Triangle singularity in the $J/\psi \to \phi\pi^+ a_0^-(\pi^-\eta)$, $\phi\pi^- a_0^+(\pi^+\eta)$ decays

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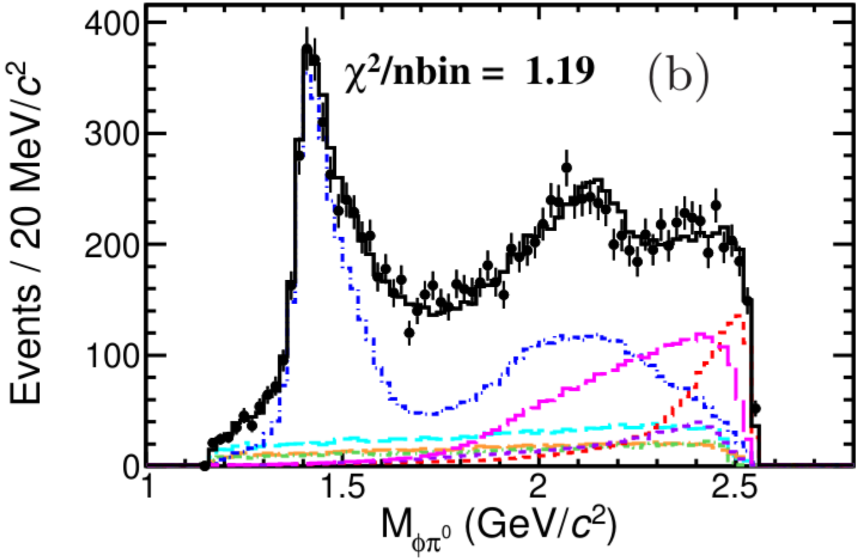
Motivation

Formalism

Results

Comparison with BESIII data



Motivation

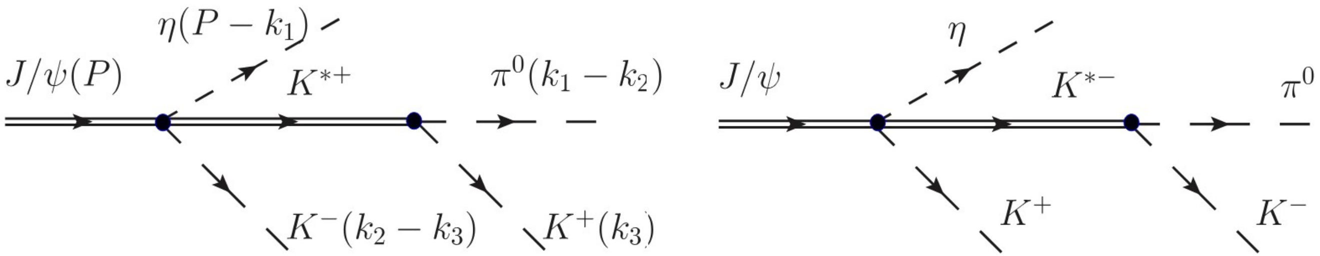
Study of the decay $J/\psi \to \phi\pi^0\eta$

BESIII   2311.07043

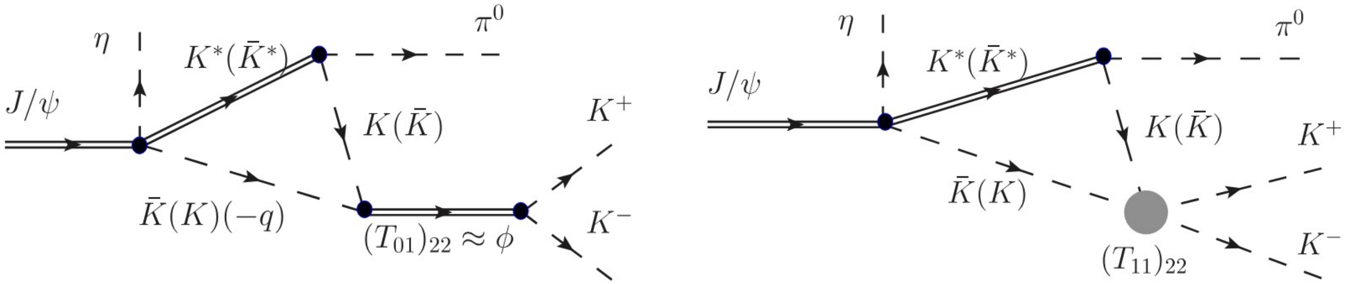
A triangle singularity develops in triangle djagrams when the three intermediate particles can be simultaneously placed on shell and are collinear

JING, SAKAI, GUO, and ZOU

PHYS. REV. D 100, 114010 (2019)



The triangle diagram develops a triangle singularity, but Schmid theorem comes into play. It says that TS can be reabsorbed into tree level with a change in the phase.



