Heavy-light Pseudoscalar Mesons:

Light-Front Wave Functions and Generalized Parton Distributions

J. Segovia[1] B. Almeida-Zamora[1,2] J. Cobos-Martínez[2] A. Bashir[3,4]
K. Raya[5] J. Quintero-Rodríguez[5]

[1]Universidad Pablo de Olavide, de Sevilla, España

[2]Universidad de Sonora, México

[3]Universidad Michoacana de San Nicolás de Hidalgo, México

[4]Thomas Jefferson National Accelerator Facility, USA

[5]Universidad de Huelva, España

...What we talk about?

- We study the **internal structure** of pseudoscalar mesons with heavy-light quark content through an **algebraic model** within the framework of **the light-front formalism**.
- We have a relation between the **light-front wave function** (LFWF) and the **parton distribution amplitude** (PDA) of hadrons.
- The purpose is the (analytic) extraction of **electromagnetic form factors** (FFs) and **parton distribution functions** (PDFs) through **generalized parton distribution functions** (GPDs).

...What we talk about?

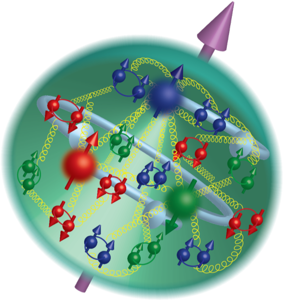
- We study the **internal structure** of pseudoscalar mesons with heavy-light quark content through an **algebraic model** within the framework of **the light-front formalism**.
- We have a relation between the **light-front wave function** (LFWF) and the **parton distribution amplitude** (PDA) of hadrons.
- The purpose is the (analytic) extraction of **electromagnetic form factors** (FFs) and **parton distribution functions** (PDFs) through **generalized parton distribution functions** (GPDs).

Open Access

Algebraic model to study the internal structure of pseudoscalar mesons with heavy-light quark content

B. Almeida-Zamora, J. J. Cobos-Martínez, A. Bashir, K. Raya, J. Rodríguez-Quintero, and J. Segovia
Phys. Rev. D **109**, 014016 – Published 17 January 2024

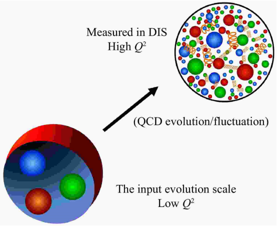
How to describe the hadron structure?



- **Non-perturbative QCD:** Unraveling the hadron structure from the fundamental d.o.f is an outstanding problem.
- **Parton Model:**
  - Hadrons can be considered as a collection of point-like constituents.
  - Is based on the concept of PDFs and used to interpret DIS data.
- **GPDs** provide a picture of the spatial and momentum distributions of quarks and gluons.

Light-Front Wave Function

Probability amplitude that a hadron with momentum P^+ consists of partons with physical momentum p^+ and $p_\perp$.

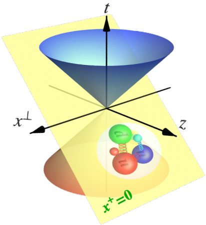


- Light-front formalism exploits dynamical evolution in the light-front time x^+.

- LFWF depend only on the relative variables:

$$x = \frac{p^+}{P^+}$$

$$k_\perp = p_\perp - x P_\perp$$



- Many distributions are related via the leading-twist LFWF
  - Distribution Amplitudes:

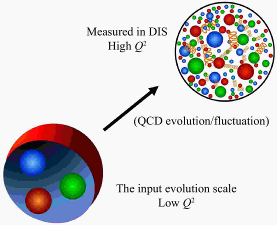
$$f_M \varphi_M^u(x,\zeta_H) = \int_{dk_\perp} \psi_M(x, k_\perp^2, \zeta_H)$$

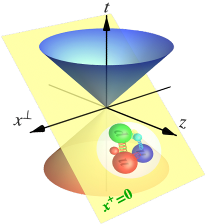
  - Distribution Functions:

$$u_M(x,\zeta_H) = \int_{dk_\perp} \left|\psi_M(x, k_\perp^2, \zeta_H)\right|^2$$

  - In the DGLAP kinematic domain, the valence-quark GPD:

$$\mathcal{H}_M^u(x,\xi,t) = \int_{dk_\perp} \psi_M^*(x^-, k_\perp^{-2}) \psi_M(x^+, k_\perp^{+2})$$

