Generalized parton distributions from Lattice QCD

Krzysztof Cichy
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Supported by the National Science Center of Poland
SONATA BIS grant No. 2016/22/E/ST2/00013 (2017-2022)
OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

Outline:

Introduction
GPDs from lattice:
– how to access
– reference frames
– twist-2
– twist-3
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao
K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee
N. Nurminen, P. Petreczky, A. Scapellato, F. Steffens, Y. Zhao

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See also plenary talk by Shohini Bhattacharya tomorrow!

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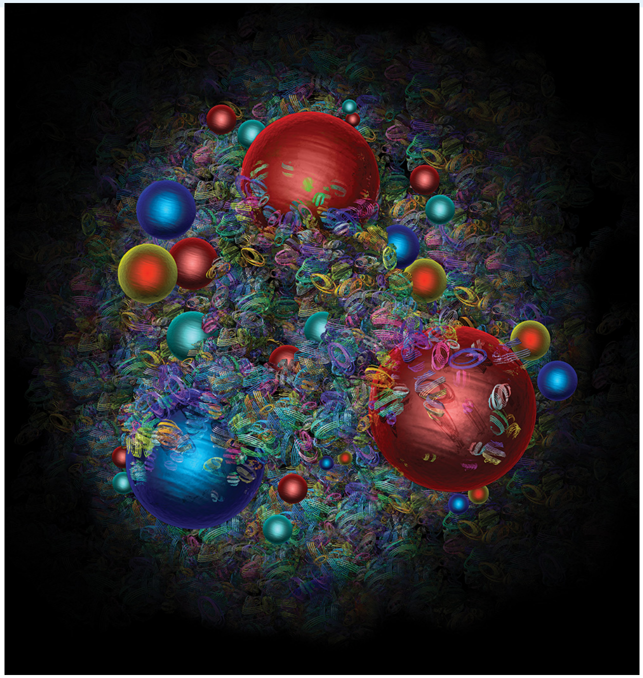
One of the central aims of hadron physics:
to understand better nucleon's 3D structure.

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise? NAS report 2018
- What are the emergent properties of dense systems of gluons?



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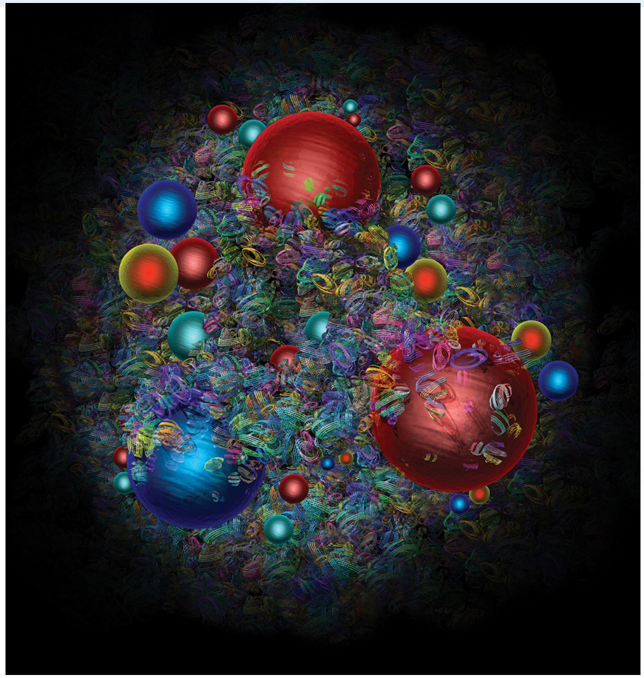

- How does the mass of the nucleon arise?
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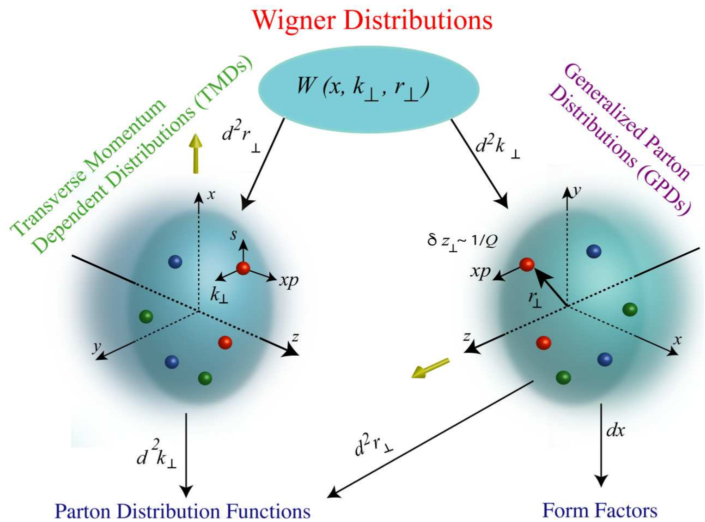
- Answering these questions is one of the crucial expectations for the upcoming years!
- For this, we need to probe the 3D structure.
- Transverse position of quarks: GPDs.
- Twist-2 GPDs as first aim, but higher-twist of growing importance.
- Both theoretical and experimental input needed.



Nucleon structure and GPDs

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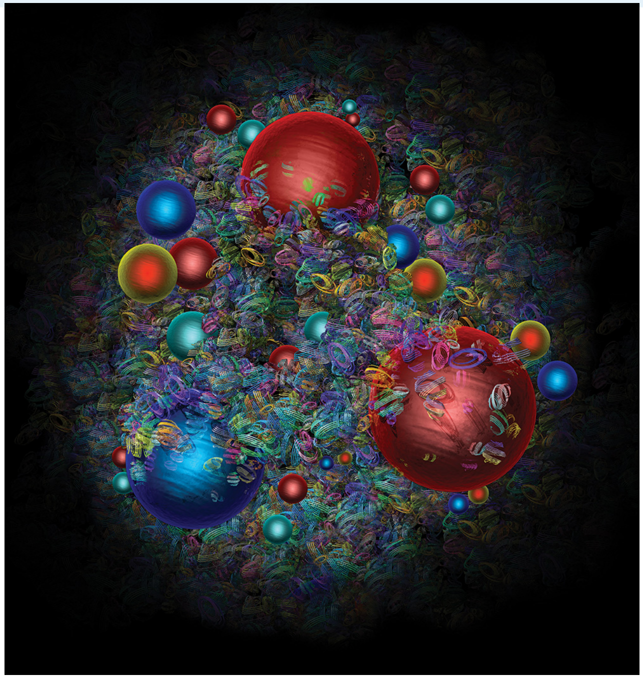

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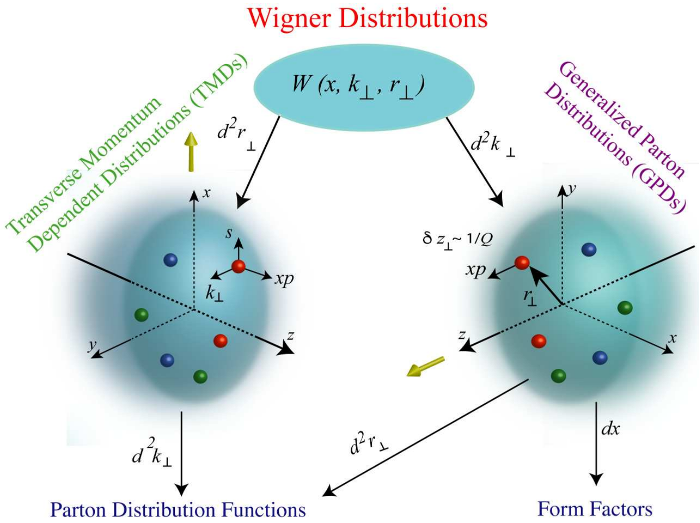
Generalized parton distributions (GPDs):

- much more difficult to extract than PDFs,
- but they provide a wealth of information:
  - ⋆ spatial distribution of partons in the transverse plane,
  - ⋆ mechanical properties of hadrons,
  - ⋆ hadron's spin decomposition,
- reduce to PDFs in the forward limit, e.g. $H(x,0,0) = q(x)$,
- their moments are form factors, e.g. $\int dx\, H(x,\xi,t) = F_1(t)$.



Partonic structure from Lattice QCD

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.

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  (experiment) cross-section = perturbative-part $*$ partonic-distribution
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution

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- Which lattice observables one can use?
- Good "lattice cross sections" [Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003]:
  - ⋆ computable on the lattice,
  - ⋆ having a well-defined continuum limit (renormalizable),
  - ⋆ perturbatively factorizable into PDFs.

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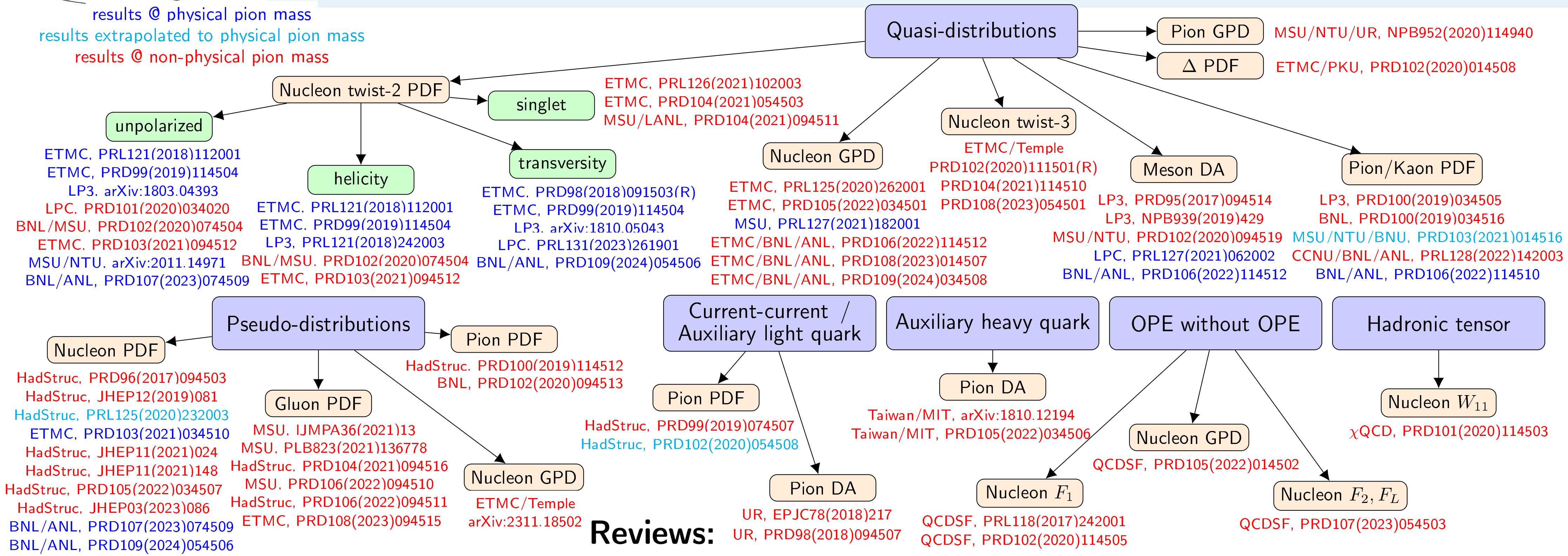
- Examples:
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Lattice PDFs/GPDs: dynamical progress



Reviews:

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

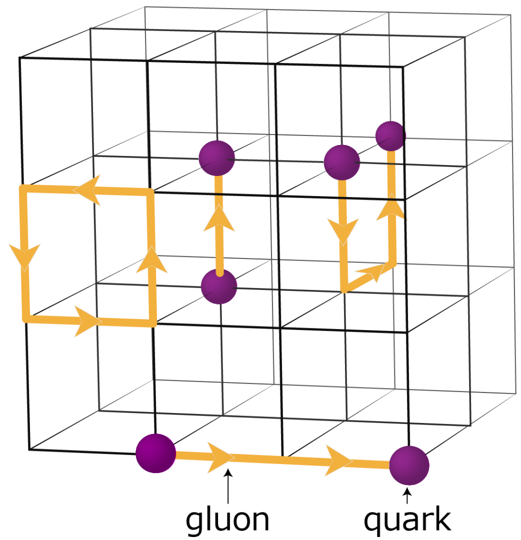
M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

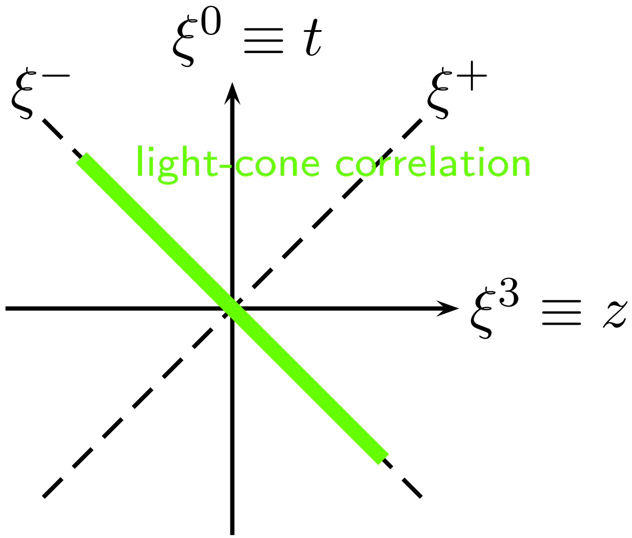
Lattice QCD – brief reminder

- QCD d.o.f.'s put on a **Euclidean** lattice
  - ⋆ quarks $\to$ sites
  - ⋆ gluons $\to$ links
- various discretizations can be used for quarks and gluons
- typical lattice parameters:
  - ⋆ $L/a = 32, 48, 64, 80, 96, 128$
  - ⋆ $a \in [0.04, 0.15]$ fm
  - ⋆ $L \in [2, 10]$ fm
  - ⋆ $m_\pi L \geq 3 - 4$
  - ⋆ $\Rightarrow$ ∞-dim path integral $\to$ $10^8 - 10^9$-dim integral



- Monte Carlo simulations to evaluate the discretized path integral
- feasible, but still requires huge computational resources of $\mathcal{O}(1 - 1000)$ million core-hours, depending on the question asked
- Lattice QCD offers a way for a careful *ab initio* study of non-perturbative aspects of QCD.
- Its huge strength: possibility to control all systematic effects: *cut-off effects, finite volume effects, renormalization quark mass effects, isospin breaking, excited states, . . .*
- Precision studies vs. exploratory studies.

