The Effective Field Theory Below the Electroweak Scale

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Low Energy Experiments: Intensity (Precision) Frontier

Proving nature for clues of NP BSM

- **Energy Frontier** — Direct measurement of high-energy physics (Bigger Accelerators, Higher Energies, etc.)
- **Intensity Frontier** — Small deviations in SM predictions due to loop-corrections from high-energy physics (More exp. & th. Precision)

Experiments with CoM energy: $s \ll M_{EW}^2 \sim (100\ \mathrm{GeV})^2$.

Most particle physics experiments up to date...

- + Lepton and Hadron Decays
- + Electric Dipole Moments
- + Anomalous Magnetic Moments
- + Neutral Particle Oscillations

Low Energy Experiments: Intensity (Precision) Frontier

These high–precision low–energy experiments also require high precision theoretical predictions (Feynman diagrams with many loops).

Working within the full SM leads to severe technical difficulties (e.g. logarithms of very different scales).

$$(M_W, M_Z \gg m_b, m_c \gg m_e)$$

Many of these issues can be solved by working with an EFT defined at these low energies!

Low-energy Effective Field Theory

(L.E.F.T.)

LEFT: Crash Course

Standard Model → Integrate out the $W^{\pm}, Z, h, t$ → Low-Energy EFT (LEFT, a.k.a. WET)

$$\sim \left(\frac{1}{\Lambda_{\rm EW}}\right)^{(d-4)}$$

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}^{(\delta m)}_{\rm fermions} + \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \textstyle\sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)} \Bigg\} \; {\rm SU}(3)_c \times {\rm U}(1)_{em}$$

$$e, \mu, \tau, \; \nu_e, \nu_\mu, \nu_\tau, \; u, c, \; d, s, b, \; G^A_{\mu\nu}, F_{\mu\nu} \quad (L, R)$$

LEFT: Crash Course

Standard Model → Integrate out the $W^{\pm}, Z, h, t$ → Low-Energy EFT (LEFT, a.k.a. WET)

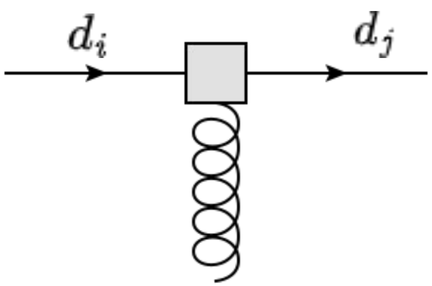
$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}^{(\delta m)}_{\text{fermions}} + \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)} \Bigg\} \ \text{SU}(3)_c \times \text{U}(1)_{em}$$

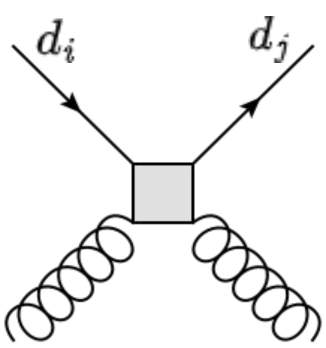
$$e, \mu, \tau, \ \nu_e, \nu_\mu, \nu_\tau, \ u, c, \ d, s, b, \ G^A_{\mu\nu}, F_{\mu\nu} \quad (L, R)$$

$$d = 5$$

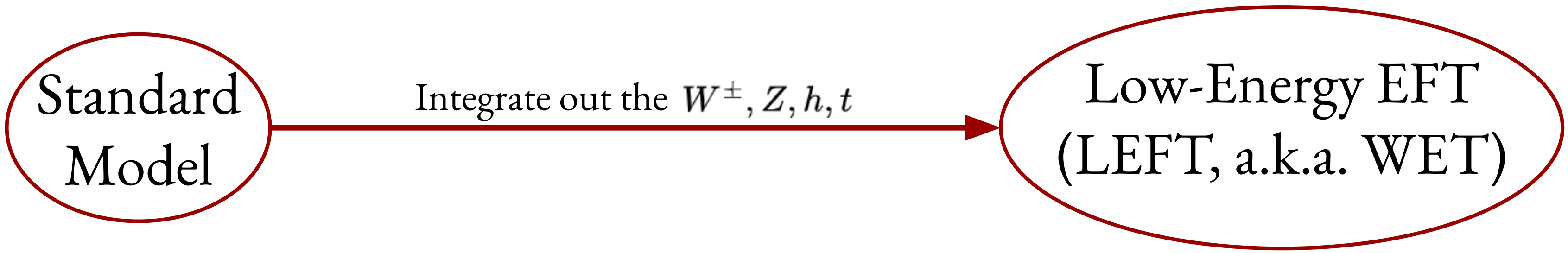
$$\sim \left(\frac{1}{\Lambda_{\text{EW}}}\right)$$

