Correlation functions for the $D_{s0}(2317)$ and N*(1535) : the inverse problem

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Construction of correlation functions

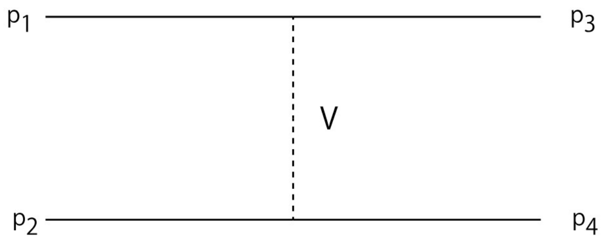
The channels in $D_{s0}(2317)$ production

The channels in the N*(1535) production

The inverse problem of getting information from the correlation functions

Discussion on experimental extraction of scattering parameters

The $D_{s0}(2317)$ state



$$D^0K^+,\ D^+K^0,\ \text{and}\ D_s^+\eta$$

$$V_{ij} = C_{ij}\, g^2 (p_1 + p_3)\cdot(p_2 + p_4);$$

$$T = [1 - VG]^{-1}V$$

$$g = \frac{M_V}{2f},\ M_V = 800\ \text{MeV},\ f = 93\ \text{MeV},$$

$$G_i(s) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3}\frac{\omega_1+\omega_2}{2\omega_1\omega_2}\frac{1}{s-(\omega_1+\omega_2)^2+i\epsilon}$$

$$C_{ij} = \begin{pmatrix} -\frac{1}{2}\left(\frac{1}{M_\rho^2}+\frac{1}{M_\omega^2}\right) & -\frac{1}{M_\rho^2} & \frac{2}{\sqrt{3}}\frac{1}{M_{K^*}^2} \\ & -\frac{1}{2}\left(\frac{1}{M_\rho^2}+\frac{1}{M_\omega^2}\right) & \frac{2}{\sqrt{3}}\frac{1}{M_{K^*}^2} \\ & & 0 \end{pmatrix}$$

Ikeno, Toledo, E. O. PLB 847, 138281

$$(p_1+p_3)\cdot(p_2+p_4) \to \frac{1}{2}\left[3s - (M^2+m^2+M'^2+m'^2) - \frac{1}{s}(M^2-m^2)(M'^2-m'^2)\right],$$

Projection in s-wave

Correlation functions

$$C(\boldsymbol{p}) = \int d^3\boldsymbol{r} S_{12}(\boldsymbol{r}) |\psi(\boldsymbol{r},\boldsymbol{p})|^2$$

$$S_{12}(r) = \frac{1}{(\sqrt{4\pi})^3 R^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

