THE GRAVITON SOFT-WALL MODEL AND THE DESCRIPTIONS OF MESONS AND EXOTIC STATES

Matteo Rinaldi[1] and Vicente Vento

[1]INFN section of Perugia

OUTLINE

- Short introduction to Glueballs

- Introduction to holographic models

- The graviton soft-wall (GSW) model and predictions for spectra of meson and glueballs

  M. R. and V. Vento, Scalar and Tensor Glueballs as Gravitons, EPJA 54 (2018)
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  M. R. and V. Vento, Scalar spectrum in a graviton soft wall model, JPG 47 (2020), 12, 125003
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- The mixing problem within the GSW model

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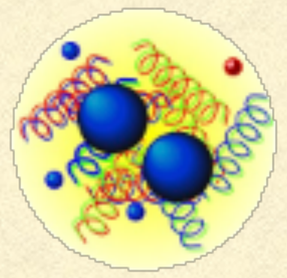
SHORT INTRODUCTION TO GLUEBALLS

From QCD:

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum \bar{\Psi}(i\gamma \cdot D - m)\Psi \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abs} A^b_\mu A^c_\nu$$

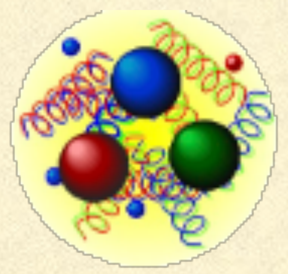
Mesons

$$\mathbf{3} \otimes \bar{\mathbf{3}}$$



Baryons

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$$



Exotic states

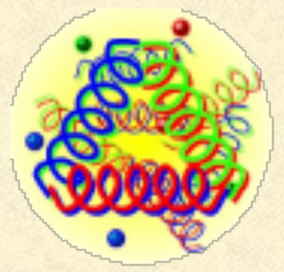
$$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$$

HYBRIDS

$$\mathbf{8} \otimes \mathbf{8}$$

$$\mathbf{8} \otimes \cdots\cdots \otimes \mathbf{8}$$

GLUEBALLS



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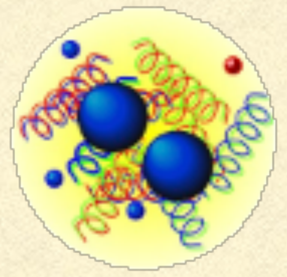

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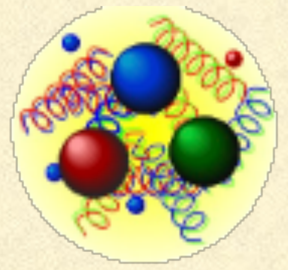

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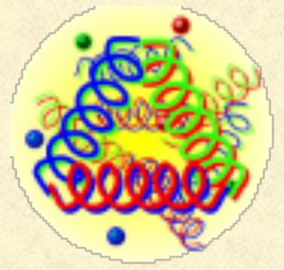

Glueball spectroscopy is a unique laboratory to test non perturbative QCD and CONFINEMENT

However :

1) several mesons have similar mass and quantum number → **MIXING**
2) Their characterization is not clear
3) Lattice calculations of decay are difficult! Models could help!

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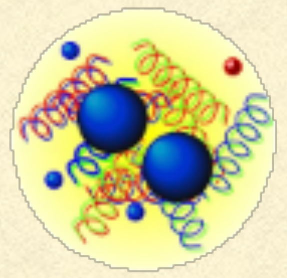

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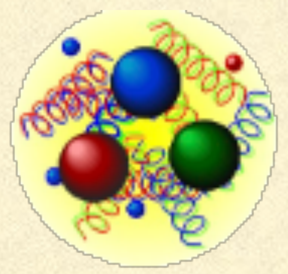

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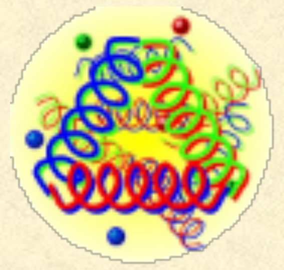

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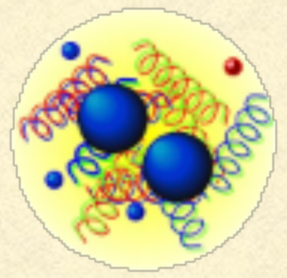

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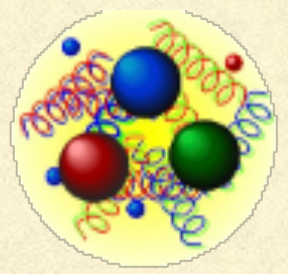

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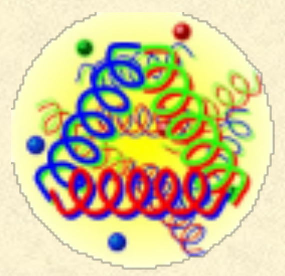

we evaluated also the spectra of hybrid candidates within our model!
M. R. and V. Vento, PRD 109 (2024) 11,11403

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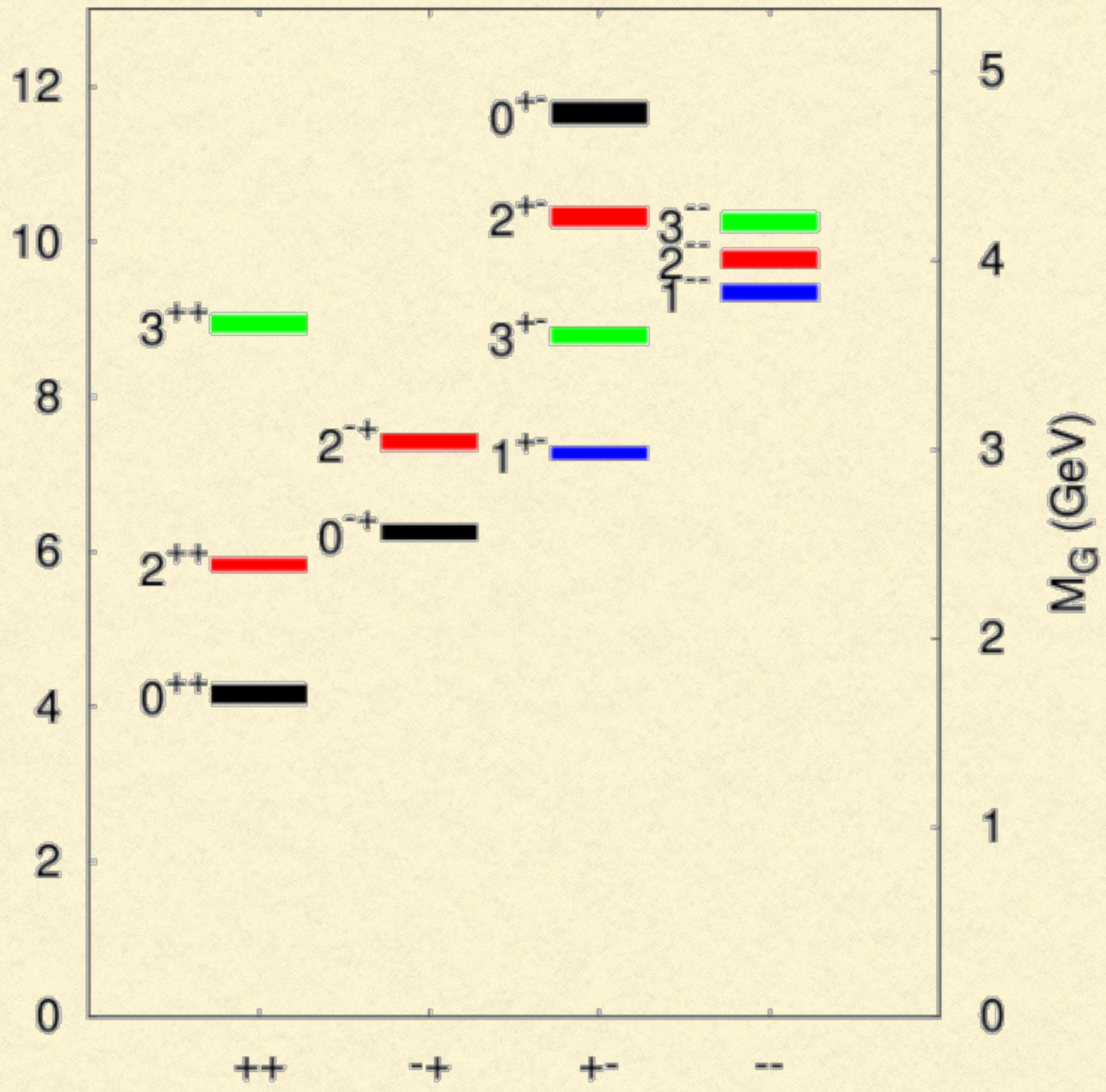
There are several calculations and lattice data:



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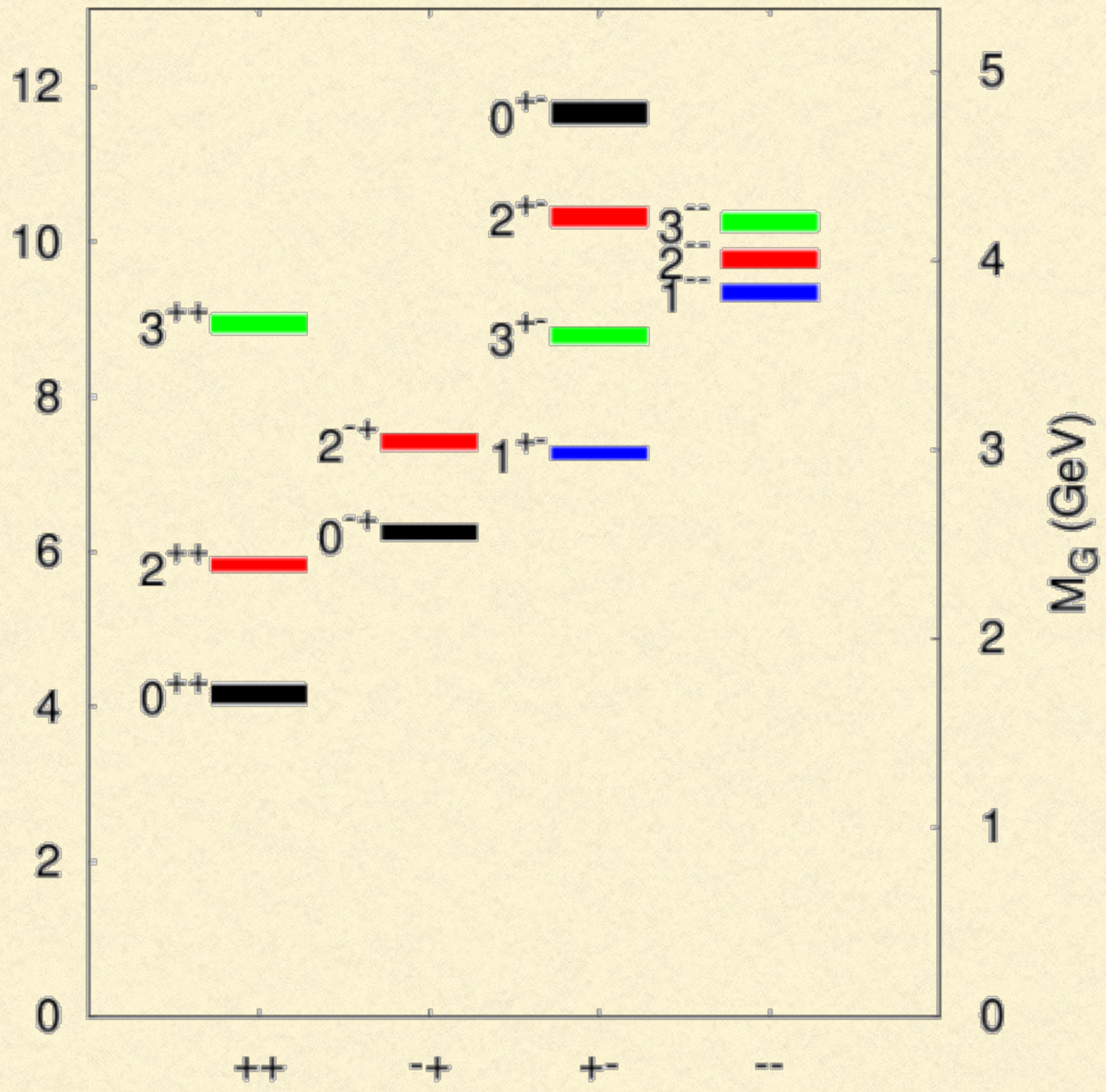

An extraction from J/ψ decay:

$$M_0 \sim 1865 \pm 25^{+10}_{-30} \text{ MeV}$$

E. Klempt et al PLB 816, 136227 (2021)

SHORT INTRODUCTION TO GLUEBALLS

Moreover, different lattice collaborations predict different masses, in particular for the ground state:

J^{PC}	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277
SDTK	$1865 \pm 25^{+10}_{-30}$					

MP: C.J. Morningstar et al, PRD 60, 034509 (1999)
YC: Y. Chen et al, PRD 73, 014516 (2006)
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Could models help in clarifying the situation?