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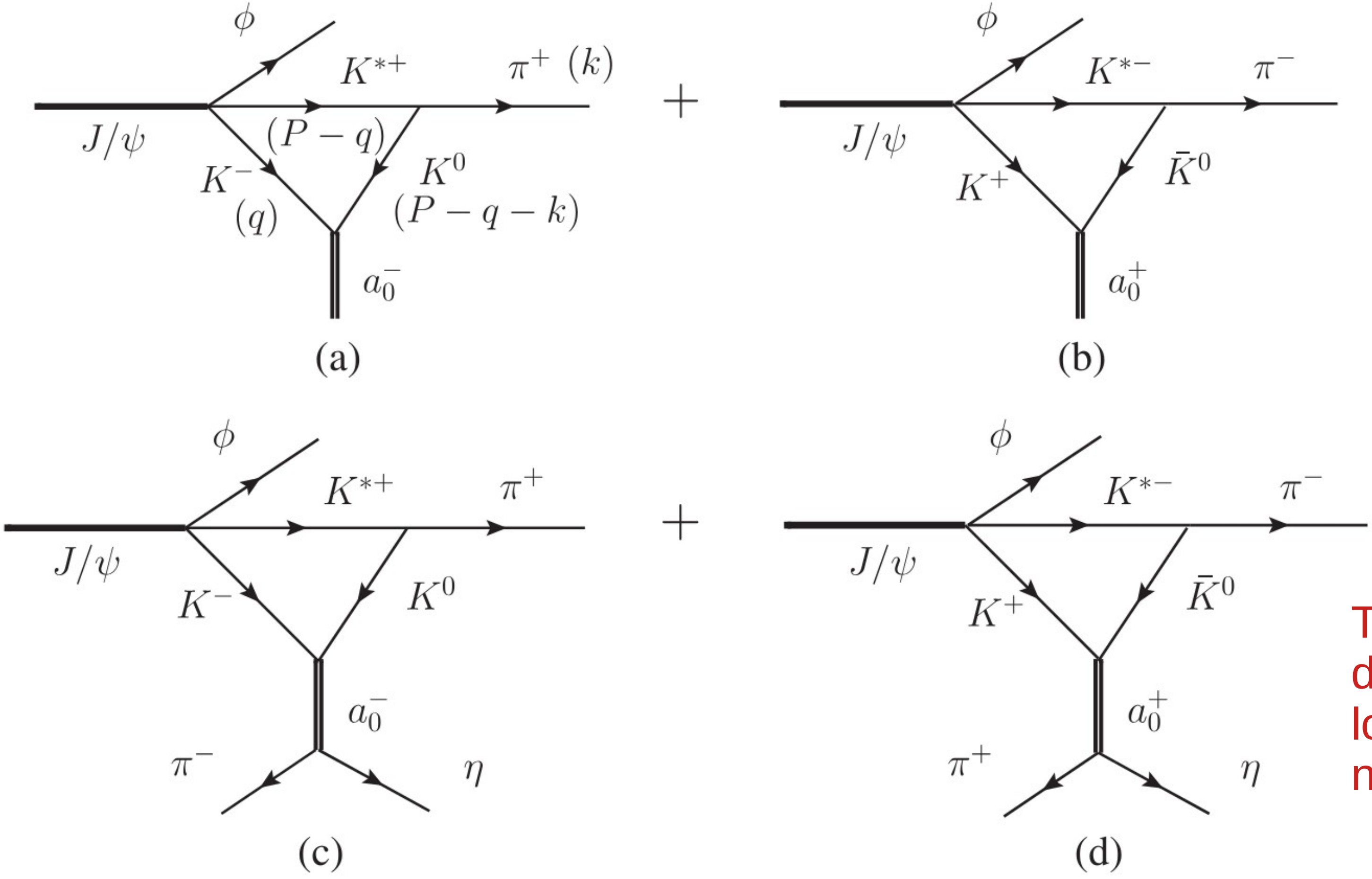
We study a clean case of TS with the $J/\psi \to \phi \pi^+ a_0^- (\pi^- \eta),\ \phi \pi^- a_0^+ (\pi^+ \eta)$ decays



Xiao, Dias, Dai, Liang, E. O.
PRD 109, 074033

The final particles are different than those of the loop. Schmid theorem does not apply

From PDG

$$\mathrm{Br}(J/\psi \to \phi K^*(892)\bar{K} + \mathrm{c.c.}) = (2.18 \pm 0.23) \times 10^{-3}$$

The K* Kbar must have C-parity positive, then

$$J/\psi \to \phi(K^{*+}K^- + K^{*0}\bar{K}^0 - K^{*-}K^+ - \bar{K}^{*0}K^0)$$

$$\mathrm{Br}(J/\psi \to \phi K^{*+}K^-) = (0.55 \pm 0.06) \times 10^{-3}$$

$$t_{J/\psi,\phi K^{*+}K^-} = C\vec{\epsilon}_{J/\psi} \cdot (\vec{\epsilon}_\phi \times \vec{\epsilon}_{K^*})$$

$$\frac{\mathrm{d}\Gamma_{J/\psi\to\phi K^{*+}K^-}}{\mathrm{d}M_{\mathrm{inv}}(K^{*+}K^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{J/\psi}^2} p_\phi \tilde{p}_{K^-} \bar{\sum}\sum |t|^2$$

$$\frac{C^2}{\Gamma_{J/\psi}} = 1.381 \times 10^{-2}\ (\mathrm{MeV}^{-1})$$

The $K^{*+} \to K^0\pi^+$ coupling is easily obtained from the standard Lagrangian,

$$\mathcal{L} = -ig\langle [P, \partial_\mu P] V^\mu \rangle$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix}$$

$$-it = -ig\epsilon_j(K^*)(2k+q)^j$$

$$t_{K^-K^0,K^-K^0}(M_{\rm inv}) = \frac{g^2_{a_0,K^-K^0}}{M^2_{\rm inv} - m^2_{a_0} + iM_{\rm inv}\Gamma_{a_0}}$$

