Transition Form Factor of η/η' at BESIII

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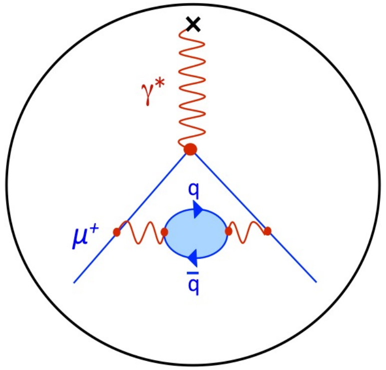
Contents

Form Factor Physics

- ✓ Describe the complex internal structure or intermediate processes
- ✓ It determines the size of hadronic quantum corrections in the calculation of the $(g-2)_\mu$

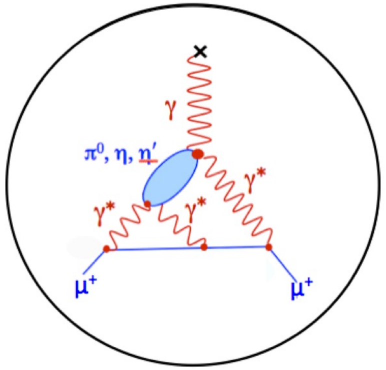
$$a_\mu = \frac{1}{2}(g-2)_\mu$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr},\ a_\mu^{hadr} = a_\mu^{HVP} + a_\mu^{HLbL}$$



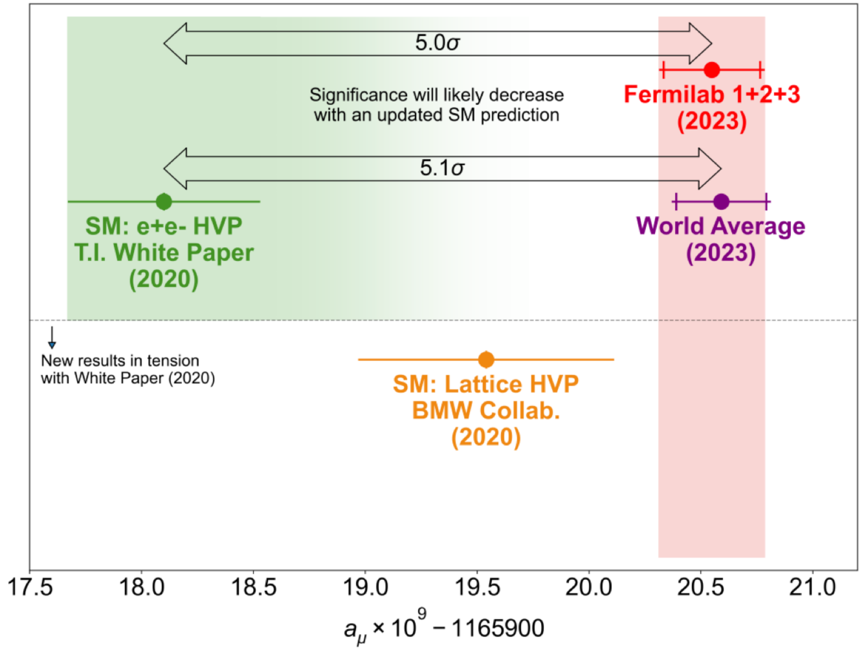
$a_\mu^{HVP} = 6845(40)\times10^{-11}$

Hadronic Vacuum Polarization(LO)



$a_\mu^{HLbL} = 92(18)\times10^{-11}$

Hadronic Light-by-Light



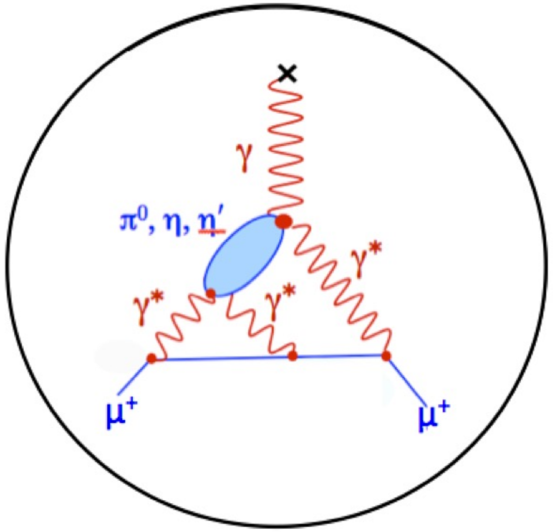
In 2021, $a_\mu^{exp} - a_\mu^{SM} \approx 4.2\sigma$

- ✓ Experimental input is needed to improve the accuracy of predictions!

Form Factor Physics

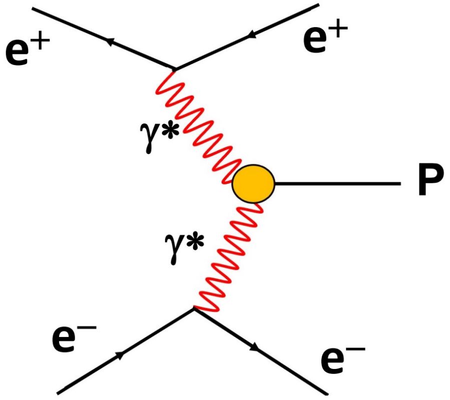
- ✓ a_μ^{HLbL} not directly related to measurable quantities.
- ✓ The coupling of π^0, η, and η' with two photons in HLbL can be described using transition form factor (TFF).

TFFs as experimental input !

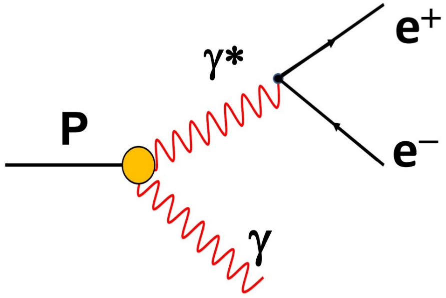


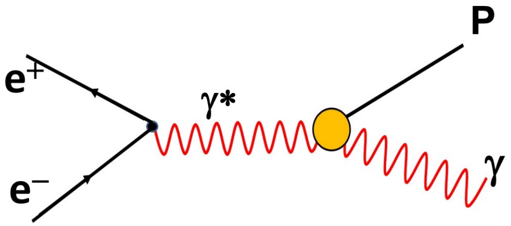
- ✓ TFFs are experimentally accessible in three different processes

