The $\Omega_c(3120)$ as a molecular state and its analogy with the $\Omega(2012)$

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R. Pavao and E. Oset, Eur. Phys. J. C78, 857 (2018).
N. Ikeno, G. Toledo, and E. Oset, Phys. Rev. D 101, 094016 (2020).
N. Ikeno, W. H. Liang, and E. Oset, Phys. Rev. D 109, 054023 (2024).

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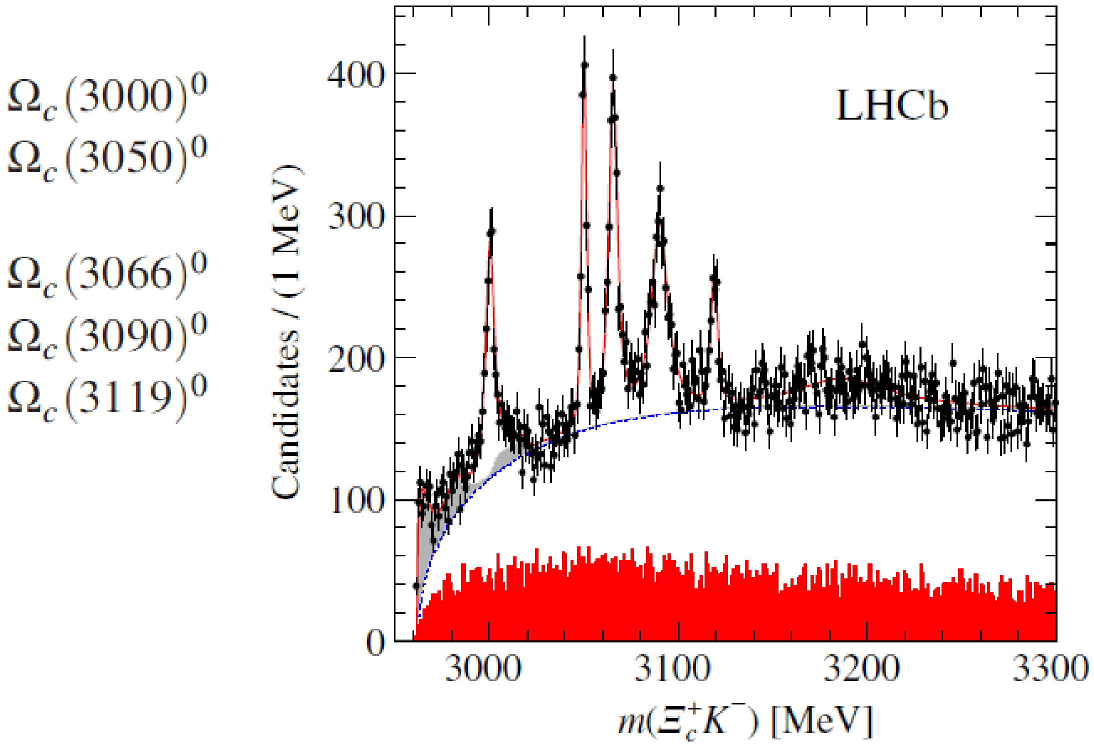
Discovery of excited Ω_c (=css) states by LHCb

- Five states of Ω_c in 2017

  Phys. Rev. Lett. 118, 182001 (2017)



- Two additional states of Ω_c in 2023

  Phys. Rev. Lett. 131, 131902 (2023)

Resonance	m (MeV)	Γ (MeV)
$\Omega_c(3000)^0$	3000.44 ± 0.07	3.83 ± 0.23
$\Omega_c(3050)^0$	3050.18 ± 0.04	0.67 ± 0.17
$\Omega_c(3065)^0$	3065.63 ± 0.06	3.79 ± 0.20
$\Omega_c(3090)^0$	3090.16 ± 0.11	8.48 ± 0.44
$\Omega_c(3119)^0$	3118.98 ± 0.12	0.60 ± 0.63
$\Omega_c(3185)^0$	3185.1 ± 1.7	50 ± 7
$\Omega_c(3327)^0$	3327.1 ± 1.2	20 ± 5

- Many theoretical studies to understand the Ω_c nature

  -Quark model, -Molecular picture, Lattice QCD,....

