Meson dominance and gravitational form factor of the pion

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Based on
Gravitational form factors of the pion and meson dominance
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Motivation and Outline

The energy momentum tensor $\Theta_{\mu\nu}$ is the conserved Noether current corresponding to the symmetry under space-time translations

$$x^\mu \to x'^\mu = x^\mu + \epsilon^\mu \implies \phi'(x') = \phi(x) \implies \delta\phi(x) = \epsilon^\mu \partial_\mu \phi$$

The invariance of the Lagrangian gives

$$\delta\mathcal{L}(x) = \epsilon^\mu \partial_\mu \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi}\delta\partial^\mu\phi = \partial^\nu \left[\frac{\partial \mathcal{L}}{\partial \partial^\nu \phi}\right]\delta\phi + \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi}\delta\partial^\mu\phi \implies \epsilon^\nu \partial^\mu \Theta_{\mu\nu} = 0\,,$$

For example for scalar theory

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)^2 - U(\phi) \implies \Theta^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\mathcal{L}$$

The canonical of Noether EMT is NOT always symmetric.
How to measure $\Theta^{\mu\nu}$? Natural way coupling to gravity via a curved space time.
We take the Hilbert or metric EMT

$$\Theta^{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}\Big|_{g^{\mu\nu}=\eta^{\mu\nu}}\,, \qquad \eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \implies \Theta^{\mu\nu} = \Theta^{\nu\mu}$$

Because of derivatives the quantum operator is badly divergent The improved EMT (Coleman+Callan+)

$$\bar\Theta^{\mu\nu} = \Theta^{\mu\nu} - \frac{1}{6}\left[\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2\right]\phi^2 \implies \Theta = \Theta^\mu_\mu$$

has the property that for $U(\phi) = m^2\phi^2/2 + g\phi^4/4!$ with $m=0$ one has scale invariance and a trace anomaly after quantization

$$\bar\Theta = 0 \implies \partial^\mu D_\mu = \Theta^\mu_\mu = \beta(g)\frac{1}{4!}\phi^4$$

Lorentz properties

$$x^\mu \to \Lambda^\mu_\alpha x^\alpha \implies \Theta^{\mu\nu} \to \Lambda^\mu_\alpha \Lambda^\nu_\beta \Theta^{\alpha\beta}$$

The (Hilbert) EMT is conserved and symmetric but not irreducible.

$$\Theta^{\mu\nu} = \Theta^{\nu\mu} . \qquad \partial_\mu \Theta^{\mu\nu} = 0$$

$\implies$ 6 independent componentes.
The trace is a scalar

$$\Theta \equiv \Theta^\mu_\nu$$

A naive decomposition

$$\Theta^{\mu\nu} = \Theta^{\mu\nu}_S + \Theta^{\mu\nu}_T \equiv \frac{1}{4} g^{\mu\nu}\Theta + \left[\Theta^{\mu\nu} - \frac{1}{4} g^{\mu\nu}\Theta\right] \implies \partial_\mu \Theta^{\mu\nu}_S = \partial^\nu \Theta \neq 0$$

A consistent decomposition where two tensor components are conserved separately.

$$\Theta^{\mu\nu} = \Theta^{\mu\nu}_S + \Theta^{\mu\nu}_T$$

with

$$\Theta^{\mu\nu}_S = \frac{1}{6}\left[g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2}\right]\Theta \implies \partial_\mu \Theta^{\mu\nu}_S = 0$$

We will analyze lattice data using the consistent decomposition.