Space-like



Time-like

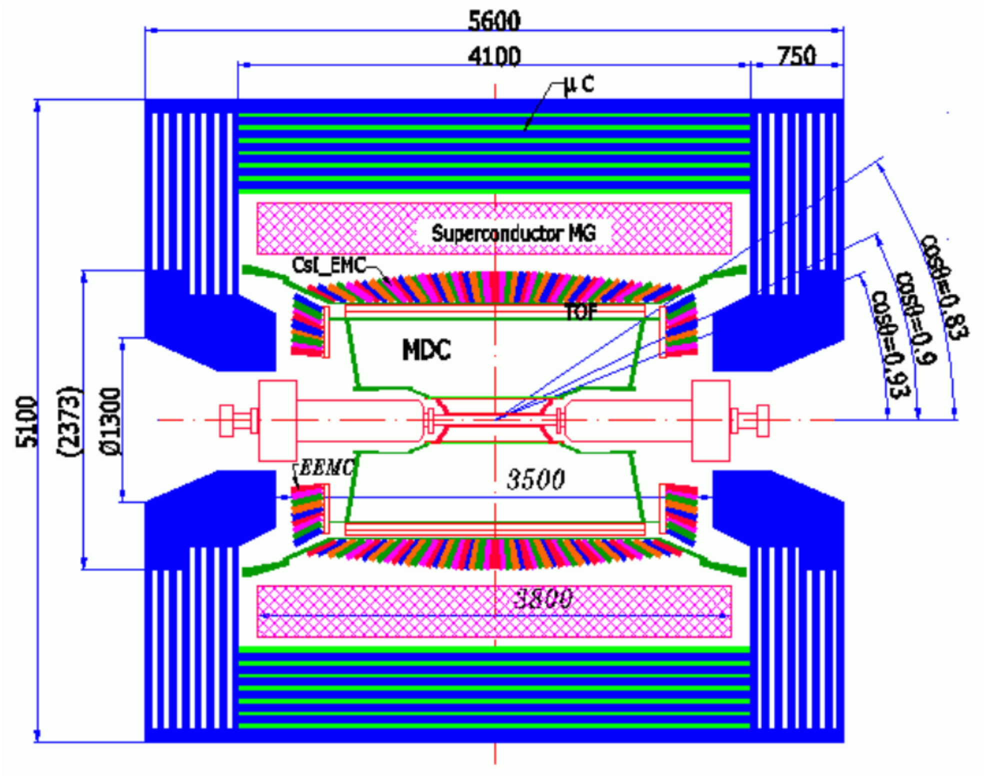




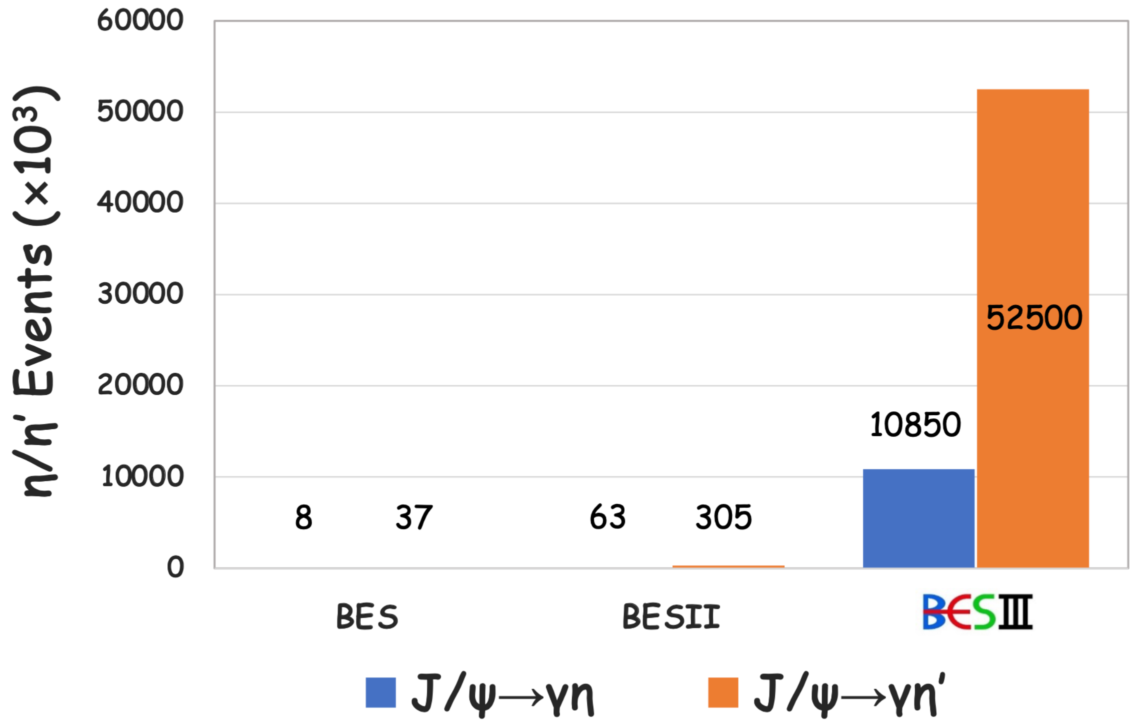
- The fusion of both photons to form a meson is described by the TFF.

BESIII Detector

- ✓ The BESIII detector records symmetric e^+e^- collisions provided by the BEPCII storage ring.
- ✓ The facility is used for studies of hadron physics and τ-charm physics.
- ✓ Collected 10 billion J/ψ Events!



An overview of the BESIII detector.



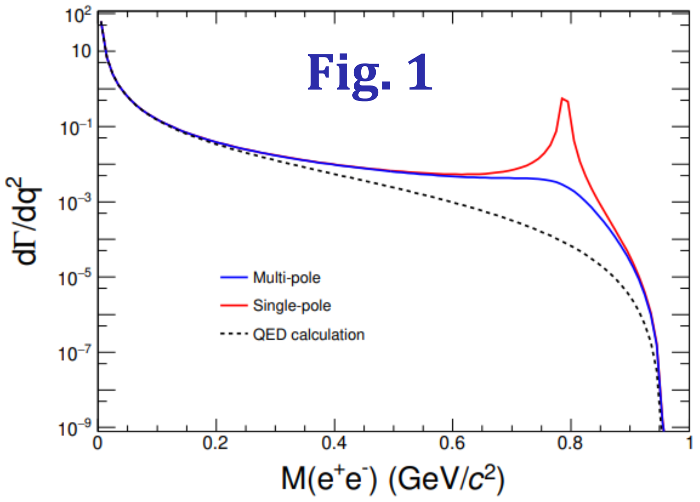
A light meson factory!

