HEAVY-LIGHT PSEUDOSCALAR MESONS:
LIGHT-FRONT WAVE FUNCTIONS AND GENERALIZED PARTON DISTRIBUTIONS

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We study the internal structure of pseudoscalar mesons with heavy-light quark content through an algebraic model within the framework of the light-front formalism.

We have a relation between the light-front wave function (LFWF) and the parton distribution amplitude (PDA) of hadrons.

The purpose is the (analytic) extraction of electromagnetic form factors (FFs) and parton distribution functions (PDFs) through generalized parton distribution functions (GPDs).
What we talk about?

- We study the **internal structure** of pseudoscalar mesons with heavy-light quark content through an **algebraic model** within the framework of **the light-front formalism**.

- We have a relation between the **light-front wave function** (LFWF) and the **parton distribution amplitude** (PDA) of hadrons.

- The purpose is the (analytic) extraction of **electromagnetic form factors** (FFs) and **parton distribution functions** (PDFs) through **generalized parton distribution functions** (GPDs).

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**Algebraic model to study the internal structure of pseudoscalar mesons with heavy-light quark content**

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*Phys. Rev. D* **109**, 014016 – Published 17 January 2024
Non-perturbative QCD: Unraveling the hadron structure from the fundamental d.o.f is an outstanding problem.

Parton Model:
- Hadrons can be considered as a collection of point-like constituents.
- Is based on the concept of PDFs and used to interpret DIS data.

GPDs provide a picture of the spatial and momentum distributions of quarks and gluons.
Light-Front Wave Function

**Probability amplitude that a hadron with momentum** $P^+$ **consists of partons with physical momentum** $p^+$ **and** $p_\perp$.

- Light-front formalism exploits dynamical evolution in the light-front time $x^+$.
- LFWF depend only on the relative variables:

$$x = \frac{p^+}{P^+}$$

$$k_\perp = p_\perp - xP_\perp$$
Many distributions are related via the leading-twist LFWF

Distribution Amplitudes:

\[ f^u_M(x, \zeta_H) = \int dk_\perp \psi^*_M(x, k_\perp^2, \zeta_H) \]

Distribution Functions:

\[ u_M(x, \zeta_H) = \int dk_\perp |\psi_M(x, k_\perp^2, \zeta_H)|^2 \]

In the DGLAP kinematic domain, the valence-quark GPD:

\[ \mathcal{H}^u_M(x, \xi, t) = \int dk_\perp \psi^*_M(x^-, k_\perp^{-2})\psi_M(x^+, k_\perp^{+2}) \]
Generalised Parton Distributions

**Probabilistic interpretation:**

*Probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction “x” of the hadron’s averaged light-cone momentum.*

“hadron-parton” amplitudes which depend on three variables $(x, \xi, t)$

- **x:** average momentum fraction carried by the active parton
- **\(\xi\):** skewness parameter
- **t:** the Mandelstam variable (four-momentum transfer)

- **Structure mapped in terms of:**
  - $b_\perp = \text{transverse position}$
  - $k_\perp = \text{transverse momentum}$
3-dimensional picture of Hadrons

Elastic Scattering

Deep Inelastic Scattering

Deeply Virtual Compton Scattering

FFs

PDFs

GPDs
Compute everything from the LFWF.

The internal dynamics of a meson can be described via its Bethe-Salpeter wave function:

$$\chi_M(k, P) = S_q(k_+) \Gamma_M(k, P) S_{\bar{h}}(k_-)$$

Quark propagator and Bethe-Salpeter amplitude:

$$S(k) = \frac{-i \gamma \cdot k + M}{k^2 + M^2},$$

$$N_M \Gamma_M(k) = i \gamma_5 \int_{-1}^{1} dw \rho(w) \left[ \frac{\Lambda_w^2}{k_w^2 + \Lambda_w^2} \right]^\nu$$
Light-Front Wave Function