Gravitational Form factors

The EMT has matrix elements between hadronic states (helicity-normality basis) $|pJ\lambda N$

$$\langle p'j'\lambda' N'|\Theta^{\mu\nu}|pj\lambda N\rangle = \sum_i \chi^\dagger_{j'\lambda'} O_i^{\mu\nu}(p',p)\chi^\dagger_{j\lambda} F_i(q^2)$$

The invariant functions $F_i(q^2)$ are the corresponding gravitational form factors.
The spin-0 particle like the pion is the simplest case

$$\langle \pi^a(p')|\Theta^{\mu\nu}(0)|\pi^b(p)\rangle = \delta_{ab}\left[2P^\mu P^\nu A(t) + \tfrac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu}q^2\right) D(t) + 2m_\pi^2 \left(\textstyle\sum_p \bar{c}_p(t) g^{\mu\nu} = 0\right)\right]$$

a, b - isospin, $P = \frac{1}{2}(p' + p)$, $q = p' + p$, $t = q^2 = -Q^2$

$$\Theta^\mu_\mu \equiv \Theta(q^2) = 2\left(m_\pi^2 - \frac{q^2}{4}\right) A(q^2) - \frac{3}{2} q^2 D(q^2). \qquad (1)$$

The rank-two tensor $\Theta^{\mu\nu}$ can be decomposed into a sum of two separately conserved irreducible tensors corresponding to well-defined total angular momentum, $J^{PC} = 0^{++}$ (scalar) and 2^{++} (tensor), namely [?]

$$\Theta^{\mu\nu} = \Theta_S^{\mu\nu} + \Theta_T^{\mu\nu}, \qquad \begin{cases} \Theta_S^{\mu\nu} = \frac{1}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Theta \\ \Theta_T^{\mu\nu} = 2\left[P^\mu P^\nu - \frac{P^2}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\right] A \end{cases}$$

Since Θ and A carry the information on good J^{PC} channels, they should be regarded as the primary objects, whereas the D-term form factor mixes the quantum numbers, with the explicit formula

$$D = -\frac{2}{3t}\left[\Theta - \left(2m_\pi^2 - \tfrac{1}{2}t\right) A\right]. \qquad (2)$$

Mechanistic interpretation

... physical meaning, "force distribution"
[M. Polyakov 2003, Polyakov, Schweitzer 2018, Ji 2021, Lorcé, Metz, Pasquini, Rodini 2021, ...]

$$\frac{\langle \pi_{\rm rest} | \int d^3r \, \Theta^{\mu\nu}(\vec{r}) | \pi_{\rm rest} \rangle}{\langle \pi_{\rm rest} | \pi_{\rm rest} \rangle} = \mathrm{diag}(m_\pi, 0, 0, 0)$$

Balance of pressure, $\int d^3r \, p(r) = 0$ ($p(r)$ must change sign), $\Theta^{00} \to$ distribution of mass

Also, $m_\pi \int d^3r \, r^2 p(r) = D(t=0)$ and for the shear forces $-\frac{4}{15} m_\pi \int d^3r \, r^2 s(r) = D(0)$

D - Druck term
Since $D(0) < 0$, $p(r)$ must change from $+$ at low r to $-$ and high r

QCD

Trace Anomaly

$$\partial^\mu D_\mu = \Theta^\mu_\mu \equiv \Theta = \frac{\beta(\alpha)}{2\alpha} G^{\mu\nu a} G^a_{\mu\nu} + \sum_q m_q \left[1 + \gamma_m(\alpha)\right] \bar{q} q. \tag{3}$$

Here $\beta(\alpha) = \mu^2 d\alpha/d\mu^2$ denotes the beta function, $\alpha = g^2/(4\pi)$ is the running coupling constant, $\gamma_m(\alpha) = d \log m/d \log \mu^2$ is the anomalous dimension of the current quark mass m_q, and $G^a_{\mu\nu}$ is the field strength tensor of the gluon field.
Breakup of hadron mass: (Ji 1995)

$$\Theta^{\mu\nu} = \Theta^{\mu\nu}_q + \Theta^{\mu\nu}_g$$

Scale dependent decomposition.

