Transverse Momentum Moments

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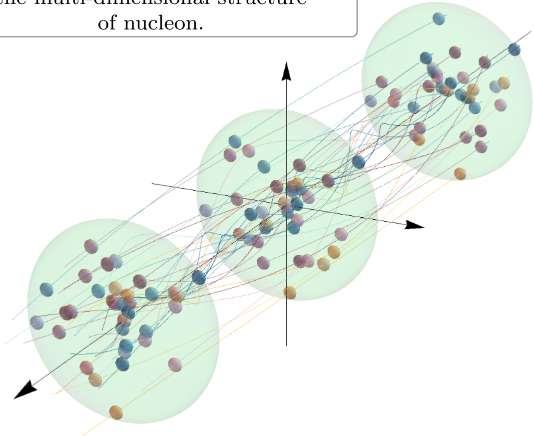
Outline

Review of **Transverse Momentum Dependent (TMD)** distributions
with an emphasis on the connection with collinear observables
via **Transverse Momentum Moments (TMM)**

- What are TMD distributions and how to determine them
- The latest extraction: ART23
- TMM (from 3D to 1D)

Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.



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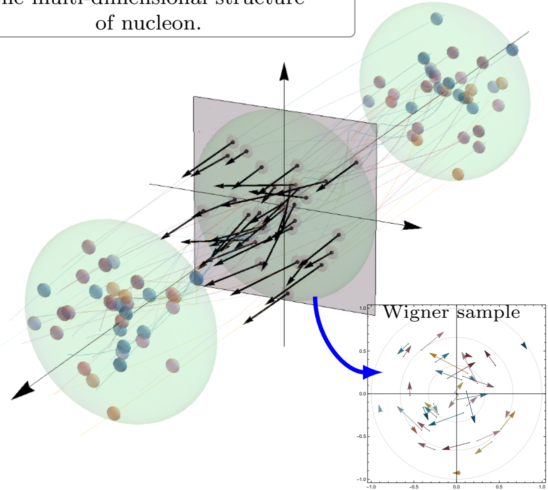

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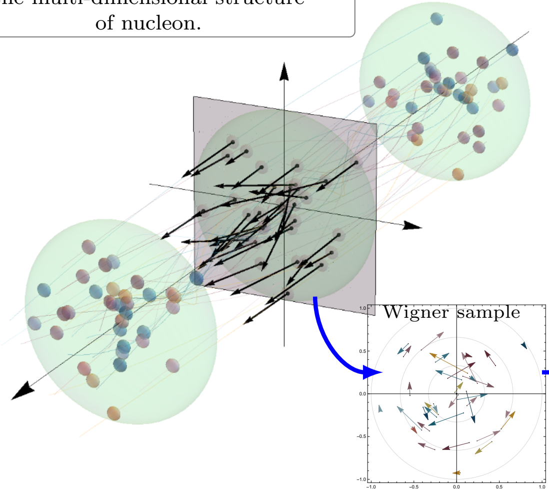
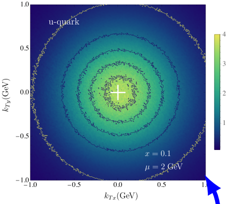
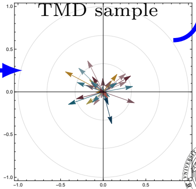


[Bury,Prokudin,AV, PRL 126 (2021)]

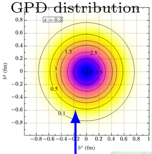


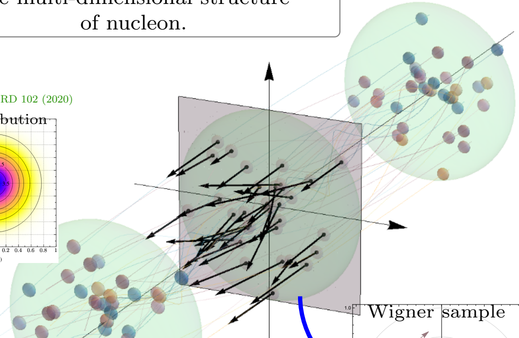


Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.