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002



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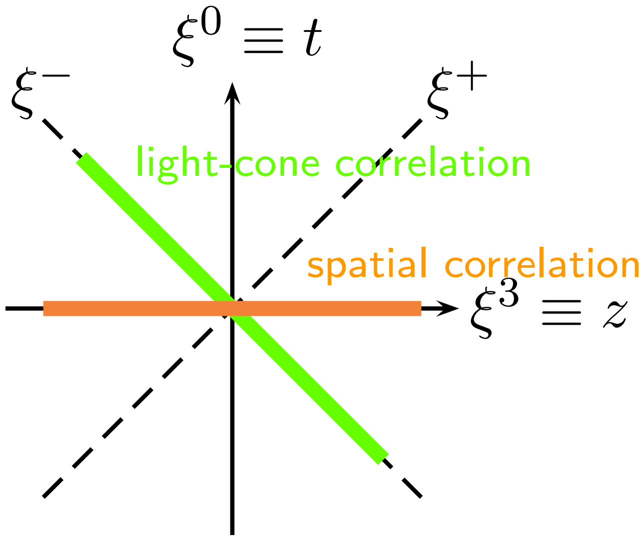

Quasi-distributions

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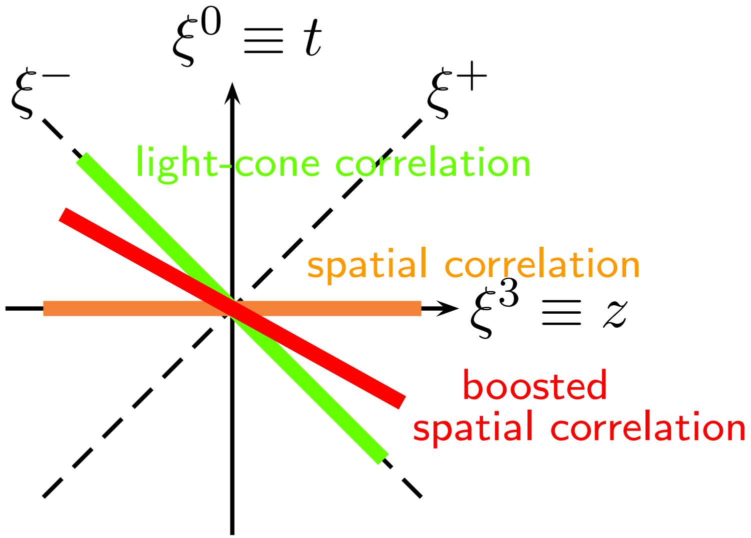

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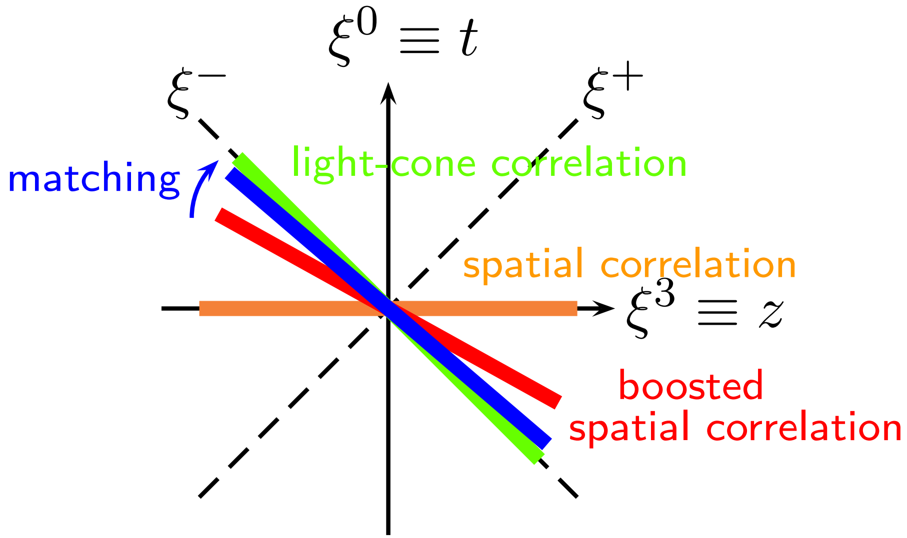

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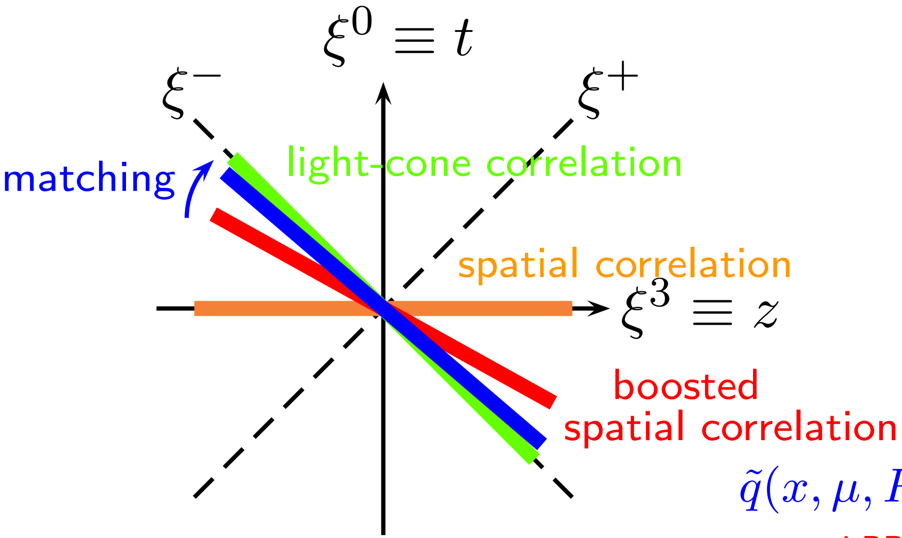

Euclidean matrix element:

$\langle P_f | \overline{\psi}(z) \Gamma \mathcal{A}(z,0) \psi(0) | P_i \rangle$

Its Fourier transform (quasi-distribution) can be matched onto the light-cone distribution:

(Large Momentum Effective Theory (LaMET))

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF — pert.kernel — PDF — higher-twist effects

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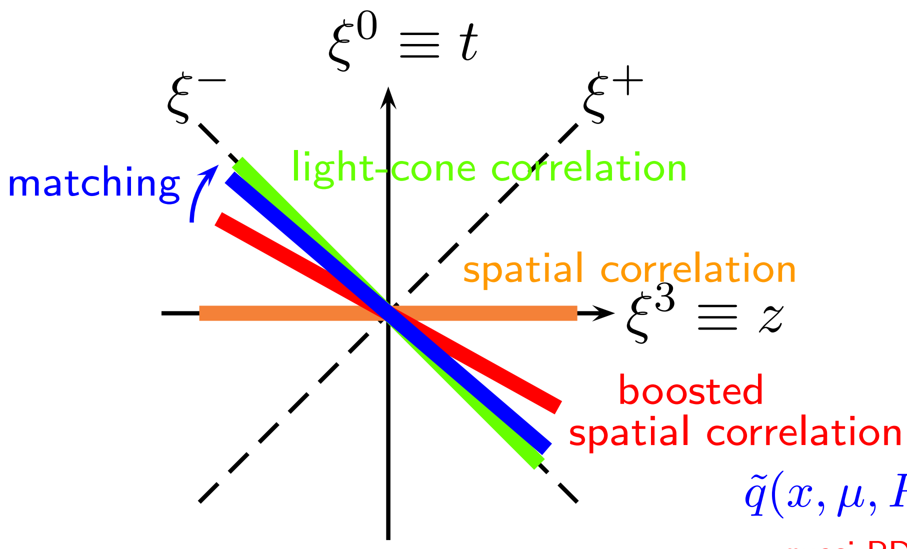

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quasi-PDF — pert.kernel — PDF — higher-twist effects

Dirac structures Γ for different GPDs:

VECTOR: γ_0, γ_3: H, E (unpolarized twist-2),
γ_1, γ_2: G_1, G_2, G_3, G_4 (vector twist-3).

AXIAL VECTOR: $\gamma_5\gamma_0, \gamma_5\gamma_3$: $\tilde{H}, \tilde{E}$ (helicity twist-2),
$\gamma_5\gamma_1, \gamma_5\gamma_2$: $\tilde{G}_1, \tilde{G}_2, \tilde{G}_3, \tilde{G}_4$ (axial vector twist-3).

TENSOR: $\gamma_1\gamma_3, \gamma_2\gamma_3$: $H_T, E_T, \tilde{H}_T, \tilde{E}_T$ (transversity twist-2),
$\gamma_1\gamma_2$: H_2', E_2' (tensor twist-3).

Need different projectors
to disentangle 2/4 GPDs

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$

POL-k: $\mathcal{P} = \frac{1+\gamma_0}{4} i\gamma_5\gamma_k$

Quasi-GPDs lattice procedure

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spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{\Delta}$, $\vec{\Delta}$ – momentum transfer
lattice computation of bare ME

different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

↓

extraction of amplitudes and/or GPDs
frame-dependent formulas

amplitudes frame-invariant
possible different definitions of GPDs

QUASI

renormalization
e.g. RI scheme

↓

reconstruction of x-dependence (F.T. in z)
e.g. Backus-Gilbert

↓

matching to light cone
in x-space

PSEUDO

renormalization
e.g. double ratio

↓

matching to light cone
in ν-space

↓

reconstruction of x-dependence (F.T. in ν)
e.g. fitting ansatz

logarithmic and power divergences
in bare MEs/GPDs

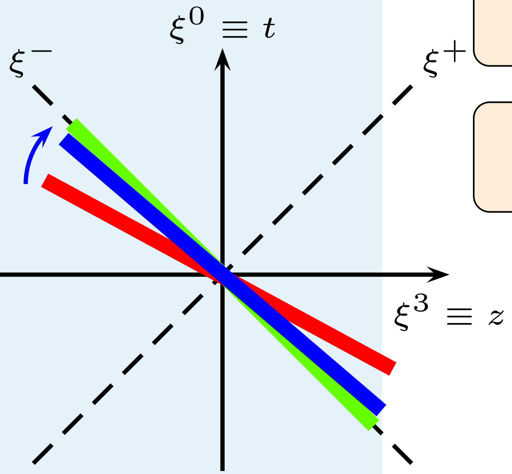
reconstruction:
non-trivial ("inverse problem")

matching:
needs large boosts
valid up to HTEs

↓

light-cone PDF

the final desired object!



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Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV^2, most data: $-t = 0.64,\ 0.69$ GeV^2,
- skewness: $\xi = 0, 1/3$.

$\mathcal{O}(20000)$ measurements (≈ 250 confs, 8 source positions, 8 permutations of $\vec{\Delta}$).

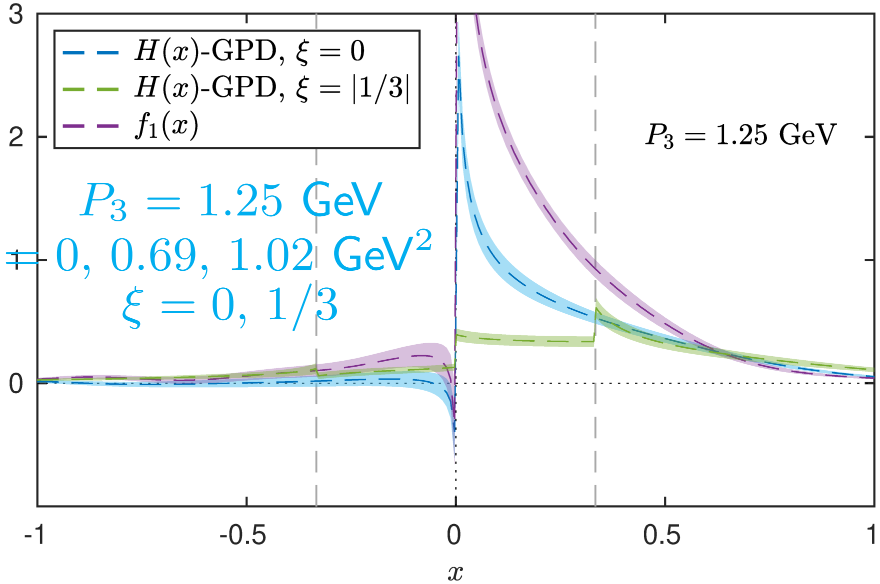
Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001
Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501
Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512
Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507
Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501
Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 109(2024)034508
Twist-2 unpolarized GPDs (pseudo-GPDs) S. Bhattacharya et al. (ETMC/Temple) arXiv:2405.04414
Twist-2 transversity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) in preparation

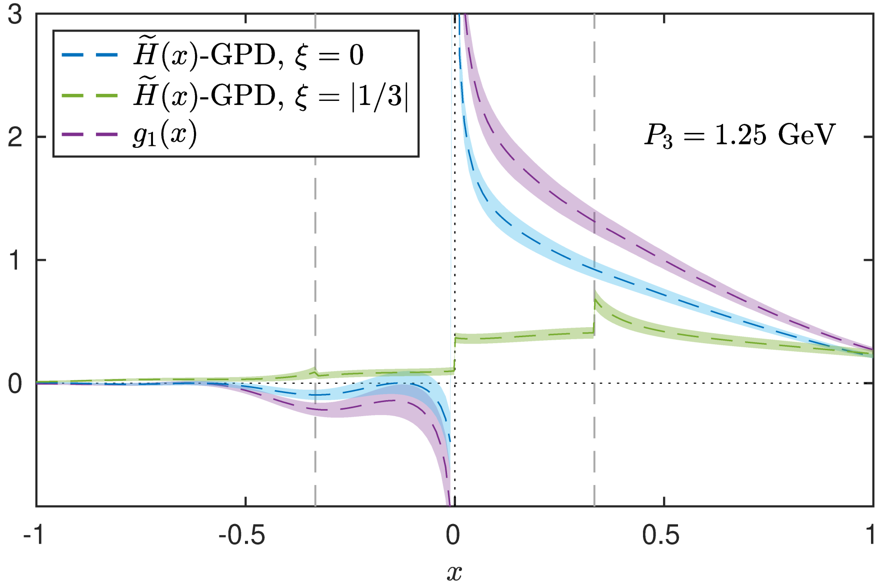
First extractions of x-dependent GPDs

unpolarized

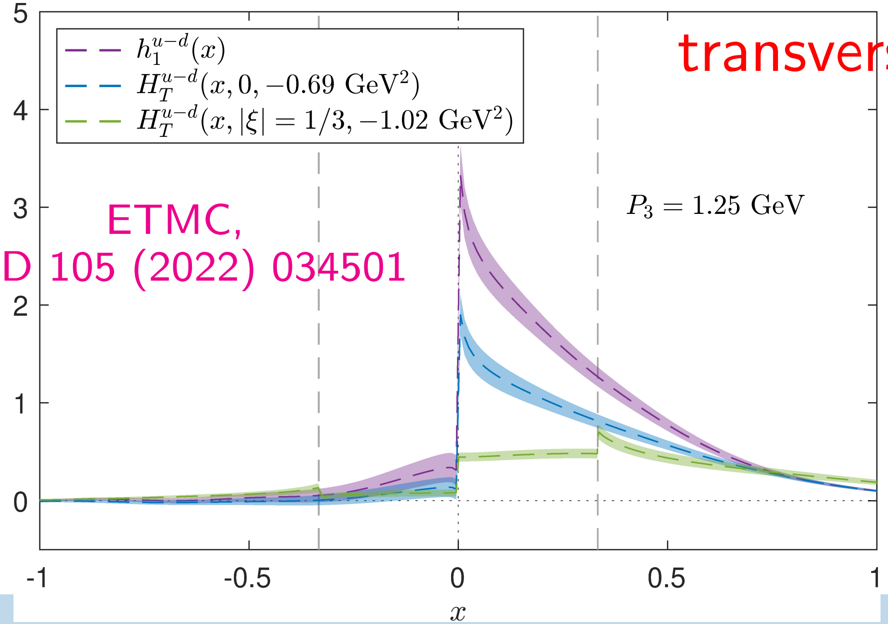
ETMC, Phys. Rev. Lett. 125 (2020) 262001

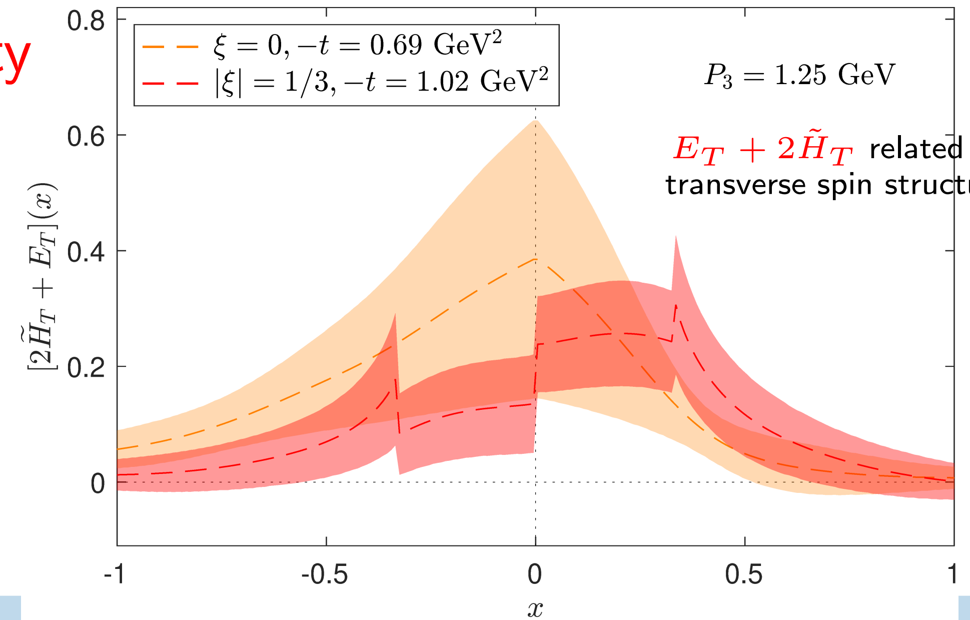
helicity





transversity





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Standard symmetric (Breit) frame:

source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,

sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

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Lattice perspective:
construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator
preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.
Hence, **separate calculation for each momentum transfer $\vec{\Delta}$!**

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Asymmetric frame:

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Lattice perspective:

Several momentum transfer vectors $\vec{\Delta}$ can be obtained within a single calculation!