SHORT INTRODUCTION TO GLUEBALLS

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J^{PC}	0^{++}
MP	1730 ± 94
YC	1719 ± 94
LTW	1475 ± 72
SDTK	$1865 \pm 25^{+10}_{-30}$

Meson	$f_0(1500)$	$f_0(1710)$
PDG	1504 ± 6	1723 ± 6

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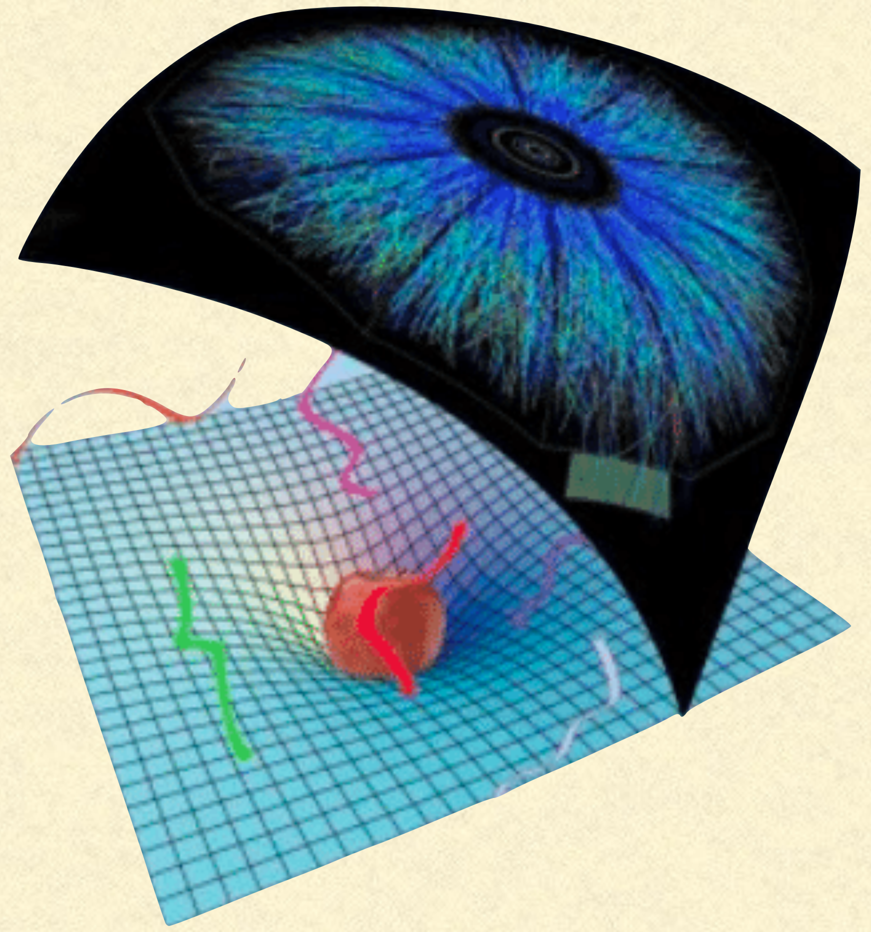
Is there a mixing between glueballs and meson states? Also in this case models could help to understand in which conditions mixing occurs or not.

SHORT INTRODUCTION TO ADS/QCD HOLOGRAPHY

Starting from the Maldacena's conjecture:

N=4 SU(N) SYM ⟷ String theory on $AdS_5 \times S_5$

Isometries between group symmetries



We can study the QFT problem in the non-perturbative regime in the gravity sector!

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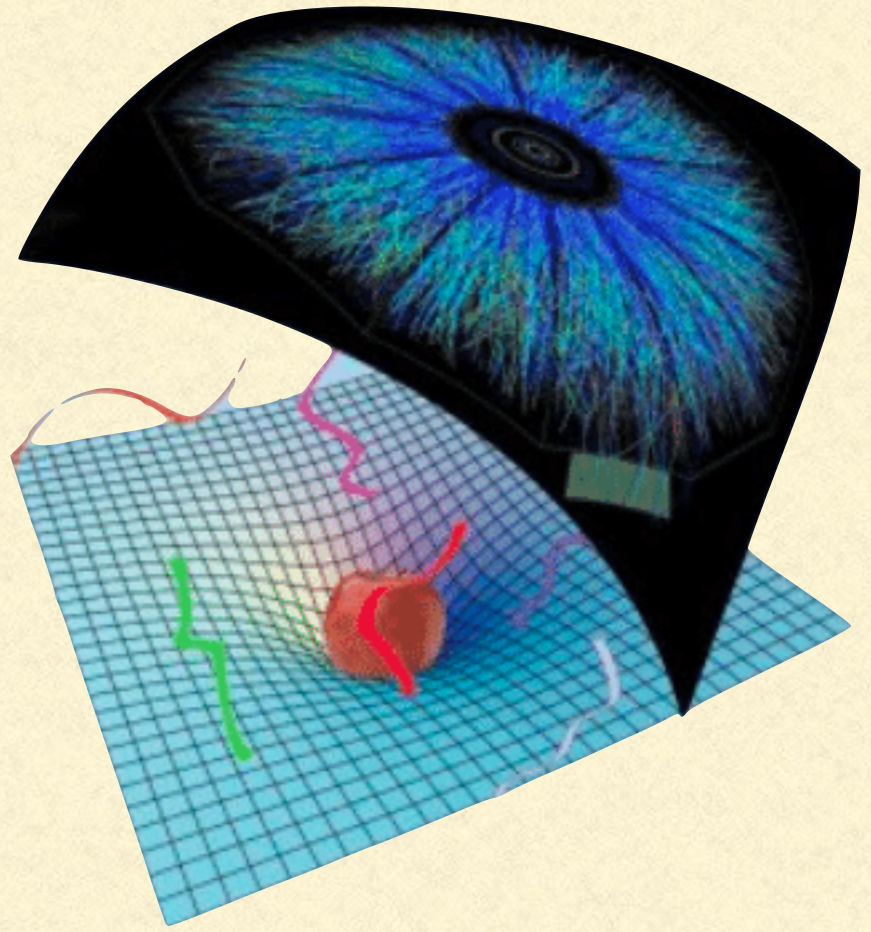

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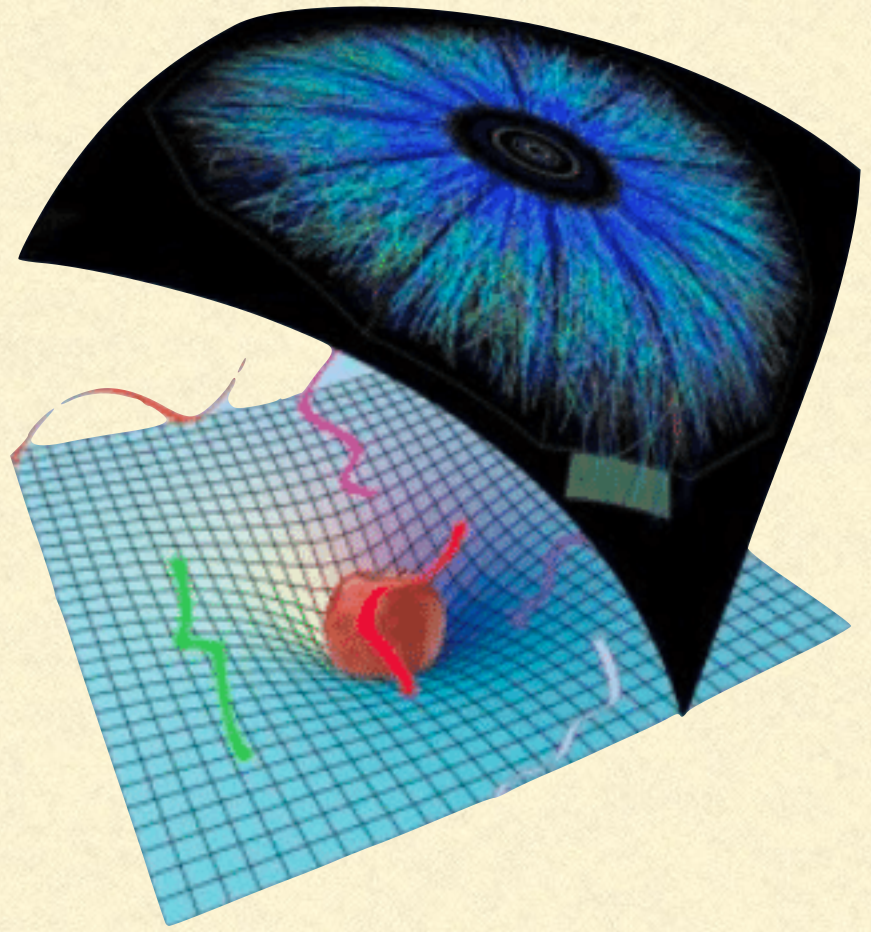
String theory on AdS_5 x S_5

- supersymmetry
- Conformal symmetry
- N is ∞
- No Confinement
- **This is NOT QCD**



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  via compattification (Witten)
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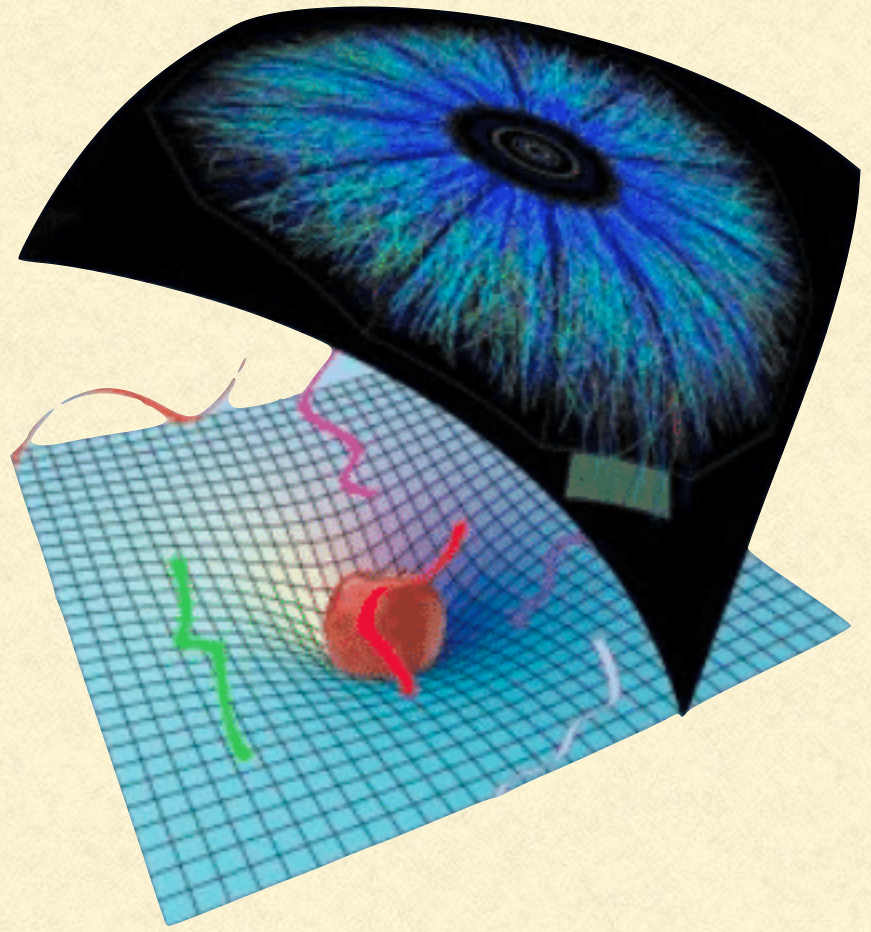

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supersymmetry ✗
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Conformal symmetry ✗
Low energy scales
Chiral limit
N is ∞

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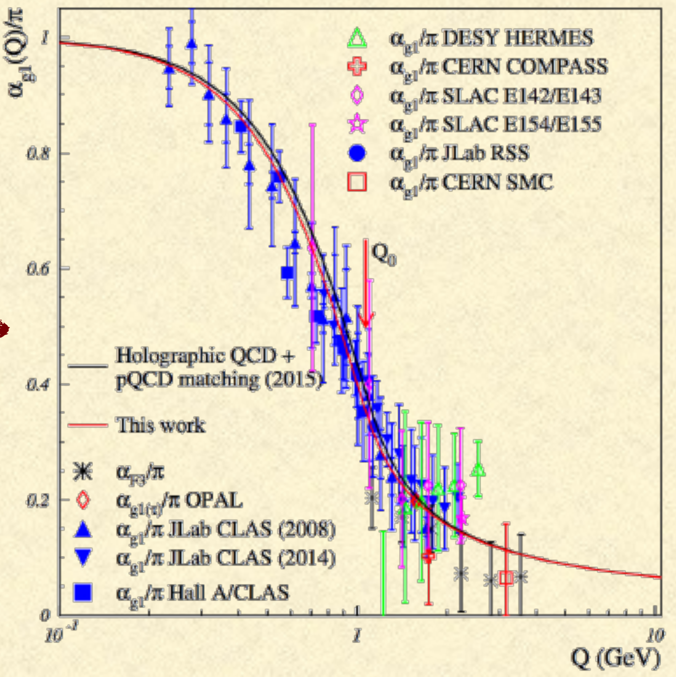

