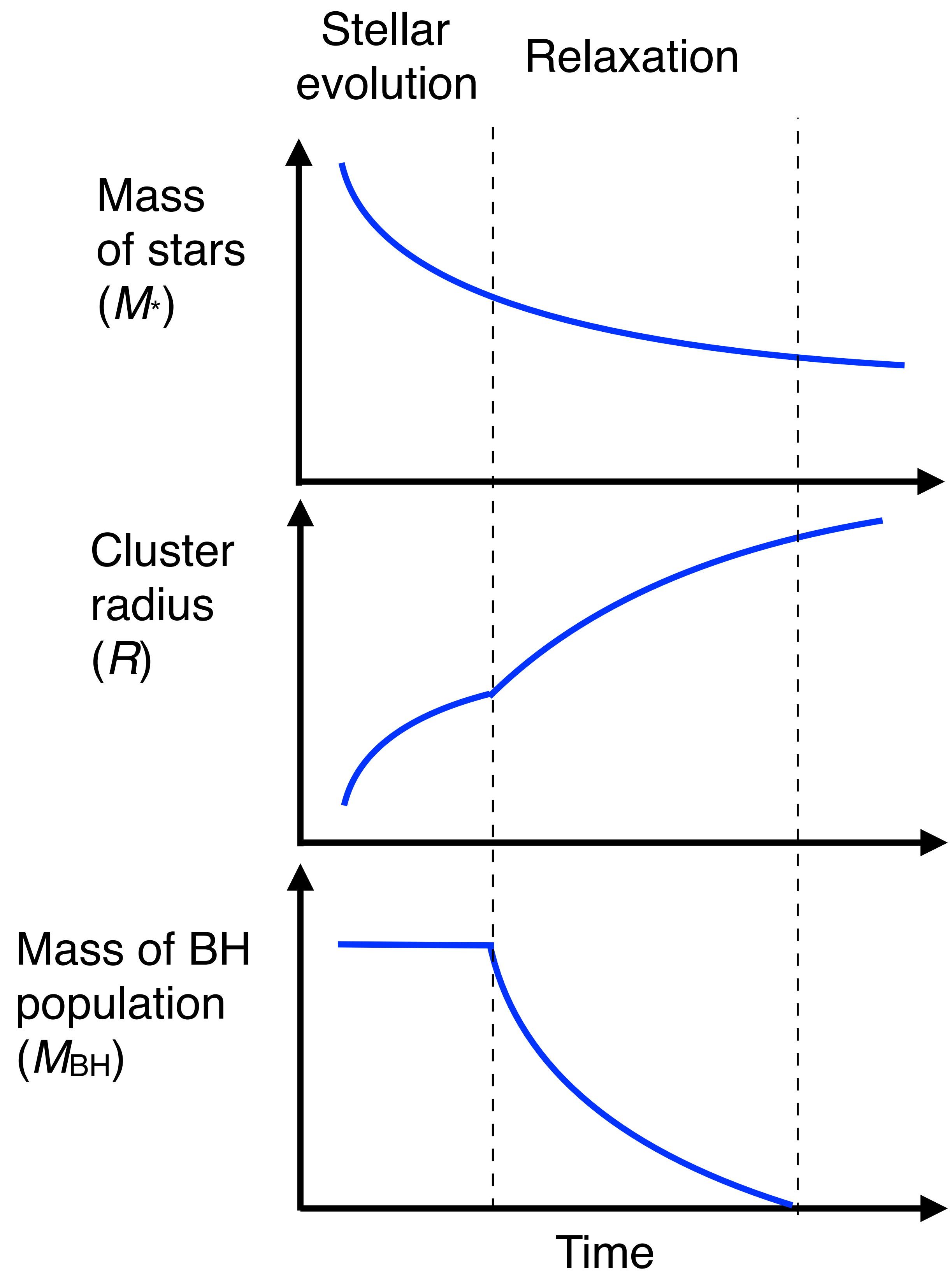
Relaxation:

$$\dot{E} \propto |E|/\tau_{\rm rh}$$ (Hénon 1961)

$$\dot{M}_{\rm BH} \propto -M/\tau_{\rm rh}$$

(Breen & Heggie 2013)

Integrate: $\dot{M}_*, \dot{R}, \dot{M}_{\rm BH}$



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Assumptions:

1. Active binary is BHB
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5. $m_1 = m_2 = m_3 = m_{max}$ *(for now!)*

Definitions:

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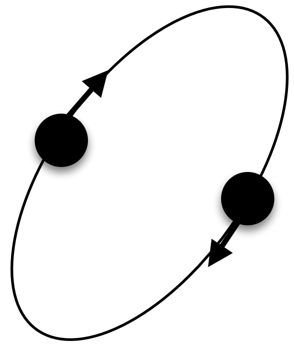

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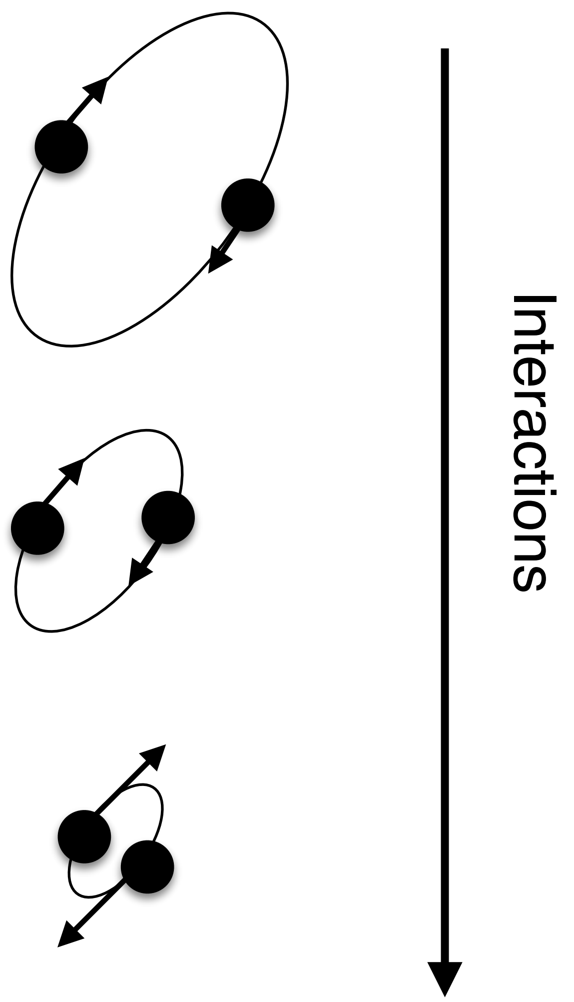

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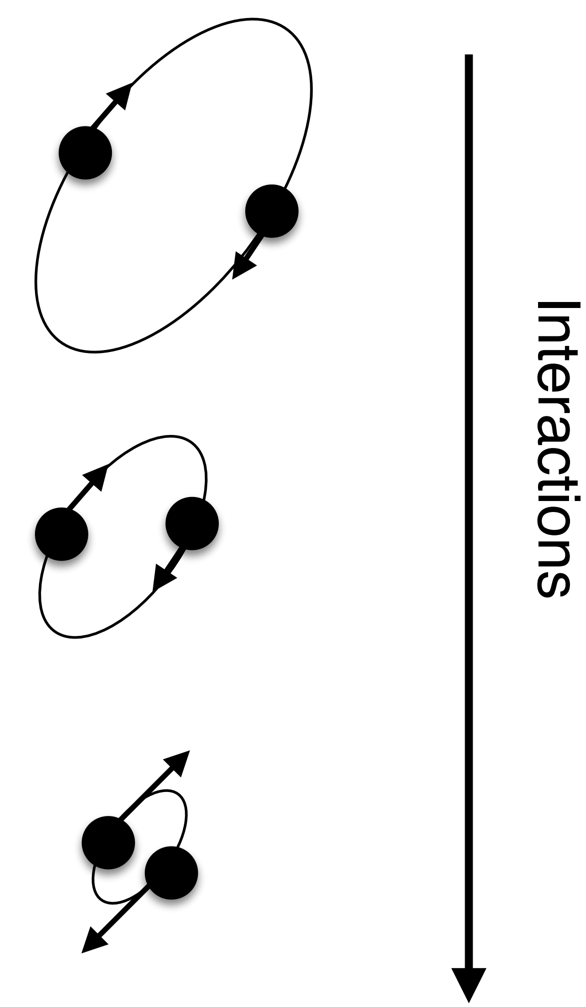
Binaries harden at rate set by global energy diffusion rate (Hénon's principle)

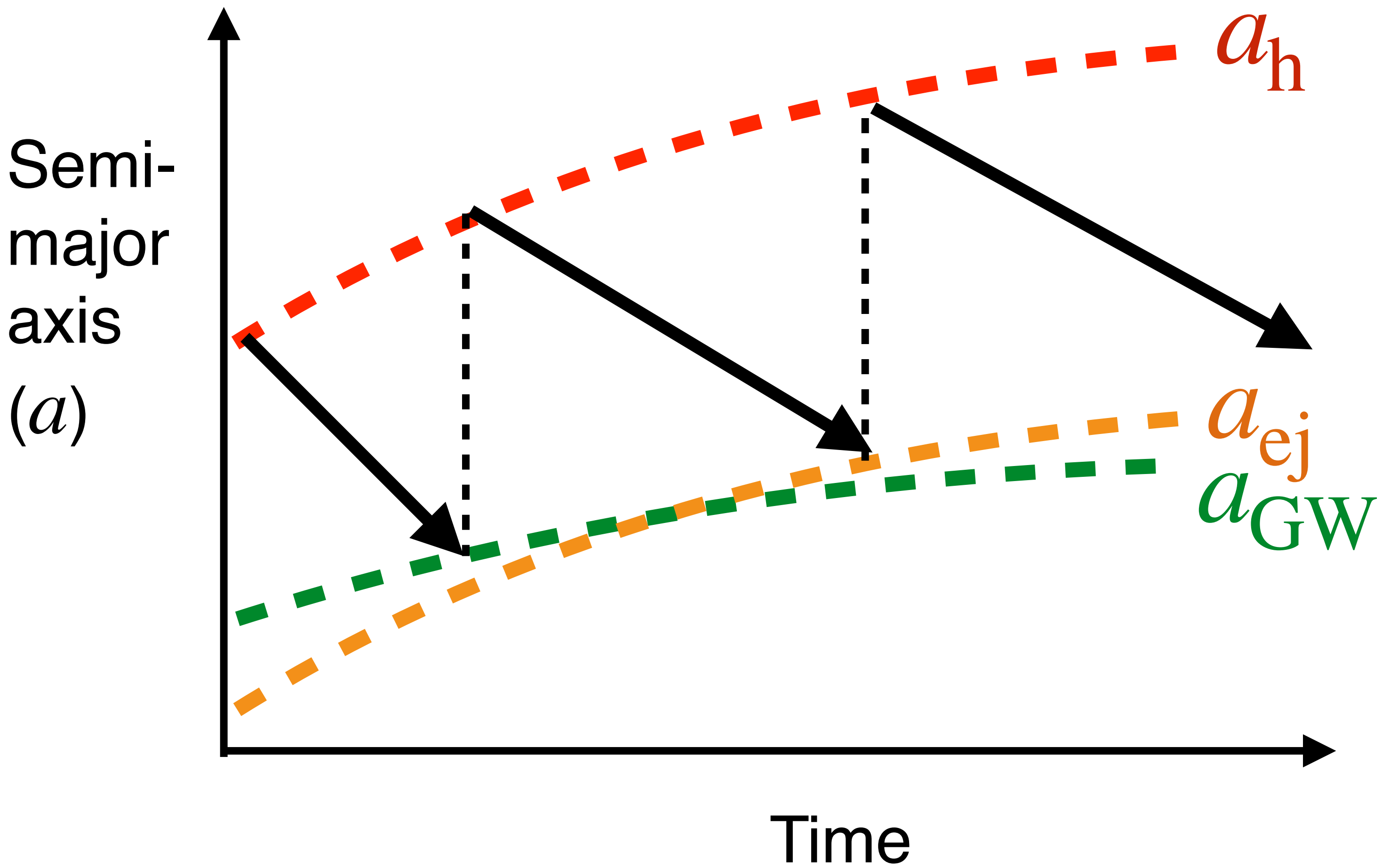
$$\dot{E}_{bin} = -\dot{E}$$

(Hénon 1961, 1965, 1975)



Three types of mergers

1. In-cluster mergers
2. Ejected mergers
3. GW captures



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3. Dimensionless angular momentum: $l^2 = 1 - e^2$

1. In-cluster mergers:

$p_{\rm GW}(a) = l_{\rm GW}^2$: probability that $\tau_{\rm GW} < \tau_3$

$$P_{\rm GW}(a_{\rm m}) = \int_{a_{\rm h}}^{a_{\rm m}} p_{\rm GW}(a) {\rm d}a, \quad a_{\rm m} = \max(a_{\rm ej}, a_{\rm GW})$$

2. GW captures:

$p_{\rm cap}(a) = N_{\rm IS} l_{\rm cap}^2, \quad l_{\rm cap}^2 \simeq \left(R_{\rm S}/a\right)^{5/7}$ Samsing 2014

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$e \sim 10^{-4}$

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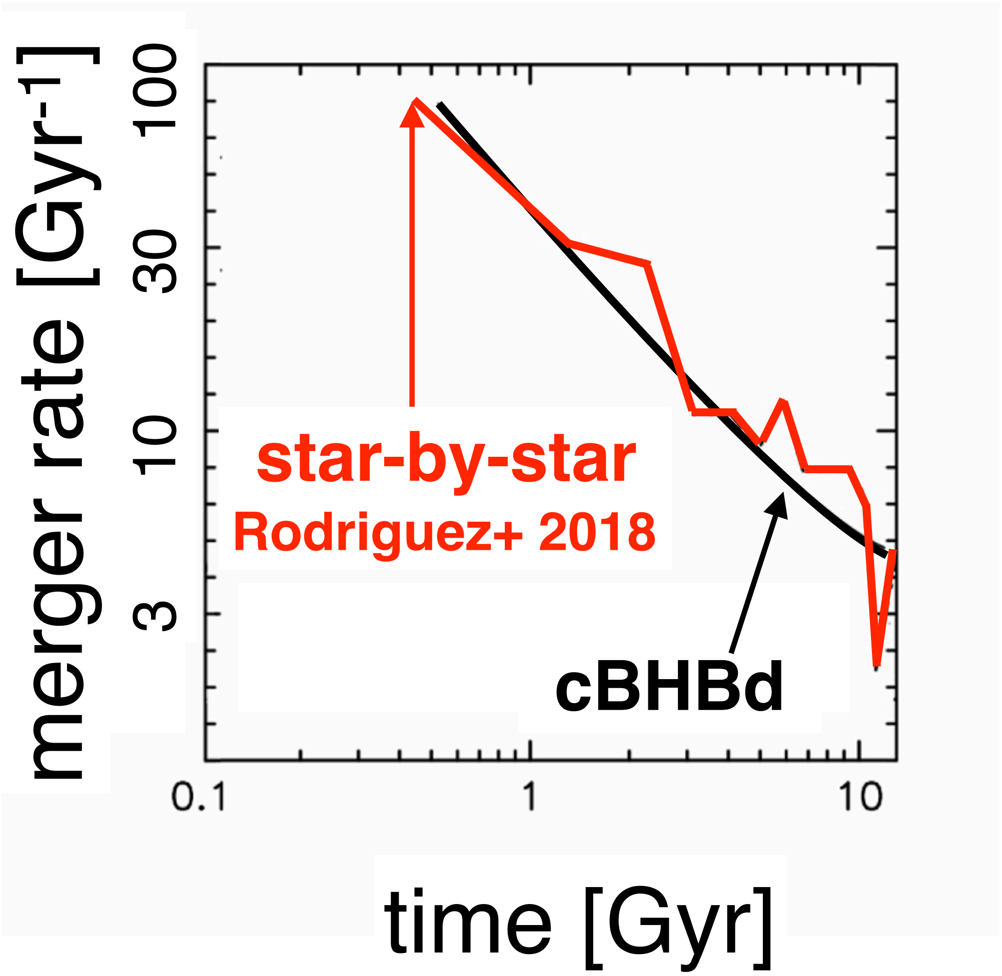
clusterBHBdynamics (cBHBd)

Orders of magnitude faster, acceptable loss of accuracy



Antonini & Gieles 2020a, MNRAS

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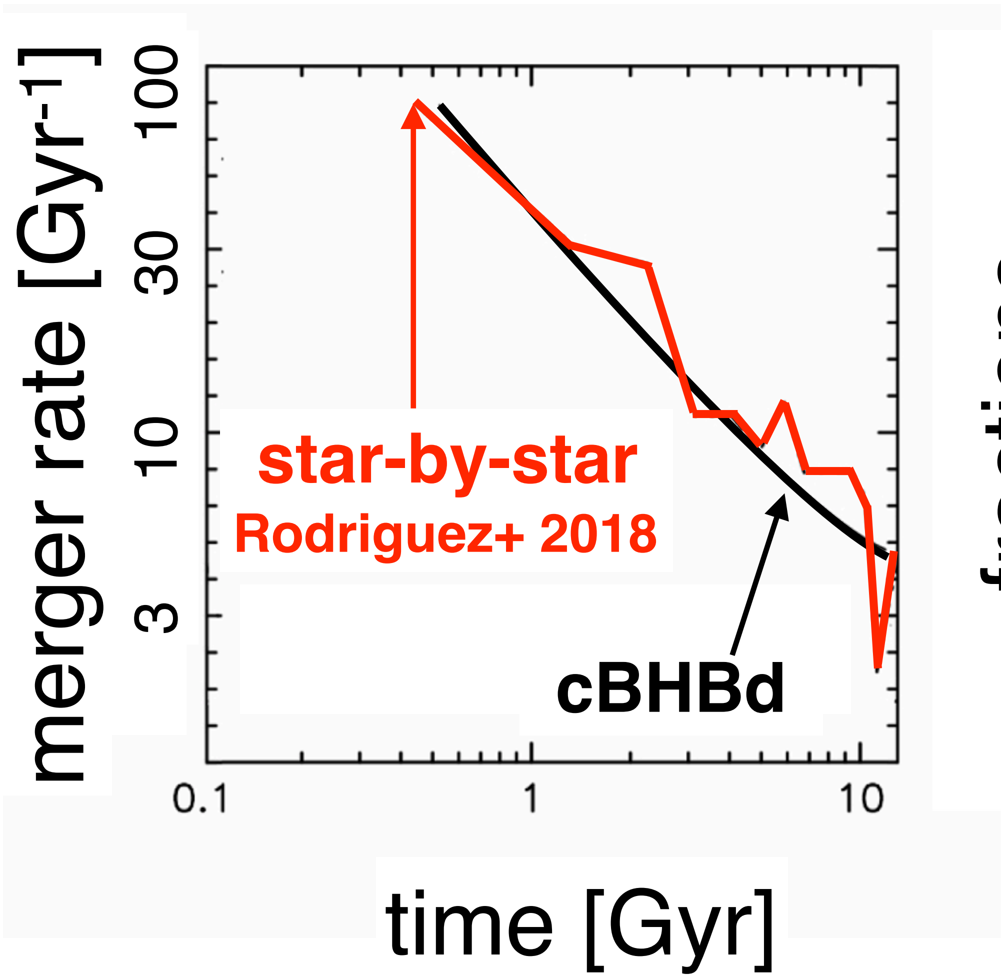

