The $D^+ \to \bar{K}^0 \pi^+ \eta$ reaction and $a_0(980)$

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N. Ikeno, J. M. Dias, W. H. Liang, and E. Oset, Eur. Phys. J. C 84, 469 (2024).

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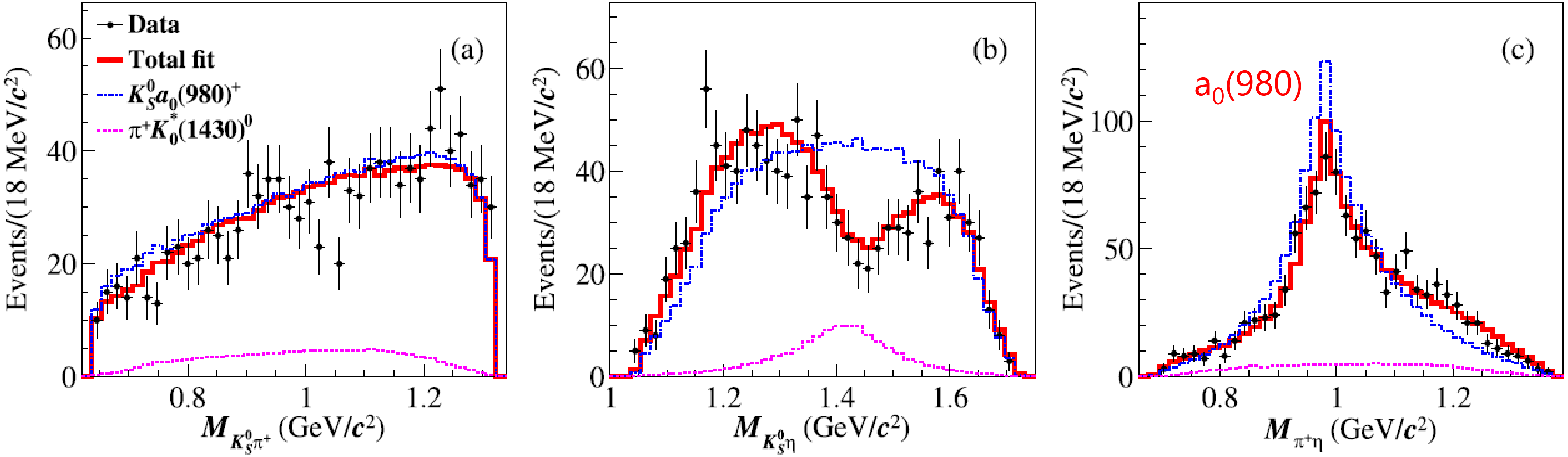
The $D^+ \to K_s^0 \pi^+ \eta$ reaction by BESIII

Phys. Rev. Lett. 132,131903 (2024)



The $a_0(980)$ was observed as a clear peak in $M_{\pi\eta}$

$\Rightarrow$ The $D^+ \to K_s^0 \pi^+ \eta$ reaction is an ideal reaction to isolate the $a_0(980)$ contribution

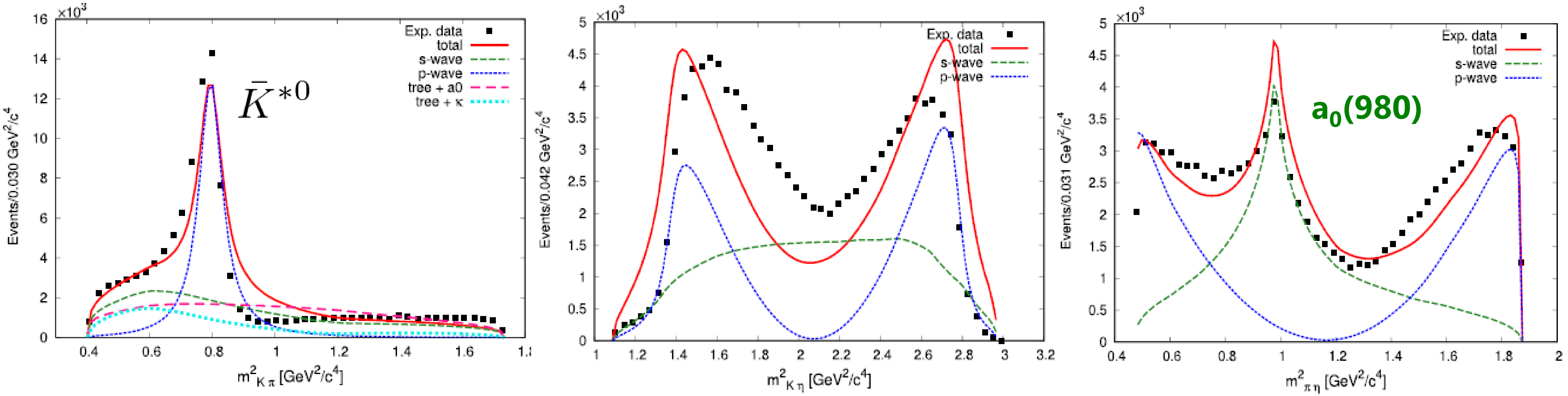
This is actually the $D^+ \to \bar{K}^0 \pi^+ \eta$ reaction since the $\bar{K}^0$ is observed as a K^0_s state.

Just a copy of the $D^0 \to K^- \pi^+ \eta$ reaction measured by Belle!?

(by changing a $\bar{d} \to \bar{u}$ quark)

The $D^0 \to K^-\pi^+\eta$ reaction

Experiment by Belle: Phys. Rev. D 102, 012002 (2020)
Theoretical study: G. Toledo, N. Ikeno and E. Oset, EPJC81, 268 (2021).



- $\bar{K}^{*0}$ excitation comes from $K^-\pi^+$ <-> No $\bar{K}^{*0}$ contribution in $D^+ \to \bar{K}^0\pi^+\eta$
- $a_0(980)$ is also identified in $M_{\pi\eta}$ but affected by $\bar{K}^{*0}$

=> The two reactions are drastically different

Our study

N. Ikeno, J. M. Dias, W. H. Liang, and E. Oset, EPJC 84, 469 (2024).

- We like to understand the spectrum of the $D^+ \to \bar{K}^0\pi^+\eta$ reaction based on the perspective of $a_0(980)$ resonance as a dynamically generated state from the interaction of the $\pi\eta$, $K\bar{K}$ channels

- The $a_0(980)$ is well described by the chiral unitary approach

- We consider the reaction mechanisms:

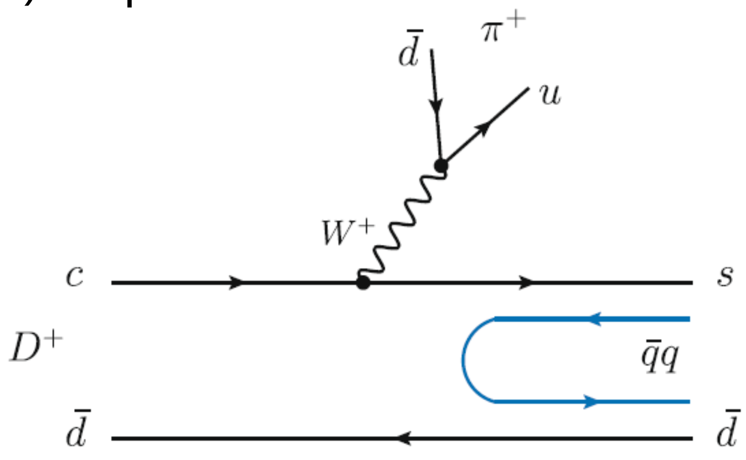
  - external and internal emission at the quark level