$\eta' \to \gamma e^+ e^-$

- Decay mode: $J/\psi \to \gamma\eta'$, $\eta' \to \gamma e^+e^-$
- In 2015, BESIII Collaboration used 1.3 billion J/ψ events with bin-by-bin method getting the results of η' form factor.
- The decay rate

$$\frac{d\Gamma(P\to\gamma l^+l^-)}{dq^2\Gamma(P\to\gamma\gamma)} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{q^2}{m_P^2}\right)^3|F(q^2)|^2$$

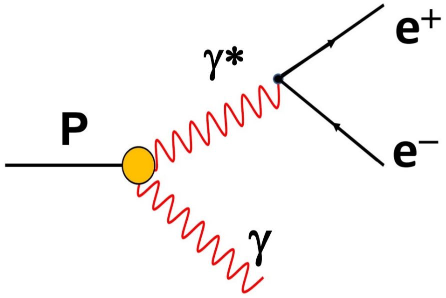
$$= [\mathrm{QED}(q^2)]\times|F(q^2)|^2$$

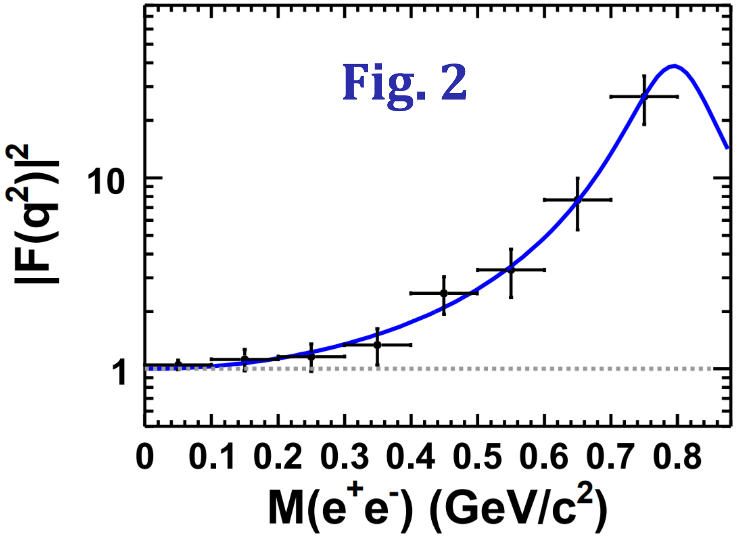


- Single-pole: $F(q^2) = \frac{1}{1-q^2/\Lambda^2}$
- Multi-pole: $|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2+\gamma^2)}{(\Lambda^2-q^2)^2+\Lambda^2\gamma^2}$

$\Lambda_{\eta'} = (0.79 \pm 0.04 \pm 0.02)$ GeV

$\gamma_{\eta'} = (0.13 \pm 0.06 \pm 0.03)$ GeV





$\eta/\eta' \to \gamma e^+e^-$

Phys. Rev. D 109, 072001 (2024)

TFF Results of $\eta' \to \gamma e^+e^-$

- 10 billion J/ψ Events available.
- Performed an unbinned fit based on the amplitude formula.
- Multi-pole formula

$$|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2+\gamma^2)}{(\Lambda^2-q^2)^2+\Lambda^2\gamma^2}$$

$$\Lambda_{\eta'} = (0.802 \pm 0.007 \pm 0.008)\ \mathrm{GeV}/c^2$$

$$\gamma_{\eta'} = (0.113 \pm 0.009 \pm 0.002)\ \mathrm{GeV}/c^2$$

- Slope parameter $b_{\eta'} = \frac{dF^2(q^2)}{dq^2}|_{q=0}$
- The root mean square (RMS) of the interaction regions,

