Heavy quarkonium in the quark-gluon plasma as an open quantum system

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QCD

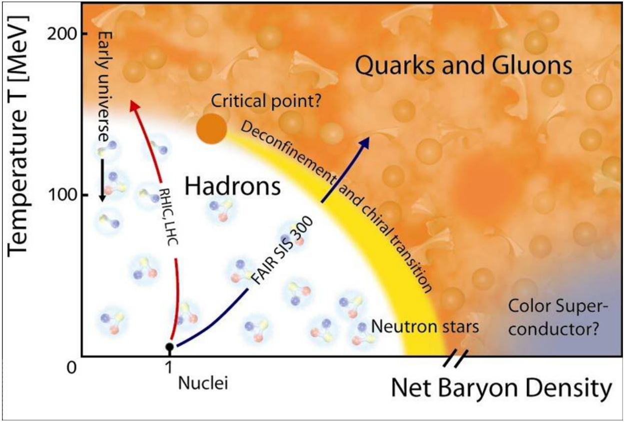
QCD is the theory of the strong interactions and it explains the origin of most of the mass observed in the universe. $m_{proton} \gg m_u, m_d$

The most relevant features of QCD are

- Confinement
- Asymptotic freedom

...or at least this is what happens in the vacuum.

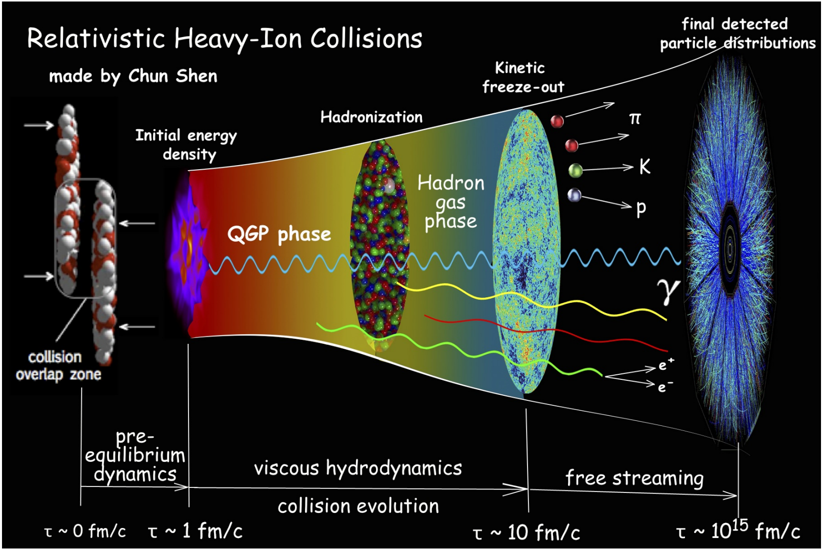
QCD phase diagram



Quark gluon plasma

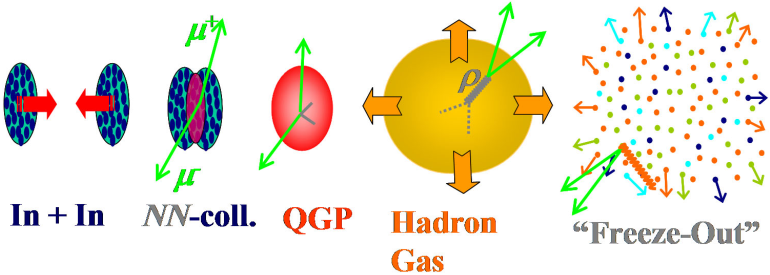
- If instead of the vacuum we consider a system with a given temperature, the mean energy of the particles is related with the temperature.
- If the temperature is very high asymptotic freedom implies that particles interact weekly, almost a free gas.
- In this way we can scape confinement. Deconfinement.
- Understanding deconfinement is a way to understand confinement. It was also a phase that was present in the early universe.