$$\mathcal{O}_{\substack{dG \\ ij}} = (\bar{d}_i \sigma^{\mu\nu} P_R T^A d_j) G^A_{\mu\nu}$$





LEFT: Crash Course



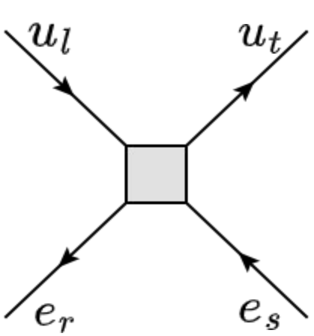
$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{fermions}}^{(\delta m)} + \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \textstyle\sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)} \Big\} \ \mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$$

$$\boxed{e, \mu, \tau,\ \nu_e, \nu_\mu, \nu_\tau,\ u, c,\ d, s, b,\ G^A_{\mu\nu}, F_{\mu\nu}} \ (L, R)$$

$$\boxed{d = 6}$$

$$\sim \left(\frac{1}{\Lambda_{\text{EW}}}\right)^2$$

$$\mathcal{O}^{V,LR}_{\substack{eu \\ rstl}} = (\bar{e}_r \gamma^\mu P_L e_s)(\bar{u}_t \gamma_\mu P_R u_l)$$



Why working with the LEFT?

$$\mathcal{L}_{\mathrm{LEFT}} = \mathcal{L}^{(\delta m)}_{\mathrm{fermions}} + \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{QED}} + \sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)}$$

This offers two important advantages…

1. **Ressumation of large logarithms of ratios of scales.**

$$(\Lambda \gg \lambda) \ : \ \log\left(\frac{\Lambda}{\lambda}\right) \longrightarrow \log\left(\frac{\mu}{\lambda}\right) - \log\left(\frac{\mu}{\Lambda}\right)$$

Matching into $L_i(\mu)$

RG–Improved Perturbation Theory in the EFT

The resulting EFT has small logarithms around $\mu \sim \lambda$!

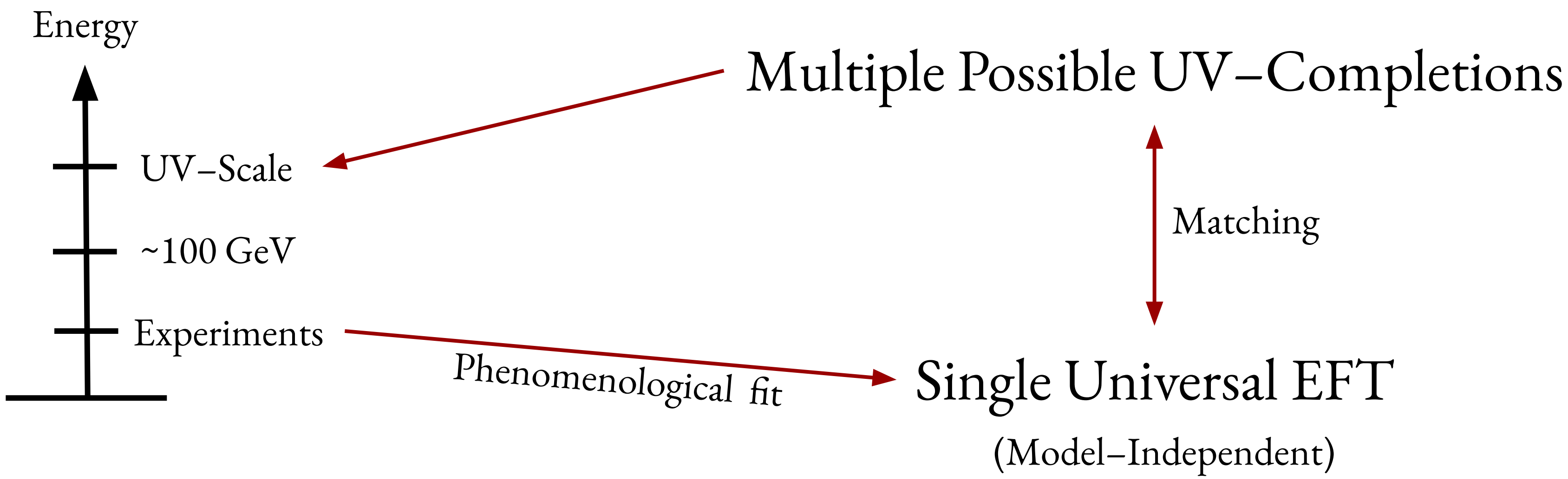
(e.g. $\log\left(\frac{M_W}{m_q}\right)$ in quark decays)