$$R_{\eta'} = \sqrt{6b_{\eta'}} = (0.596 \pm 0.005 \pm 0.006)\ \mathrm{fm}$$

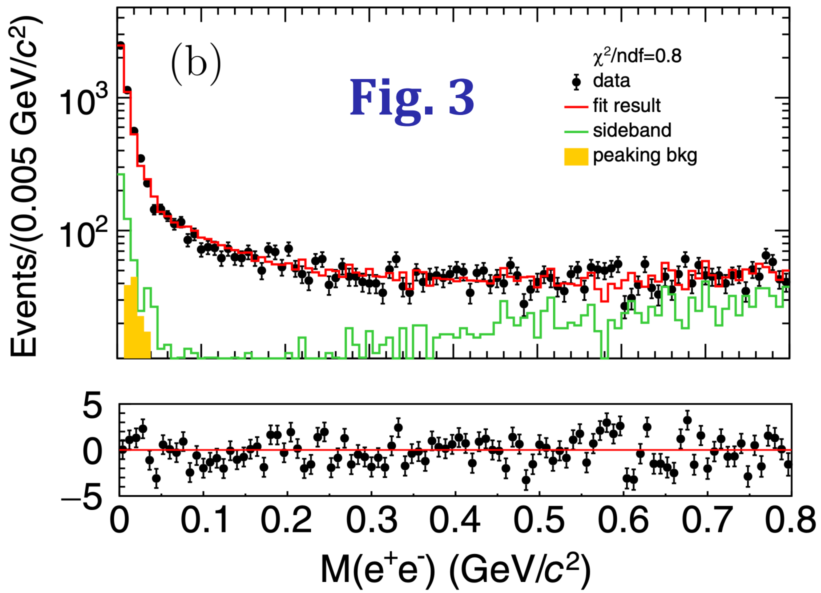


Fig. 3

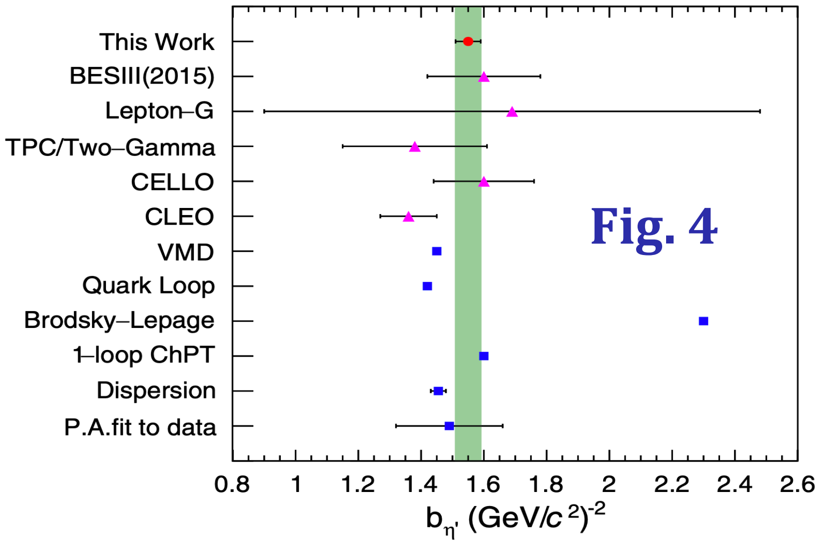


Fig. 4

$\eta/\eta' \to \gamma e^+e^-$

TFF Results of $\eta \to \gamma e^+e^-$

- Single-pole is sufficient to describe the data

$$F(q^2) = \frac{1}{1 - q^2/\Lambda^2}$$

$$\Lambda_\eta = (0.749 \pm 0.026 \pm 0.008)\ \mathrm{GeV}/c^2$$

- Slope parameter $b_\eta = \frac{dF^2(q^2)}{dq^2}\big|_{q=0} = \Lambda^{-2}$
- The RMS of the interaction regions,

$$R_\eta = \sqrt{6b_\eta} = (0.645 \pm 0.022 \pm 0.007)\ \mathrm{fm}$$

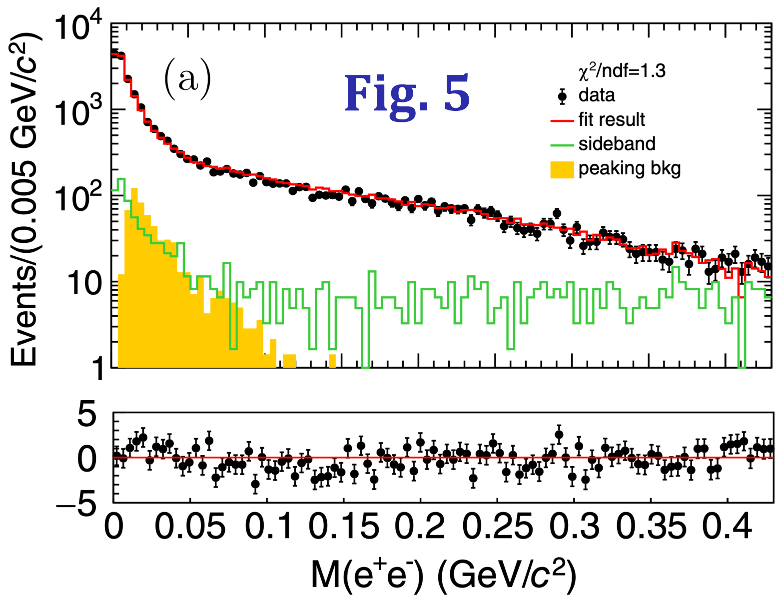


Fig. 5

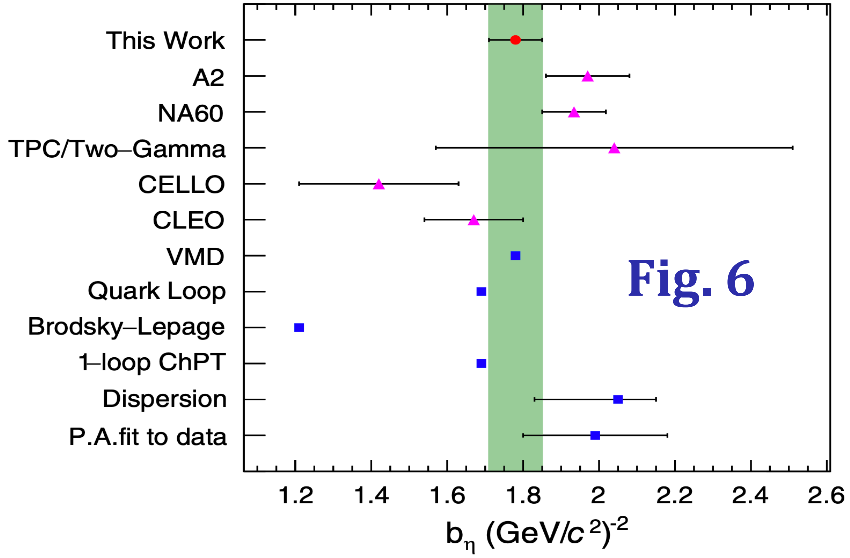


Fig. 6

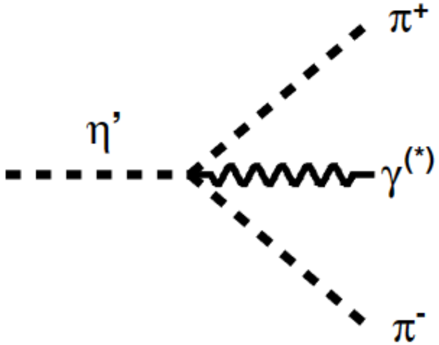
η' → $\pi^+\pi^-l^+l^-$

- Decay mode: $J/\psi \to \gamma\eta'$, $\eta' \to \pi^+\pi^+l^+l^+$ (l=e or μ)
- Decay amplitude

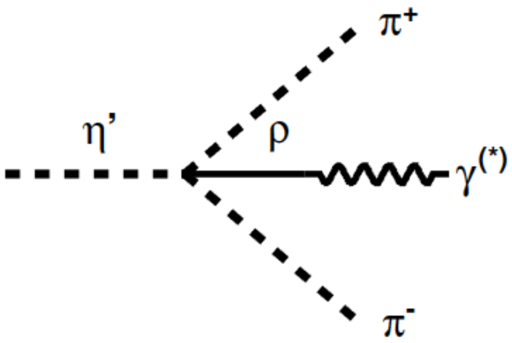
$$\overline{\left|\mathcal{A}_{\eta'\to\pi^+\pi^-l^+l^-}\right|}^2(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_1, \phi) = \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2\times\lambda\left(m_{\eta'}^2, s_{\pi\pi}, s_{ll}\right)\times[1-\beta_1^2\sin^2\theta_1\sin^2\phi]s_{\pi\pi}\beta_\pi^2\sin^2\theta_\pi$$

- $M(s_{\pi\pi}, s_{ll}) = \mathcal{M}_{mix}\times VMD(s_{\pi\pi}, s_{ll})$ contains the information of the decaying particle and the VMD input.
- Within the VMD model, TFF can be parameterized into three separate parts