  - hadronization of the $\bar{q}q$ components into two mesons

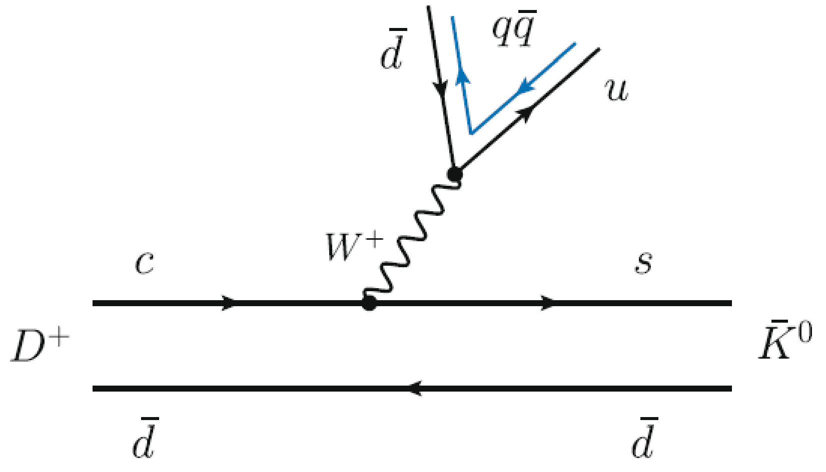
  - final-state interaction between these mesons

External emission: hadronization

(a) $s\bar{d}$ pair is hadronized



(b) $u\bar{d}$ pair is hadronized



$$s\bar{d} \rightarrow \sum_i s\ \bar{q}_i q_i\ \bar{d} = \sum_i \mathcal{P}_{3i}\ \mathcal{P}_{i2} = \left(\mathcal{P}^2\right)_{32}$$

$$= K^-\pi^+ - \frac{1}{\sqrt{2}}\pi^0\bar{K}^0,$$

the $\bar{K}^0\eta$ component has cancelled

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}$$

$$u\bar{d} \rightarrow \sum_i u\ \bar{q}_i q_i\ \bar{d} = \sum_i \mathcal{P}_{1i}\ \mathcal{P}_{i2} = \left(\mathcal{P}^2\right)_{12}$$

$$= \frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0$$

the $\pi^0\pi^+$ component has cancelled

Together with the π^+

=> Does not lead to the final state $\bar{K}^0\eta\pi^+$

we disregarded the possible rescattering

$$K^-\pi^+ \rightarrow \bar{K}^0\eta,\ \bar{K}^0\pi^0 \rightarrow \bar{K}^0\eta$$

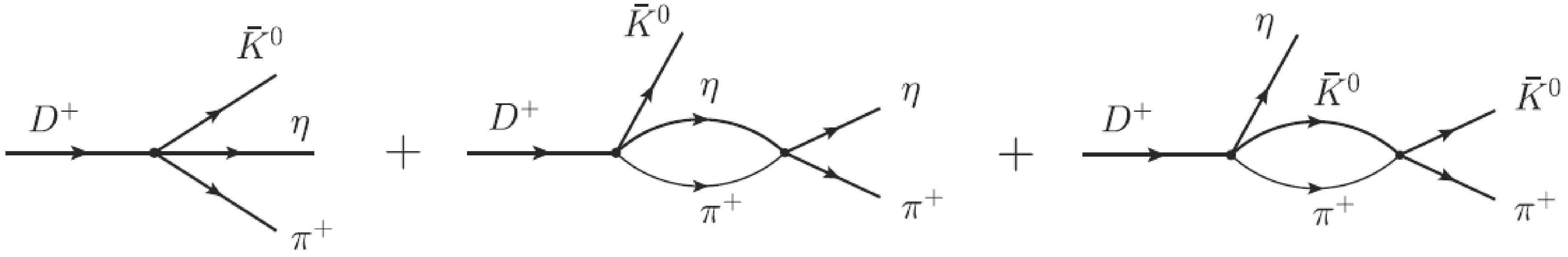
Together with the $\bar{K}^0$

$$H = \left(\frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0\right)\bar{K}^0$$

disregard

External emission: final-state interaction

$$H = \frac{2}{\sqrt{3}} \eta \pi^+$$



(a) (b) (c)

tree level | $\eta\pi^+$ rescattering | $\bar{K}^0\pi^+$ rescattering

$$t^{(ee)} = \mathcal{C}\Big\{ h_{\eta\pi^+\bar{K}^0} + h_{\eta\pi^+\bar{K}^0}\big[G_{\eta\pi^+}(M_{\rm inv}(\eta\pi^+)) \cdot t_{\eta\pi^+,\,\eta\pi^+}(M_{\rm inv}(\eta\pi^+)) + G_{\bar{K}^0\pi^+}(M_{\rm inv}(\bar{K}^0\pi^+)) \cdot t_{\bar{K}^0\pi^+,\bar{K}^0\pi^+}(M_{\rm inv}(\bar{K}^0\pi^+))\big] \Big\}$$

with

$$h_{\eta\pi^+\bar{K}^0} = \frac{2}{\sqrt{3}}$$

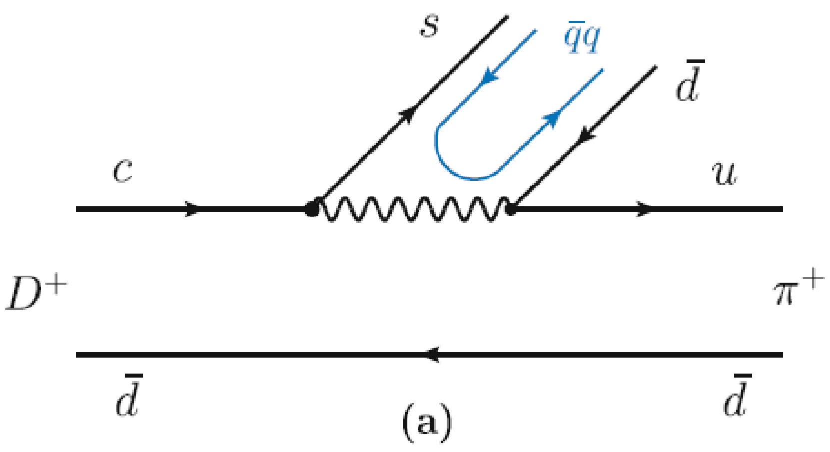
G_i : the loop functions of two mesons

t_{ij} : the scattering matrix for the transition of channel i to channel j

C: a global constant that will be used to get the normalization of the data

Internal emission: hadronization

(a) $s\bar{d}$ pair is hadronized

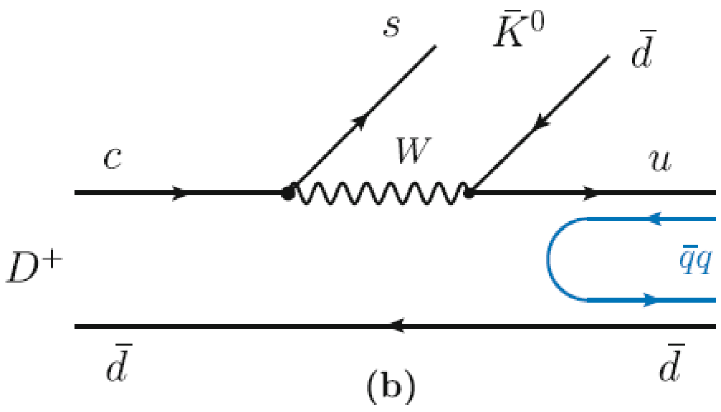


$$s\bar{d} \to \sum_i s\ \bar{q}_i q_i\ \bar{d} = \sum_i \mathcal{P}_{3i}\ \mathcal{P}_{i2} = \left(\mathcal{P}^2\right)_{32}$$