Modified Kookin Pratt formalism

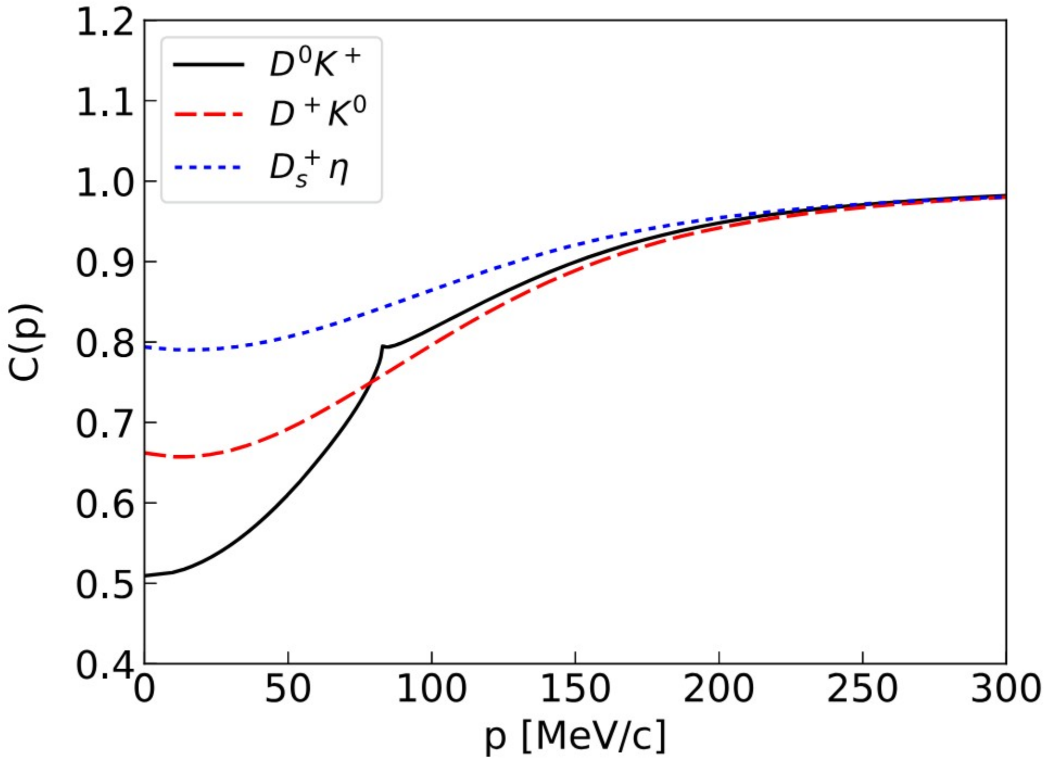
I.~Vidana, A.~Feijoo, M.~Albaladejo, J.~Nieves and E.~Oset Phys.Lett.B 846 (2023) 138201

$$C_{D^0K^+}(p_{K^+}) = 1 + 4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\, \theta(q_{\max} - p_{K^+}) \left\{ \left| j_0(p_{K^+} r) + T_{11}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2 + \omega_2 \left| T_{21}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 + \omega_3 \left| T_{31}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2 - j_0^2(p_{K^+} r) \right.$$

$$C_{D^+K^0}(p_{K^0}) = 1 + 4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\, \theta(q_{\max} - p_{K^0}) \left\{ \left| j_0(p_{K^0} r) + T_{22}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 + \omega_1 \left| T_{12}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2 + \omega_3 \left| T_{32}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2 - j_0^2(p_{K^0} r) \right\}$$

$$C_{D_s\eta}(p_\eta) = 1+4\pi \int_0^{+\infty} dr r^2 S_{12}(r)\ \theta(q_{\max} - p_\eta)$$

$$\left\{ \left| j_0(p_\eta r) + T_{33}(\sqrt{s})\, \widetilde{G}^{(3)}(s,r) \right|^2 + \omega_1 \left| T_{13}(\sqrt{s})\, \widetilde{G}^{(1)}(s,r) \right|^2 + \omega_2 \left| T_{23}(\sqrt{s})\, \widetilde{G}^{(2)}(s,r) \right|^2 - j_0^2(p_\eta r) \right\}$$



C(p) constructed with R=1m

$$\widetilde{G}^{(i)}(s,r) = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1^{(i)}(q)+\omega_2^{(i)}(q)}{2\omega_1^{(i)}(q)\,\omega_2^{i}(q)} \cdot \frac{j_0(qr)}{s - \left[\omega_1^{(i)}(q)+\omega_2^{(i)}(q)\right]^2 + i\epsilon}$$

Inverse problem

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ & V_{22} & V_{23} \\ & & 0 \end{pmatrix}$$

$$|DK, I=0\rangle = \frac{1}{\sqrt{2}}(D^+K^0 + D^0K^+)$$

$$|DK, I=1, I_3=0\rangle = \frac{1}{\sqrt{2}}(D^+K^0 - D^0K^+)$$

we will assume that the potential has isospin symmetry

we impose that $\langle I=0|V|I=1\rangle = 0$

$$V_{11} = V_{22}, \quad V_{13} = V_{23},$$

$$\langle DK, I=0|V|DK, I=0\rangle = V_{11} + V_{12}$$

$$\langle DK, I=1|V|DK, I=1\rangle = V_{11} - V_{12}$$

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ & V_{11} & V_{13} \\ & & 0 \end{pmatrix}$$