Works based on molecular perspective of Ω_c

G. Montana, A. Feijoo and A. Ramos, Eur. Phys. J. A 54, 64 (2018)
V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Phys. Rev. D97, 094035 (2018)

- Coupled channels of Meson-Baryon interaction
- Vector meson exchange interaction based on local hidden gauge approach

- Ω_c(3050), Ω_c(3090):

$J^P = 1/2^-$ states by both works

- **Ω_c(3119):**

Not obtained as a $J^p=1/2^-$ state by both works

$J^P = 3/2^-$ state by V. R. Debastiani et al.
- Couples mostly to $\Xi_c^*\bar{K}, \Omega_c^*\eta,$
- The mass was around 3125 MeV, and the width was zero

TABLE I. $J = 1/2$ states chosen and threshold mass in MeV.

States	$\Xi_c\bar{K}$	$\Xi_c'\bar{K}$	ΞD	$\Omega_c\eta$	ΞD^*	$\Xi_c\bar{K}^*$	$\Xi_c'\bar{K}^*$
Threshold	2965	3074	3185	3243	3327	3363	3472

TABLE II. $J = 3/2$ states chosen and threshold mass in MeV.

States	$\Xi_c^*\bar{K}$	$\Omega_c^*\eta$	ΞD^*	$\Xi_c\bar{K}^*$	$\Xi^* D$	$\Xi_c'\bar{K}^*$
Threshold	3142	3314	3327	3363	3401	3472

V. R. Debastiani et al., PRD 97, 094035 (2018)

Discovery of Ω(2012) by Belle: Excited state of Ω (=sss)

In 2018, Belle reported a new state Ω(2012) state: PRL121(2018)052003

$$M_{\Omega(2012)} = 2012.4 \pm 0.7 \pm 0.6 \text{ MeV}$$

$$\Gamma_{\Omega(2012)} = 6.4^{+2.5}_{-2.0} \pm 1.6 \text{ MeV}$$

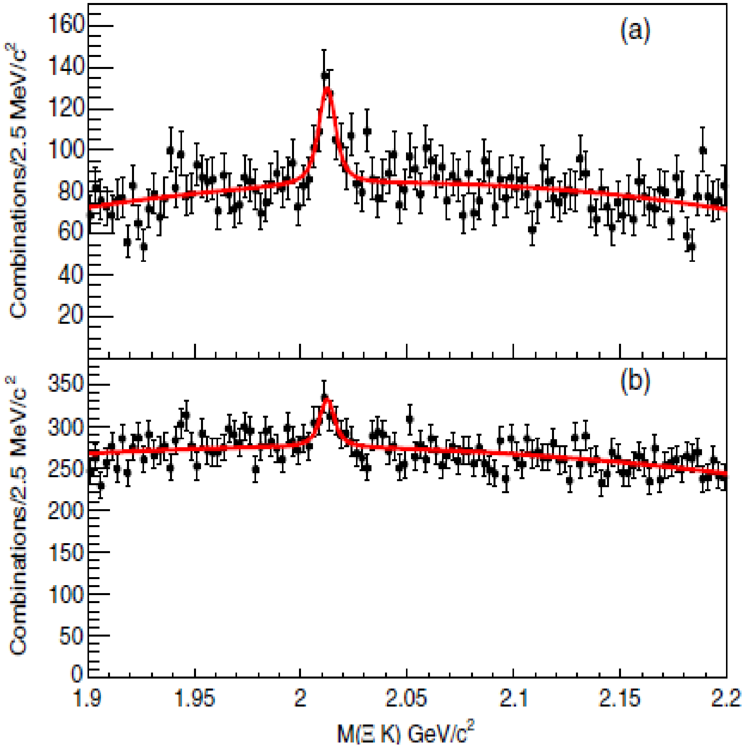


FIG. 2. The (a) $\Xi^0 K^-$ and (b) $\Xi^- K^0_S$ invariant mass distributions in data taken at the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ resonance energies.