Example – helicity GPDs

Lorentz-covariant parametrization of matrix elements (axial vector case):

$$F^{[\gamma^\mu\gamma_5]} = \bar{u}(p',\lambda')\left[\frac{i\epsilon^{\mu P z \Delta}}{m}A_1 + \gamma^\mu\gamma_5 A_2 + \gamma_5\left(\frac{P^\mu}{m}A_3 + m z^\mu A_4 + \frac{\Delta^\mu}{m}A_5\right) + m\not{z}\gamma_5\left(\frac{P^\mu}{m}A_6 + m z^\mu A_7 + \frac{\Delta^\mu}{m}A_8\right)\right]u(p,\lambda)$$

S. Bhattacharya et al., PRD109(2024)034508

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Two definitions of $\widetilde{H}$:

standard ($\gamma_5\gamma_3$ operator): $F_{\tilde{H}} = A_2 + zP_3A_6 - m^2z^2A_7$,
another ($\gamma_5\gamma_i$ operators, $i = 0, 1, 2$): $F_{\tilde{H}} = A_2 + zP_3A_6$.

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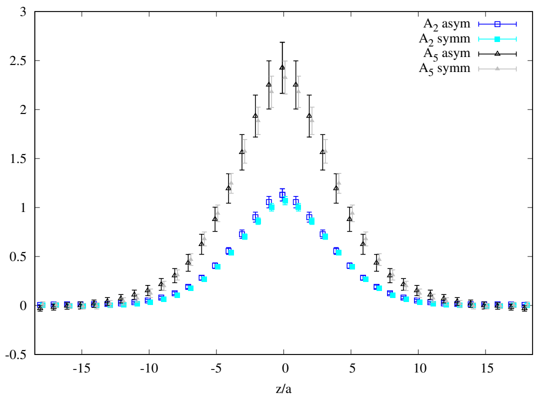
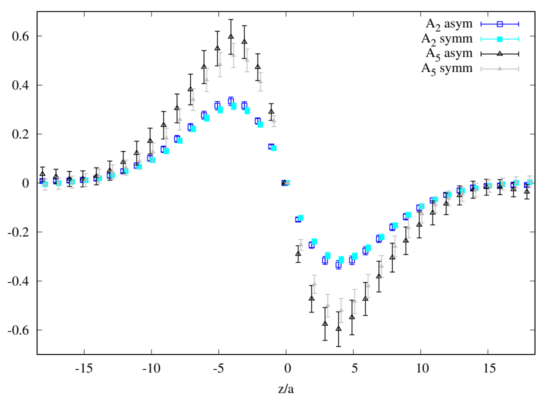
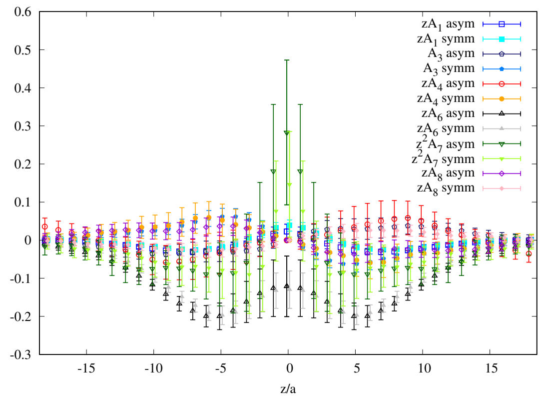
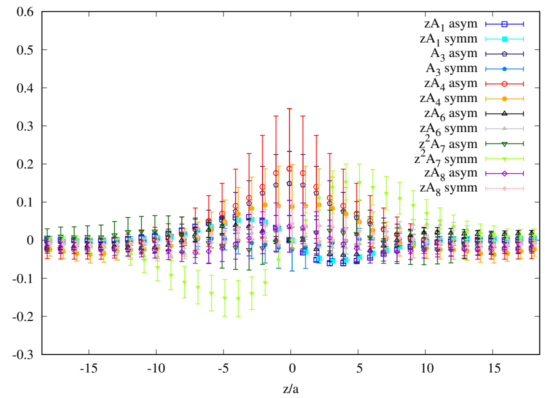
A_2, A_5 (leading ones)





zA_1, A_3, zA_4, zA_6, z^2A_7, zA_8 (suppressed ones)



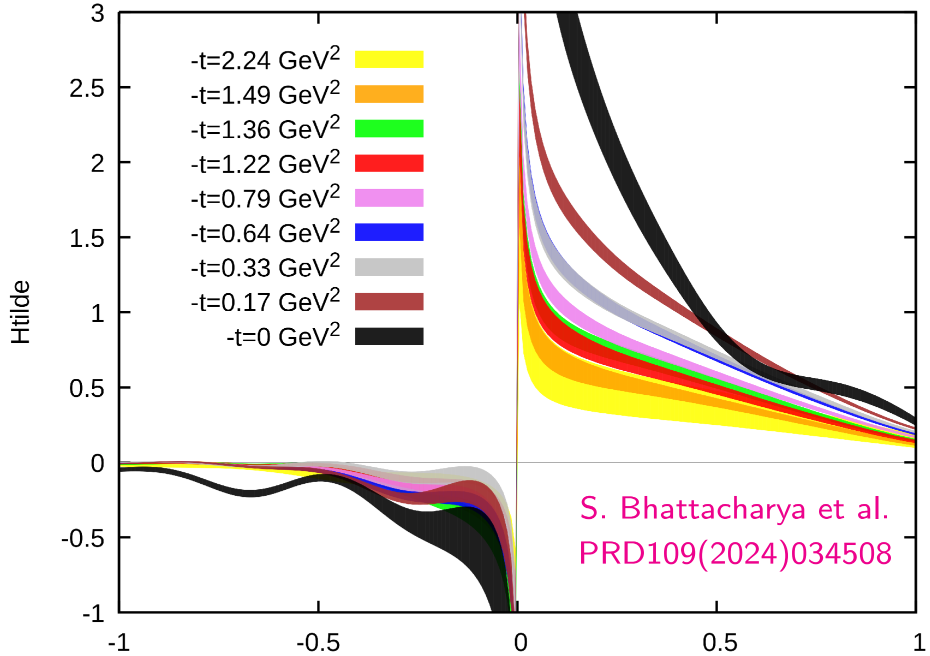


MEs are **frame-dependent**, but the A_i's are **frame-invariant**

Introduction

Results
First extraction
Reference frames
Quasi
Quasi vs. pseudo
Pseudo
GPDs moments
Twist-3

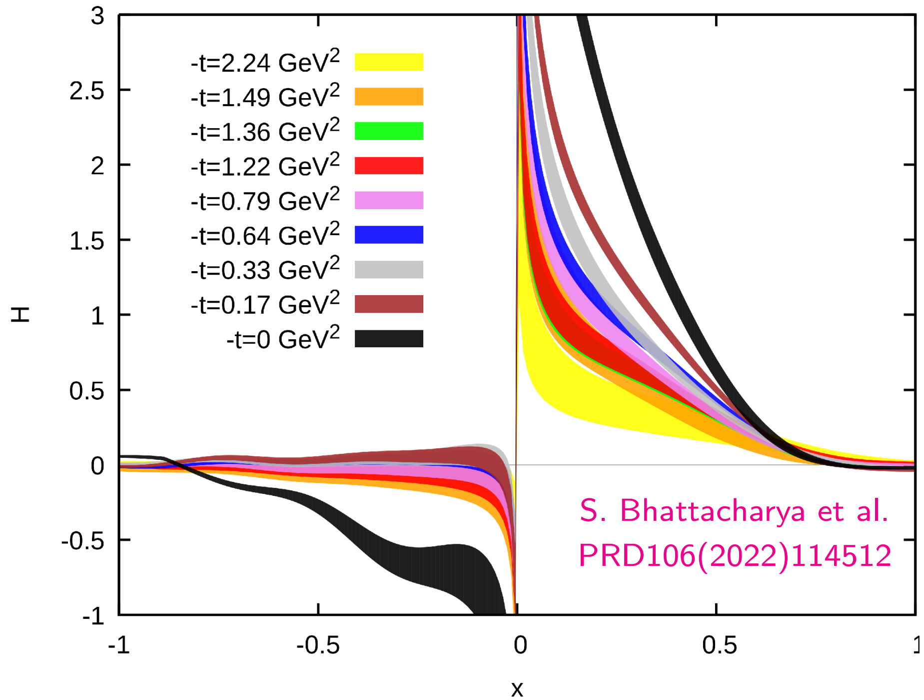
Summary

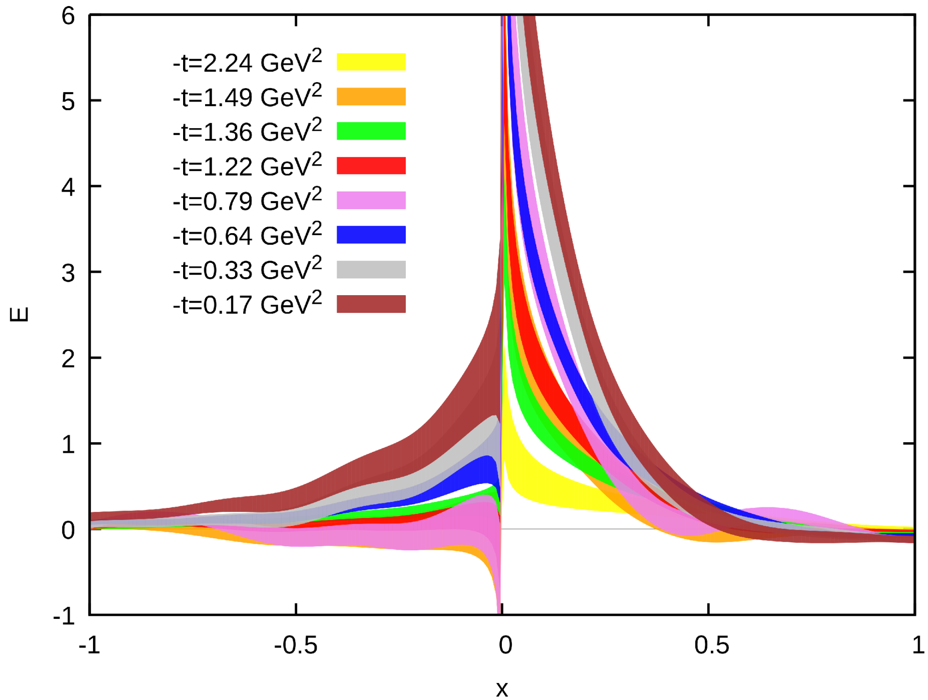


$\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2
$\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2
$\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2
$\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2
$\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2
$\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2
$\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2
$\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2

Impact parameter distribution:

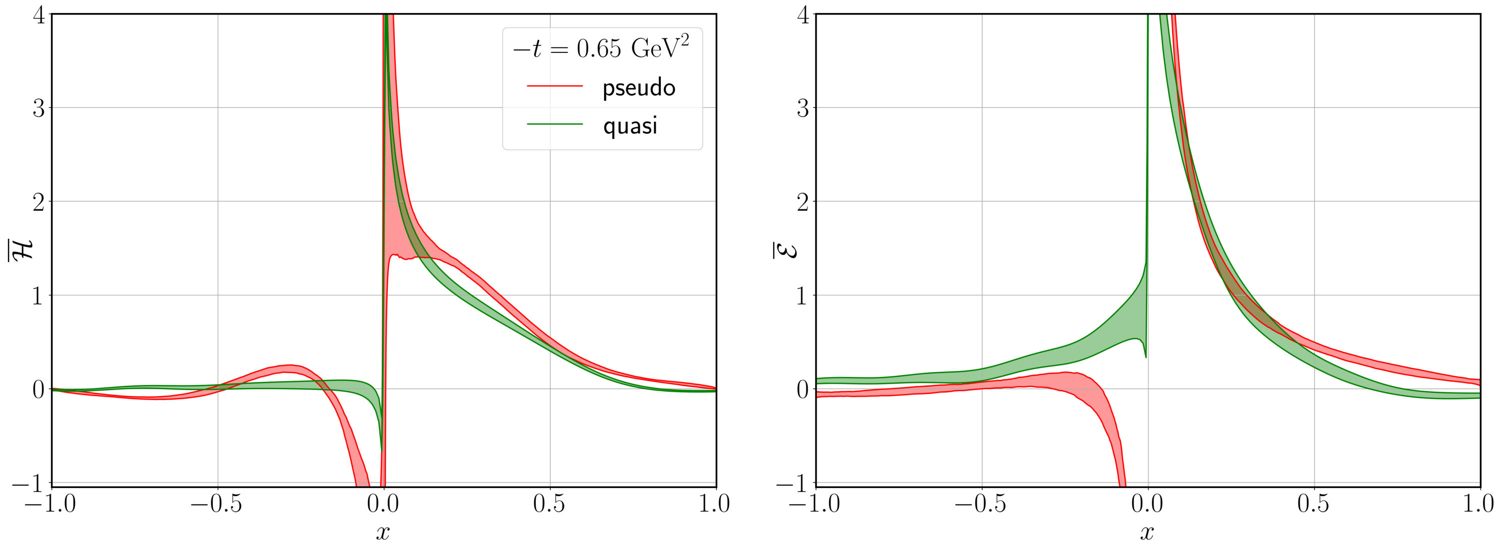
$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} GPD(x,t)$$





H GPD from quasi vs. pseudo, $-t = 0.65$ GeV^2

S. Bhattacharya et al., 2405.04414

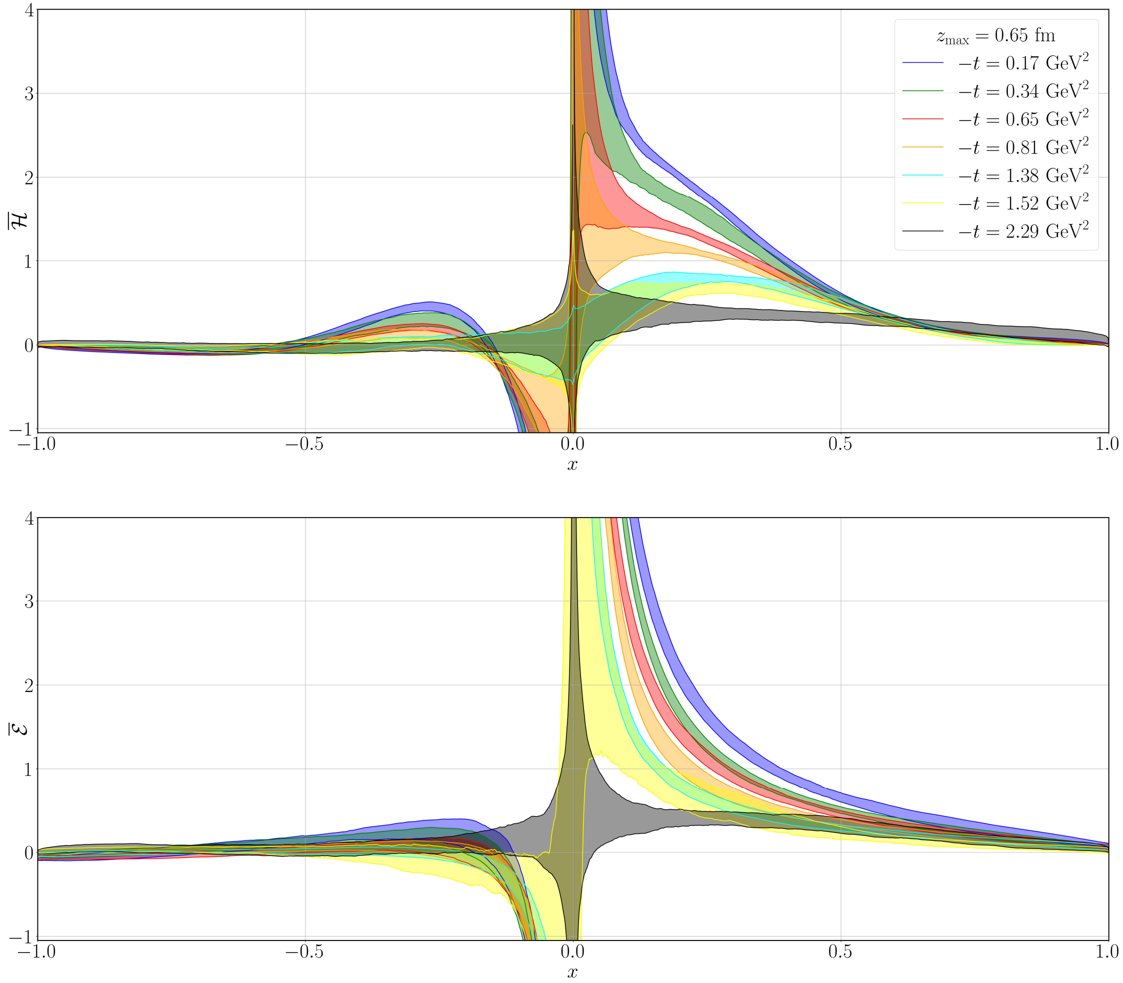


The same lattice data can also be analyzed within the framework of pseudo-distributions.