$$V_{11} = V'_{11} + \frac{\alpha}{M_V^2}(s-\bar{s}),$$

$$V_{12} = V'_{12} + \frac{\beta}{M_V^2}(s-\bar{s}),$$

$$V_{13} = V'_{13} + \frac{\gamma}{M_V^2}(s-\bar{s}),$$

Free parameters V'_{11}, V'_{12}, V'_{13}, α, β, γ, $q_{\rm max}$, and R

We do many fits to the data with the resampling technique to evaluate errors in the observables, assuming errors in the correlation functions of the order of 0,02

$$q_{\max} = 689.03 \pm 103.37 \text{ MeV}$$

$$R = 0.984 \pm 0.040 \text{ fm}\,.$$

We get a pole at $E = 2314.2 \pm 21.0$ MeV

$$T_{ij} \sim \frac{g_i g_j}{s - s_0}$$

$$g_1^2 = \lim_{s \to s_0} (s - s_0)\, T_{11}; \quad g_j = g_1 \lim_{s \to s_0} \frac{T_{1j}}{T_{11}}$$

$$P_i = -g_i^2 \frac{\partial G_i}{\partial s}|_{s=s_0}$$

$$T \equiv -8\pi \sqrt{s} f^{QM} \approx -8\pi \sqrt{s} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

$$-\frac{1}{a} = -8\pi \sqrt{s} T^{-1}|_{s=s_{\text{th}}},$$

$$r_0 = \frac{\partial}{\partial k^2}\, 2(-8\pi \sqrt{s}\; T^{-1} + ik)$$

$$= \frac{\sqrt{s}}{\mu} \frac{\partial}{\partial s}\, 2(-8\pi \sqrt{s} T^{-1} + ik)|_{s=s_{\text{th}}}$$

Table 1

Values of the couplings, probabilities, scattering lengths, and effective ranges.

channel i	$1: D^0 K^+$	$2: D^+ K^0$	$3: D_s^+ \eta$
g_i [MeV]	8556.08 ± 2707.16	8571.21 ± 2710.52	-6161.84 ± 6307.93
P_i	0.357 ± 0.133	0.306 ± 0.119	0.083 ± 0.070
a_i [fm]	0.720 ± 0.131	$(0.518 \pm 0.051) - i\,(0.120 \pm 0.030)$	$(0.213 \pm 0.014) - i\,(0.054 \pm 0.025)$
$r_{0,i}$ [fm]	-2.479 ± 0.824	$(-0.162 \pm 0.778) - i\,(2.520 \pm 0.329)$	$(-0.165 \pm 1.677) - i\,(0.171 \pm 0.663)$

The probabilities are similar as in the lattice work, A. Martínez Torres, E. Oset, S. Prelovsek, A. Ramos, J. High Energy Phys. 05 (2015)

Table 2

Same as Table 1 except with the use of the two correlation functions of $D^0 K^+$ and $D^0 K^0$.

channel i	$1: D^0 K^+$	$2: D^+ K^0$	$3: D_s^+ \eta$
g_i [MeV]	7773.42 ± 3462.55	7789.64 ± 3483.53	-5716.45 ± 5659.24
P_i	0.353 ± 0.198	0.301 ± 0.184	0.080 ± 0.134
a_i [fm]	0.707 ± 0.060	$(0.504 \pm 0.034) - i\,(0.110 \pm 0.015)$	$(0.259 \pm 0.067) - i\,(0.055 \pm 0.036)$
$r_{0,i}$ [fm]	-3.139 ± 1.299	$(-0.665 \pm 1.020) - i\,(2.386 \pm 0.341)$	$(0.336 \pm 0.858) - i\,(0.081 \pm 0.447)$

The equal couplings for $D^0 K^+$ and $D^+ K^0$ indicate that we have a D K isospin I=0 state

The chiral unitary approach for the $N^*(1535)$

Kaiser, Siegel and Weise
Phys.Lett.B 362 (1995) 23

$$K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \pi^+ n, \pi^0 p, \eta p$$

$$V_{ij} = -\frac{1}{4f^2} C_{ij}(k^0 + k'^0); \qquad f = 93 \text{ MeV} \qquad T = [1 - VG]^{-1} V$$

TABLE I. C_{ij} coefficients of Eq. (3).