$$= K^-\pi^+ - \frac{1}{\sqrt{2}}\pi^0\bar{K}^0,$$

Together with the π^+

(b) $u\bar{d}$ pair is hadronized



$$u\bar{d} \to \sum_i u\ \bar{q}_i q_i\ \bar{d} = \sum_i \mathcal{P}_{1i}\ \mathcal{P}_{i2} = \left(\mathcal{P}^2\right)_{12}$$

$$= \frac{2}{\sqrt{3}}\eta\pi^+ + K^+\bar{K}^0,$$

Together with the $\bar{K}^0$

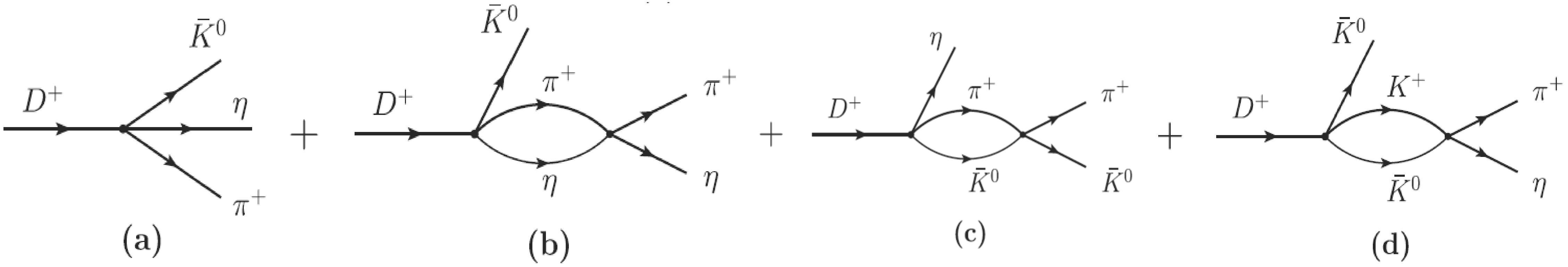
Summing the two terms,

$$H' = K^-\pi^+\pi^+ - \frac{1}{\sqrt{2}}\pi^0\pi^+\bar{K}^0 + \frac{2}{\sqrt{3}}\eta\pi^+\bar{K}^0 + K^+\bar{K}^0\bar{K}^0$$

Internal emission: final-state interaction

$$H' = K^-\pi^+\pi^+ - \frac{1}{\sqrt{2}}\pi^0\pi^+\bar{K}^0 + \frac{2}{\sqrt{3}}\eta\pi^+\bar{K}^0 + K^+\bar{K}^0\bar{K}^0$$

we disregarded the possible rescattering $K^-\pi^+ \to \bar{K}^0\eta,\ \bar{K}^0\pi^0 \to \bar{K}^0\eta,\ \bar{K}^0\eta \to \bar{K}^0\eta$



tree level; $\pi^+\eta$ rescattering; $\pi^+\bar{K}^0$ rescattering; $K^+\bar{K}^0$ rescattering

$$t^{(ie)} = \beta C \Big\{ \bar{h}_{\eta\pi^+\bar{K}^0} + \bar{h}_{\eta\pi^+\bar{K}^0} \big[G_{\eta\pi^+}(M_{\rm inv}(\eta\pi^+)) \cdot t_{\eta\pi^+,\,\eta\pi^+}(M_{\rm inv}(\eta\pi^+)) + G_{\pi^+\bar{K}^0}(M_{\rm inv}(\pi^+\bar{K}^0)) \cdot t_{\pi^+\bar{K}^0,\,\pi^+\bar{K}^0}(M_{\rm inv}(\pi^+\bar{K}^0)) \big] + 2\,\bar{h}_{K^+\bar{K}^0\bar{K}^0}\, G_{K^+\bar{K}^0}(M_{\rm inv}(\pi^+\eta)) \cdot t_{K^+\bar{K}^0,\pi^+\eta}(M_{\rm inv}(\pi^+\eta)) \Big\}$$

with

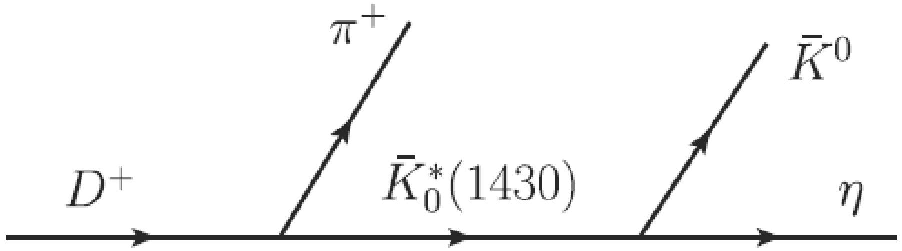
$$\bar{h}_{\eta\pi^+\bar{K}^0} = \frac{2}{\sqrt{3}} \qquad \bar{h}_{K^+\bar{K}^0\bar{K}^0} = 1$$

β is the relative weight of the internal to external emission and is expected to be the order of 1/Nc

$K_0^*(1430)$ contribution

In Exp., the scalar $K_0^*(1430)\,[I\,(J^P) = \frac{1}{2}\,(0^+)]$ contribution showed up in $M_{K\eta}$

We take into account the $K_0^*(1430)$ contribution phenomenologically



$$t^* = \mathcal{D}\, e^{i\phi}\, \frac{M_D^2}{s_{13} - M_{K_0^*}^2 + i\, M_{K_0^*}\, \Gamma_{K_0^*}},$$

D and ϕ: free parameters $\quad s_{13} = (p_{\bar{K}^0} + p_\eta)^2$

The final amplitude: $t = t^{(ee)} + t^{(ie)} + t^*$

Mass distribution:

$$\frac{d^2\Gamma}{ds_{12}\, ds_{23}} = \frac{1}{(2\pi)^3}\, \frac{1}{32\, M_D^3}\, |t|^2 \qquad \bar{K}^0(1), \pi^+(2), \eta(3)$$

We can integrate over the limits of the PDG formula
to get d/ds12 integrating over s23

Scattering amplitudes $t_{\eta\pi^+,\,\eta\pi^+}$, $t_{\bar{K}^0\pi^+,\,\bar{K}^0\pi^+}$, $t_{\bar{K}^0K^+,\,\pi^+\eta}$

By evaluating the coupled channels, T matrix

$$T = [1 - VG]^{-1}\, V$$

- Scattering amplitude in $K^+K^-(1),\, K^0\bar{K}^0(2),\, \pi^0\eta(3)$

J.X. Lin, J.T. Li, S.J. Jiang, W.H. Liang, E. Oset, Eur. Phys. J. C 81, 1017 (2021)

$$t_{\eta\pi^+,\,\eta\pi^+} = t_{\eta\pi^0,\,\eta\pi^0}$$

$$t_{\bar{K}^0K^+,\,\pi^+\eta} = \sqrt{2}\, t_{K^+K^-,\,\pi^0\eta}$$

=> $a_0(980)$ is dynamically generated from the channels

- Scattering amplitude in $\pi^-K^+(1),\, \pi^0K^0(2),\, \eta K^0(3)$

G. Toledo, N. Ikeno, E. Oset, Eur. Phys. J. C 81, 268 (2021)

$$t_{\bar{K}^0\pi^+,\,\bar{K}^0\pi^+} = \frac{2}{3}\, T_{22} + \frac{1}{3}\, T_{11} + \frac{2\sqrt{2}}{3}\, T_{12}$$