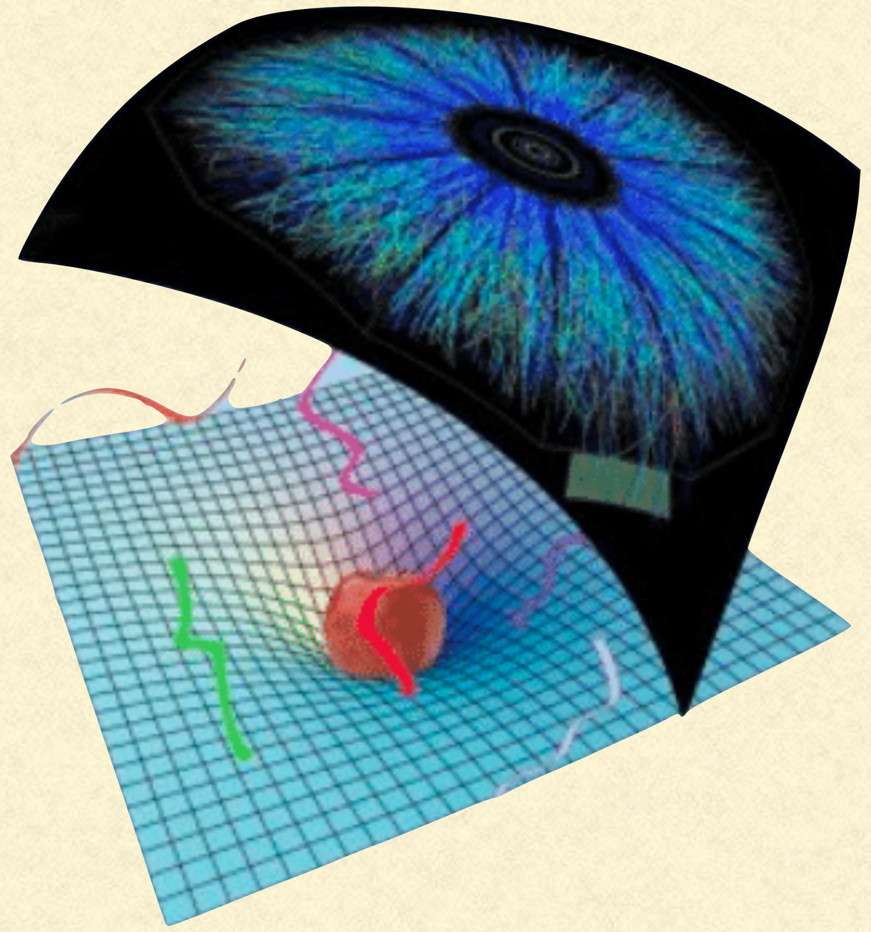

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We can calculate ~ N^0

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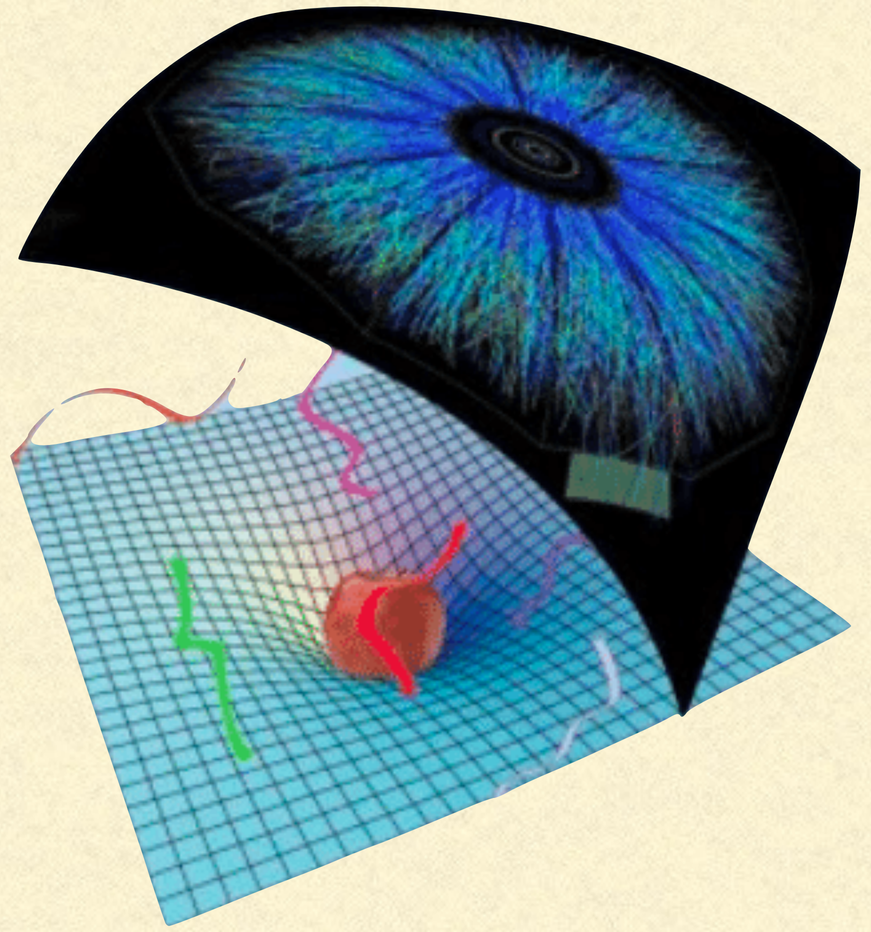

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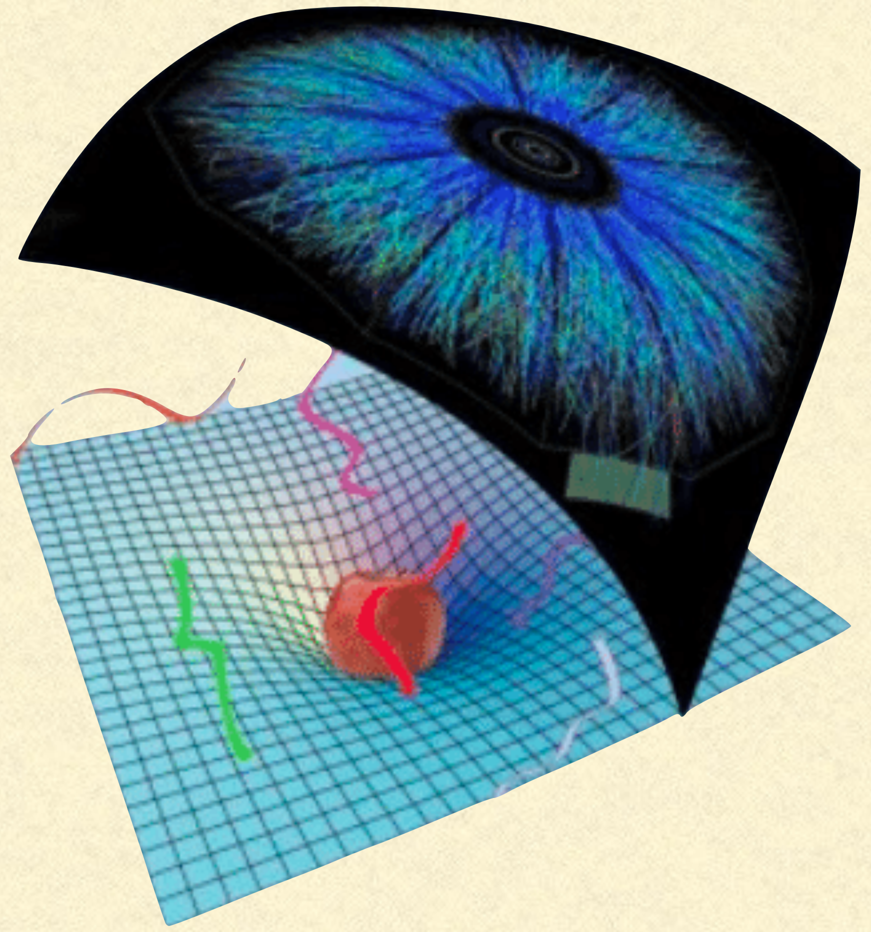
No Confinement
Buttom-up approach: we modify
The gravity theory to provide it

This is ALMOST QCD



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SHORT INTRODUCTION TO ADS/QCD HOLOGRAPHY

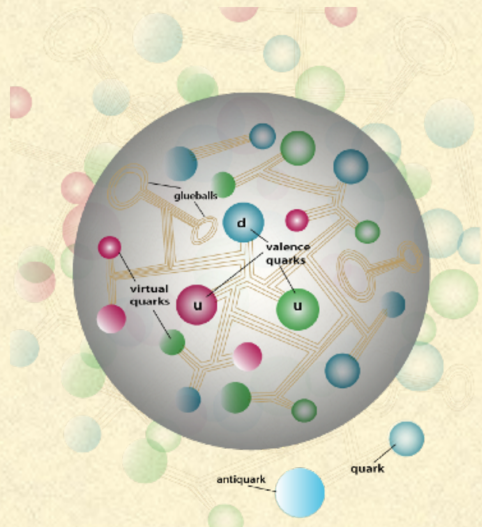
The dream is to understand QCD using its dual gravity theory!

- ❖ No supersymmetry
- ❖ Confinement
- ❖ Conformal symmetry broken
- ❖ N is finite

Top-down Approach:
Find a gravitational theory dual to QCD

Bottom-up Approach:
Starts from QCD and attempts to construct a five dimensional holographic dual

Confinement



Confinement

Hard-wall model
Compactification of the 5° dimension by hand. AdS geometry cut by two branes: UV and IR.

Soft-wall model
Soft cutoff of AdS space by introducing a dilaton field.

SHORT INTRODUCTION TO ADS/QCD HOLOGRAPHY

Qualitatively agreement with data of hadron spectroscopy and hadron parton distributions:

HADRON SPECTRUM:

S.J. Brodsky et al, Phys. Rep. 584 (2015)
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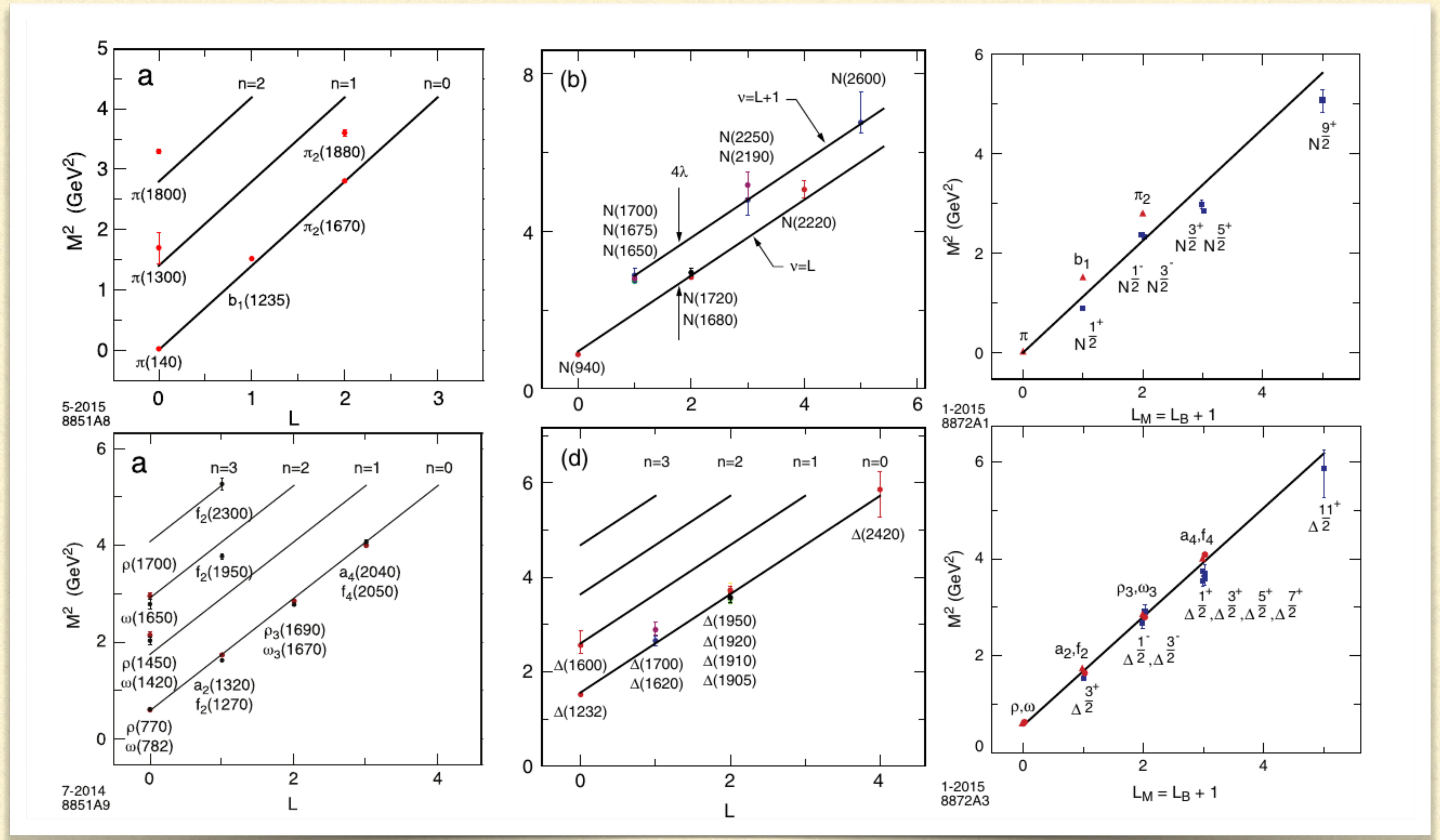

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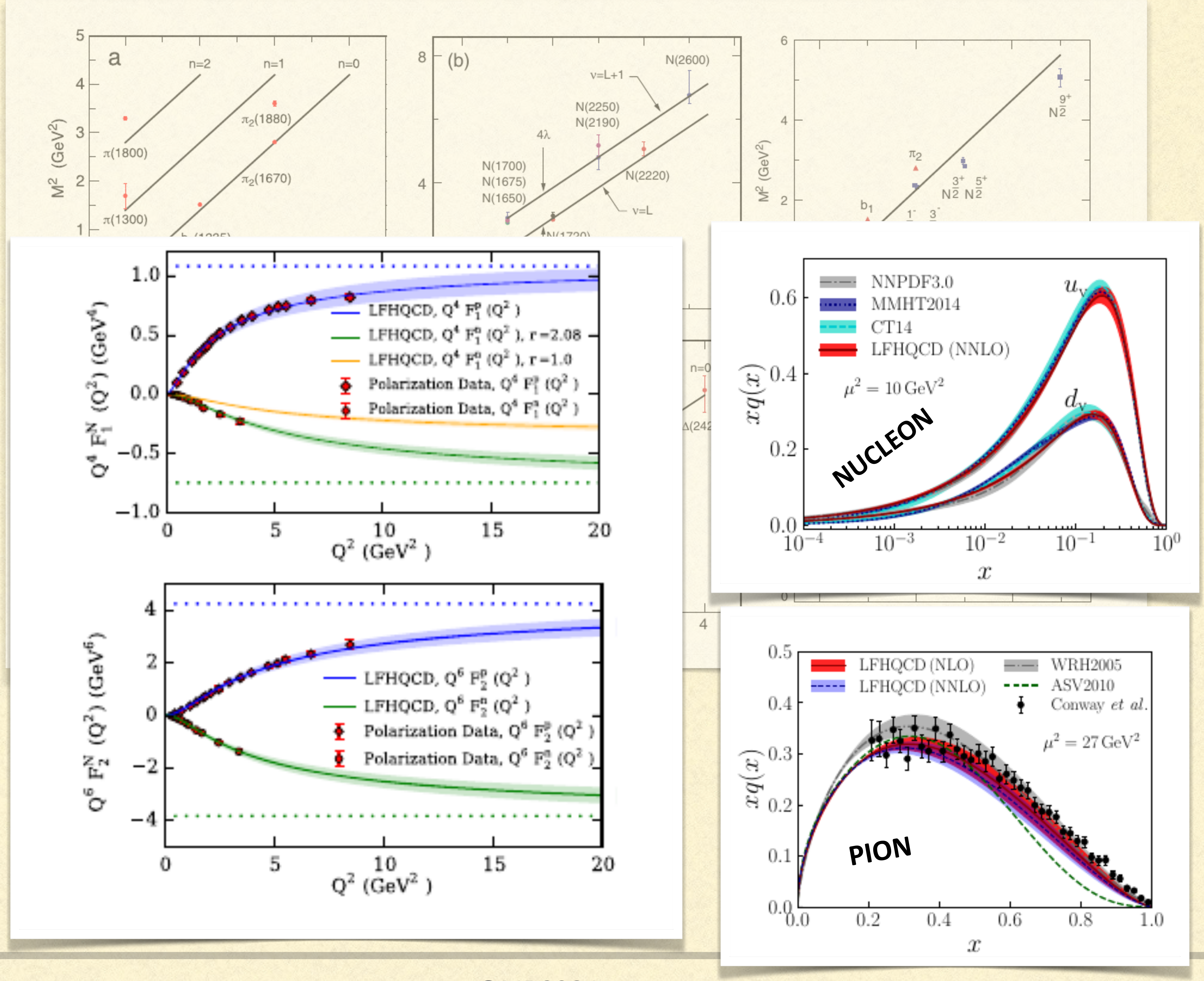