C_{ij}	$K^0\Sigma^+$	$K^+\Sigma^0$	$K^+\Lambda$	$\pi^+ n$	$\pi^0 p$	ηp
$K^0\Sigma^+$	1	$\sqrt{2}$	0	0	$\frac{1}{\sqrt{2}}$	$-\sqrt{\frac{3}{2}}$
$K^+\Sigma^0$		0	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$K^+\Lambda$			0	$-\sqrt{\frac{3}{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$
$\pi^+ n$				1	$\sqrt{2}$	0
$\pi^0 p$					0	0
ηp						0

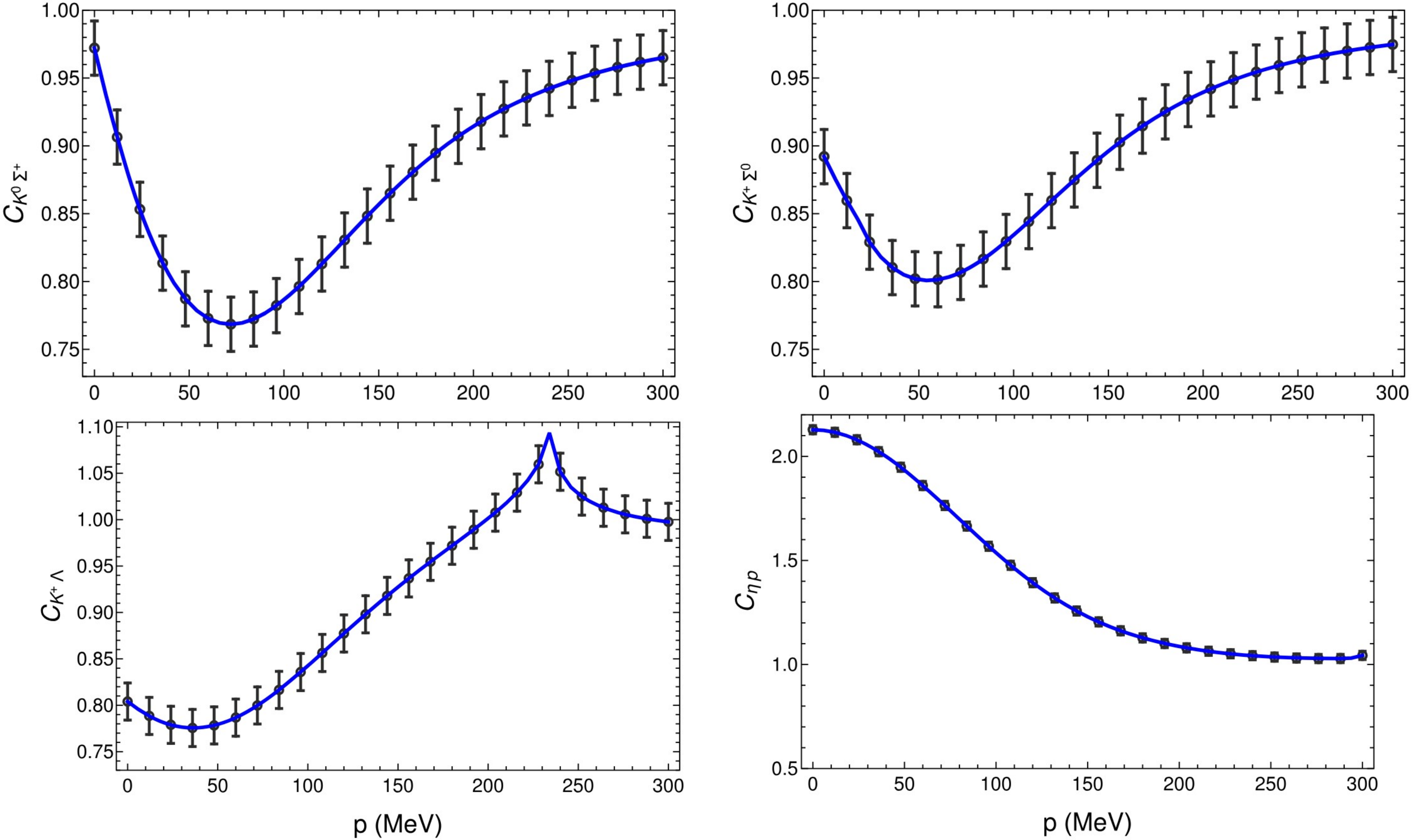
Correlation functions: Molina, Xiao, Liang, E. O. PRD 109, 054002