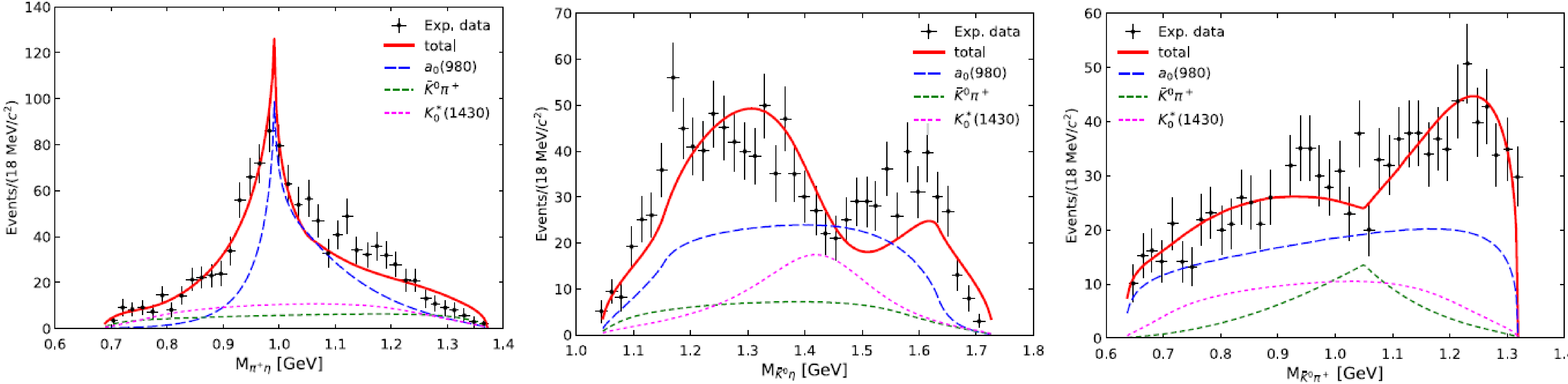
Numerical results

Four parameters in our model

=> we perform a best fit to the three mass distributions

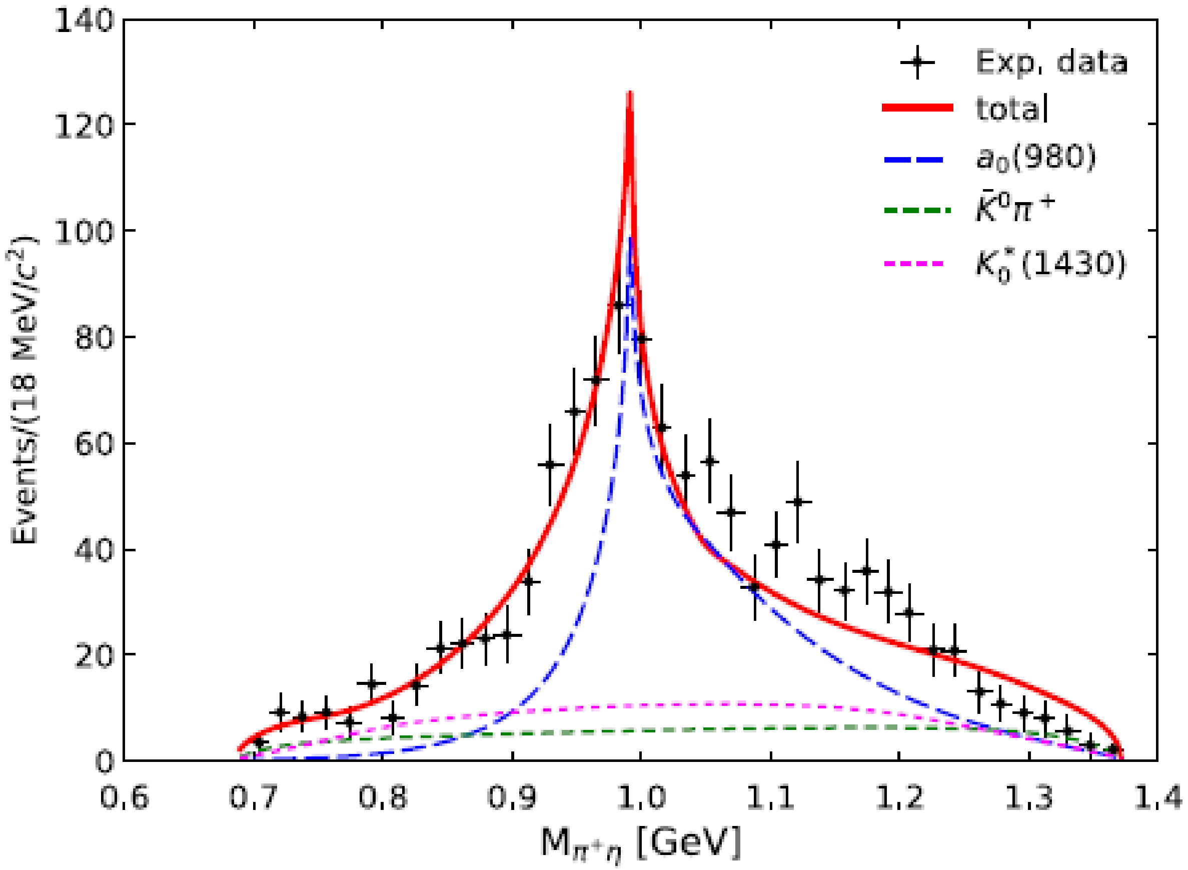
Parameters	
$\mathcal{C}$	486.90
$\mathcal{D}$	63.94
β	0.70
ϕ	-2.16 radians

- C: global normalization
- D: $K_0^*(1430)$ contribution
- β: relative weight of the internal to external emission
- Phase exp(iϕ): interference between the $K_0^*(1430)$ and others



Our theoretical calculations closely reproduce the experimental data.

Calculated $\pi^+\eta$ mass distribution



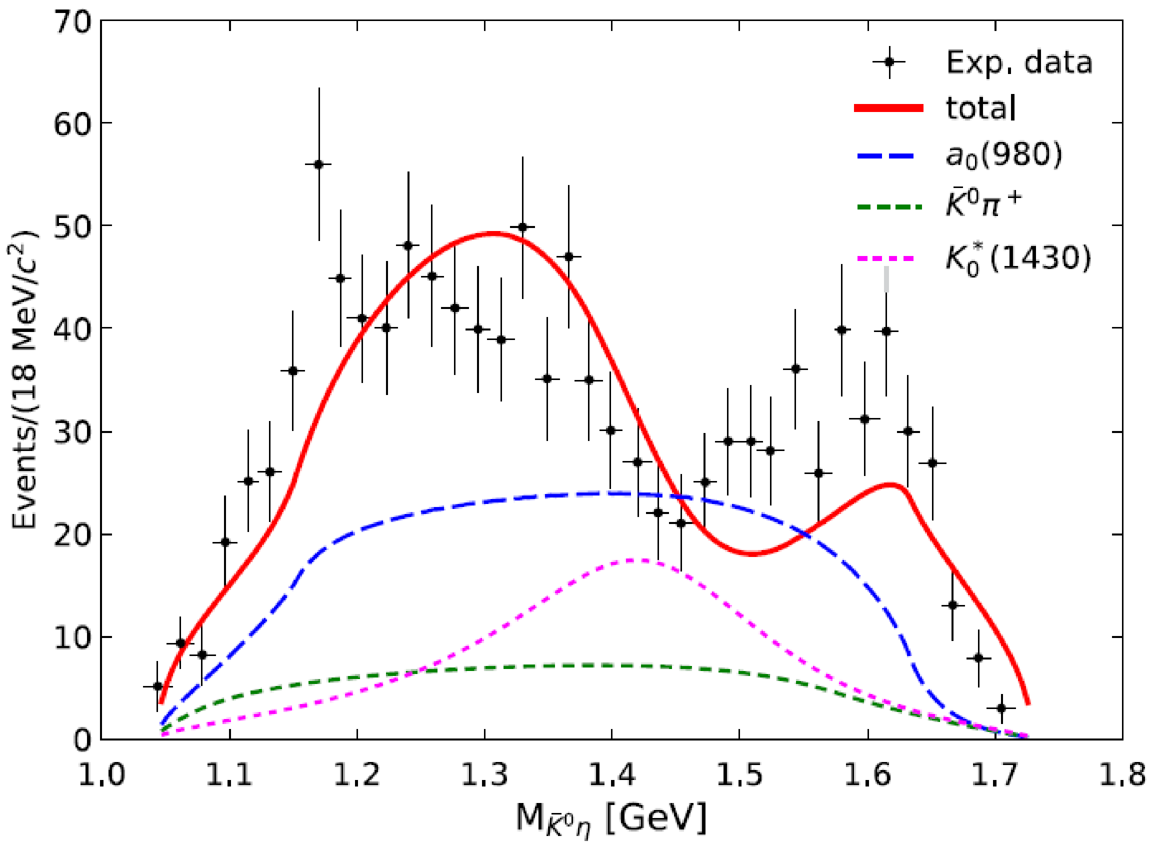
We can see a clear peak around 1.0 GeV, corresponding to the $a_0(980)$ resonance