- Main difference:
  quasi = factorization in x-space (LaMET),
  pseudo = short-distance factorization (SDF) in ν-space.
- Practical difference: reconstruction of x-dependence
  quasi = Backus-Gilbert, pseudo = fitting ansatz.

t-dependence of H/E GPDs (pseudo)

S. Bhattacharya et al., 2405.04414



Qualitatively similar picture to the one from quasi-GPDs.
Quantitative conclusions after careful estimation of systematics!

Short-distance factorization (SDF) can also be used to extract moments of GPDs.

GPDs moments from OPE of non-local operators

Short-distance factorization (SDF) can also be used to extract moments of GPDs.

For ratio-renormalized H/E: $\mathcal{F}^{\overline{\rm MS}}(z,P,\Delta) = \sum_{n=0} \frac{(-izP)^n}{n!} C_n^{\overline{\rm MS}}(\mu^2 z^2)\langle x^n\rangle + \mathcal{O}(\Lambda^2_{\rm QCD} z^2)$,

$C_n^{\overline{\rm MS}}(\mu^2 z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

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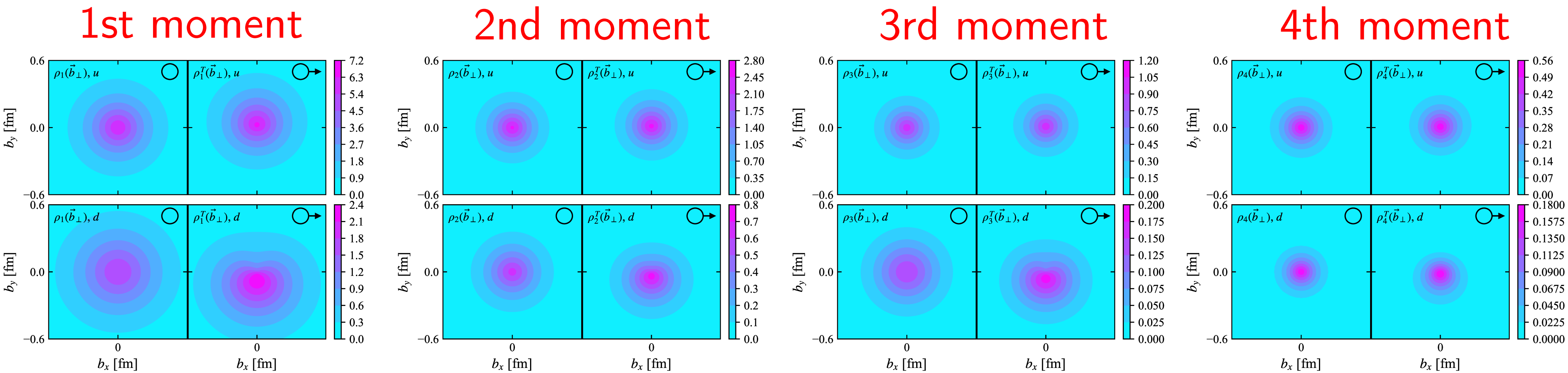
$C_n^{\overline{\text{MS}}}(\mu^2 z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}^2_\perp) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}^2_\perp) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}^2_\perp)] e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}.$$



S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

Twist-3 GPDs

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

Twist-2 – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3 – no density interpretation, contain important information about qgq correlations, appear in QCD factorization theorems for a variety hard scattering processes, interesting connections with TMDs, important for JLab12 and EIC, but difficult to measure.

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Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., PRD102(2020)034005, PRD102(2020)114025
  BC-type sum rules S. Bhattacharya, A. Metz, PRD105(2022)054027
  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, JHEP 05(2021)086, 11(2021)087
- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
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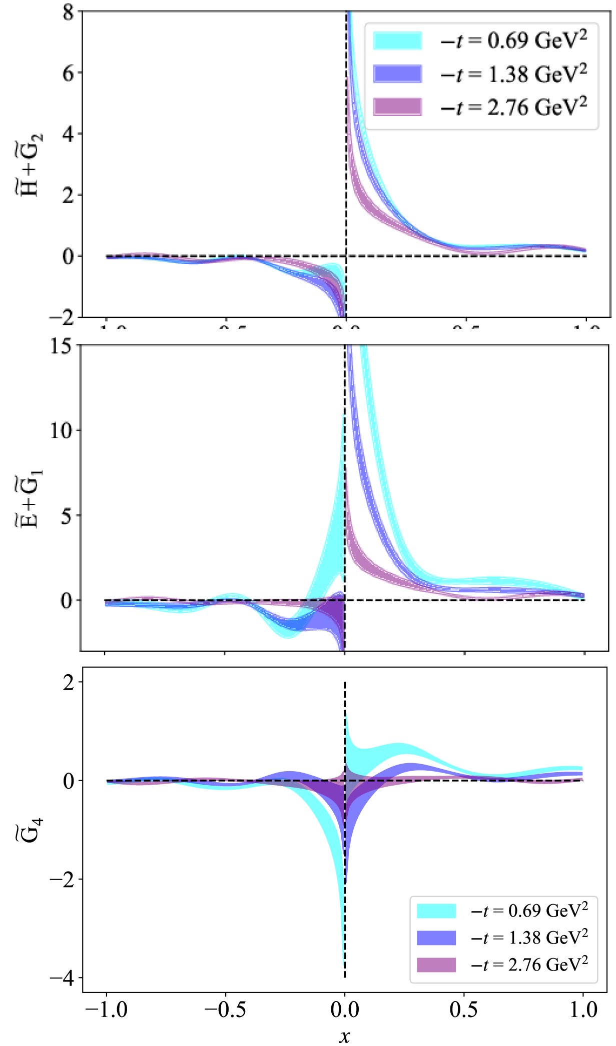

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  S. Bhattacharya et al., PRD102(2020)111501(R), PRD104(2021)114510
- first exploration of twist-3 axial GPDs: $\widetilde{G}_1, \widetilde{G}_2, \widetilde{G}_3, \widetilde{G}_4$
  S. Bhattacharya et al., PRD108(2023)054501

$$\mathcal{F}^{[\gamma_j \gamma_5]} = -i\frac{\Delta_j\,\gamma_5}{2m}\,F_{\widetilde{E}+\widetilde{G}_1} + \gamma_j\,\gamma_5\;F_{\widetilde{H}+\widetilde{G}_2} + \frac{\Delta_j\,\gamma_3\gamma_5}{P_3}\,F_{\widetilde{G}_3} - \frac{\mathrm{sign}[P_3]\,\varepsilon_\perp^{j\,\rho}\,\Delta_\rho\,\gamma_3}{P_3}\,F_{\widetilde{G}_4}$$

Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- A lot of follow-up work in progress: transversity GPDs, pion and kaon GPDs, other twist-3 GPDs, extension of kinematics.
- Obviously, GPDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Quantification of systematics very laborious, but crucial.
- Consistent progress will ensure complementary role to pheno!

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Thank you for your attention!

Many more interesting things in the plenary talk by Shohini Bhattacharya tomorrow!

Introduction
Results
Summary
Backup slides
Definitions
Convergence
Twist-3
GPDs moments
GPDs moments

Backup slides

Lorentz-covariant parametrization

Main theoretical tool: S. Bhattacharya et al., PRD106(2022)114512

Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

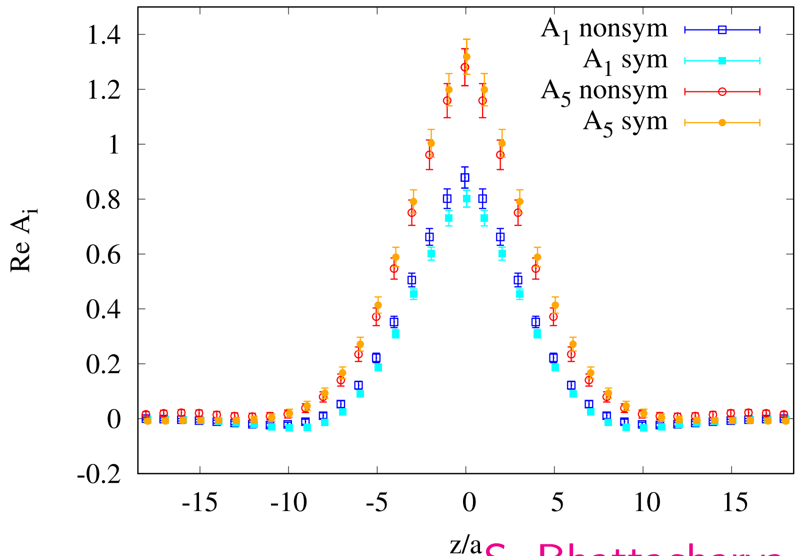
asymmetric frame:

$$\Pi_0^a(\Gamma_0) = C\left(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3 z}{4m}A_4\right.$$
$$\left.+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_f P_3(E_f-E_i)^2 z}{2m^3}A_8\right).$$

- matrix elements $\Pi_\mu(\Gamma_\nu)$ are **frame-dependent**,
- but the amplitudes A_i are **frame-invariant**.

Proof of concept (comparison between frames)

A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (suppressed ones)



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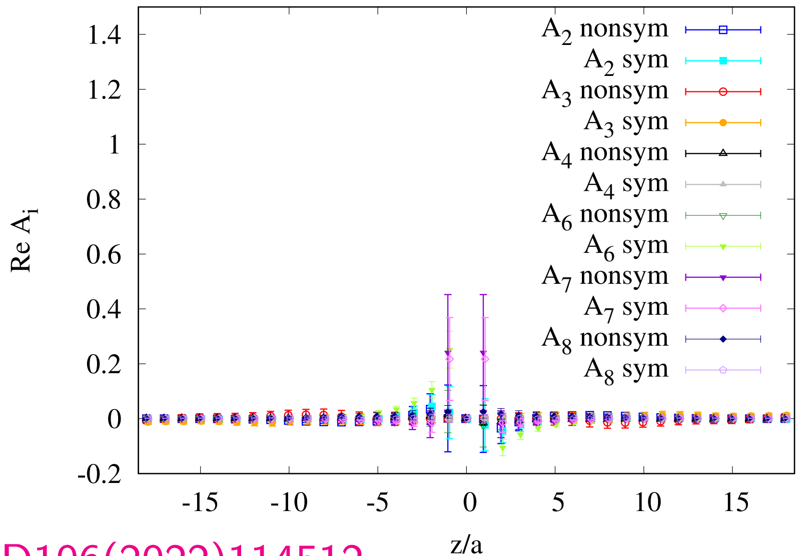
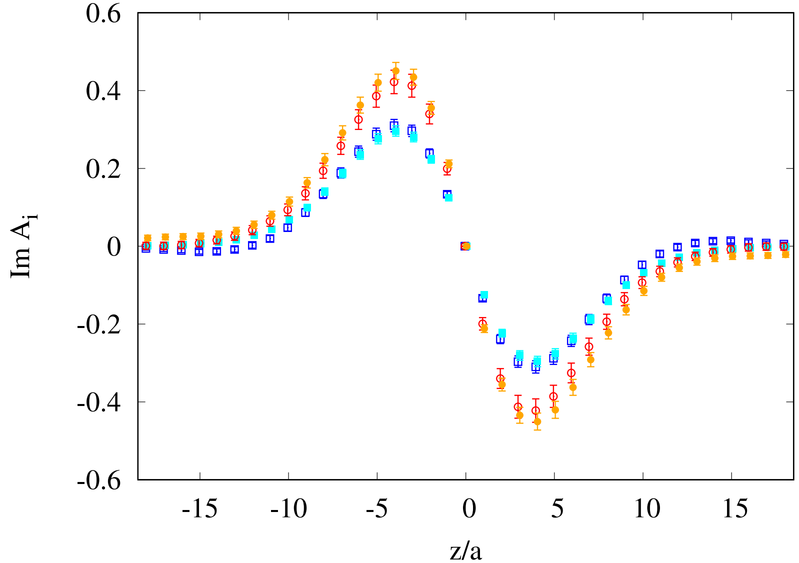


S. Bhattacharya et al., PRD106(2022)114512



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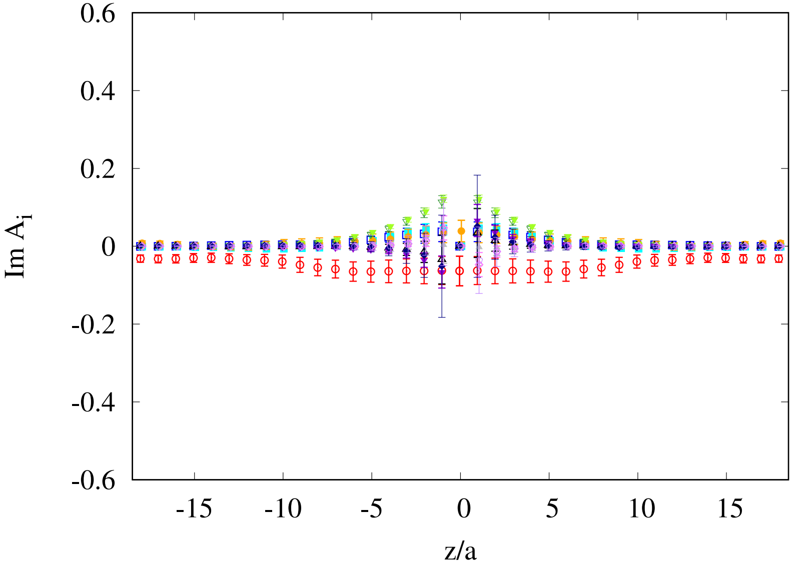


H and E GPDs – possible definitions

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,,$$

$$F_{E^{(0)}} = -A_1 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6 \,.$$

ASYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8 \,,$$

$$F_{E^{(0)}} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8 \,.$$

One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

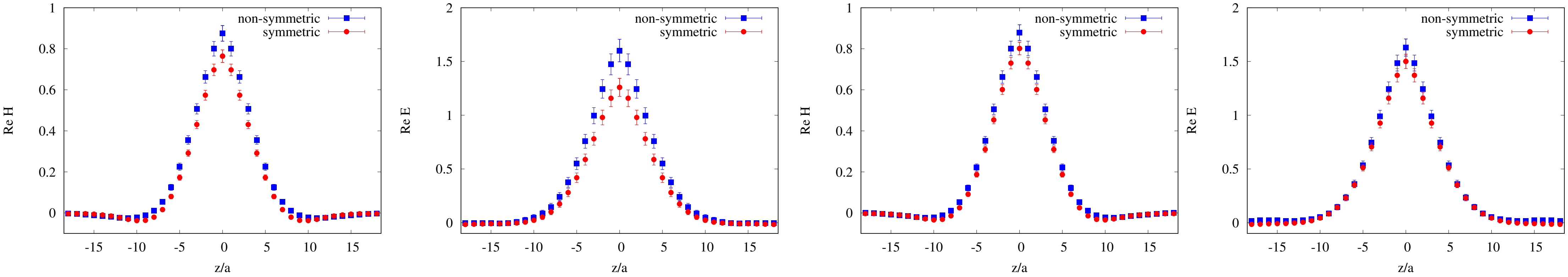
LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