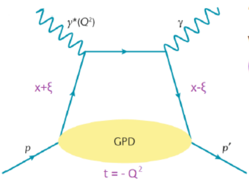




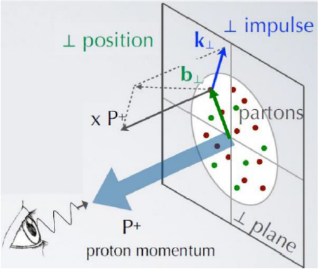
Generalised Parton Distributions

Probabilistic interpretation:
Probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction "x" of the hadron's averaged light-cone momentum.

Deeply Virtual Compton Scattering



"hadron-parton" amplitudes which depend on three variables **(x, ξ, t)**

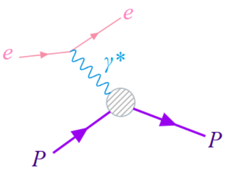


Structure mapped in terms of:
$b_\perp$ = transverse position
$k_\perp$ = transverse momentum

x: average momentum fraction carried by the active parton
ξ: skewness parameter
t: the Mandelstam variable (four-momentum transfer)

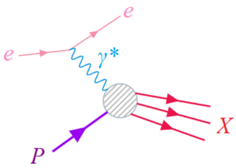
3-dimensional picture of Hadrons

Elastic Scattering



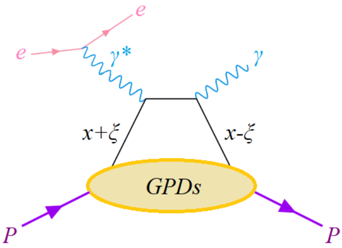
FFs

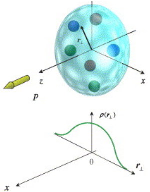
Deep Inelastic Scattering

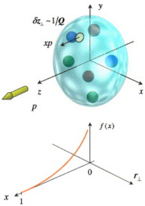


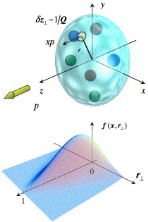
PDFs

Deeply Virtual Compton Scattering

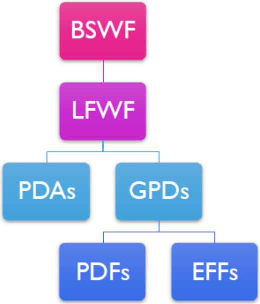








Algebraic Model



- Compute everything from the LFWF.
- The internal dynamics of a meson can be described via its Bethe-Salpeter wave function:

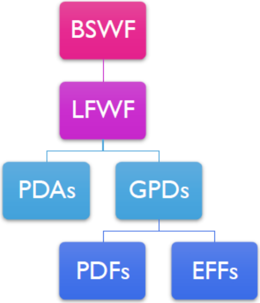
$$\chi_M(k,P) = S_q(k_+)\Gamma_M(k,P)S_{\bar{h}}(k_-)$$

- Quark propagator and Bethe-Salpeter ampltiude:

$$S(k) = \frac{-i\gamma\cdot k + M}{k^2 + M^2},$$

$$N_M \Gamma_M(k) = i\gamma_5 \int_{-1}^{1} \mathrm{d}w\ \rho(w) \left[\frac{\Lambda_w^2}{k_w^2 + \Lambda_w^2}\right]^\nu$$

Light-Front Wave Function



- Bethe-Salpeter Amplitude Phys. Rev. 130, 1230

$$N_M \Gamma_M(k) = i\gamma_5 \int_{-1}^{1} \mathrm{d}w \; \rho(w) \left[\frac{\Lambda_w^2}{k_w^2 + \Lambda_w^2} \right]^{\nu}$$

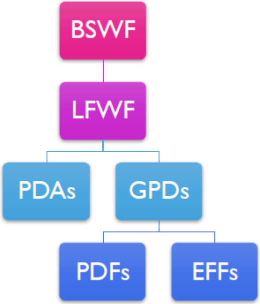
- **Algebraic Model** $\Lambda \to \Lambda(w)$**:** Phys. Rev. D 106, 034003

$$\Lambda_w^2 = M_q^2 + \frac{1}{4}(1-w^2)P^2 + \frac{1}{2}(1-w)(M_h^2 - M_q^2)$$