This prompted many theoretical studies of the Ω(2012) nature

- Quark model pictures
- Molecular pictures based on the meson-baryon interaction

Ω(2012) is dynamically generated as a molecular state from the $\bar{K}\Xi^*$ and ηΩ coupled channels interaction

J^P = 3/2⁻ state

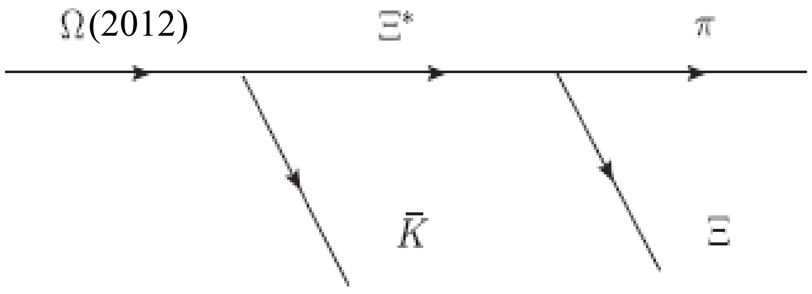
To test the molecular nature of Ω(2012), the Belle performed some tests, particularly looking at the decay into $\bar{K}\pi\Xi$, a signal of the $\bar{K}\Xi^*$ component of the state.

Belle experiment of Ω(2012)

- In **2019**, Belle reported a ratio of the Ω(2012) decay: PRD100(2019)032006

$$\mathcal{R}^{\Xi\pi K}_{\Xi K} = \frac{\mathcal{B}(\Omega(2012)\to\Xi(1530)(\to\Xi\pi)K)}{\mathcal{B}(\Omega(2012)\to\Xi K)} < 11.9\%$$

$\Omega(2012)\to\pi\bar{K}\Xi$

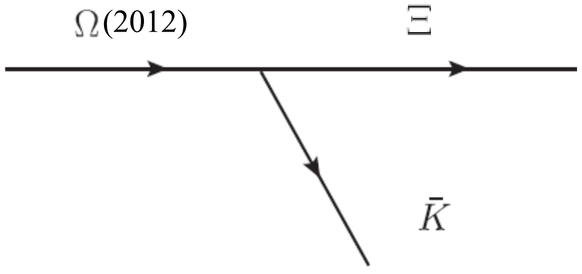


Three-body ΞπK decay width is significantly smaller than that of two-body ΞK decay width

⇒ Challenging result for the molecular picture nature, although not necessarily, as explained in

N. Ikeno, G. Toledo, E. Oset, PRD(2020)

J. Lu, C. Zeng, E. Wang, J. Xie, and L. Geng, EPJC(2020)



- In **2022**, a reanalysis of data (different cut): arXiv:2207.03090

$$\mathcal{R}^{\Xi\pi\bar{K}}_{\Xi\bar{K}} = 0.97 \pm 0.24 \pm 0.07$$

⇒ Consistent with the molecular interpretation e.g. R. Pavao and E. Oset, EPJC78(2018)