- **Bethe-Salpeter Amplitude**  
  \[ N_M \Gamma_M(k) = i\gamma_5 \int_{-1}^{1} dw \rho(w) \left[ \frac{\Lambda_w^2}{k_w^2 + \Lambda_w^2} \right]^{\nu} \]

- **Algebraic Model** \( \Lambda \rightarrow \Lambda(w) \):  
  \[ \Lambda_w^2 = M_q^2 + \frac{1}{4}(1 - w^2)P^2 + \frac{1}{2}(1 - w)(M_h^2 - M_q^2) \]

- \( \Lambda \) is a mass scale.
- The LFWF can be obtained by projecting the meson BSWF:

  \[ \psi_M(x, q_{\perp}^2) = \text{Tr} \int \frac{d^2 q_{\parallel}}{\pi} \delta_{n}^{x} \gamma_5 \gamma \cdot n \chi_M(q, P) \]
In the light-cone formalism the PDA can be expressed as

\[ f_M \phi_M(x) = \frac{1}{16\pi^3} \int_{dq_\perp} \psi_M(x, q_\perp^2) \]

Relation between the LFWF and the PDA

\[ \psi^q_M(x, k_\perp^2) = \frac{16\pi^2 f_M \nu (\Lambda_{1-2x}^2)^\nu}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi^q_M(x) \]

No need to construct spectral density \( \rho(w) \)

The PDA is an input to compute de LFWFs
- **Valence-quark PDF:**
  - EHM-induced dilated distributions
  - Soft end-point behavior
  - Dilation and narrowness of PDA are filtered into PDFs.

- **Charge radii:**
  \[ r_{\pi}^2 = -6 \frac{dF_{\pi}(t)}{dt} \bigg|_{t=0} = 0.65 \text{ fm} \]
  - Experimentally \( r_{\pi}^2 \sim 0.66 \text{ fm} \).
In the chiral limit, $m_{0-} = 0$, and quark-antiquark flavor symmetry, $M_q = M_Q$, one has

$$\psi_{0-}^{\text{chiral}}(x, p_{\perp}^2) = \left[ 16\pi^2 f_0 - \frac{\nu M_q^{2\nu}}{(p_{\perp}^2 + M_q^2)^{\nu+1}} \right] \phi_{0-}(x)$$

hence, the $x$ and $p_{\perp}^2$ dependence of the LFWF is completely factorized.

Contrary, as captured in

$$\psi_{M}^q(x, k_{\perp}^2) = \frac{16\pi^2 f_M \nu (\Lambda_{1-2x}^2)^\nu}{(k_{\perp}^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_{M}^q(x)$$

a non-zero meson mass and quark-antiquark flavor asymmetry, i.e. $m_{0-}^2 \neq 0$ and $M_q \neq M_Q$, yield a LFWF which correlates $x$ and $p_{\perp}^2$.

One should expect an increasingly dominant role of $x$ and $p_{\perp}^2$ correlations in heavy-light systems.
**Heavy-light Pseudoscalar Mesons’ PDAs**

- **Inputs:** D and B meson’s PDAs
  - Physics Letters B Volume 790 (2019), 257-262

- Asymmetry: Induced by Higgs.

- The PDAs become increasingly asymmetric.

- More sharply peaked as the disparity grows between the current-masses of the meson’s valence-quarks.
Compute LFWFs of heavy-light pseudoscalar mesons

- Use the uniqueness of Mellin moments to compute LFWF

\[
\langle x^m \rangle_{\psi^q_M} = \text{Tr} \int_0^1 dx \int \frac{d^2 k_\parallel}{\pi} x^m \delta(n \cdot k - x n \cdot P) \gamma_5 \gamma \cdot n \chi_M(k_-, P)
\]
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<th>( \langle (2x - 1)^m \rangle )</th>
<th>( m = 0 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>6.82</td>
<td>-4.24</td>
<td>2.97</td>
<td>-2.15</td>
<td>1.62</td>
<td>-1.25</td>
<td>0.99</td>
<td>-0.79</td>
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<td>2.10</td>
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<tr>
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<td>-1.47</td>
<td>0.99</td>
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<td>-0.40</td>
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<td>1.72</td>
<td>-1.14</td>
<td>0.84</td>
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<td>0.49</td>
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<td>0.90</td>
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</tbody>
</table>

