FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

VIRGO

TEOBResumS:

an advanced waveform model for O4

R. Gamba for the TEOBResumS (coding) team

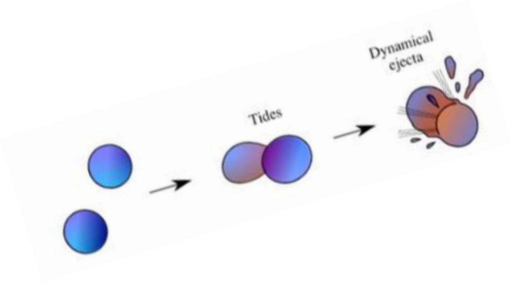
UB, 10.10.2022





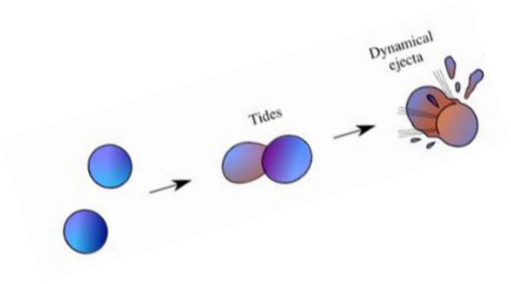


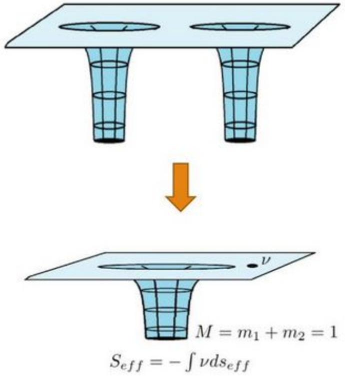

TEOBResumS



TEOBResumS

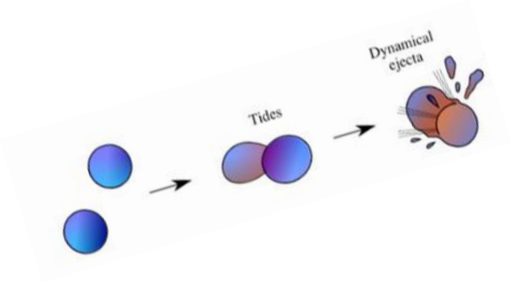
Tidal

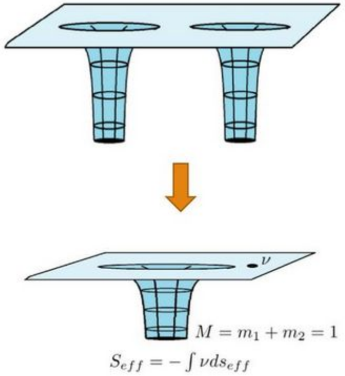




TEOBResumS

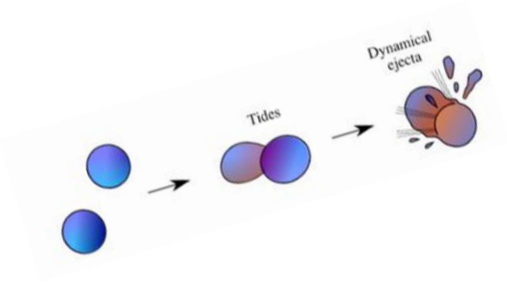
Effective One Body

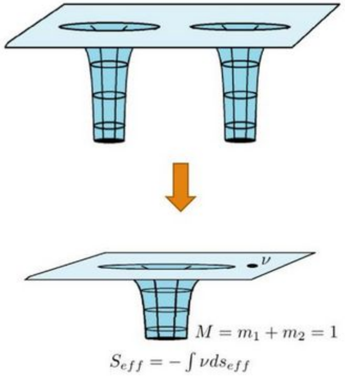


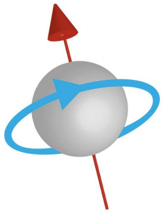


TEOBResumS

(Padé) Resummed

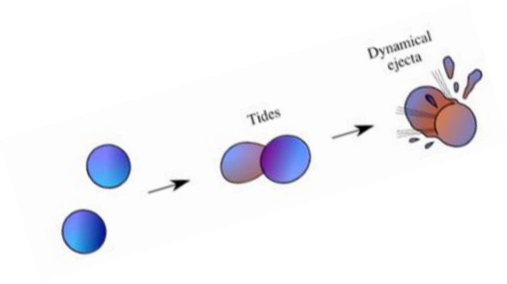


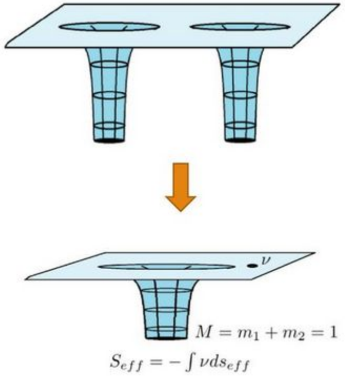


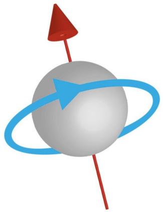


TEOBResumS

Spinning

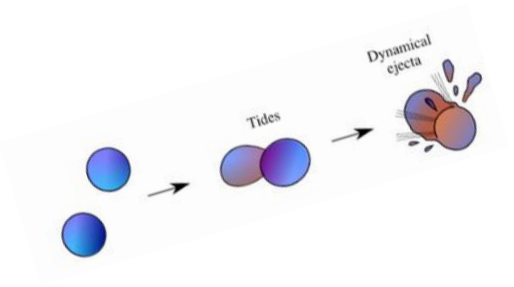


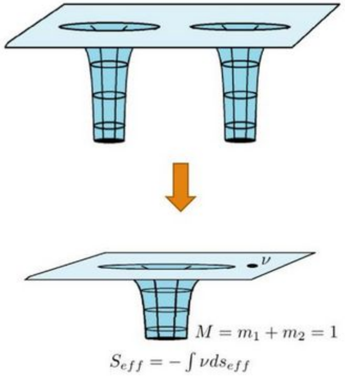


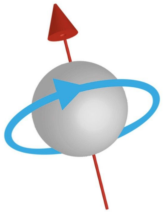


TEOBResumS

GIOTTO/DALI’



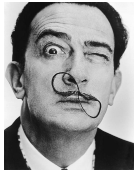


TEOBResumS

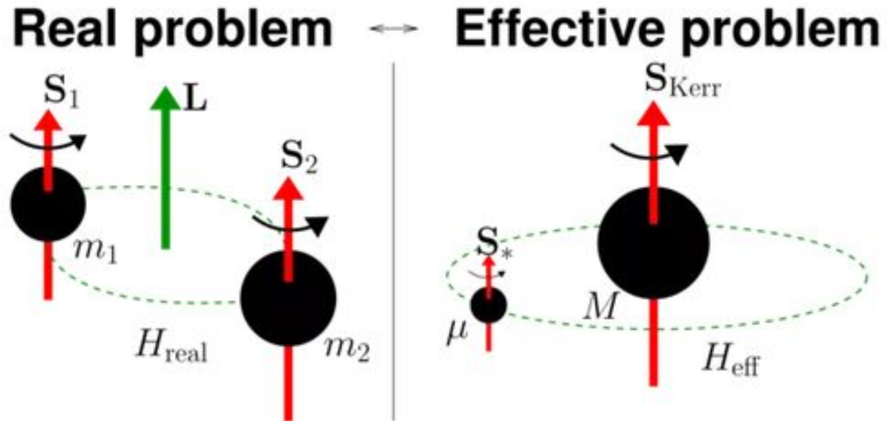
GIOTTO/DALI’

Quasi-circular/Eccentric



Effective One Body

- Newtonian gravity: two body problem → one body moving in effective potential
- General relativity: two body problem → one test particle moving in an **effective metric**



$$g_{\mu\nu}^{\mathrm{ext}}\, dx^\mu\, dx^\nu = -A(R)\, c^2\, dT^2 + B(R)\, dR^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Polar coordinates (r, φ) and associated momenta (p_{r*}, p_φ)

Effective One Body

Hamiltonian (conservative motion)

$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)}, \quad \hat{H}_{\rm eff} = \sqrt{p_{r*}^2 + A(r)(1+p_\varphi^2/r^2)}$$

Resummed

Radiation reaction (dissipation)

$$\mathcal{F}_\varphi$$

Factorized & Resummed

Waveform at infinity

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\rm eff}^{(\epsilon)} \hat{h}_{\ell m}^{\rm tail} f_{\ell m} \hat{h}_{\ell m}^{\rm NQC}$$