Why working with the LEFT?

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}^{(\delta m)}_{\rm fermions} + \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)}$$

This offers two important advantages…

2. **Very efficient method for characterizing new physics.**



Why working with the LEFT?

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}^{(\delta m)}_{\text{fermions}} + \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \sum_{d>4} L_i^{(d)} \mathcal{O}_i^{(d)}$$

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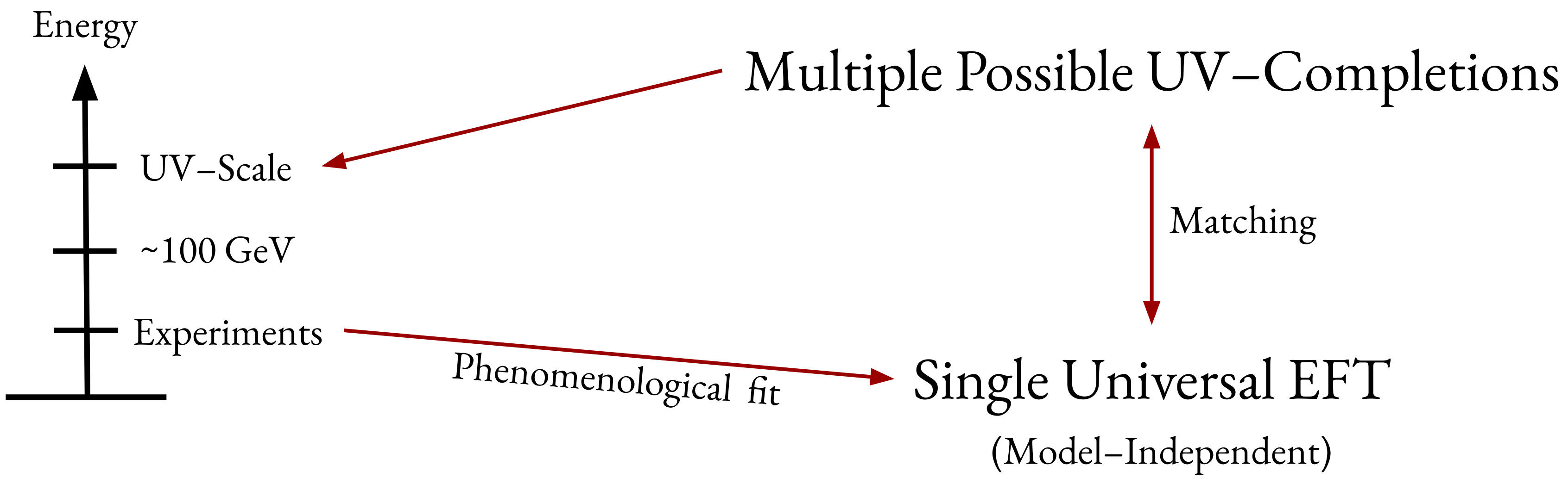


But also… every QFT is an EFT, even the SM!