H and E GPDs – comparison of definitions

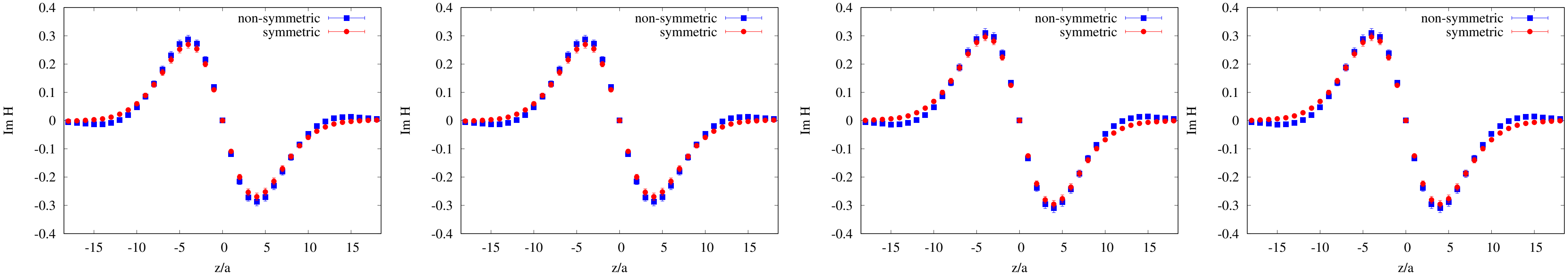
STANDARD DEFINITION

LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD



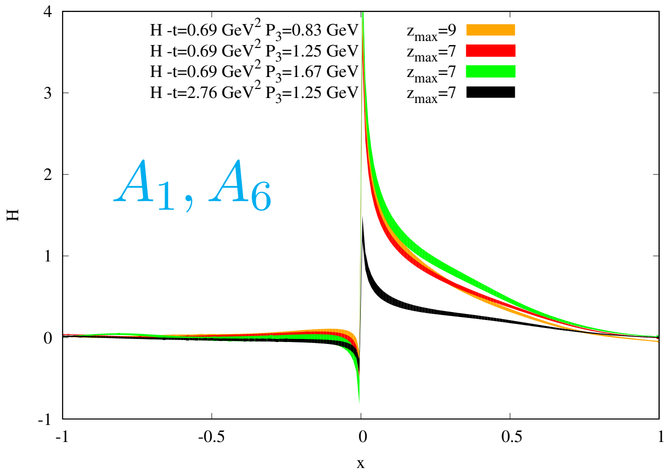
S. Bhattacharya et al., PRD106(2022)114512

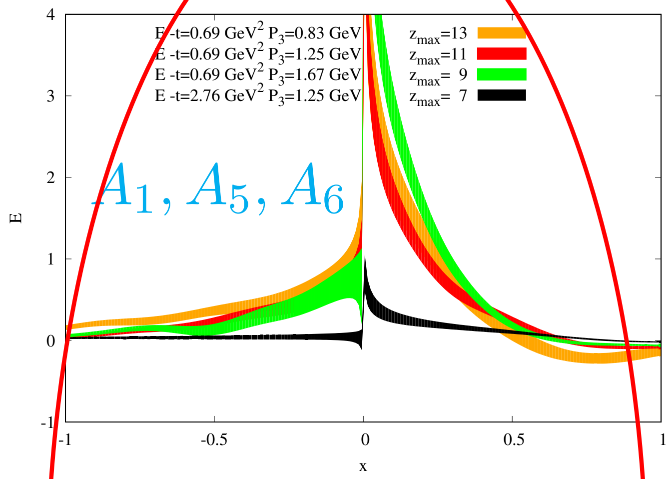


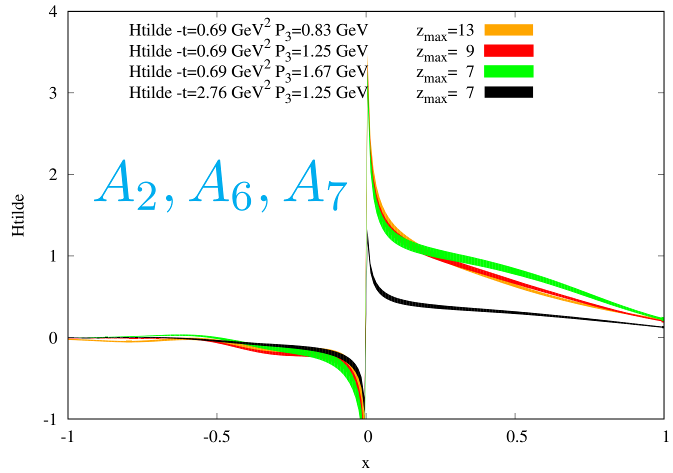
Convergence of alternative definitions of $\tilde{H}/H/E$

UNPOLARIZED | HELICITY

STANDARD





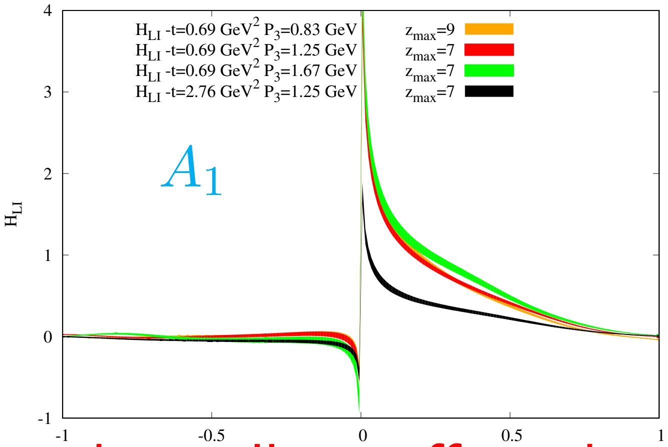


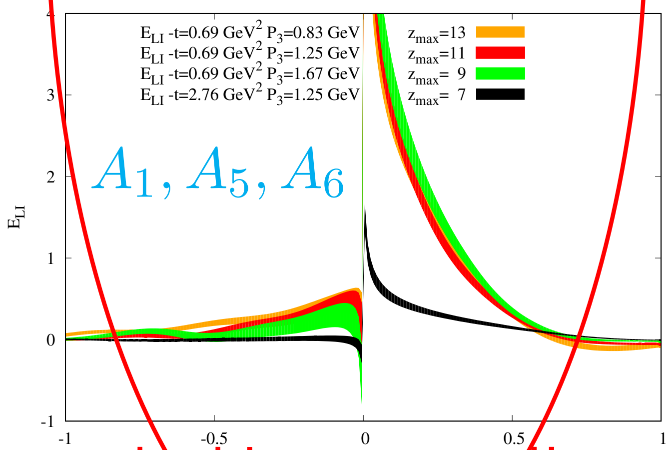
γ_0 operator (non-LI) | $\gamma_5\gamma_3$ operator (LI)

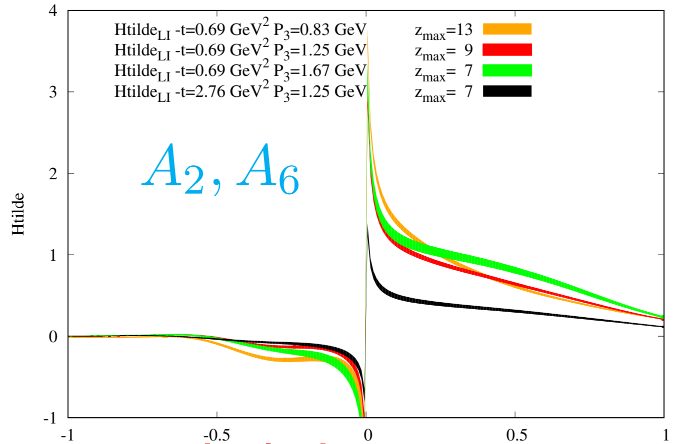
H-GPD | E-GPD | $\widetilde{H}$-GPD

γ_0, γ_T operators (LI) | $\gamma_5\gamma_0$, $\gamma_5\gamma_T$ operators (LI)

ALTERNATIVE



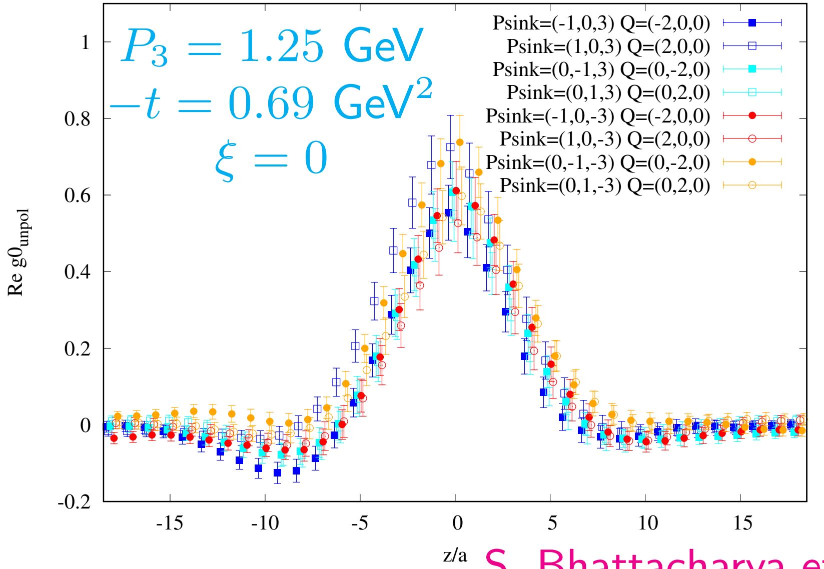




basically unaffected | highly-improved! | slightly worse

Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame | non-symmetric frame



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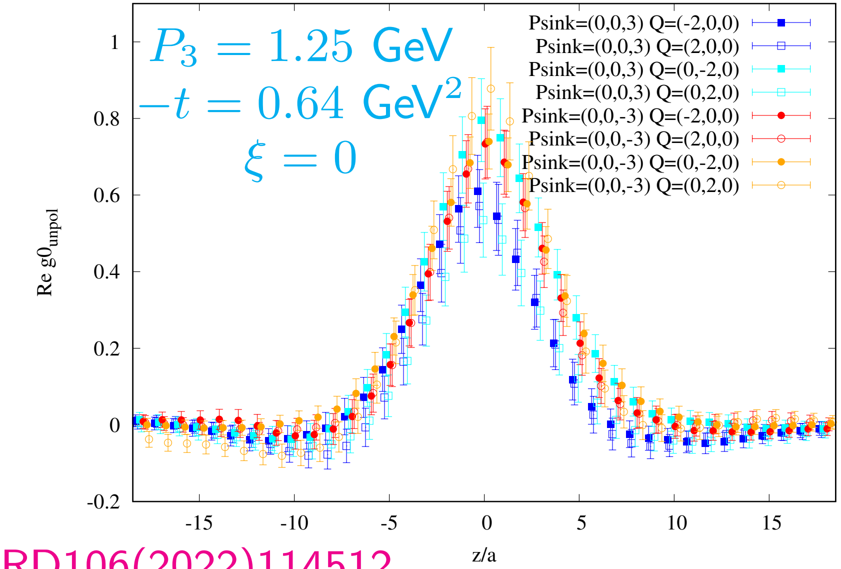
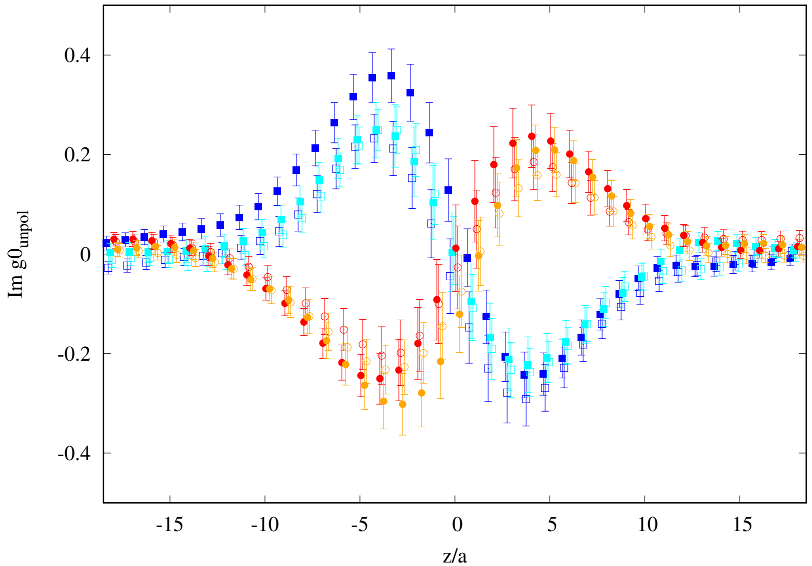


S. Bhattacharya et al., PRD106(2022)114512



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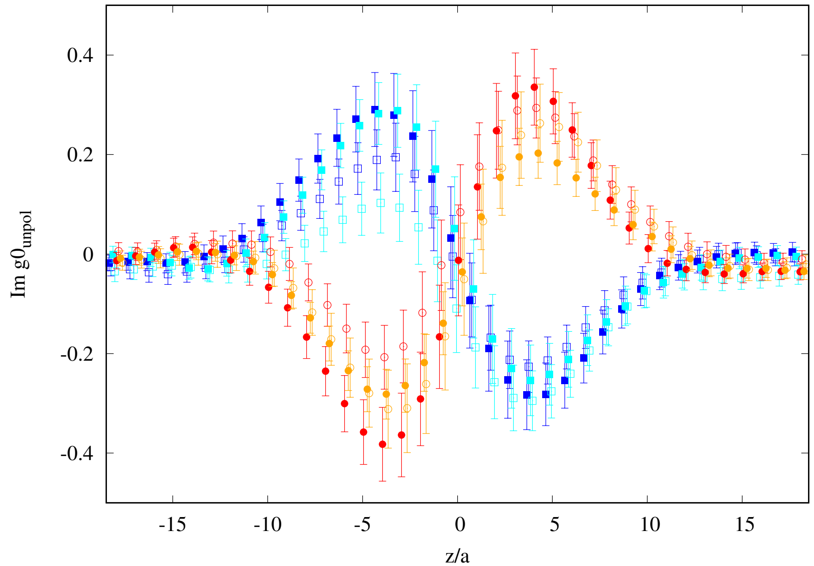
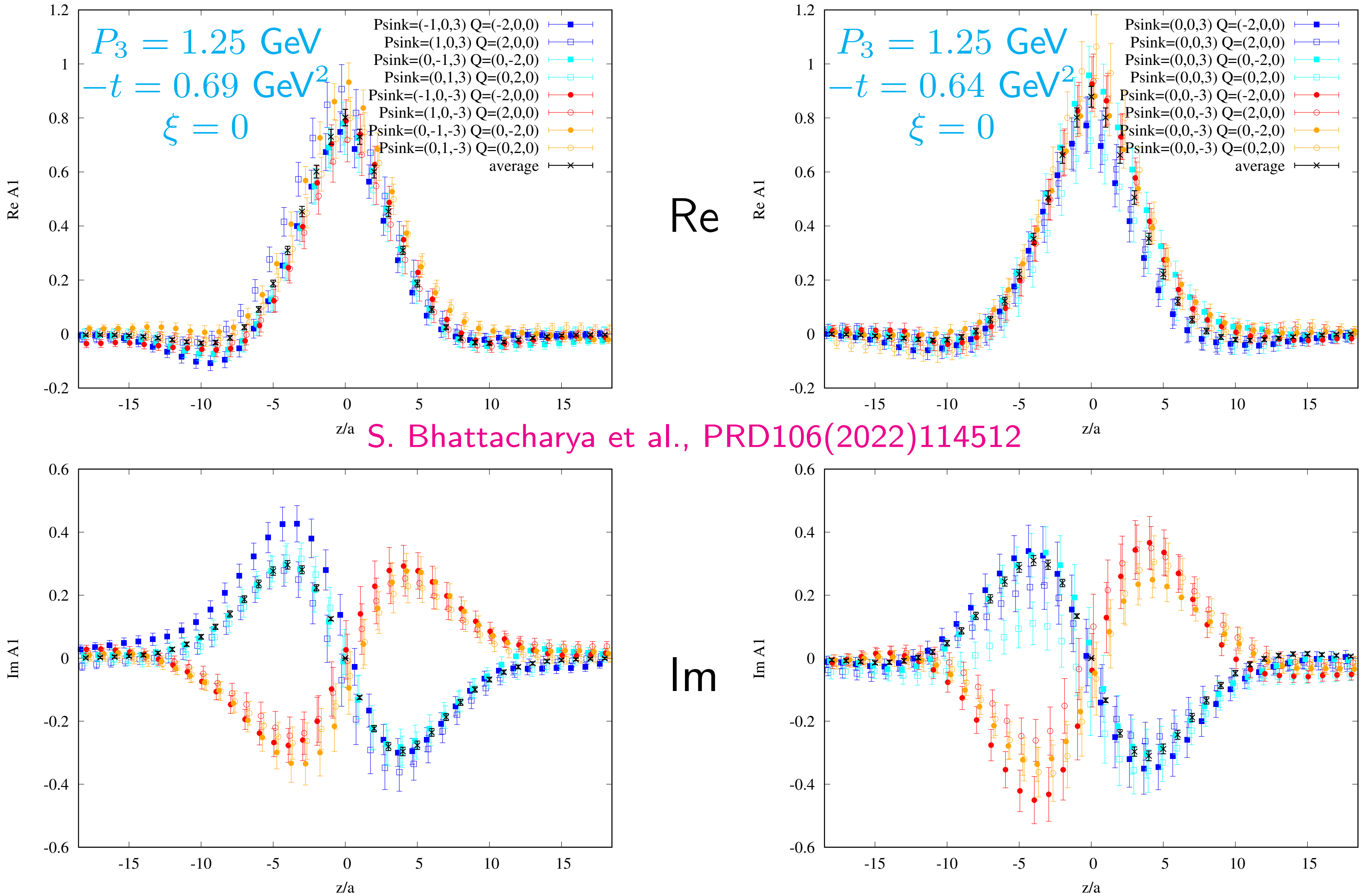