Factorized & Resummed

EOB dynamics

$$\frac{d\varphi}{dt} = \Omega = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi,$$

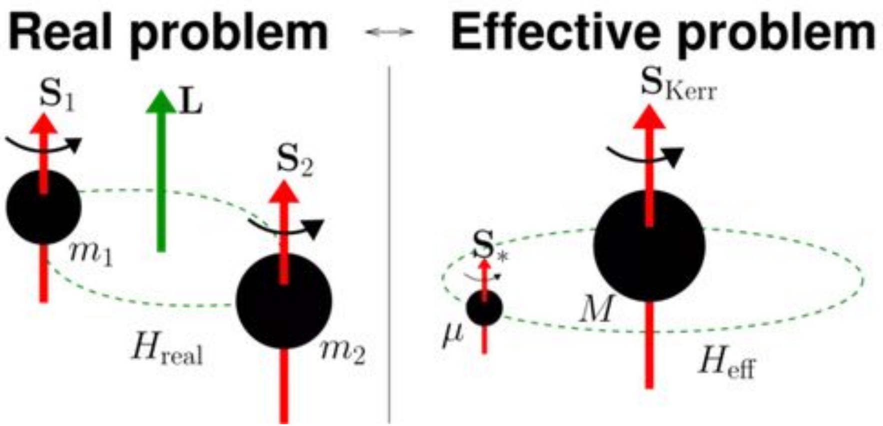
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$

BH Perturbation theory

EOB Waveform

See Alessandro's talk later!

TEOBResumS – GIOTTO



$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)}, \quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1+\frac{p_\varphi^2}{r_c^2}+Q(r_c,p_{r_*})\right)} + p_\varphi(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{S}_{\rm eff}^{(\epsilon)}\hat{h}_{\ell m}^{\rm tail} f_{\ell m}\hat{h}_{\ell m}^{\rm NQC}$$

TEOBResumS – GIOTTO

$$A_{4\mathrm{PN}}(u,\nu) = 1 - 2u + 2\nu u^3 + \nu\left(\frac{94}{3} - \frac{41}{32}\pi^2\right)u^4 + \nu\left(a_5^c(\nu) + a_5^{\ln}(\nu)\ln u\right)u^5$$

4 PN analytical

$$D(u,\nu) \equiv \frac{1}{1 + 6\nu u^2 + 2(26 - 3\nu)\nu u^3}$$

3 PN analytical

flm also use high order PN information *and* test mass info

$$H_{\mathrm{EOB}} = M\sqrt{1 + 2\nu(\hat{H}_{\mathrm{eff}} - 1)}, \quad \hat{H}_{\mathrm{eff}} = \sqrt{p_{r_*}^2 + A(r_c)\left(1 + \frac{p_\varphi^2}{r_c^2} + Q(r_c, p_{r_*})\right)} + p_\varphi(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{S}_{\mathrm{eff}}^{(\epsilon)}\hat{h}_{\ell m}^{\mathrm{tail}} f_{\ell m}\hat{h}_{\ell m}^{\mathrm{NQC}}$$

TEOBResumS – GIOTTO

Spin-Spin

$$r_c^2 = r^2 + \hat{a}_0^2\left(1+\frac{2}{r}\right) + \delta\hat{a}^2,$$

$$\delta\hat{a}^2 = \frac{1}{r}\left\{\frac{5}{4}(\tilde{a}_A-\tilde{a}_B)\hat{a}_0 X_{AB} - \left(\frac{5}{4}+\frac{\nu}{2}\right)\hat{a}_0^2 + \left(\frac{1}{2}+2\nu\right)\tilde{a}_A\tilde{a}_B\right\}.$$

Spin-Orbit

$$S = S_A + S_B,$$

$$S_* = \frac{M_B}{M_A}S_A + \frac{M_A}{M_B}S_B$$

$$G_{\hat{S}_*}\hat{S}_* + G_{\hat{S}}\hat{S}$$

$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)},\quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1+\frac{p_\varphi^2}{r_c^2} + Q(r_c,p_{r_*})\right)} + p_\varphi\left(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*\right)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{S}_{\rm eff}^{(\epsilon)}\hat{h}_{\ell m}^{\rm tail} f_{\ell m}\hat{h}_{\ell m}^{\rm NQC}$$

Spins (aligned). Twist for precession!

TEOBResumS – GIOTTO

$$A = A_0 + A_T^{(+)} \quad A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2} \hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1 - r_{\rm LR}u} + \frac{X_A \tilde{A}_1^{(2^+)1\rm SF}}{(1 - r_{\rm LR}u)^{7/2}} + \frac{X_A^2 \tilde{A}_2^{(2^+)2\rm SF}}{(1 - r_{\rm LR}u)^p}$$

Metric

$$\hat{a}_Q^2 = C_{QA}\tilde{a}_A^2 + 2\tilde{a}_A\tilde{a}_B + C_{QB}\tilde{a}_B^2$$

Spin-spin

$$H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)}, \quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1 + \frac{p_\varphi^2}{r_c^2} + Q(r_c, p_{r_*})\right)} + p_\varphi(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{S}_{\rm eff}^{(\epsilon)}\hat{h}_{\ell m}^{\rm tail} f_{\ell m}\hat{h}_{\ell m}^{\rm NQC} + \hat{h}_{\ell m}^N \hat{h}_{\ell m}^T$$

Matter effects

TEOBResumS – GIOTTO

$$\hat{h}^{\rm NQC}_{\ell m} = \left(1 + a_1^{\ell m} n_1^{\ell m} + a_2^{\ell m} n_2^{\ell m}\right) e^{{\rm i}\left(b_1^{\ell m} n_3^{\ell m} + b_2^{\ell m} n_4^{\ell m}\right)},$$

NQCs

$$\bar{h}(\tau) = A_{\bar{h}}(\tau) e^{{\rm i}\phi_{\bar{h}}(\tau)},$$

$$A_{\bar{h}}(\tau) = c_1^A \tanh\left(c_2^A \tau + c_3^A\right) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = - c_1^\phi \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi}\right)$$

Ringdown

$$H_{\rm EOB} = M\sqrt{1 + 2\nu(\hat{H}_{\rm eff} - 1)}, \quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1 + \frac{p_\varphi^2}{r_c^2} + Q(r_c, p_{r_*})\right)} + p_\varphi\left(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*\right)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)} \hat{S}_{\rm eff}^{(\epsilon)} \hat{h}_{\ell m}^{\rm tail} f_{\ell m} \hat{h}_{\ell m}^{\rm NQC} + h_{\ell m}^{\rm RD}$$

(explicit) NR info

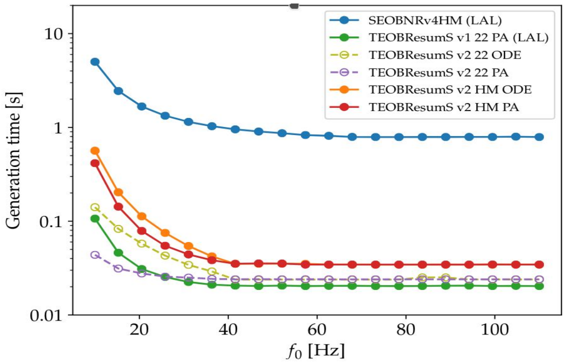
TEOBResumS – GIOTTO

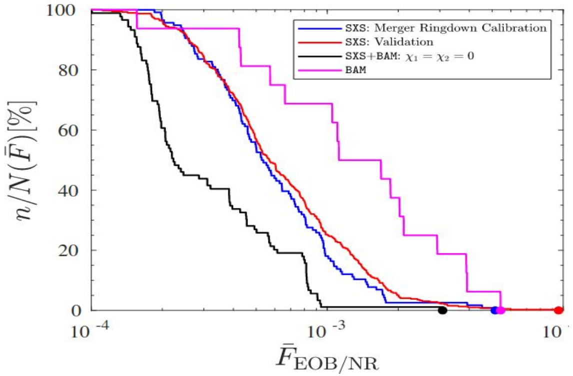
Quasi-circular EOB model for generic-spins [2111.03675] :

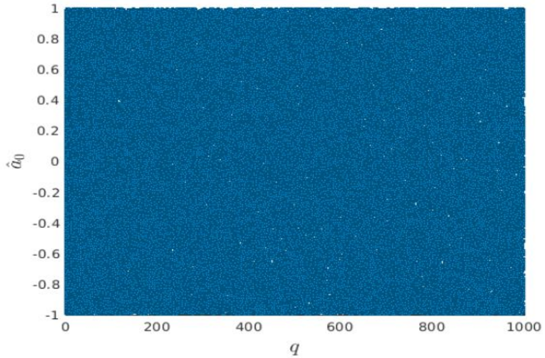
- **BNS** [1812.02744,1812.07923]:
  - Electric ell=2,..,8 interactions, magnetic ell=2
  - GSF resummation ell=2,3
  - f-mode resonance model
  - Spin-quadrupole terms included (@NNLO)
  - Higher modes (up to ell = 8)
  - Frequency Domain model (SPA)
- **BBH** [2001.09082, 2104.07533]:
  - Inspiral-Merger-Ringdown
  - Higher modes, with MR up to ell=5
  - NR-informed NQCs and merger/ringdown
  - High-consistency between radiation reaction and waveform

Takeaway: TEOBResumS is one of the most physically complete models available!