The little bang

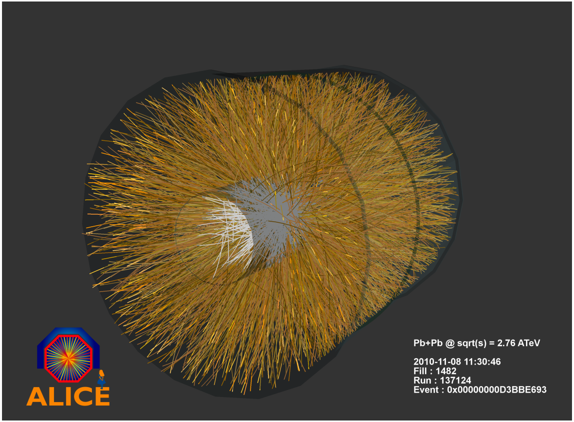


Heavy ion collisions

They are performed nowadays at LHC and RHIC



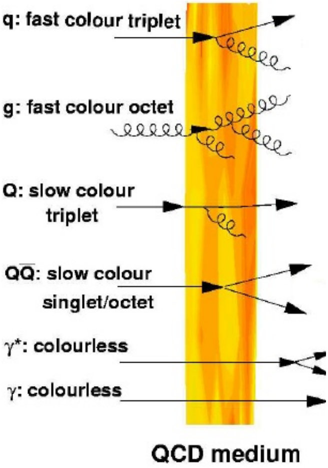
Difficulties in heavy ion collisions



Difficulties in heavy ion collisions

- We want to understand QGP.
- This is just one of the many stages that takes place in heavy ion collisions.
- At the end what we see in the detectors is the same type of particles as in proton-proton collisions, but a lot of them.

Hard probes

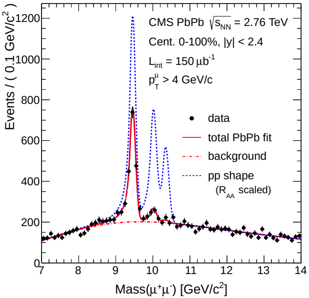


Probes that are created at the beginning of the collision (typically because its creation needs a high energy) that get modified in a substantial way and that are relatively easy to detect.

- Jet quenching.
- Heavy quark diffusion.
- Quarkonium suppression.
- Photon production.

Picture taken from d'Enterria (2007)

Quarkonium suppression



- It was predicted by Matsui and Satz that quarkonium melts in heavy-ion collisions and that this is a signal of the formation of a quark-gluon plasma.
- One of the most interesting quantities is the nuclear modification factor, R_{AA}.

$$R_{AA} = \frac{Number\ of\ quarkonia\ in\ AA\ collisions}{Number\ of\ nucleon\text{-}nucleon\ col \cdot Number\ of\ quarkonia\ in\ pp\ col}$$

What enters into R_{AA}?

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- *Medium effects*. We need to know
  - How quarkonium interact with the medium. Focus of this talk.
  - The properties of the medium (Temperature, etc...) at any point inside the medium. Relativistic hydrodynamics.

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- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

The mechanisms of dissociation

Screening

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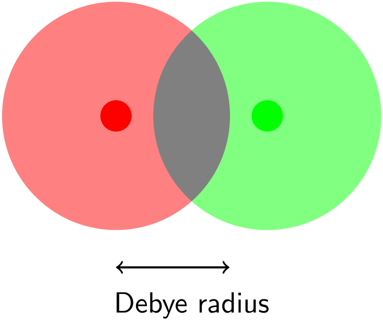
$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



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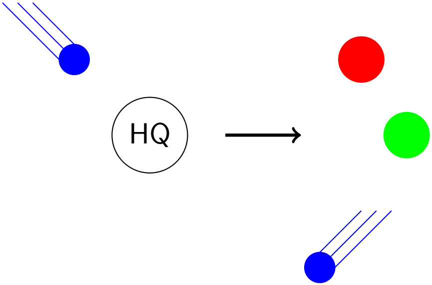
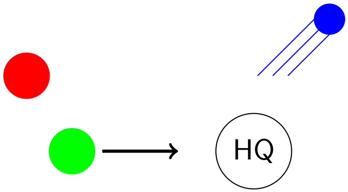


Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

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- When studying screening, we need to know if for a given potential a bound state solution exists. We need quantum mechanics to describe this.
- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

Quarkonium as an Open quantum system

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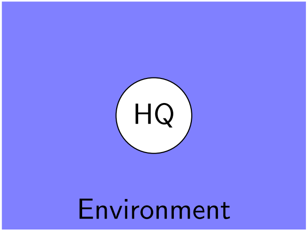

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

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In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

Problems of the Lindblad equation

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- *Computational cost*. Since the state of the system is represented by a density matrix, if we use a lattice with the double of points, the cost is multiplied by four.
- *Black box*. From the Lindblad equation it is difficult to get a physical picture of what is the leading effect.
- *Solution*. Use the Quantum Trajectories method (Dalibard, Castin and Molmer, 1992). Monte Carlo technique to solve the Lindblad equation using wave functions instead of the density matrix.

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k \left(C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa) C_k(\kappa), \rho\} \right)$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\frac{\Gamma}{2}$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle \Psi(t) | \Gamma | \Psi(t) \rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle \Psi(t) | \Gamma_n | \Psi(t) \rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n |\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

How does the quantum trajectory method encode each effect?