⇒ Strong support for the molecular picture

Analogy of $\Omega_c(3119)$ [$\Omega_c(3120)$ in PDG] and $\Omega(2012)$

Transition potential from coupled channels

- $\Omega_c(3120)$ with $J^p=3/2^-$

$$
\begin{array}{ccc}
 & \begin{array}{cc} \Xi_c^*\bar{K} & \Omega_c^*\eta \end{array} & \\
V = & \left(\boxed{\begin{array}{cc} F & \frac{4}{\sqrt{3}}F \\ \frac{4}{\sqrt{3}}F & 0 \end{array}}\right) & \begin{array}{c} \Xi_c^*\bar{K} \\ \Omega_c^*\eta \end{array}
\end{array}
$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0)$$

k^0, k'^0 the energies of initial and final states

V. R. Debastiani, J. M. Dias, W. H. Liang, E. Oset, PRD97 (2018)

- $\Omega(2012)$ with $J^p=3/2^-$

$$
\begin{array}{ccc}
 & \begin{array}{ccc} \bar{K}\Xi^* & \eta\Omega & \bar{K}\Xi \end{array} & \\
V = & \left(\begin{array}{ccc} 0 & 3F & \alpha q_{\rm on}^2 \\ 3F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{array}\right) & \begin{array}{c} \bar{K}\Xi^* \\ \eta\Omega \\ \bar{K}\Xi \end{array}
\end{array}
$$

- s-wave potentials between $\bar{K}\Xi^*$ and $\eta\Omega$:
  S. Sarkar, E. Oset, M.J. Vicente Vacas, NPA750(2005) 294

- $K\Xi$ channel is considered after the discovery of $\Omega(2012)$
  R. Pavao and E. Oset, EPJC78(2018)

=> In the $\Omega_c(3120)$ case, we can introduce the $\Xi_c\bar{K}$ channel in the D-wave by analogy with what was done in the $\Omega(2012)$ case

Our study of $\Omega_c(3120)$

N. Ikeno, W. H. Liang, and E. Oset, PRD109, 054023 (2024)

- We like to understand the nature of $\Omega_c(3120)$ based on the molecular picture

- Apply the analogy of the $\Omega(2012)$ studies to $\Omega_c(3120)$

- We retake the work by V. R. Debastiani et al., PRD 97 (2018), and we introduce the $\Xi_c \bar{K}$ channel in the D-wave

- We evaluate the mass, width of $\Omega_c(3120)$, and the partial decay widths into $\Xi_c \bar{K}$ and $\pi \Xi_c \bar{K}$

Coupled channels approach

3 channels: $\Xi_c^* \bar{K}, \Omega_c^* \eta$ (s-wave), $\Xi_c \bar{K}$ (d-wave)

- Transition potential: Ω_c^* $J^P=3/2^-$

$$
\begin{array}{cc}
 & \begin{array}{ccc} \Xi_c^* \bar{K} & \Omega_c^* \eta & \Xi_c \bar{K} \end{array} \\
V = & \left(\begin{array}{ccc} F & \frac{4}{\sqrt{3}} F & \alpha q_{\rm on}^2 \\ \frac{4}{\sqrt{3}} F & 0 & \beta q_{\rm on}^2 \\ \alpha q_{\rm on}^2 & \beta q_{\rm on}^2 & 0 \end{array} \right) \begin{array}{c} \Xi_c^* \bar{K} \\ \Omega_c^* \eta \\ \Xi_c \bar{K} \end{array}
\end{array}
$$

$$F = -\frac{1}{4f^2}(k^0 + k'^0) \qquad q_{\rm on} = \frac{\lambda^{1/2}(s, m_{\bar{K}}^2, m_{\Xi_c}^2)}{2\sqrt{s}}$$

k^0, k'^0 the energies of initial and final states

Attractive in the $\Xi_c^* \bar{K}$ channel

(↔ Diagonal potential is null in the $\Omega(2012)$ case)