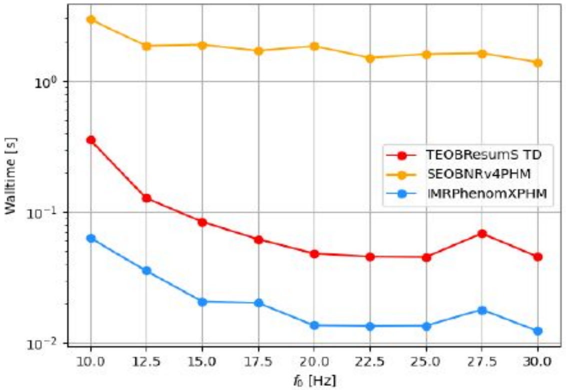
TEOBResumS – GIOTTO

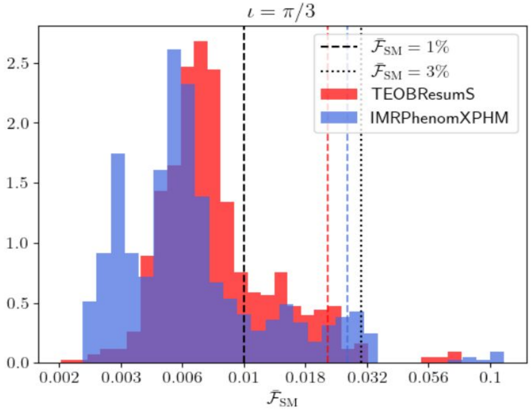


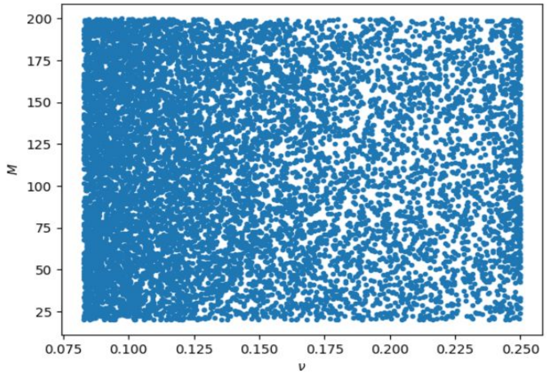




Fast **Faithful** **Robust**

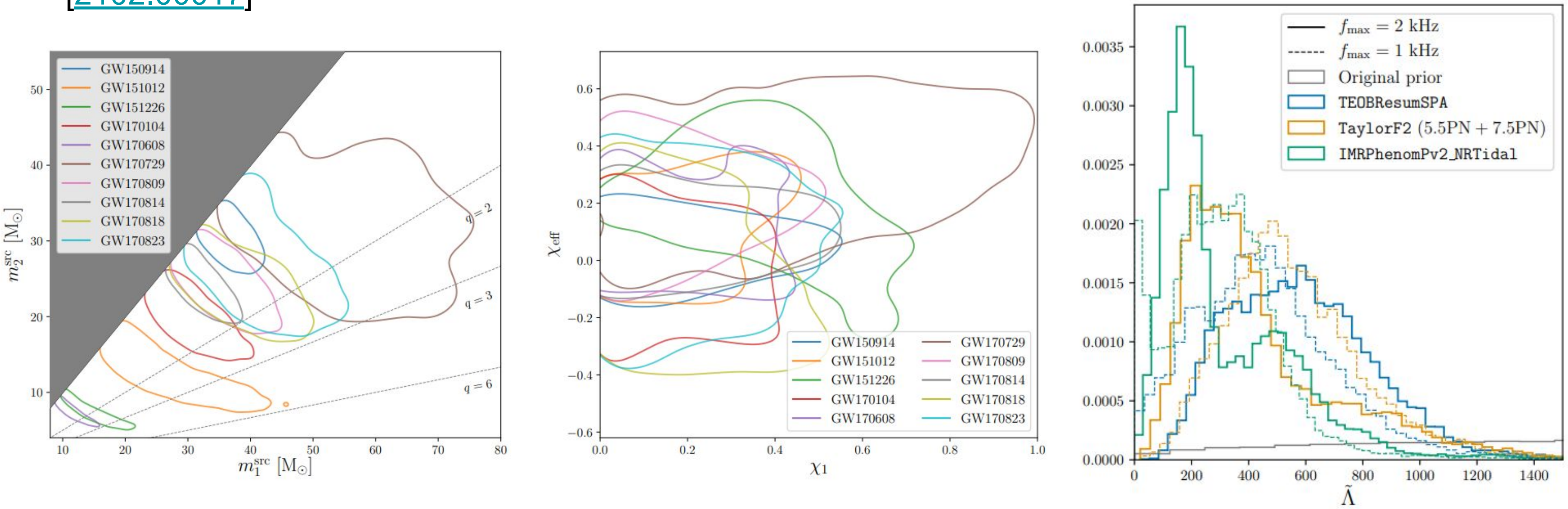






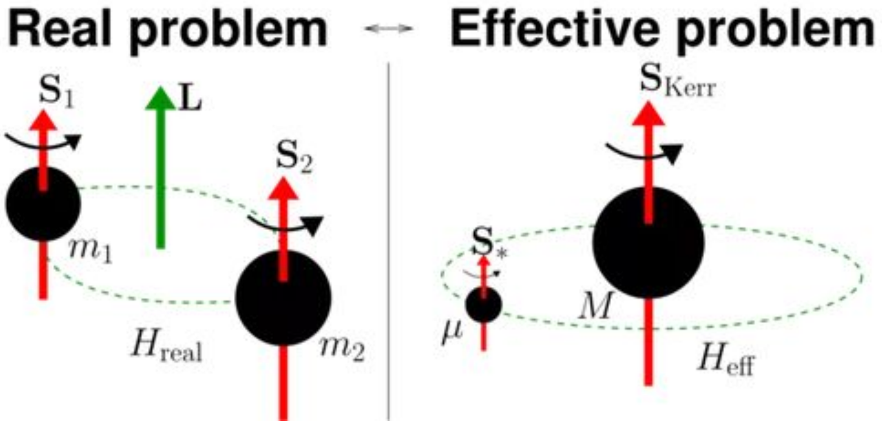
TEOBResumS – GIOTTO

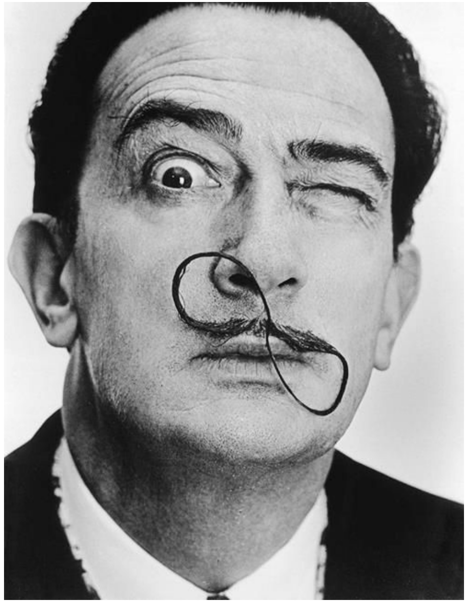
[2102.00017]



Fast enough to be used in real PE!