- Screening. Through the Hermitian part of the Hamiltonian. If there are no bound states the heavy quark and antiquark will separate.
- Decay width. Through the Non-hermitian part of the Hamiltonian. Possibility to take a quantum jump to an unbound state.
- Recombination. Through jump operator. Finite probability to jump back to a bound state.

What is the difference between MCWF method and Boltzmann-like equations?

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- In a more general master equation (non-Markovian) the decay width would depend on the whole history of the wave-function.

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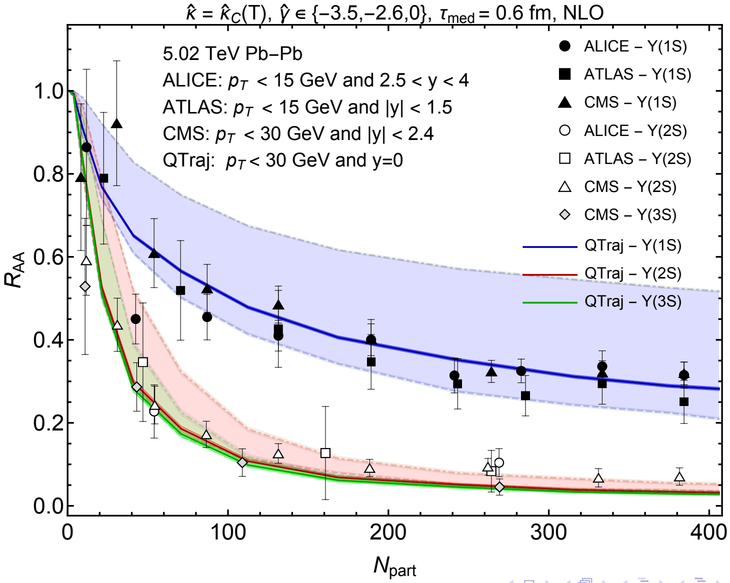
- At early times, the wave packet is very small and, therefore, the decay width is small.
- Later, if no quantum jump occurs, the wave packet will be of the size of a bound state.
- Boltzmann-like approaches model this introducing the concept of formation time (Blaizot and Ollitrault, 1987). In the open quantum system approach it comes out naturally.

Results

Brambilla et al. (2022)



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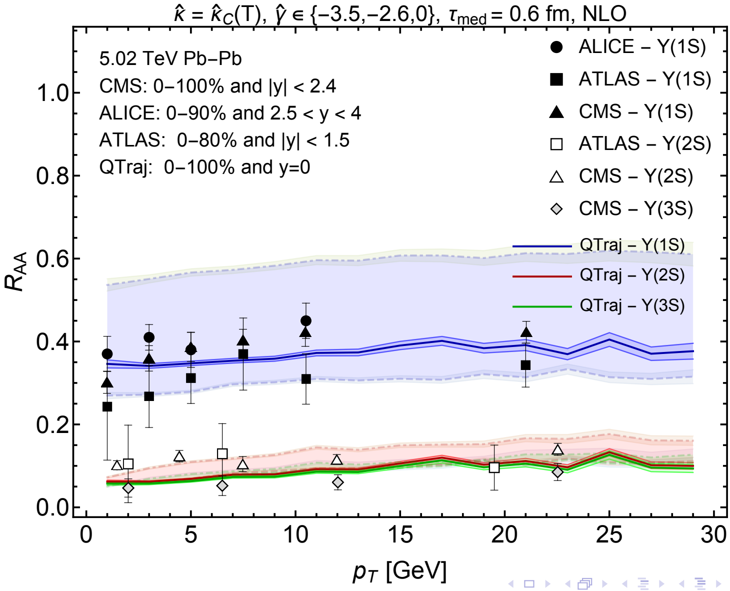

Remarks

- Many ingredients enter phenomenological predictions:
  - Initial conditions of the medium and quarkonium.
  - Hydrodynamical evolution of the medium.
  - Feed-down. Probability of transition to less excited state after freeze-out.
  - Thermal lattice QCD computations.
- We assume the hierarchy $1/r \gg T \gg E$.
- We do not include yet CNM effects and possible corrections from the hadron gas phase.

Conclusions and remarks

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- The same equation can accommodate the three phenomena that change the number of bound states in a medium.
- Good agreement with data until now.
- More precise lattice QCD measurements will lead to more precise predictions in our model.

Future prospects

- Relax approximations, explore more hierarchy of scales.
- Apply the framework to exotic quarkonium states, also measurable in heavy-ion collisions.
- Similar techniques can be applied to problems in astrophysics when finite temperature effects are important.