[Hashamipour,et al, PRD 102 (2020)]





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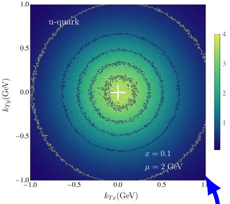
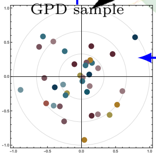
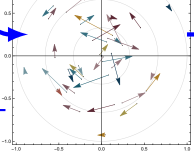
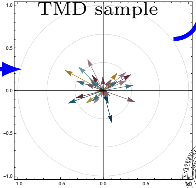






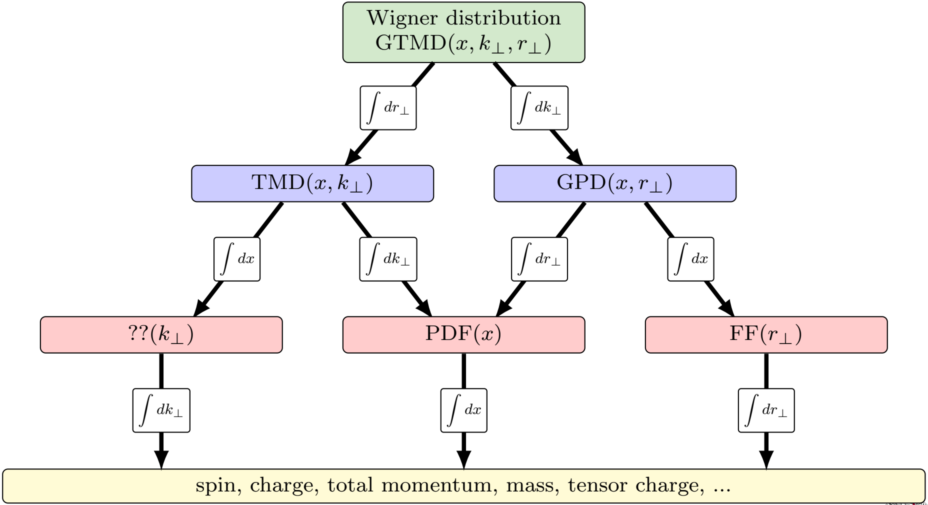


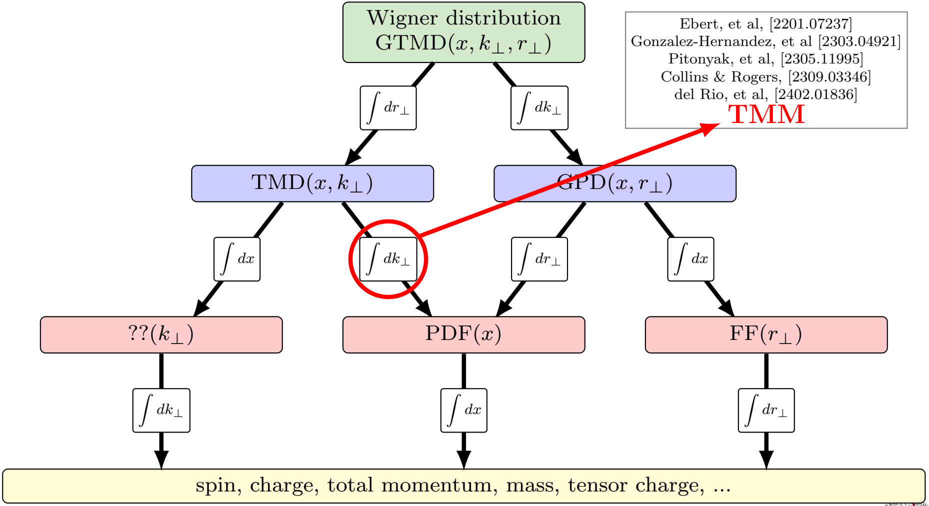
		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ Unpolarized		$h_1^{\perp}(x,k_T^2)$ - Boer-Mulders
	L		$g_1(x,k_T^2)$ - Helicity	$h_{1L}^{\perp}(x,k_T^2)$ - Kozinian-Mulders, "worm" gear
	T	$f_{1T}^{\perp}(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Kozinian-Mulders, "worm" gear	$h_1(x,k_T^2)$ - Transversity $h_{1T}^{\perp}(x,k_T^2)$ - Pretzelosity

$$F(x,k_T)=\int \frac{d\lambda d^2b}{(2\pi)^3} e^{-ix\lambda p_+ - i(bk)_T} \langle p,s|\bar q(\lambda n+b)[\text{staple Wilson line}]\gamma^+ q(0)|p,s\rangle$$

Interpretation as parton densities is “naive”

- TMDs are not positive definite (not theoretically, nor practically)
- Double-scale evolution complicates a lot