- Λ is a mass scale.
- The LFWF can be obtained by projecting the meson BSWF:

$$\psi_M(x, q_\perp^2) = \mathrm{Tr} \int \frac{d^2 q_\parallel}{\pi} \delta_n^x \gamma_5 \gamma \cdot n \chi_M(q, P)$$

Parton Distribution Amplitude



- In the light-cone formalism the PDA can be expressed as

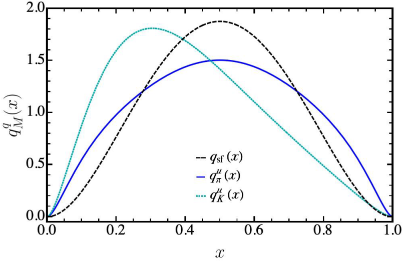
$$f_M \phi_M(x) = \frac{1}{16\pi^3} \int_{dq_\perp} \psi_M(x, q_\perp^2)$$

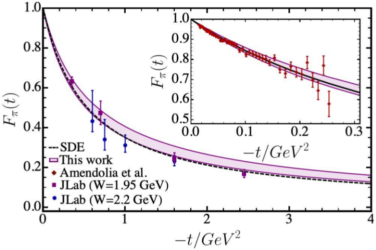
- Relation between the LFWF and the PDA

$$\psi_M^q(x, k_\perp^2) = \frac{16\pi^2 f_M \nu (\Lambda_{1-2x}^2)^\nu}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_M^q(x)$$

- No need to construct spectral density $\rho(w)$
- The PDA is an input to compute de LFWFs

Backup: Lightest Pseudoscalar Meson





- **Valence-quark PDF**:
  EHM-induced dilated distributions
- Soft end-point behavior
- Dilation and narrowness of PDA are filtered into PDFs.

- **Charge radii:**

$$r_\pi^2 = -6 \frac{\mathrm{d}F_\pi(t)}{\mathrm{d}t}\Big|_{t=0} = 0.65 \text{ fm}$$

- Experimentally $r_\pi^2 \sim 0.66$ fm.

Backup: Lightest Pseudoscalar Meson

- In the chiral limit, $m_{0^-} = 0$, and quark-antiquark flavor symmetry, $M_q = M_{\bar{Q}}$, one has

$$\psi_{0^-}^{\text{chiral}}(x, p_\perp^2) = \left[16\pi^2 f_{0^-} \frac{\nu M_q^{2\nu}}{(p_\perp^2 + M_q^2)^{\nu+1}}\right] \phi_{0^-}(x)$$

hence, the x and $p_\perp^2$ dependence of the LFWF is completely factorized.

- Contrary, as captured in

$$\psi_M^q(x, k_\perp^2) = \frac{16\pi^2 f_M \nu (\Lambda_{1-2x}^2)^\nu}{(k_\perp^2 + \Lambda_{1-2x}^2)^{\nu+1}} \phi_M^q(x)$$

a non-zero meson mass and quark-antiquark flavor asymmetry, *i.e.* $m_{0^-}^2 \neq 0$ and $M_q \neq M_{\bar{Q}}$, yield a LFWF which correlates x and $p_\perp^2$.

One should expect an increasingly dominant role of x and $p_\perp^2$ correlations in heavy-light systems.

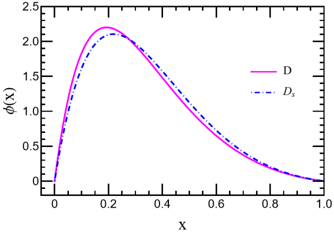
Heavy-light Pseudoscalar Mesons' PDAs

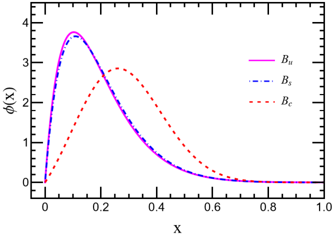
- **Inputs**: D and B meson's PDAs
  Physics Letters B Volume 790 (2019), 257-262

- Asymmetry: Induced by Higgs.

- The PDAs become increasingly asymmetric.

- More sharply peaked as the disparity grows between the current-masses of the meson's valence-quarks.