- Even moments are positive and odd ones are negative but, in absolute value, they systematically fall-off towards zero.
- Higher order moments have lower values for all mesons being in general the last reported moment an order of magnitude smaller than the first one.
- The value of a given moment decreases as \( p_\perp^2 \) increases for any meson, however, once such a value is small enough it remains nearly constant.
DGLAP kinematic domain: GPD from the overlap representation of the LFWF

\[ \mathcal{H}^q_M(x, \xi, t) = \int_{dk_\perp} \psi^*_M(x^-, (k^-)^2) \psi^q_M(x^+, (k^+)^2) \]

- zero-th Mellin’s moments:
  Electromagnetic Form Factor

\[ F^q_M(t) = \int_{-1}^{1} dx \mathcal{H}^q_M(x, 0, t) \]

- Forward limit (\(\xi = 0\)):
  Parton Distribution Functions

\[ \mathcal{H}^q_M(x, 0, 0) = q_M(x) \]
The $D$-meson GPD presents a very sharp behavior with respect to the momentum transfer; in fact, it is almost zero beyond $-t \approx 1 \text{ GeV}^2$.

The $x$-dependence of the $D$-meson GPD is weighted at $x \lesssim 0.5$, with a maximum at around $x = 0.2$ and presenting negligible values for $x \gtrsim 0.5$. 

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PDFs: Hadron Scale $\zeta_H$

- The PDF is obtained from the forward limit of the GPD.
- $\zeta_H$: all the momentum is carried by the valence-quarks.
- The PDF becomes sharper as the mass of the valence-quark decreases.
- The $x$ value moves towards larger values as the light valence-quark is getting heavier.
Electromagnetic Form Factors

- EFFs is obtained from the t-dependence of the GPD’s 0-th moment.
- Decreases asymptotically at the same rate for the same heavy-quark sector.
- Falls off with respect to the transferred momentum more smoothly when the mass difference of its valence quarks is smaller.
- Similar features can be deduced from other results:
There is a general trend of decreasing charge radii as the mass of the constituent quark increases

\[ r_{d\bar{c}} > r_{s\bar{c}}, \]
\[ r_{u\bar{b}} > r_{s\bar{b}} > r_{c\bar{b}} \]

We know experimentally that \( r_\pi \sim 0.66 \text{ fm}, \ r_K \sim 0.56 \text{ fm}. \)

We need more information to confirm or refute our results.
IPS-GPDs: $D$ and $D_s$ mesons

The heavy antiquark is almost fixed at the center of transverse momentum.

The highest probability of finding the light quark in the transverse plane is at a distance:

- $0.60 \times r_D$ for the $D$-meson.
- $0.14 \times r_{D_s}$ for the $D_s$-meson.
IPS-GPDs: $B$, $B_s$, $B_c$-mesons

The highest probability of finding the light quark in the transverse plane is at a distance:

- $0.65 \times r_B$ for the $B$-meson
- $0.13 \times r_{B_s}$ for the $B_s$-meson.
- $0.035 \times r_{B_c}$ for the $B_c$-meson.

As the constituent quark mass is larger, the distribution is wider in $x$ but less extended in $b_\perp$, with their maximum smaller.
The proposal $\Lambda \rightarrow \Lambda(w)$ and the Nakanishi Integral Representation of the BSA make possible to calculate analytically the LFWF of mesons with heavy-light content.

This method only needs the PDAs, and from them

- We can calculate EFFs, PDFs, and IPS-GPDs.
- The model shows reasonable agreement with other results.

GPDs, accessible by high energy scattering processes, encode important information on hadron’s 3D structure—distributions as well as motions of quarks and gluons.

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