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...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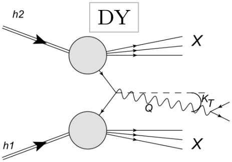
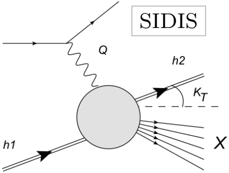
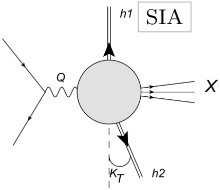
spin, charge, total momentum, mass, tensor charge, ...
Ebert, et al, [2201.07237]
Gonzalez-Hernandez, et al [2303.04921]
Pitonyak, et al, [2305.11995]
Collins & Rogers, [2309.03346]
del Rio, et al, [2402.01836]
TMM

TMD factorization theorem

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$







Main scales:
The invariant mass of photon: $|q^2| = Q^2$
Transverse component of photon momentum: q_T

$$Q \gg \Lambda \qquad Q \gg q_T$$

TMD factorization theorem

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N^{4}LO
(for all structure functions)

Nucleon Polarization	Quark Polarization: Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
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Evolution
N^{4}LO
(for all distributions!)

Double-scale evolution

$$\mu^2 \frac{d}{d\mu^2} \ln F = \gamma_F$$

$$\zeta \frac{d}{d\zeta} \ln F = -\mathcal{D}(b) = \frac{K(b)}{2}$$

$\mathcal{D}$=Collins-Soper kernel

nonperturbative

TMD factorization theorem

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{d\sigma}{dq_T} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bq_T)} C\left(\frac{Q}{\mu}\right) R[\mathcal{D}](b;\mu) F(x_1, b) F(x_2, b)$$

Double-scale evolution

$$\mu^2 \frac{d}{d\mu^2} \ln F = \gamma_F$$

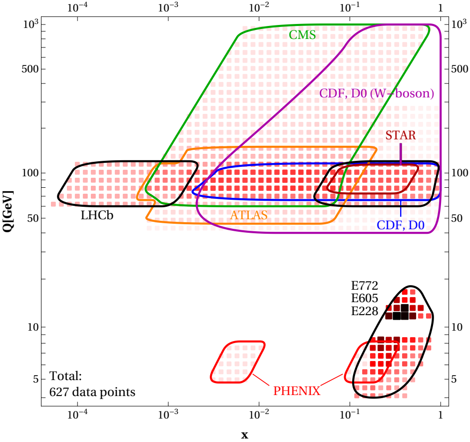
$$\zeta \frac{d}{d\zeta} \ln F = -\mathcal{D}(b) = \frac{K(b)}{2}$$

$\mathcal{D}$=Collins-Soper kernel

nonperturbative

Each point of cross-section is a composition of **three** nonperturbative elements

Unpolarized TMD PDF is the most studied case



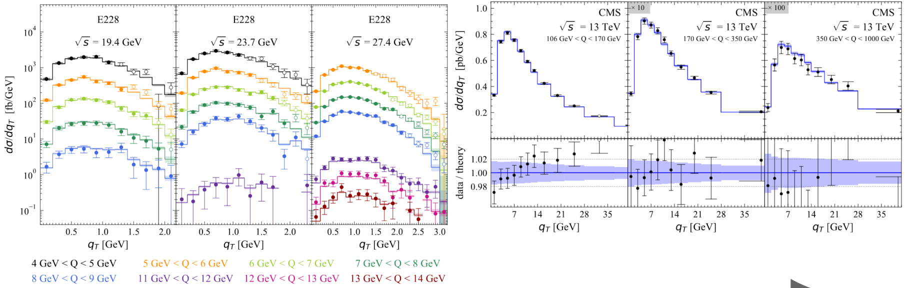
[V.Moos, I.Scimemi,AV,P.Zurita JHEP 05 (2024) 036]

ART23 extraction

- Data
  - Large energy span: $4 < Q < 1000$GeV
  - $q_T < 0.25Q$
- Theory=N^{4}LL
  - N^{4}LO evolution
  - N^{3}LO collinear matching
- Output
  - CS kernel
  - un.TMDPDF
- **artemide**

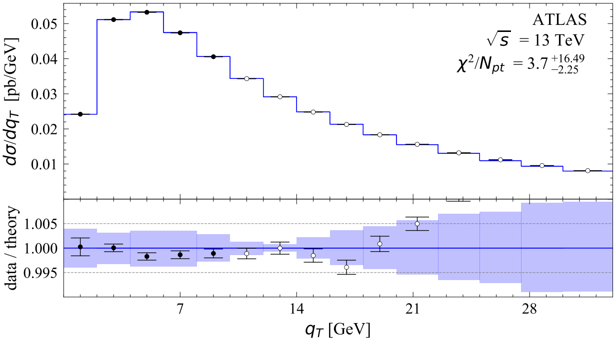
Also **MAP24**, **MAP22**, **SV19** (N^{3}LL +SIDIS)