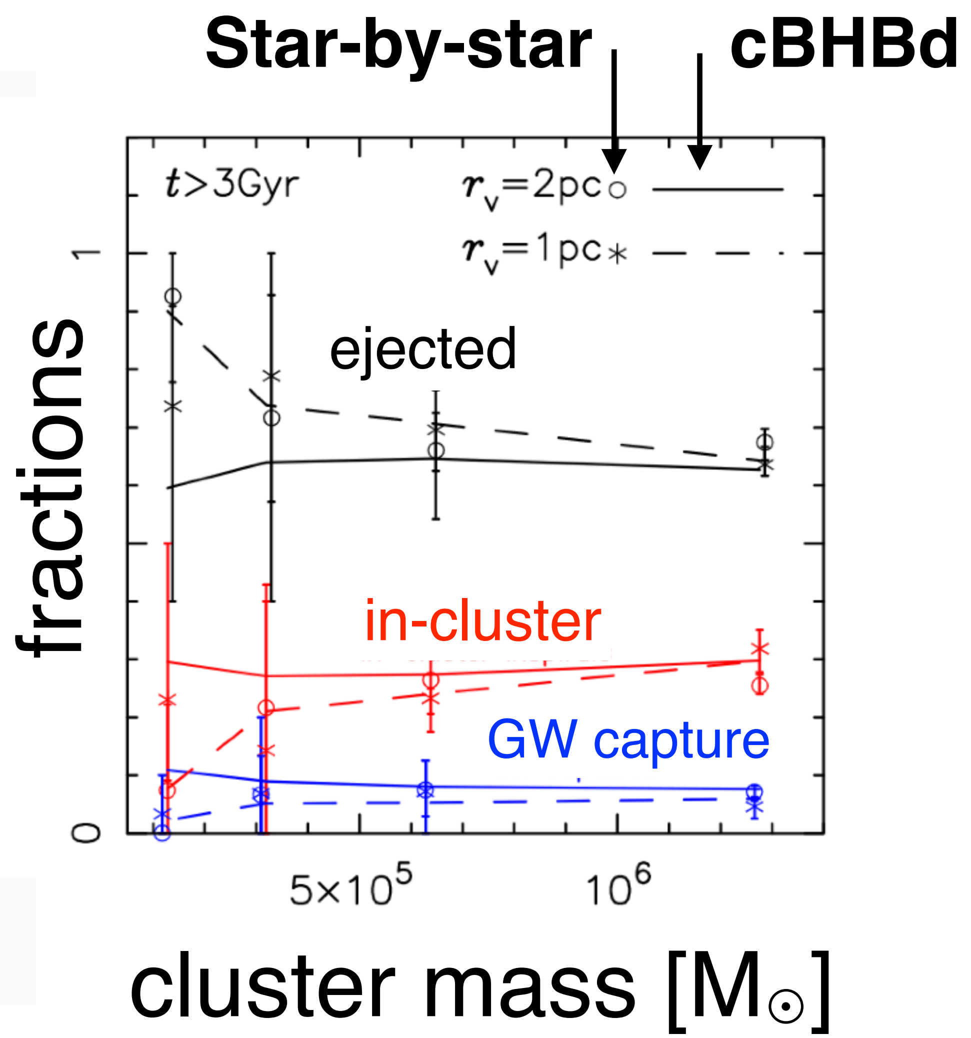

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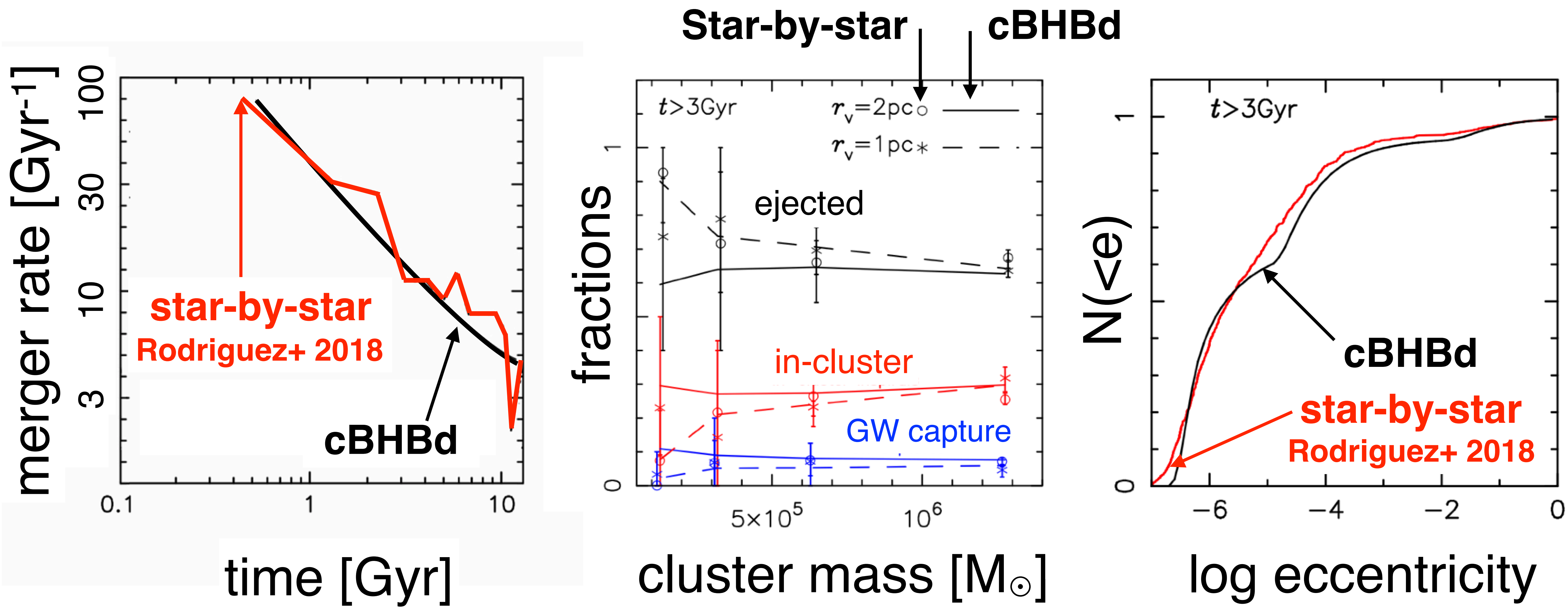

Antonini & Gieles 2020a, MNRAS

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Population modelling and compare to GWTCs

Ongoing work

Population synthesis

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$\rho_{\rm GC} \propto \rho_{\rm DM}$ Harris+ 2013, 2015, 2017

$$= (7.3 \pm 2.6) \times 10^{14}\, M_\odot\, {\rm Gpc}^{-3}$$

Antonini & Gieles 2020b, PRD

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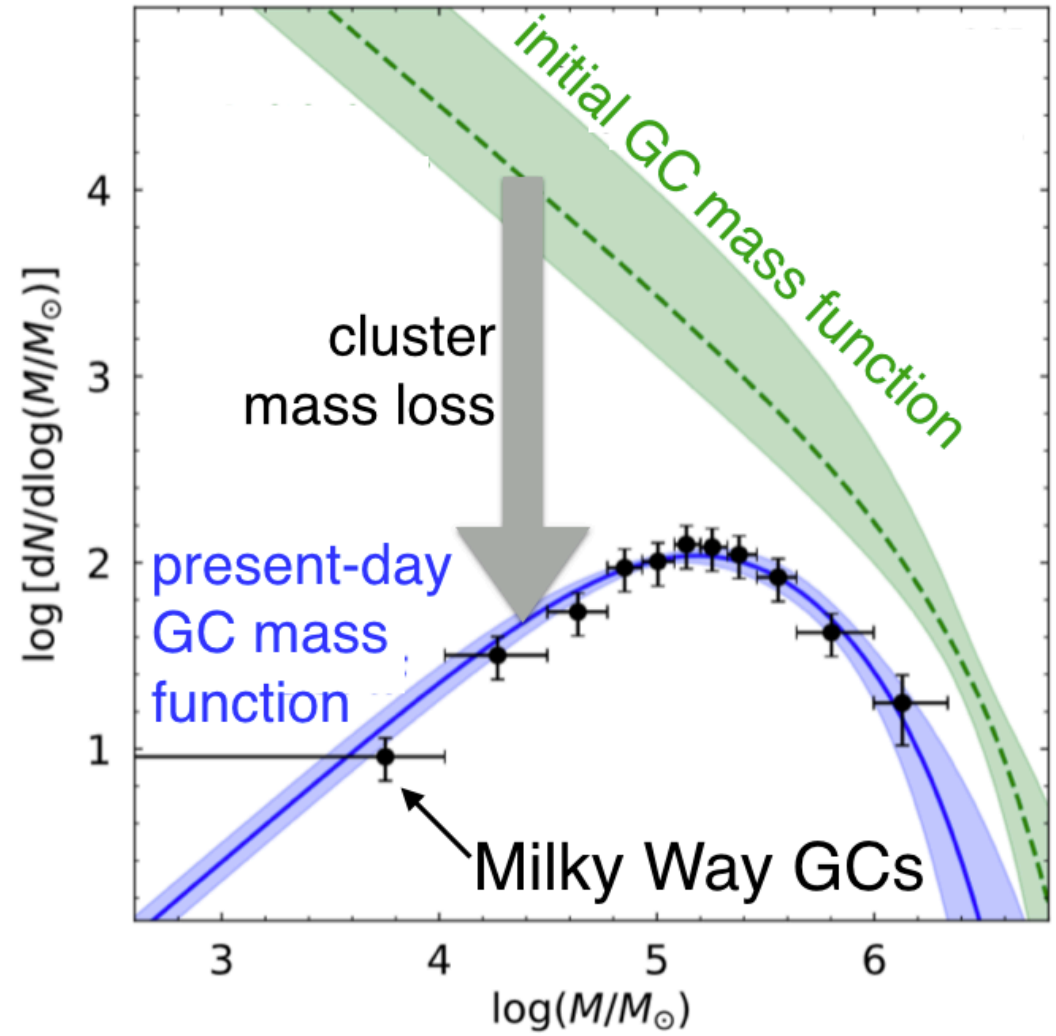

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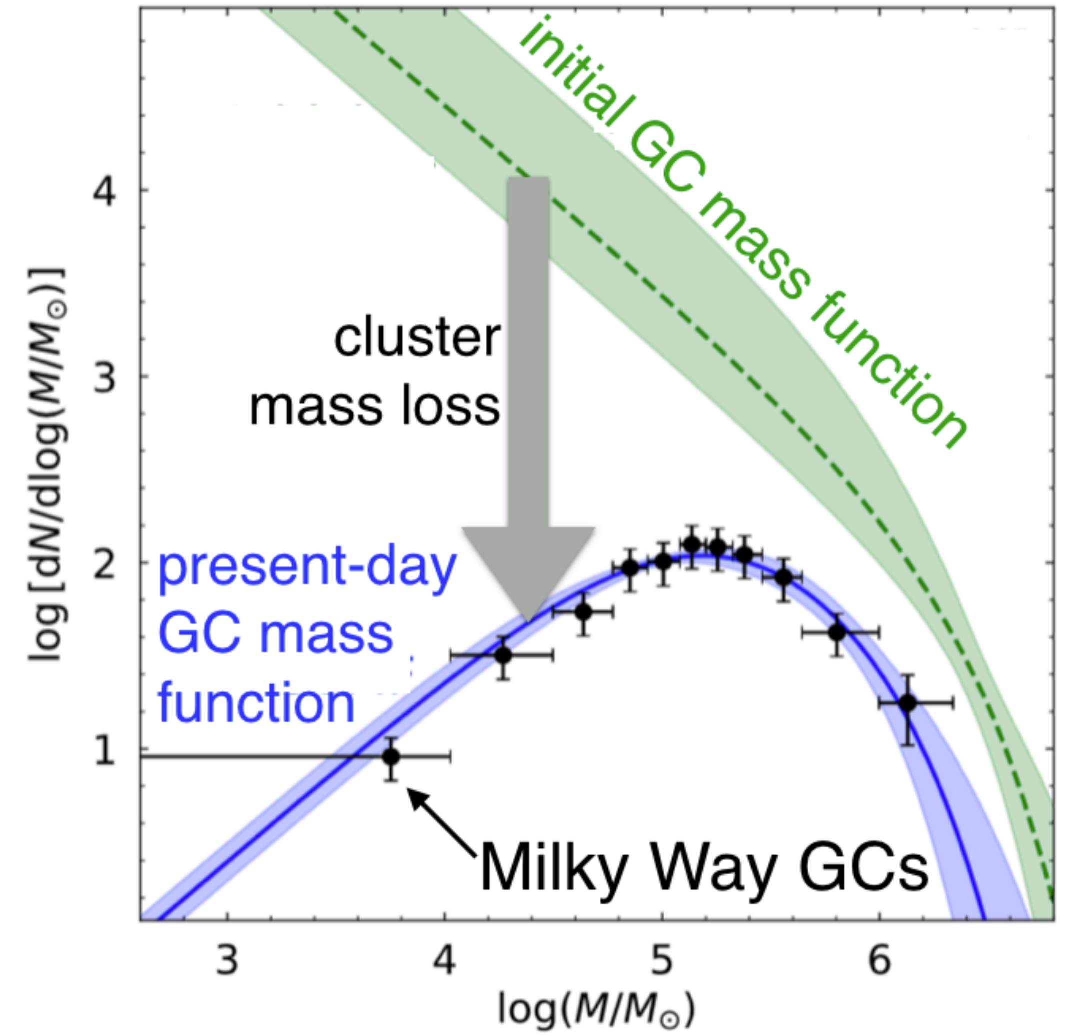

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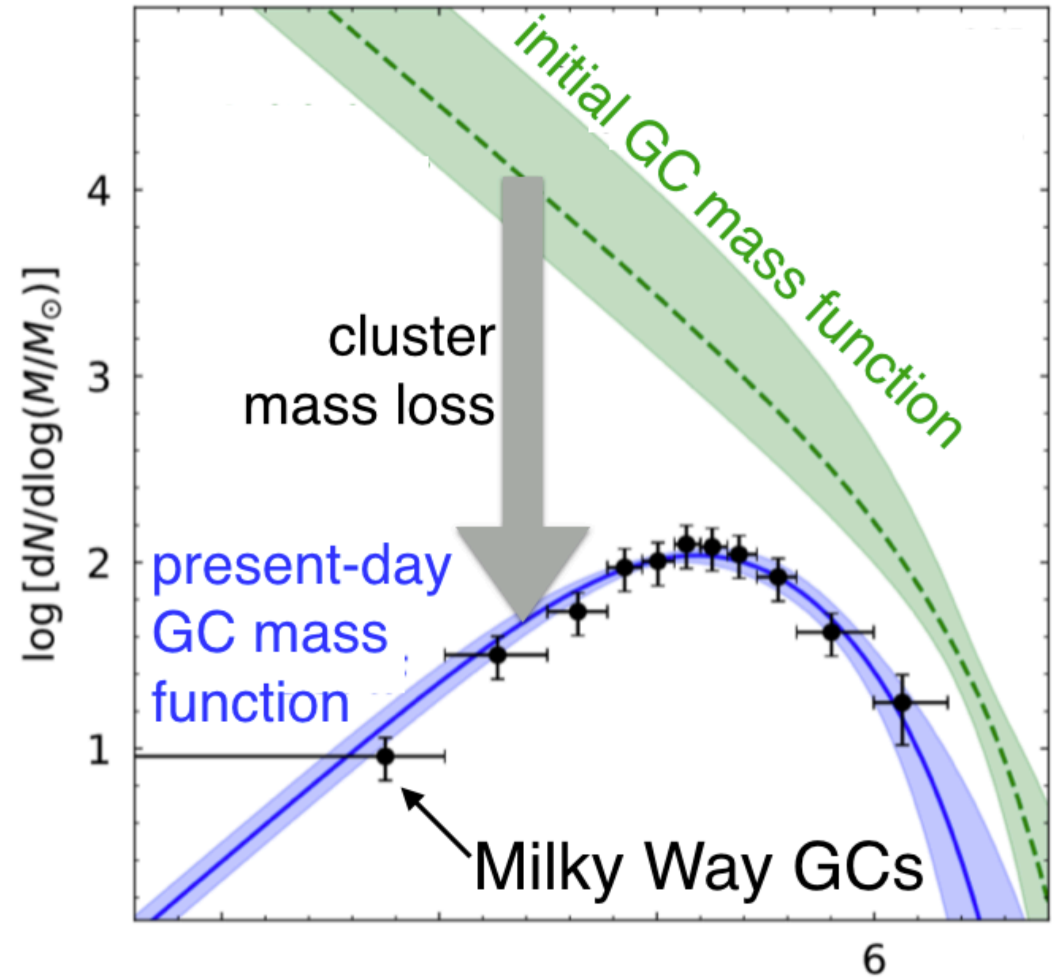