TEOBResumS – DALI'





$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)},\quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1+\frac{p_\varphi^2}{r_c^2}+Q(r_c,p_{r_*})\right)} + p_\varphi(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*)$$

a6c c3

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi,$$

$$\dot{p}_{r_*} = \sqrt{\frac{A}{B}}\left(-\partial_r \hat{H}_{\rm EOB} + \hat{\mathcal{F}}_r\right)$$

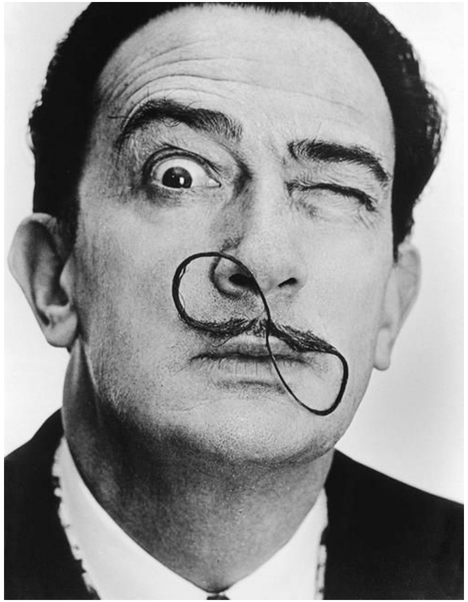
$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\hat{S}_{\rm eff}^{(\epsilon)}\hat{h}_{\ell m}^{\rm tail} f_{\ell m}\hat{h}_{\ell m}^{\rm NQC}$$

Also, new initial conditions!

TEOBResumS – DALI’

EOB model for generic orbit, aligned spins :

- **BNS**:
  - All features of GIOTTO, except…
  - ~~Frequency Domain model (SPA)~~
- **BBH** [2001.11736 , 2108.02043] :
  - All features of GIOTTO, but…
  - New treatment of dissipative terms
  - Different initial conditions
  - New determinations of a6c and c3



TEOBResumS – DALI' : Waveforms

$$\mathcal{S}(t) = \frac{1}{1 + e^{\alpha(t-t_0)}}$$

The same sigmoid activates the NQCs

"Standard" QC newtonian prefactor

Sigmoid

$$h_{22}^{(N,0)} = -8\sqrt{\frac{\pi}{5}}\nu(r_\omega\Omega)^2\left(1 + \mathcal{S}(t)\hat{h}_{22}^{\rm nc}\right)e^{-2i\varphi}$$

Similar expression for other multipoles with (l,m) > (2,2)

Non-circular term

$$\hat{h}_{22}^{\rm nc} = -\frac{1}{2}\left(\frac{\dot{r}^2}{(r\Omega)^2} + \frac{\ddot{r}}{r\Omega^2}\right) + {\rm i}\left(\frac{2\dot{r}}{r\Omega} + \frac{\dot{\Omega}}{2\Omega^2}\right)$$

TEOBResumS – DALI' : Fluxes

$$\hat{\mathcal{F}}_r = \frac{32}{3}\nu p_{r_*} u^4 P_2^0 \left[\hat{f}_r^{2\mathrm{PN}}(u)\right]$$

Radial radiation reaction (not zero!)

$$\mathcal{F}_\varphi^{\mathrm{EOB}_{\mathrm{Newt_{nc}}}} = -\frac{32}{5}\nu^2 r_\omega^4 \Omega^5 \hat{f}_\varphi^{\mathrm{Newt_{nc}}} \hat{f}(\Omega)$$

"Standard" QC newtonian prefactor

Non-circular term

$$\hat{f}_\varphi^{\mathrm{Newt_{nc}}} = 1 + \frac{3}{4}\frac{\dot{r}^2}{r^2\Omega^4} - \frac{\ddot{\Omega}}{4\Omega^3} + \frac{3\dot{r}\dot{\Omega}}{r\Omega^3} \tag{7}$$
$$+ \frac{4\ddot{r}^2}{r^2\Omega^2} + \frac{\ddot{\Omega}\dot{r}^2}{8r^2\Omega^5} + \frac{3}{4}\frac{\dot{r}^3\dot{\Omega}}{r^3\Omega^5} + \frac{3}{4}\frac{\dot{r}^4}{r^4\Omega^4} + \frac{3}{4}\frac{\dot{\Omega}^2}{\Omega^4}$$
$$- \dddot{r}\left(\frac{\dot{r}}{2r^2\Omega^4} + \frac{\dot{\Omega}}{8r\Omega^5}\right) + \ddot{r}\left(-\frac{2}{r\Omega^2} + \frac{\ddot{\Omega}}{8r\Omega^5} + \frac{3}{8}\frac{\dot{r}\dot{\Omega}}{r^2\Omega^5}\right),$$

TEOBResumS – DALI' : Initial conditions

$$(e_0, \Omega_0, \xi_0) \rightarrow (r_0, p_\varphi^0, p_{r_*}^0)$$

- Assume:

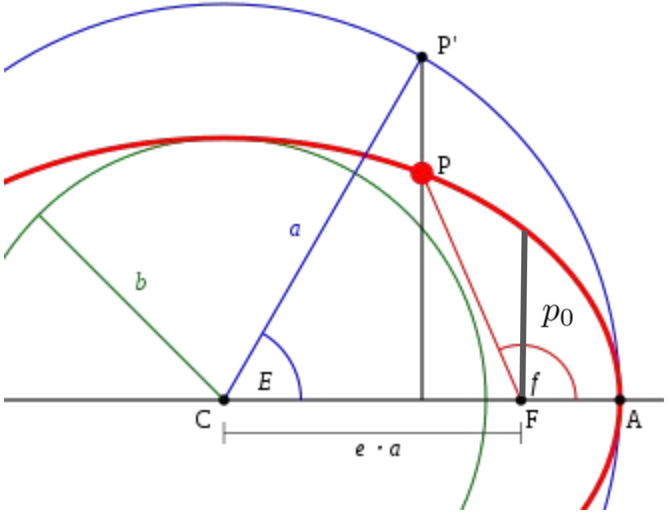
$$r_0 = p_0/(1+e_0) \quad \text{or} \quad r_0 = p_0/(1-e_0)$$

- Solve (numerically)

$$\partial_{p_\varphi} H(r_0(p_0), j_0(p_0), p_{r_*} = 0) = \Omega_0$$

$$\hat{H}^0_{\rm eff}(p_0, j_0, \xi = 0) = \hat{H}^0_{\rm eff}(p_0, j_0, \xi = \pi)$$