[A. Bacchetta, et al, JHEP 10 (2022) 127]
[A. Bacchetta, et al, 2405.13833]
[I.Scimemi,AV, JHEP 06 (2020) 137]



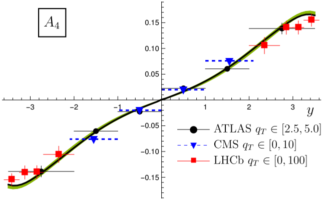
4GeV 1000GeV

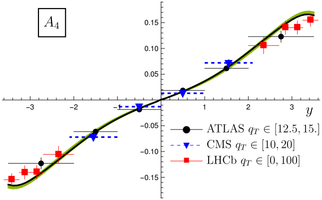
Very presice test of TMD evolution

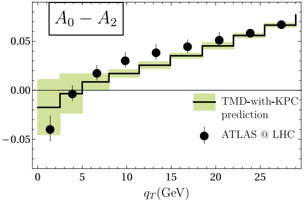


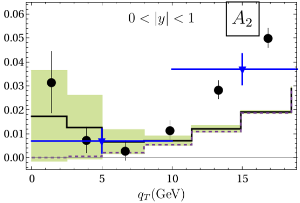
$$\text{TOTAL } (N_{\text{pt}} = 627): \quad \chi^2/N_{\text{pt}} = 0.96^{+0.09}_{-0.01}$$

Z-boson angular distribution at LHC



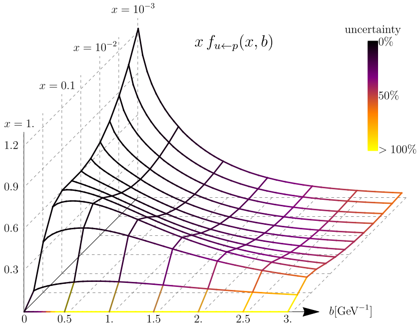






ART23

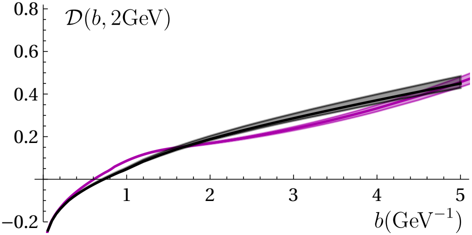
[V.Moos, I.Scimemi, AV, P.Zurita, 2305.07473]



Extra features of analyses:

- Flavor dependent NP-ansatz
  - 2 parameters per flavor
  - u, d, $\bar{u}$, $\bar{d}$, rest
- New parametrization for Collins-Soper kernel (3 parameters)
- Consistent inclusion of the PDF uncertainty
- *artemide*

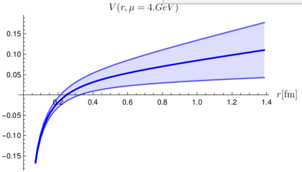
Nonperturbative evolution (Collins-Soper kernel)
Window to the QCD vacuum



CS kernel

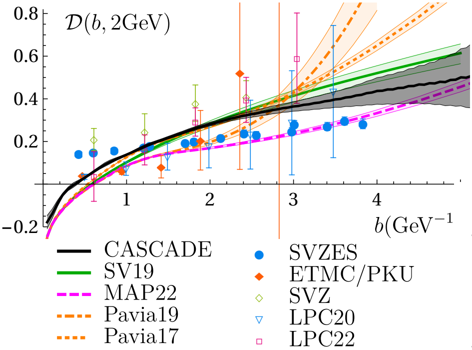
- Perturbative at $b \to 0$
  - N^3LO (4-loops)
- Exclusively sensitive to the QCD vacuum
  - $\sim b^2$ term $\sim \langle GG \rangle$
  - Could be related to the inter-quark potential
  - Interpretation still unclear

AV, Phys.Rev.Lett. 125 (2020)]



$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^\infty \frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2 - r^2}} + \ldots$$

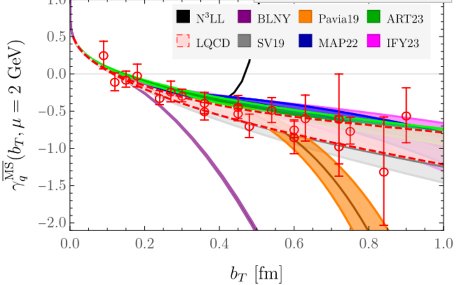
A.Martinez, AV, 2206.01105]