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

But a_0 is a cusp, no clear couplings

$$g^2_{a_0,K^-K^0} = -\frac{1}{\pi}\int {\rm d}M^2_{\rm inv}\,{\rm Im}\,t_{K^-K^0,K^-K^0}(M_{\rm inv})$$

From the chiral unitary approach

$$\frac{{\rm d}^2\Gamma_{J/\psi,\to\phi\pi^+a_0^-(\pi^-\eta)}}{{\rm d}M_{\rm inv}(\pi^-\eta){\rm d}M_{\rm inv}(\pi^+a_0^-)} = -\frac{1}{\pi}2M_{\rm inv}(\pi^-\eta)\,{\rm Im}\,t_{K^-K^0,K^-K^0}(M_{\rm inv}(\pi^-\eta)) \times \frac{1}{(2\pi)^3}\frac{1}{4M^2_{J/\psi}}p_\phi \tilde{p}'_{\pi^+}\bar{\sum}|\tilde{t}_{\rm TS}|^2,$$

$$\frac{1}{q^2-m_K^2+i\varepsilon}=\frac{1}{2\omega(\vec{q})}\left(\frac{1}{q^0-\omega_K(\vec{q})+i\varepsilon}-\frac{1}{q^0+\omega_K(\vec{q})-i\varepsilon}\right)$$

Since for the TS the particles are placed on shell, we need only the part of positive energy

$$\begin{aligned}-i\tilde{t}_{\rm TS} &= -iC\int\frac{d^4q}{(2\pi)^4}\varepsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)\epsilon_l(K^*)(-i)g\epsilon_m(K^*)\\ &\times(2k+q)_m(-i)\frac{1}{2\omega_{K^-}(\vec{q})}\frac{1}{2\omega_{K^0}(\vec{q}+\vec{k})}\\ &\times\frac{1}{2\omega_{K^{*+}}(\vec{q})}\frac{i}{q^0-\omega_{K^-}(\vec{q})+i\varepsilon}\\ &\times\frac{i}{P^0-q^0-\omega_{K^{*+}}(\vec{q})+i\frac{\Gamma_{K^*}}{2}}\\ &\times\frac{i}{P^0-q^0-k^0-\omega_{K^0}(\vec{q}+\vec{k})+i\varepsilon},\end{aligned}\qquad(24)$$

$$\begin{aligned}\tilde{t}_{\rm TS} &= gC\varepsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)\int\frac{d^3q}{(2\pi)^3}(2k+q)_l\\ &\times\frac{1}{2\omega_{K^-}(\vec{q})}\frac{1}{2\omega_{K^{*+}}(\vec{q})}\frac{1}{2\omega_{K^0}(\vec{q}+\vec{k})}\\ &\times\frac{i}{P^0-\omega_{K^-}(\vec{q})-\omega_{K^{*+}}(\vec{q})+i\frac{\Gamma_{K^*}}{2}}\\ &\times\frac{i}{P^0-k^0-\omega_{K^-}(\vec{q})-\omega_{K^0}(\vec{q}+\vec{k})+i\varepsilon}.\end{aligned}$$

Factor in integral

q_{max}=600 MeV

$\theta(q_{\rm max}-|\vec{q}^{\,*}|)$ where $\vec{q}^{\,*}$ is the K^- momentum in the $\pi^-\eta$ rest frame