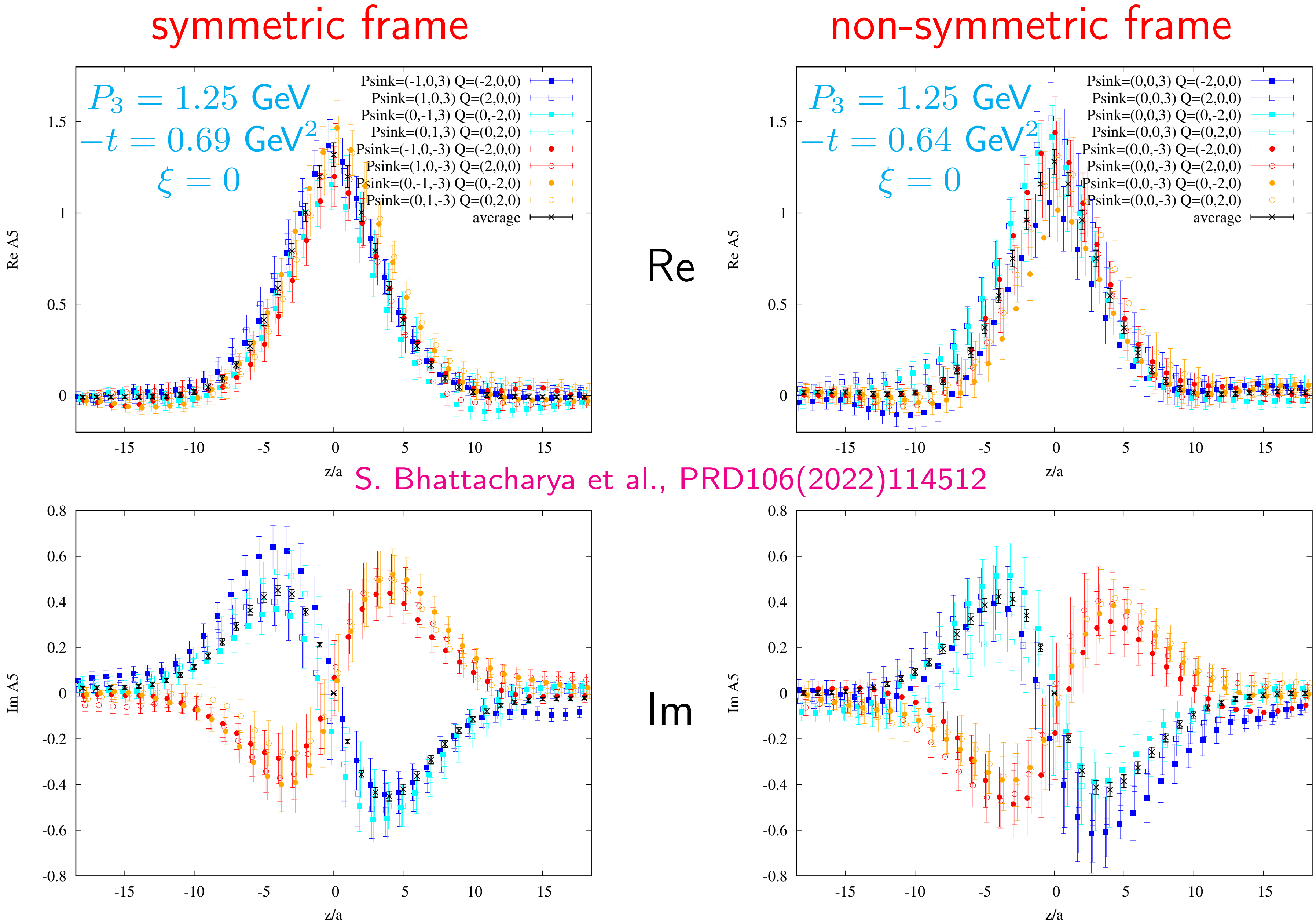
Example amplitude A_1

symmetric frame | non-symmetric frame



S. Bhattacharya et al., PRD106(2022)114512

Example amplitude A_5

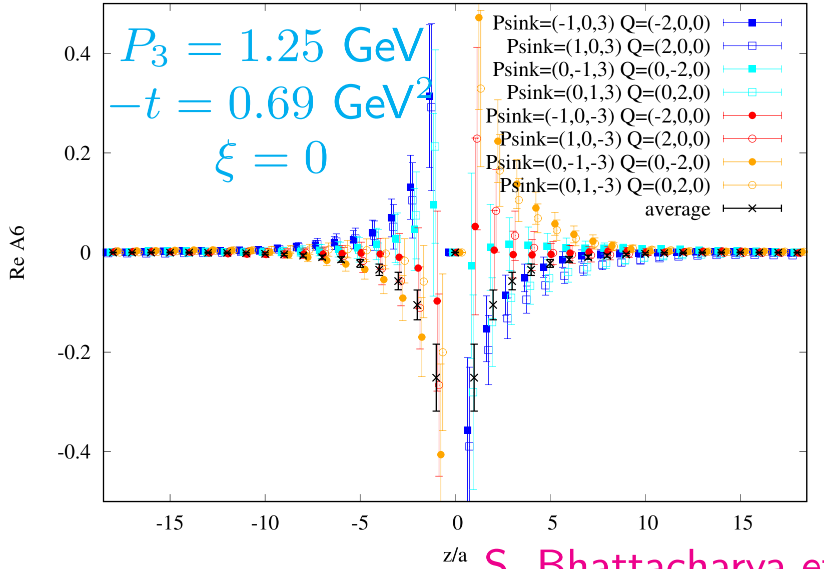


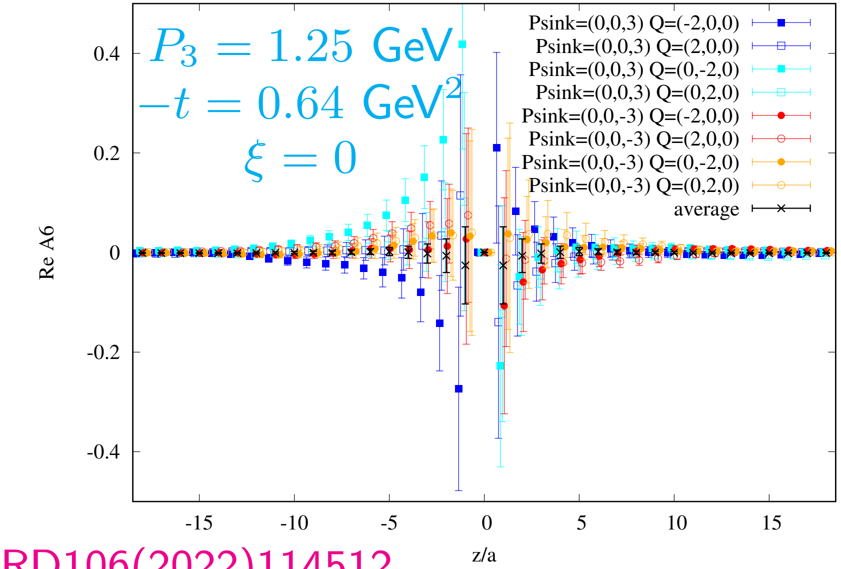
S. Bhattacharya et al., PRD106(2022)114512

Example amplitude A_6

symmetric frame

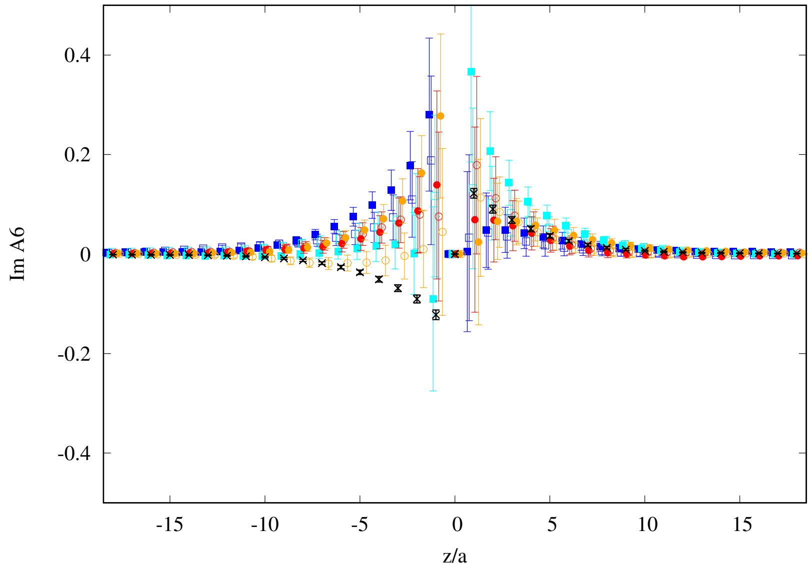
non-symmetric frame

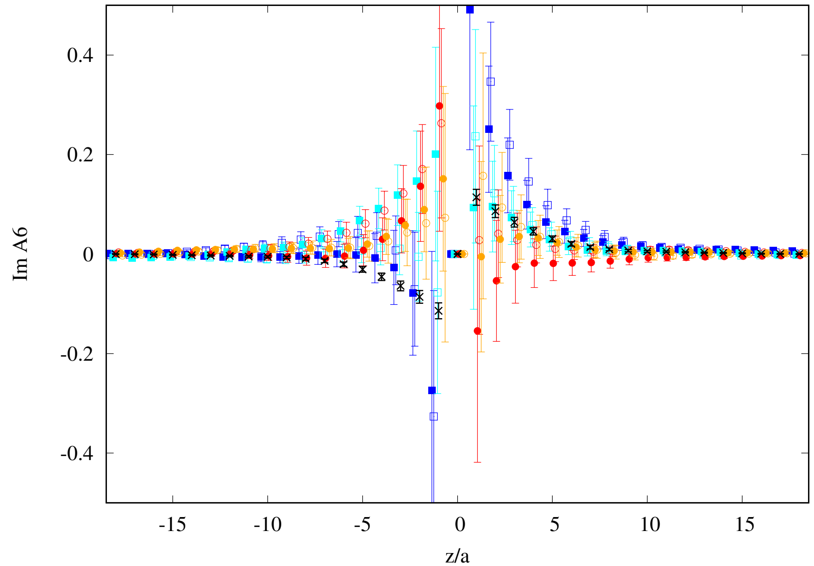




Re

S. Bhattacharya et al., PRD106(2022)114512





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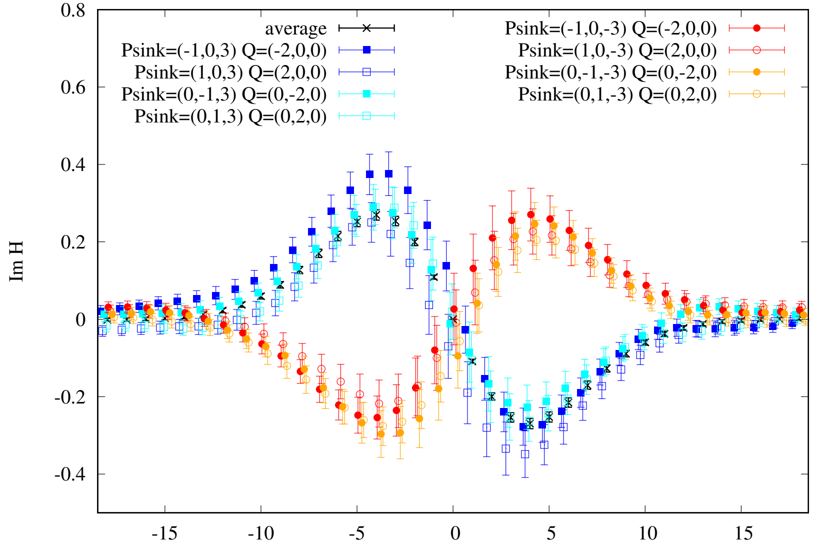
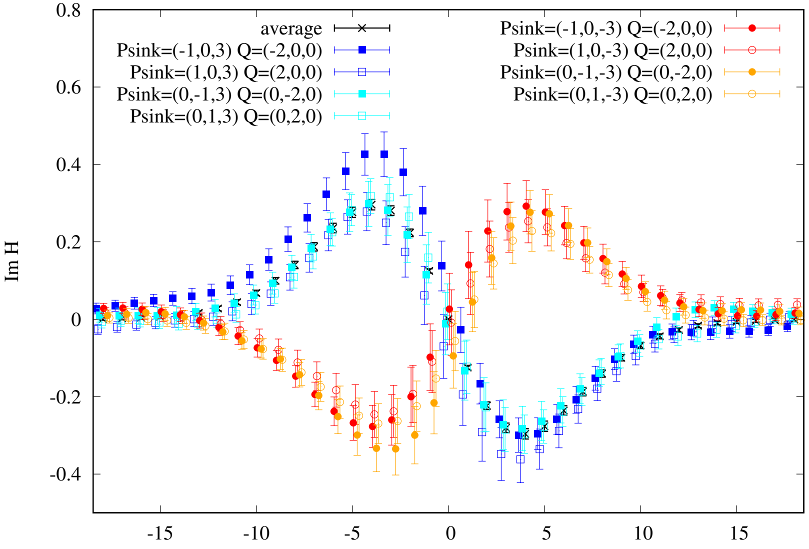
H and E GPDs – signal improvement