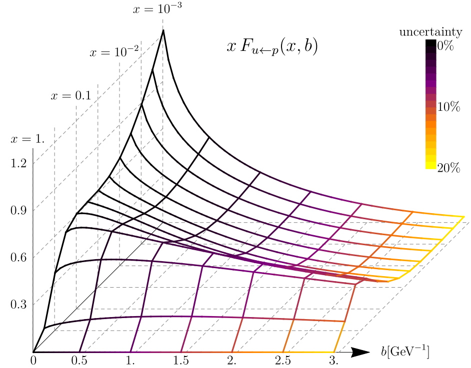
CS kernel

- Perturbative at $b \to 0$
  - N^3LO (4-loops)
- Exclusively sensitive to the QCD vacuum
- Lattice computations

A.Avkhadiev, et al, 2402.06725]



Unpolarized TMDPDF



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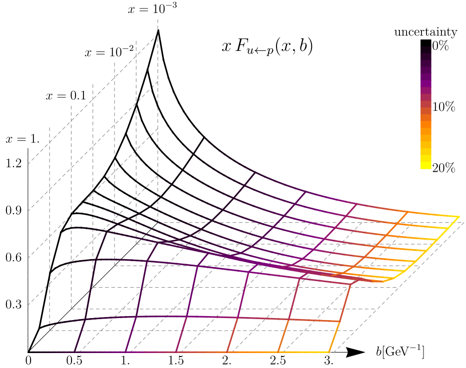

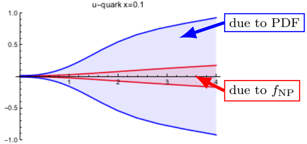

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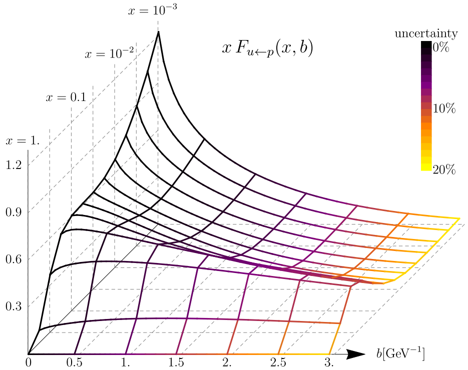
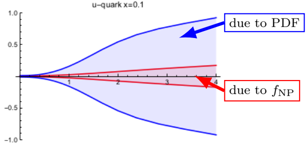
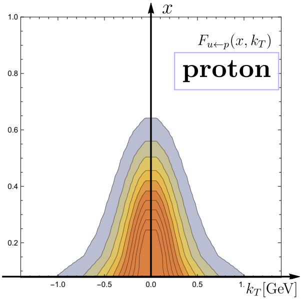
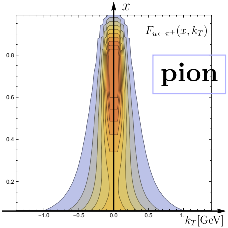
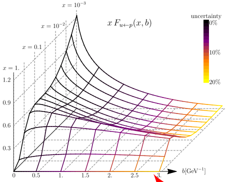




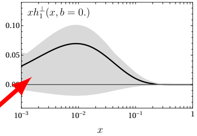


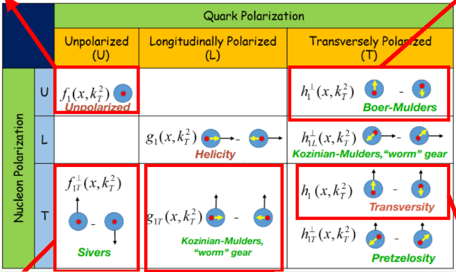




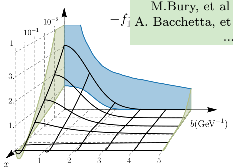
MAP24 [2405.13833]
ART23 [2305.07473]
MAP22 [2206.07598]
SV19 [1912.06532]
...

S.Piloneta,AV



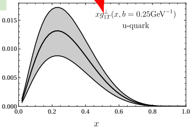


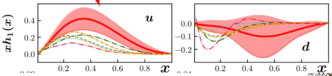
JAM [2002.08384]
M.Radici, et al [1802.05212]



$-f_1$ M.Bury, et al [2103.03270]
A. Bacchetta, et al [2004.14278]
...