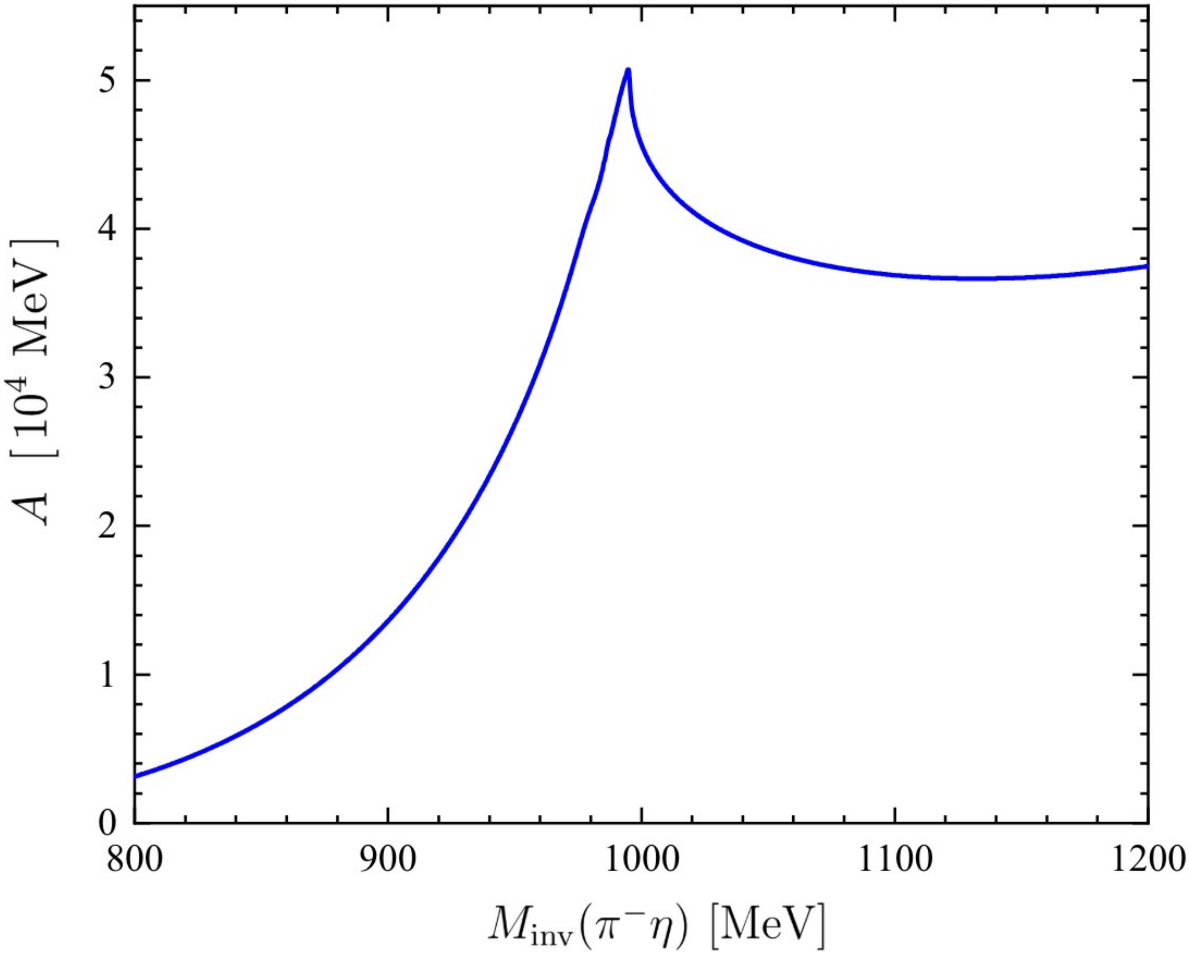


FIG. 3. Factor $A \equiv [-\frac{2}{\pi} M_{\rm inv}(\pi^-\eta) {\rm Im}\, t_{K^-K^0,K^-K^0}(M_{\rm inv}(\pi^-\eta))]$ as a function of $M_{\rm inv}(\pi^-\eta)$.