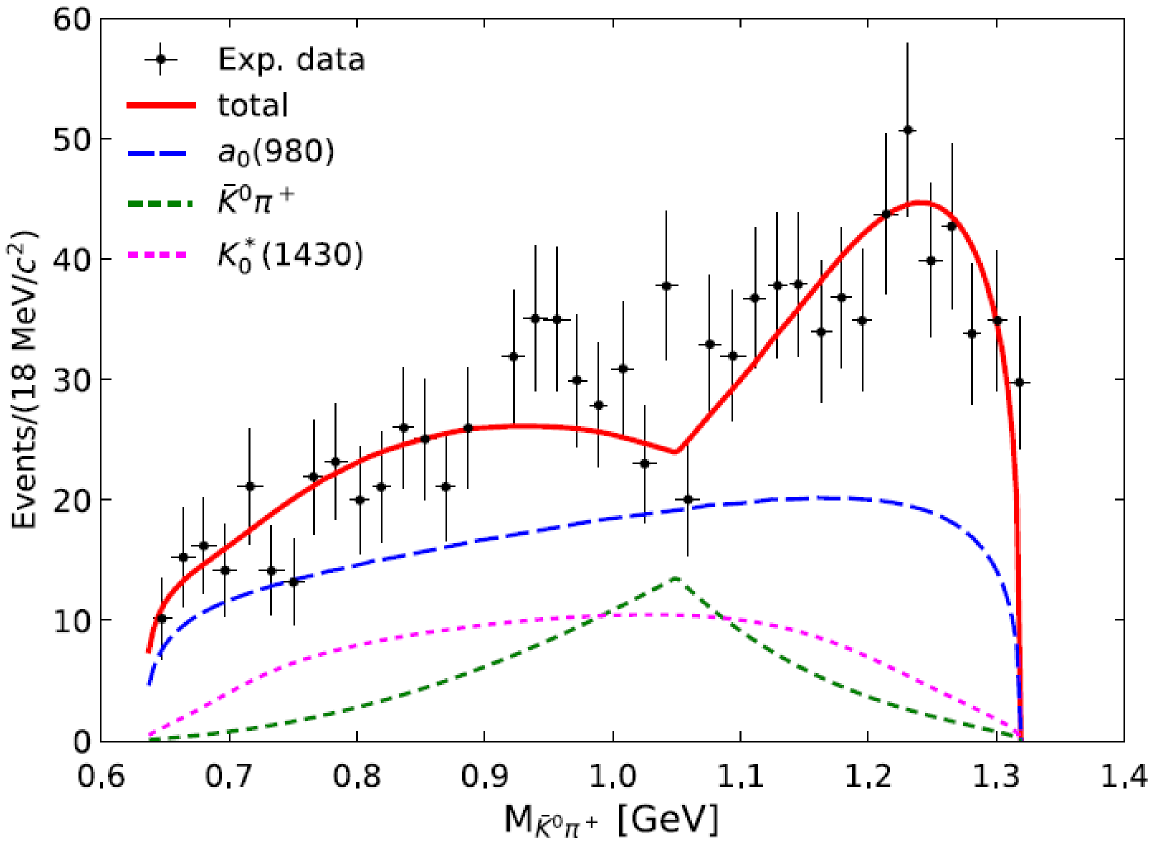
$K_0^*(1430)$ contribution: relatively small

We can claim that the peak observed in the experiment can be identified as the $a_0(980)$ state.

Note that the lineshape of the $a_0(980)$ is broader than those observed in other reactions
<= Due to interference with other contributions, particularly with the tree level

Calculated $\bar{K}^0\eta$, $\bar{K}^0\pi^+$ mass distributions





- We reproduce a double hump structure: the interference between the $K_0^*(1430)$ and $a_0(980)$

- No distinct peak structure.
- $\bar{K}^0\pi^+$ spectrum has a discontinuity at 1.05 GeV because of the cut-off mass

Effect of the cut mass M_{cut}

We use the prescription in high-energies because of the limit of chiral Unitary approach

$$Gt(M_{\rm inv}) = Gt(M_{\rm cut})\ e^{-\alpha(M_{\rm inv}-M_{\rm cut})} \quad \text{for} \quad M_{\rm inv} > M_{\rm cut},$$

- Results for the considered distributions with M_{cut} fixed at 1150 MeV.

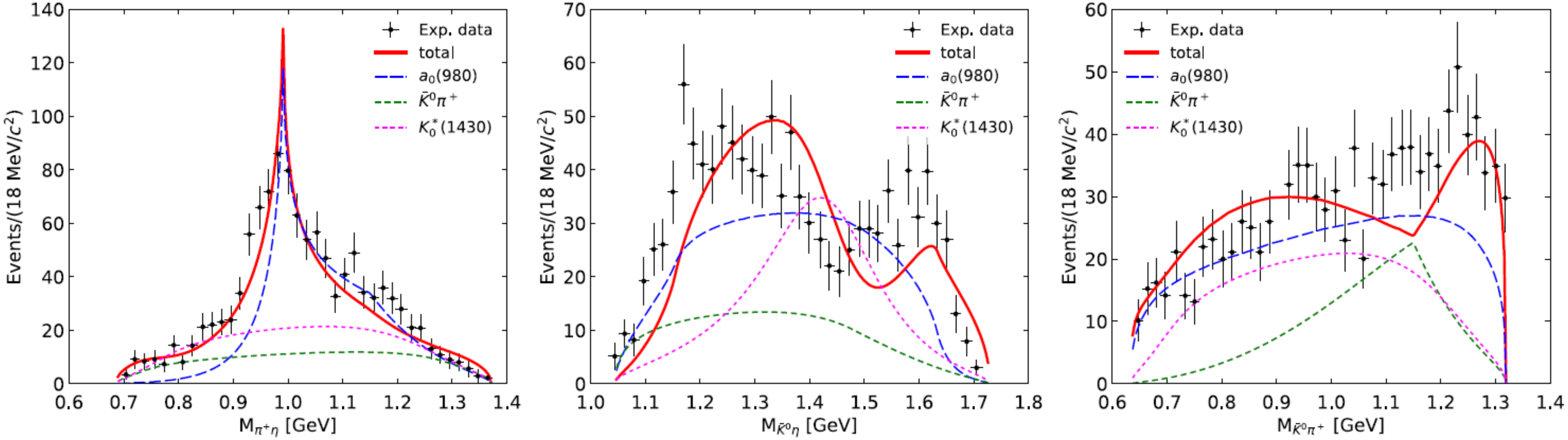


Fig. 13 The mass distributions of $\pi^+\eta$ (left), $\bar{K}^0\eta$ (middle), and $\bar{K}^0\pi^+$ (right) with fixed $M_{\rm cut} = 1150$ MeV. The parameters $\mathcal{C} = 532.04$, $\mathcal{D} = 90.28$, $\beta = 0.70$, and $\phi = -2.07$ radians are used

The dip in the $\bar{K}^0\pi^+$ spectrum has shifted to 1150 MeV ← directly influenced by M_{cut}

⇒ we can conclude that the dip in our model is not physical, and we have a smooth curve in that region.