$$\langle p|\Theta^{\mu\nu}|p\rangle = 2p^\mu p^\nu \left[\langle x\rangle_{\rm q} + \langle x\rangle_{\rm g}\right] \implies \langle x\rangle_{\rm q} + \langle x\rangle_{\rm g} = 1$$

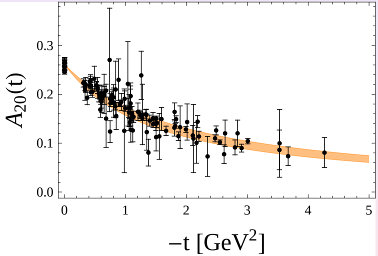
In Deep Inelastic Scattering we have

$$\langle x\rangle^\pi_{\rm val} = \langle x\rangle^N_{\rm val} \sim 0.6 \qquad \mu = 2{\rm GeV}$$

We will not analyze the separate contributuions here

Early estimates

Chiral quark models: $\langle r_2 \rangle_A = \frac{1}{2} \langle r_2 \rangle_{EM}$ - mass distribution more compact than charge [WB, ERA, 2008]

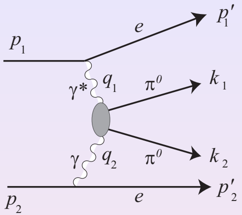


Lattice [Brommel 2007] vs meson dominance [Masjuan, ERA, WB, Phys.Rev.D 87 (2013) 1, 014005]

$(A_{20}(t) \equiv \frac{1}{2} A_q(t)$ - quark part)

At that time $D_q(t)$ very noisy, no gluons

Determination from the Belle data



[Belle, 2015]

[Kumano, Song, Teryaev, 2015] (GDAs, quark parts only) $\rightarrow$

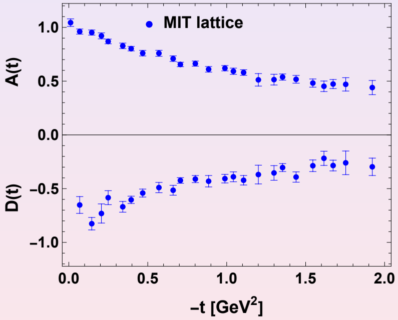
$$\langle r^2\rangle_A = (0.32 - 0.39 \text{ fm})^2$$
$$\langle r^2\rangle_D = (0.82 - 0.88 \text{ fm})^2$$

recall $\langle r^2\rangle_{EM} = (0.656 \pm 0.005 \text{ fm})^2$ (PDG 2021)

(case of A in line with our earlier quark model estimate)

MIT data

[Phys.Rev.D 108 (2023) 11, 114504 & D. Pefkou, PhD Thesis]
Unprecedented accuracy, both quarks and gluons, $m_\pi = 170$ MeV
(below the total q+g used, as it corresponds to the conserved current $\rightarrow$ renorm invariant)



Accuracy allows for more stringent tests and general understanding

Dispersion relations

gffs satisfy dispersion relations. Once-subtracted form:

$$A(s) = 1 + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'}\frac{\mathrm{Im}A(s')}{s'-s-i\epsilon} \qquad D(s) = D(0) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{s}{s'}\frac{\mathrm{Im}D(s')}{s'-s-i\epsilon}$$

pQCD: $\lim_{Q^2\to\infty} A(-Q^2) = \lim_{Q^2\to\infty} D(-Q^2) = 0$ (vanish as $1/Q^2$ mod logs) $\to$

$$0 = 1 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}A(s')}{s'} \qquad 0 = D(0) - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}D(s')}{s'}$$

Correspondingly,

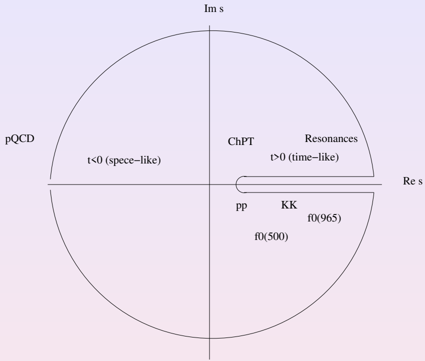
$$\lim_{Q^2\to\infty} \Theta(-Q^2) = 2m_\pi^2 - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\Theta(s')}{s'}$$