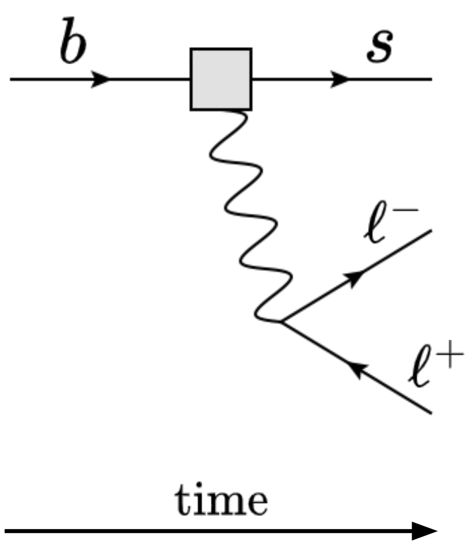
LEFT Example I: B-decays

$$b \to s\ell^+\ell^- \Big\} \quad \text{Involved in B-meson decays:} \quad B \to K\ell^+\ell^-$$

$$(R_K \;\; \& \;\; R_{K^*})$$

Operators contributing at tree level:

- $O(\frac{1}{\Lambda})$: $\mathcal{L}_{b\to s\ell^+\ell^-} = L^*_{\substack{d\gamma \\ bs}} \times \underbrace{(\bar{s}\sigma^{\mu\nu}P_L b)F_{\mu\nu}}_{\mathcal{O}^\dagger_{\substack{d\gamma \\ bs}}} + L_{\substack{d\gamma \\ sb}} \times \underbrace{(\bar{s}\sigma^{\mu\nu}P_R b)F_{\mu\nu}}_{\mathcal{O}_{\substack{d\gamma \\ sb}}}$



LEFT Example I: B-decays

$b \to s\ell^+\ell^-$ } Involved in B-meson decays: $B \to K\ell^+\ell^-$

$(R_K \;\; \& \;\; R_{K^*})$

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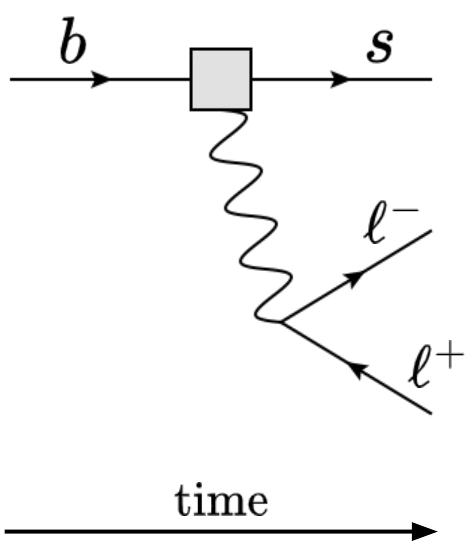


$$-i\frac{2e}{q^2}(\bar{u}_\ell\gamma^\mu v_\ell)\left[\underset{bs}{L^*_{d\gamma}}\mathcal{F}^{T,L}_\mu + \underset{sb}{L_{d\gamma}}\mathcal{F}^{T,R}_\mu\right]$$

$* \, q^\mu = p^\mu_{\ell^+} + p^\mu_{\ell^-} *$

$$(\bar{u}_s q^\nu \sigma_{\nu\mu} P_{L,R} u_b)$$

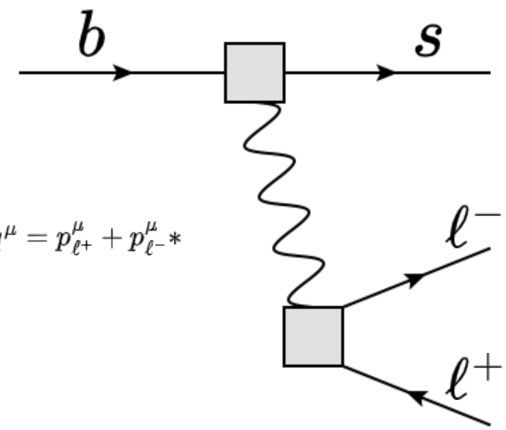
{ Tensor Form Factor $\langle K|\bar{s}q^\nu\sigma_{\nu\mu}P_{L,R}b|B\rangle$ }