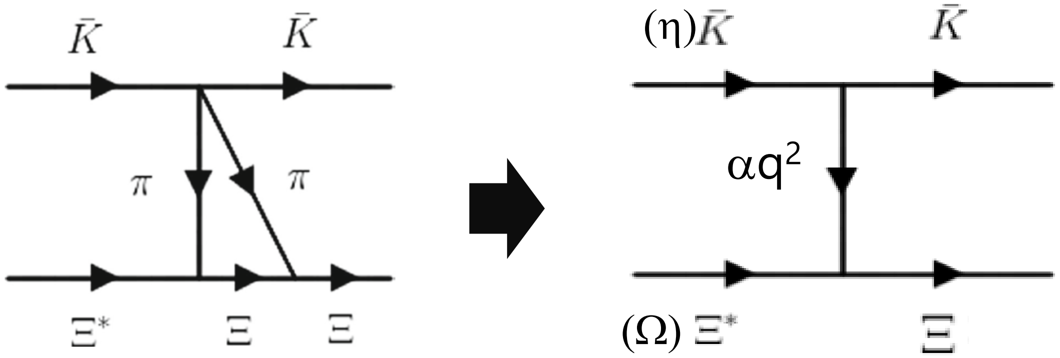
- Bethe-Salpeter equation:

$$T = [1 - VG]^{-1} V$$

- s-wave potentials between $\Xi_c^* \bar{K}, \Omega_c^* \eta$ taken from V. R. Debastiani, J. M. Dias, W. H. Liang, E. Oset, PRD97 (2018)

- d-wave potential between $\Xi_c \bar{K}$ and $\Xi_c^* \bar{K}$ or $\Omega_c^* \eta$ described in terms of α, β: free parameters

Similar way to R. Pavao and E. Oset, EPJC78(2018)



A possible d-wave diagram for the $\bar{K}\Xi^*$ -> $\bar{K}\Xi$ transition

We do not make a model

Effect of Ξ_c^* decay width

- Meson-Baryon loop function G: $\mathrm{diag}(G_{\Xi_c^*\bar{K}}, G_{\Omega_c^*\eta}, G_{\Xi_c\bar{K}})$

q_{max}: cut-off parameter

For s-wave channel of $\Xi_c^*\bar{K}, \Omega_c^*\eta$:

$$G_i(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2\omega_i(q)} \frac{M_i}{E_i(q)} \frac{1}{\sqrt{s}-\omega_i(q)-E_i(q)+i\varepsilon}$$

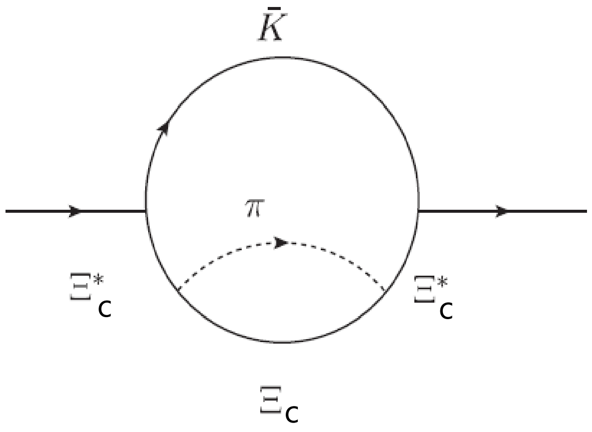
For d-wave channel of $\Xi_c\bar{K}$:

$$G_{\Xi_c\bar{K}}(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \left(\frac{q}{q_{\mathrm{on}}}\right)^4 \frac{1}{2\omega_{\bar{K}}(q)} \frac{M_{\Xi_c}}{E_{\Xi_c}(q)} \frac{1}{\sqrt{s}-\omega_{\bar{K}}(q)-E_{\Xi_c}(q)+i\varepsilon}$$

