Comparison of molecular and compact states for the Tcc(3875) and X(3872)

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1) Dai, Song & Oset , Evolution of genuine states to molecular ones: The Tcc(3875) case, arXiv: 2306.01607, PLB846(2023)138200

2) Song, Dai & Oset , Evolution of compact states to molecular ones with coupled channels: The X(3872) case, arXiv: 2307.02382, PRD108(2023)114017

1. Tcc(3875)

Dai, Song & Oset, Evolution of genuine states to molecular ones: The Tcc(3875) case, PLB846 (2023) 138200

Motivation

The dilemma between molecular states and compact (genuine) quark states is the subject of a **continuous debate** in hadron physics.

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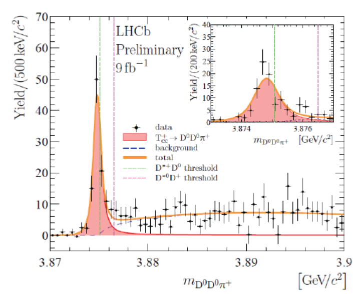
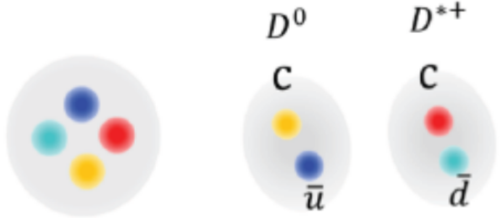
here we take $T_{cc}(3875)$ as an example

LHCb experiment

Nature Physics 18 (2022) 751;
Nature Communication 13 (2022) 3351

$T^+_{cc}(3875)$ $(cc\bar{u}\bar{d})$





Its mass and width:

$M_{T_{cc}} = M_{D^{*+}D^0} + \delta m_{\text{exp}}$

$\Gamma = 48 \pm 2^{+0}_{-14}$ keV

$M_{D^{*+}D^0} = 3875.09$ MeV

$\delta m_{\text{exp}} = -360 \pm 40^{+4}_{-0}$ keV

compact (genuine) states? **molecular states?**

mixture?

We can see the debate ...

Various models for Tcc(3875)

Molecular state

PLB826(2022)136897; CTP73(2021)125201;
PRD104(2021)114015; PRD104(2021)116010;
AHEP2022(2022)9103031; EPJC82(2022)581;
PLB829(2022)137052; PRD105(2022)014024;
PLB833(2022)137290; EPJC82(2022)313;
PRD105(2022)054015; EPJC82(2022)144;
JHEP06(2022)057; EPJA58 (2022)131;
Phys Rep 1019 (2023)1;
NPB985(2022)115994;PLB833(2022)137391;
EPJC82(2022)724;PRD105(2022)034028;
PLB841(2023)137918
. . . . . .

Compact state

PRD37(1988)744; ZPC57(1993)273;
ZPC61(1994)271; PLB393(1997)119;
PLB123(1983)449; ZPC30(1986)457;
PRD105(2022)014021; EPJA58(2022)110;
. . . . . .

a mixture

PRD105(2022)014007;
Few Body Syst 35 (2004)175;
. . . . . .

debate $\Longrightarrow$ the nature of molecular or compact or mixture?

In the present work

- We develop the general formalism in single channel calculation

- Application to Tcc(3875)

We start with a compact state proving that in the limit of small binding the state becomes purely molecular.

The conclusions are general.

develop general formalism

in single-channel calculation

Dai, Song, Oset, PLB846(2023)138200 [arXiv: 2306.01607]

- assume a hadronic state of bare mass m_R (original compact state)
- simplify $\Longrightarrow$ consider an $I = 0$ state in the single-channel calculation ($D^{*+}D^0$).

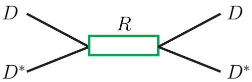


Fig. 1. DD^* amplitude based on the genuine resonance R.

$$\tilde{t}_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R} \quad (0.1)$$

This amplitude is not unitarity.