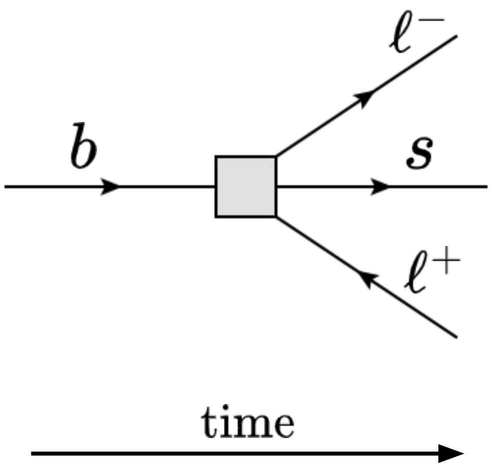
LEFT Example I: B-decays

- $O(\frac{1}{\Lambda^2})$:

$$\mathcal{L}_{b\to s\ell^+\ell^-} = L^*_{\substack{d\gamma\\bs}}\mathcal{O}^\dagger_{\substack{d\gamma\\bs}} + L_{\substack{d\gamma\\sb}}\mathcal{O}_{\substack{d\gamma\\sb}} + L_{\substack{e\gamma\\\ell\ell}}\mathcal{O}_{\substack{e\gamma\\\ell\ell}} + L^{V,LL}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,LL}_{\substack{ed\\\ell\ell sb}}$$
$$+ L^{V,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,RR}_{\substack{ed\\\ell\ell sb}} + L^{V,LR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,LR}_{\substack{ed\\\ell\ell sb}} + L^{V,LR}_{\substack{de\\sb\ell\ell}}\mathcal{O}^{V,LR}_{\substack{de\\sb\ell\ell}}$$
$$+ L^{S,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{S,RR}_{\substack{ed\\\ell\ell sb}} + L^{S,RR\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{S,RR\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{S,RL}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{S,RL}_{\substack{ed\\\ell\ell sb}}$$
$$+ L^{S,RL\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{S,RL\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{T,RR\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{T,RR\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{T,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{T,RR}_{\substack{ed\\\ell\ell sb}}$$

$* q^\mu = p^\mu_{\ell^+} + p^\mu_{\ell^-} *$





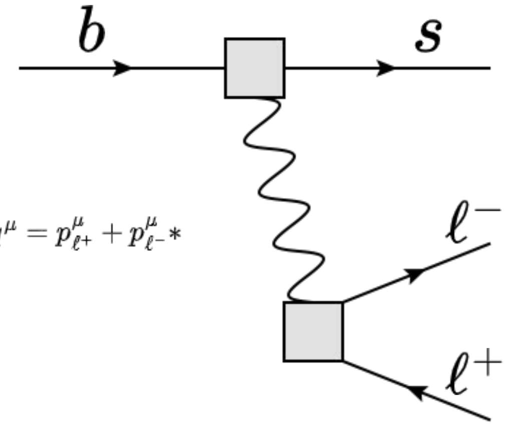
LEFT Example I: B-decays

- $O(\frac{1}{\Lambda^2})$: $\mathcal{L}_{b\to s\ell^+\ell^-} = L^*_{\substack{d\gamma\\bs}}\mathcal{O}^\dagger_{\substack{d\gamma\\bs}} + L_{\substack{d\gamma\\sb}}\mathcal{O}_{\substack{d\gamma\\sb}} + L_{\substack{e\gamma\\\ell\ell}}\mathcal{O}_{\substack{e\gamma\\\ell\ell}} + L^{V,LL}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,LL}_{\substack{ed\\\ell\ell sb}}$