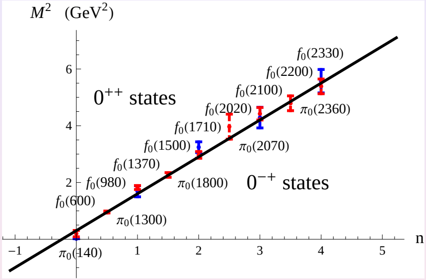
Asymptotically, it goes to a constant (!) mod $\log(Q^2)$ [X.-B. Tong, J.-P. Ma, F. Yuan 2021, 2022]

(but sign change in $D(-Q^2)$)

Absorptive parts

Genericaly





Question of modeling/using the spectral density

$$\rho(s) = \begin{cases} \rho_{\rm ChPT}(s) & 4m_\pi^2 \le s \le 16m_\pi^2 \\ \rho_{\rm R}(s) & 16m_\pi^2 \le s \le \Lambda^2_{pQCD} \\ \rho_{\rm pQCD}(s) & \Lambda^2_{pQCD} \le s \end{cases}$$

Chiral Perturbation Theory

At leading order

$$\Theta(s) = 2m_\pi^2 + s + \mathcal{O}(p^4) \implies \Theta(-Q_0^2) = 0$$

At NLO χPT, the spectral function above the 2π production threshold is (Donoghue:1990xh)

$$\frac{1}{\pi}\mathrm{Im}\Theta(s) = \frac{\sqrt{1-\frac{4m_\pi^2}{s}}\left(2m_\pi^2+s\right)\left(2s-m_\pi^2\right)}{32\pi^2 f_\pi^2} > 0, \tag{4}$$

Contribution to the mass sum rule

$$-\frac{1}{\pi}\int_{4m_\pi^2}^{16m_\pi^2} ds\frac{\mathrm{Im}\Theta(s)}{s} = -0.03\ \mathrm{GeV}^2, \tag{5}$$

which is about 50% of the value at the origin, $\Theta(0) = 2m_\pi^2$, and small compared to the values reached at higher Q^2.

Finite widths at space-like momenta

Energy dependent Breit-Wigner parametrization

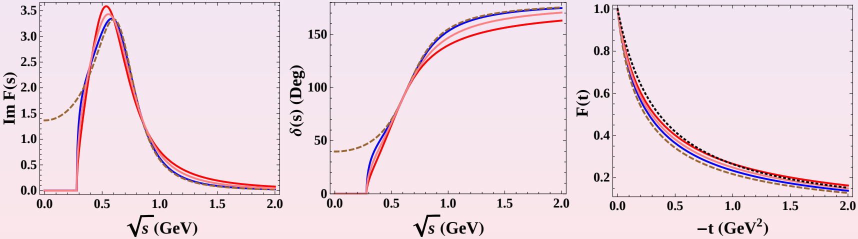
$$N(s) = M^2 - s + i\Gamma M \Gamma(s) \qquad S(s) = e^{2i\delta(s)} = \frac{N(s)}{N(s)^*} \implies \delta(M^2) = \frac{\pi}{2}$$

Resonance = Pole in the second Riemann sheet

$$1/S_{II}(s_R) = S_I(s_R) = 0$$

Omnes representation complies with Watson's theorem

$$F(t) = \exp\left[\frac{t}{\pi}\int_{4m_\pi^2} \frac{ds}{s}\frac{\delta(s)}{s-t}\right] \qquad F(0) = 1 \implies \frac{F(t+i0)}{F(t-i0)} = e^{2i\delta(s)}$$