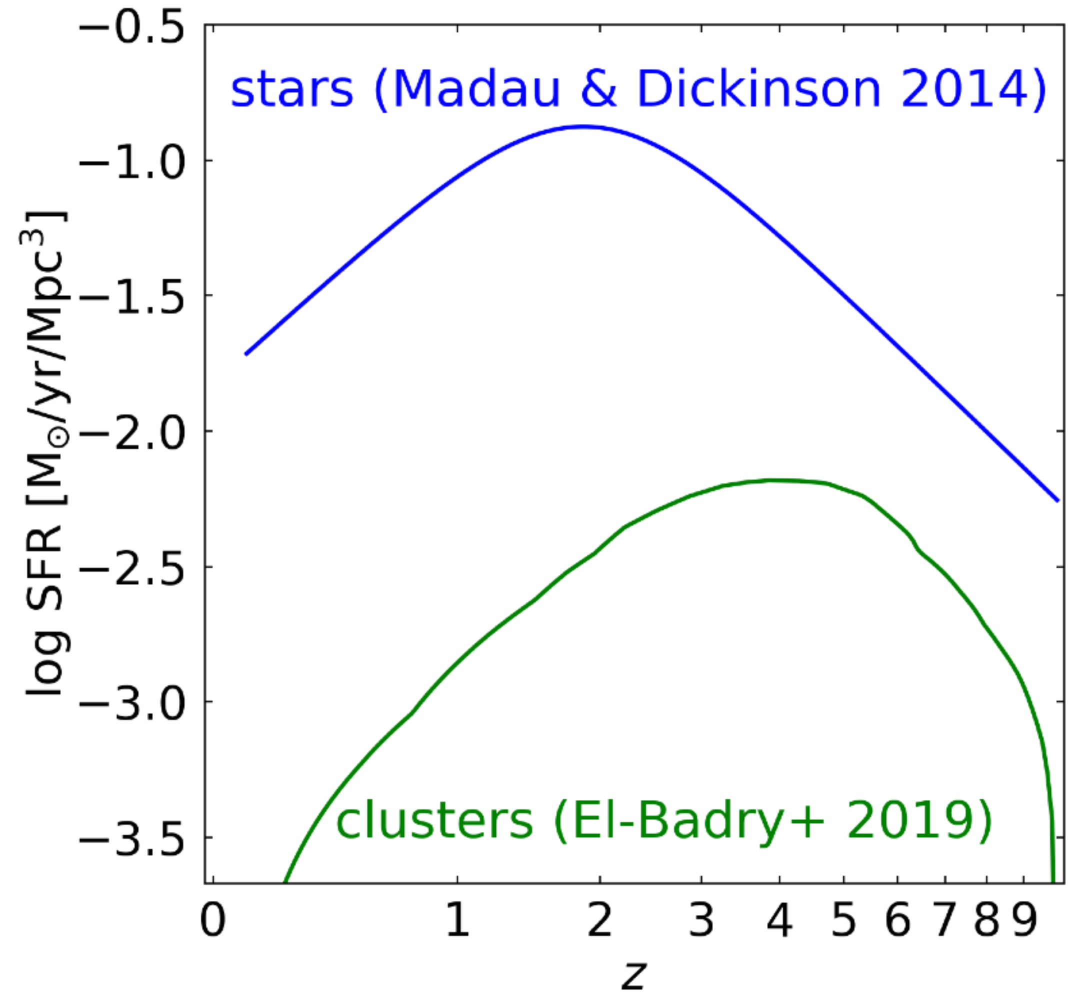

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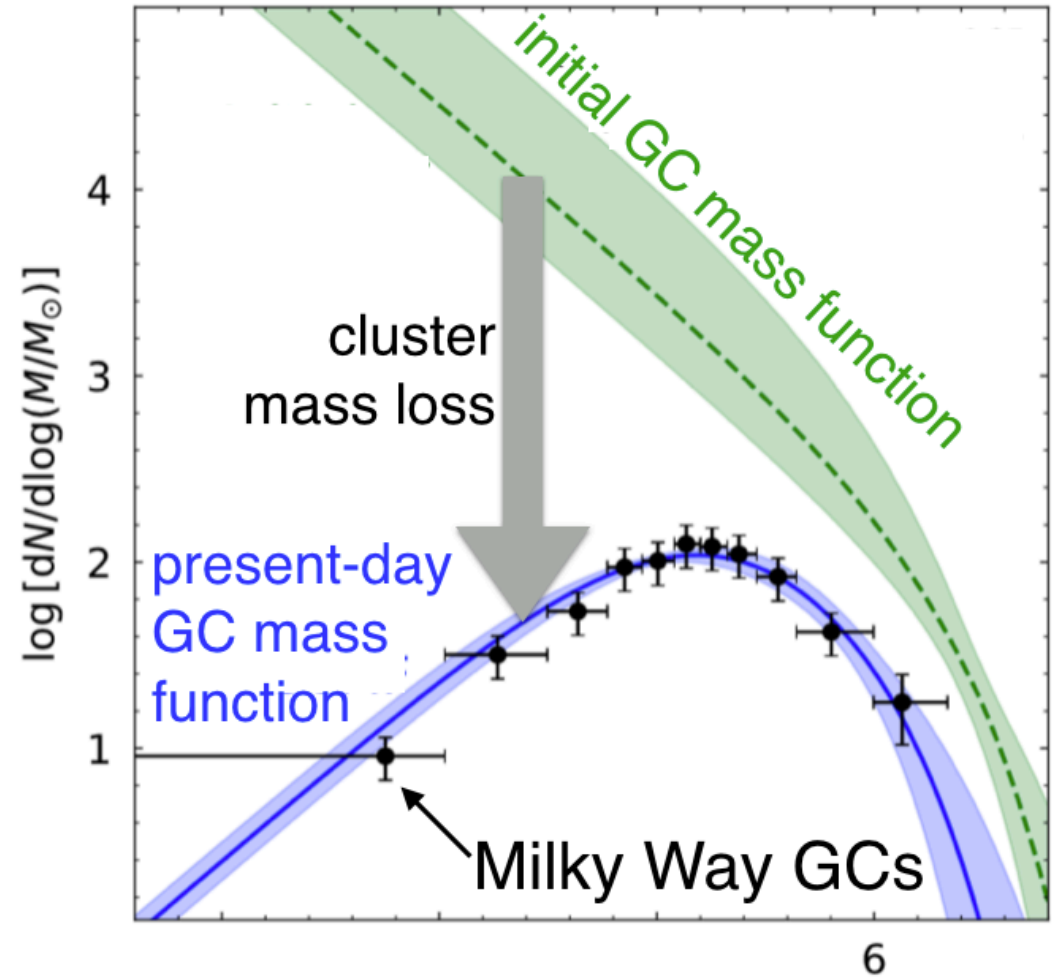
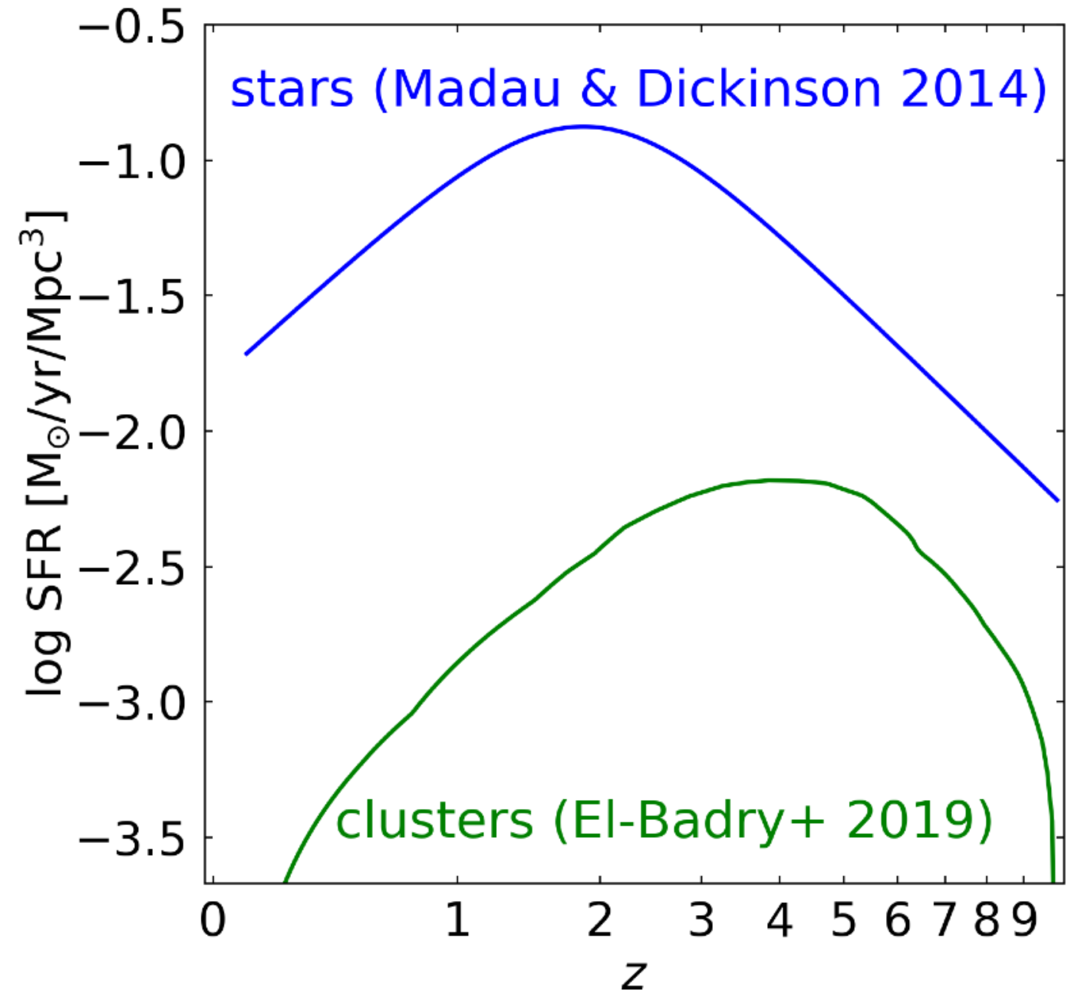
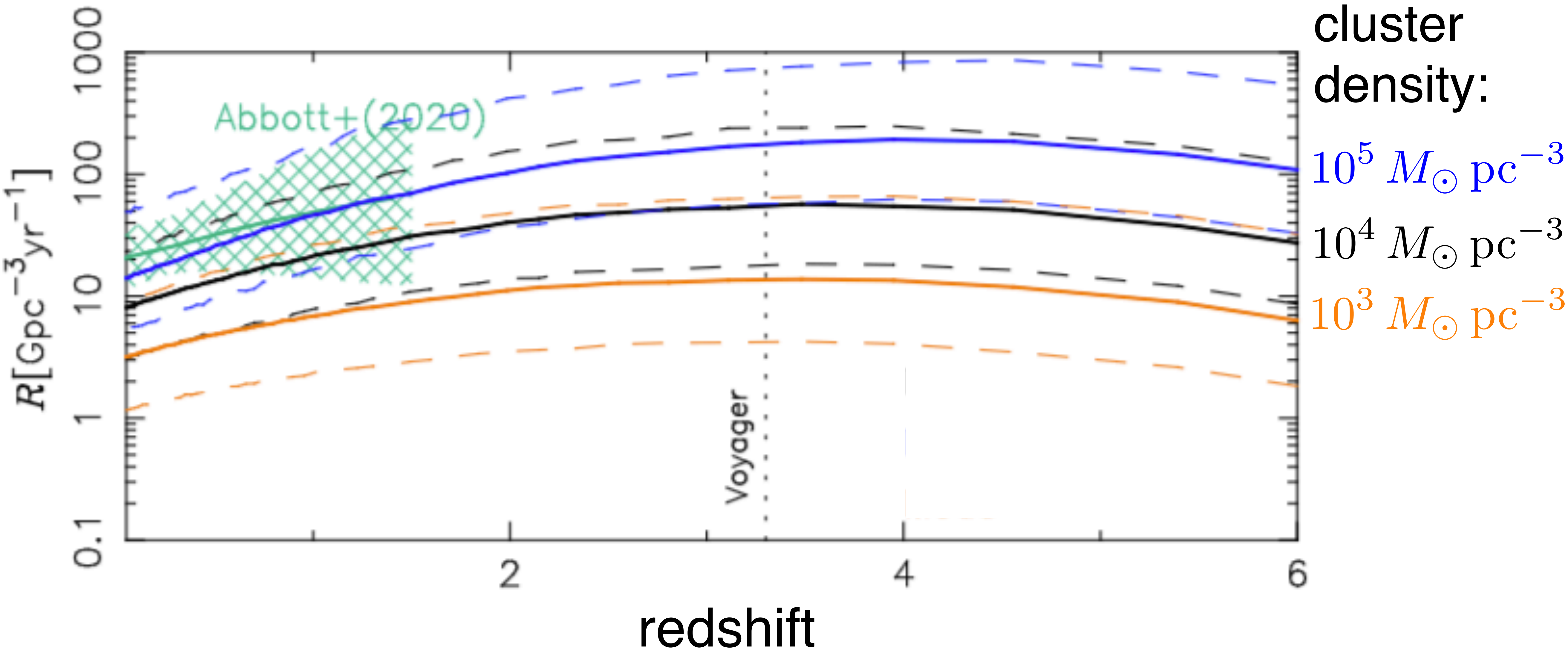
5. evolve!
   Monte Carlo
   (20 million clusters)





Antonini & Gieles 2020b, PRD

Comparison to GWTC-2

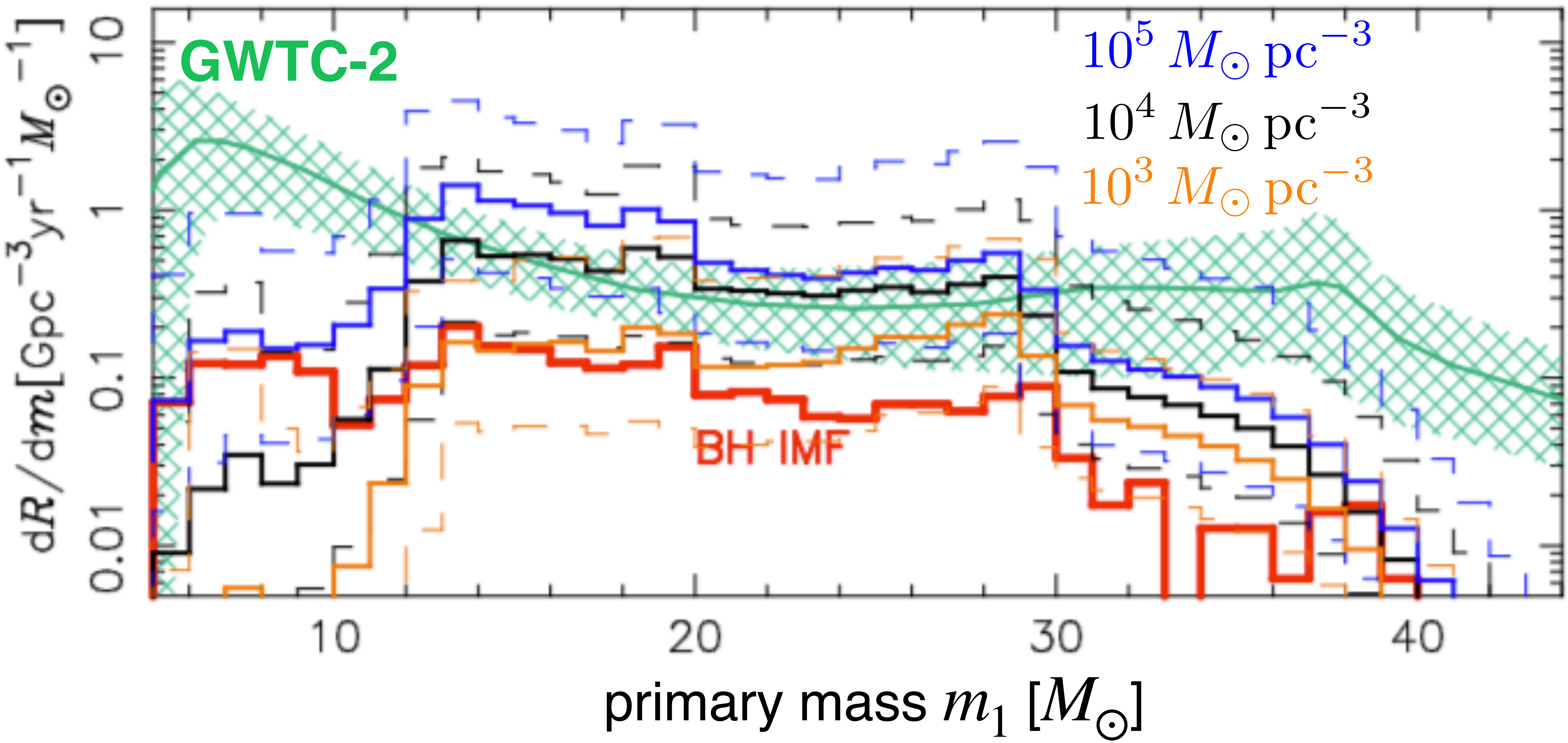


$$\mathcal{R}_0 = 7.2^{+21.5}_{-5.5}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

GWTC-2: $19.1^{+16.2}_{-9.0}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$

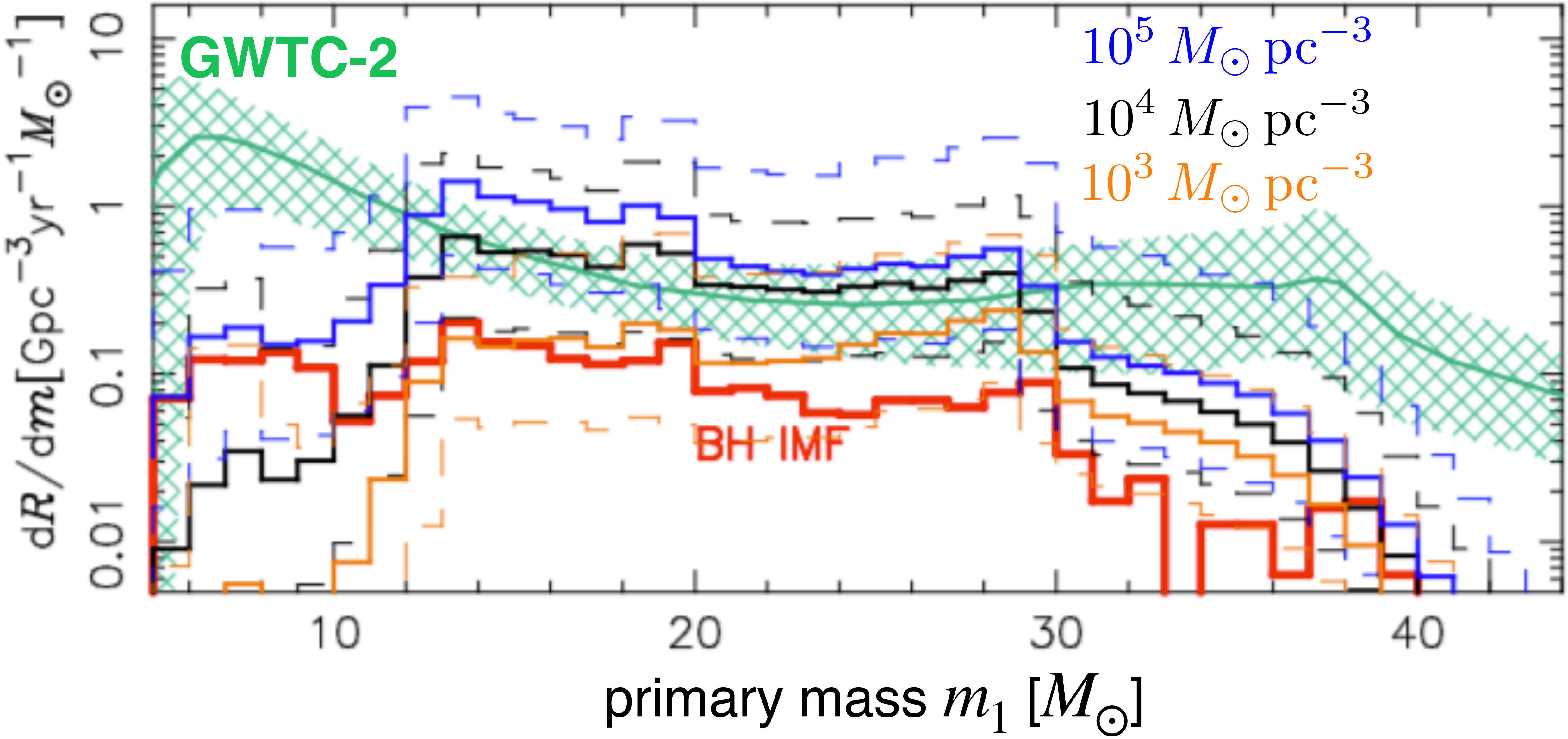
Antonini & Gieles 2020b, PRD

Mass-dependent rate



Antonini & Gieles 2020b, PRD

Mass-dependent rate

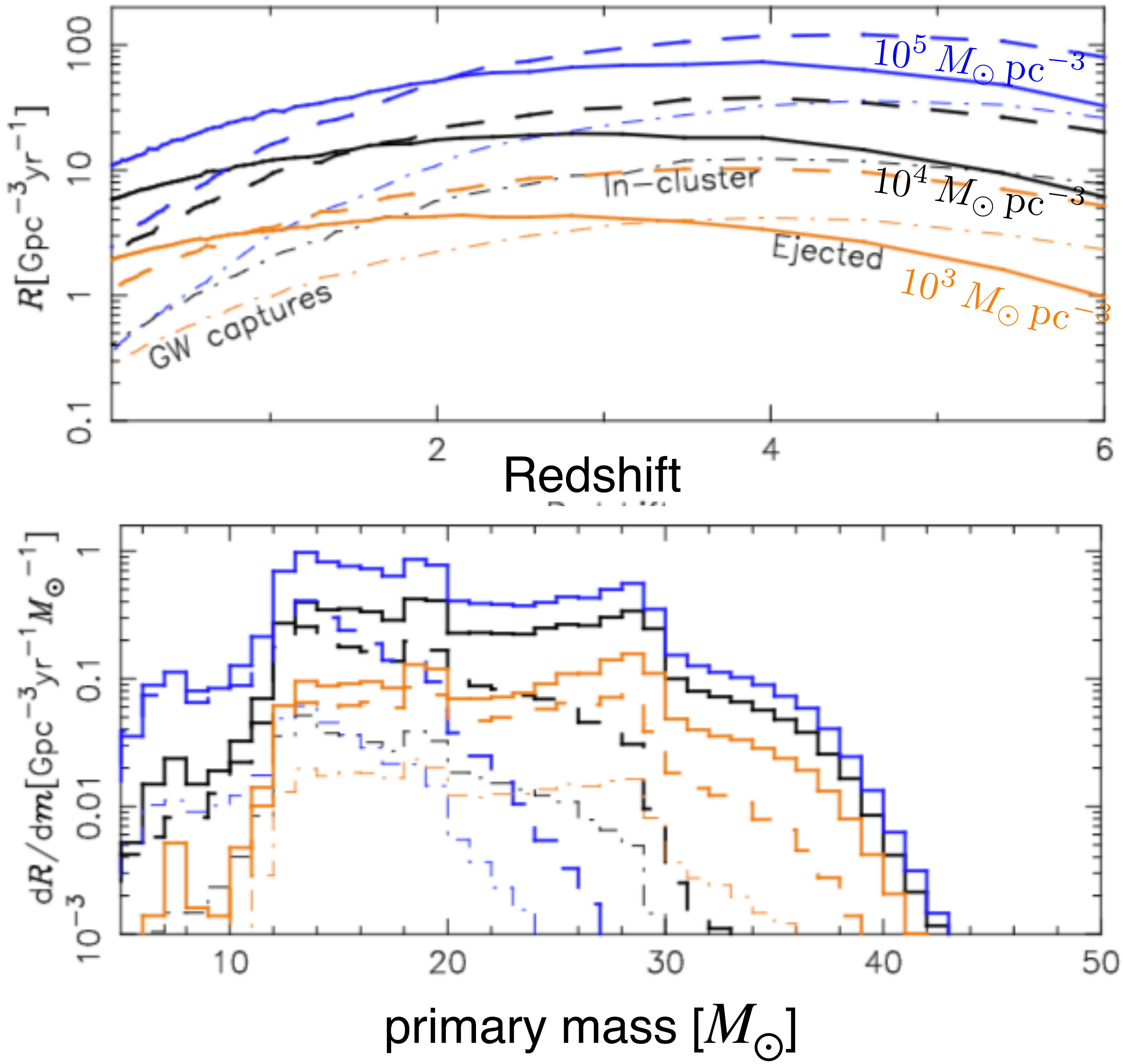


$\rho_0 = 10^5\ M_\odot\ \mathrm{pc}^{-3}$ reproduces the total rate

$\rho_0 = 10^4\ M_\odot\ \mathrm{pc}^{-3}$ reproduces the ***mass-dependent*** rates between 13-30 $M_\odot$