- We take into account Ξ_c^* decay width for $\Xi_c^* \to \pi\Xi_c$ decay

$$\tilde{G}_{\Xi_c^*\bar{K}}(\sqrt{s}) = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{1}{2\omega_{\bar{K}}(q)} \frac{M_{\Xi_c^*}}{E_{\Xi_c^*}(q)} \frac{1}{\sqrt{s}-\omega_{\bar{K}}(q)-E_{\Xi_c^*}(q)+i\frac{\sqrt{s'}}{2E_{\Xi_c^*}(q)}\Gamma_{\Xi_c^*}(\sqrt{s'})}$$

$$\Gamma_{\Xi_c^*}(M_{\mathrm{inv}}) = \frac{M_{\Xi_c^*}}{M_{\mathrm{inv}}} \left(\frac{q'}{q'_{\mathrm{on}}}\right)^3 \Gamma_{\mathrm{on}}\theta(M_{\mathrm{inv}}-m_\pi-M_{\Xi_c})$$



- Result with Ξ_c^* decay width: Accounts for $K\Xi_c$ and $\pi K\Xi_c$ decays
- Result without Ξ_c^* decay width: Only for $K\Xi_c$ decay

$\Rightarrow$ Estimate $\Omega_c(3120)$ decay width into $K\Xi_c$ and $\pi K\Xi_c$ decay channels

Calculated mass and width of $\Omega_c(3120)$

We make a fit to the experimental data by changing the α, β, q_{max} parameters.

Exp. data:

$$M_{\Omega_c(3120)} = 3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}\ \mathrm{MeV},$$
$$\Gamma_{\Omega_c(3120)} = 1.1 \pm 0.8 \pm 0.4\ \mathrm{MeV}.$$

We get a good fit to the data with the parameters

$$q_{\max} = 674.6\ \mathrm{MeV}, \quad \alpha = 2.6 \times 10^{-8}\ \mathrm{MeV}^{-3},$$
$$\beta = 2.0 \times 10^{-9}\ \mathrm{MeV}^{-3}.$$

Pole position appears at $(3119.13 + i0.54)$ MeV

$\Rightarrow$ The width is 1.08 MeV in agreement with the data

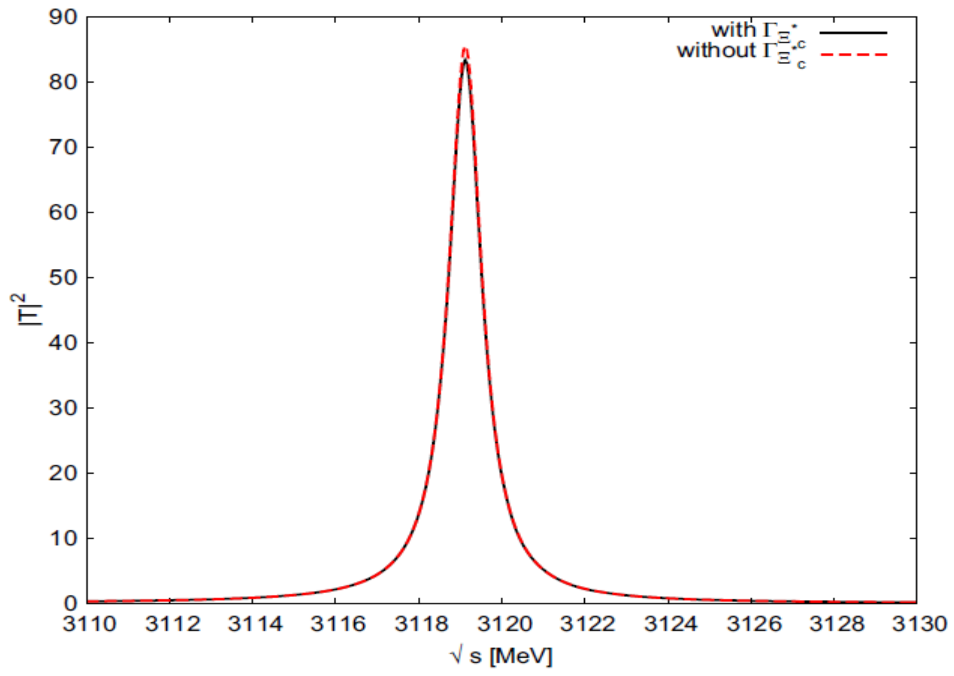


FIG. 2. $|T_{\Xi_c^* \bar{K}}|^2$ as a function of $\sqrt{s}$ in the cases with $\Gamma_{\Xi_c^*}$ and without $\Gamma_{\Xi_c^*}$, respectively.