standard

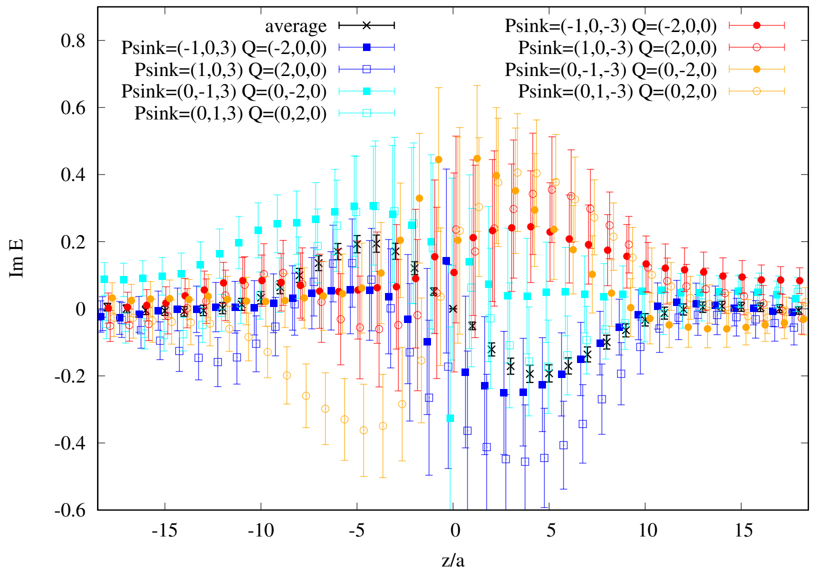


Lorentz-invariant

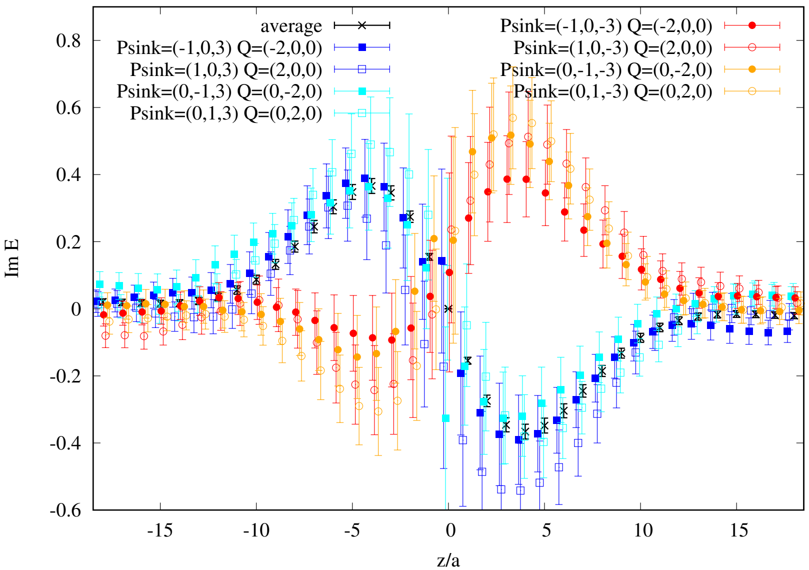


Im H

S. Bhattacharya et al., PRD106(2022)114512

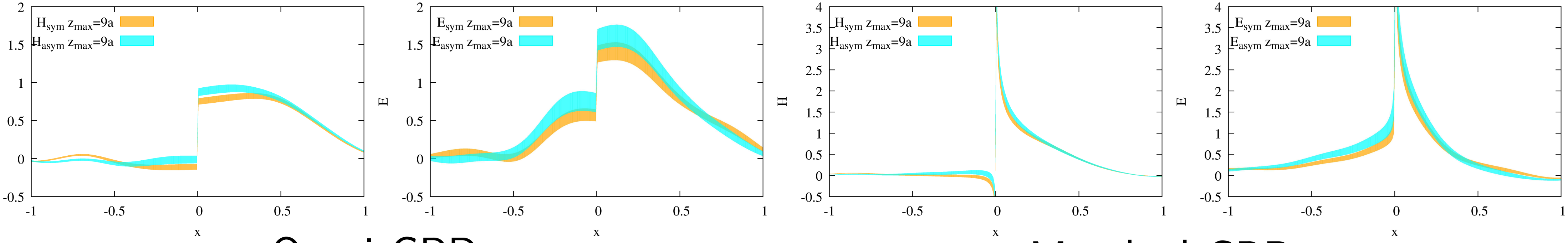


Im E



Quasi- and matched H and E GPDs

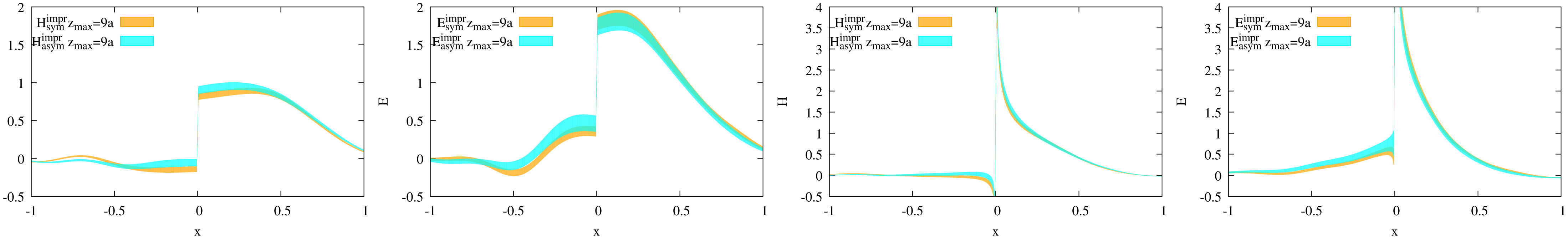
STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



Twist-3

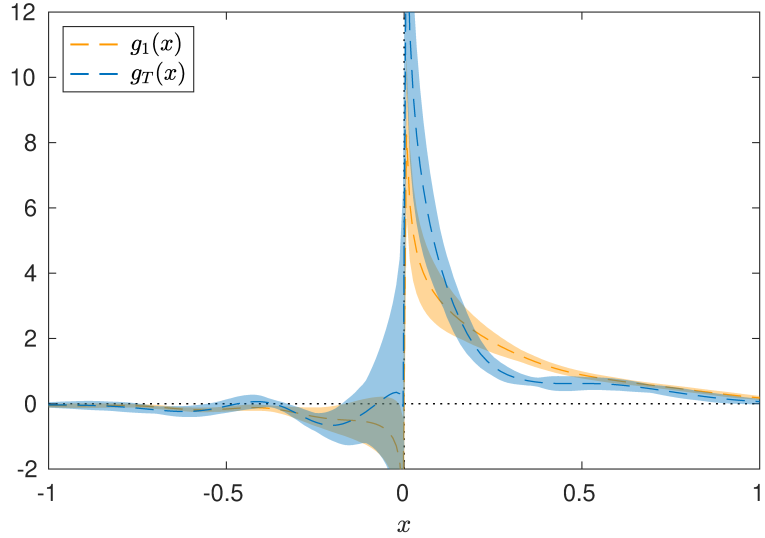
PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.



Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025
  BC-type sum rules S. Bhattacharya, A. Metz, Phys. Rev. D105 (2022) 054027
  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, JHEP 05(2021)086, 11(2021)087
- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
  + test of Wandzura-Wilczek approximation
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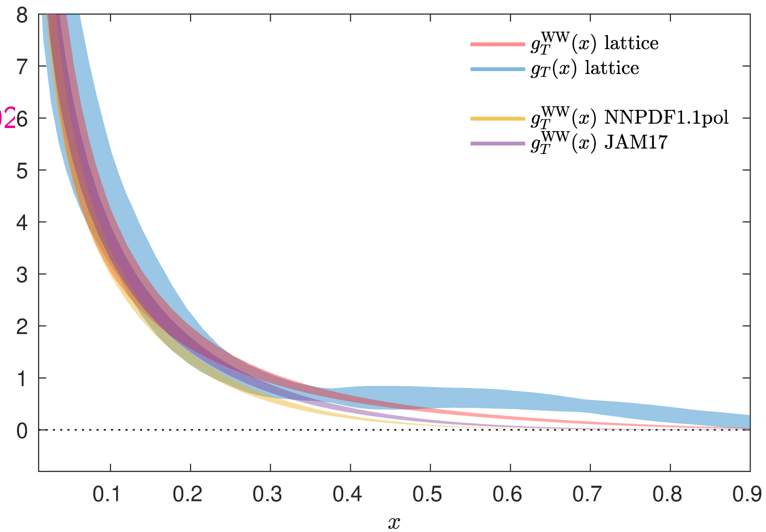
  S. Bhattacharya et al., Phys. Rev. D104 (2021) 114510



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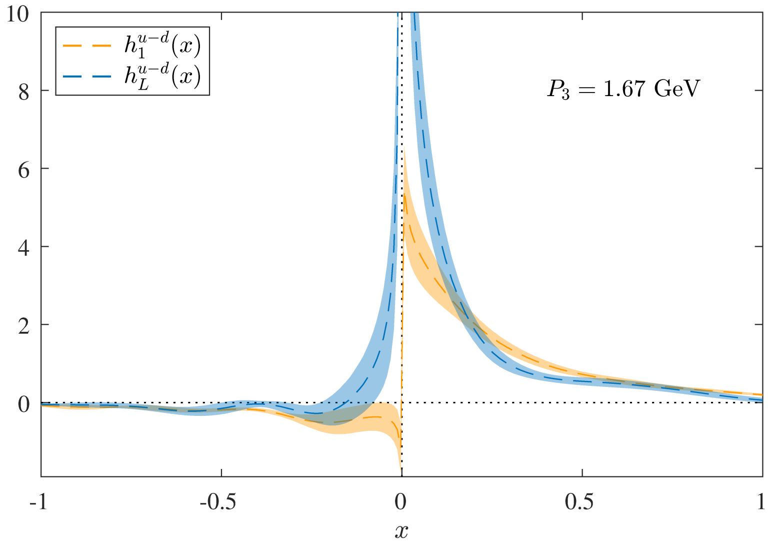

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  BC-type sum rules S. Bhattacharya, A. Metz, Phys. Rev. D105 (2022) 05402
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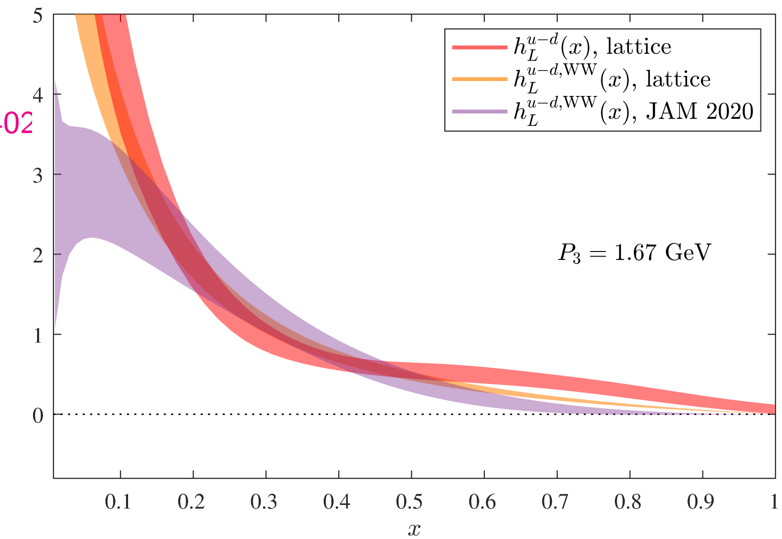
  S. Bhattacharya et al., Phys. Rev. D104 (2021) 114510
- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2306.05533



Twist-3 axial GPDs

Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., PRD108(2023)054501

Twist-3 axial GPDs: $\widetilde{G}_1$, $\widetilde{G}_2$, $\widetilde{G}_3$, $\widetilde{G}_4$

$$\mathcal{F}^{[\gamma_j \gamma_5]} = -i \frac{\Delta_j \gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1} + \gamma_j \gamma_5 \; F_{\widetilde{H}+\widetilde{G}_2} + \frac{\Delta_j \gamma_3 \gamma_5}{P_3} F_{\widetilde{G}_3} - \frac{\mathrm{sign}[P_3]\, \varepsilon_\perp^{j\,\rho} \Delta_\rho \gamma_3}{P_3} F_{\widetilde{G}_4}$$

Contributions from different insertions and projectors ($\vec{\Delta} = (\Delta_1, 0, 0)$):

$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

Twist-3 GPDs in coordinate space

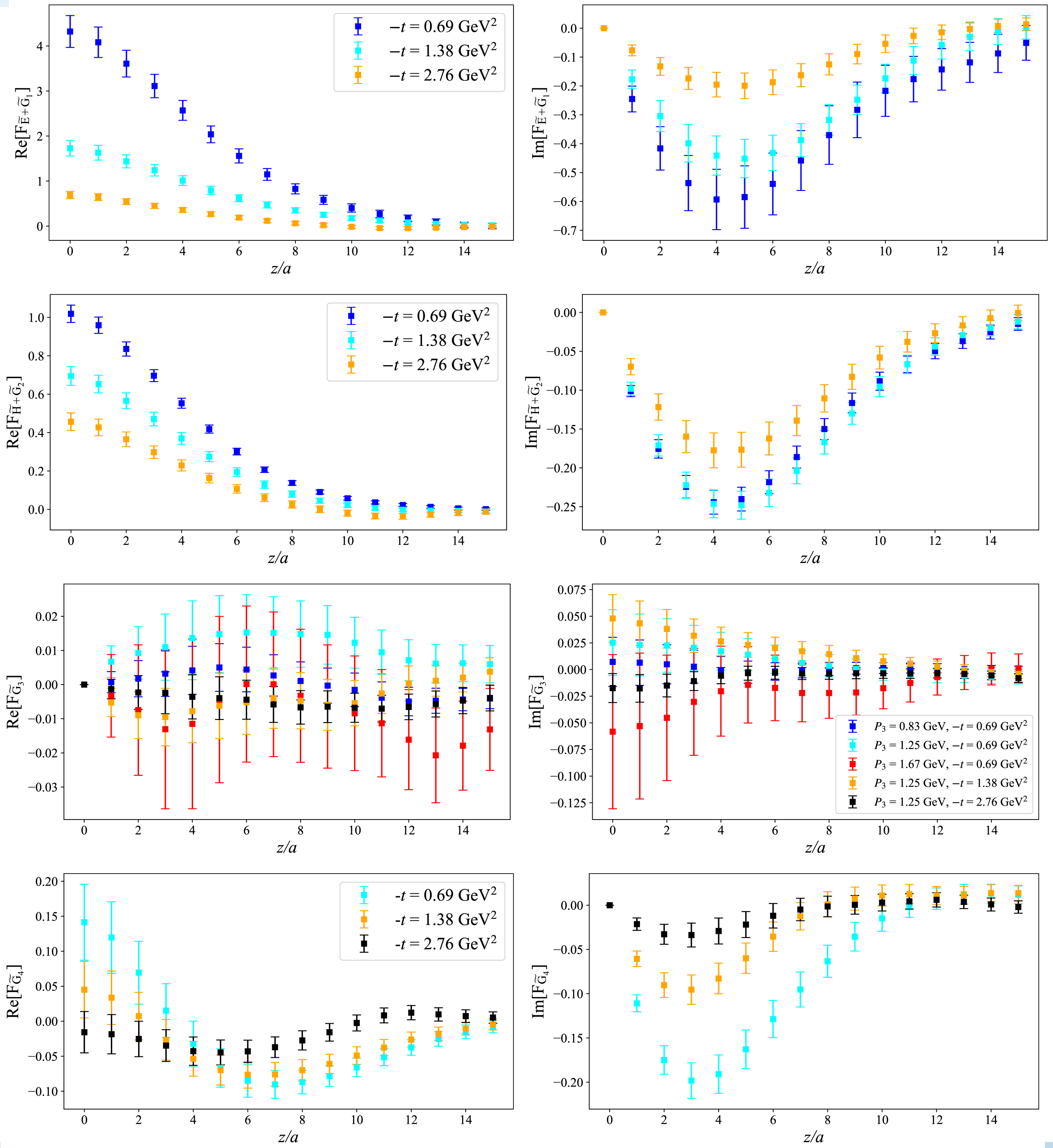
S. Bhattacharya et al.
PRD108(2023)054501