Effect of the parameter β

β: the relative weight of the internal emission mechanism to the external emission, and is expected the order of 1/Nc

we restrict the value of the β within [−0.33:0.33]

- Results with β = 0.33

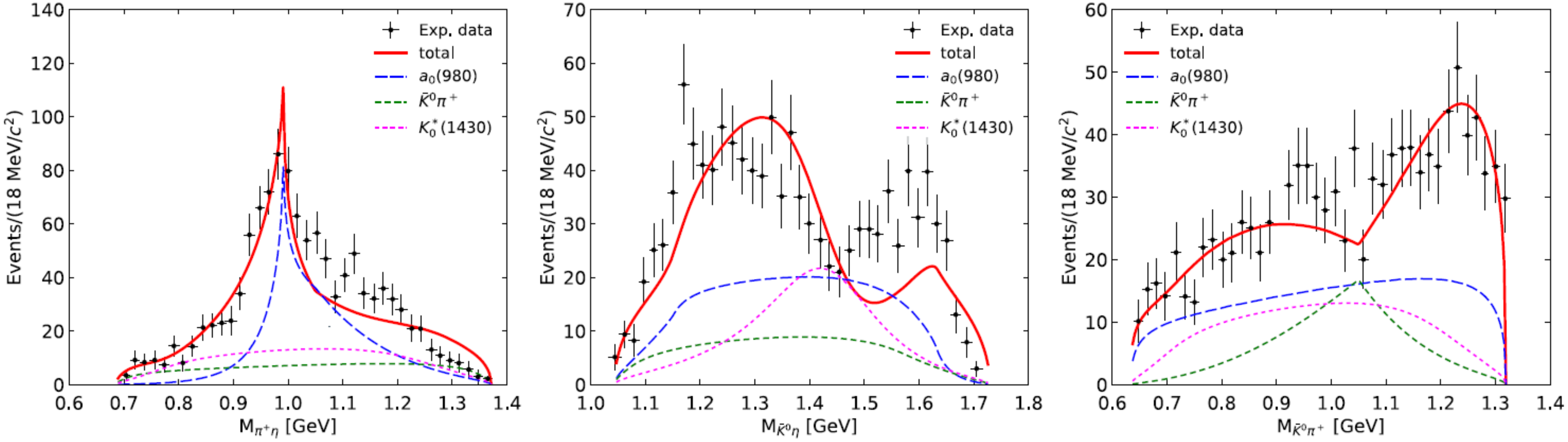


Fig. 14 The mass distributions of $\pi^+\eta$ (left), $\bar{K}^0\eta$ (middle), and $\bar{K}^0\pi^+$ (right) with fixed $M_{\rm cut}$ = 1050 MeV. The parameters $\mathcal{C}$ = 691.80, $\mathcal{D}$ = 71.29, β = 0.33, and ϕ = −2.29 radians are used

we see that the changes are not that big.

Effect of the $K_0^*(1430)$ mass

$K_0^*(1430)$ mass has a relatively large uncertainty $M_{K_0^*} = 1425 \pm 50$ MeV

We take $M_{K_0^*} = 1385$ MeV within error bar

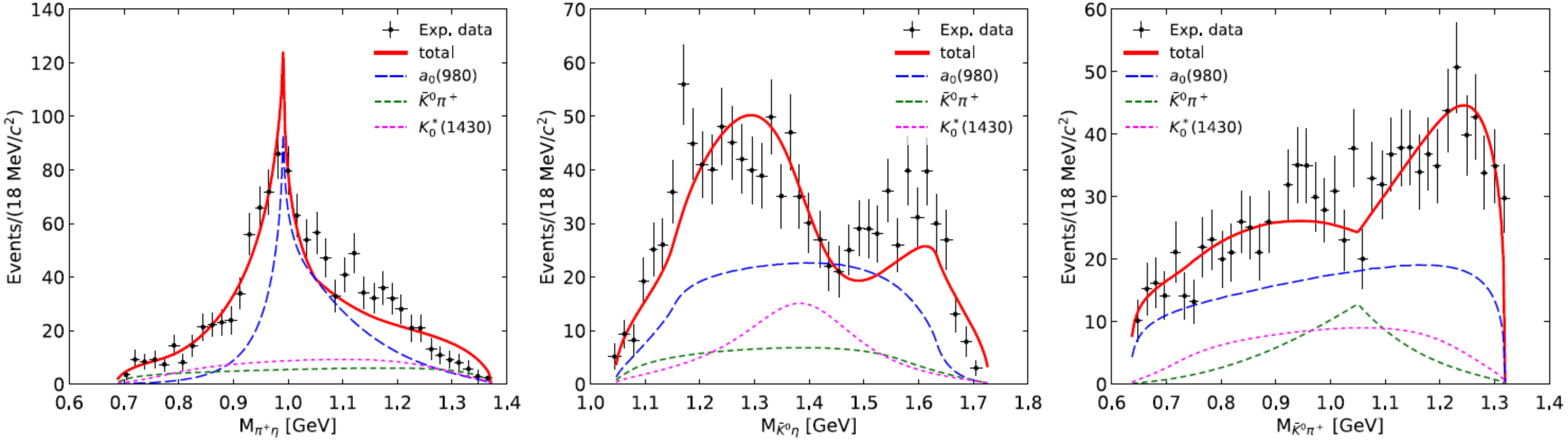


Fig. 15 The mass distributions of $\pi^+\eta$ (left), $\bar{K}^0\eta$ (middle), and $\bar{K}^0\pi^+$ (right) with fixed $M_{K_0^*} = 1385$ MeV and fixed $M_{\text{cut}} = 1050$ MeV. The parameters $\mathcal{C} = 473.34$, $\mathcal{D} = 57.27$, $\beta = 0.70$, and $\phi = -2.39$ radians are used

The resulting calculation of the $\overline{K}^0\eta$ mass distribution is in better agreement with the data because the peak position of $K_0^*(1430)$ has moved a bit to the left from before.

In conclusion, we can see the clear peak of the $a_0(980)$ contribution even considering the uncertainties.

Summary

- We have studied the $D^+ \to \bar{K}^0\pi^+\eta$ reaction based on the picture of $a_0(980)$ resonance as a dynamically generated state from the interaction of the $\pi\eta$, $K\bar{K}$ channels

- We showed that this reaction is drastically different from the apparently analogous one $D^0 \to K^-\pi^+\eta$
  - $D^+ \to \bar{K}^0\pi^+\eta$ reaction: absence of $\bar{K}^{*0}$ contribution
  - $D^0 \to K^-\pi^+\eta$ reaction: $\bar{K}^{*0}$ contribution is the driving term

=> The absence of $\bar{K}^{*0}$ contribution leads to a much cleaner signal of the $a_0(980)$ excitation as seen in the experiment

- We obtained a fair reproduction of the three mass distributions.
  While the $a_0(980)$ production is the dominant term, we also find other terms in the reaction that interfere with this production mode.