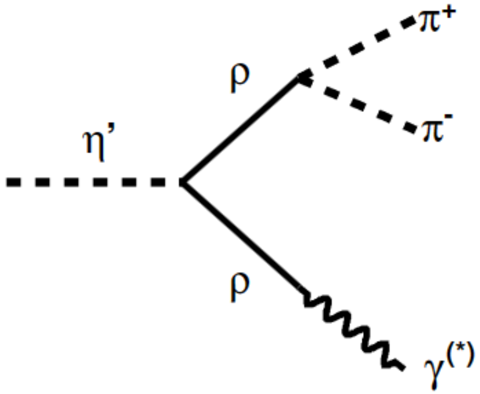
$$VMD(s_{\pi\pi}, s_{ll}) = 1-\frac{3}{4}(c_1-c_2+c_3) + \frac{3}{4}(c_1-c_2-c_3)\frac{m_V^2}{m_V^2-s_{ll}-im_V\Gamma(s_{ll})} + \frac{3}{2}c_3\frac{m_V^2}{m_V^2-s_{ll}-im_V\Gamma(s_{ll})}\frac{m_{V,\pi}^2}{m_{V,\pi}^2-s_{\pi\pi}-im_{V,\pi}\Gamma(s_{\pi\pi})}$$



Axial anomaly



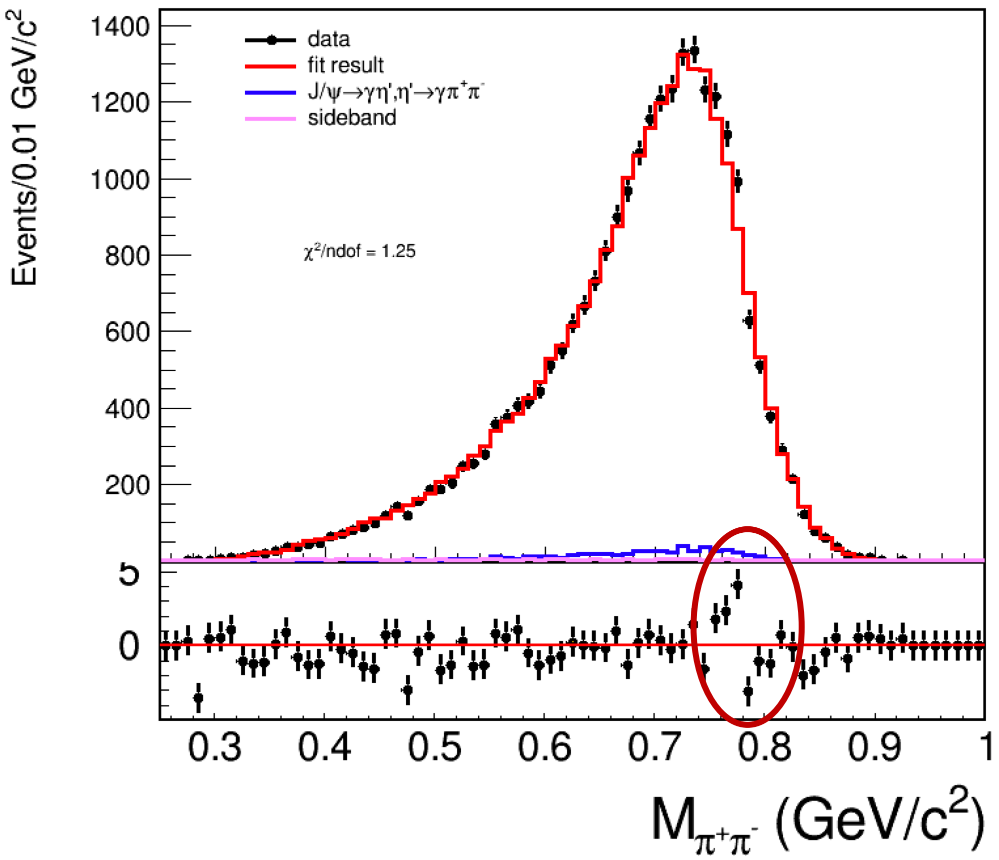
VMD contribution



VMD contribution

η' → π⁺π⁻l⁺l⁻

- By adjusting the values of the c_i-parameters, we can switch between the various VMD models.
  - I. Hidden gauge model: $\boldsymbol{c_1 - c_2 = c_3 = 1}$
  - II. Full VMD model: $\boldsymbol{c_1 - c_2 = \frac{1}{3}, c_3 = 1}$
  - III. Modified VMD: $\boldsymbol{c_1 - c_2 \neq c_3}$



- For $\eta' \to \pi^+\pi^-e^+e^-$ decay
  - ✓ ρ^0 only can not describe data well.
  - ✓ $\omega \to \pi^+\pi^-$ decay is necessary!

$$\frac{m_{V,\pi}^2}{m_{V,\pi}^2 - s_{\pi\pi} - i m_{V,\pi}\Gamma(s_{\pi\pi})} + \beta e^{i\theta}\frac{m_\omega^2}{m_\omega^2 - s_{\pi\pi} - i m_\omega \Gamma(s_{\pi\pi})}$$

η' → π⁺π⁻l⁺l⁻

arXiv:2402.01993 (hep-ex)