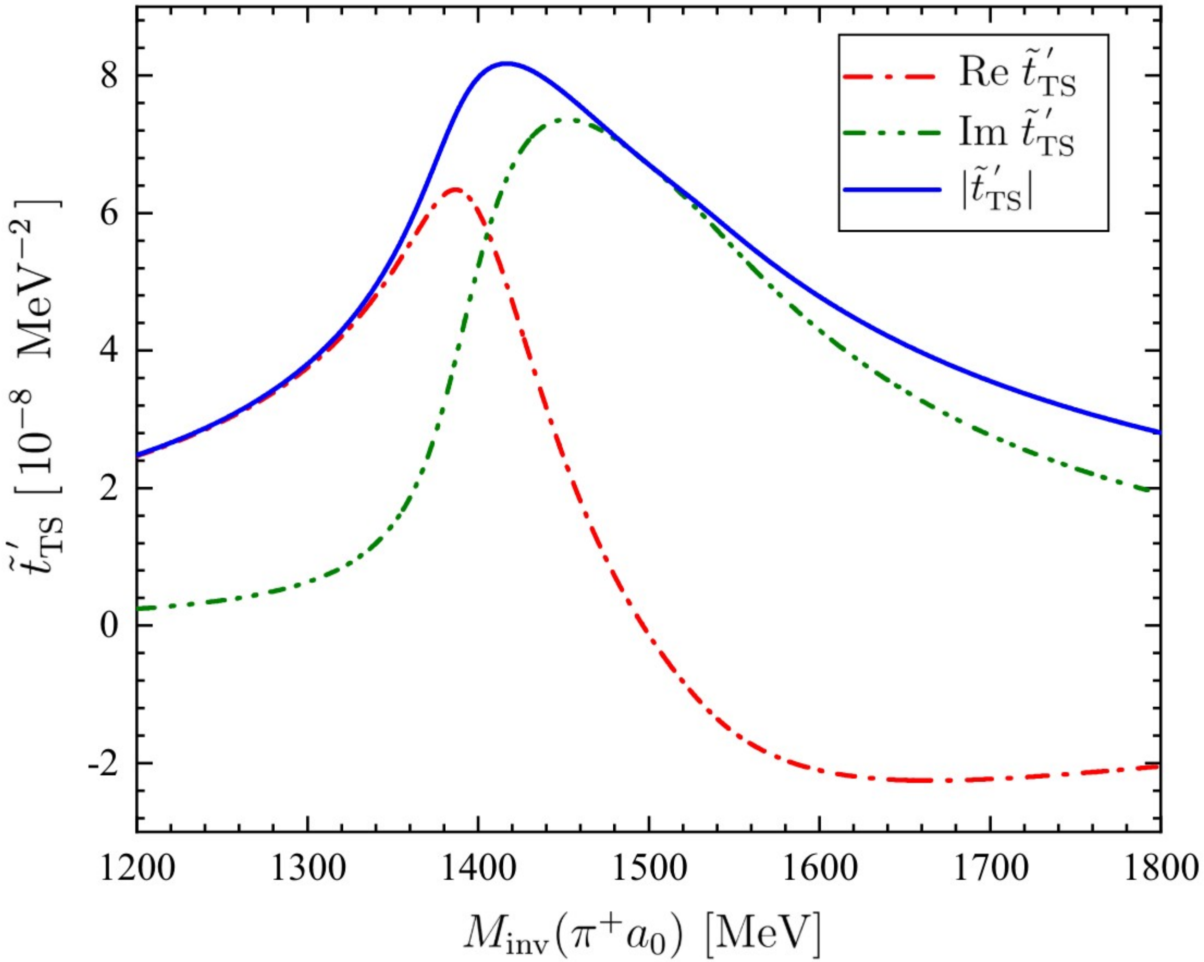


FIG. 4. $\tilde{t}'_{\rm TS}$ given by Eq. (29) as a function of $M_{\rm inv}(\pi^+ a_0^-)$ when fixing $M_{\rm inv}(\pi^- \eta) = m_{a_0}$.

$$\tilde{t}_{\rm TS} = gC\epsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)k_l\tilde{t}'_{\rm TS}$$

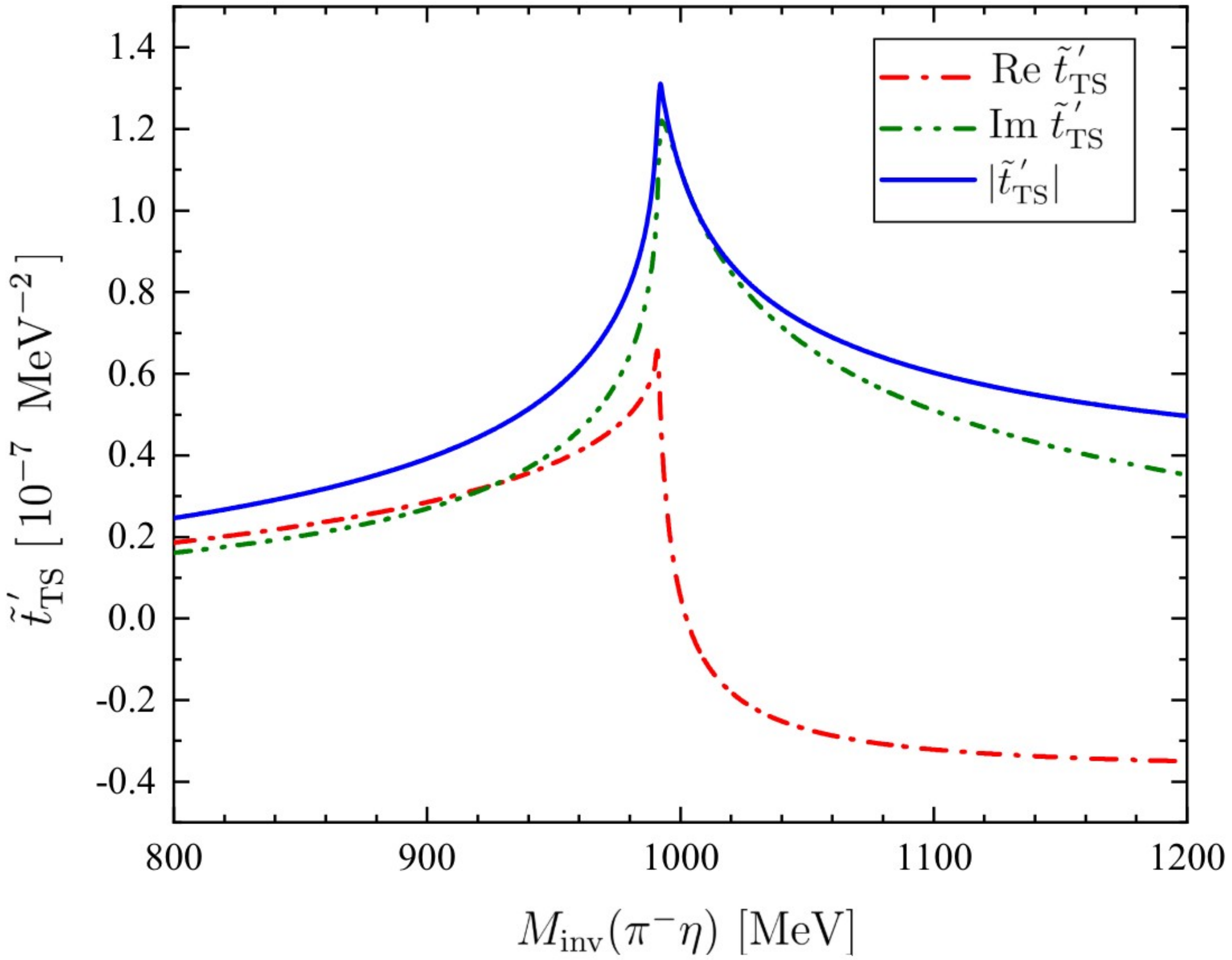


FIG. 6. $\tilde{t}'_{\rm TS}$ given by Eq. (29) as a function of $M_{\rm inv}(\pi^-\eta)$ when fixing $M_{\rm inv}(\pi^+ a_0^-) = 1416$ MeV.