Even for a broad S-wave resonance the Form Factor resembles a monopole for space-like momenta

$$F(t) \sim \frac{M^2}{M^2 - t}$$

Large N_c

Resonance Saturation with narrow states

$$\langle \pi\pi | \Theta^{\mu\nu} | 0 \rangle = \sum_R \langle \pi\pi | R \rangle \frac{1}{m_R^2 - q^2} \langle R | \Theta^{\mu\nu} | 0 \rangle, \qquad (6)$$

Problems with subtractions for higher spin $J \geq 2$ particles.
We take the absortive part and use the disersion relation with pertinent subtractions

$$\frac{1}{\pi} \mathrm{Im} \langle \pi\pi | \Theta^{\mu\nu} | 0 \rangle = \sum_R \langle \pi\pi | R \rangle \langle R | \Theta^{\mu\nu} | 0 \rangle \delta(m_R^2 - s), \qquad (7)$$

$$\begin{aligned} \langle S | \Theta^{\mu\nu} | 0 \rangle &= f_S (g^{\mu\nu} q^2 - q^\mu q^\nu)/3, \\ \langle T | \Theta^{\mu\nu} | 0 \rangle &= f_T m_T^2 \epsilon_\lambda^{\mu\nu}, \end{aligned} \qquad (8)$$

where $\epsilon_\lambda^{\mu\nu}$ is the spin-2 polarization tensor, which is symmetric $\epsilon_\lambda^{\mu\nu} = \epsilon_\lambda^{\nu\mu}$, traceless $g_{\mu\nu} \epsilon_\lambda^{\mu\nu} = 0$, and transverse $q_\mu \epsilon_\lambda^{\mu\nu} = 0$. The extra factor 3 in the definition is conventional such that $\langle S | \Theta | 0 \rangle = f_S m_S^2$. The *on-shell* couplings of the resonances to the $\pi\pi$ continuum are taken as

$$\begin{aligned} \langle S | \pi\pi \rangle &= g_{S\pi\pi}, \\ \langle T | \pi\pi \rangle &= g_{T\pi\pi} \epsilon_\lambda^{\alpha\beta} P^\alpha P^\beta = g_{T\pi\pi} \epsilon_\lambda^{\alpha\beta} p'^\alpha p^\beta. \end{aligned} \qquad (9)$$

Thus, we get

$$\frac{1}{\pi} \mathrm{Im} \langle \pi\pi | \Theta^{\mu\nu} | 0 \rangle = \sum_S \frac{g_{S\pi\pi} f_S}{3} \delta(m_S^2 - q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + \sum_{T,\lambda} \epsilon_\lambda^{\alpha\beta} P^\alpha P^\beta \epsilon_\lambda^{\mu\nu} g_{T\pi\pi} f_T \delta(m_T^2 - q^2), \qquad (10)$$

which naturally complies with separate conservation for each contribution when contracting with q^μ.

Large N_c (II)

The sum over tensor polarizations is given by

$$\sum_\lambda \epsilon_\lambda^{\alpha\beta} \epsilon_\lambda^{\mu\nu} = \frac{1}{2}\left(X^{\mu\alpha}X^{\nu\beta} + X^{\nu\alpha}X^{\mu\beta}\right) - \frac{1}{3}X^{\mu\nu}X^{\alpha\beta}, \quad (11)$$

with $X^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$, hence the on-shell condition $P \cdot q = 0$ implies $P_\alpha X^{\alpha,\beta} = P^\beta$ and we get

$$\sum_\lambda \epsilon_\lambda^{\alpha\beta} P_\alpha P_\beta \epsilon_\lambda^{\mu\nu} = P^\mu P^\nu - \frac{1}{3}(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})P^2 \quad (12)$$

(cf. the tensor structure in Eq. (**??**)). Therefore, in the narrow resonance, large-N_c motivated approach

$$\begin{aligned} \frac{1}{\pi}\mathrm{Im}A(s) &= \frac{1}{2}\sum_T g_{T\pi\pi} f_T \delta(m_T^2 - q^2), \\ \frac{1}{\pi}\mathrm{Im}\Theta(s) &= \sum_S g_{S\pi\pi} f_S m_S^2 \delta(m_S^2 - q^2), \end{aligned} \quad (13)$$

where, as expected, A and Θ get contributions exclusively from the 2^{++} and 0^{++} states, respectively.