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It is rendered **unitary** immediately by **iterating** the diagram of Fig. 1 as shown in Fig. 2

insert the DD^* selfenergy in the propagator

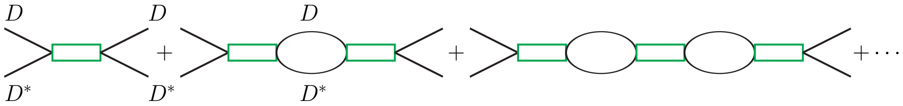


Fig. 2. implementing unitarity of the DD^* amplitude.

$$t_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R - \tilde{g}^2 G_{DD^*}(s)} \tag{0.2}$$

$\Downarrow$

selfenergy

we choose to regularize with a sharp cutoff

$$G_{DD^*}(s) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\,\omega_1\,\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon} \tag{0.3}$$

where $\omega_i = \sqrt{\boldsymbol{q}^2 + m_i^2}$. The selfenergy is negative, we take $s_R = m_R^2$ above the DD^* threshold.

The condition that a pole appears at s_0 (the square of the mass of the physical state) below the threshold

$$s_0 - s_R - \tilde{g}^2 G_{DD^*}(s_0) = 0 \tag{0.4}$$

$$\Downarrow$$

the value of $\tilde{g}^2$ can be obtained

Molecular probability

PRD81(2010)014029; IJMPA28(2013)1330045

$$P = -g^2 \frac{\partial G}{\partial s}\Big|_{s=s_0}, \qquad \widetilde{g}^2 = \lim_{s\to s_0}(s-s_0)\frac{\widetilde{g}^2}{s - s_R - \widetilde{g}^2 G_{DD^*}(s)} = \frac{\widetilde{g}^2}{1-\widetilde{g}^2\frac{\partial G}{\partial s}}\Big|_{s=s_0},$$

Thus the molecular probability is

$$P = -\frac{\widetilde{g}^2\frac{\partial G}{\partial s}}{1-\widetilde{g}^2\frac{\partial G}{\partial s}}\Big|_{s=s_0} \tag{0.5}$$

Several limits:

1) $\tilde{g}^2 \to 0$, $P \to 0$, the compact state survives
2) $\tilde{g}^2 \to \infty$, $P \to 1$, the state becomes pure molecular
3) $s_0 \to s_{\rm th}$, $P \to 1$, the state becomes pure molecular which is interesting

$\Downarrow$

It is a consequence of unitarity and analyticity of the t and G functions.

When the binding energy goes to zero, the state becomes fully molecular, the compact component has been fagocitated by the molecular component.

3. Results scenario 1 ($\beta = 0$)

for molecular probability $\sqrt{s_R} = \sqrt{s_{th}} + \Delta\sqrt{s_R}$

$\sqrt{s_0} \Rightarrow$ assumed value of the energy of the bound state

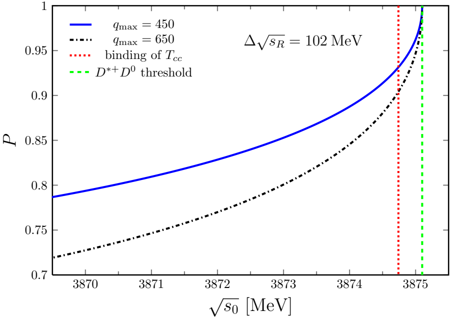


Fig. 3. as a function of $\sqrt{s_0}$ with $\Delta\sqrt{s_R} = 102$ MeV [PRL119(2017)202002]

1) when $\sqrt{s_0} \to \sqrt{s_{th}}$, $P \to 1$.