Effect of Ξ_c^* decay width is very small in Figure

$\Rightarrow$ Unlike in the case of the $\Omega(2012)$, we cannot determine the $\Omega_c(3120) \rightarrow \Xi_c^*\bar{K} \rightarrow \Xi_c\pi\bar{K}$ in this way

$\Rightarrow$ We use the other way

Couplings g_i, wf at the origin(g_iG_i), probability $-g_i^2 \frac{\partial G_i}{\partial\sqrt{s}}$

- $\Omega_c(3120)$

	$\Xi_c^*\bar{K}$	$\Omega_c^*\eta$	$\Xi_c\bar{K}$
g_i	$2.06 - i0.02$	$2.09 - i0.01$	-0.138
g_iG_i	$-36.77 + i0.17$	$-17.64 + i0.06$	
$-g_i^2 \frac{\partial G_i}{\partial\sqrt{s}}$	0.63	0.10	

- $\Omega(2012)$ R. Pavao and E. Oset, EPJC78(2018)

	$\Xi^*\bar{K}$	$\Omega\eta$	$\Xi\bar{K}$
g_i	$2.01 + i0.02$	$2.84 - i0.01$	-0.29
g_iG_i	$-37.11 + i0.55$	$-24.95 + i0.38$	
$-g_i^2 \frac{\partial G_i}{\partial\sqrt{s}}$	0.64	0.16	

The couplings are defined at the pole as

$$g_i g_j = \lim_{\sqrt{s}\to\sqrt{s_p}} (\sqrt{s} - \sqrt{s_p}) T_{ij},$$

with $\sqrt{s_\text{p}}$ the energy of the pole

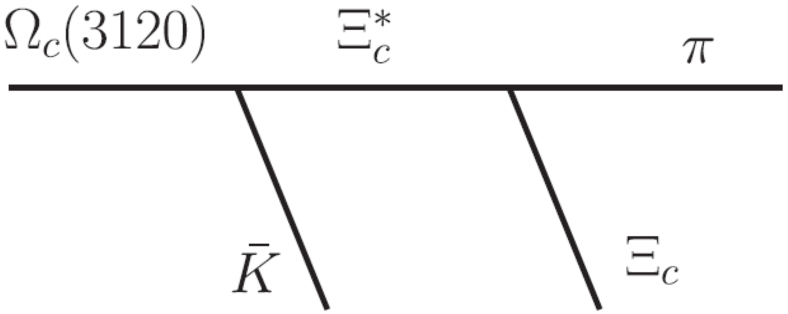
$\Xi_c^*\bar{K}$ has the largest probability of around 63% and $\Omega_c^*\eta$ around 10%

$\Rightarrow$ Largely molecular state

The results of $\Omega_c(3120)$ are similar to those of $\Omega(2012)$.

Partial decay widths into $\Xi_c \bar{K}\pi$ and $\Xi_c \bar{K}$

- Mechanism for $\Omega_c(3120)$ to decay into $\Xi_c\bar{K}\pi$ via primary decay into $\Xi_c^*\bar{K}$