Antonini & Gieles 2020b, PRD

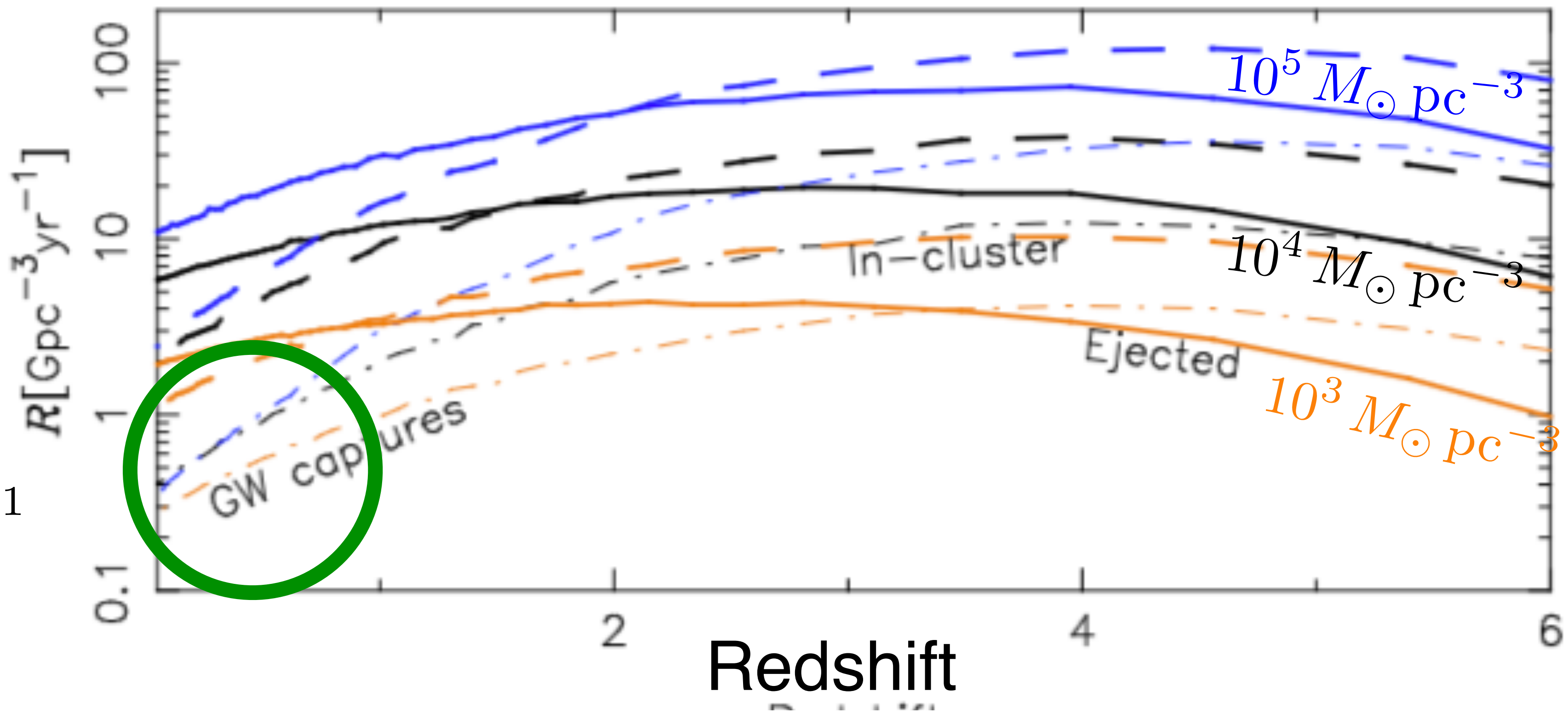
In-cluster / ejected / GW captures



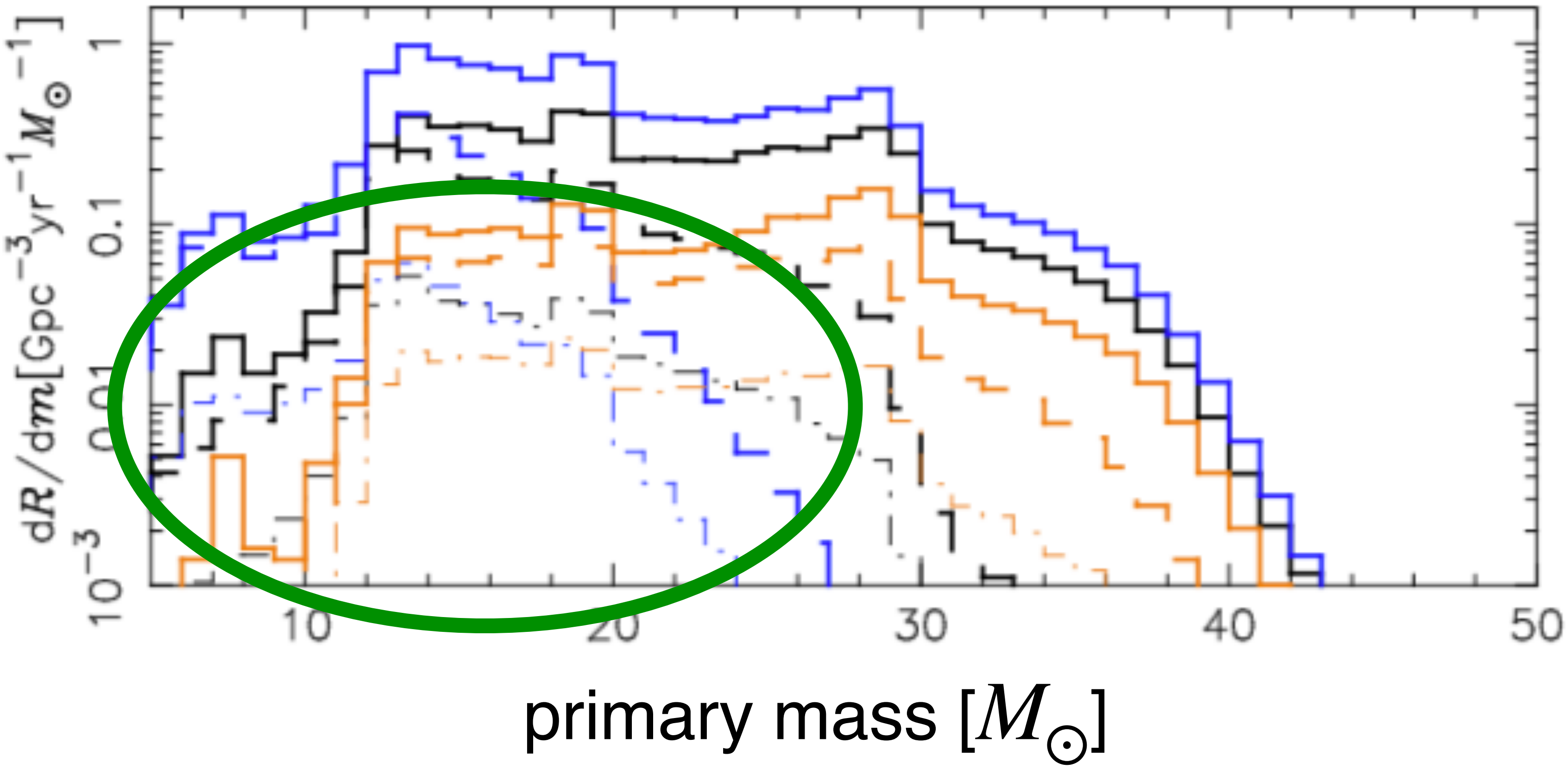
In-cluster / ejected / GW captures

$\mathcal{R}(e > 0.1) \simeq 0.4\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$

1. insensitive to ρ_0
2. steep *z* dependence
3. ~4-10%



3. low mass



eBBHs

model predictions

Globular clusters:

$$\mathcal{R}(e > 0.1) \simeq 0.4\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$$

5-10% of all mergers

Antonini & Gieles 2021b, Zevin+ 2019

Young massive clusters:

$$\mathcal{R}(e > 0.1) \simeq 5\ \mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$$

Banerjee 2021

observations

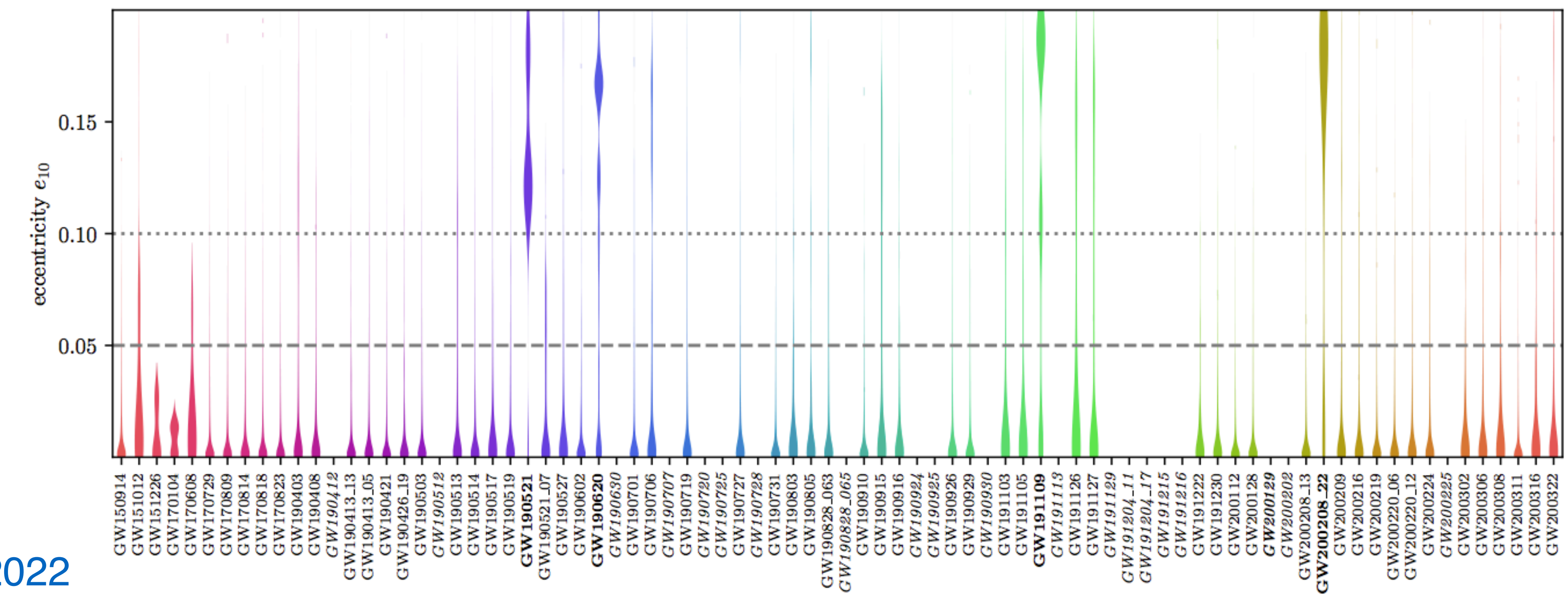
After O1+O2:

$$\mathcal{R}(e > 0.1) < 100\ \mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$$

Abbott+ 2019

GW190521

Abbott+ 2020; Gayathri+ 2020;
Calderón Bustillo+ 2021



Romero Shaw+ 2019, 2022

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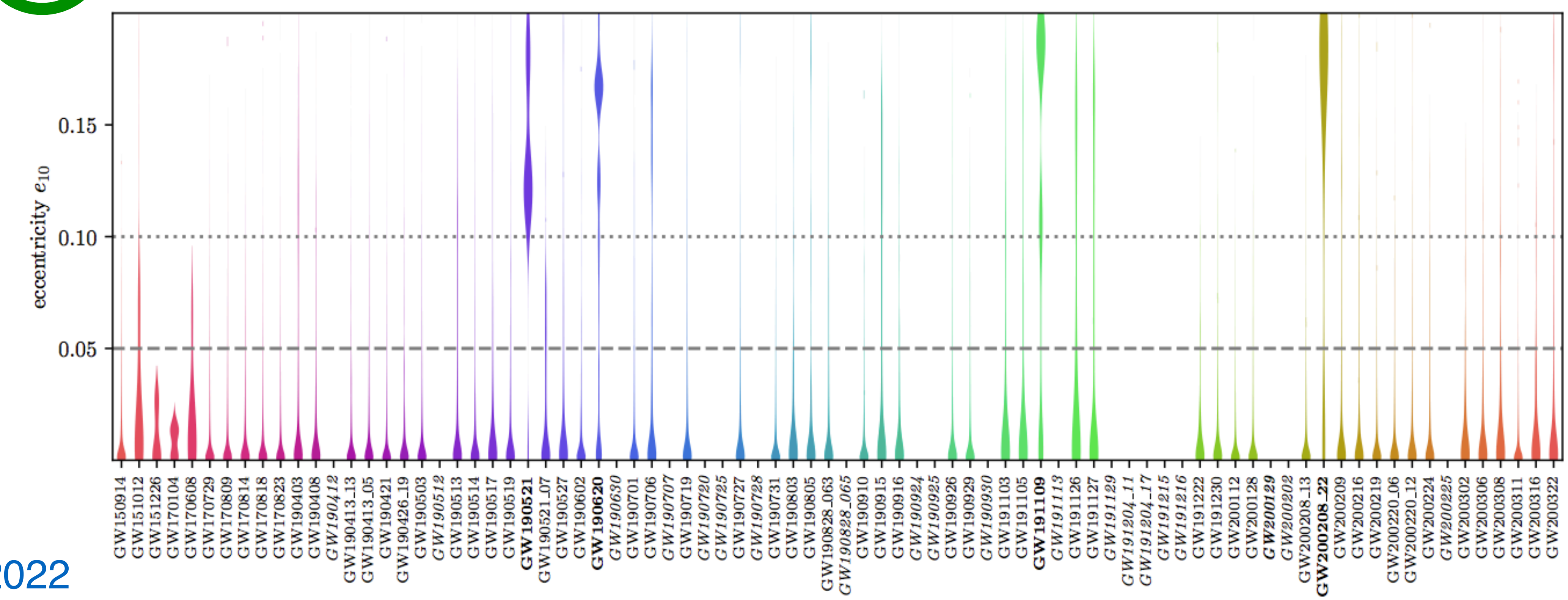
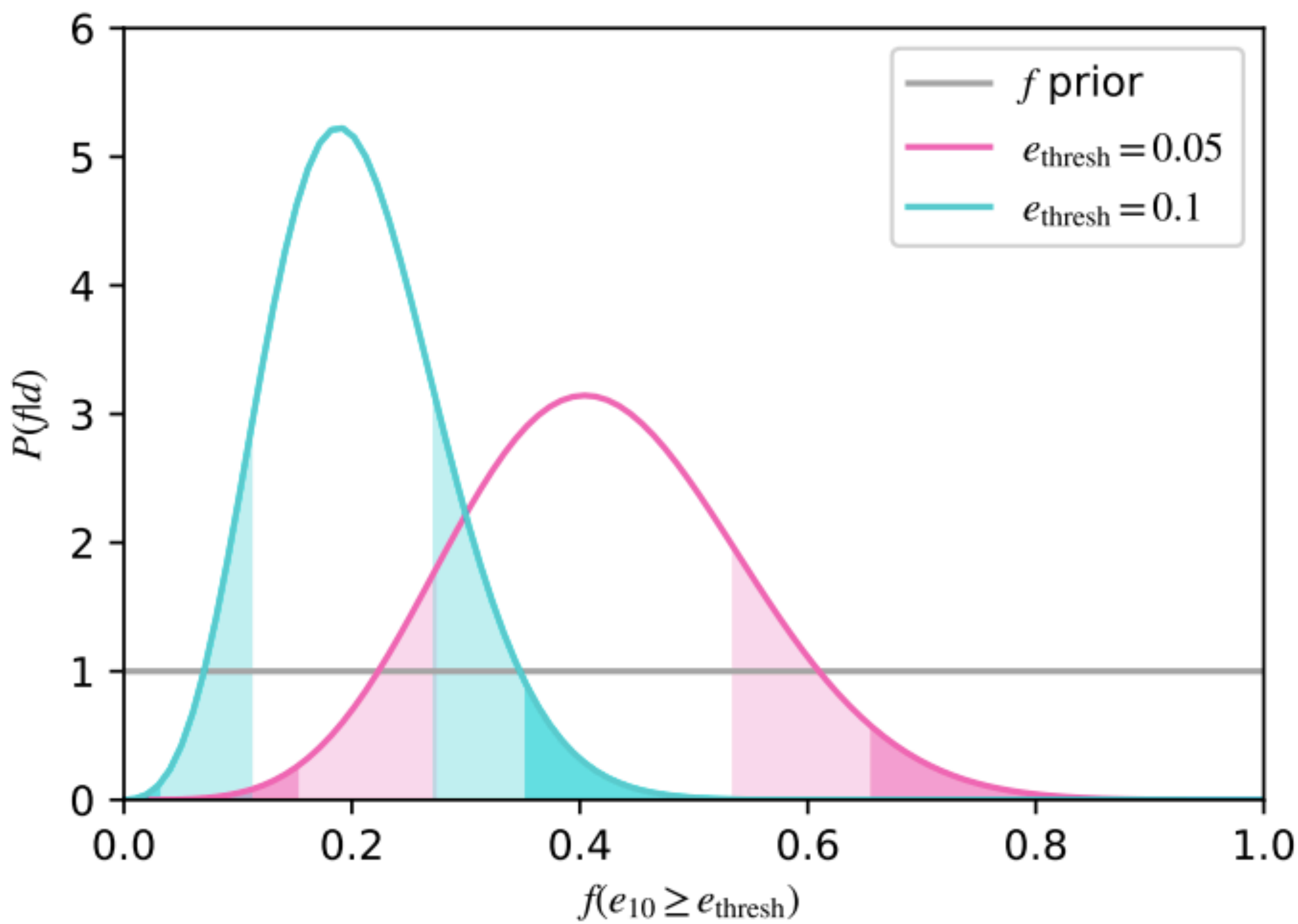


Romero Shaw+ 2019, 2022

Small fraction of eBBH detectable, all BBH dynamical?



Romero Shaw+ 2022

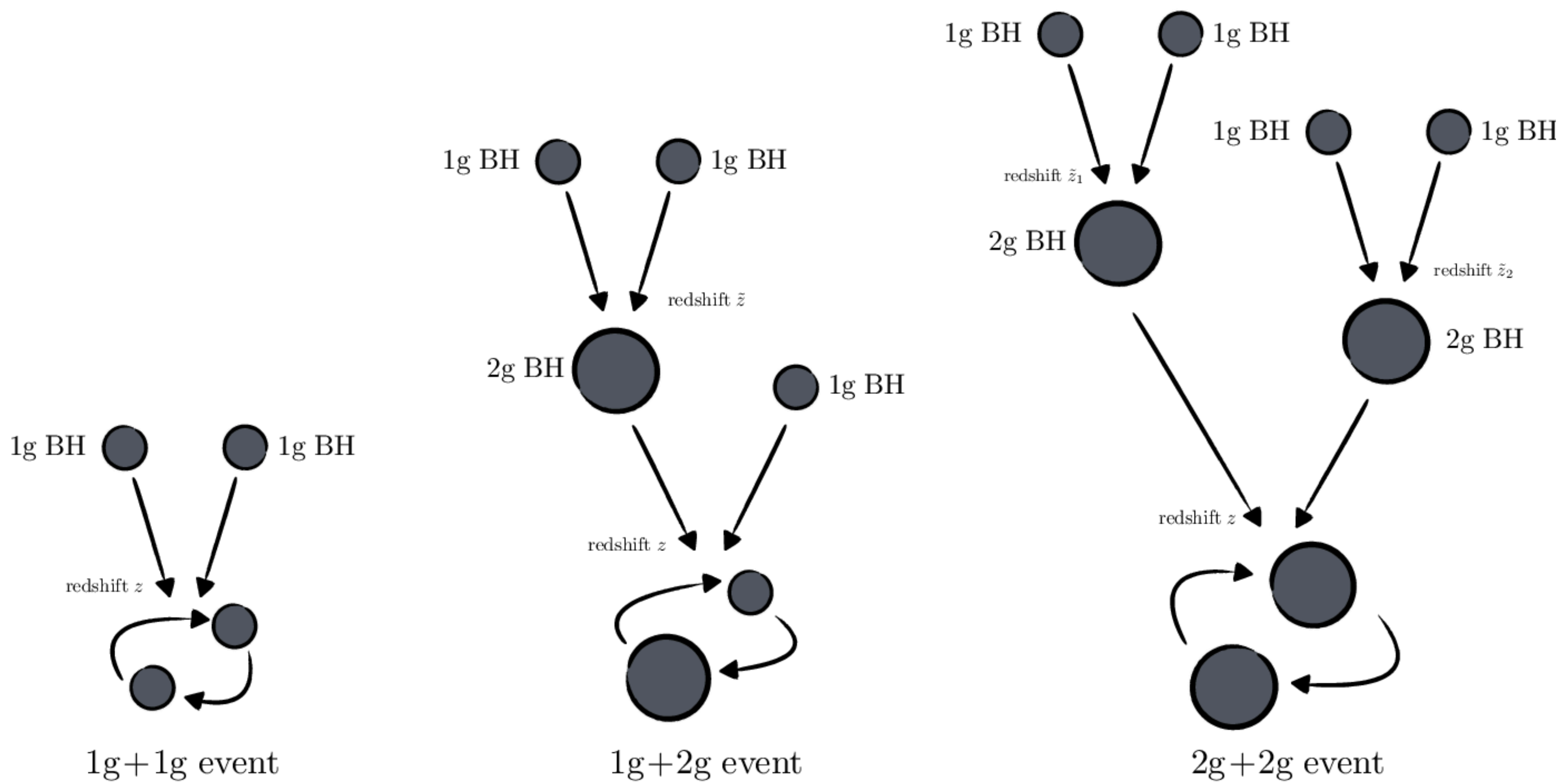
Fast model for dynamical BHB mergers

Population modelling and compare to GWTCs

Ongoing work

Recent improvements: 1. Hierarchical mergers

Merger retained if $v_{\rm esc} > v_{\rm GW,kick}$ (Rezzolla+ 2008)



Gerosa & Berti 2017

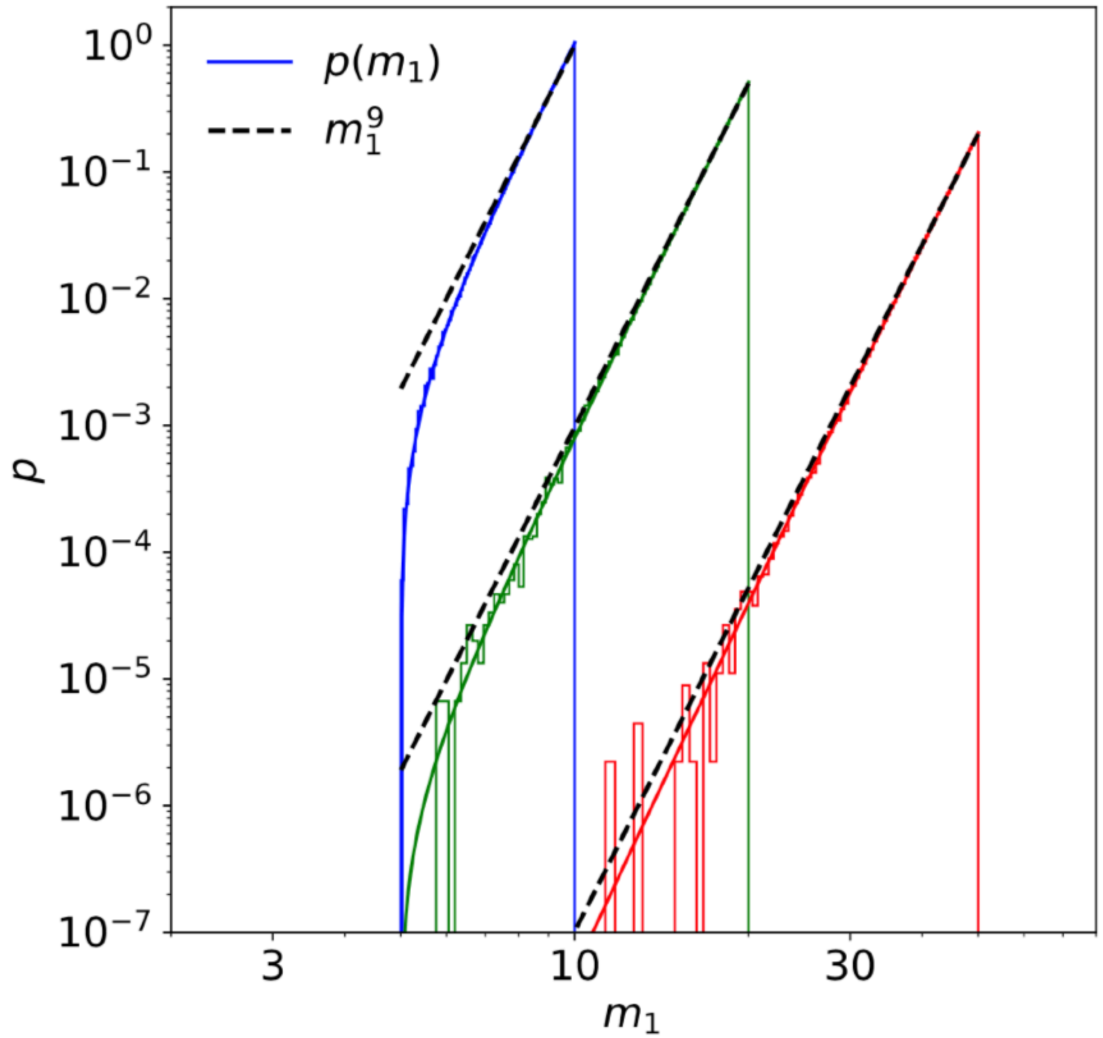
Recent improvements: 2. Sample m_1, q, m_3