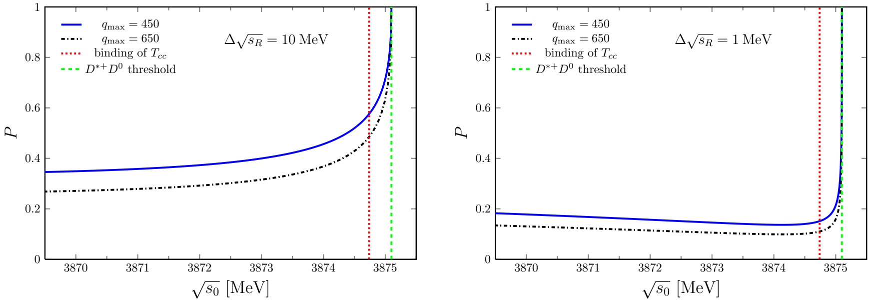
2) for $q_{max} = 450$ MeV, at $s_0^{exp} = \sqrt{s_{th}} - 0.36$ MeV, $P \sim 0.9$ $\Rrightarrow$ indicating that the original compact state has evolved to become practically a molecular state.

"scale" with $\Delta\sqrt{s_R} = 10$ MeV and $\Delta\sqrt{s_R} = 1$ MeV

$\Downarrow$

blue curve for $q_{\rm max} = 450$ MeV

It is seen that the "scale" shows up clearly.



1) when $\sqrt{s_0} \to \sqrt{s_{\rm th}}$, molecular probability $P \to 1$, the same trend.
2) at $s_0^{\rm exp}$: $\Delta\sqrt{s_R} = 1$ MeV, $P \sim 0.15 \Rightarrow$ indicating that the state remains mostly nonmolecular.
3) It can be seen that $\Delta\sqrt{s_R} = 10$ MeV, $P \sim 0.55 \Rightarrow$ as $\Delta\sqrt{s_R}$ becomes smaller, P is decreasing.

The binding energy by itself cannot give a proof of the nature of the state.

So what other magnitudes can really tell us about the nature of the state?

- **Scattering length**
- **Effective range**

For scattering length & effective range

The unitarity of the t_{DD^*,DD^*} amplitude

$$\mathrm{Im}\, t^{-1} = \mathrm{Im}\left(\frac{s-s_R}{\tilde{g}^2} - G_{DD^*}(s)\right) = -\mathrm{Im}\, G_{DD^*}(s) = \frac{k}{8\pi\sqrt{s}} \tag{0.6}$$

with k the meson-meson on shell momentum.
The relationship with the f^{QM} [Quantum Mechanics]

$$t = -8\pi\sqrt{s} f^{\mathrm{QM}} \simeq -8\pi\sqrt{s}\,\frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik} \tag{0.7}$$

It is easy to induce

$$-\frac{1}{a} = \frac{s_{\mathrm{th}} - s_R}{\tilde{g}^2} - \mathrm{Re}\, G_{DD^*}(s_{\mathrm{th}}) \tag{0.8}$$

$$r_0 = 2\frac{\sqrt{s}}{\mu}\frac{\partial}{\partial s}\left\{(-8\pi\sqrt{s})\left(\frac{s-s_R}{\tilde{g}^2} - \mathrm{Re}\, G_{DD^*}(s)\right)\right\}\Big|_{s=s_{\mathrm{th}}} \tag{0.9}$$

scattering length and effective range

$$q_{\max} = 450 \text{ MeV at } s_0^{\exp} = \sqrt{s_{th}} - 0.36 \text{ MeV}$$

$\Delta\sqrt{s_R}$ [MeV]	a [fm]	r_0 [fm]
0.1	0.87	-114.07
0.3	1.19	-79.33
1	2.10	-38.20
5	4.62	-9.26
10	5.74	-4.51
50	7.25	-0.47
70	7.39	-0.17
102	7.51	0.06

It can be seen that as $\Delta\sqrt{s_R}$ becomes smaller (decreasing the P), a becomes smaller and smaller and r_0 grows indefinitely.