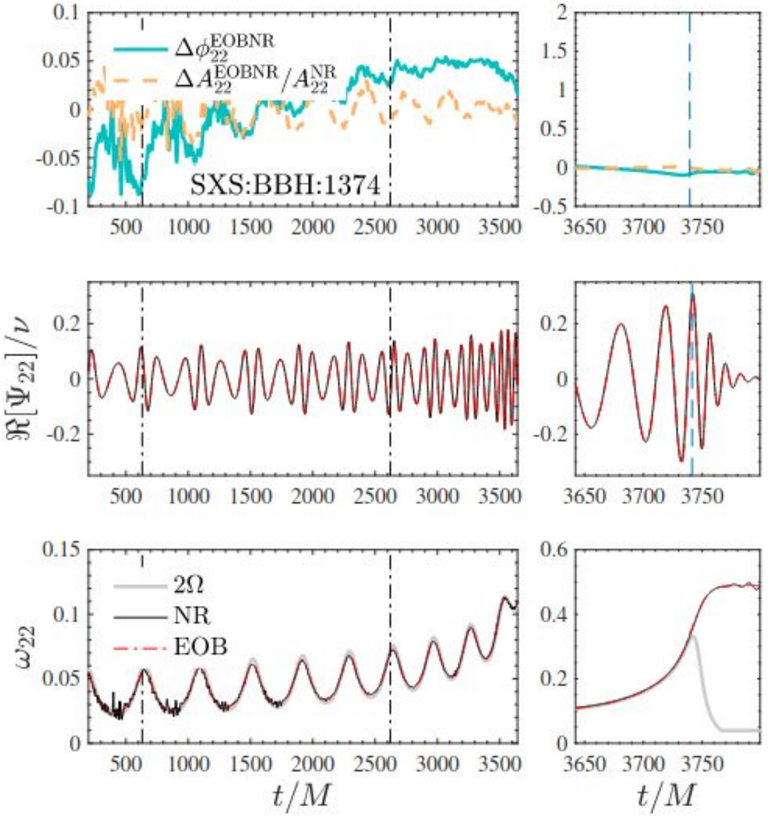
$$r_0 = p_0/(1 + e_0 \cos \xi_0)$$

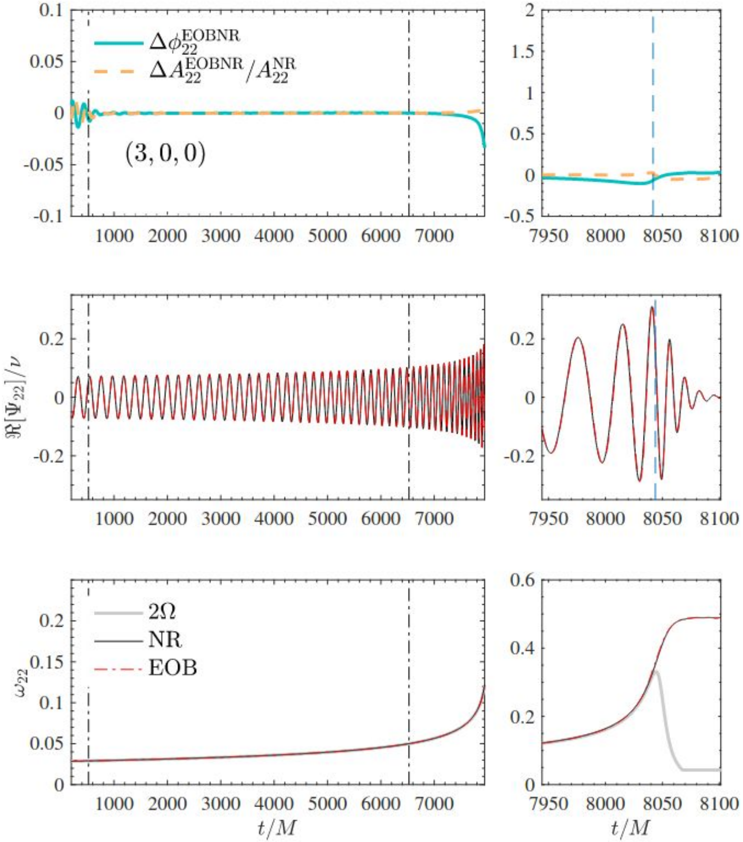


This procedure works, but is **only adiabatic**! (Giotto: 2PA)

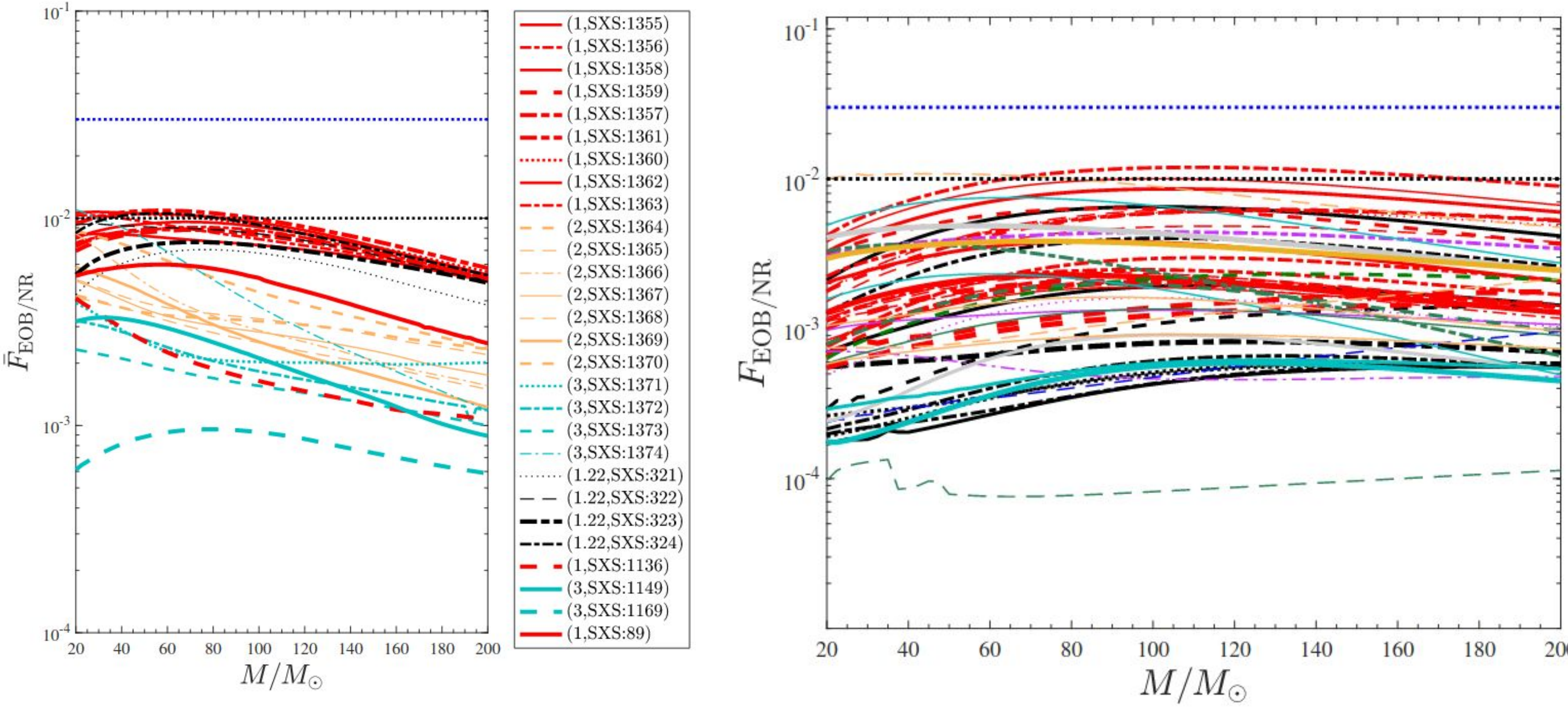
Alternative way of giving initial conditions: via $(p_\varphi, E_{\rm EOB}, r_0)$ → Works for scatterings and encounters!