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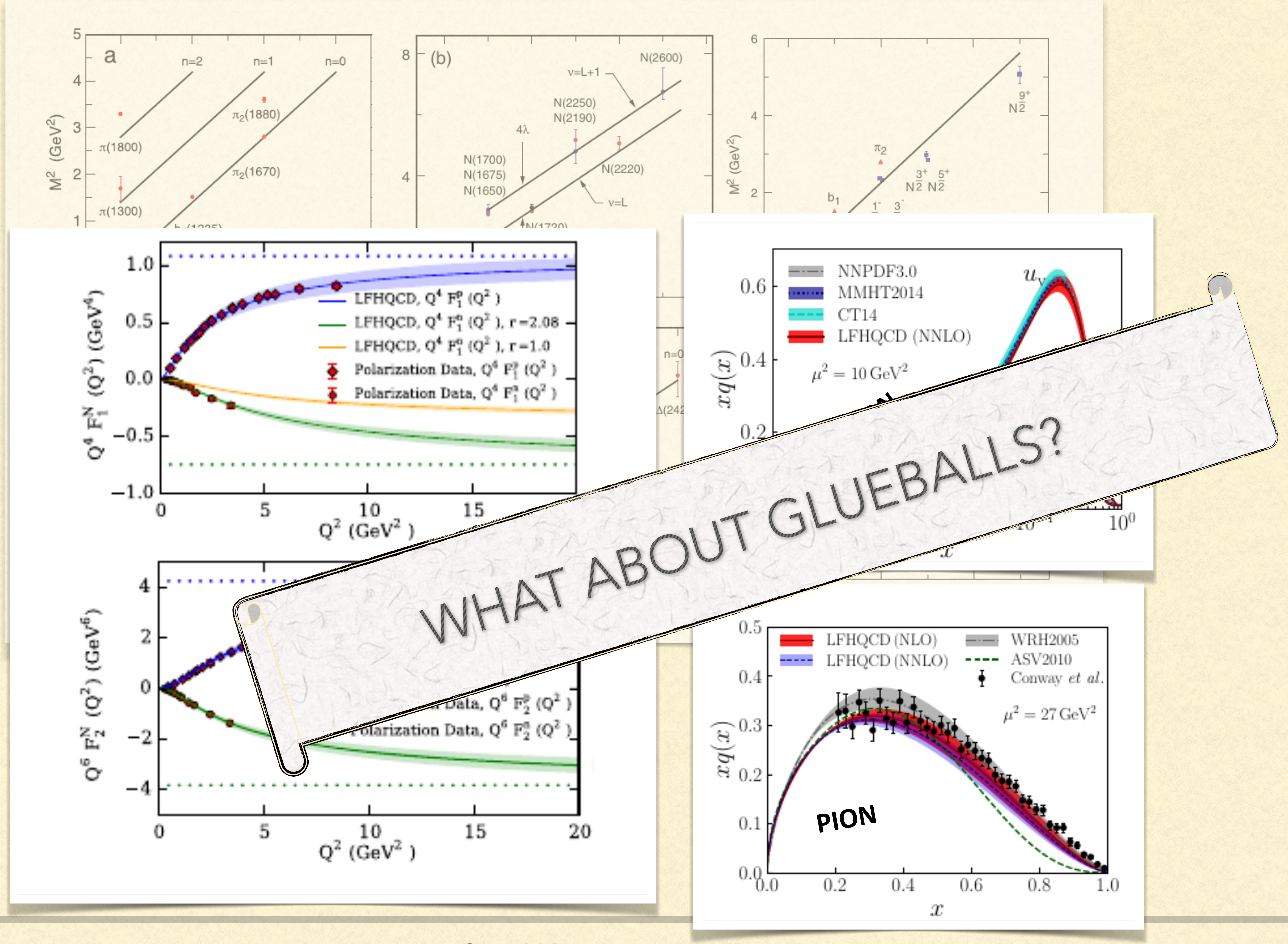
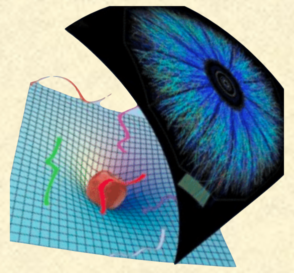


GLUEBALLS IN SOFT-WALL ADS/QCD



karch et al, PRD 74, 015005 (2006)

The gravitational theory is based on Anti-De Sitter space in (4+1) dimensions:

$$g_{MN}dx^M dx^N = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

The confinement can be implemented by adding a **Dilaton** in the action:

$$I = \int d^4x dz \sqrt{-g}\, e^{\varphi(z)} \mathscr{L}$$

$$\varphi(z) = k^2 z^2$$

the only free parameter of the model

From the Eulero-Lagrngian equation for scalars, vectors… we get the mode functions and the spectrum

Successful in describing the Regge behavior of the spectrum: $M^2_{n,J} \sim n + j, \qquad j \geq 0$

WHAT ABOUT GLUEBALLS?

GLUEBALLS IN SOFT-WALL ADS/QCD

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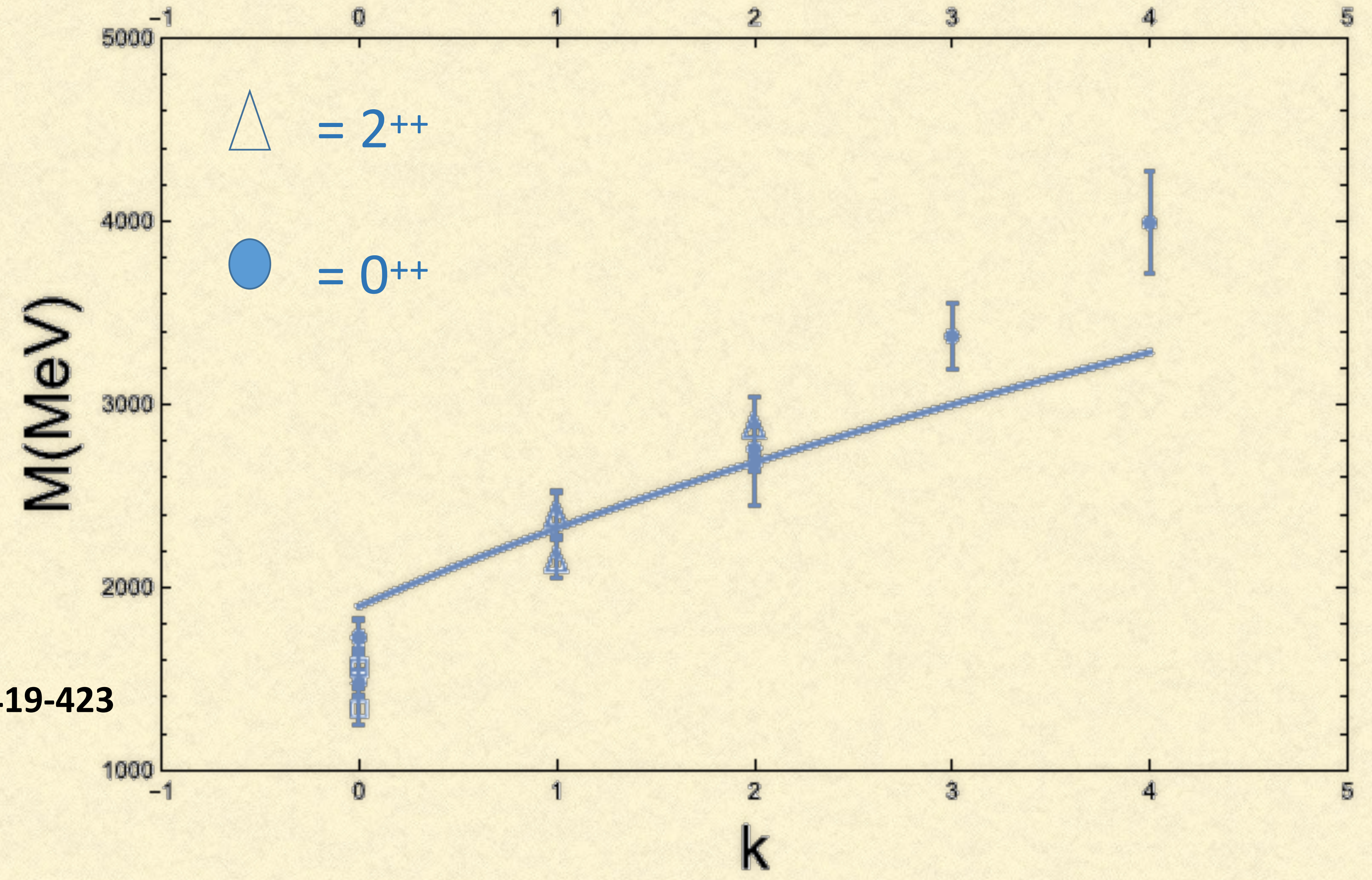
5° dimensional mass ≠ the physical one

We have the field equations (we start with the scalar case): $I = \int d^5x \, \sqrt{g}\, e^{-\varphi(z)}\left[g^{MN}\partial_M \mathcal{G} \partial_N \mathcal{G} + M_5^2 \mathcal{G}^2\right]$



Eduardo Folco Capossoli et al, PLB 753 (2019) 419-423

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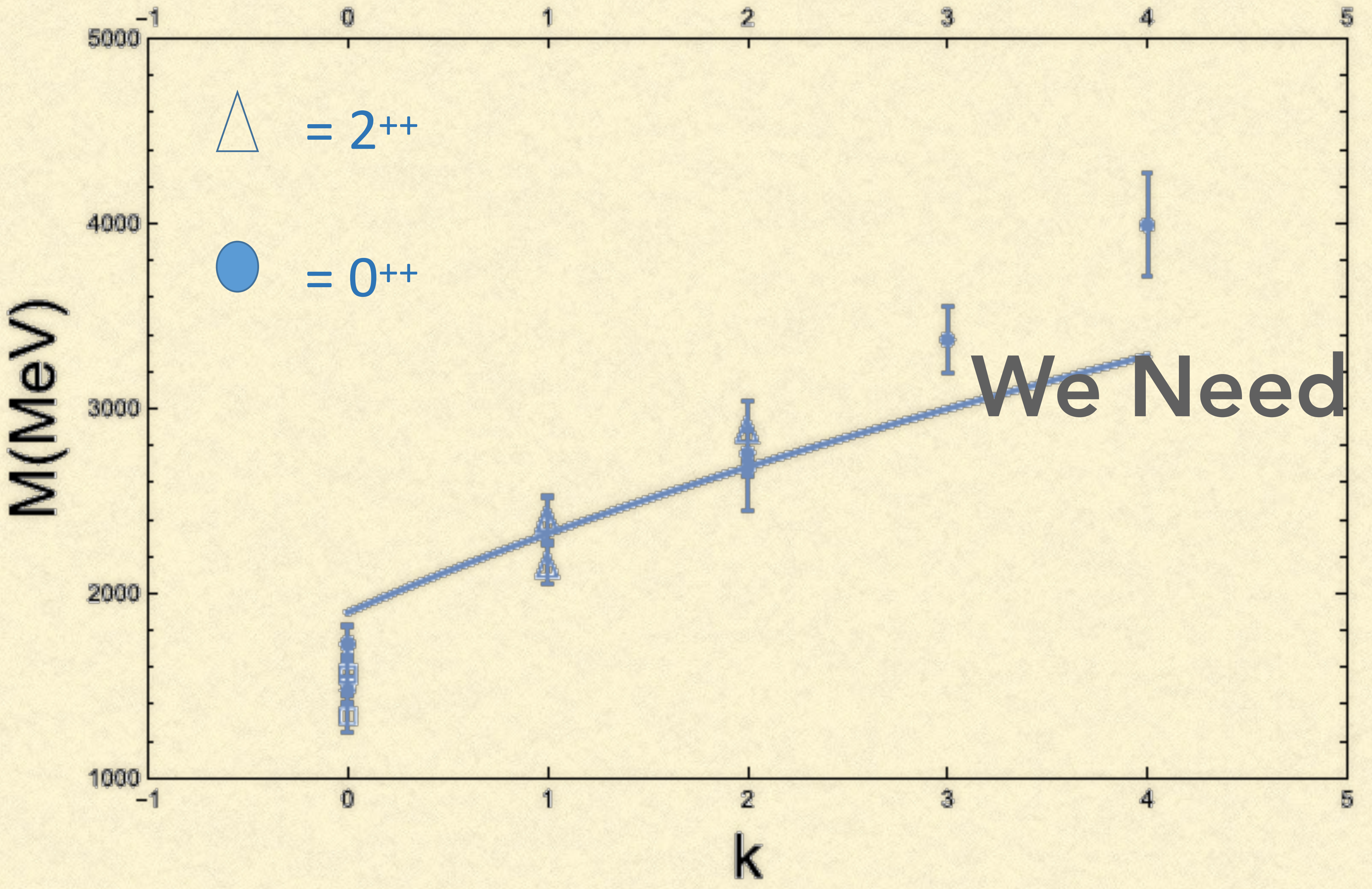

We Need To Improve The Model!