The lesson we draw is the a and r_0 are very **useful** to determine the molecular probability of the state.

scenario 2 (hybrid)

assume a **mixture** of the compact state and the molecular one, by taking a potential

$$V' = V + \frac{\tilde{g}^2}{s - s_R} \tag{0.10}$$

It is easy to generalize the probability

$$P = -\frac{\left[\tilde{g}^2 + (s - s_R)V\right] \frac{\partial G}{\partial s}}{1 - \left[\tilde{g}^2 + (s - s_R)V\right] \frac{\partial G}{\partial s} - VG}\Big|_{s=s_0} \tag{0.11}$$

The pole at s_0 appears when

$$s_0 - s_R - \left[\tilde{g}^2 + (s_0 - s_R)V\right] G(s_0) = 0 \tag{0.12}$$

scenario 3 (direct interaction)

just a test for short of binding, we take a potential

$$1 - VG(s_{\text{th}}) = 0\,, \qquad V = \beta V_{\text{LHG}} \tag{0.13}$$

where V_{LHG} is the attractive potential from the local hidden gauge approach [Phys. Rep. 164, 217; Phys. Rep. 381, 1; Phys. Rep. 161, 213; Phys. Rev. D 79, 014015]

$\Delta\sqrt{s_R}$ [MeV]	$\beta = 0$	$\beta = 0.74$
10	0.58	0.94
20	0.73	0.97
50	0.87	0.99

There is some attractive interaction, the molecular probability increases appreciably.

Extension to X(3872)

Song, Dai, Oset, Evolution of compact states to molecular ones with coupled channels: The case of the X(3872), PRD108(2023)114017

Develop general formalism (coupled-channel)

Same as above in single-channel for Tcc(3875), we start with a bare mass m_R in coupled-channel for X(3872)

$$|D^*\bar{D},\ I=0\rangle = \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^{*+}D^-) \quad (0.14)$$

$$t_{D^*\bar{D}}(I=0) = \frac{\tilde{g}^2}{s - s_R} \quad (0.15)$$

If we decide to have a bound state at s_0, once given s_R, we can obtain $\tilde{g}^2$ as

$$\tilde{g}^2 = \left.\frac{s - s_R}{\frac{1}{2}G_1 + \frac{1}{2}G_2}\right|_{s_0}. \quad (0.16)$$

The loop functions G_i of $i = 1$ for $\bar{D}^0 D^{*0}$ and $i = 2$ for $D^- D^{*+}$.

Couplings and probabilities

$$g_1^2 = \lim(s - s_0)T_{11}; \qquad g_2^2 = \lim(s - s_0)T_{22} \tag{0.17}$$

$$g_2 = g_1 \lim(s - s_0)\frac{T_{21}}{T_{11}}$$

By using L'Hospital's rule we easily find

$$g_1^2 = \left.\frac{\frac{1}{2}\tilde{g}^2}{1 - \frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1 + G_2)}\right|_{s_0}; \qquad g_2 = g_1 \tag{0.18}$$

$$P_1 = -g_1^2\frac{\partial G_1}{\partial s}|_{s_0} = -\left.\frac{\frac{1}{2}\tilde{g}^2\frac{\partial G_1}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1 + G_2)}\right|_{s_0} \tag{0.19}$$

$$P_2 = -g_2^2\frac{\partial G_2}{\partial s}|_{s_0} = -\left.\frac{\frac{1}{2}\tilde{g}^2\frac{\partial G_2}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1 + G_2)}\right|_{s_0}$$

The X(3872) is closer to the $D^{*0}\bar{D}^0$ $(i = 1)$, we find

1) when $\tilde{g}^2 \to 0$, $P_1 \to 0$, $P_2 \to 0$, compact state.

2) when $\tilde{g}^2 \to \infty$, $P_1 + P_2 = 1$, completely molecular.

3) when $s_0 \to s_{\text{th}1}$, $P_1 \to 1$, $P_2 \to 0$, completely molecular state dominated by the $D^{*0}\bar{D}^0$ (i=1) component.