$$C_{K^0\Sigma^+}(p_{K^0}) = 1 + 4\pi\theta(q_{\max} - p_{K^0}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^0}r) + T_{K^0\Sigma^+,K^0\Sigma^+}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2$$
$$+ |T_{K^+\Sigma^0,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,K^0\Sigma^+}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{\eta p,K^0\Sigma^+}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^0}r)\},$$

$$C_{K^+\Sigma^0}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^+}r) + T_{K^+\Sigma^0,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,K^+\Sigma^0}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Lambda,K^+\Sigma^0}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{\eta p,K^+\Sigma^0}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+}r)\},$$

$$C_{K^+\Lambda}(p_{K^+}) = 1 + 4\pi\theta(q_{\max} - p_{K^+}) \int drr^2 S_{12}(r) \cdot \{|j_0(p_{K^+}r) + T_{K^+\Lambda,K^+\Lambda}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,K^+\Lambda}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,K^+\Lambda}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2$$
$$+ |T_{\eta p,K^+\Lambda}(E)\tilde{G}^{(\eta p)}(r;E)|^2 - j_0^2(p_{K^+}r)\},$$

$$C_{\eta p}(p_\eta) = 1 + 4\pi\theta(q_{\max} - p_\eta) \int drr^2 S_{12}(r) \cdot \{|j_0(p_\eta r) + T_{\eta p,\eta p}(E)\tilde{G}^{(\eta p)}(r;E)|^2$$
$$+ |T_{K^0\Sigma^+,\eta p}(E)\tilde{G}^{(K^0\Sigma^+)}(r;E)|^2 + |T_{K^+\Sigma^0,\eta p}(E)\tilde{G}^{(K^+\Sigma^0)}(r;E)|^2 + |T_{K^+\Lambda,\eta p}(E)\tilde{G}^{(K^+\Lambda)}(r;E)|^2 - j_0^2(p_\eta r)\}$$



R=1fm

Isospin symmetry

$$(K^+, K^0), (-\pi^+, \pi^0, \pi^-), (-\Sigma^+, \Sigma^0, \Sigma^-)$$

$$\left|K\Sigma, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}K^0\Sigma^+ + \sqrt{\frac{1}{3}}K^+\Sigma^0,$$

$$\left|K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right\rangle = -\sqrt{\frac{1}{3}}K^0\Sigma^+ + \sqrt{\frac{2}{3}}K^+\Sigma^0.$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V\left|K\Sigma, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = 0,$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V|K^+\Lambda\rangle = 0,$$

$$\left\langle K\Sigma, I=\frac{3}{2}, I_3=\frac{1}{2}\right|V|\eta p\rangle = 0.$$

channels $K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$

$$V_{ij} = -\frac{1}{4f^2}\tilde{C}_{ij}(k^0 + k'^0)$$

$$V_{ij} = \begin{pmatrix} V_{11} & \sqrt{2}(V_{11} - V_{22}) & V_{13} & V_{14} \\ & V_{22} & \frac{1}{\sqrt{2}}V_{13} & \frac{1}{\sqrt{2}}V_{14} \\ & & V_{33} & V_{34} \\ & & & V_{44} \end{pmatrix}$$