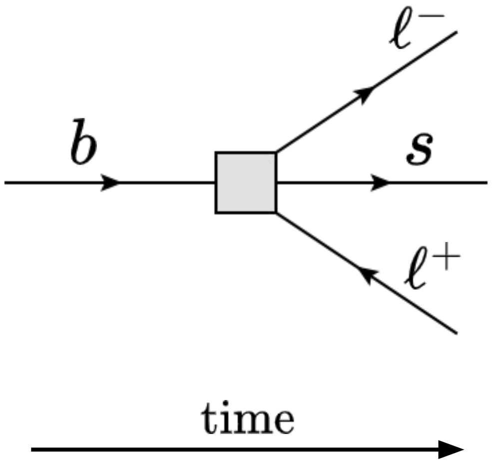
$$+ L^{V,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,RR}_{\substack{ed\\\ell\ell sb}} + L^{V,LR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{V,LR}_{\substack{ed\\\ell\ell sb}} + L^{V,LR}_{\substack{de\\sb\ell\ell}}\mathcal{O}^{V,LR}_{\substack{de\\sb\ell\ell}}$$

$$+ L^{S,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{S,RR}_{\substack{ed\\\ell\ell sb}} + L^{S,RR\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{S,RR\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{S,RL}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{S,RL}_{\substack{ed\\\ell\ell sb}}$$

$$+ L^{S,RL\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{S,RL\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{T,RR\,*}_{\substack{ed\\\ell\ell bs}}\mathcal{O}^{T,RR\,\dagger}_{\substack{ed\\\ell\ell bs}} + L^{T,RR}_{\substack{ed\\\ell\ell sb}}\mathcal{O}^{T,RR}_{\substack{ed\\\ell\ell sb}}$$



$* q^\mu = p^\mu_{\ell^+} + p^\mu_{\ell^-} *$



$$\frac{4i}{q^2}\left[L^*_{\substack{d\gamma\\bs}}L_{\substack{e\gamma\\\ell\ell}}\mathcal{F}^{T,L}_{\mu}\mathcal{F}^{T\,\mu} + L_{\substack{d\gamma\\sb}}L_{\substack{e\gamma\\\ell\ell}}\mathcal{F}^{T,R}_{\mu}\mathcal{F}^{T\,\mu}\right]$$

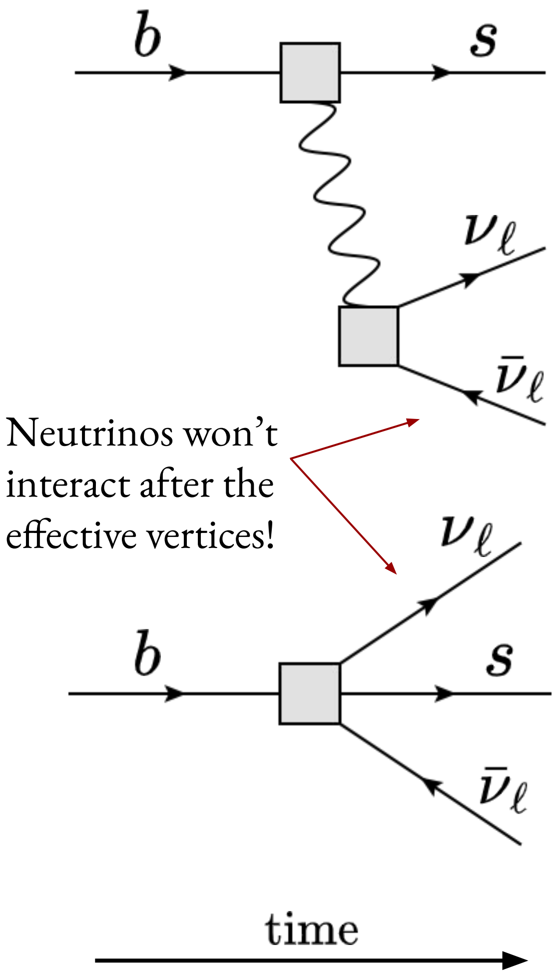
$$+L^{V,LR}_{\substack{de\\sb\ell\ell}}\left\langle\mathcal{O}^{V,LR}_{\substack{de\\sb\ell\ell}}\right\rangle^{(\text{tree})} + \sum L^{(\cdots)}_{\substack{ed\\\ell\ell sb}}\left\langle\mathcal{O}^{(\cdots)}_{\substack{ed\\\ell\ell sb}}\right\rangle^{(\text{tree})}$$

$$+ \sum L^{(\cdots)\,*}_{\substack{ed\\\ell\ell bs}}\left\langle\mathcal{O}^{(\cdots)\,\dagger}_{\substack{ed\\\ell\ell bs}}\right\rangle^{(\text{tree})}$$