GLUEBALLS IN GRAVITON SOFT-WALL ADS/QCD

We propose to modify the metric to properly describe the glueball dynamics:

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M. R. and V. Vento JPG 47 (2020), 5, 055104
M. R. and V. Vento JPG 47 (2020), 12, 125003
M. R. and V. Vento, PRD 104 (2021) 3,034016

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right) \qquad \varphi(z) = k^2 z^2$$

We keep the dilaton in the action: $\tilde{\mathcal{I}} = \int d^5x \sqrt{-\tilde{g}}\, e^{-\beta\varphi(x)} \mathcal{L}$ apparently we have two parameters

But we fix β to have the same kinetic term of the SW model.

Modified Soft-Wall model in e.g.:

E. F. Capossoli et al, PLB 753, 419-423 (2006)
O. Andreev, PRD 100 (2019) 2, 026013
E. F. Capossoli et al, Chin. Phys. C 44 (2020) 6, 064194
W. de Paula et al, PRD 79, 075019 (2009)
S. Afonin et al, JPG, 49 (2022) 10, 105003

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In our model we consider a graviton, propagating in this modified space, as dual for the **glueball**:

$$-\frac{1}{2}\tilde{h}^{;c}_{ab;c} - \frac{1}{2}\tilde{h}^{c}_{c;ab} + \frac{1}{2}\tilde{h}^{;c}_{ac;b} + \frac{1}{2}\tilde{h}^{;c}_{bc;a} + 4\tilde{h}_{ab} = 0$$

$$\Psi''(t) + V_G(t)\Psi(t) = \Lambda^2 \Psi(t)$$

with:

$$\begin{cases} t = i\alpha z/\sqrt{2} \\ \Lambda^2 = \dfrac{M^2}{\alpha^2} \\ V_G(t) = \dfrac{e^{2t^2}}{t^2} - \dfrac{17}{4t^2} + 14 - 15t^2 \end{cases}$$

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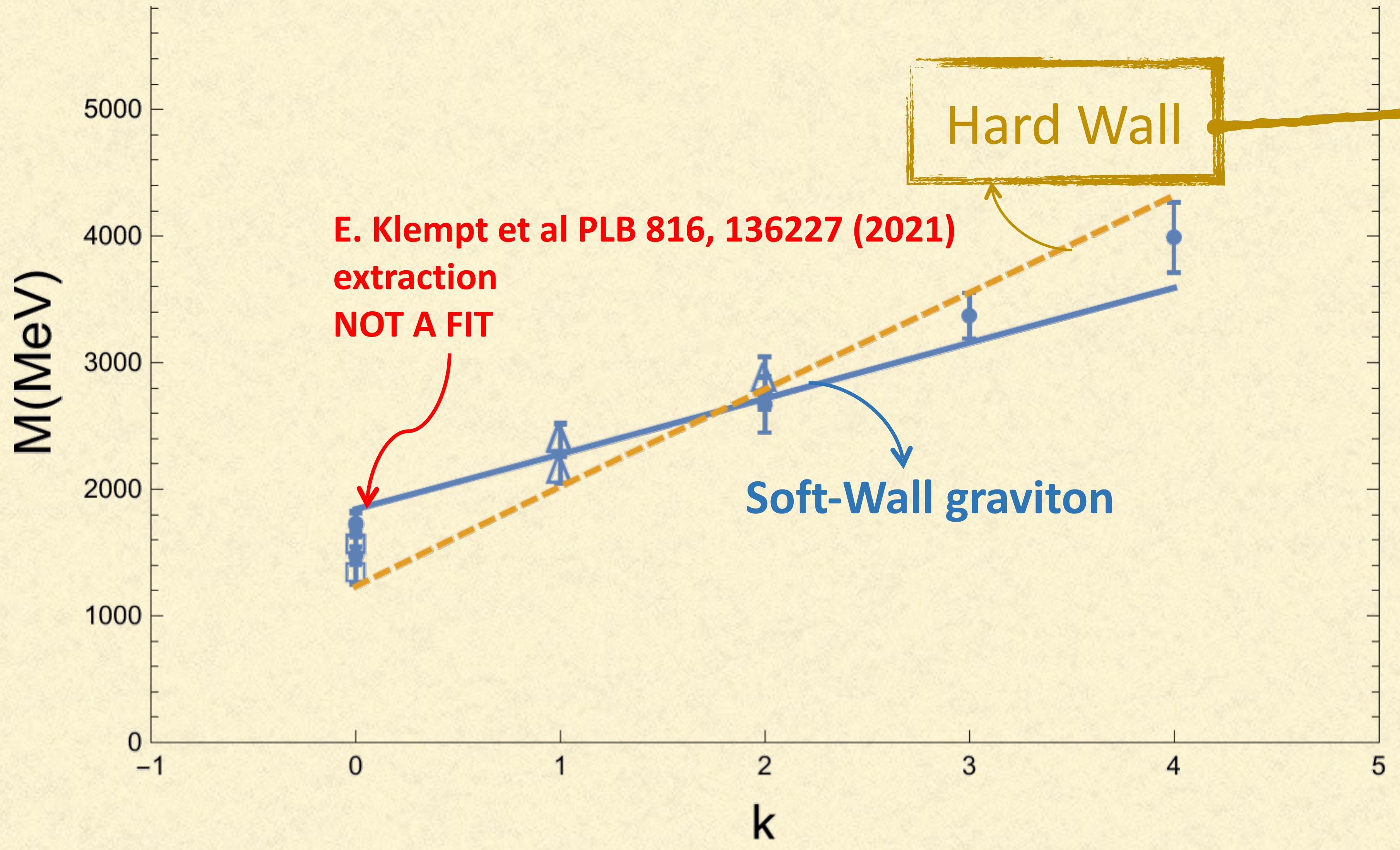

no good for meson spectra

$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$

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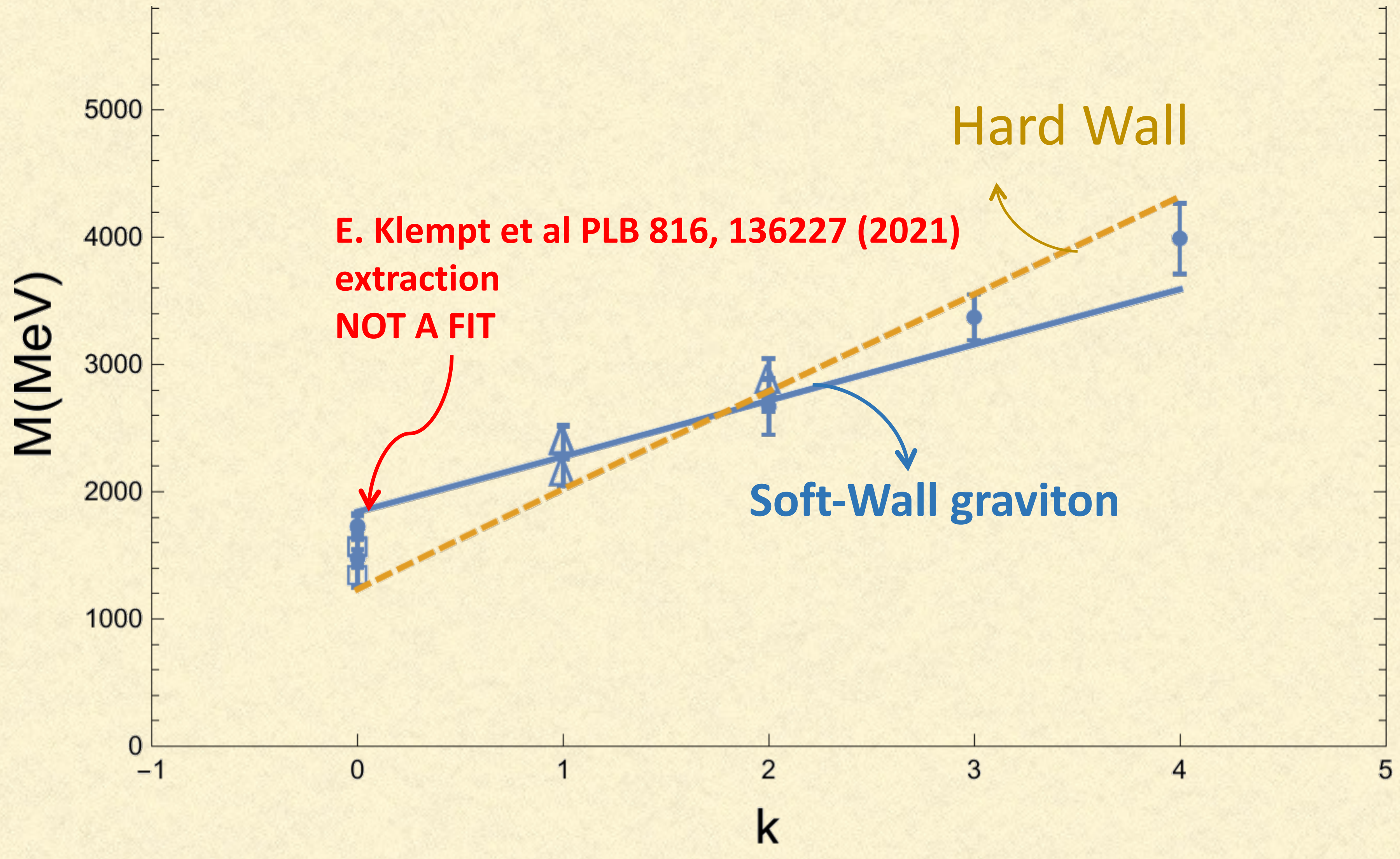
In c… …on, propagating in this modified space, as dual for the **glueball**:

$- \frac{1}{2} \ldots = 0$

	SW	GSW
Meson	V	?
Glueball	X	V



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Usual dilaton

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Metric correction

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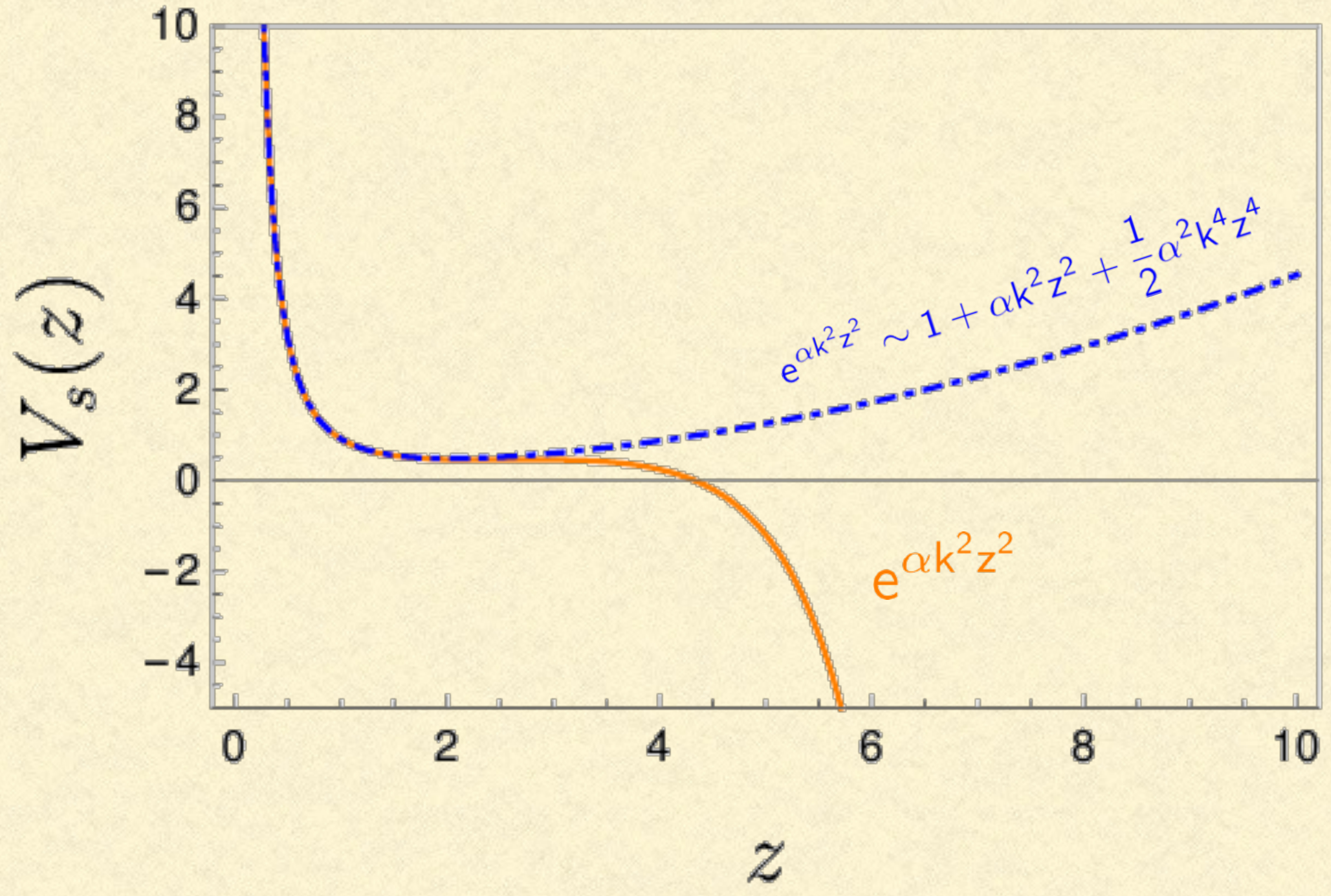
Additional dilaton