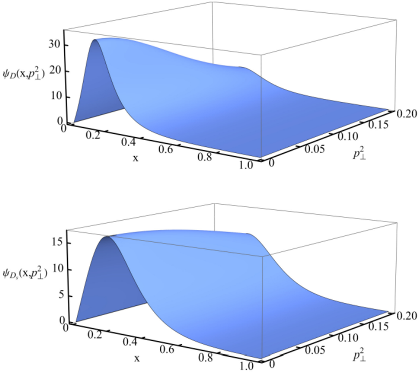




Compute LFWFs of heavy-light pseudoscalar mesons

- Use the uniqueness of Mellin moments to compute LFWF

$$\langle x^m \rangle_{\psi_M^q} = \text{Tr} \int_0^1 \text{d}x \int \frac{\text{d}^2 k_\parallel}{\pi} x^m \delta(n \cdot k - x n \cdot P) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$



$\langle(2x-1)^m\rangle$		$m=0$	1	2	3	4	5	6	7	8	9	10
D	$p_\perp^2=0.0$	6.82	-4.24	2.97	-2.15	1.62	-1.25	0.99	-0.79	0.65	-0.54	0.45
	$p_\perp^2=0.1$	5.07	-3.03	2.10	-1.51	1.14	-0.88	0.70	-0.56	0.46	-0.38	0.32
	$p_\perp^2=0.2$	2.72	-1.47	0.99	-0.69	0.52	-0.40	0.32	-0.26	0.21	-0.18	0.15
D_s	$p_\perp^2=0.0$	5.49	-2.64	1.72	-1.14	0.84	-0.62	0.49	-0.38	0.31	-0.26	0.22
	$p_\perp^2=0.1$	5.09	-2.42	1.58	-1.05	0.77	-0.57	0.45	-0.35	0.29	-0.24	0.20
	$p_\perp^2=0.2$	4.13	-1.91	1.25	-0.82	0.61	-0.45	0.36	-0.28	0.23	-0.19	0.16
B	$p_\perp^2=0.0$	6.28	-5.24	4.46	-3.84	3.35	-2.94	2.60	-2.31	2.07	-1.85	1.67
	$p_\perp^2=0.1$	5.10	-4.19	3.53	-3.02	2.61	-2.27	2.00	-1.77	1.58	-1.41	1.27
	$p_\perp^2=0.2$	3.23	-2.56	2.10	-1.75	1.48	-1.27	1.10	-0.96	0.85	-0.75	0.67
B_s	$p_\perp^2=0.0$	5.38	-4.13	3.30	-2.69	2.24	-1.88	1.61	-1.38	1.20	-1.05	0.93
	$p_\perp^2=0.1$	5.08	-3.88	3.08	-2.51	2.08	-1.75	1.49	-1.28	1.11	-0.98	0.86
	$p_\perp^2=0.2$	4.30	-3.25	2.56	-2.07	1.71	-1.43	1.22	-1.04	0.90	-0.79	0.69
B_c	$p_\perp^2=0.0$	5.10	-2.53	1.47	-0.92	0.61	-0.42	0.30	-0.23	0.17	-0.13	0.11
	$p_\perp^2=0.1$	5.02	-2.49	1.45	-0.90	0.60	-0.42	0.30	-0.22	0.17	-0.13	0.11
	$p_\perp^2=0.2$	4.79	-2.37	1.38	-0.86	0.57	-0.40	0.29	-0.21	0.16	-0.13	0.10

- Even moments are positive and odd ones are negative but, in absolute value, they systematically fall-off towards zero.
- Higher order moments have lower values for all mesons being in general the last reported moment an order of magnitude smaller than the first one.
- The value of a given moment decreases as $p_\perp^2$ increases for any meson, however, once such a value is small enough it remains nearly constant.