Use: 1. BH mass function : $p(m) \propto m^{\alpha}$

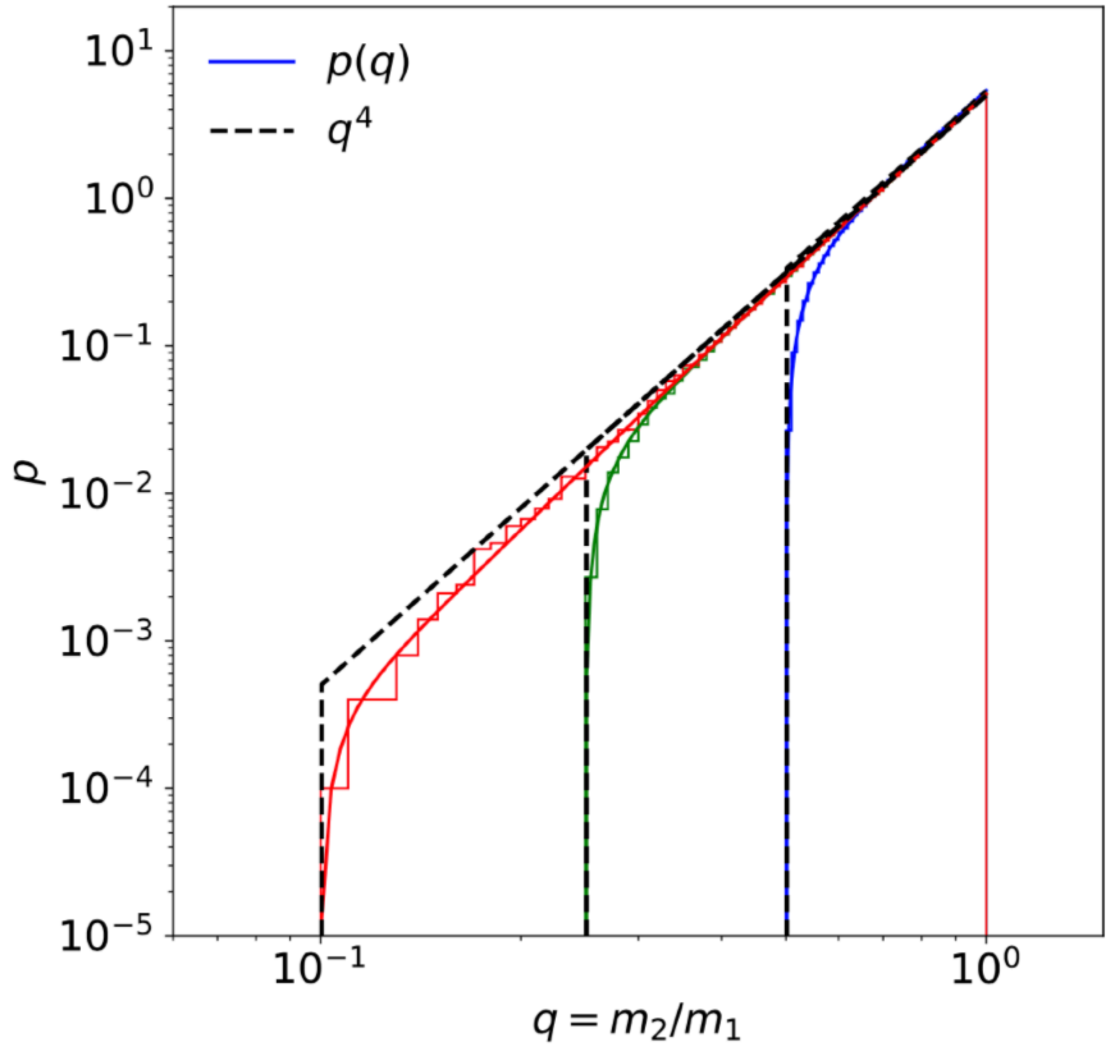
2. Heggie's 3-body formation rate: $\Gamma_{3b}(m_1, m_2, m_3)$

3. Cross section for masses of interacting single BHs

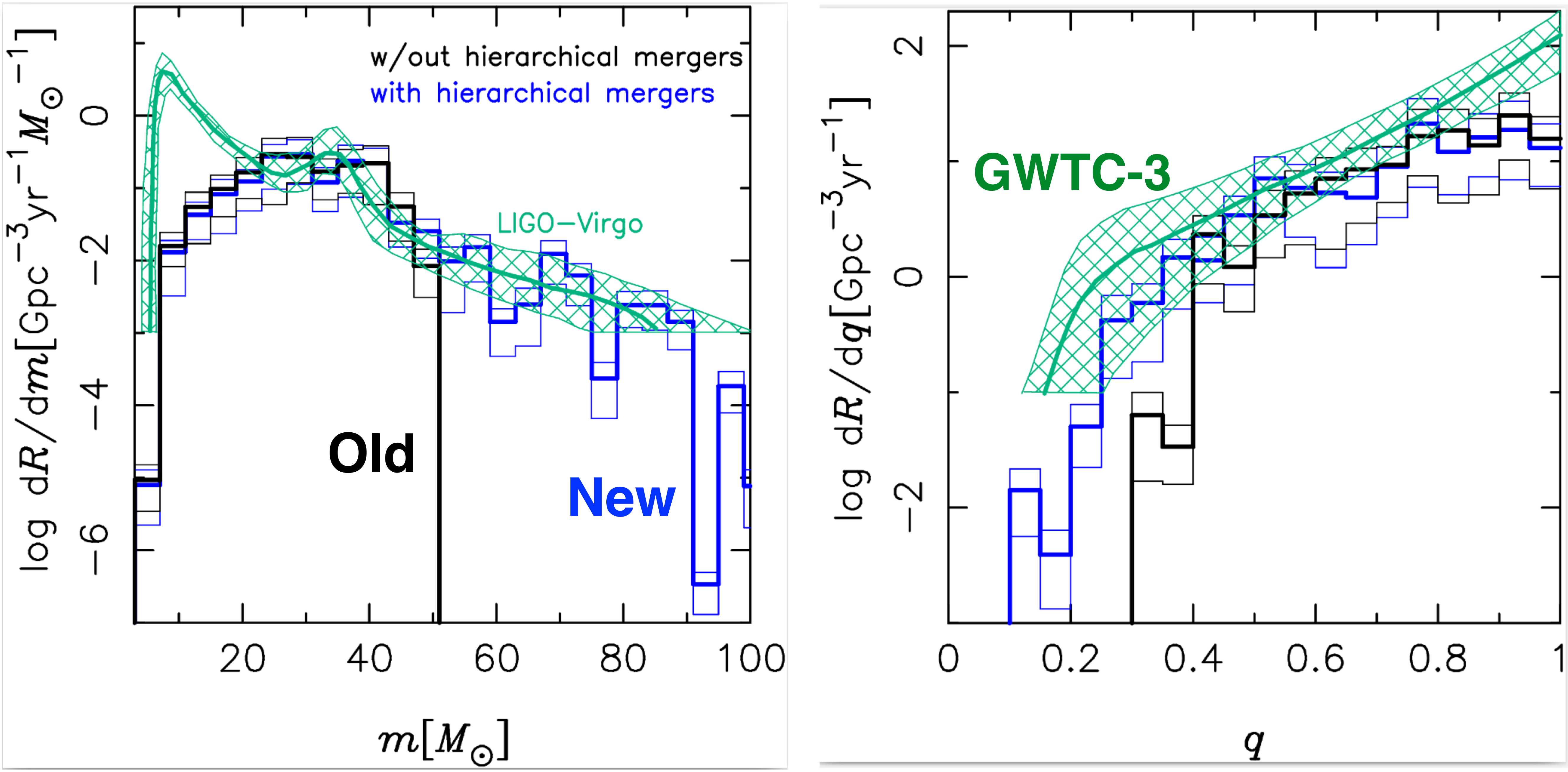
$$p(m_1) \propto m_1^{2\alpha+8}$$



$$p(q) \propto q^{\alpha+3.5}$$

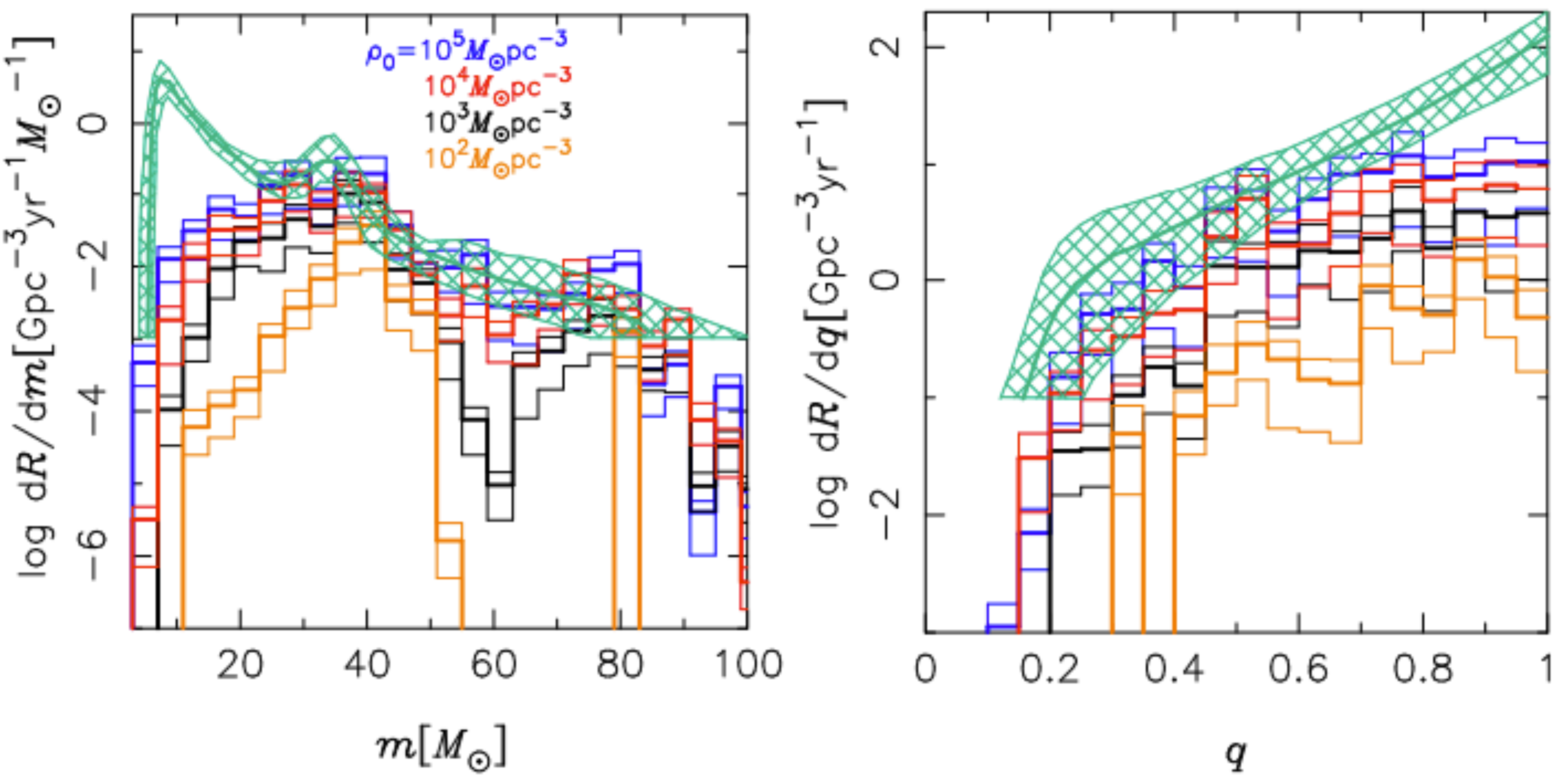


Hierarchical mergers and m_{BH} sampling



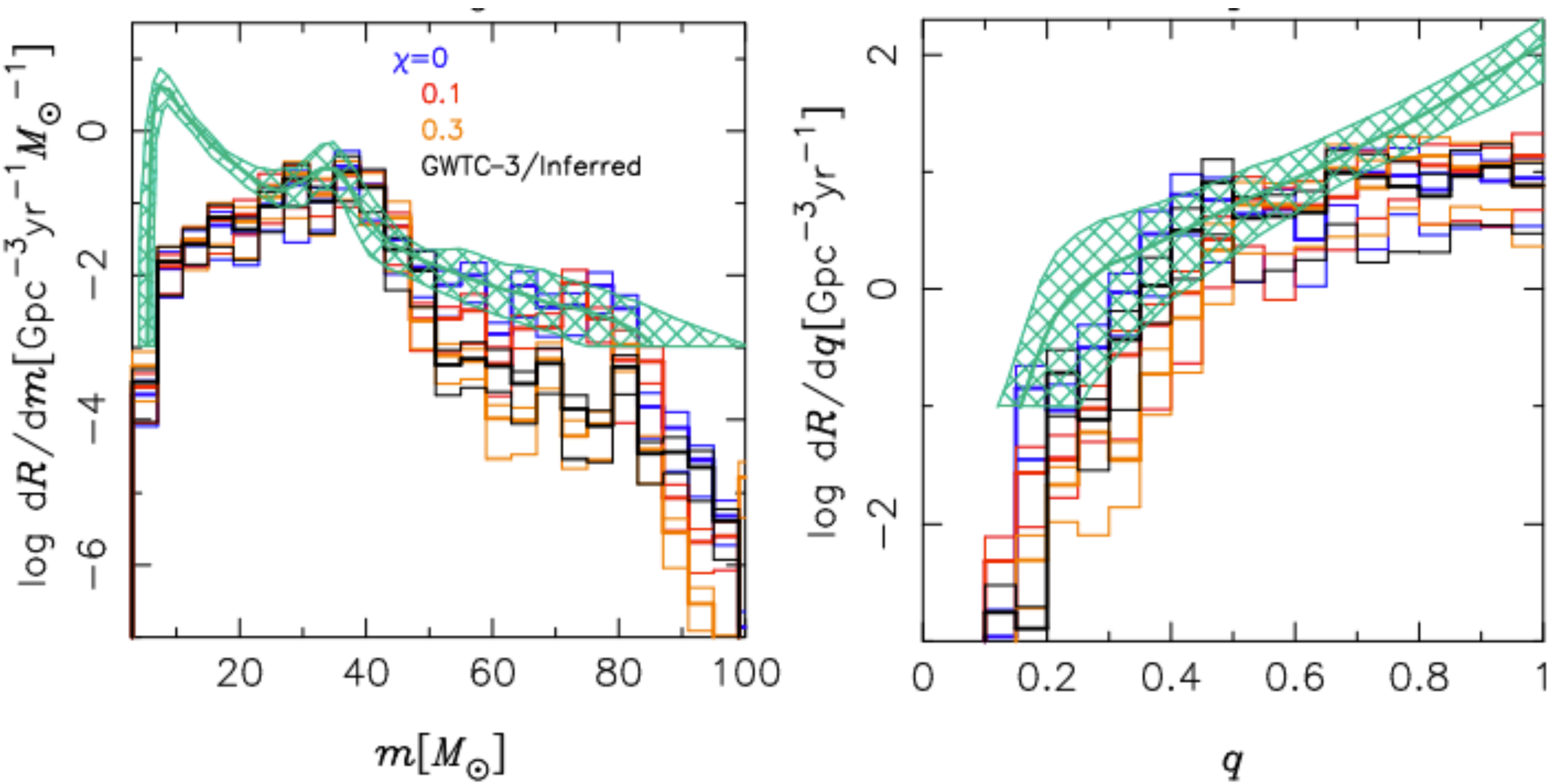
Antonini, Gieles+ 2022, arXiv:2208.01081

Cluster density



Antonini, Gieles+ 2022, arXiv:2208.01081

BH spin

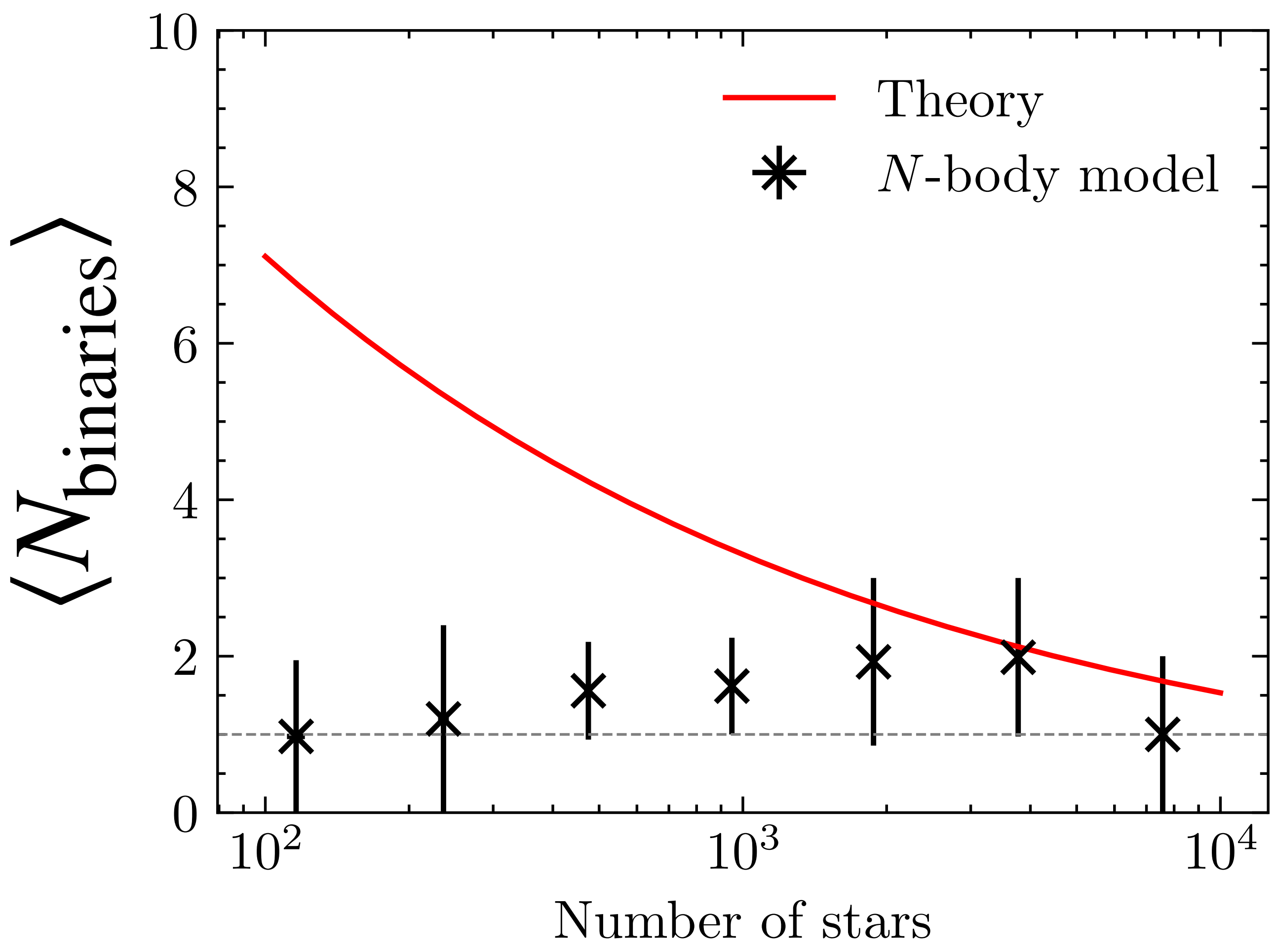


Antonini, Gieles+ 2022, arXiv:2208.01081

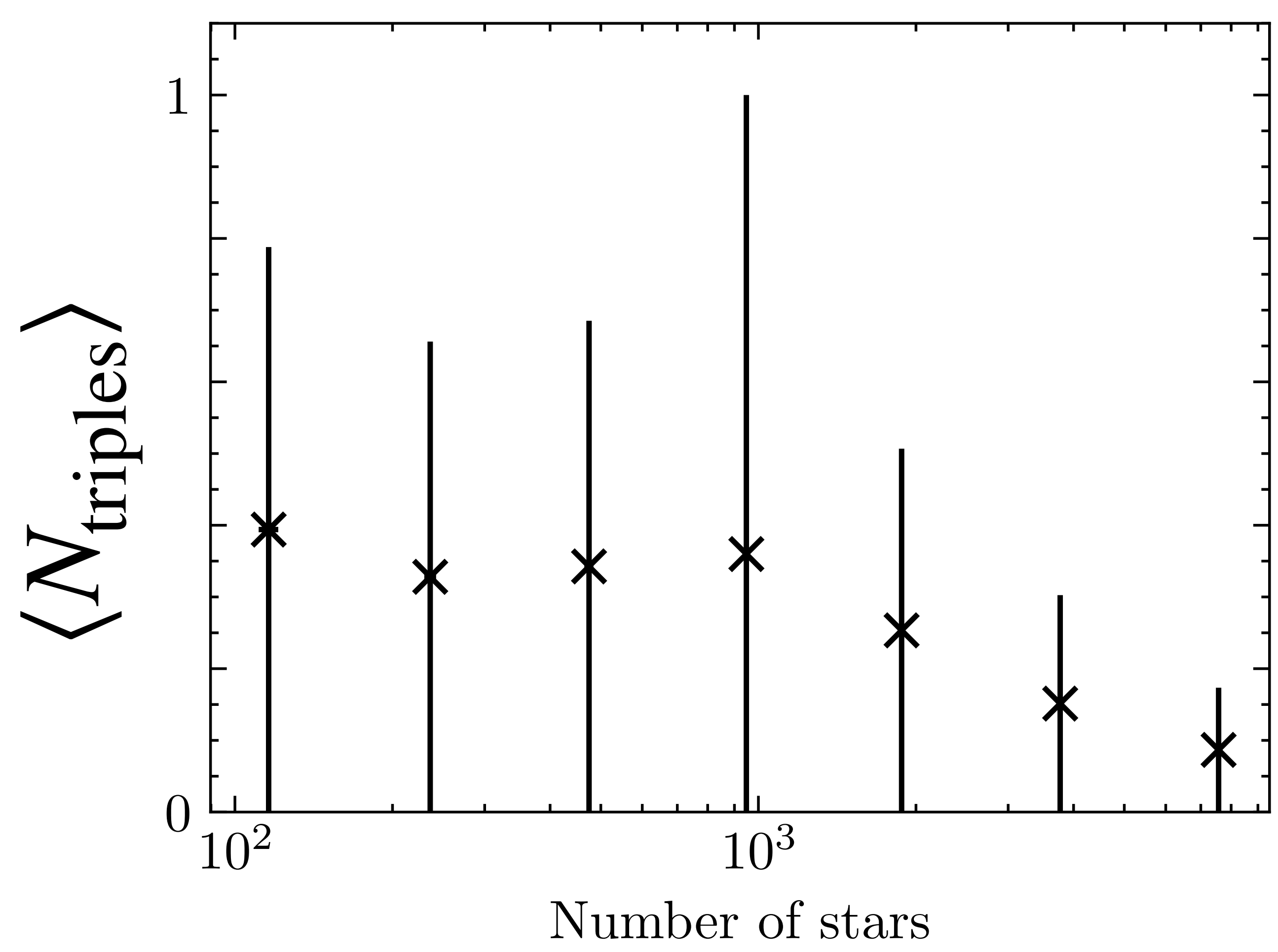
Hierarchical eBBHs?