$(\bar{u}_s\gamma^\mu P_L u_b)(\bar{u}_\ell\gamma_\mu P_R v_\ell)$

LEFT Example I*: B-decays with neutrinos

Let's consider now: $B \to K\, \nu_\ell\, \bar{\nu}_\ell$ } Very similar lagrangian... $(\ell \leftrightarrow \nu_\ell)$



- The leading order in this case is $O\left(\frac{1}{\Lambda^2}\right)$.

$$\frac{4i}{q^2} L_{\substack{\nu\gamma \\ \ell\ell}} (\bar{u}_{\nu_\ell} q^\eta \sigma_{\eta\mu} P_R v_{\nu_\ell}) \times$$

$$\times \left[L^*_{\substack{d\gamma \\ bs}} \langle K | \bar{s} q^\nu \sigma_{\nu\mu} P_L b | B \rangle + L_{\substack{d\gamma \\ sb}} \langle K | \bar{s} q^\nu \sigma_{\nu\mu} P_R b | B \rangle \right]$$

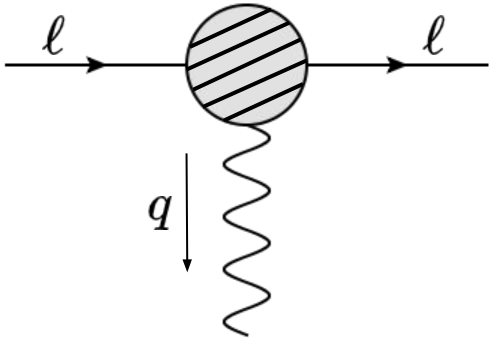
Given that neutrinos aren't affected by neither QED nor QCD, they will factorize out of the amplitude at any loop-order!

LEFT Example II: Lepton Dipole Moments

Anomalous Magnetic Moment (**g–2**) } Famous 4.2σ discrepancy with the SM in the case of the muon

Electric Dipole Moment (EDM) } Break parity (P) and time reversal (T) symmetries } CP Breaking

The dipole moments of leptons are defined through the vertex:



$$= \quad -ie(\bar{u}_\ell \Gamma^\mu u_\ell)$$

$$\Gamma^\mu(q) = \gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_M(q^2) + \frac{\sigma^{\mu\nu}q_\nu}{2m_\ell}\gamma^5 F_D(q^2) + \frac{k^2\gamma^\mu - k^\mu k_\nu \gamma^\nu}{m_\ell^2}\gamma^5 F_A(k^2)$$

$$\boxed{g - 2 = 2F_M(0)}$$

$$\boxed{d_{\text{EDM}} = -\frac{e}{2m_\ell}F_D(0)}$$

LEFT Example II: Lepton Dipole Moments

The LEFT has a tree level contribution to both magnitudes...

- $O\left(\frac{1}{\Lambda}\right)$: $\mathcal{L}_{\text{dipole}} = L_{\substack{e\gamma \\ \ell\ell}}(\bar{\ell}\sigma^{\mu\nu}P_R\ell)F_{\mu\nu} + L_{\substack{\nu\gamma \\ \ell\ell}}(\bar{\nu}_\ell\sigma^{\mu\nu}P_R\nu_\ell)F_{\mu\nu}$

$$g-2 = 8m_\ell L_{\substack{e\gamma \\ \ell\ell}}$$
$$d_{\text{EDM}} = -8eL_{\substack{e\gamma \\ \ell\ell}}$$

$$g-2 = 8m_\ell L_{\substack{\nu\gamma \\ \ell\ell}}$$
$$d_{\text{EDM}} = -8eL_{\substack{\nu\gamma \\ \ell\ell}}$$

This is just an example, the actually interesting contributions start appearing further in the perturbative expansion (one-loop order and beyond)...

LEFT: Renormalization and Mixing

When going beyond tree level in the perturbative expansion, we have to renormalize the theory...

Renormalization in EFTs is achievable and consistent if done order by order in the mass–dimension expansion $(1/\Lambda^n)$.

↓

Usually, the renormalization of a **d > 4** operator will require counterterms coming from other operators.