- **DGLAP** kinematic domain: GPD from the overlap representation of the LFWF

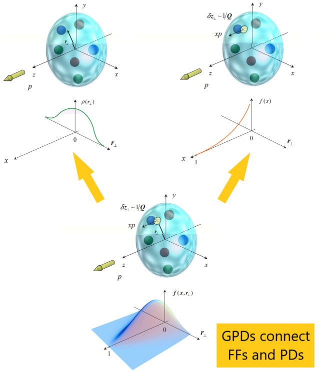
$$\mathcal{H}_M^q(x,\xi,t) = \int_{dk_\perp} \psi_M^{q*}[x^-,(k_\perp^-)^2]\psi_M^q[x^+,(k_\perp^+)^2]$$

- zero-th Mellin's moments:
  Electromagnetic Form Factor

$$F_M^q(t) = \int_{-1}^{1} dx \mathcal{H}_M^q(x,0,t)$$

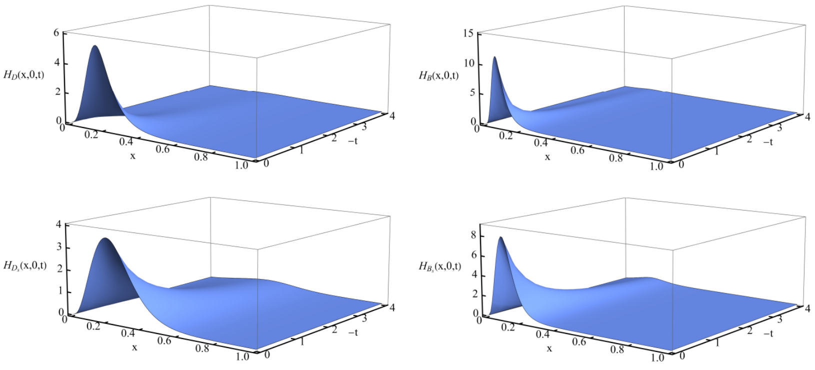
- Forward limit ($\xi = 0$):
  Parton Distribution Functions

$$\mathcal{H}_M^q(x,0,0) = q_M(x)$$



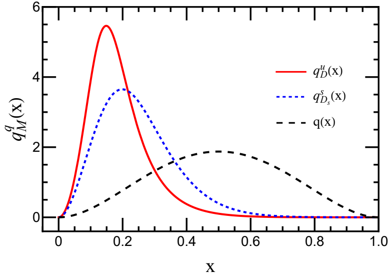
GPDs connect FFs and PDs

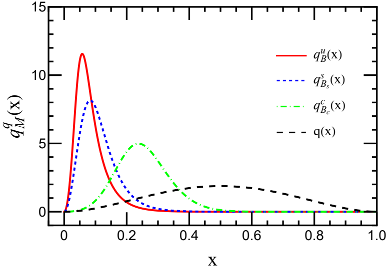
GPDs from the overlap of the LFWF



- The D-meson GPD presents a very sharp behavior with respect to the momentum transfer; in fact, it is almost zero beyond $-t \approx 1\,\text{GeV}^2$.
- The x-dependence of the D-meson GPD is weighted at $x \lesssim 0.5$, with a maximum at around $x = 0.2$ and presenting negligible values for $x \gtrsim 0.5$.

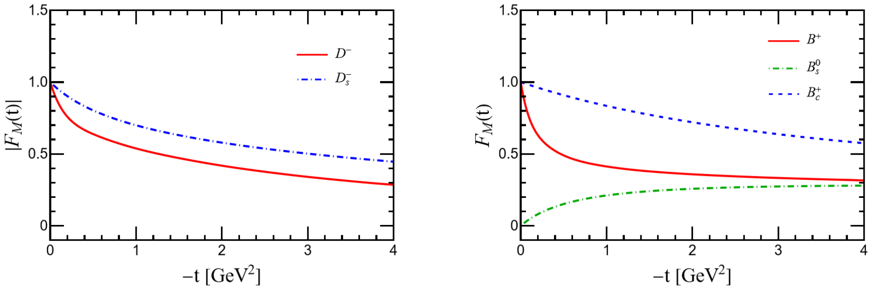
PDFs: Hadron Scale ζ_H





- The PDF is obtained from the forward limit of the GPD.
- ζ_H: all the momentum is carried by the valence-quarks.
- The PDF becomes sharper as the mass of the valence-quark decreases.
- The x value moves towards larger values as the light valence-quark is getting heavier.