Testing model assumptions: Single binary?

N-body model: efficient binary disruption → binary-binary encounters → eccentric mergers Zevin+ 2019



Or: number of BHs



Or: number of BHs

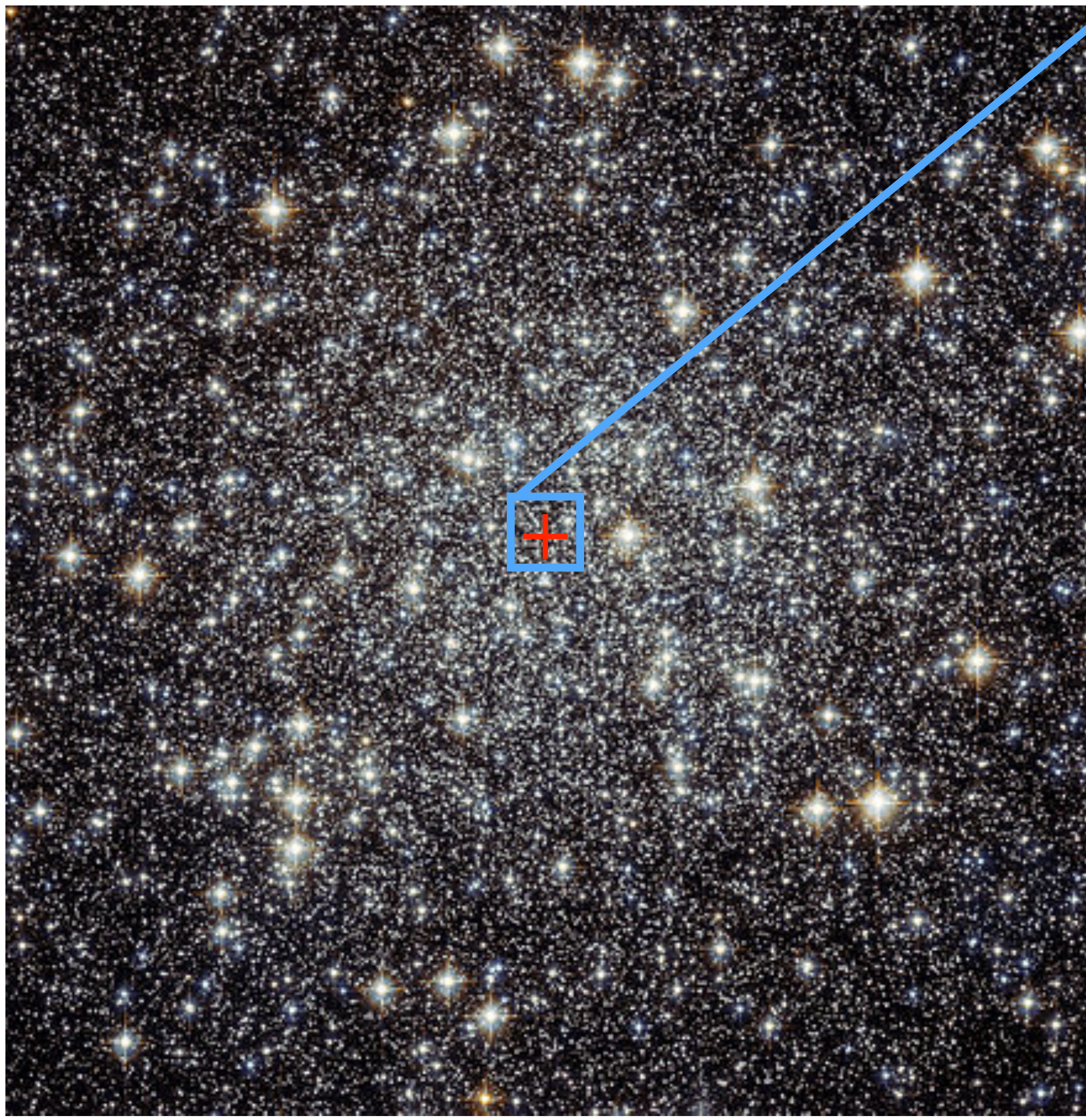
Marín Pina+ in prep

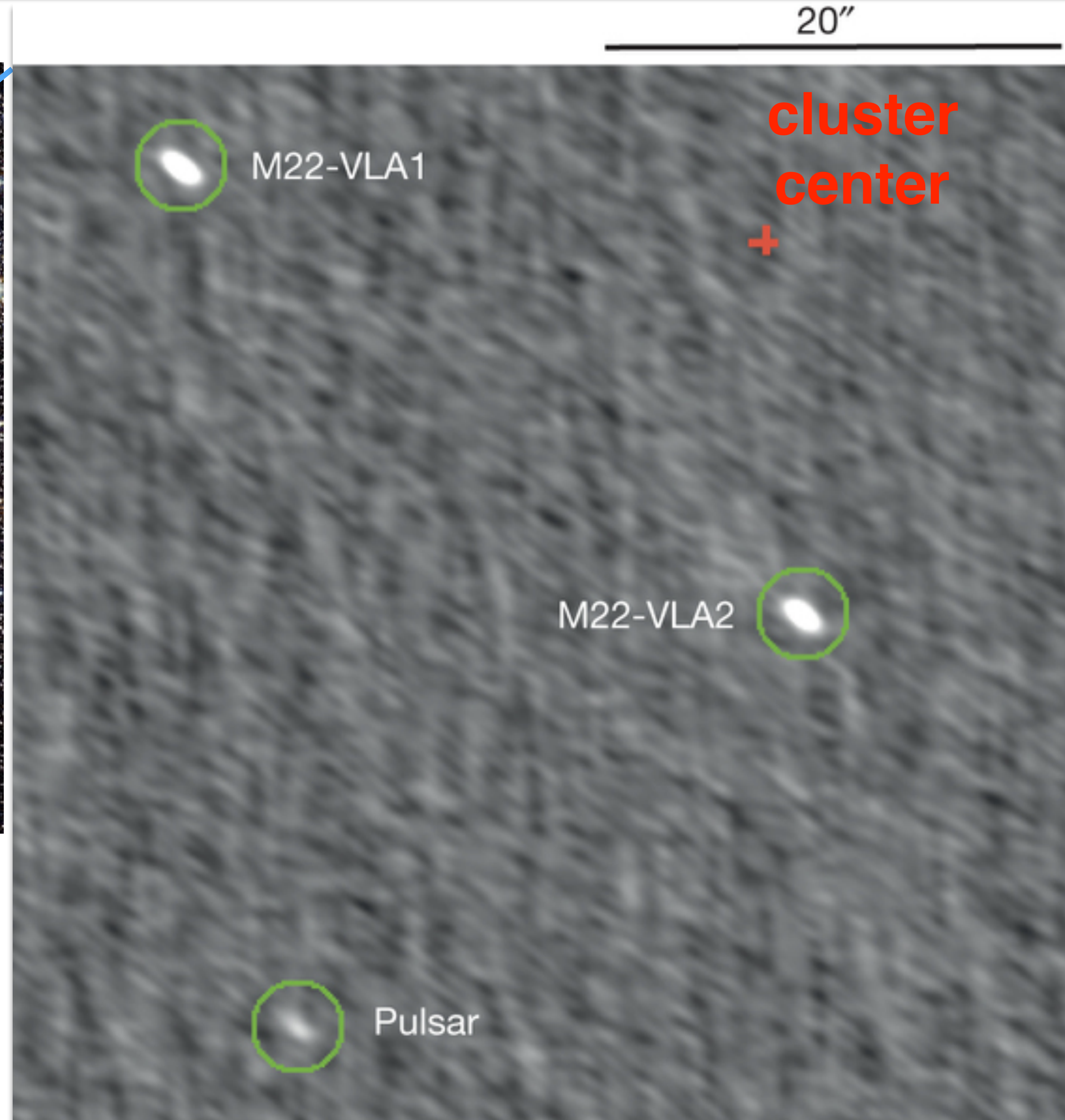
Dynamics can explain mergers $m_1 \gtrsim 20\,\mathrm{M}_\odot$

5-10% of dynamical BBHs are eccentric $(e \gtrsim 0.05)$

Interpretation of detected eBBHs challenging: most eBBHs likely missed

(Accreting) stellar-mass BHs in GCs

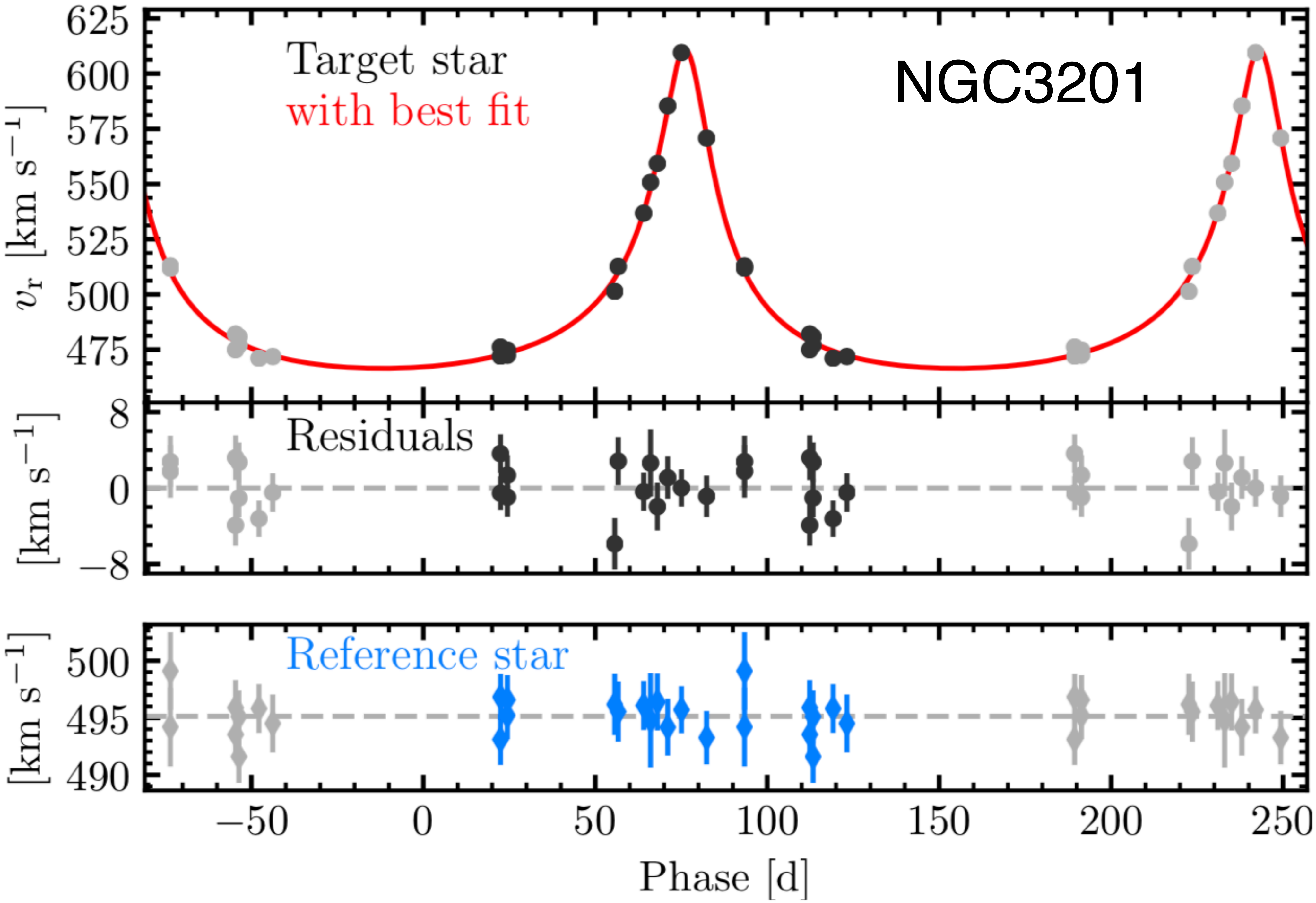




Strader+ 2012; Tremou+ 2018

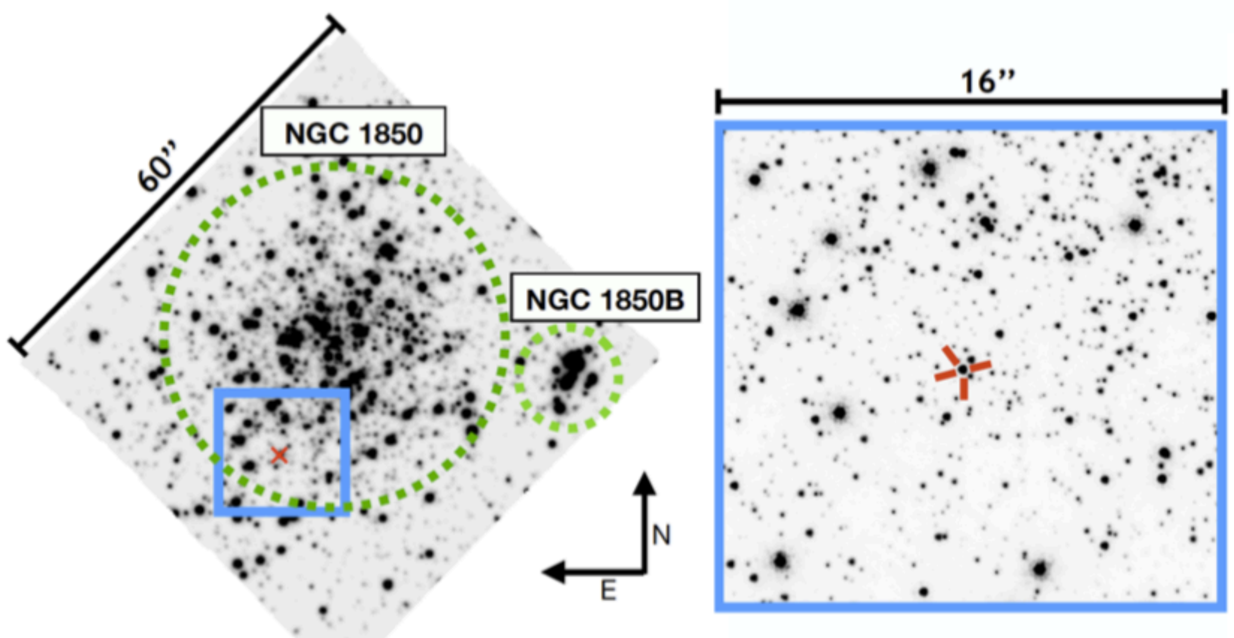
also: Chomiuk+ 2013 (M62);
Miller Jones+ 2015 (47 Tuc)