TEOBResumS – DALI' : NR comparisons

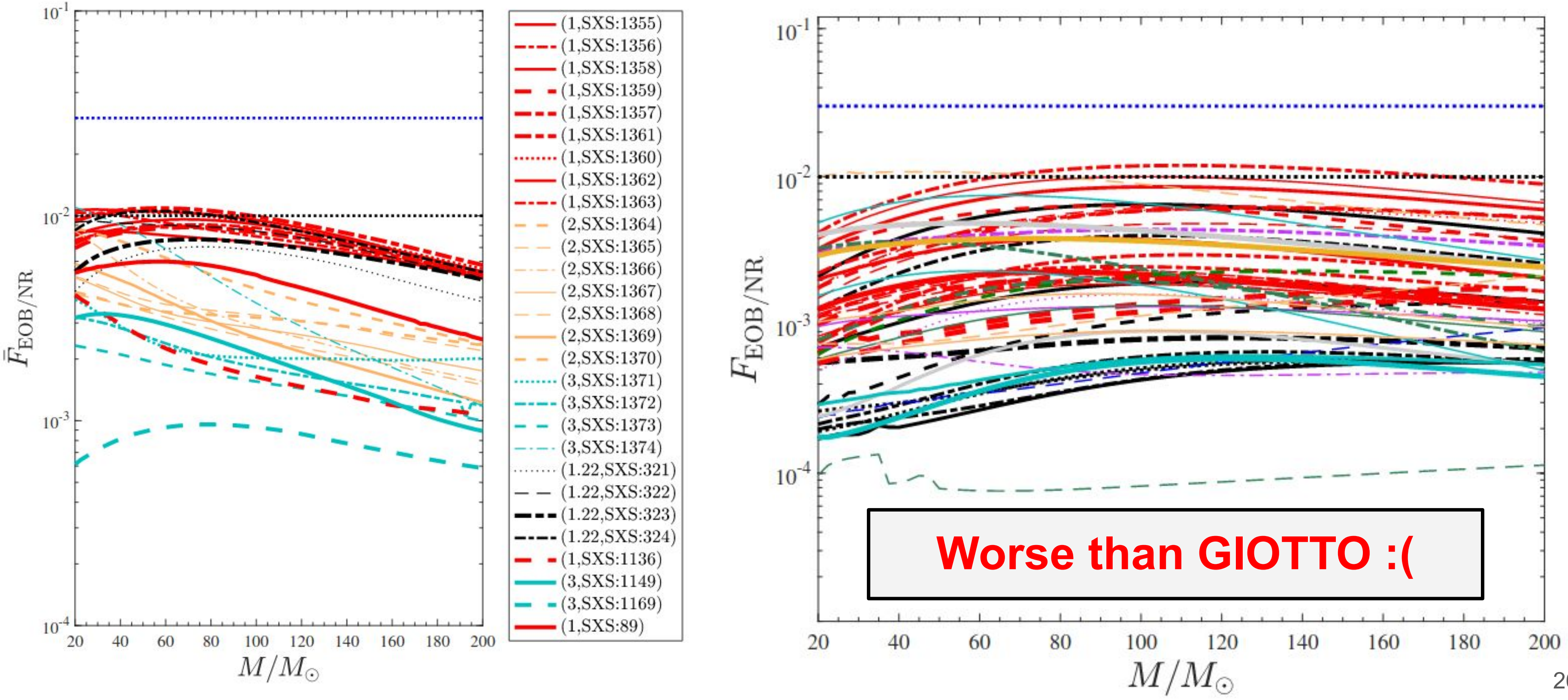




TEOBResumS – DALI' : NR comparisons

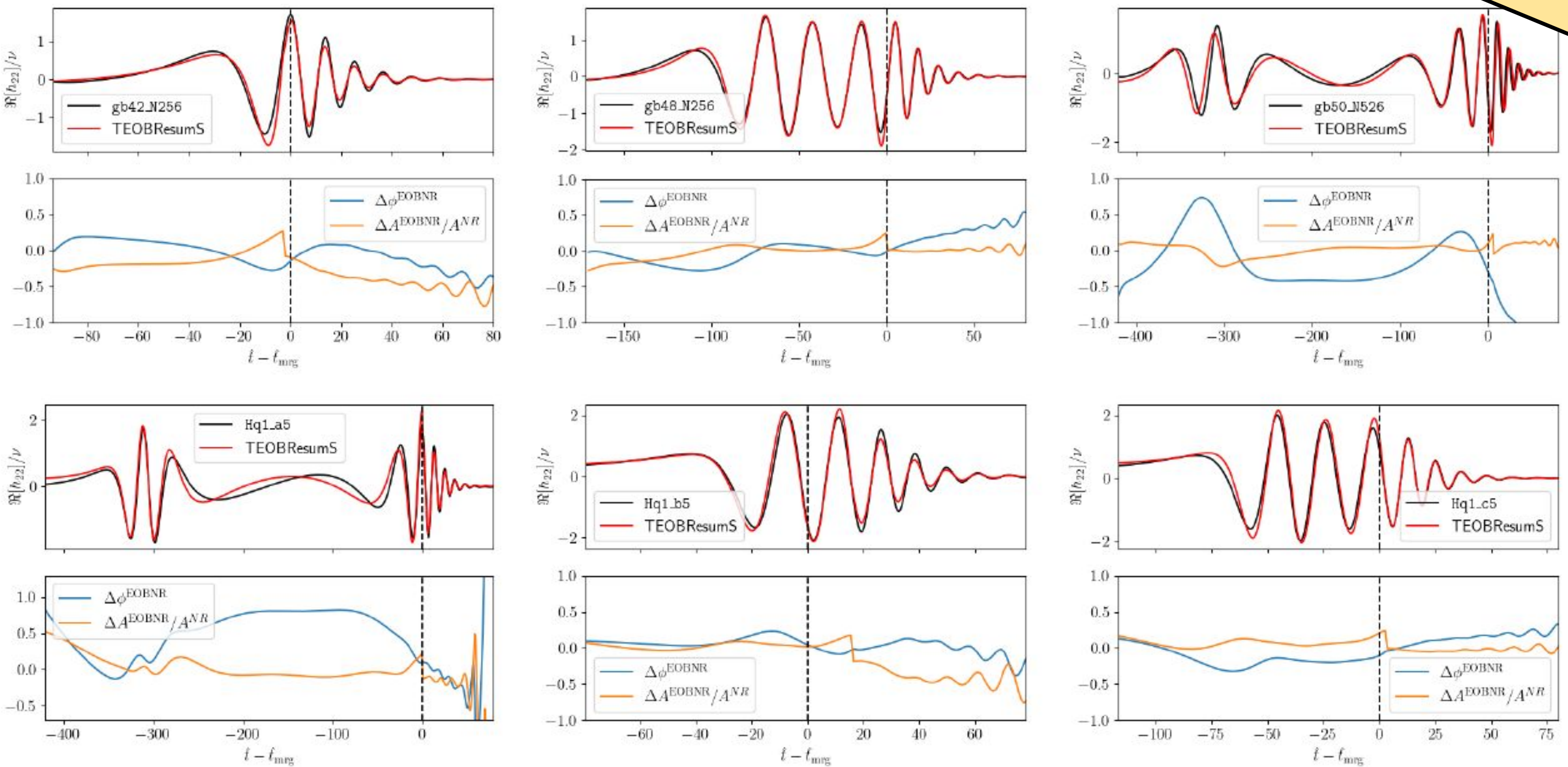


TEOBResumS – DALI’ : NR comparisons

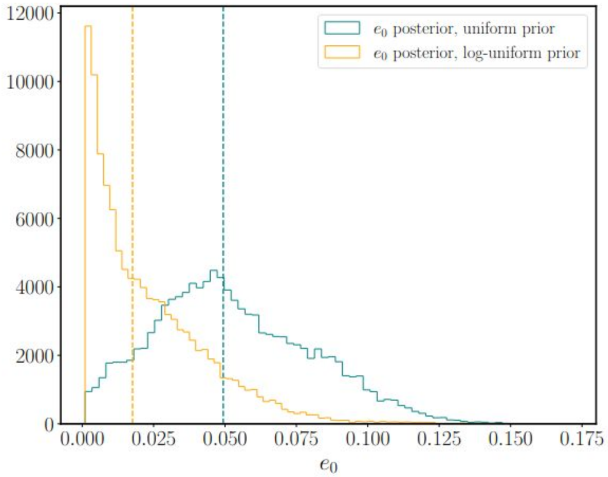


TEOBResumS – DALI’ : NR comparisons

See Tomas’ talk tomorrow!



TEOBResumS – DALI’ : NR comparisons



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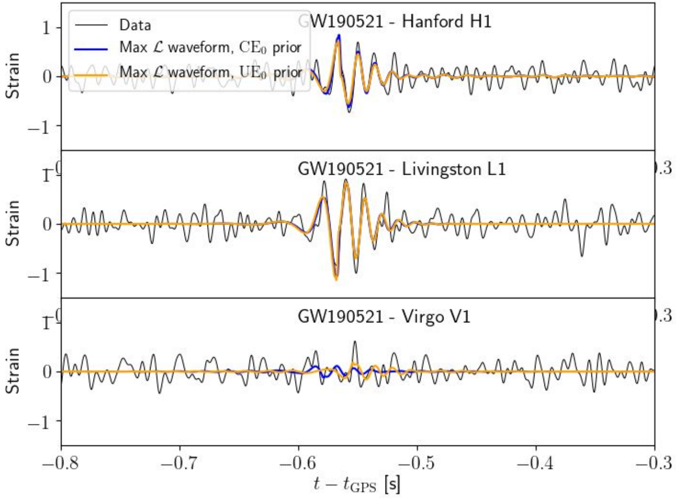

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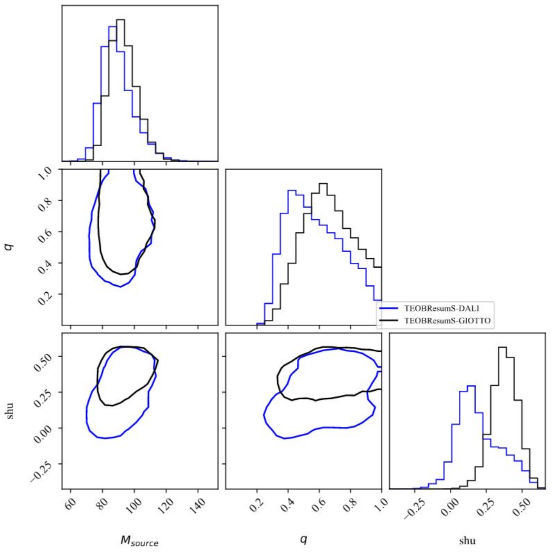
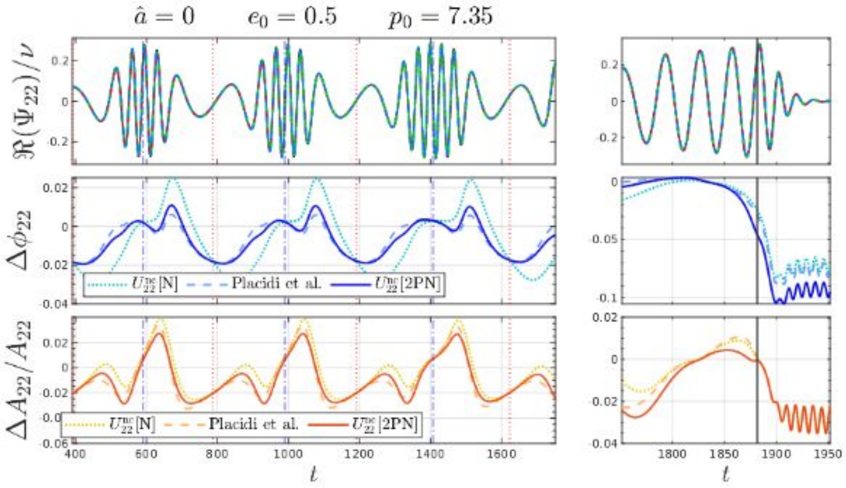