Other reactions

$\chi_{c1} \to \eta\pi^+\pi^-$

W. H. Liang, J. J. Xie and E. Oset,
Eur. Phys. J. C 76, 700 (2016)

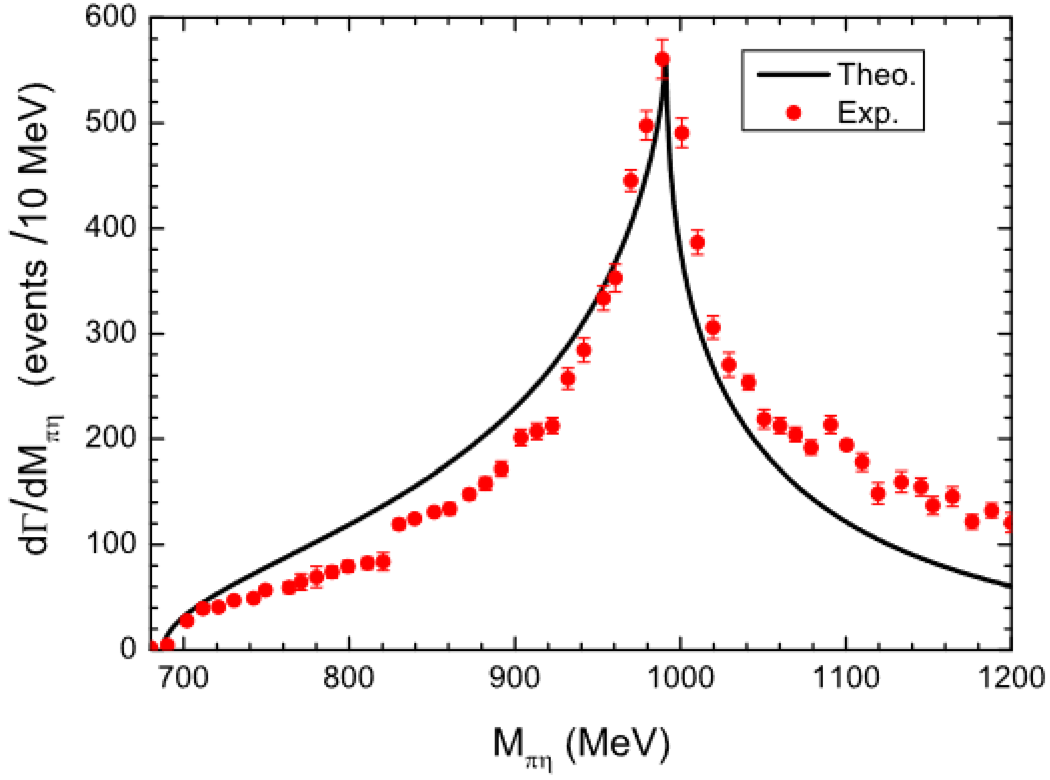


FIG. 6: $\pi\eta$ invariant mass distribution for the $\chi_{c1} \to \eta\pi^+\pi^-$ decay. Preliminary BESIII data from Ref. [3].

$\eta_c \to \eta\pi^+\pi^-$

V. R. Debastiani, W. H. Liang, J. J. Xie and E. Oset,
Phys. Lett. B 766, 59-64 (2017)

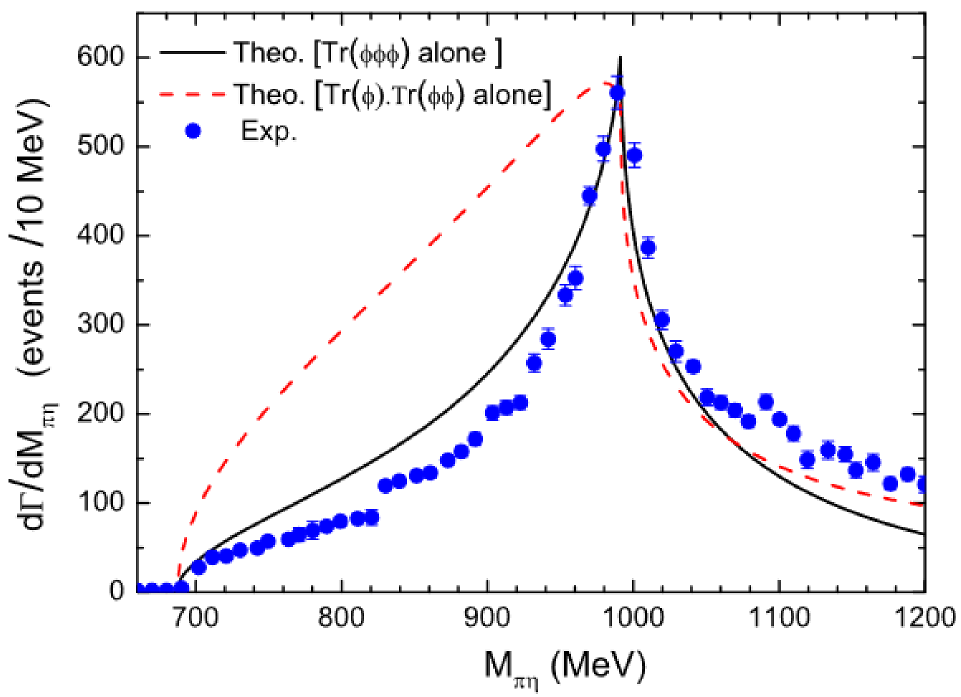
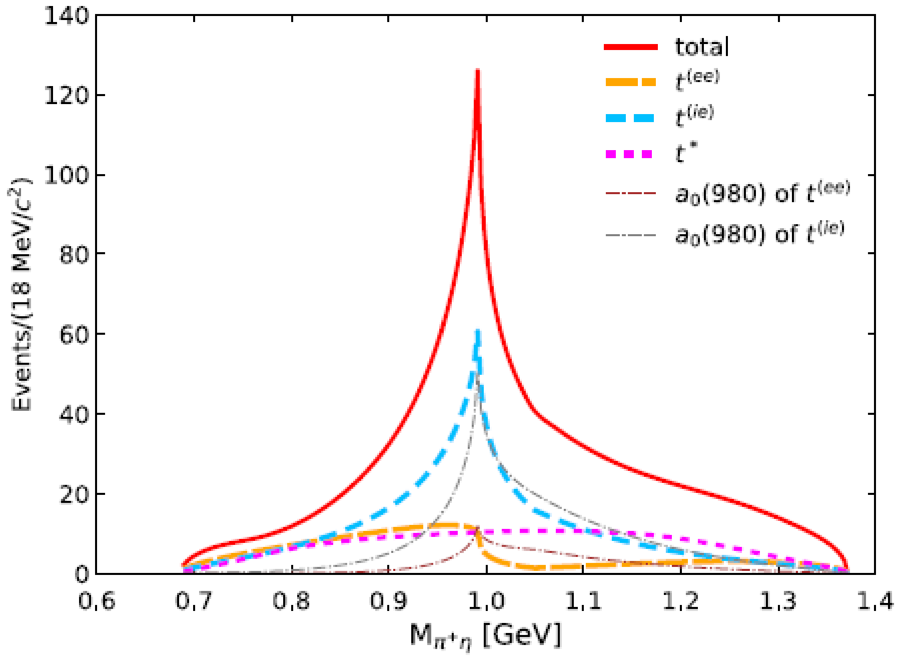
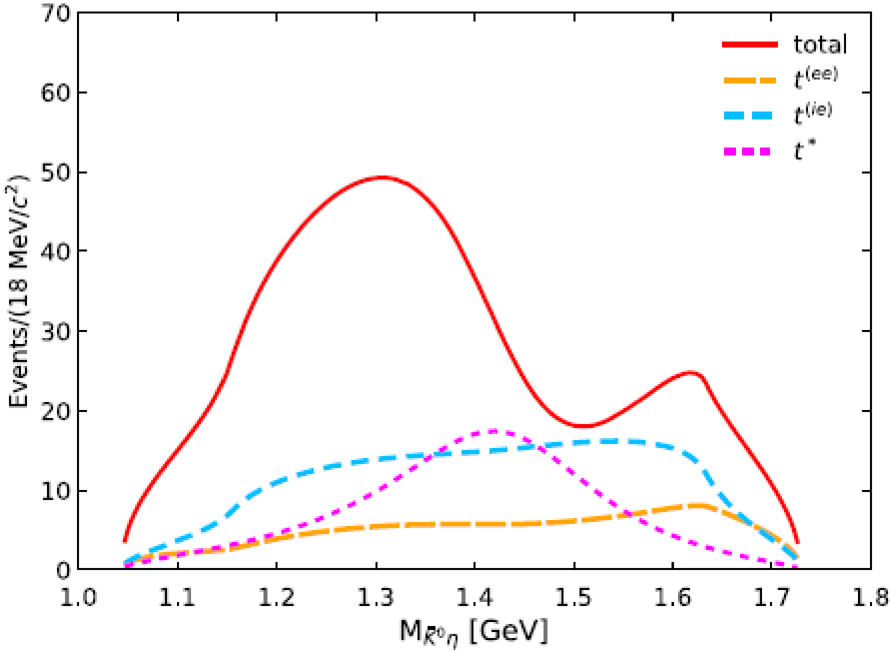