7 Cij free parameters plus qmax, and R

q_{max} (MeV)	R (fm)
637 ± 72	1.02 ± 0.02

TABLE III. Scattering lengths for channel i (in units of fm).

a_1	a_2
$(0.46 \pm 0.04) - (0.64 \pm 0.03)i$	$(0.32 \pm 0.01) - (0.35 \pm 0.02)i$
a_3	a_4
$(0.30 \pm 0.02) - (0.22 \pm 0.04)i$	$(-0.780 \pm 0.013) + (0 \pm 0)i$

$$\mathcal{P}_1 \simeq 0.12 - 0.23i, \qquad \mathcal{P}_2 \simeq 0.06 - 0.12i,$$
$$\mathcal{P}_3 \simeq 0.22 - 0.28i, \qquad \mathcal{P}_4 \simeq -0.34 - 0.24i$$

$$|\mathcal{P}_1| = 0.26, \qquad |\mathcal{P}_2| = 0.13,$$
$$|\mathcal{P}_3| = 0.35, \qquad |\mathcal{P}_4| = 0.42.$$

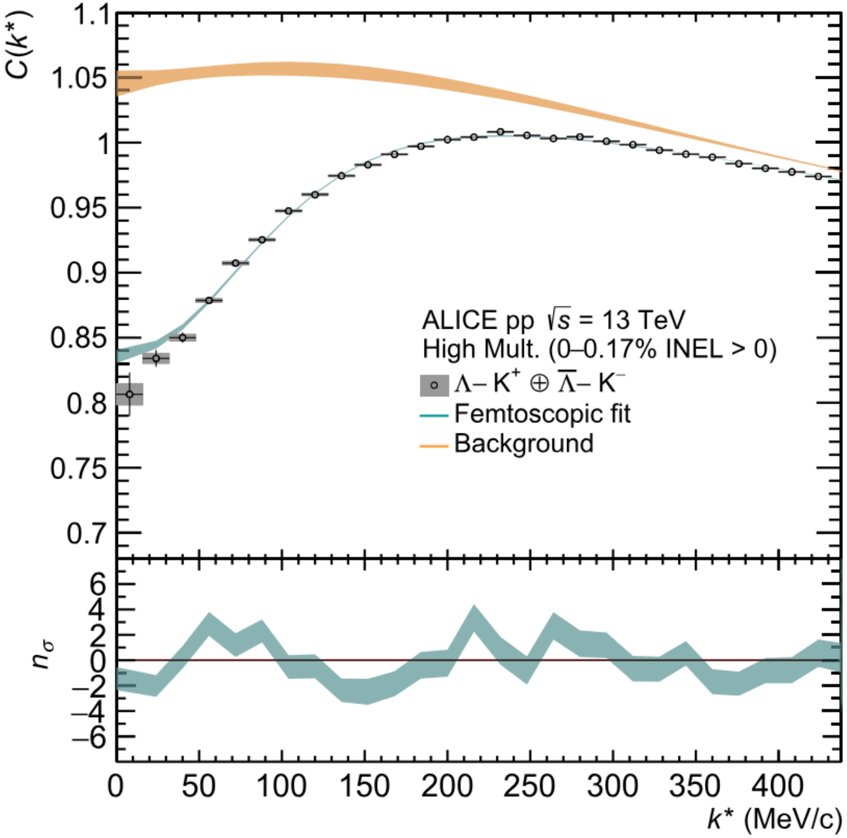
TABLE IV. Effective range parameters for channel i (in units of fm).