M.Horstamnn, et al [2210.07268]
S.Bhattacharya, et al [2110.10253]



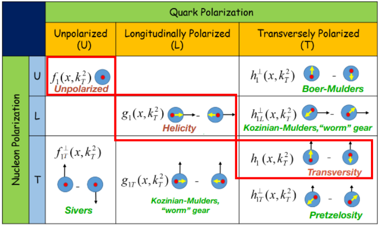


TMDs are 3D

The question is how can we related them to 1D (collinear) distributions

Very Naively:

$$\int d^2k_T F(x, k_T) = f(x)$$



Collinear distribution from TMDs

Naively

$f_1(x, \mathbf{k}_T)$ unpol.	$\int dk_T$ →	$q(x)$ unpol.
$h_1(x, \mathbf{k}_T)$ transver.	$\int dk_T$ →	$\Delta q(x)$ transver.
$f_1^{\perp}(x, \mathbf{k}_T)$ Sivers	$\int dk_T$ →	$T(-x, 0, x)$ Qiu-Sterman
$h_1^{\perp}(x, \mathbf{k}_T)$ Boer-Mulders	$\int dk_T$ →	$E(-x, 0, x)$ e(x)
$\text{TMD}(x, \mathbf{k}_T)$	$\int dk_T$ →	$\text{PDF}(x)$

Collinear distribution from TMDs
Properly

$f_1(x,\mathbf{k}_T,\mu,\zeta)$ unpol. → OPE → $C(x,k_\perp;\mu,\zeta;\mu')\otimes$ N^{3}LO — $q(x,\mu')$ unpol.

$h_1(x,\mathbf{k}_T,\mu,\zeta)$ transver. → OPE → $C(x,k_\perp;\mu,\zeta;\mu')\otimes$ N^{2}LO — $\Delta q(x,\mu')$ transver.

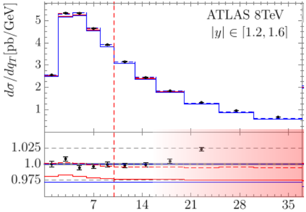
$f_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Sivers → OPE → $C(x,k_\perp;\mu,\zeta;\mu')\otimes$ NLO — $T(-x,0,x,\mu')$ Qiu-Sterman

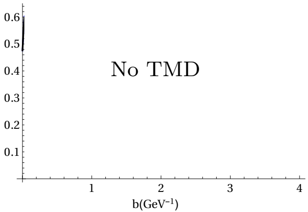
$h_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Boer-Mulders → OPE → $C(x,k_\perp;\mu,\zeta;\mu')\otimes$ NLO — $E(-x,0,x,\mu')$ e(x)

$\text{TMD}(x,\mathbf{k}_T,\mu,\zeta)$ → OPE → $C(x,k_\perp;\mu,\zeta;\mu')\otimes$ — $\text{PDF}(x,\mu')$

1. Coefficient function $\sim \ln^n(\mathbf{k}^2)/\mathbf{k}^2$
2. Three scales: (μ,ζ) in TMD, μ' in collinear PDF

This is also reflected in the data



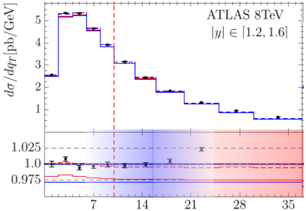


Kinematic ranges:

- Power corrections $q_T \sim Q$

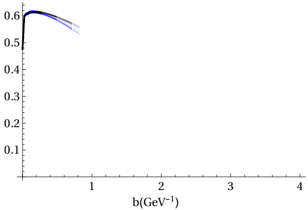
fixed order

$$F(x,b) \sim f(x) \to f(x)\delta(k_T)$$



Kinematic ranges:

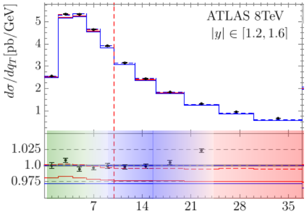
- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$

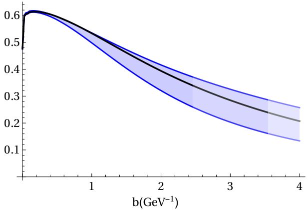


$$F(x,b) = C(x, \ln(b)) \otimes f(x) + b^2 \ldots$$

[Moos,AV,2008.01744]