To avoid that the potential does not bind



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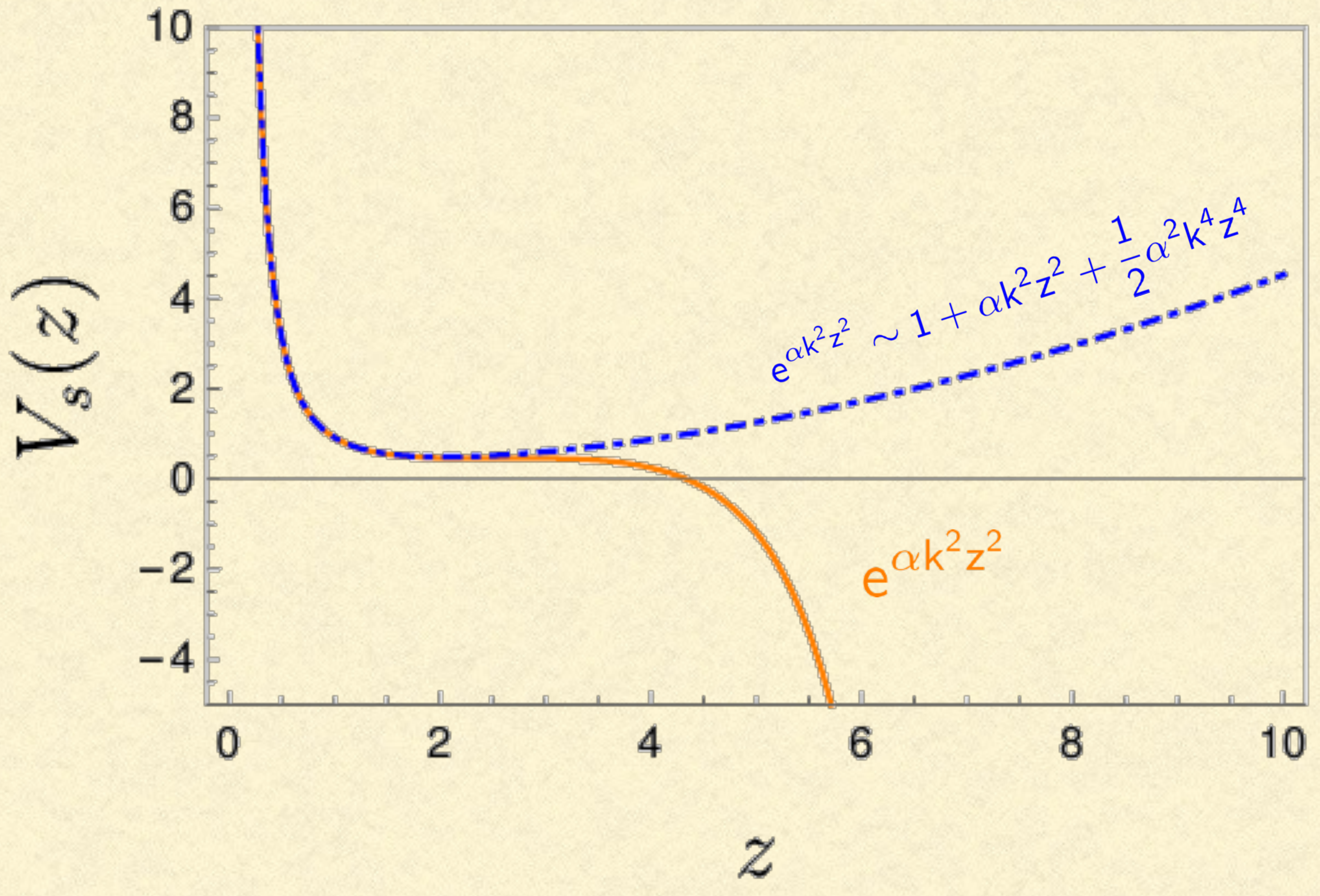
To avoid that the potential does not bind

$$e^{\alpha k^2 z^2} \sim 1 + \alpha k^2 z^2 + \frac{1}{2}\alpha^2 k^4 z^4$$

Phenomenological approximation:

1) leads to a binding potential
2) contains gluo dynamics described through the the metric deformation



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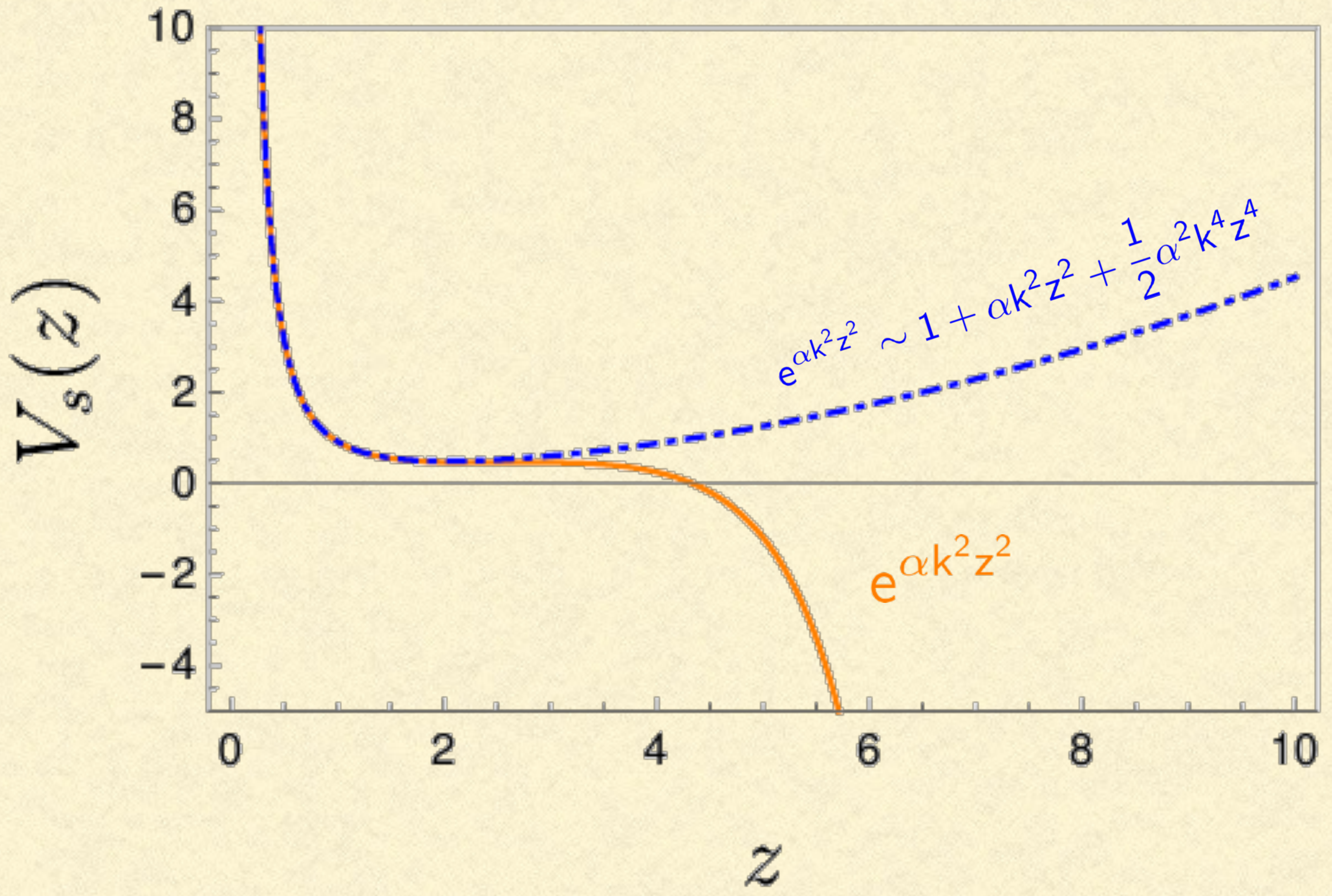
To avoid that the potential does not bind

$$-\frac{\varphi_n''(z)}{2} + \varphi_n'(z)\left(\frac{3}{2z} + k^2 z\right) + \frac{\varphi_n'(z)^2}{4} + \frac{M_5^2R^2}{z^2}\left[e^{\alpha k^2 z^2} - 1 - \alpha k^2 z^2 - \frac{1}{2}\alpha^2 k^4 z^4\right] = 0$$

- The additional dilaton guaranties that the potential is binding
- No free parameters!
- We only needs that the above differential equation has a solution.



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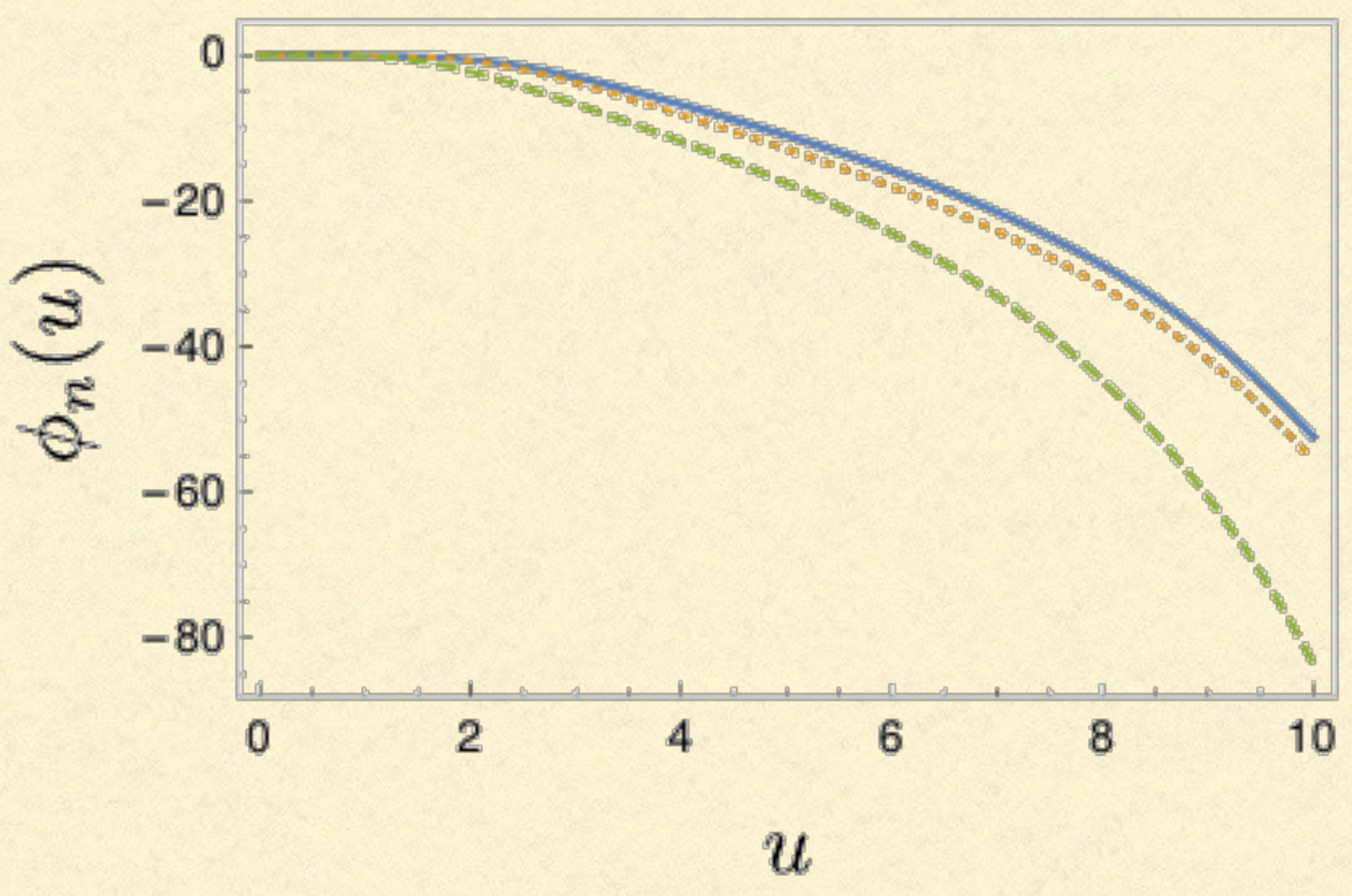

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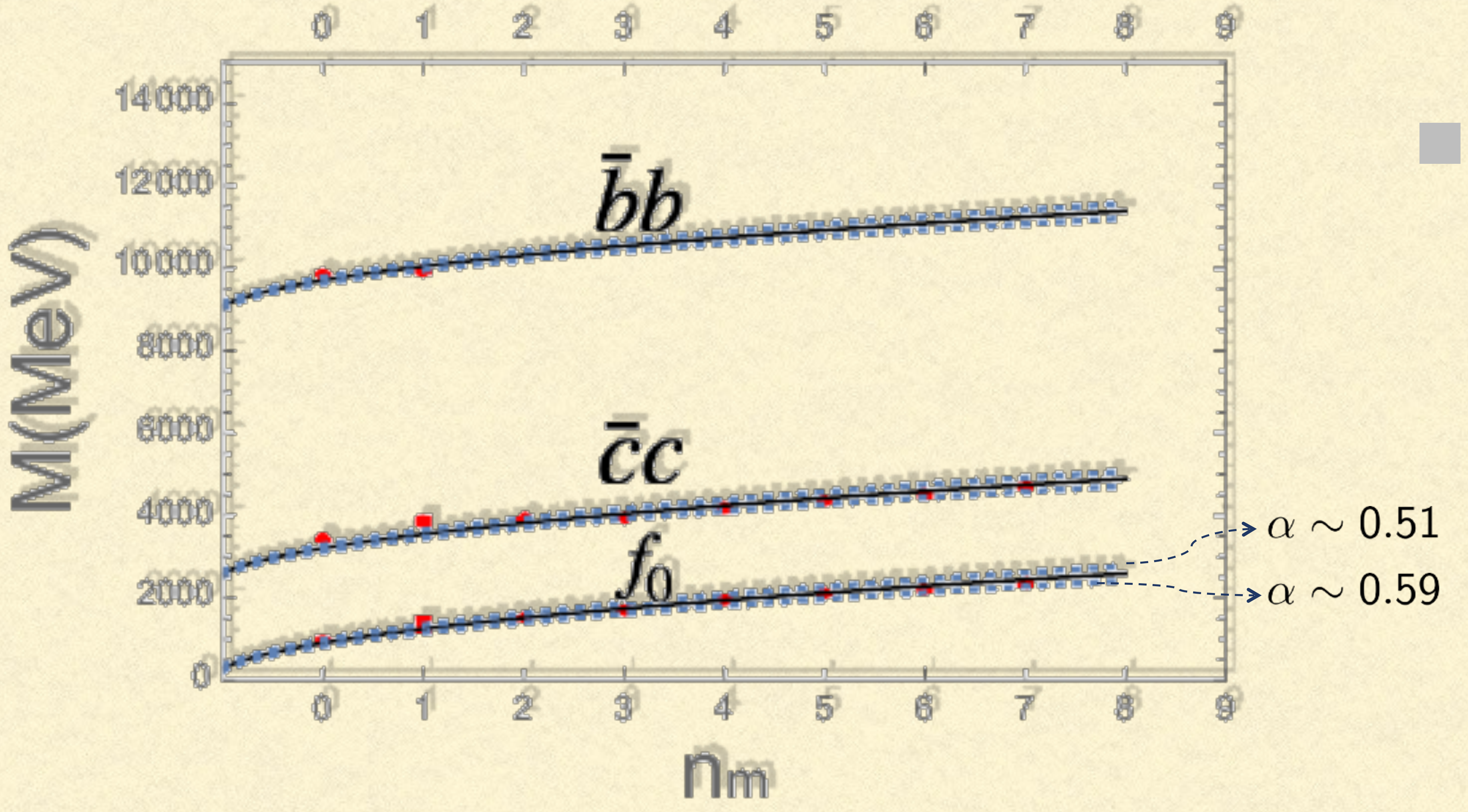