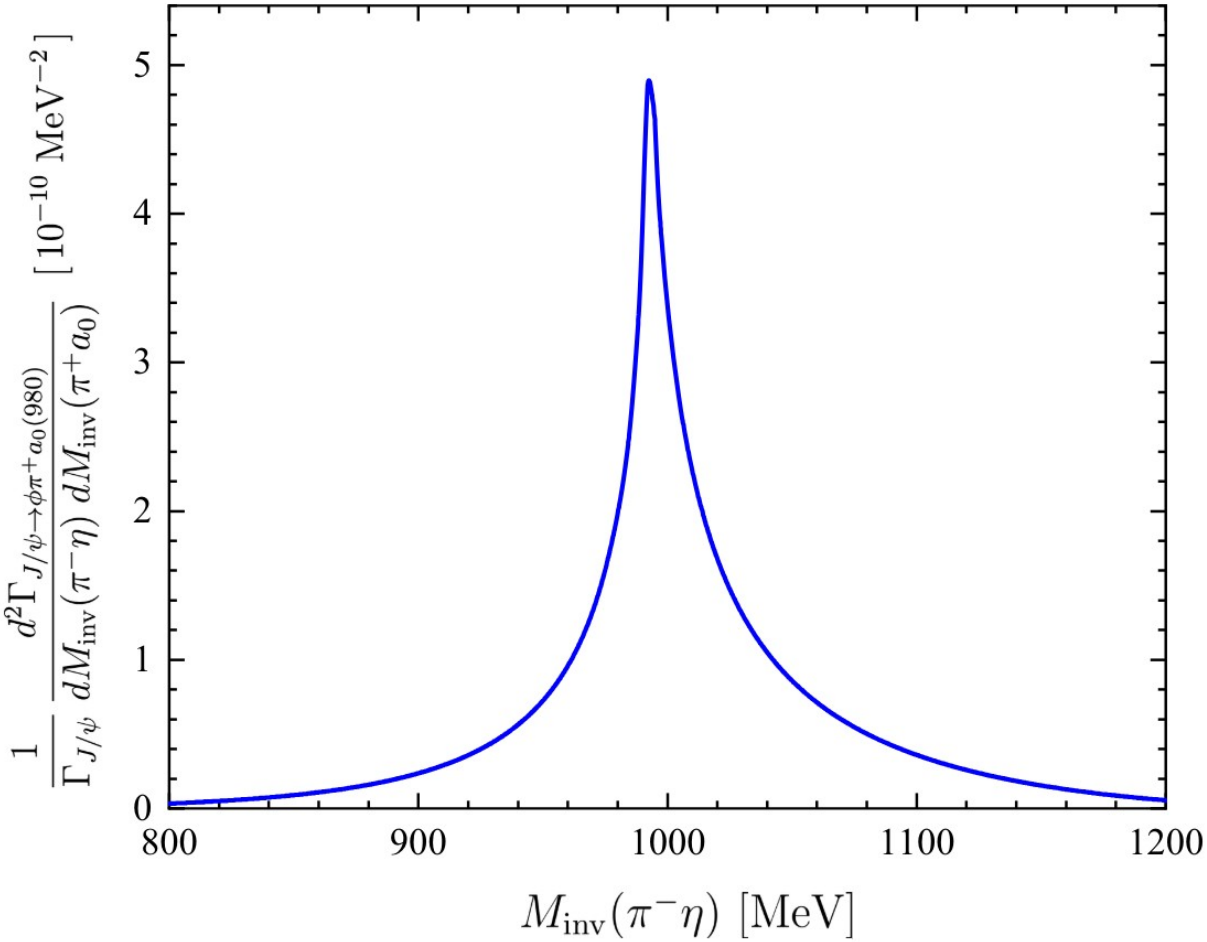


FIG. 7. $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{dM_{\rm inv}(\pi^-\eta) dM_{\rm inv}(\pi^+ a_0^-)}$ as a function of $M_{\rm inv}(\pi^-\eta)$ when fixing $M_{\rm inv}(\pi^+ a_0^-) = 1416$ MeV.