More BHs in GCs: 3 detached binaries with $M_\bullet > 4\ M_\odot$

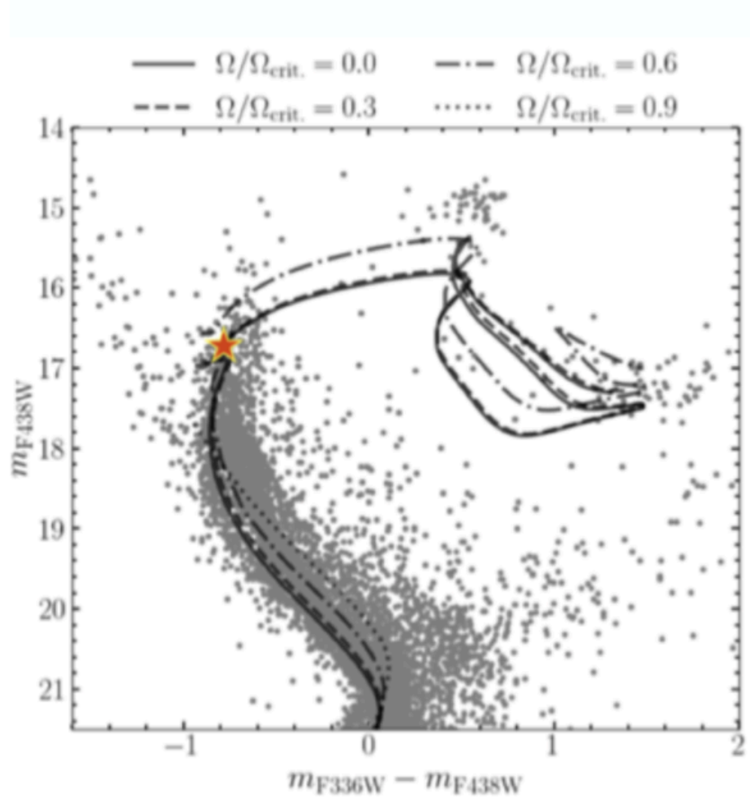


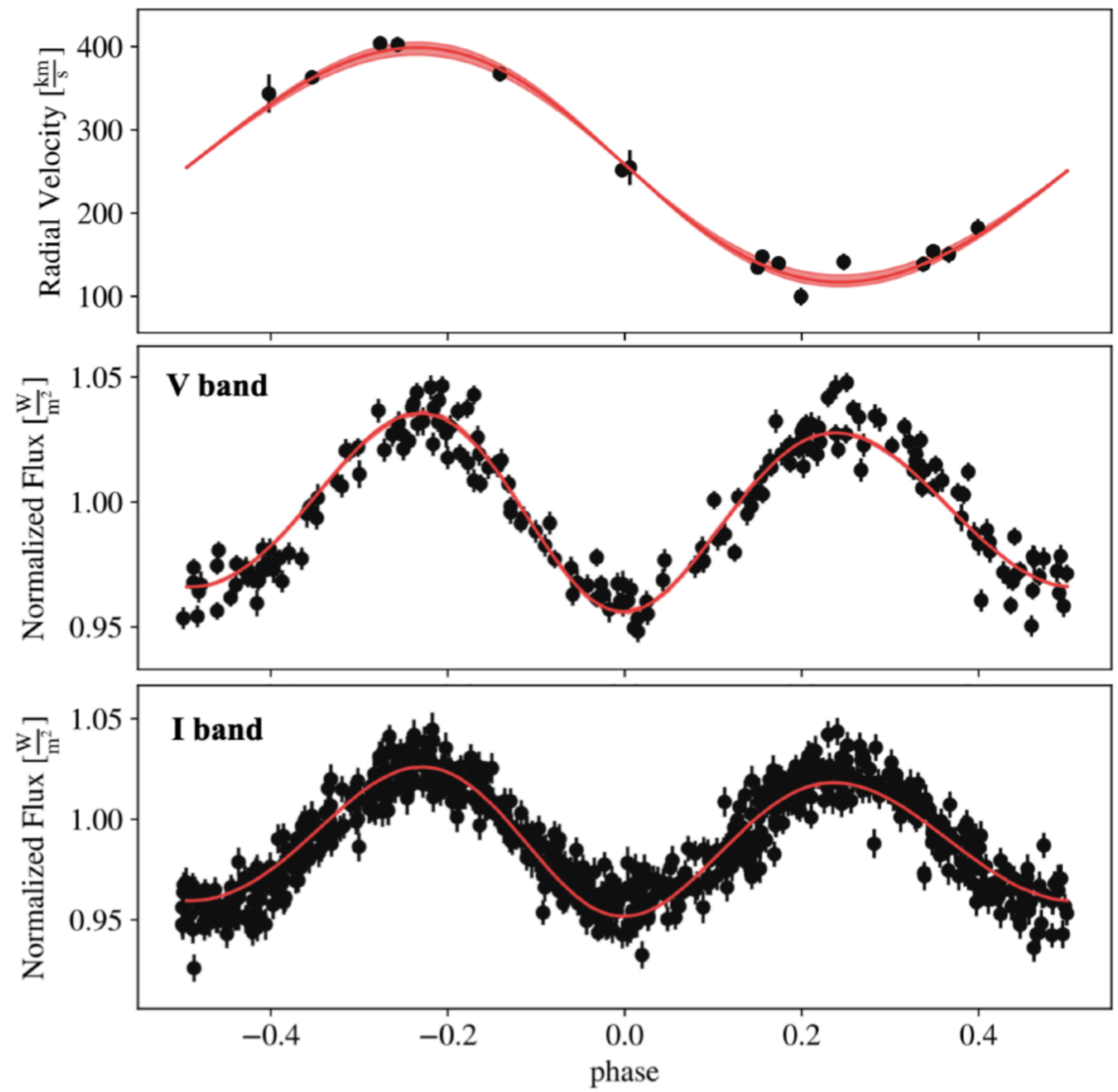
Giesers+ 2018, 2019

A semi-detached binary in 100 Myr cluster: $M_\bullet \simeq 11\ M_\odot$



(a) MUSE (b) HST





Saracino+ 2021

But see: El-Badry & Burdge 2022, but wait for Saracino+ in prep

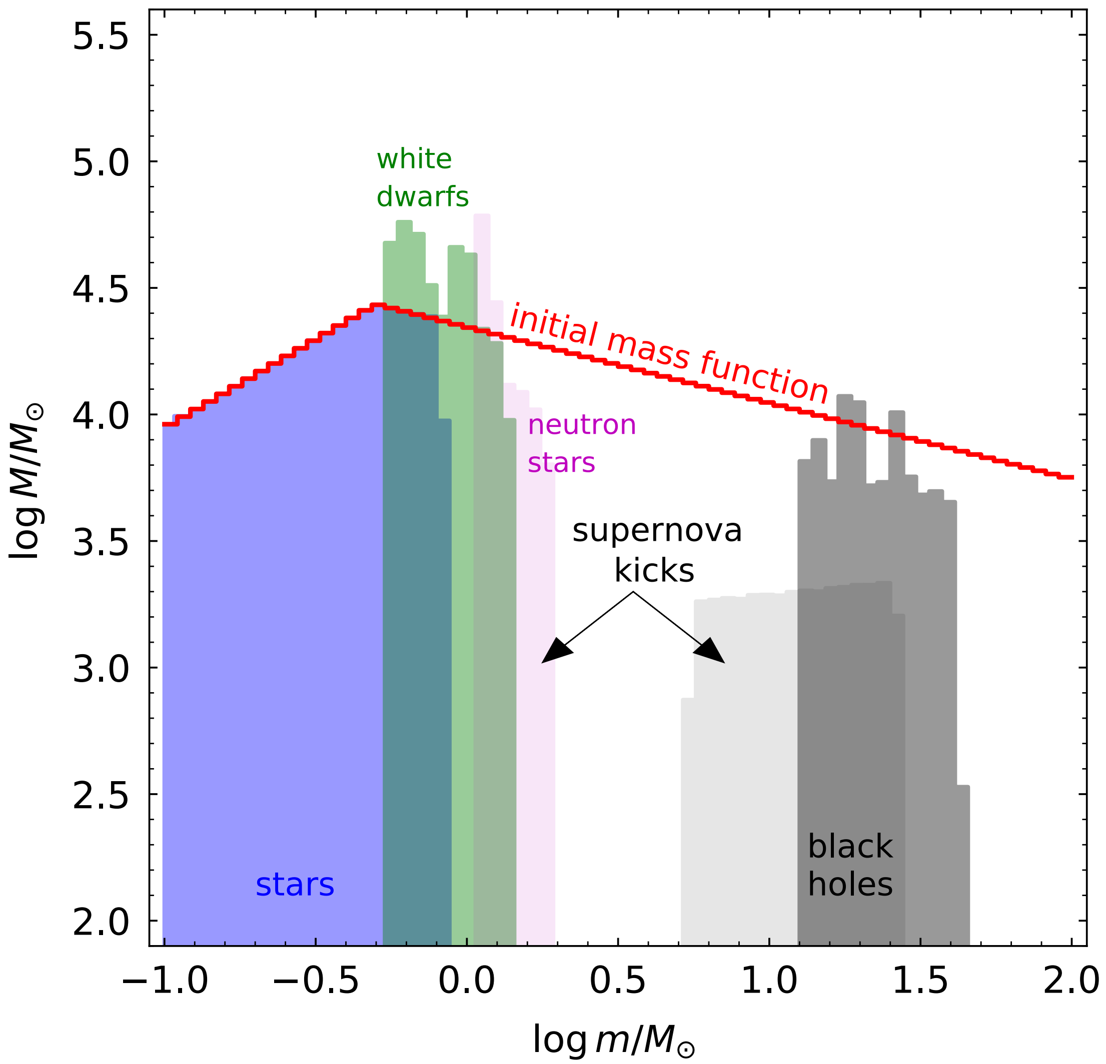
How many BHs can we expect?

IMF = Kroupa 2001

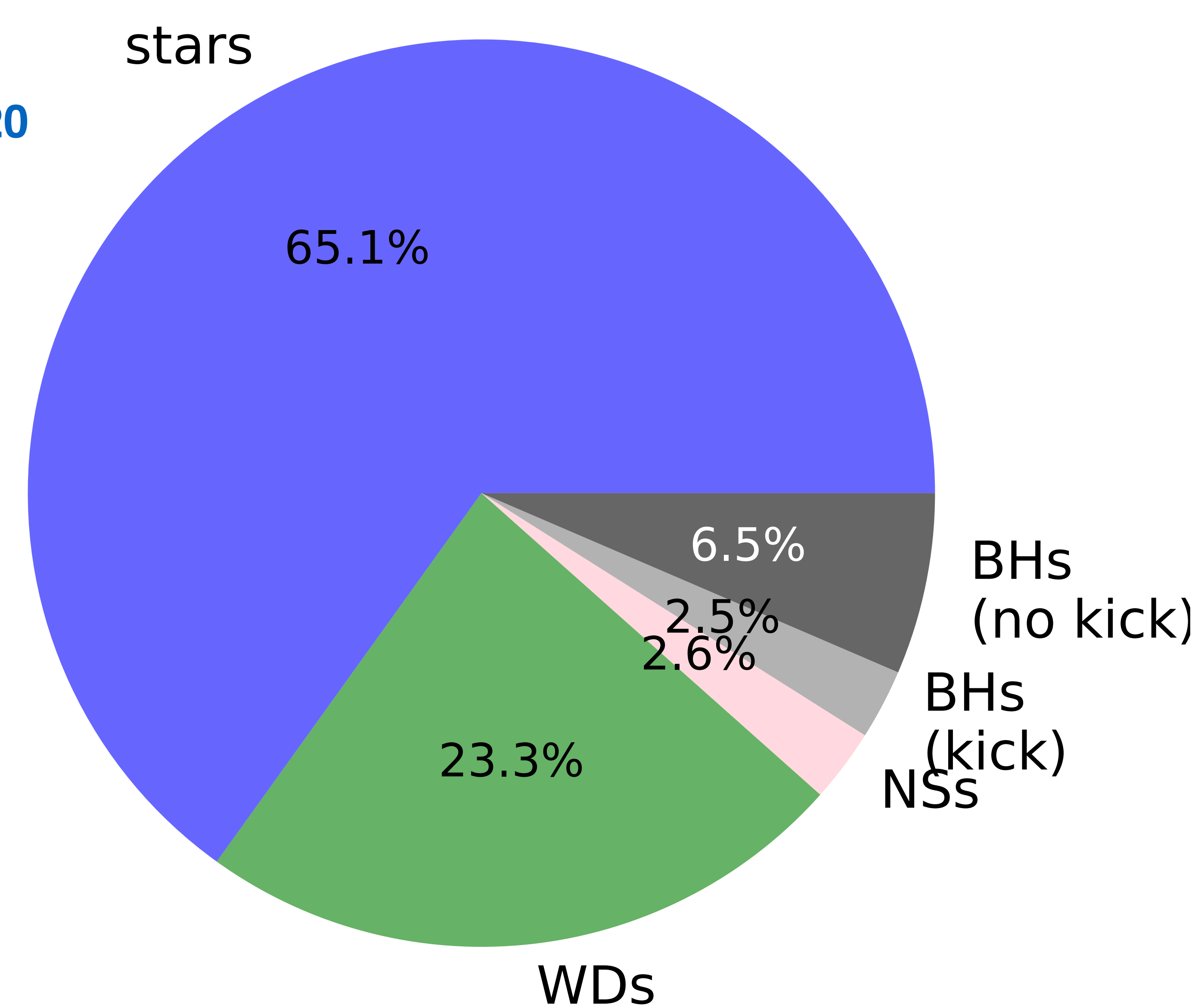
Mass = $10^6\ M_\odot$

[Fe/H] = -1.5

Evolve with SSE **Hurley+ 2000; Banerjee 2020**



mass fractions @12 Gyr

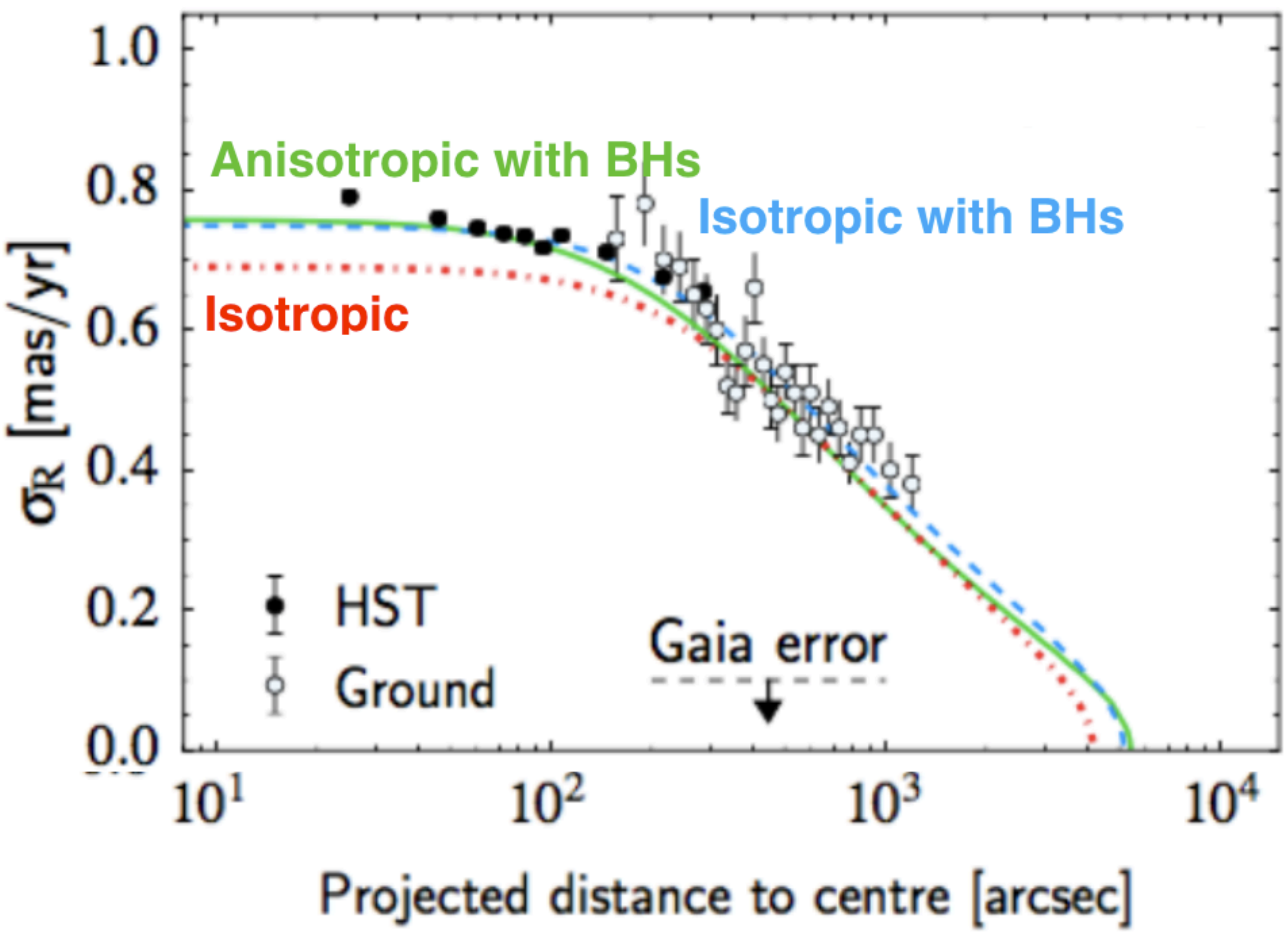


10^5 M$_\odot$ BH population in Omega Centauri (ω Cen)

5% of total mass!

velocity dispersion

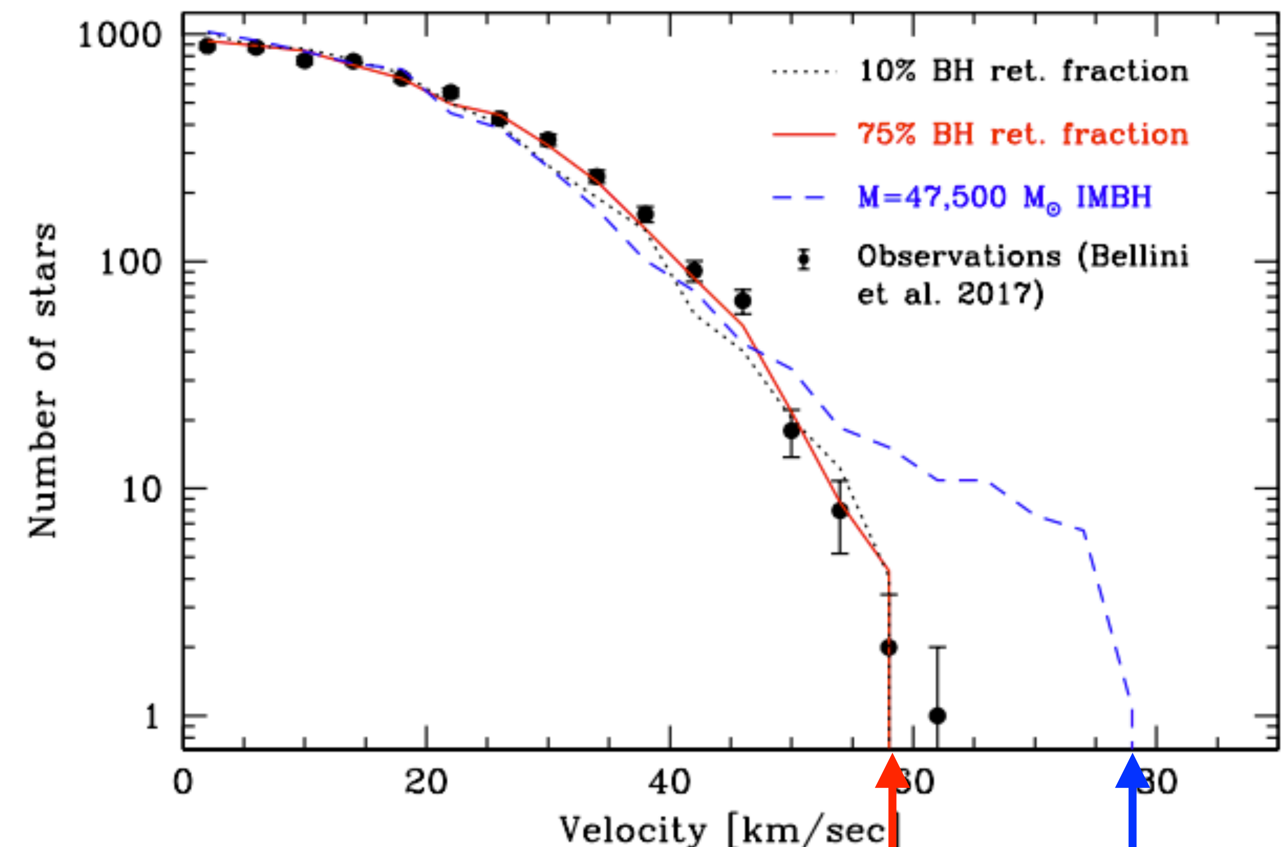
DF-based models (`limepy`)