$\widetilde{E} + \widetilde{G}_1$

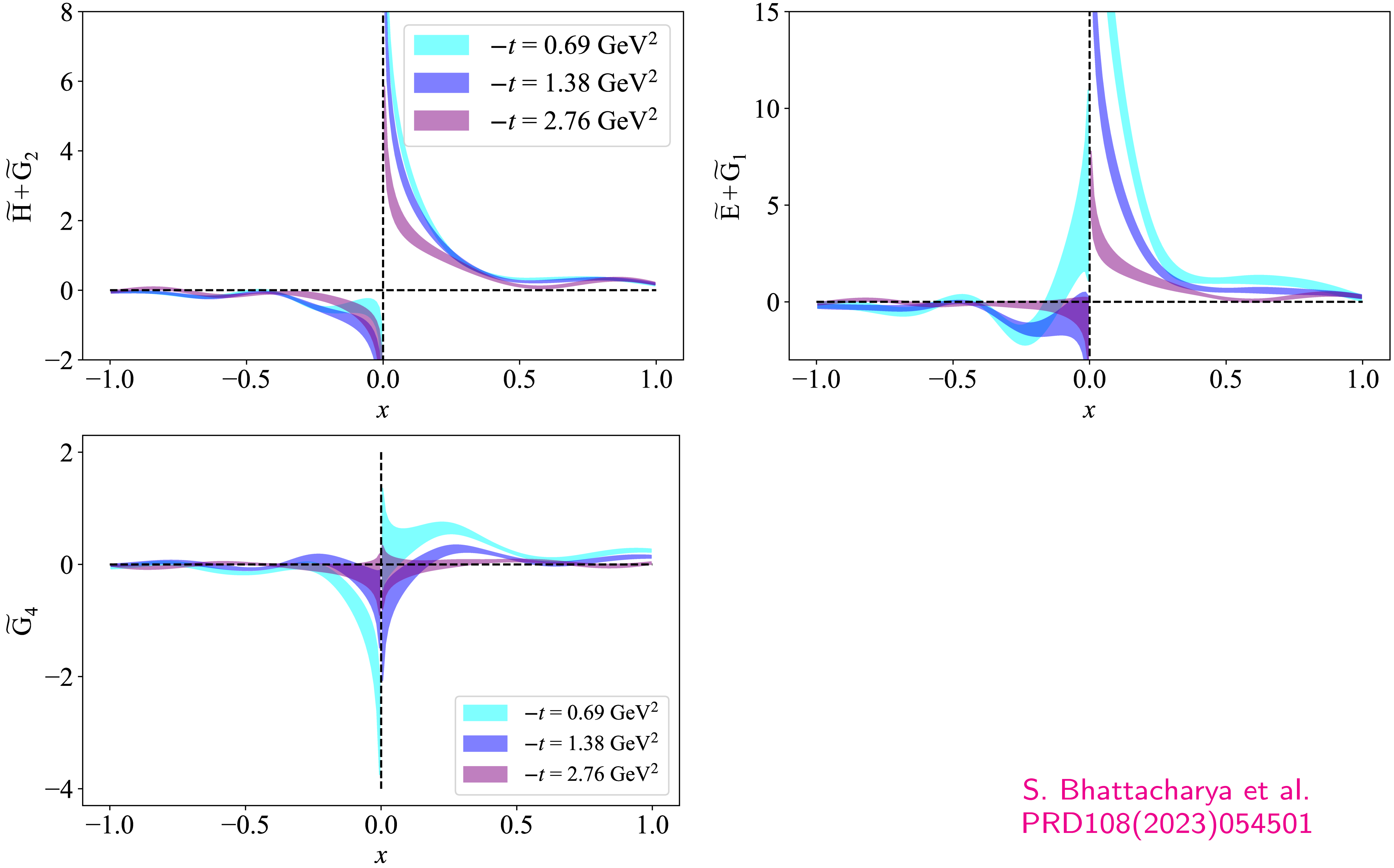
$\widetilde{H} + \widetilde{G}_2$

$\widetilde{G}_3$

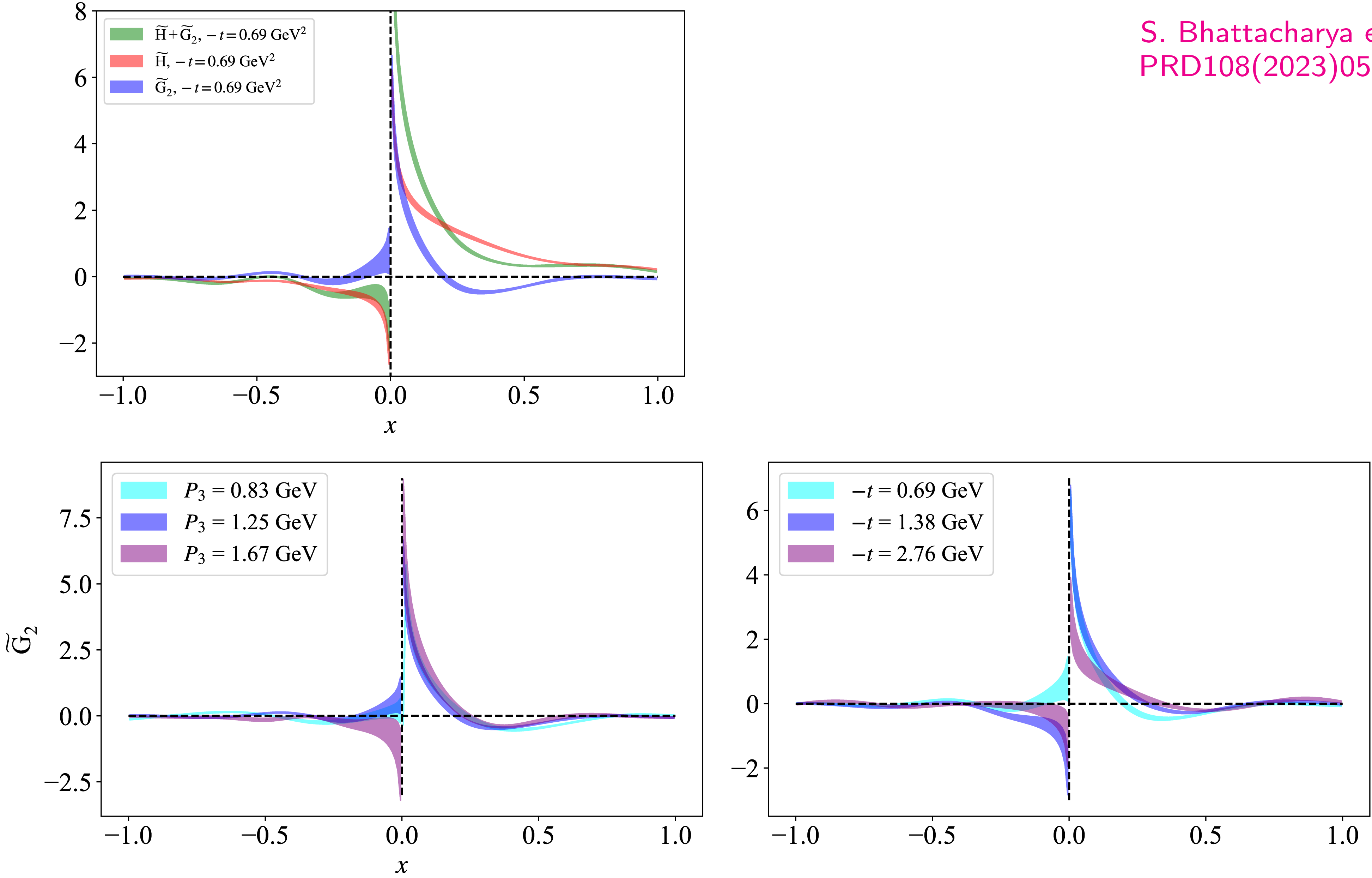
$\widetilde{G}_4$



Twist-3 GPDs in x-space



S. Bhattacharya et al.
PRD108(2023)054501



S. Bhattacharya et al.
PRD108(2023)054501

Consistency checks

Burkhardt-Cottingham-type sum rules:

$$G_P(t) = \int_{-1}^{1} dx \left(\widetilde{E}(x,\xi,t) + \widetilde{G}_1(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t)$$

$$G_A(t) = \int_{-1}^{1} dx \left(\widetilde{H}(x,\xi,t) + \widetilde{G}_2(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t)$$

$$\Rightarrow \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0$$

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

- satisfied for $\widetilde{H} + \widetilde{G}_2$ – same local limit and norm as $\widetilde{H}$,
- cannot be tested for $\widetilde{E} + \widetilde{G}_1$ – $\widetilde{E}$ inaccessible at $\xi = 0$.
- norms of $\widetilde{G}_2$ and $\widetilde{G}_4$ close to vanishing.

Efremov-Leader-Teryaev-type sum rules:

$$\int dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4} G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E(t)\,.$$

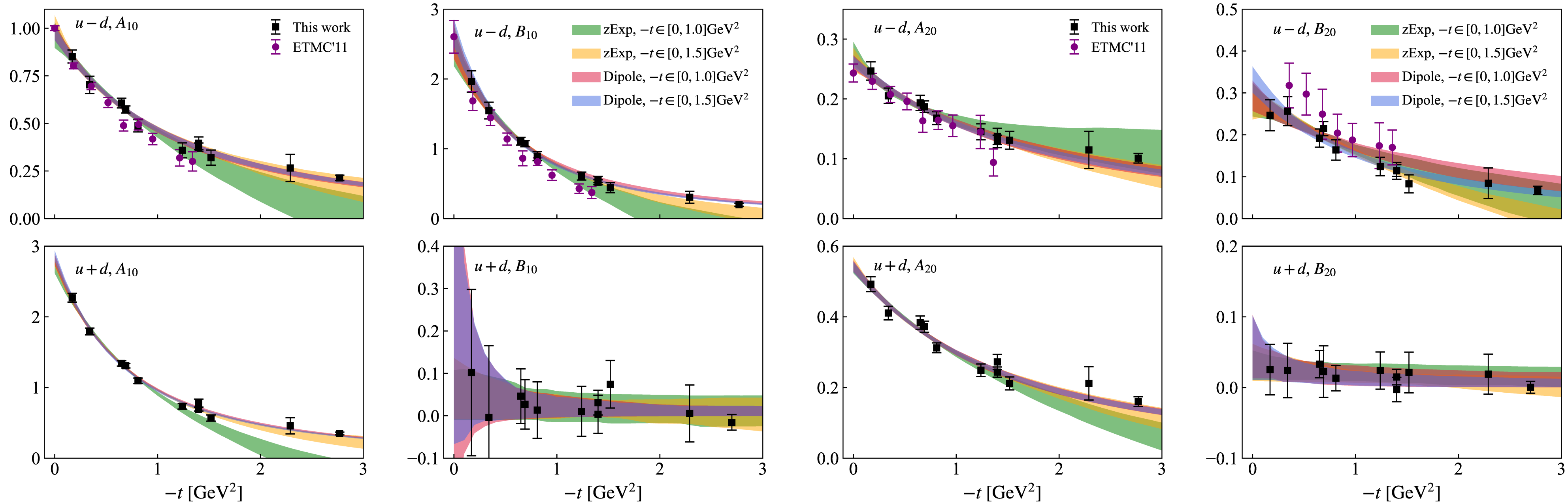
- $\widetilde{G}_3$ indeed vanishes at $\xi = 0$,
- $\widetilde{G}_4$ non-vanishing and small.

GPDs moments from OPE of non-local operators

Short-distance factorization of ratio-renormalized H/E:

$$\mathcal{F}^{\overline{\mathrm{MS}}}(z,P,\Delta)=\sum_{n=0}\frac{(-izP)^n}{n!}C_n^{\overline{\mathrm{MS}}}(\mu^2z^2)\langle x^n\rangle+\mathcal{O}(\Lambda^2_{\mathrm{QCD}}z^2),$$

$C_n^{\overline{\mathrm{MS}}}(\mu^2z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)



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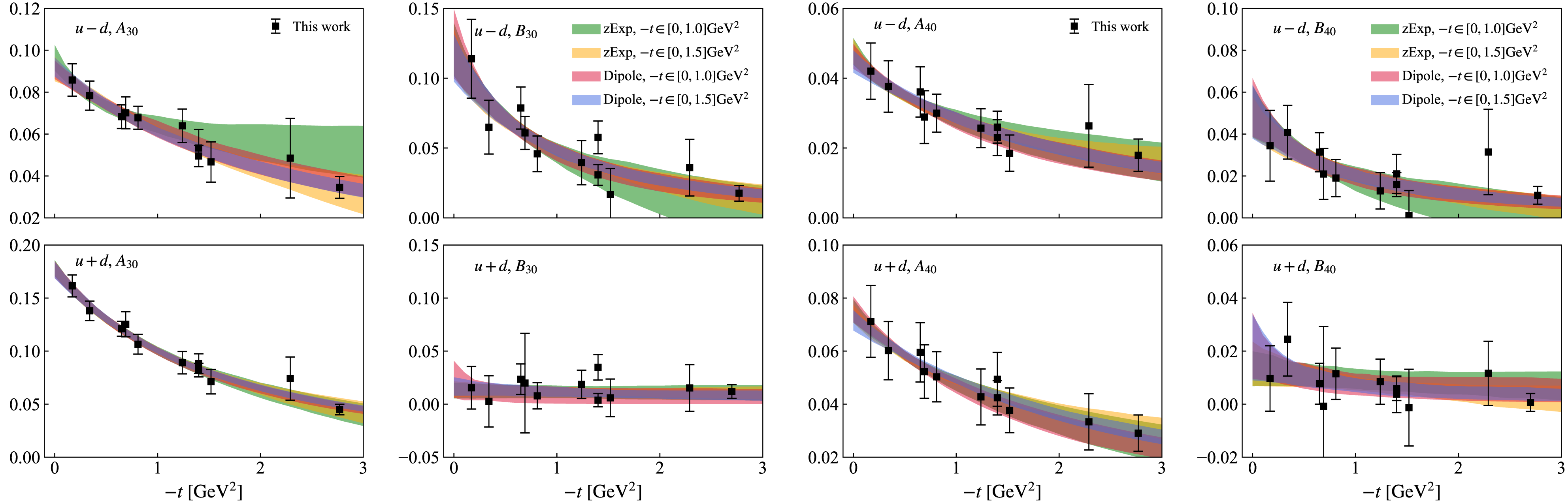
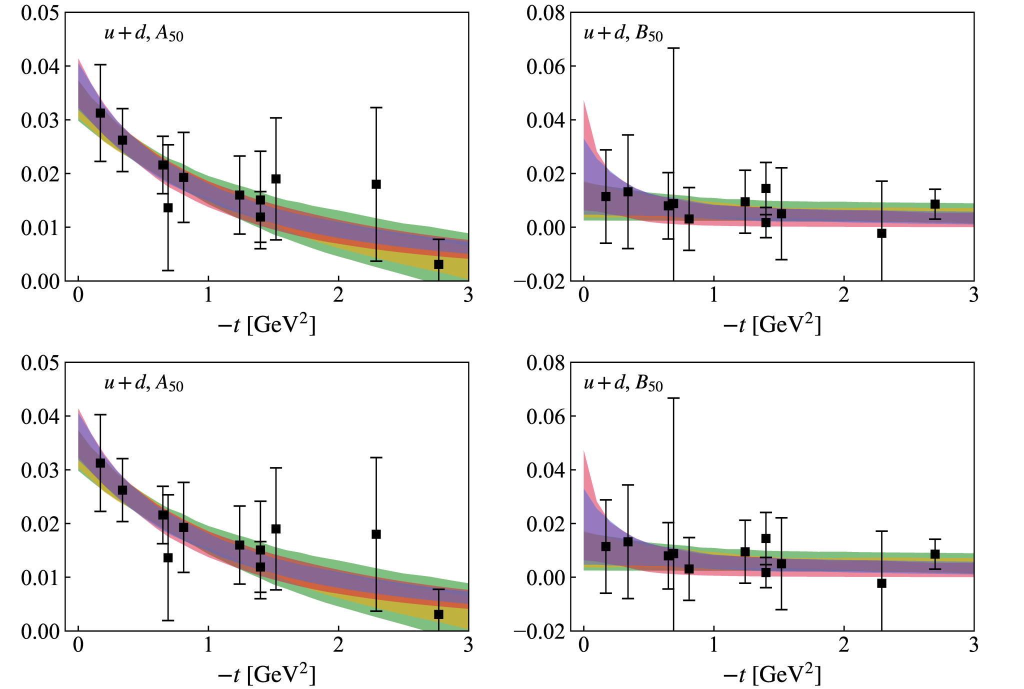
GPDs moments from OPE of non-local operators



Also
higher moments!



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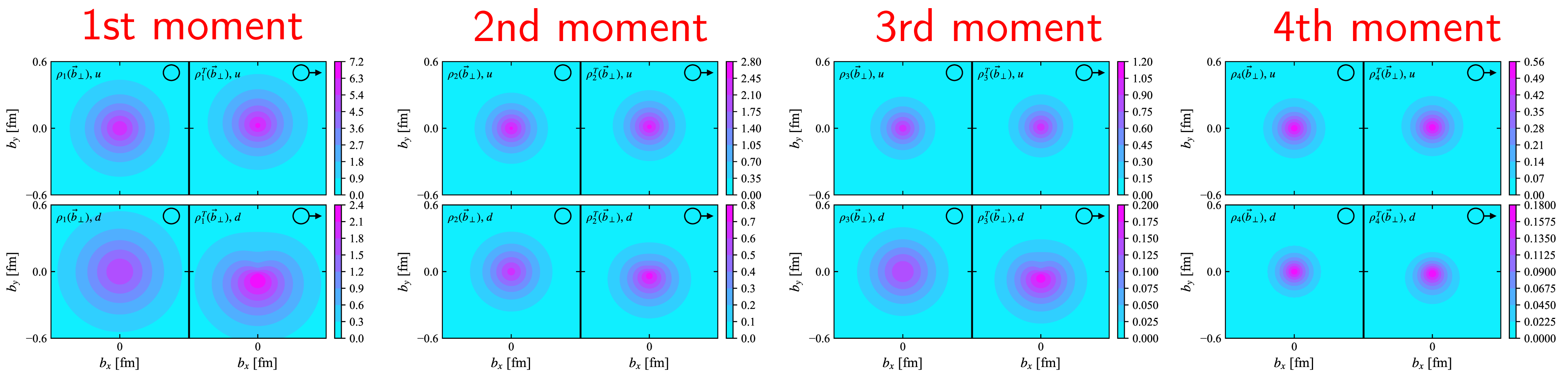
(ETMC/BNL/ANL)
PRD 108(2023)014507

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}_\perp^2) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}_\perp^2)] e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp}.$$



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