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  - I. Hidden gauge model: $c_1 - c_2 = c_3 = 1$
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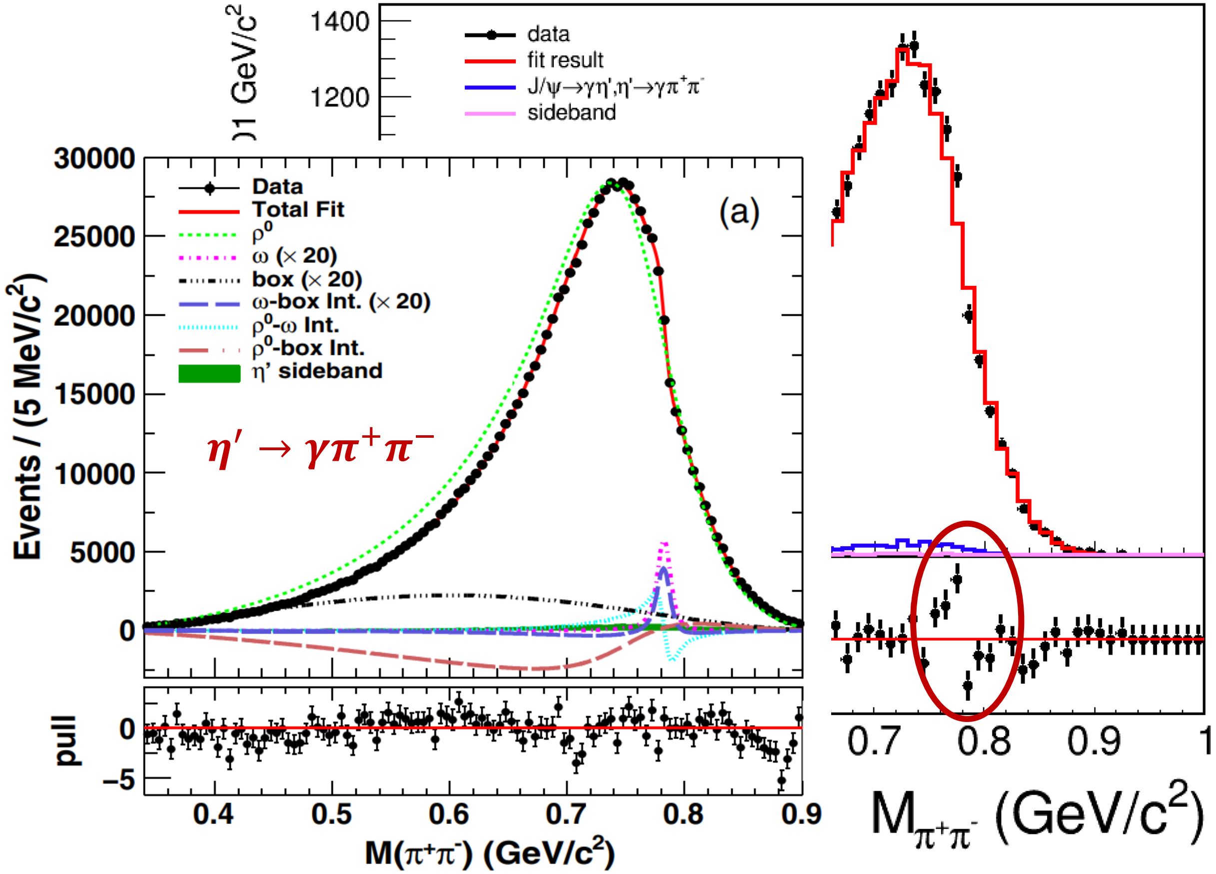
$$\frac{m_{V,\pi}^2}{m_{V,\pi}^2 - s_{\pi\pi} - im_{V,\pi}\Gamma(s_{\pi\pi})} + \beta e^{i\theta}\frac{m_\omega^2}{m_\omega^2 - s_{\pi\pi} - im_\omega\Gamma(s_{\pi\pi})}$$



box-anomaly?

BESIII: PRL120,242003(2018)

$\eta' \to \pi^+\pi^- l^+ l^-$

arXiv:2402.01993 (hep-ex)