Name	Function	Twist of leading matching	Twist-2 distributions in matching	Twist-3 distributions in matching	Order of leading power coef.function
unpolarized	$f_1(x,b)$	tw-2	$f_1(x)$	–	N^{3}LO (α_s^3)
Sivers	$f_{1T}^{\perp}(x,b)$	tw-3	–	$T(-x,0,x)$	NLO (α_s^1)
helicity	$g_{1L}(x,b)$	tw-2	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s^1)
worm-gear T	$g_{1T}(x,b)$	tw-2/3	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s)
transversity	$h_1(x,b)$	tw-2	$h_1(x)$	$\mathcal{T}_h(x)$	NNLO (α_s^2)
Boer-Mulders	$h_1^{\perp}(x,b)$	tw-3	–	$\delta T_\epsilon(-x,0,x)$	NLO (α_s)
worm-gear L	$h_{1L}^{\perp}(x,b)$	tw-2/3	$h_1(x)$	$\mathcal{T}_h(x)$	NLO (α_s)
pretzelosity	$h_{1T}^{\perp}$	tw-3/4	–	$\mathcal{T}_h(x)$	LO (α_s^0)





Kinematic ranges:

- Power corrections $q_T \sim Q$
- Resummation $\Lambda \gg q_T \gg Q$
- Nonperturbative $q_T \lesssim \Lambda \sim 2 - 4\text{GeV}$

$$F(x, b) = C(x, \ln(b)) \otimes f(x) f_{\text{NP}}(b)$$

f_{NP} to fit

Most part of high-energy data
is "collinear"

Most part of low-energy data
is "non-perturbative TMD"

We know how to insert "collinear" into TMD
but how extract "collinear" from TMD?

Why would on need it?

- Only unpolarized PDF is well-known, many other PDFs are less-known or unreachable
- TMD can set further constraints on PDF (joined TMD+PDF fits ?)
- How to determine higher moments?

The main problem is the evolution.
TMD distribution obeys CSS-evolution

$$\frac{d}{d\ln\mu}F(x,b;\mu,\zeta)=\gamma_F(\mu,\zeta)F(x,b;\mu,\zeta),\qquad \frac{d}{d\ln\zeta}F(x,b;\mu,\zeta)=\mathcal{D}(b,\mu)F(x,b;\mu,\zeta)$$

PDF obeys DGLAP-evolution

$$\frac{d}{d\ln\mu}f(x,\mu)=\int_x^1\frac{dy}{y}P(y,\mu)f\Big(\frac{x}{y},\mu\Big)$$

$$\int_{-\infty}^{\infty} d^2\mathbf{k}_T F(x,\mathbf{k}_T;\mu,\zeta) = \text{UV divergent}$$

$$\sim \frac{1}{\mathbf{k}_T^2}$$

$$\int_{-\infty}^{\infty} d^2\mathbf{k}_T F(x,\mathbf{k}_T;\mu,\zeta) = \text{UV divergent}$$

$$\sim \frac{1}{\mathbf{k}_T^2}$$

$$\int_{-\infty}^{\infty} d^2\mathbf{k}_T \theta(|\mathbf{k}_T| < \mu') F(x,\mathbf{k}_T;\mu,\zeta) = \tilde{f}(x,\mu,\zeta,\mu')$$

$$\frac{d}{d\ln\mu'}\tilde{f}(x,\mu,\zeta,\mu') = (\text{DGLAP} + (\mu,\zeta))\tilde{f}(x,\mu,\zeta,\mu') + \mathcal{O}(\mu'^{-2})$$

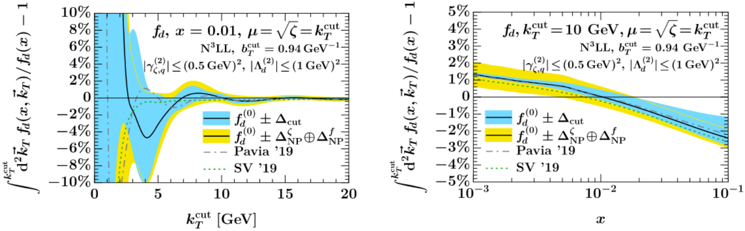
1. $\zeta = \mu^2 = \mu'^2$
2. ζ-prescription

proven at all-orders of PT
[O. del Rio, et al, 2402.01836]

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) \simeq q(x,\mu)$$

[Ebert, et al 2201.07237]
[Conzalez-Hernandez, et al, 2205.05750]