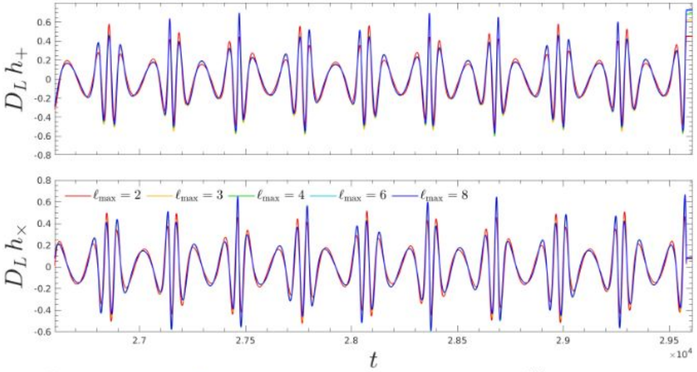
GW190620

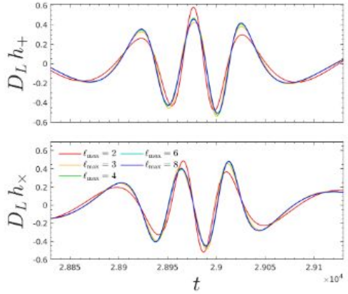
See Juan’s talk tomorrow!

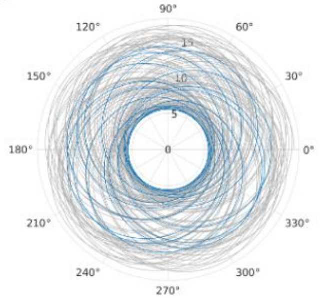
TEOBResumS – DALI’ : future perspective

- 2PN non-circular waveform corrections [2112.05448, 2203.16286]
- New resummations of metric potential
- GSF-informed metric potentials (large q) [2207.14002]



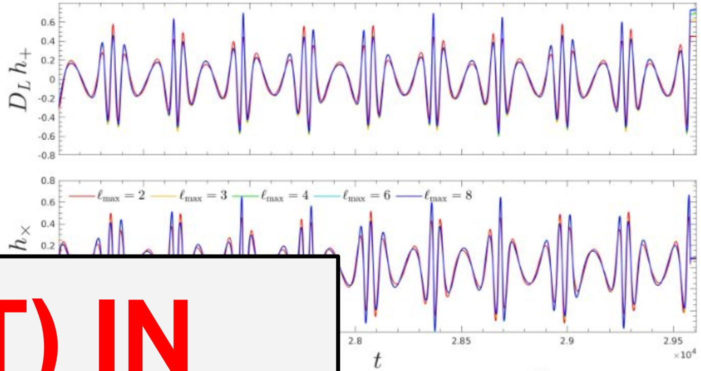




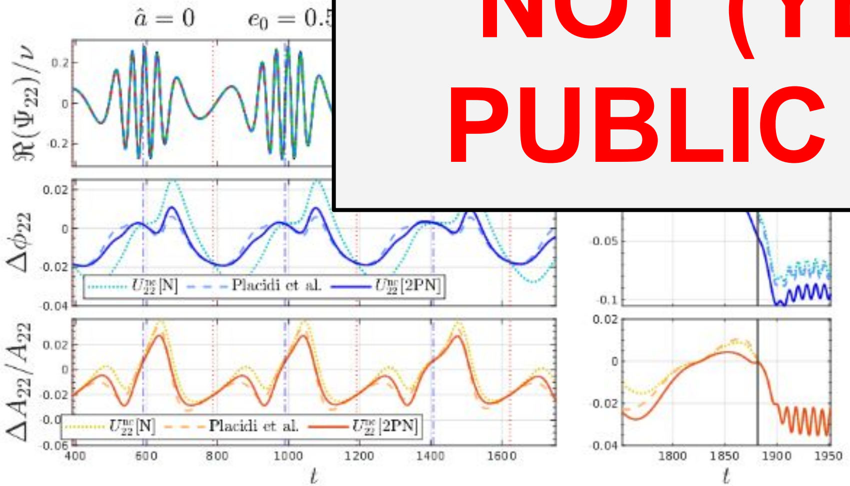


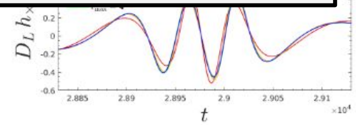
TEOBResumS – DALI' : future perspective

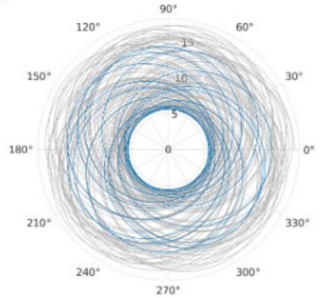
- 2PN non-circular waveform corrections [2112.05448, 2203.16286]
- New resummations of metric potential
- GSF-informed metric potentials (large q) [2207.14002]



NOT (YET) IN PUBLIC CODE

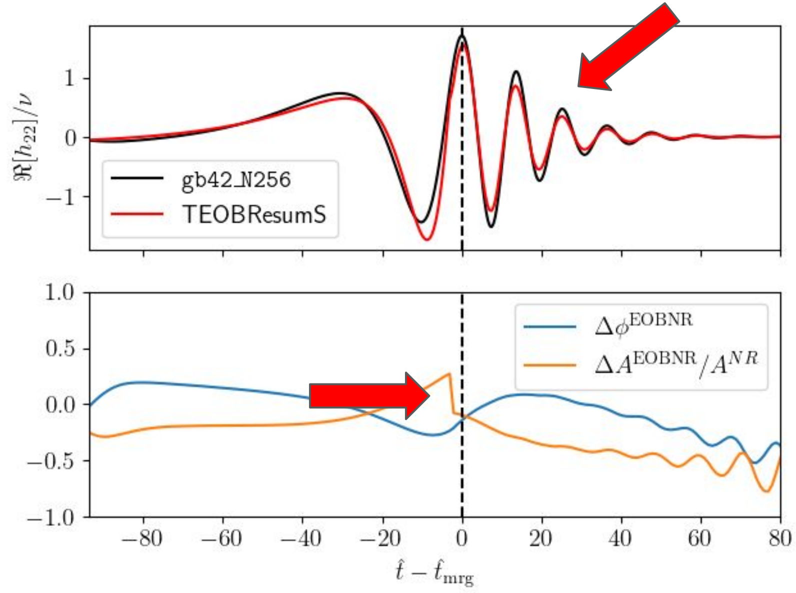


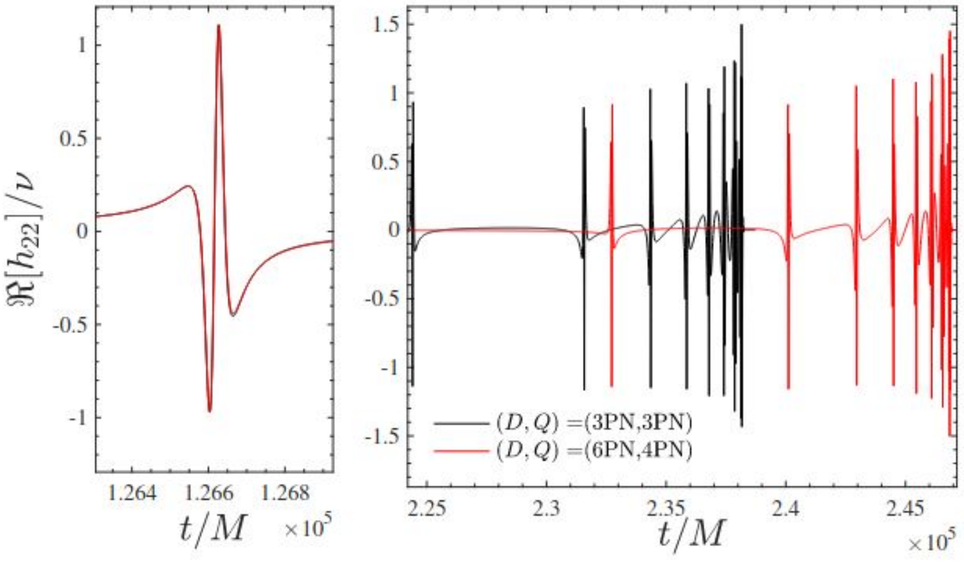




TEOBResumS – DALI' : future perspective

- Improvements **needed:**
  - Ringdown and NQCs
  - Quasi-circular limit
  - Precession
- Additional **testing** for:
  - Spins (aligned and precessing!)
  - high mass ratio
  - Choice of resummation, radiation reaction





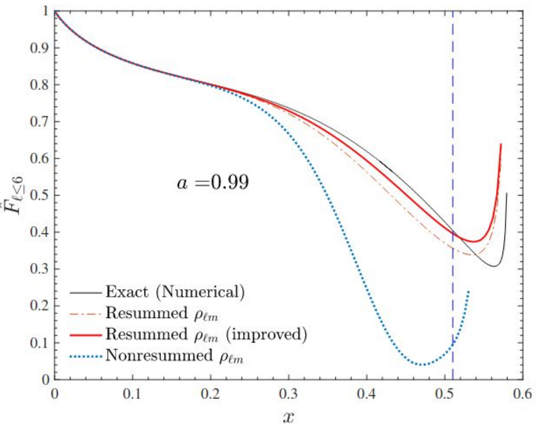
Conclusions

- **TEOBResumS** is an advanced EOB waveform approximant which is **fast, faithful, and robust**
- Both **GIOTTO and DALI**' are publicly available here , and have been **used in real PE** of LIGO-Virgo sources
- Physical completeness (BBH, BNS, BHNS, eccentricity, precession, bound and unbound dynamics) is at reach!