Electromagnetic Form Factors



- EFFs is obtained from the t-dependence of the GPD's 0-th moment.
- Decreases asymptotically at the same rate for the same heavy-quark sector.
- Falls off with respect to the transferred momentum more smoothly when the mass difference of its valence quarks is smaller.
- Similar features can be deduced from other results:
  Phys. Rev. D 104, 096020 (2021), Eur. Phys. J. C 23, 585 (2002), Phys. Rev. D 107, 054002 (2023)

Charge Radii Results

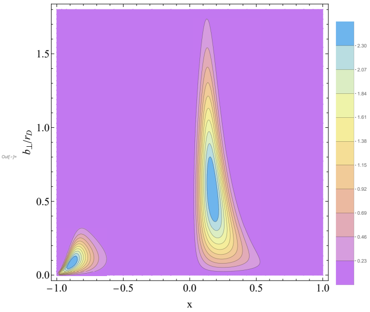
	$D^-(d\bar{c})$	$D_s^-(s\bar{c})$	$B^+(u\bar{b})$	$B_s^0(s\bar{b})$	$B_c^+(c\bar{b})$
Our results	0.680	0.372	0.926	0.345	0.217
Lattice QCD	0.450(24)	0.465(57)	-	-	-
Covariant CQM	0.505	0.377	-	-	-
LFQM	0.429	0.352	0.615	0.345	0.208
SDE-CI	-	0.260	0.340	0.240	0.170

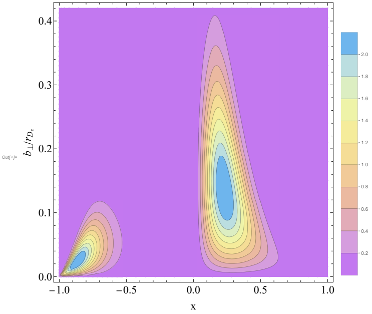
- There is a general trend of decreasing charge radii as the mass of the constituent quark increases

$$r_{d\bar{c}} > r_{s\bar{c}},$$
$$r_{u\bar{b}} > r_{s\bar{b}} > r_{c\bar{b}}$$

- We know experimentally that $r_\pi \sim 0.66$ fm, $r_K \sim 0.56$ fm.
- We need more information to confirm or refute our results.

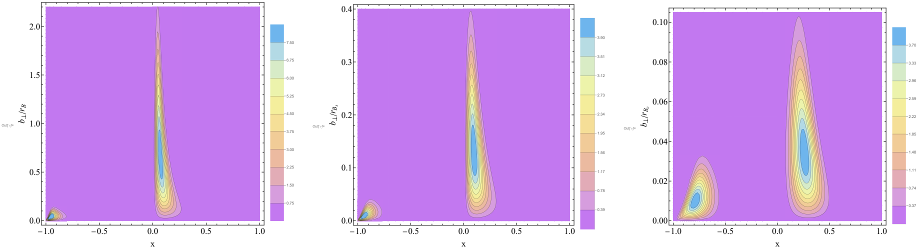
IPS-GPDs: D and D_s mesons





- The heavy antiquark is almost fixed at the center of transverse momentum.
- The highest probability of finding the light quark in the transverse plane is at a distance:
  - $0.60 \times r_D$ for the D-meson.
  - $0.14 \times r_{D_s}$ for the D_s-meson.

IPS-GPDs: B-, B_s-, B_c-mesons



- The highest probability of finding the light quark in the transverse plane is at a distance:
  - $0.65 \times r_B$ for the B-meson
  - $0.13 \times r_{B_s}$ for the B_s-meson.
  - $0.035 \times r_{B_c}$ for the B_c-meson.
- As the constituent quark mass is larger, the distribution is wider in x but less extended in $b_\perp$, with their maximum smaller.

Summary

- The proposal $\Lambda \to \Lambda(w)$ and the Nakanishi Integral Representation of the BSA make possible to calculate analytically the LFWF of mesons with heavy-light content.
- This method only needs the PDAs, and from them
  - We can calculate EFFs, PDFs, and IPS-GPDs.
  - The model shows reasonable agreement with other results.
- GPDs, accessible by high energy scattering processes, encode important information on hadron's 3D structure–distributions as well as motions of quarks and gluons.

Thanks

Summary

- The proposal $\Lambda \to \Lambda(w)$ and the Nakanishi Integral Representation of the BSA make possible to calculate analytically the LFWF of mesons with heavy-light content.
- This method only needs the PDAs, and from them
  - We can calculate EFFs, PDFs, and IPS-GPDs.
  - The model shows reasonable agreement with other results.
- GPDs, accessible by high energy scattering processes, encode important information on hadron's 3D structure–distributions as well as motions of quarks and gluons.

Thanks