$\Uparrow$

interesting case

We should stress that even if $P_1 \to 1$, $P_2 \to 0$, in strong interaction of zero range what matters is the wave function at the origin and the $D^{*0}\bar{D}^0$ and $D^{*+}D^-$ components become equally important [PRD80(2009)014003; PRD81(2009)014029]

Inclusion of direct interaction

In the local hidden gauge approach the interaction comes from the exchange of vector mesons [Phys. Rept. 164 (1988) 217; Phys. Rept. 381 (2003)1; Phys. Rept. 61 (1988) 213; PRD79(2009)014015]

$$\frac{1}{2}V = -g'^2 \frac{4\, m_{D^{*0}} m_{D^0}}{m_V^2}. \qquad (0.20)$$

with $g' = \frac{m_{\rm V}}{2f_\pi}$, $m_{\rm V} = 800$ MeV, $f_\pi = 93$ MeV.

$$\frac{\tilde{g}^2}{s - s_R} \to \frac{\tilde{g}^2}{s - s_R} + \beta V \qquad (0.21)$$

Scattering length and effective range

At first threshold

$$-\frac{1}{a_1} = (-8\pi\sqrt{s})\left[\frac{s-s_R}{\frac{1}{2}[\tilde{g}^2+\beta V(s-s_R)]} - \mathrm{Re}G_1 - G_2\right]\Bigg|_{s_{th1}}, \tag{0.22}$$

$$r_{0,1} = 2\frac{\sqrt{s}}{\mu_1}\frac{\partial}{\partial s}\left\{(-8\pi\sqrt{s})\left[\frac{s-s_R}{\frac{1}{2}[\tilde{g}^2+\beta V(s-s_R)]} - \mathrm{Re}G_1 - G_2\right]\right\}\Bigg|_{s_{th1}}, \tag{0.23}$$

At second threshold

$$-\frac{1}{a_2} = (-8\pi\sqrt{s})\left[\frac{s-s_R}{\frac{1}{2}[\tilde{g}^2+\beta V(s-s_R)]} - \mathrm{Re}G_2 - G_1\right]\Bigg|_{s_{th2}}, \tag{0.24}$$

$$r_{0,2} = 2\frac{\sqrt{s}}{\mu_2}\frac{\partial}{\partial s}\left\{(-8\pi\sqrt{s})\left[\frac{s-s_R}{\frac{1}{2}[\tilde{g}^2+\beta V(s-s_R)]} - \mathrm{Re}G_2 - G_1\right]\right\}\Bigg|_{s_{th2}}, \tag{0.25}$$

with μ_i the reduced mass of the channel.

$\Delta\sqrt{s_R} = 100$ MeV, Molecular probability of P_1 and P_2

scenario 1 ($\beta = 0$) [red lines: $D^0 D^{*0}$ threshold]

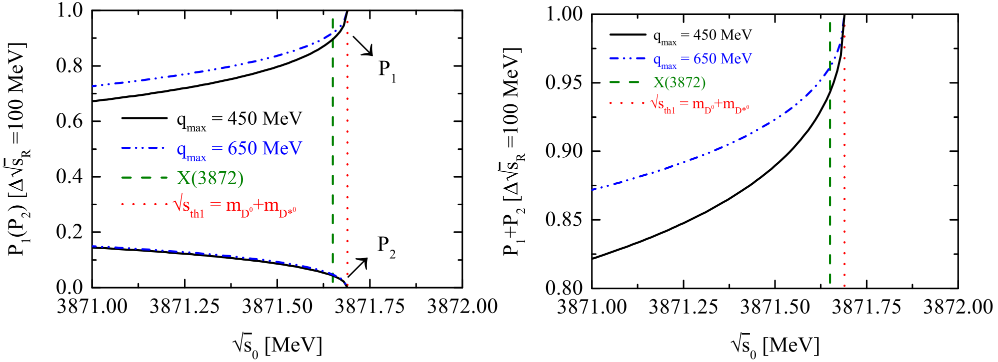
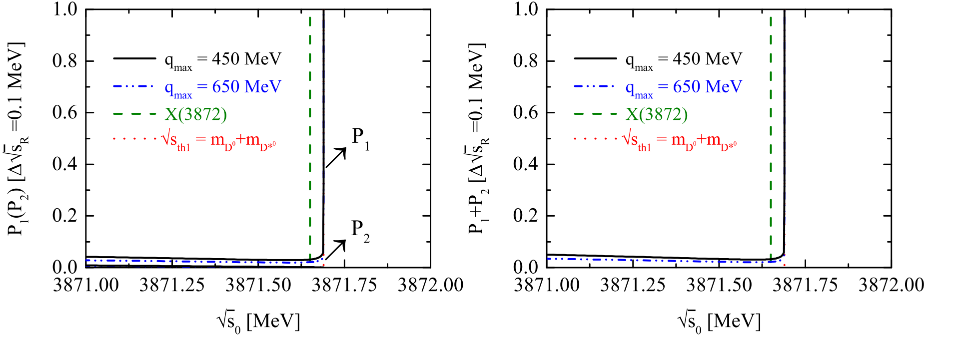