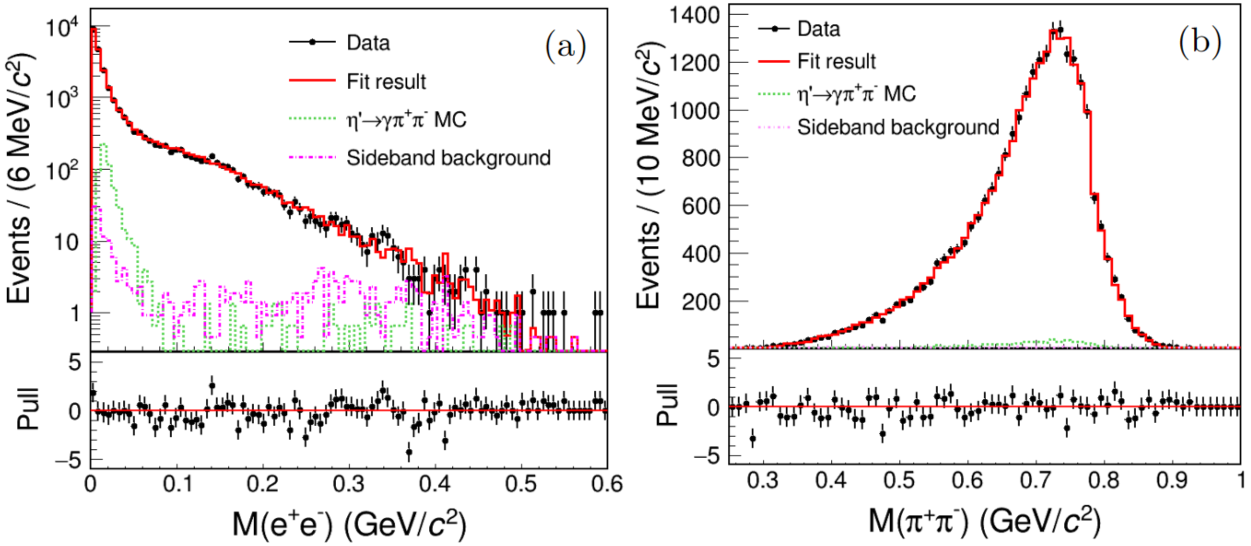
TFF Results

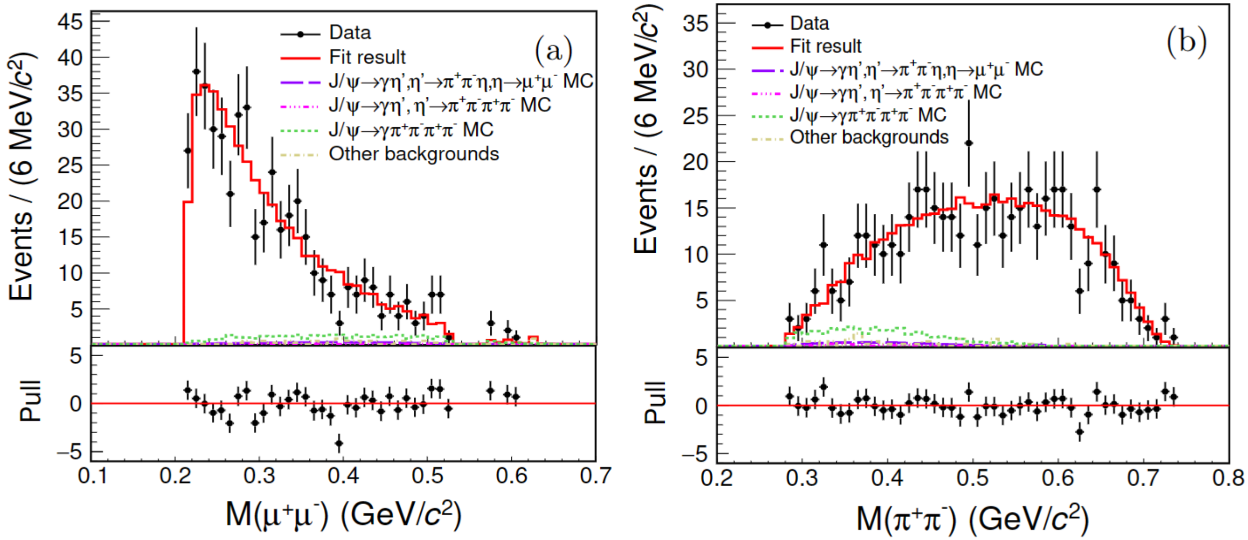
$\eta' \to \pi^+\pi^- e^+e^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V\,(\mathrm{MeV}/c^2)$	$954.3 \pm 82.5 \pm 36.4$	857.4 ± 74.3	787.5 ± 137.9
$m_{V,\pi}\,(\mathrm{MeV}/c^2)$	$765.3 \pm 1.1 \pm 20.2$	765.4 ± 1.1	764.8 ± 1.3
$m_\omega\,(\mathrm{MeV}/c^2)$	$778.7 \pm 1.3 \pm 17.3$	778.7 ± 1.3	778.7 ± 1.4
$\beta(10^{-3})$	$8.5 \pm 1.4 \pm 0.7$	8.5 ± 1.4	8.1 ± 1.4
θ	$1.4 \pm 0.3 \pm 0.1$	1.4 ± 0.3	1.4 ± 0.4
$c_1 - c_2$	1	1/3	-0.03 ± 0.87
c_3	1	1	1.03 ± 0.02
$\chi^2/ndof(e^+e^-, \pi^+\pi^-)$	65.3/82.0, 44.5/65.0	66.1/82.0, 44.3/65.0	66.8/82.0, 42.2/65.0
$b_{\eta'}\,(\mathrm{GeV}/c^2)^{-2}$	$1.10 \pm 0.19 \pm 0.07$	1.36 ± 0.24	1.61 ± 0.56

$\eta' \to \pi^+\pi^- \mu^+\mu^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V\,(\mathrm{MeV}/c^2)$	$649.4 \pm 52.3 \pm 35.6$	601.6 ± 24.0	589.6 ± 24.2
$m_{V,\pi}\,(\mathrm{MeV}/c^2)$	$757.3 \pm 22.6 \pm 18.0$	765.4 ± 17.6	774.4 ± 40.7
$c_1 - c_2$	1	1/3	0.01 ± 0.42
c_3	1	1	0.98 ± 0.38
$\chi^2/ndof(\mu^+\mu^-, \pi^+\pi^-)$	36.1/34.0, 30.4/46.0	36.1/34.0, 30.4/46.0	37.4/35.0, 29.9/46.0
$b_{\eta'}\,(\mathrm{GeV}/c^2)^{-2}$	$2.37 \pm 0.38 \pm 0.27$	2.76 ± 0.22	2.88 ± 0.24

All the three models provide good description of data.

✧ Hidden gauge model (Model I): $c_1 - c_2 = c_3 = 1$





$\eta' \to \pi^+\pi^-l^+l^-$

arXiv:2402.01993 (hep-ex)

TFF Results

$\eta' \to \pi^+\pi^- e^+e^-$	Model I	Model II	Model III
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$m_V\,(\mathrm{MeV}/c^2)$	$954.3 \pm 82.5 \pm 36.4$	857.4 ± 74.3	787.5 ± 137.9
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$\eta' \to \pi^+\pi^- \mu^+\mu^-$	Model I	Model II	Model III
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$b_{\eta'}(\mathrm{GeV}/c^2)^{-2}$	$2.37 \pm 0.38 \pm 0.27$	2.76 ± 0.22	2.88 ± 0.24

- Limited statistics at the high mass region of e^+e^-

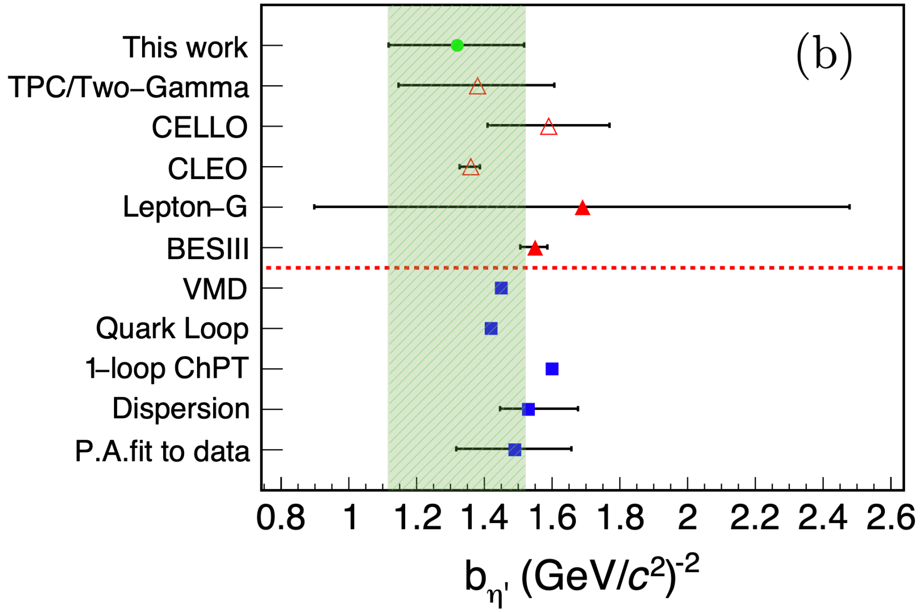
→ **Large statistical uncertainty of m_V and $c_1 - c_2$**

- A test with $c_1 - c_2 = c_3$ gives

$$c_1 - c_2 = c_3 = 1.03 \pm 0.02$$

- Provide a weighted average of the slope parameter for $\eta'\to\pi^+\pi^-e^+e^-$ and $\eta'\to\pi^+\pi^-\mu^+\mu^-$ based on Model I.