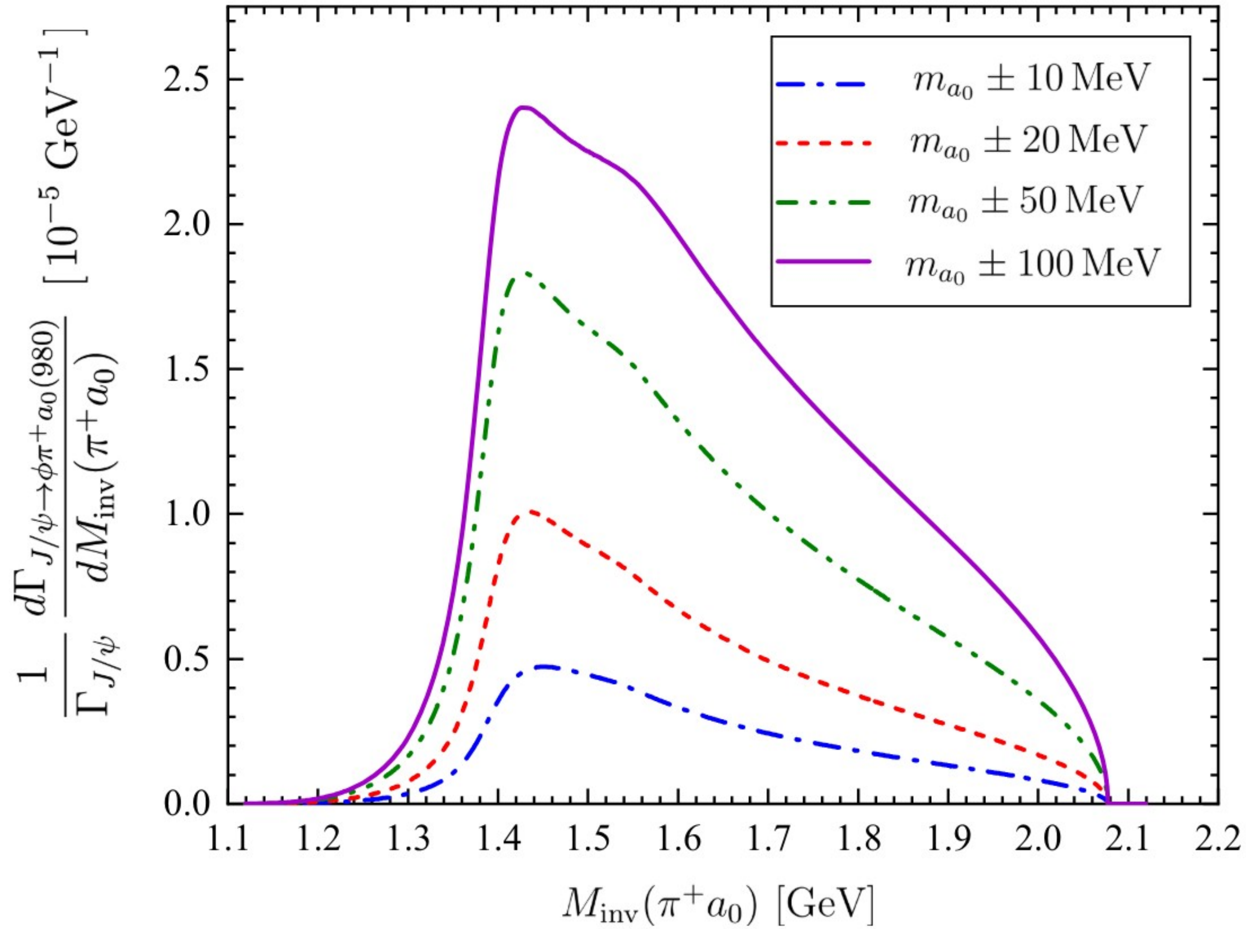


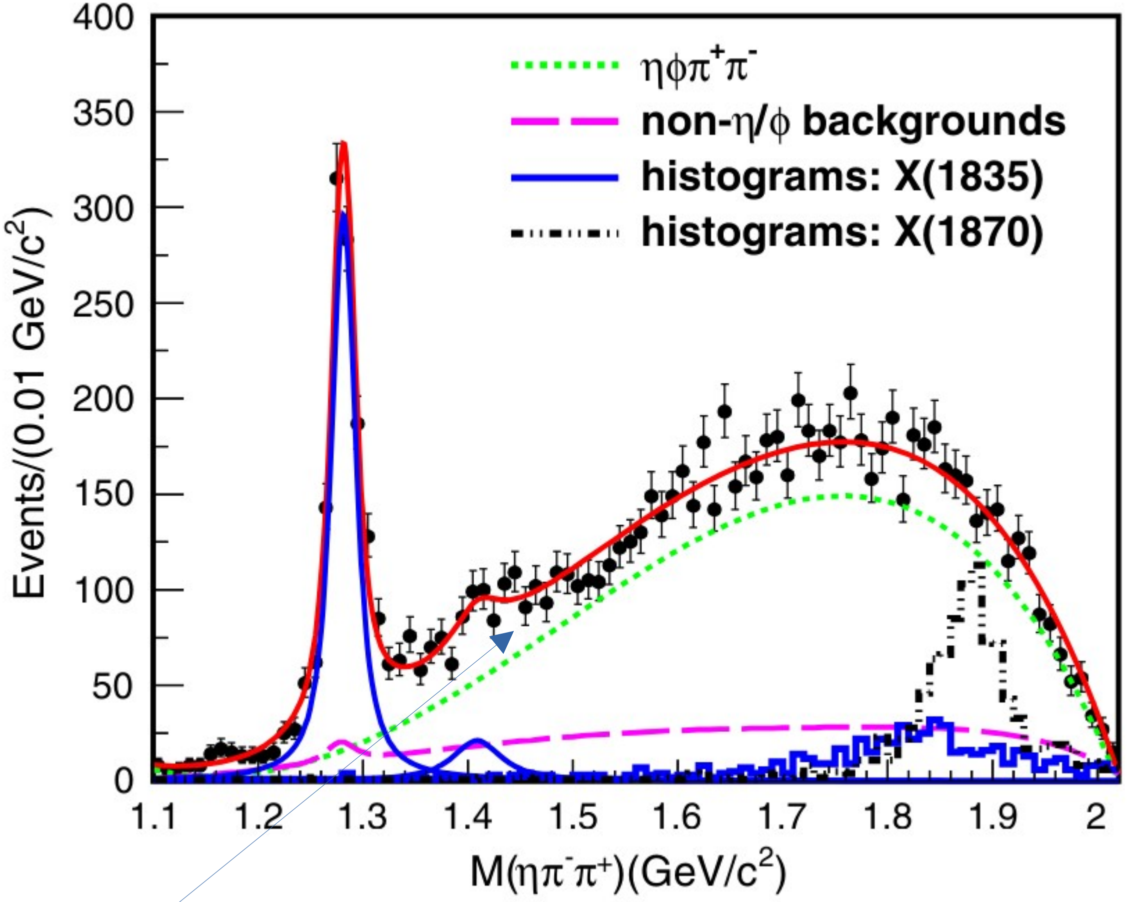
FIG. 8. $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{{\rm d}M_{\rm inv}(\pi^-\eta){\rm d}M_{\rm inv}(\pi^+ a_0^-)}$ as a function of $M_{\rm inv}(\pi^+ a_0^-)$ when integrating over $M_{\rm inv}(\pi^-\eta)$ in the ranges $m_{a_0} \pm 10$ MeV, $m_{a_0} \pm 20$ MeV, $m_{a_0} \pm 50$ MeV and $m_{a_0} \pm 100$ MeV.