Fig. 4. **as a function of** $\sqrt{s_0}$.

1) when $\sqrt{s_0} \to s_{th1}$, $P_1 \to 1$, $P_2 \to 0$, $P_1 + P_2 \to 1$
2) **at the energy of** $X(3872)$, **the probability** $P_1 \sim 0.9$ **and** $P_2 \sim 0.05$, $P_1 + P_2 \sim 0.95$

It is also seen that the "scale" shows up clearly.

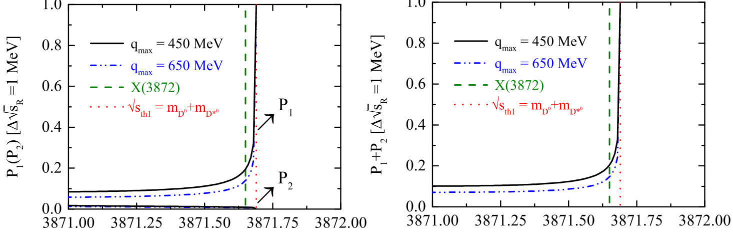
$\Delta\sqrt{s_R} = 0.1$ MeV ($\beta = 0$)



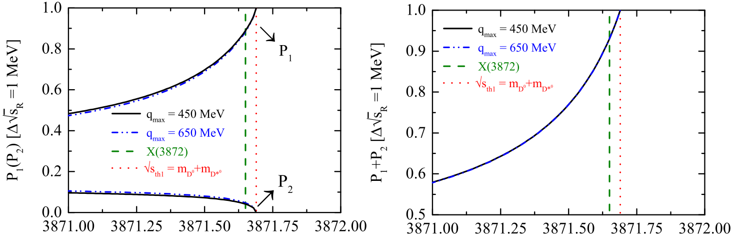
For $\Delta\sqrt{s_R} = 0.1$ MeV, we see that the $P_1 + P_2$ is around 0.02, indicating that the induced molecular component is negligible.

The conclusion: The binding energy by itself does not give us the molecular probability. It is possible to have a very small binding and still have a negligible molecular component.

$\Delta\sqrt{s_R} = 1$ MeV ($\beta = 0$)



$\Delta\sqrt{s_R} = 1$ MeV and $\beta \neq 0$ (mixture) $\Longleftarrow$ by adding the direct interaction



The presence of a reasonable direct meson-meson interaction has as a consequence a drastic increase in the molecular probability of the state.