↓

Counterterm Matrix

LEFT: Renormalization and Mixing

After renormalization, one can use the Renormalization Group to evolve the Wilson coefficients with energy (logarithm resummation)...

$$\frac{d}{d\log\mu} L_i(\mu) = \sum_j \hat{\gamma}_{ji} L_j(\mu)$$

Operator Mixing

Anomalous Dimension Matrix

The ADM is related to the matrix of counterterms (simple poles $1/\epsilon$), and can be computed in perturbation theory, as an expansion in powers of the QCD coupling (α_s) and the QED one (α_{em}).

LEFT: Evanescent Operators

In **D = 4** dimensions, Fierz identities establish relations between different Dirac structures, e.g. $(\bar{s}\gamma_\mu\gamma_\nu\gamma_\rho P_L d)(\bar{s}\gamma^\mu\gamma^\nu\gamma^\rho P_L d) = 4(\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d)$.

When doing dimensional regularization these identities cannot be used anymore, as they are only valid in four dimensions...

↓

Fierz identities out of **D = 4** define the evanescent operators!

$$D = 4 - 2\epsilon \ : \quad (\bar{s}\gamma_\mu\gamma_\nu\gamma_\rho P_L d)(\bar{s}\gamma^\mu\gamma^\nu\gamma^\rho P_L d) - (4 + 4\epsilon)(\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d)$$

These operators appear in the amplitude of diagrams with insertions of EFT operators. They are renormalized to vanish in the finite results, but the choice of evanescent basis has an impact.

It impacts the finite amplitudes, and beyond two-loop order they also impact the ADM!!

LEFT: Two-Loop ADM

We are working on the calculation of the two-loop ADM of the LEFT up to dimension six, but trying to avoid having to compute two-loop diagrams.

Our method is...

- Compilation of known two-loop results in other EFTs and/or operator bases, and posterior transformation into the LEFT.

$$\hat{\gamma}'^{(n)} = \hat{R}\hat{\gamma}^{(n)}\hat{R}^{-1}$$

If the transformation involves evanescent operators, this simple change of basis introduces a finite counterterm that shifts the renormalization out of the $\overline{\text{MS}}$ scheme.

→ **This shift must be corrected!**

LEFT: Two-Loop ADM

The shift in renormalization scheme starts affecting the ADM at two–loop order, so $\hat{\gamma}^{(0)}$ does not need a correction.

$$\hat{\gamma}'^{(0)} = \hat{R}\hat{\gamma}^{(0)}\hat{R}^{-1}$$

At two–loop order, and in the simpler case of both operator bases sharing the same basis of evanescent operators, the transformation becomes...

$$\hat{\gamma}'^{(1)} = \hat{R}\hat{\gamma}^{(1)}\hat{R}^{-1} - 2\beta_0 \Delta\hat{r}^{(0)} - \left[\Delta\hat{r}^{(0)}, \hat{\gamma}'^{(0)}\right]$$

Difference between finite amplitudes in both bases

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Difference between finite amplitudes in both bases

- Use of flavour symmetries to connect different sectors of the ADM. These symmetries can also be used to verify the consistency of the results.

Thanks to the structure of the operators in the LEFT, and to the fact that the fermion masses don not affect the ADM.

LEFT: Two-Loop ADM

To complicate things even further, the LEFT has 5631 different operators up to $\mathbf{d = 6}$ (with their corresponding Wilson coefficients), and hence the ADM could be a LARGE matrix, excruciatingly tedious to compute.

Luckily, it is distributed in independent subsectors of operators connected by the RGE, such that the problem can be reduced to the computation of many different smaller ADMs. Some examples on the four-quark sector…

- ΔF=1 with four different flavours: $\mathcal{O}_i \sim (\bar{u}\,\Gamma\,c)(\bar{s}\,\Gamma'\,b)$ } 2×10 Operators
- ΔF=1 with a quark-antiquark pair: $\mathcal{O}_i \sim (\bar{q}\,\Gamma\,q)(\bar{s}\,\Gamma'\,b)$ } 2×40 Operators
- ΔF=2: $\mathcal{O}_i \sim (\bar{s}\,\Gamma\,b)(\bar{s}\,\Gamma'\,b)$ } 8 Operators

Thank You

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