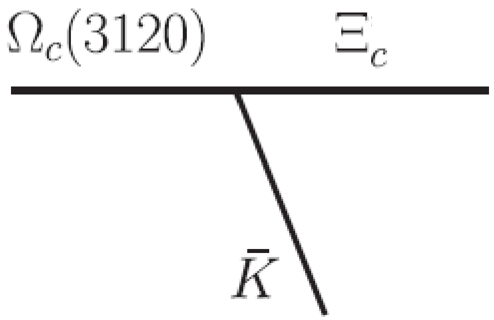


$$\frac{d\Gamma_{\Omega_c}}{dM_{\text{inv}}(\pi\Xi_c)} = \frac{1}{(2\pi)^3}\frac{M_{\Xi_c}}{M_{\Omega_c}} p_{\bar{K}}\tilde{p}_\pi |t_{\Omega_c\to\pi\bar{K}\Xi_c}|^2$$

$$t_{\Omega_c\to\pi\bar{K}\Xi_c} = g_{\Omega_c,\bar{K}\Xi_c^*}\frac{1}{M_{\text{inv}}(\pi\Xi_c) - M_{\Xi_c^*} - i\Gamma_{\Xi_c^*}/2} g_{\Xi_c^*,\pi\Xi_c}\tilde{p}_\pi$$

$$\Gamma_{\Omega_c\to\Xi_c\pi\bar{K}} = \underline{0.03\ \text{MeV}}$$

- $\Omega_c(3120)$ to decay into $\Xi_c\bar{K}$



$$\Gamma_{\Omega_c\to\Xi_c\bar{K}} = \frac{1}{2\pi}\frac{M_{\Xi_c}}{M_{\Omega_c}} g^2_{\Omega_c,\Xi_c\bar{K}} p'_{\bar{K}} \quad = \underline{0.90\ \text{MeV}}$$

=> Sum of them is **$\Gamma_{\Omega c}$ ~ 1 MeV**

The width of $\Omega_c(3120)$ decay to $\Xi_c\bar{K}\pi$ is much smaller than the $\bar{K}\pi\Xi$ in the case of the $\Omega(2012)$. The small ratio of 3% is challenging in the present experimental errors

Scattering length a_j and effective range $r_{0,j}$

- $\Omega_c(3120)$

	a_j	$r_{0,j}$
$\Xi_c^* \bar{K}$	$1.45 - i0.07$	$-0.08 - i0.01$
$\Omega_c^* \eta$	$0.44 - i0.09$	$0.26 + i0.01$

- $\Omega(2012)$

	a_j	$r_{0,j}$
$\Xi^* \bar{K}$	$1.69 - i0.17$	$-0.37 - i0.01$
$\Omega \eta$	$0.51 - i0.09$	$0.25 - i0.03$

Scattering length

$$-\frac{1}{a_j} = -\frac{8\pi\sqrt{s}}{2M_j}(T_{jj})^{-1}\Big|_{\sqrt{s_{\mathrm{th},j}}}$$

Effective range

$$r_{0,j} = \frac{1}{\mu_j}\frac{\partial}{\partial\sqrt{s}}\left[-\frac{8\pi\sqrt{s}}{2M_j}(T_{jj})^{-1} + ik_j\right]_{\sqrt{s_{\mathrm{th},j}}}$$

These magnitudes are determined experimentally, something feasible nowadays, for instance, measuring correlation functions

Discussion on the nature of Tcc and X(3872) from these information

J. Song, L. R. Dai and E. Oset, PRD 108,114017 (2023): L. R. Dai, J. Song and E. Oset, PLB846, 138200 (2023)

Summary

- We have studied the $\Omega_c(3120)$ based on the molecular picture
- The $\Omega_c(3120)$ mostly couples to $\Xi_c^* \bar{K}, \Omega_c^* \eta$ channels
- The state with $J^P = 3/2^-$ decays to $\Xi_c \bar{K}$ in the D-wave and we included this decay channel in our approach
- Evaluation of the fraction of the $\Omega_c(3120)$ width that goes into $\Xi_c \bar{K} \pi$ by the analogous analysis of $\Omega(2012)$ to see the nature of the molecular state

$\Rightarrow$ Small ratio of about 3% is obtained due to a relatively big binding, compared to its analogous $\Omega(2012)$ state

- As an alternative, the scattering length, and effective length, together with BE, and width of $\Omega_c(3120)$ will help to understand the nature of $\Omega_c(3120)$