Perturbative QCD

At large momenta one has

$$\langle \pi(p')| \frac{\beta(\alpha)}{4\alpha} G^{\mu\nu 2}(0) |\pi(p)\rangle \sim 16\pi\beta\left(\alpha(-Q^2)\right) f_\pi^2, \qquad (14)$$

$$\alpha(s) \equiv \alpha(s+i\epsilon) = \left(\frac{4\pi}{\beta_0}\right)\frac{1}{L-i\pi}, \qquad (15)$$

yielding a positive imaginary part $\mathrm{Im}\,\alpha(s+i\epsilon)^2 = (4\pi/\beta_0)^2 2\pi L/(L^2+\pi^2)^2$, hence

$$\frac{1}{\pi}\mathrm{Im}\Theta(s) = -\left(\frac{4\pi}{\beta_0}\right)^2 \frac{4\beta_0 L f_\pi^2}{(L^2+\pi^2)^2} + \mathcal{O}(\alpha^3). \qquad (16)$$

This implies that $\mathrm{Im}\Theta(s) < 0$ at large energies, which is the desired result. After computing the integral, the contribution to sum rule (**??**) is

$$-\frac{1}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\Theta(s)}{s} = 4\beta_0[\alpha(-\Lambda^2)]^2 f_\pi^2 + \mathcal{O}(\alpha^3), \qquad (17)$$

which for $\Lambda^2 \sim 20\Lambda_{\mathrm{QCD}}^2 \sim 1\mathrm{GeV}^2$ is about $5f_\pi^2 \sim 0.04\ \mathrm{GeV}^2$, small

Large N_c and meson dominance

t Hooft, Witten: At large-N_c, amplitudes are saturated by tree-level diagrams with towers of mesons in intermediate states $\rightarrow$

$$\mathrm{Im}A(s) = \sum_T c_T M_T^2 \pi \delta(M_T^2 - s), \quad \mathrm{Im}\Theta(s) = \sum_S c_S M_S^4 \pi \delta(M_S^2 - s)$$

$+$ dispersion relations $\rightarrow$

$$A(-Q^2) = 1 - \sum_T \frac{c_T Q^2}{M_T^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \sum_S \frac{c_S Q^2 M_S^2}{M_S^2 + Q^2}$$

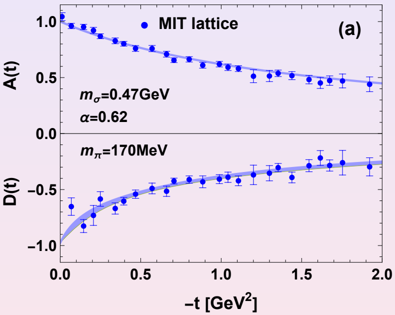
$\sum_T c_T = 1$ (since $A(-\infty) = 0$ and (in the chiral limit) $\sum_S c_S = 1$ ($\Theta'(0) = 1$)

We take one tensor meson, $f_2(1275)$, and two scalar mesons, $f_0(975)$ and σ [see talk by R. Kamiński]

$$A(-Q^2) = \frac{m_{f_2}}{m_{f_2}^2 + Q^2}, \quad \Theta(-Q^2) = 2m_\pi^2 - \alpha \frac{Q^2 m_\sigma^2}{m_\sigma^2 + Q^2} - (1-\alpha)\frac{Q^2 m_{f_0}^2}{m_{f_0}^2 + Q^2}$$

Formula for $D(-Q^2)$ follows. Only m_σ and α are fitted.