Supplementary Slides

TEOBResumS – GIOTTO

$$R(x) = \frac{\sum_{j=0}^{m} a_j x^j}{1+\sum_{k=1}^{n} b_k x^k} = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m}{1 + b_1 x + b_2 x^2 + \cdots + b_n x^n}$$



$$H_{\rm EOB} = M\sqrt{1+2\nu(\hat{H}_{\rm eff}-1)}, \quad \hat{H}_{\rm eff} = \sqrt{p_{r_*}^2 + A(r_c)\left(1+\frac{p_\varphi^2}{r_c^2} + Q(r_c, p_{r_*})\right)} + p_\varphi\left(G_{\hat{S}}\hat{S} + G_{\hat{S}_*}\hat{S}_*\right)$$

$$\dot{p}_\varphi = \mathcal{F}_\varphi$$

$$h_{\ell m} = h_{\ell m}^{(N,\epsilon)}\,\hat{S}_{\rm eff}^{(\epsilon)}\,\hat{h}_{\ell m}^{\rm tail}\,f_{\ell m}\,\hat{h}_{\ell m}^{\rm NQC}$$

Resummation

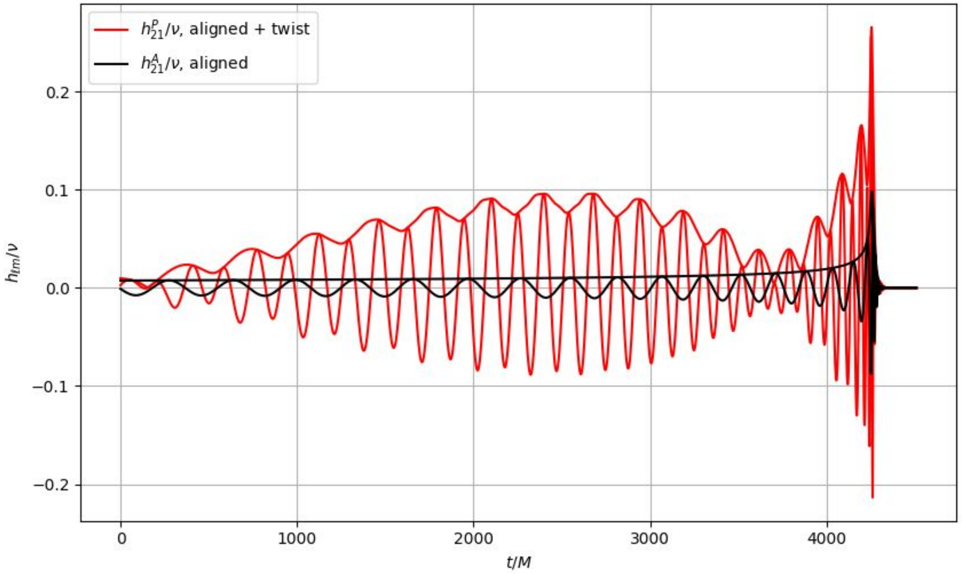
Precession

- If spins are not (mis)aligned to L
  → Orbital plane is not fixed

- In the <u>non inertial</u> frame which "follows" the precession of L (co-precessing frame), the WFs can be approximated with aligned spins waveforms

- Precessing waveforms in an inertial frame can be obtained **<u>twisting</u>** co-precessing (aligned) WFs

h^P_{21} vs h^A_{21}



$$h^P_{\ell m} = \sum_{m'=-m}^{m} h^A_{\ell m'} D^{(\ell)}_{m' m}(-\gamma, -\beta, -\alpha)$$

Euler angles: describe the evolution of L w.r.t inertial frame

NQC corrections

$$\hat{h}^{\rm NQC}_{\ell m} = \left(1 + a_1^{\ell m} n_1^{\ell m} + a_2^{\ell m} n_2^{\ell m}\right) e^{\mathrm{i}\left(b_1^{\ell m} n_3^{\ell m} + b_2^{\ell m} n_4^{\ell m}\right)},$$

Define then

$$t^{\rm EOB}_{\rm NQC} \equiv t^{\rm peak}_{\Omega} - \Delta t_{\rm NQC} \qquad t^{\rm EOB}_{\rm NQC} \leftrightarrow t^{\rm NR}_{\rm NQC} \equiv t^{\rm NR}_{A^{\rm max}_{22}} + 2. \qquad t^{\rm NR}_{{\rm NQC}-\ell m} \equiv t^{\rm NR}_{A^{\rm max}_{\ell m}} + 2. \qquad t^{\rm EOB}_{A^{\rm max}_{22}} \equiv t^{\rm EOB}_{\rm NQC} - 2,$$

With these definitions, one can solve the following system of equations to compute a1, a2, b1, b2

$$\begin{aligned}
A^{\rm EOB}_{\ell m}\left(t^{\rm EOB}_{\rm NQC} + \Delta t^{\rm NR}_{\ell m}\right) &= A^{\rm NR}_{\ell m}\left(t^{\rm NR}_{{\rm NQC}-\ell m}\right), \\
\dot{A}^{\rm EOB}_{\ell m}\left(t^{\rm EOB}_{\rm NQC,} + \Delta t^{\rm NR}_{\ell m}\right) &= \dot{A}^{\rm NR}_{\ell m}\left(t^{\rm NR}_{{\rm NQC}-\ell m}\right), \\
\omega^{\rm EOB}_{\ell m}\left(t^{\rm EOB}_{\rm NQC} + \Delta t^{\rm NR}_{\ell m}\right) &= \omega^{\rm NR}_{\ell m}\left(t^{\rm NR}_{{\rm NQC}-\ell m}\right), \\
\dot{\omega}^{\rm EOB}_{\ell m}\left(t^{\rm EOB}_{\rm NQC} + \Delta t^{\rm NR}_{\ell m}\right) &= \dot{\omega}^{\rm NR}_{\ell m}\left(t^{\rm NR}_{{\rm NQC}-\ell m}\right).
\end{aligned}$$

Note: NQCs affect the waveform amplitude, and therefore the radiation reaction → different NQCs, different inspiral!

After computing them once, one needs to use those values and rerun the entire model. Iterative procedure.

Ringdown template

$$\bar{h}(\tau) = A_{\bar{h}}(\tau) e^{i\phi_{\bar{h}}(\tau)},$$

$$A_{\bar{h}}(\tau) = c_1^A \tanh\left(c_2^A \tau + c_3^A\right) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi}\right)$$

$$c_2^A = \frac{1}{2}\alpha_{21},$$

$$c_4^A = \hat{A}^{\rm peak} - c_1^A \tanh\left(c_3^A\right),$$

$$c_1^A = \hat{A}^{\rm peak}\alpha_1 \frac{\cosh^2\left(c_3^A\right)}{c_2^A},$$

$$c_1^\phi = \Delta\omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi\left(c_3^\phi + 2c_4^\phi\right)},$$

$$c_2^\phi = \alpha_{21},$$

Each multipole depends on 10 parameters fitted from NR:

- Two quantities at merger (A_peak, omega_peak)
- QNMs fundamental complex frequency (alpha_1 + i omega_1) and alpha of the first overtone (alpha_2) → alpha_21 = alpha_2 - alpha_1. These depend on the final mass and spin of the remnant BH
- Three model parameters (c3A, c3phi, c4phi)
- Remnant mass and spin

TEOBResumS vs GW190521