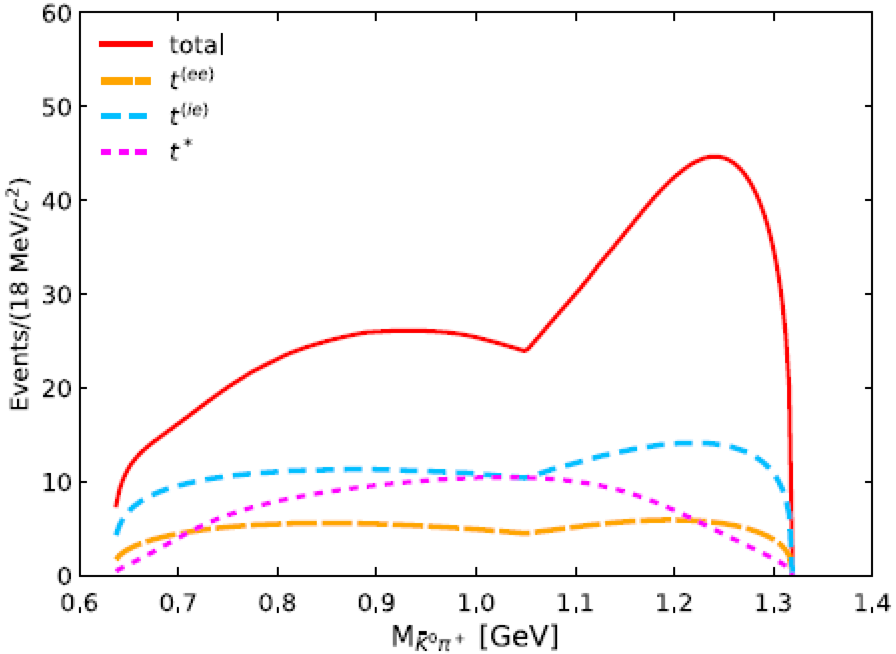
Fig. 2. (Color online.) Results for the $\pi\eta$ mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction. Data from Ref. [23]. Solid curve: results from Ref. [24] using Trace($\phi\phi\phi$). Dashed line: results using Trace(ϕ)Trace($\phi\phi$) normalized to the peak of the distribution.



total
$t^{(ee)}$
$t^{(ie)}$
t^*
$a_0(980)$ of $t^{(ee)}$
$a_0(980)$ of $t^{(ie)}$
Events/(18 MeV/c^2)
$M_{\pi^+\eta}$ [GeV]

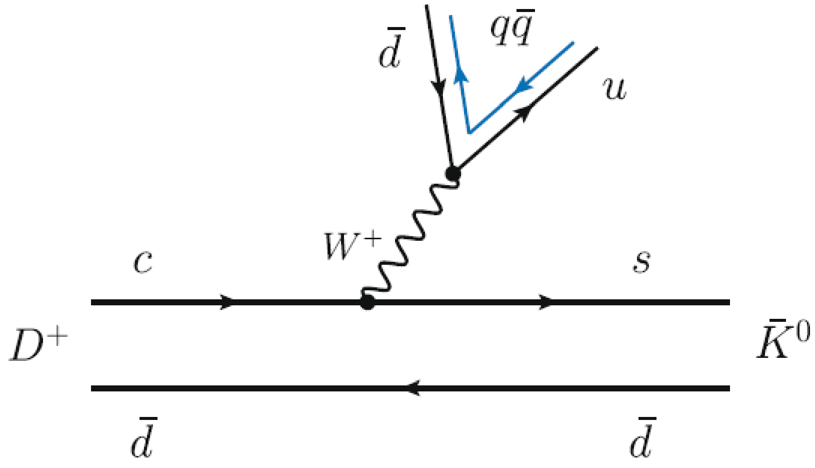


total
$t^{(ee)}$
$t^{(ie)}$
t^*
Events/(18 MeV/c^2)
$M_{\bar{K}^0\eta}$ [GeV]



total
$t^{(ee)}$
$t^{(ie)}$
t^*
Events/(18 MeV/c^2)
$M_{\bar{K}^0\pi^+}$ [GeV]

(b) $u\bar{d}$ pair is hadronized



However, there is a subtlety here concerning the $K^+\bar{K}^0$ production (see discussion in page 3 of Ref. [12]) because for dynamical reasons the $WK^+\bar{K}^0$ vertex goes as the difference of energies of $K^+\bar{K}^0$ which vanishes in the average. However, due to the different masses of $\eta\pi^+$ this cancellation does not occur, and consequently we keep the $\eta\pi^+$ term and disregard the $K^+\bar{K}^0$ one.

Fig. 3a, c. The effective WPP (P pseudoscalar meson) vertex can be evaluated with effective chiral Lagrangians $W^\mu\langle[P, \partial_\mu P]T_-\rangle$ with W^μ the W field and T_- a matrix related to the Cabibbo–Kobayashi–Maskawa elements [28, 29]. If we wish to get the two pseudoscalar mesons in s-wave, which we need to produce the scalar resonances, we get such a contribution with this Lagrangian with $\mu = 0$, which produces a vertex proportional to $p^0 - p'^0$ in the rest frame of W, and hence vanishes for particles with equal mass. This is

The $D^0 \to K_s^0\pi^+\pi^-$, $\bar{K}_s^0\pi^0\eta$

J.J. Xie, L.R. Dai, E. Oset, Phys. Lett. B 742, 363–369 (2015)

They studied prior to the experiments, paying attention to the π+ π− and π0η mass distributions, predicting that a clear signal of the a0(980) should be seen in these experiments