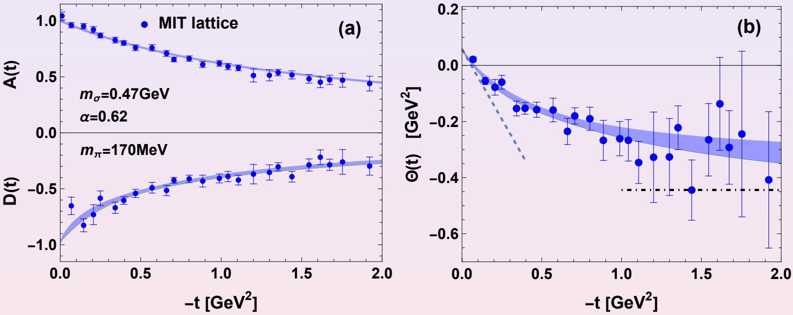
Result of the fit



band width in D - 68% CL

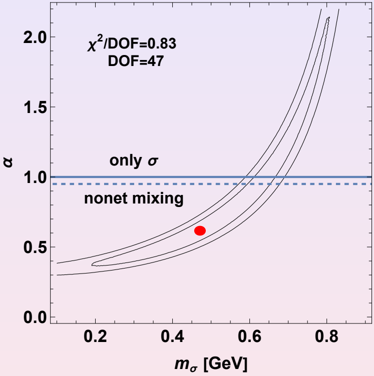
Result of the fit

(our way of taking advantage of the MIT data)



for Θ, errors added in quadrature, lines: $2m_\pi^2 + t$ and asymptotics (constant!)

Fit parameters



Strong correlation between m_σ and $\alpha \rightarrow$ ambiguity, more information needed

Properties of the model

$$D(0) = -1 + \frac{4m_\pi^2}{2m_{f_2}^2} + \mathcal{O}(m_\pi^2), \quad \langle r^2 \rangle_A = \frac{6}{m_{f_2}^2}, \quad \langle r^2 \rangle_D = \frac{4}{m_{f_2}^2} + \frac{2\alpha}{m_\sigma^2} + \frac{2(1-\alpha)}{m_{f_0}^2} + \mathcal{O}(m_\pi^2)$$

Numerically (at the lattice value $m_\pi = 170$ MeV)

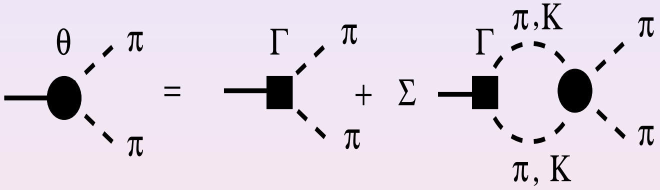
$$D(0) = -0.976, \quad \langle r^2 \rangle_A = (0.38\ \mathrm{fm})^2, \quad \langle r^2 \rangle_D = (0.74\ \mathrm{fm})^2$$

(radii in line with Kumano et al.)

Hadronic representacion

For two coupled channels

$$\begin{pmatrix}\Theta_\pi(s)\\ \Theta_K(s)\end{pmatrix} = \begin{pmatrix}\Gamma_\pi(s)\\ \Gamma_K(s)\end{pmatrix} + \begin{pmatrix}T_{\pi\pi\to\pi\pi}(s) & T_{\pi\pi\to KK}(s)\\ T_{KK\to\pi\pi}(s) & T_{KK\to KK}(s)\end{pmatrix} + \begin{pmatrix}\Delta_{\pi\pi}(s) & 0\\ 0 & \Delta_{KK}(s)\end{pmatrix}\begin{pmatrix}\Gamma_\pi(s)\\ \Gamma_K(s)\end{pmatrix}$$