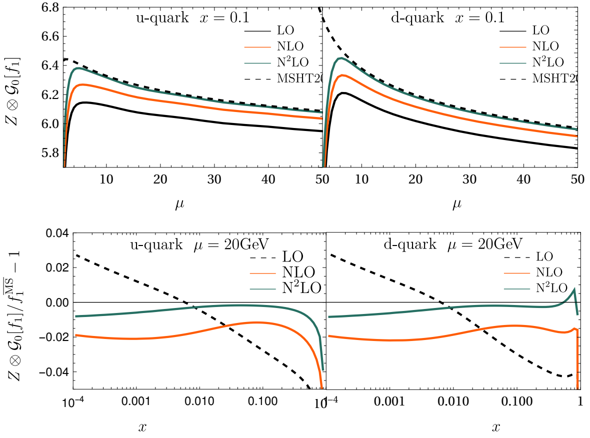


One can restore (tw2) collinear PDF up to few %. **Can we do better?**

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu)\otimes q(x,\mu)$$

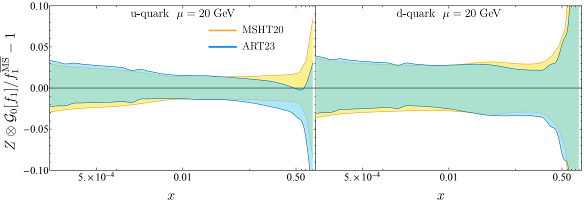
[O. del Rio, et al, 2402.01836]



Collinear distribution from TMDs

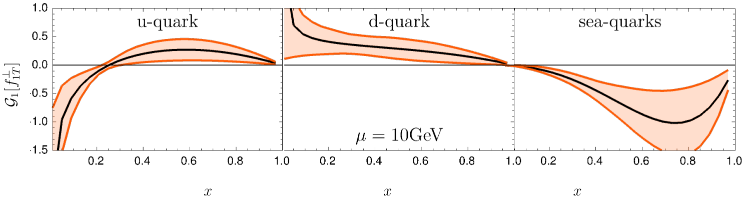
$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu)\otimes q(x,\mu)$$

[O. del Rio, et al, 2402.01836]



0'th TMM ⇒ twist-2
1'th TMM ⇒ twist-3

$T(-x,0,x)$ (from Sivers function [2103.03270])



New theoretically rigorous method to extract twist-3 distributions.

0'th TMM $\Rightarrow$ twist-2
1'th TMM $\Rightarrow$ twist-3
2'th TMM $\Rightarrow$ twist-4

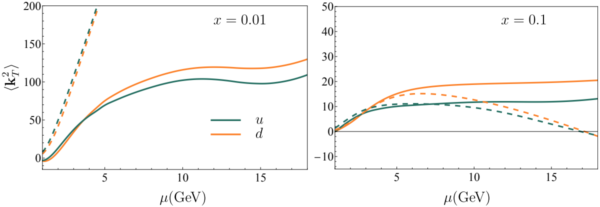
$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x, \mathbf{k}_T; \mu, \mu^2) \simeq \int e^{-ix\lambda p_+} d\lambda \langle p, s|\bar{q}[\lambda n, \infty n] \overleftrightarrow{\mathcal{D}}^2 \gamma^+ [\infty n, 0] q(0)|p, s\rangle$$

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x, \mathbf{k}_T; \mu, \mu^2) \simeq \text{power divergent}$$

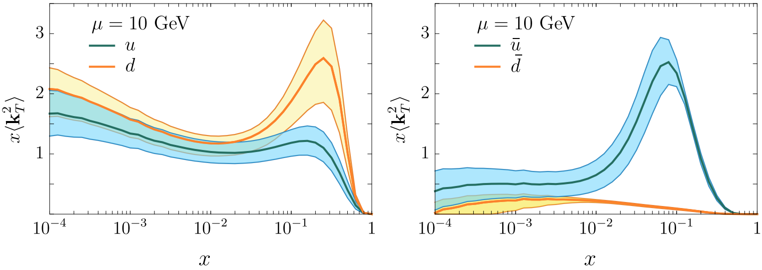
0'th TMM ⇒ twist-2
1'th TMM ⇒ twist-3
2'th TMM ⇒ twist-4

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T;\mu,\mu^2) \simeq \int e^{-ix\lambda p_+} d\lambda \langle p,s|\bar q[\lambda n,\infty n]\overleftrightarrow{\mathcal{D}}^2\gamma^+[\infty n,0]q(0)|p,s\rangle$$

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T;\mu,\mu^2) - \underbrace{\mu^2\text{subtraction term}}_{\text{analytic}} = \langle \bar q D^2 q\rangle_{\overline{\text{MS}}}$$



[O. del Rio, et al, 2402.01836]



$$\int \langle \mathbf{k}^2 \rangle (x)\ dx = \langle \mathbf{k}^2 \rangle$$

$$\langle \mathbf{k}^2 \rangle_{\text{val.q}} = 1.45 \pm 0.55 \text{GeV}^2$$

Conclusion

TMD physics progressed a lot, and became precise science.

It open a window to a set of observables, that were not accesible before.