- Nice results for $\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$ and $0.51 \leq \alpha \leq 0.59$

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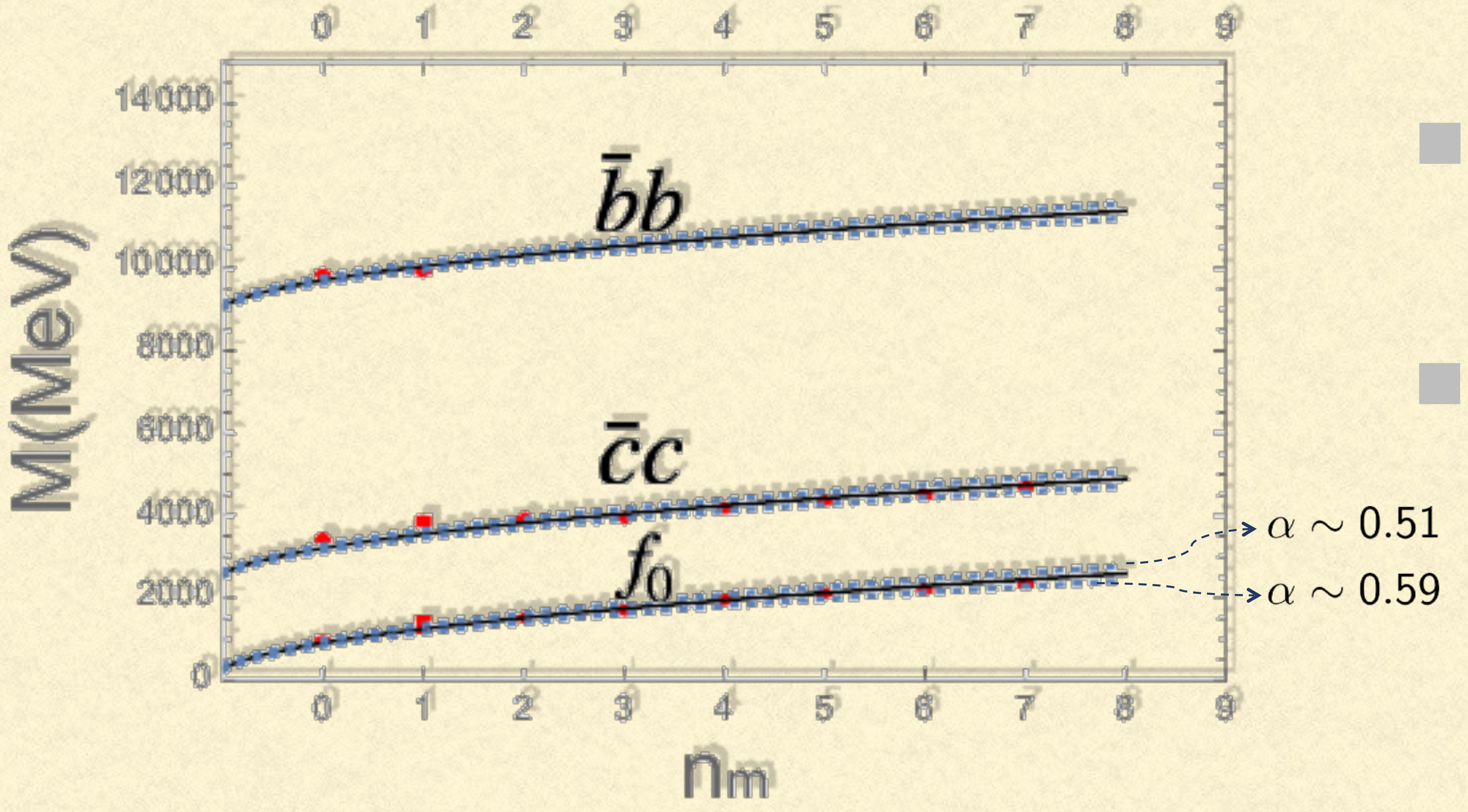

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S. S. Afonin et al, Phys. Lett. B726, 283 (2013)
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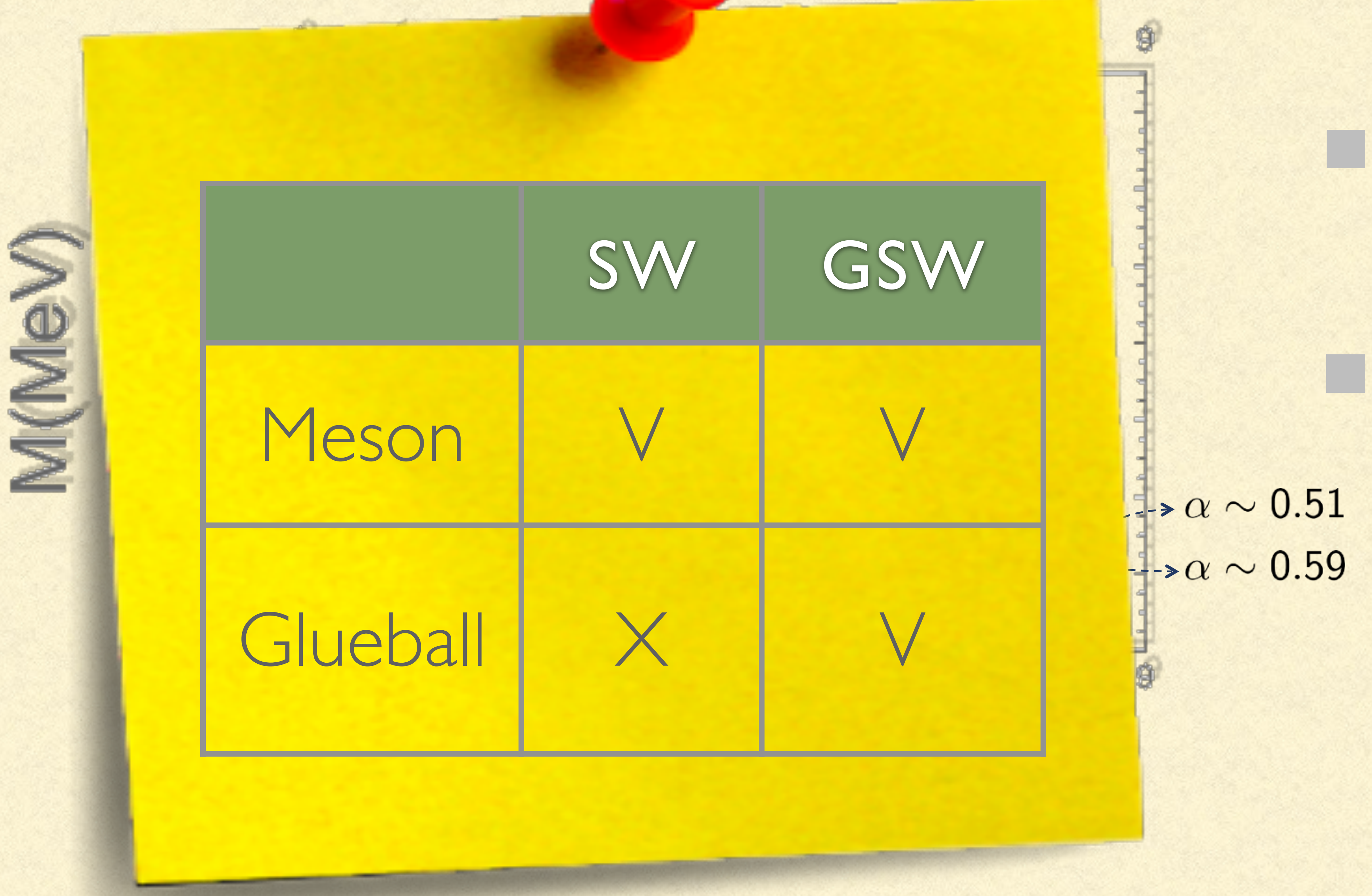

	SW	GSW
Meson	V	V
Glueball	X	V

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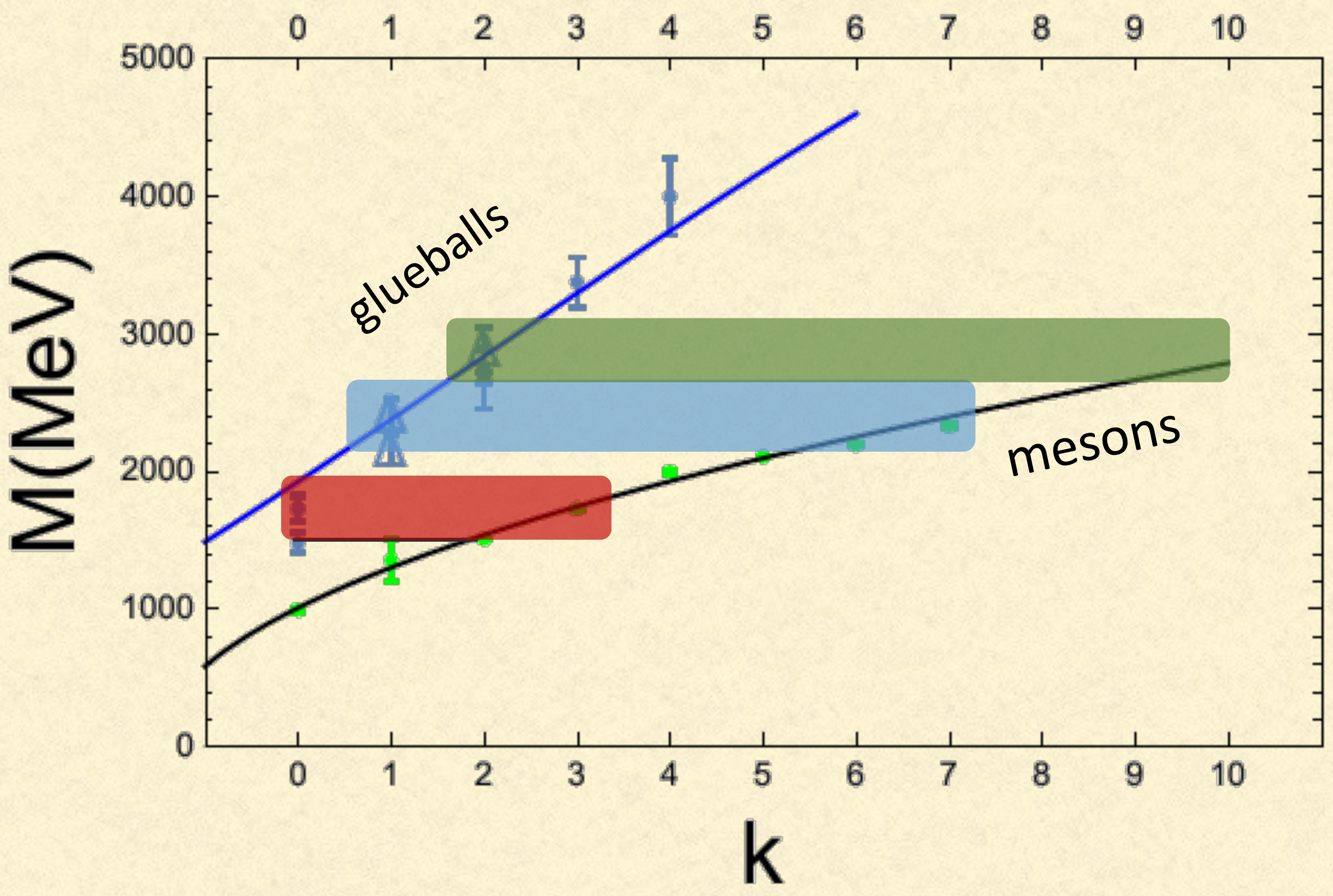
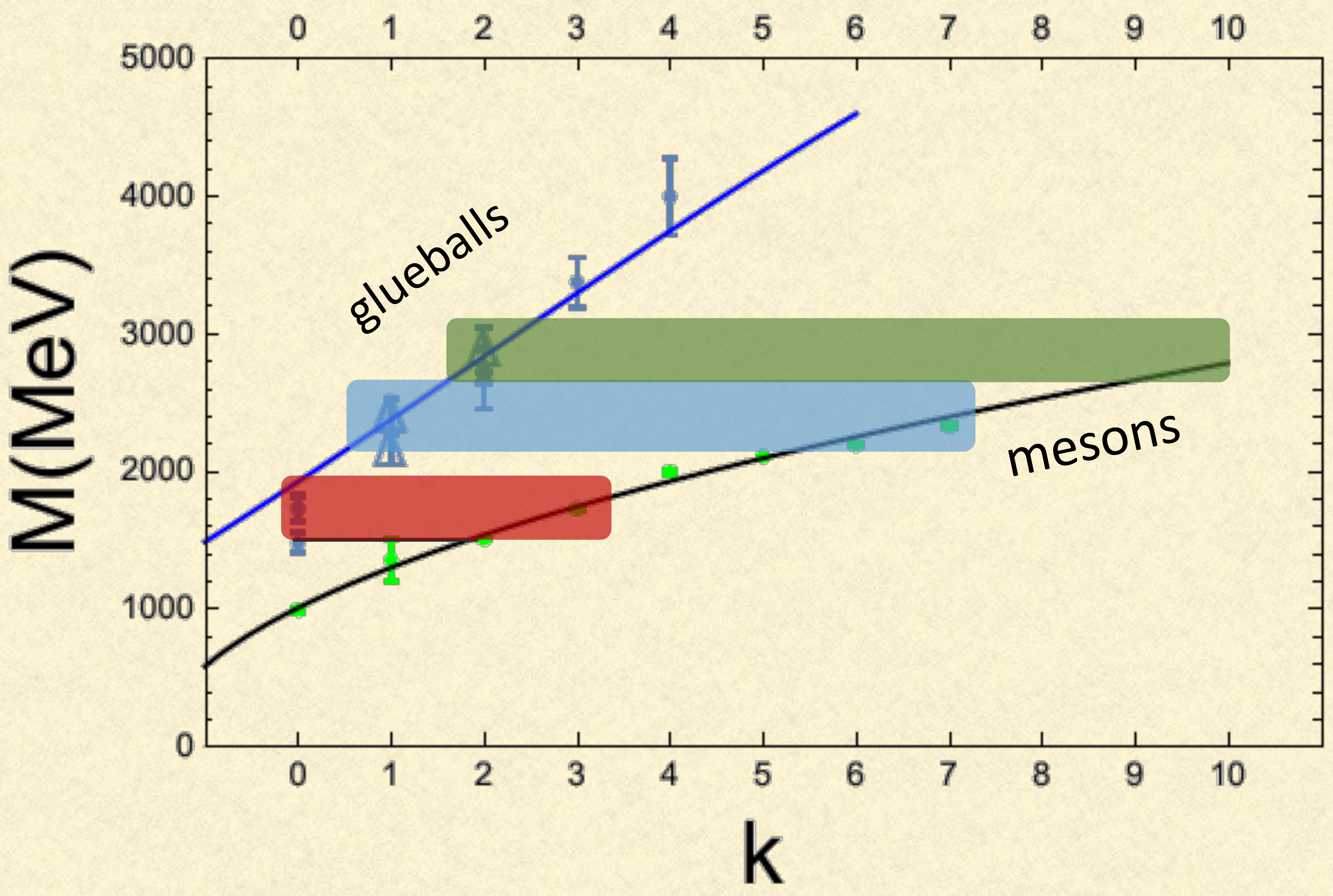
MIXING IN GRAVITON SOFT-WALL ADS/QCD