What happens for scattering length and effective range

Table 1: $q_{max} = 450$ MeV $(\beta = 0)$

$\Delta\sqrt{s_R}$	a_1 [fm]	$r_{0,1}$ [fm]	a_2 [fm]	$r_{0,2}$ [fm]
0.1	1.42	-663.61	$0.0073 - i\,0.00003$	$-664.79 - i\,1.56$
0.3	3.16	-273.51	$0.0176 - i\,0.00020$	$-273.04 - i\,1.56$
1	7.48	-89.71	$0.0530 - i\,0.00180$	$-88.46 - i\,1.56$
10	18.45	-9.68	$0.3957 - i\,0.10756$	$-8.10 - i\,1.56$
50	21.35	-2.29	$0.7558 - i\,0.58190$	$-0.68 - i\,1.56$
100	21.78	-1.37	$0.7818 - i\,0.78157$	$0.25 - i\,1.56$

1) $r_{0,1} = -5.34$ fm LHCb data in PRD102(2020)092005
$-2.78 \text{ fm} < r_{0,1} < 1 \text{ fm}$, $a_1 \approx 28$ fm in PLB833(2022)137290
2) $\Delta\sqrt{s_R} = 0.1$ MeV, a_1, a_2 become small, and most important, $r_{0,1}$, $r_{0,2}$ become extremely large, where we had a negligible molecular component. $\Longrightarrow$ enough to discard this scenario.
3) $\Delta\sqrt{s_R} = 100$ MeV, would be basically acceptable, but $P \to 1$.

scenario 2 (hybrid)

Table 2: $q_{\max} = 450$ MeV ($\beta \neq 0$)

$\Delta\sqrt{s_R}$	a_1[fm]	$r_{0,1}$[fm]	a_2[fm]	$r_{0,2}$[fm]
0.1	15.60	-24.97	$0.7068 - i\,1.116$	$1.17 - i\,1.56$
0.3	19.65	-7.13	$0.7060 - i\,1.118$	$1.16 - i\,1.56$
1	21.38	-2.30	$0.7024 - i\,1.125$	$1.14 - i\,1.56$
10	22.13	-0.63	$0.7818 - i\,0.780$	$-3.62 - i\,1.56$
100	22.21	-0.47	$0.7385 - i\,1.038$	$1.15 - i\,1.56$

$r_{0,1} = -5.34$ fm LHCb data in PRD102(2020)092005

-2.78 fm $< r_{0,1} < 1$ fm, $a_1 \approx 28$ fm in PLB833(2022)137290

1) $\Delta\sqrt{s_R} = 0.1$ MeV, a_1 and $r_{0,1}$ are still unacceptable.
2) $\Delta\sqrt{s_R} = 1$ MeV, acceptable with the current uncertainty in the experimental values
$\Longrightarrow$ This scenario with $P_1 + P_2 \sim 0.95$ can not be discarded.

scenario 3

Table 3: $\tilde{g}^2 = 0$ and $\Delta\sqrt{s_R} = 1$ MeV at threshold in different $q_{\max}$

$q_{\max}$ [MeV]	a_1 [fm]	$r_{0,1}$ [fm]	a_2 [fm]	$r_{0,2}$ [fm]
450	22.22	-0.449	$0.736 - i\,1.04$	$1.17 - i\,1.56$
650	22.07	-0.763	$0.765 - i\,0.94$	$0.82 - i\,1.56$

1) the $r_{0,1}$ (-0.449 fm) is appreciably different here versus -2.30 fm in Table 2.
$\Longrightarrow$ It is thus clear that an improvement in the measured value of $r_{0,1}$ can shed further light on the issue.
2) There is extra information from a_2 and $r_{0,2}$, which are **drastically different** from those in Table 1 (only the compact state)

All this is telling us that the precise values of a_1, $r_{0,1}$ and a_2, $r_{0,2}$ are crucial to pin down the precise nature of the $X(3872)$.

Summary

We develop the general formalisms in single-channel and coupled-channel calculations.

As an application, we make the comparison of molecular and compact states for the Tcc(3875) and X(3872) in three different scenarios.

main conclusion:

The binding energy itself does not determine the compositeness of a state, but the additional information of the scattering length and effective range can provide an answer.

Thank you （谢谢）