For the case where $M_{\rm inv}(\pi^-\eta) \in [m_{a_0} - 100, m_{a_0} + 100]$ MeV, integrating over $M_{\rm inv}(\pi^+ a_0^-)$ in the range $[m_{\pi^+} + m_{a_0}, M_{J/\psi} - m_\phi]$ gives the branching ratio

$$\mathrm{Br}(J/\psi \to \phi\pi^+ a_0^-) = 1.07 \times 10^{-5}, \qquad (35)$$

This branching ratio is easily reachable in BESIII, where branching ratios of 10^{-7} can be reached

We get double Br if we sum Br($J/\psi \to \varphi\pi^- a_0^+$) $(2.14 \pm 0.64) \times 10^{-5}$



M. Ablikim et al. (BESIII Collaboration), Study of J/ψ → ηφπ+π− at BESIII,
Phys. Rev. D 91, 052017 (2015).

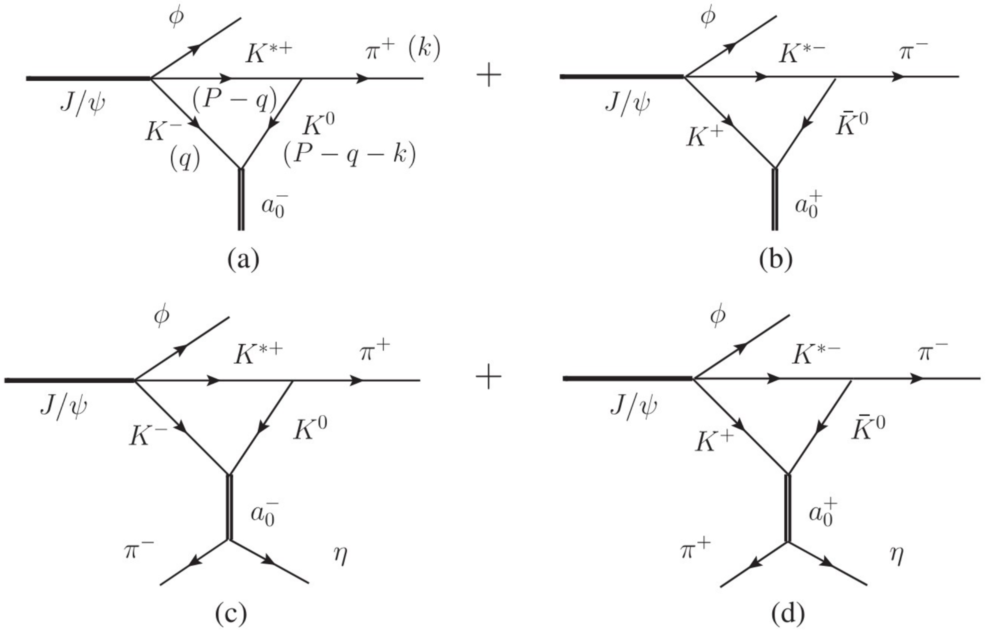
Br for this peak $(2.01 \pm 0.58 \pm 0.82) \times 10^{-5}$

Compatible with our estimate for the TS

In BESIII paper this peak is associated to η(1405) excitation

Conclusions

We have studied the $J/\psi \to \phi\pi^+ a_0^-(\pi^-\eta),\ \phi\pi^- a_0^+(\pi^+\eta)$ **decays**



And have shown that the triangle mechanisms

produce a triangle singularity visible
in the $\pi\ a_0(980)$ invariant mass

The results obtained are consistent with
a peak seen in a recent BESIII experiment