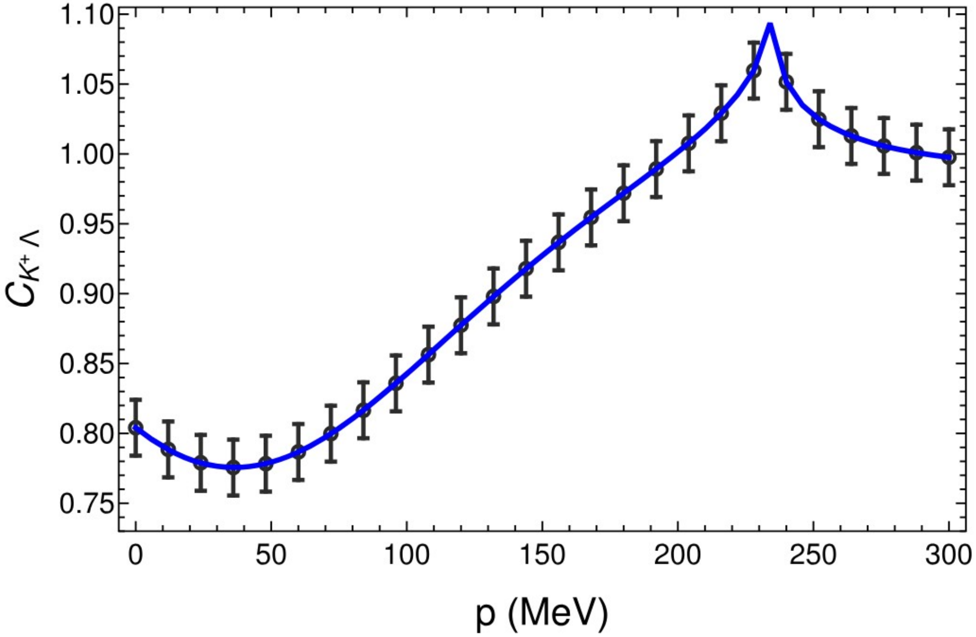
r_1	r_2	r_3	r_4
$(-1.1 \pm 0.2) - (2.7 \pm 0.2)i$	$(-6.2 \pm 1.4) + (8.8 \pm 0.5)i$	$(-2.8 \pm 0.3) - (0.3 \pm 0.6)i$	-1.48 ± 0.13

The couplings g1, g2 indicate I=1/2 state

TABLE V. Pole position and couplings (in units of MeV).

$\sqrt{s_p}$	g_1	g_2
$(1515 \pm 6) - (89 \pm 9)i$	$(3.7 \pm 0.3) - (1.04 \pm 0.13)i$	$(2.6 \pm 0.2) - (0.74 \pm 0.10)i$
	g_3	g_4
	$(3.6 \pm 0.2) - (0.28 \pm 0.05)i$	$(-2.68 \pm 0.13) + (1.4 \pm 0.2)i$





Experimental analysis of a, r, done with single channel

MESSAGE: the analysis must be done with coupled channels.

	Pair	Λ–K^+
$-a_3$	$\Re f_0$ (fm)	$-0.61 \pm 0.03(\text{stat}) \pm 0.03(\text{syst})$
	$\Im f_0$ (fm)	$0.23 \pm 0.06(\text{stat}) \pm 0.04(\text{syst})$
r_0	d_0 (fm)	$0.80 \pm 0.19(\text{stat}) \pm 0.18(\text{syst})$