Watson's final state theorem

$$F = \Gamma + VG_0F = \Gamma + TG_0\Gamma \implies \mathrm{Im}F(s) = \mathrm{Im}\left[T(s)G_0(s)\right]\Gamma(s) \implies F(t) = F(0) + \frac{1}{\pi}\int_{s_0}^{\infty} ds\frac{t}{s}\frac{\mathrm{Im}F(s)}{s-t}$$

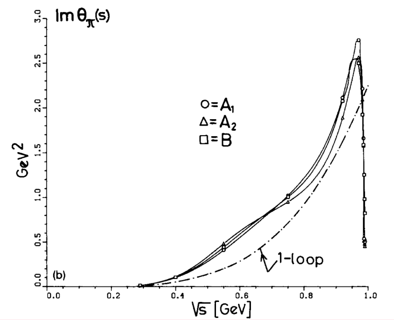
The poles of the FF in the second Riemann sheet coincide with the resonances of the S-matrix.

$$\Theta_{\mathrm{II}}(s) = S_{\mathrm{II}}(s)\Theta_{\mathrm{I}}(s)$$

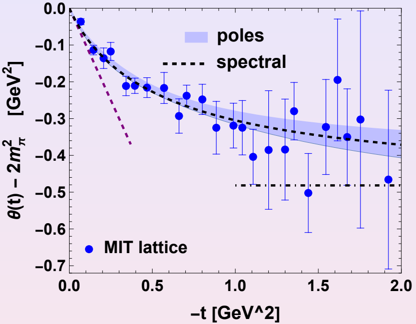
Scalar spectral function

$$\Theta(-Q^2) = 2m_\pi^2 - \frac{Q^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\Theta(s)}{s(s+Q^2)}$$

Use, e.g., the spectral density from physical (CERN-Munich) phase shifts [Donoghue, Gasser, Leutwyler, Nucl.Phys.B 343(1990)341, Fig. 3] (Watson's theorem, Omnès-Muskhelishvili coupled equations)

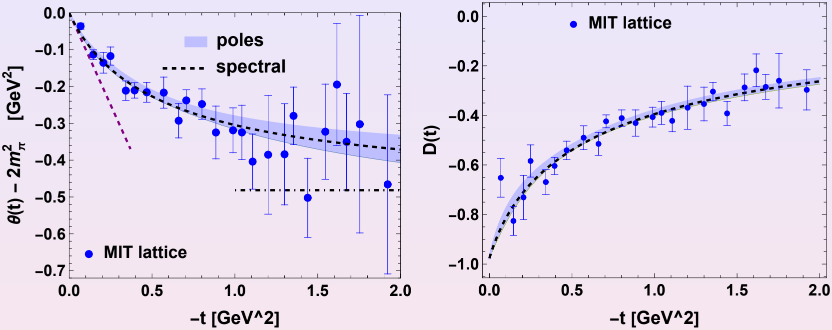


Spectral modeling vs lattice



(here we subtract $2m_\pi^2$, as $m_\pi = 170$ MeV for the data and 140 MeV for the spectral modeling)

Spectral modeling vs lattice



Lattice consistent with "meson physics", also and in particular in the scalar channel

Conclusions

1. Lattice results for gravitational ff of the pion fully compatible with meson dominance at “intermediate” values of Q^2
2. Important to look at the data in good spin channels - all expected features satisfied: $\Theta(0) = 2m_\pi^2$, $\Theta'(0) = 1 + \mathcal{O}(m_\pi^2)$, or $D(0) = -1 + \mathcal{O}(m_\pi^2)$
3. $D(t)$ (the Druck term) is a combination of good spin form factors
4. Tensor channel: just $f_2(1275)$
5. Scalar channel: $\Theta(-Q^2)$ goes to a negative constant at accessible values of Q^2. Higher Q^2 desired approach pQCD ... Modeling involves the broad σ meson!
6. For the nucleon similar story expected, cf. [Masjuan, ERA, WB, 2013]

One sees mesons all over the lattice!