Zocchi+ 2019

LOS velocity distribution

N-body models

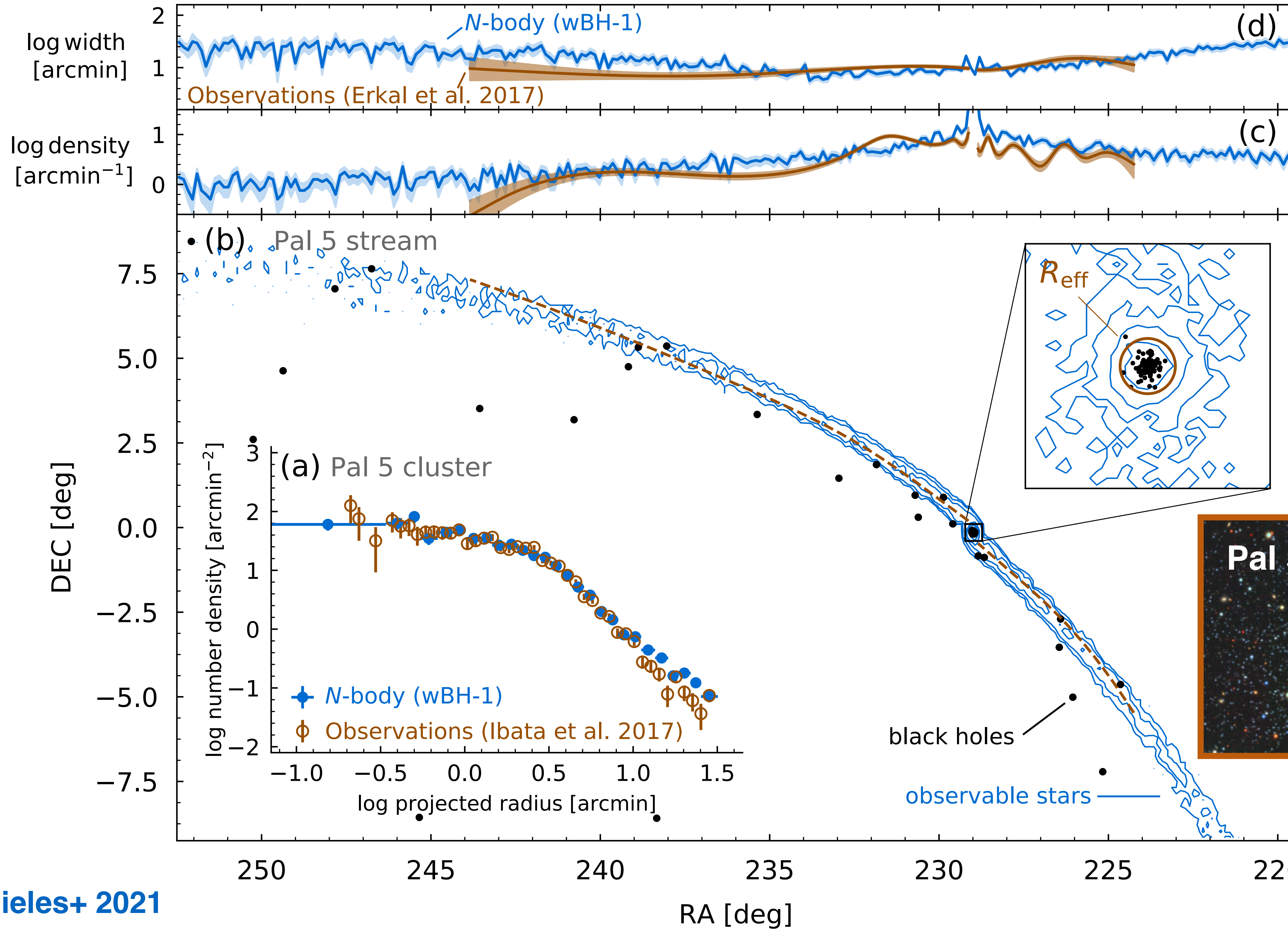


Baumgardt+ 2019

model with stellar-mass BHs

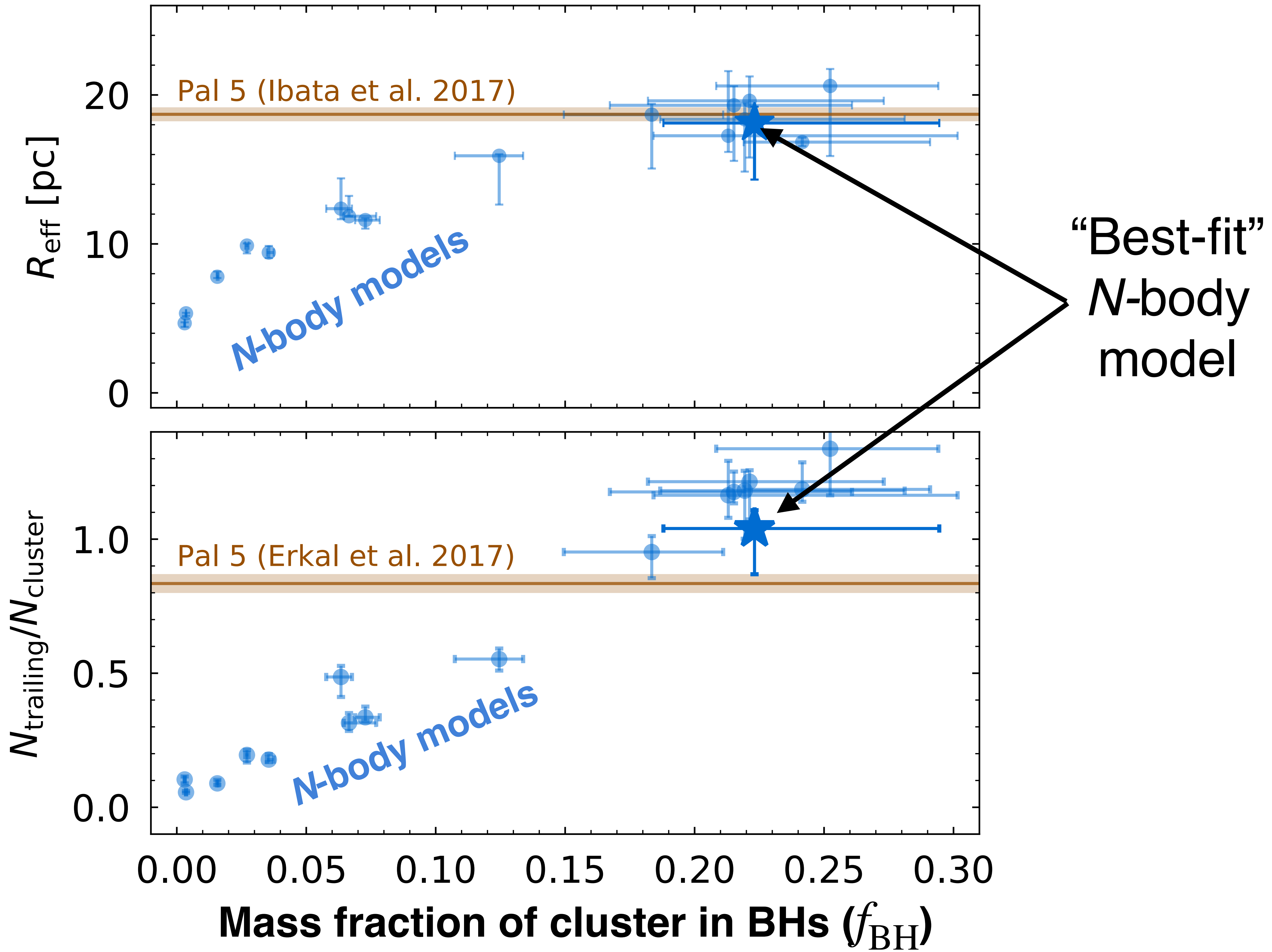
model with IMBH

BHs needed to explain radius and tidal tails of Pal 5

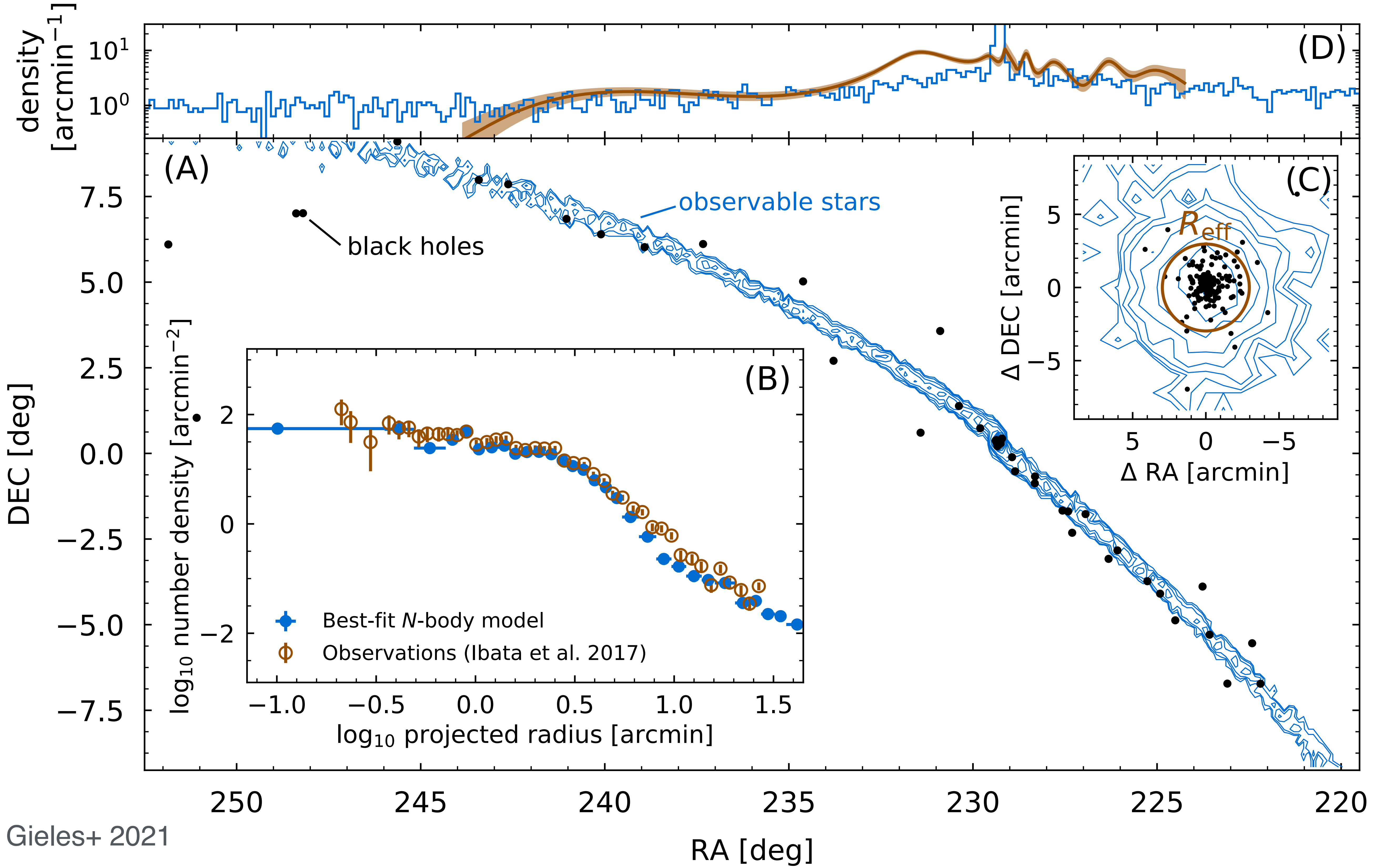


Gieles+ 2021

$R_{\rm eff}$ and tidal tails as a proxy for BHs

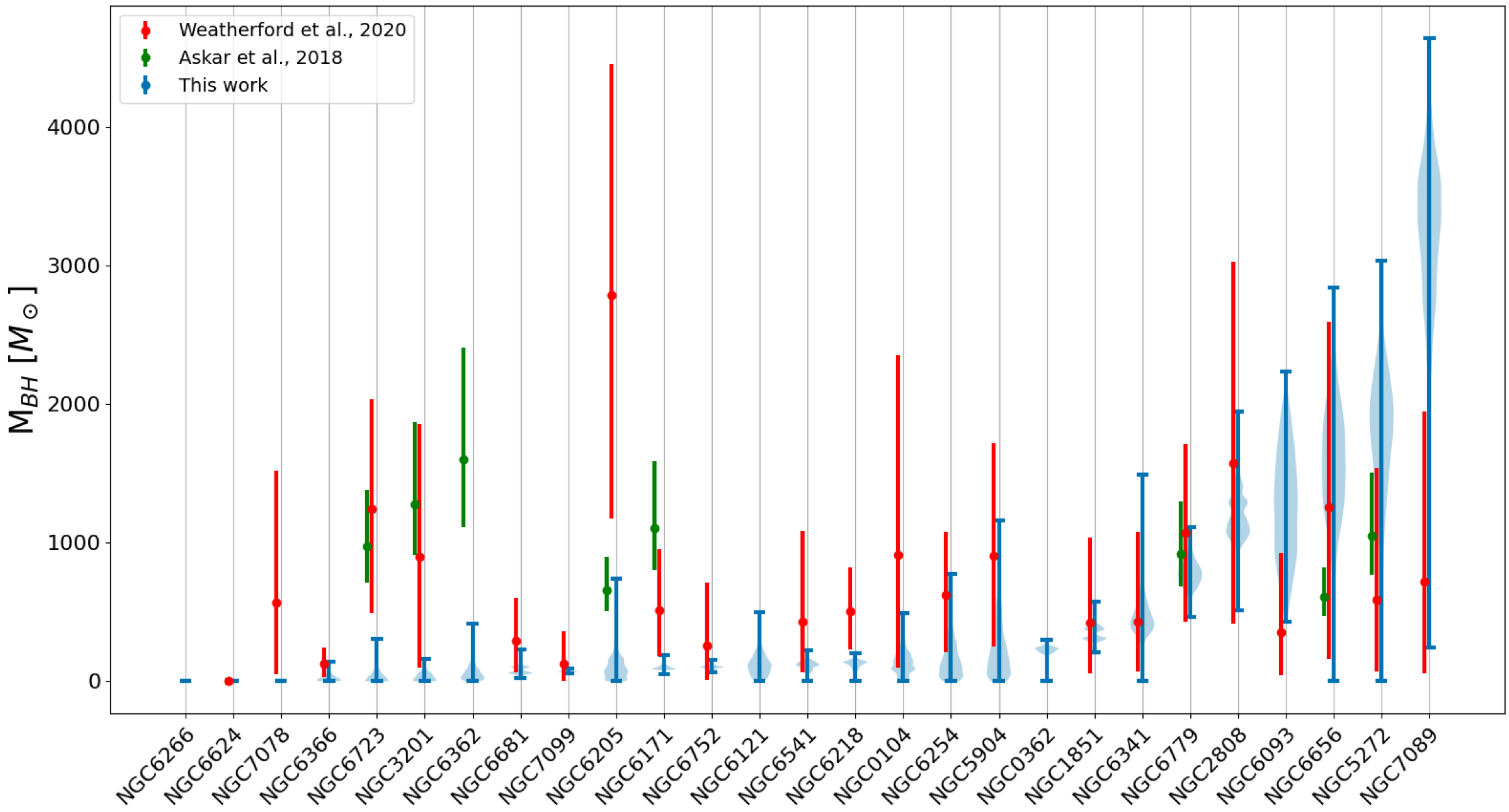


N-body model Pal 5: 20% of mass in BHs

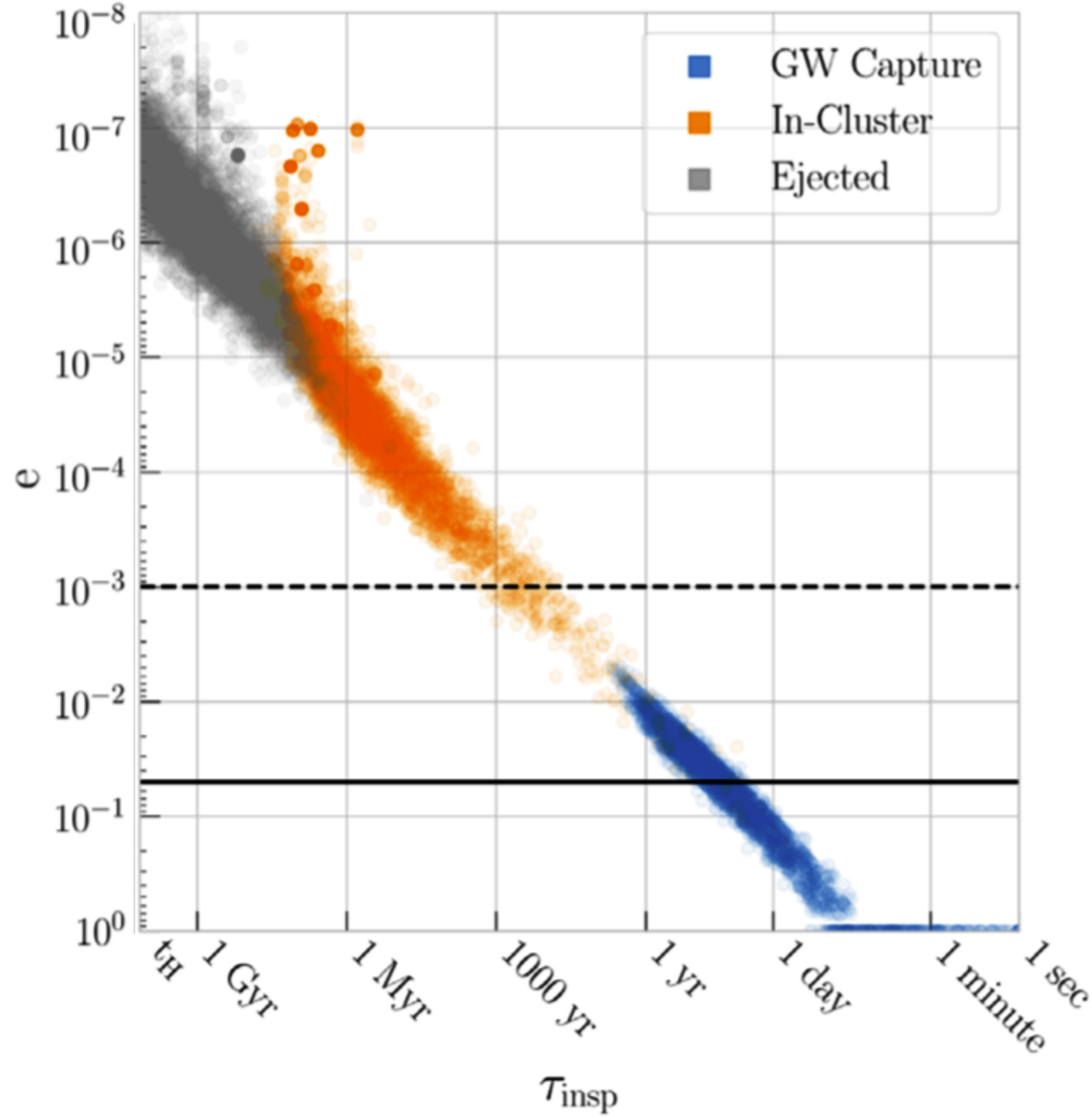


Gieles+ 2021

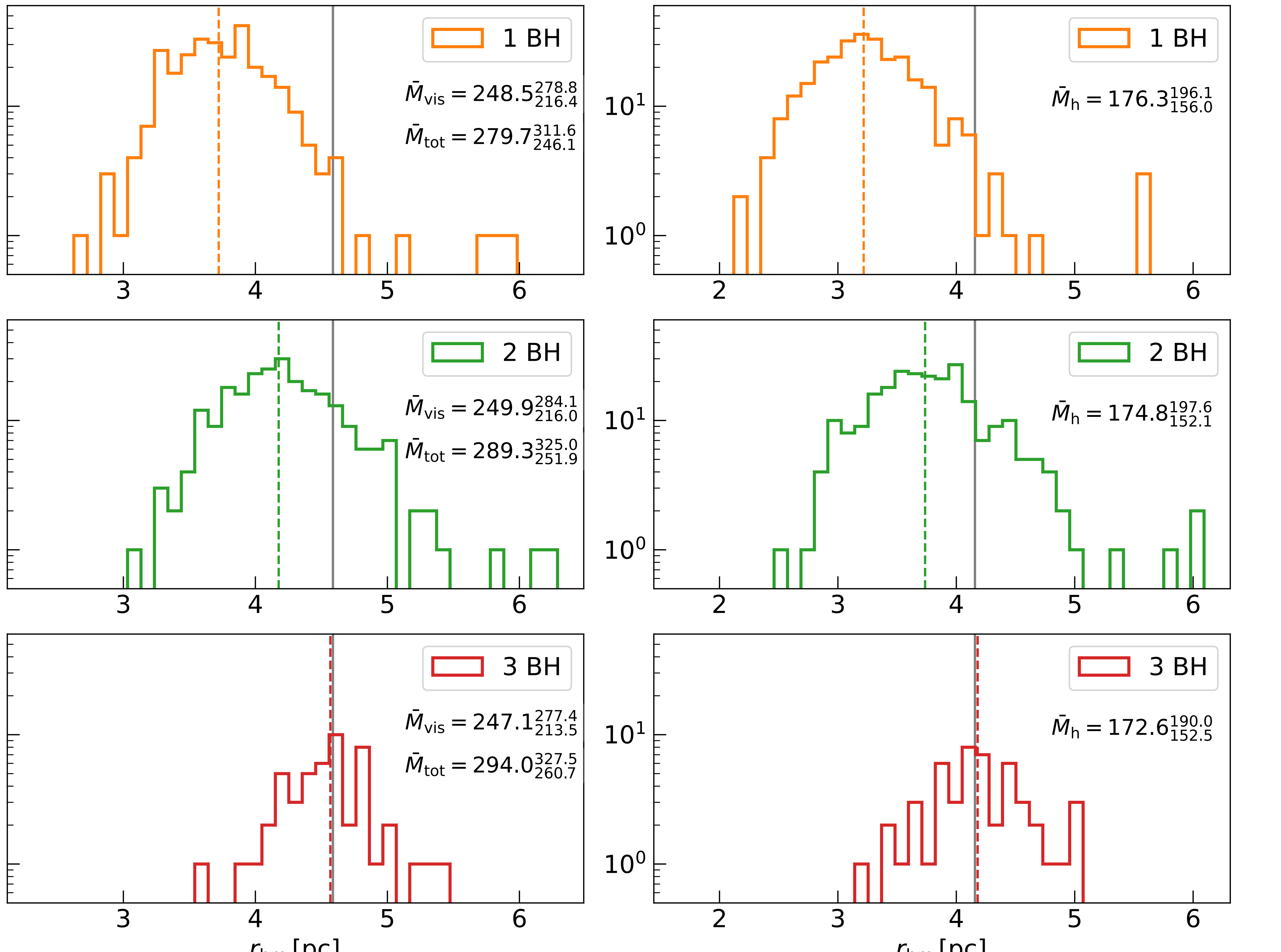
Mass modelling



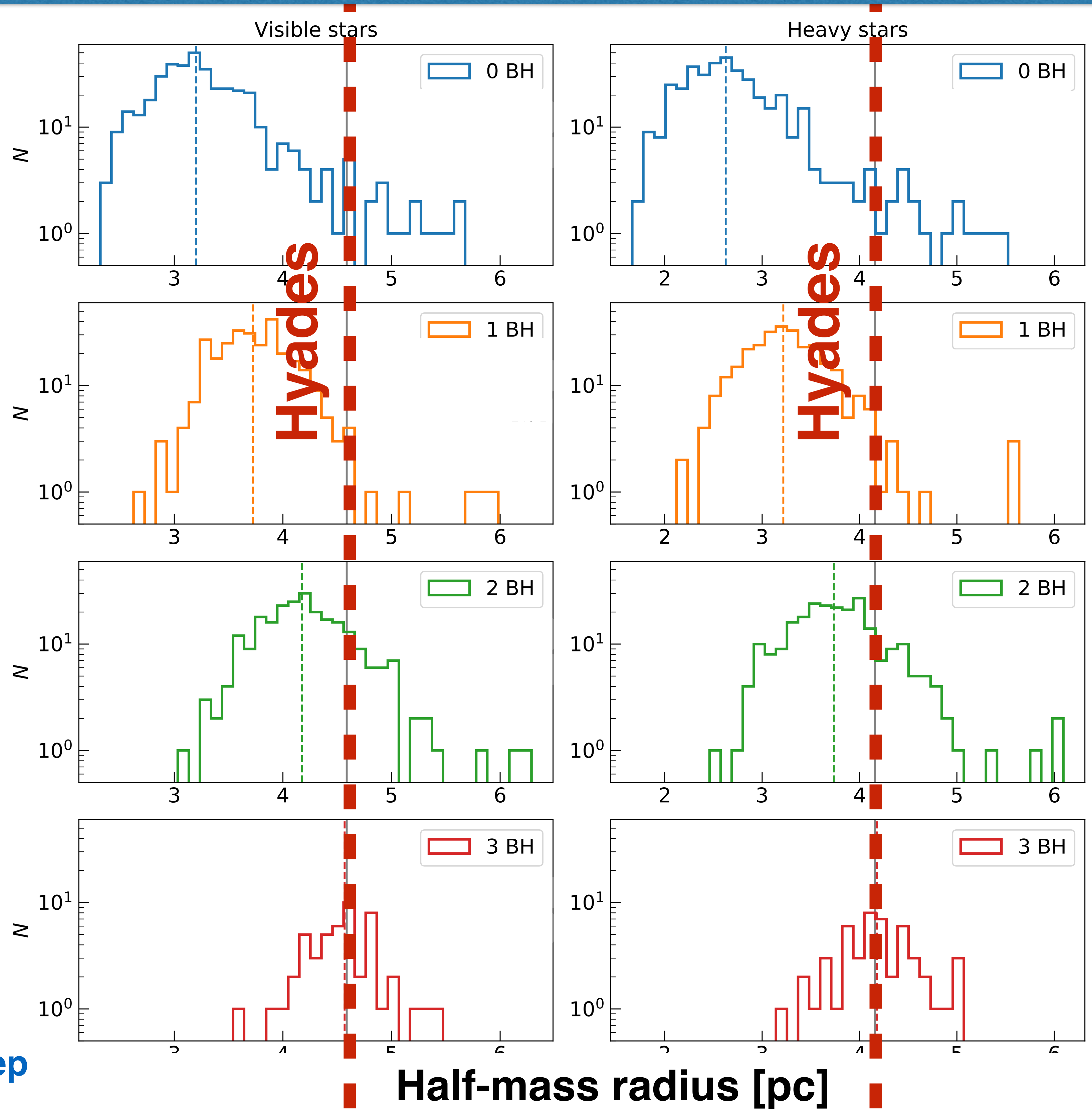
Dickson+ in prep


GW Capture
In-Cluster
Ejected
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
e
t_H
1 Gyr
1 Myr
1000 yr
1 yr
1 day
1 minute
1 sec
τ_{insp}

Zevin+ 2019


1 BH
$\bar{M}_{\rm vis} = 248.5^{278.8}_{216.4}$
$\bar{M}_{\rm tot} = 279.7^{311.6}_{246.1}$
$\bar{M}_{\rm h} = 176.3^{196.1}_{156.0}$
2 BH
$\bar{M}_{\rm vis} = 249.9^{284.1}_{216.0}$
$\bar{M}_{\rm tot} = 289.3^{325.0}_{251.9}$
$\bar{M}_{\rm h} = 174.8^{197.6}_{152.1}$
3 BH
$\bar{M}_{\rm vis} = 247.1^{277.4}_{213.5}$
$\bar{M}_{\rm tot} = 294.0^{327.5}_{260.7}$
$\bar{M}_{\rm h} = 172.6^{190.0}_{152.5}$
10^1
10^0
$r_{\rm hm}$ [pc]

Hyades: radius proxy of (tiny!) BH population



0 BH

1 BH

2 BHs

3 BHs

Torniamenti+ in prep