$$b_{\eta'} = 1.30 \pm 0.19\,(\mathrm{GeV}/c^2)^{-2}$$



$\eta' \to \pi^+\pi^-\pi^+\pi^-$

Phys. Rev. D 109, 032006 (2024)

- Decay mode: $J/\psi \to \gamma\eta'$, $\eta' \to \pi^+\pi^+\pi^+\pi^+$
- To investigate the doubly virtual isovector contribution.
- Decay amplitude is constructed with the combination of the Chiral Perturbation Theory (ChPT) and VMD model.

$$F(s_{12}, s_{34}) = \left[\frac{s_{12}}{D_\rho(s_{12})} + \frac{s_{34}}{D_\rho(s_{34})} - \frac{s_{14}}{D_\rho(s_{14})} - \frac{s_{23}}{D_\rho(s_{23})}\right]$$

$$+ \frac{c_3}{c_1 - c_2}\left[\frac{m_\rho^2(s_{12} - s_{34})}{D_\rho(s_{12})D_\rho(s_{34})} - \frac{m_\rho^2(s_{14} - s_{23})}{D_\rho(s_{14})D_\rho(s_{23})}\right]$$

$$D_\rho(s) = M_\rho^2 - s - iM_\rho\Gamma_\rho(s)$$

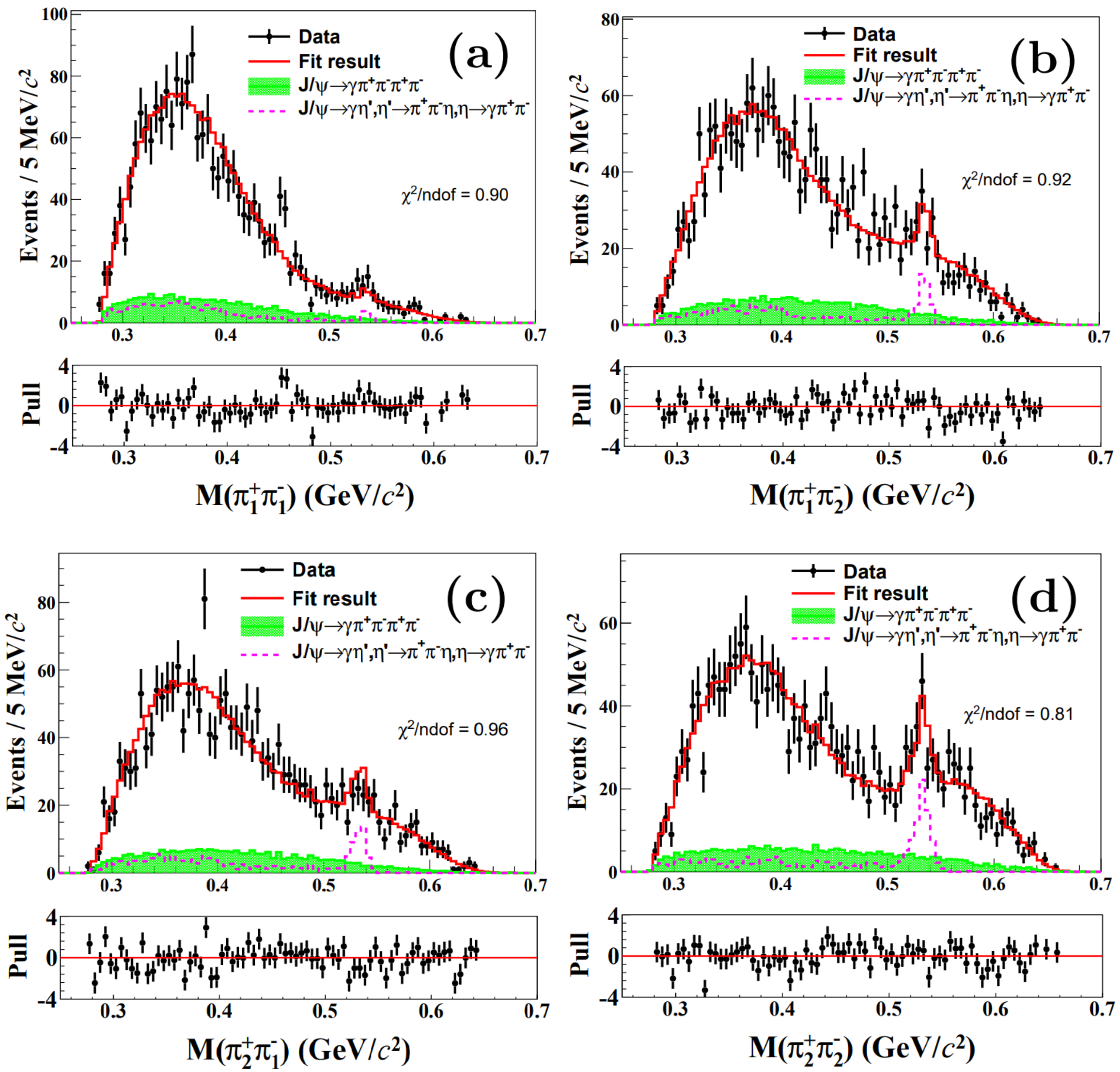
➡ The inverse ρ propagator

$$\Gamma_\rho(s) = \frac{M_\rho}{\sqrt{s}}\left(\frac{s - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2}\right)^{\frac{3}{2}}\Gamma_\rho$$

- To simplify the model for validation, we assuming $c_1 - c_2 = 1$, the fit yields

$$\boldsymbol{c_3 = 1.22 \pm 0.29 \pm 0.04},$$

which is consistent with the theoretical expectation of $c_3 = 1$



Summary

- Precise TFF of η/η' are needed as input to $(g-2)_\mu$.
- BESIII is a light meson factory, and many studies on the η/η' TFF have been conducted recently
  - $\eta/\eta' \to \gamma e^+e^-$ — Phys. Rev. D 92, 012001 (2015), Phys. Rev. D 109, 072001 (2024)
  - $\eta' \to \pi^+\pi^- l^+ l^-$ — arXiv:2402.01993 (hep-ex)
  - $\eta' \to \pi^+\pi^-\pi^+\pi^-$ — Phys. Rev. D 109, 032006 (2024)
- There are also many ongoing studies on TFFs, and more results are expected to come soon!
  - $\eta' \to e^+e^-\omega$
  - $\eta' \to \pi^+\pi^-\eta, \eta \to \gamma l^+ l^-$
  - $\eta \to \pi^+\pi^- l^+ l^-$
  - $e^+e^- \to e^+e^-\eta/\eta'$

  ……

Thank You !