a_3

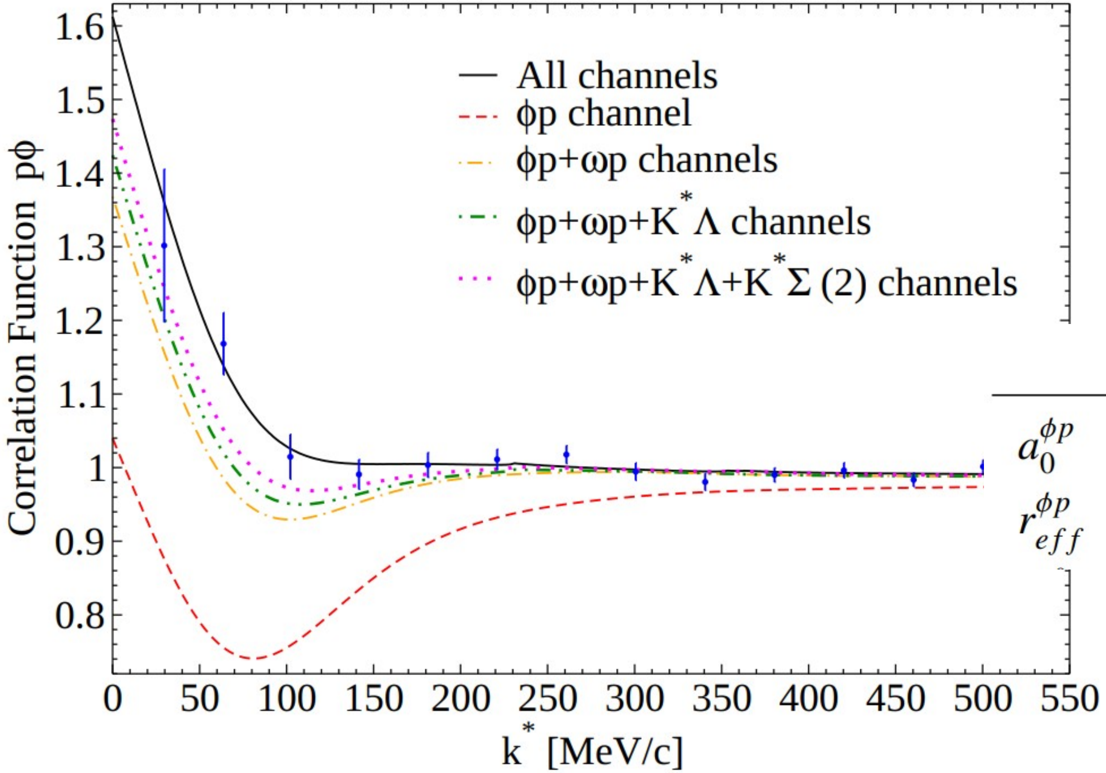
$(0.30 \pm 0.02) - (0.22 \pm 0.04)i$

r_3

$(-2.8 \pm 0.3) - (0.3 \pm 0.6)i$

Relevance of coupled channel analysis stressed in a recent paper

A.~Feijoo, M.~Korwieser and L.~Fabbietti,
%``Relevance of the coupled channels in the ϕp and ρ^0p Correlation Functions,''
[arXiv:2407.01128 [hep-ph]].



(from single channel analysis)

$$a_0^{\phi p} = (0.85 \pm 0.48) + i\,(0.16 \pm 0.19)\,\text{fm}$$

$$r_{eff}^{\phi p} = 7.85 \pm 1.80\,\text{fm}.$$

(coupled channels)

	Pure theoretical	Bootstrap
$a_0^{\phi p}$	$0.272 + i\,0.189$	$(-0.034 \pm 0.035) + i\,(0.57 \pm 0.09)$
$r_{eff}^{\phi p}$	$-7.20 - i\,0.09$	$(-8.06 \pm 2.57) + i\,(0.05 \pm 0.53)$

Conclusions

We explore the inverse problem of getting a,r_0, bound states associated, molecular probabilites

From the correlation functions of D^0K^+, D^+K^0, and $D_s^+\eta$
we find the existence of the $D_{s0}(2013)$ state

From the correlation functions of the channels $K^0\Sigma^+, K^+\Sigma^0, K^+\Lambda, \eta p$
we find the existence of the N*(1535) state

a, r_0 for all the channels are obtained with high precision.

ONE MUST AVOID USING SINGLE CHANNEL ANALYSIS TO DETERMINE a AND r_0