Mixing (?) for
very different mode numbers

MIXING IN GRAVITON SOFT-WALL ADS/QCD



Mixing (?) for
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We define the probability for NO MIXING as: $P_{mg} \equiv 1 - |\langle \Psi^g | \Psi^m \rangle|^2$

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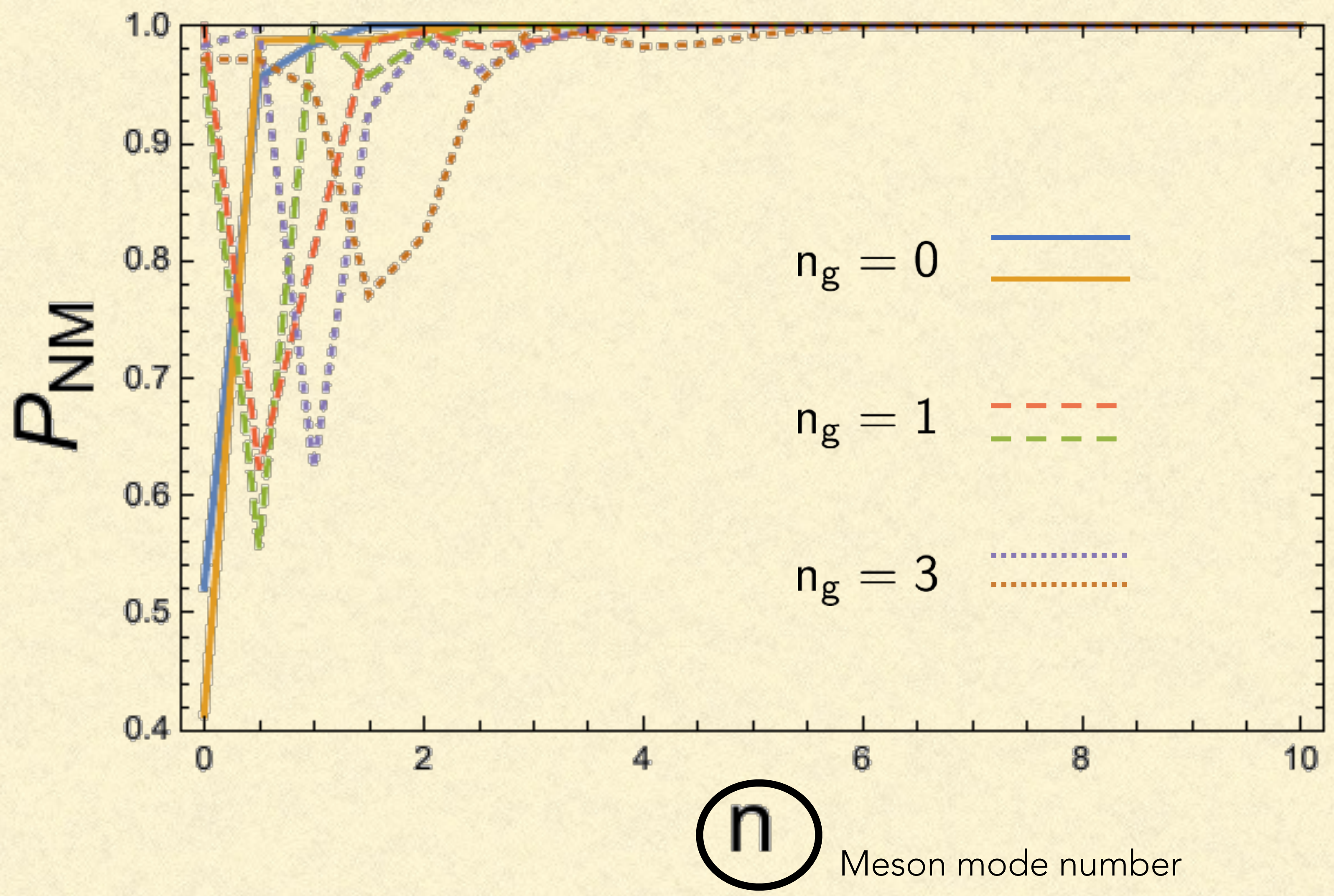


2 lines for:

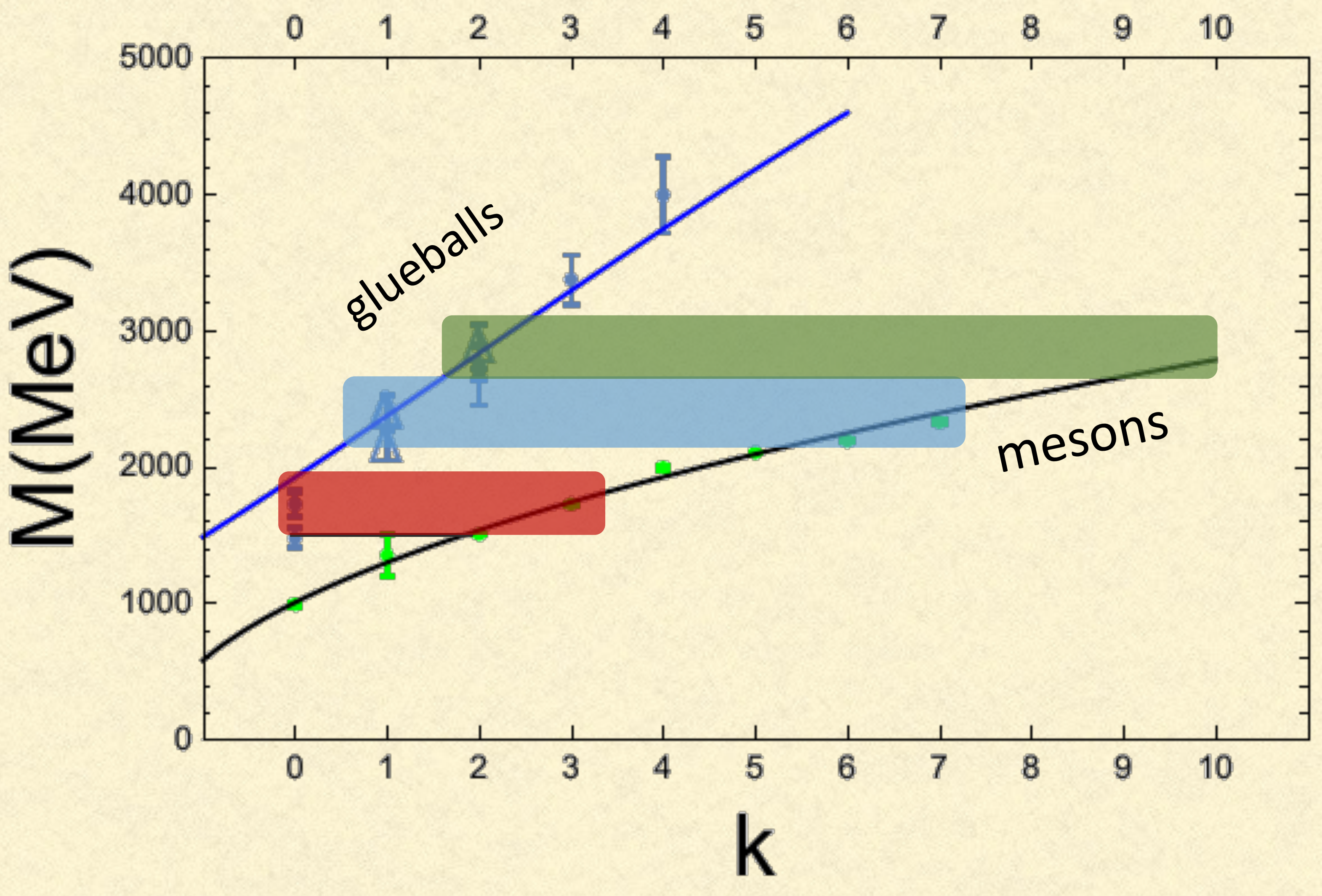
$\alpha = 0.51$

$\alpha = 0.59$

(here the parameter of the model)

Mixing for similar mode numbers

MIXING IN GRAVITON SOFT-WALL ADS/QCD



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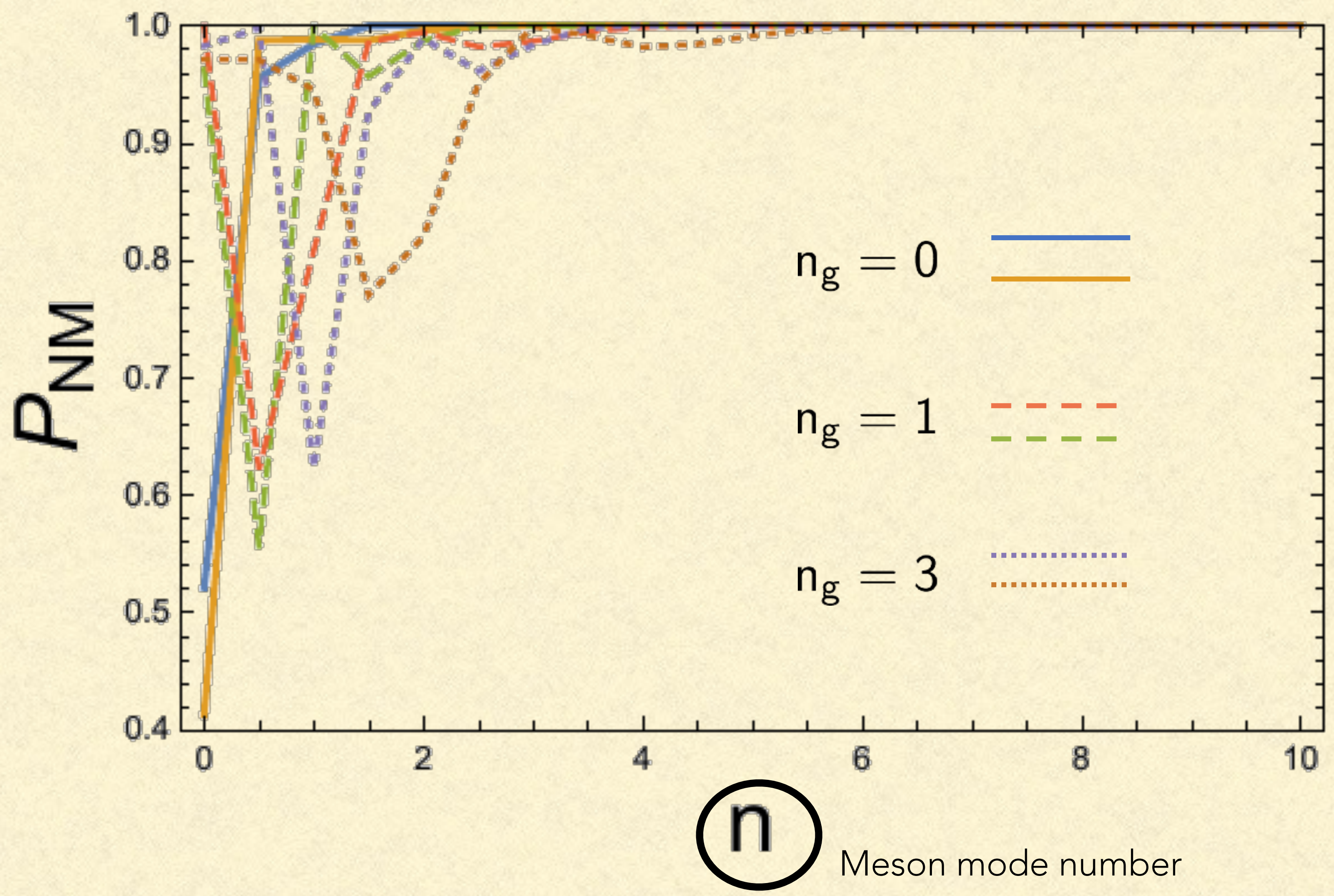

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(here the parameter of the model)

Mixing for similar mode numbers

Within the GSW AdS/QCD models (standard and with graviton) **pure glueballs in the scalar sector exist in the mass range above 2 GeV**!

PSUEDO-SCALARS IN GRAVITON SOFT-WALL ADS/QCD

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M.R. and V. Vento, PRD 104 (2021) 3, 034016
M.R. and V. Vento, JPG 47 (2020), 12, 125003

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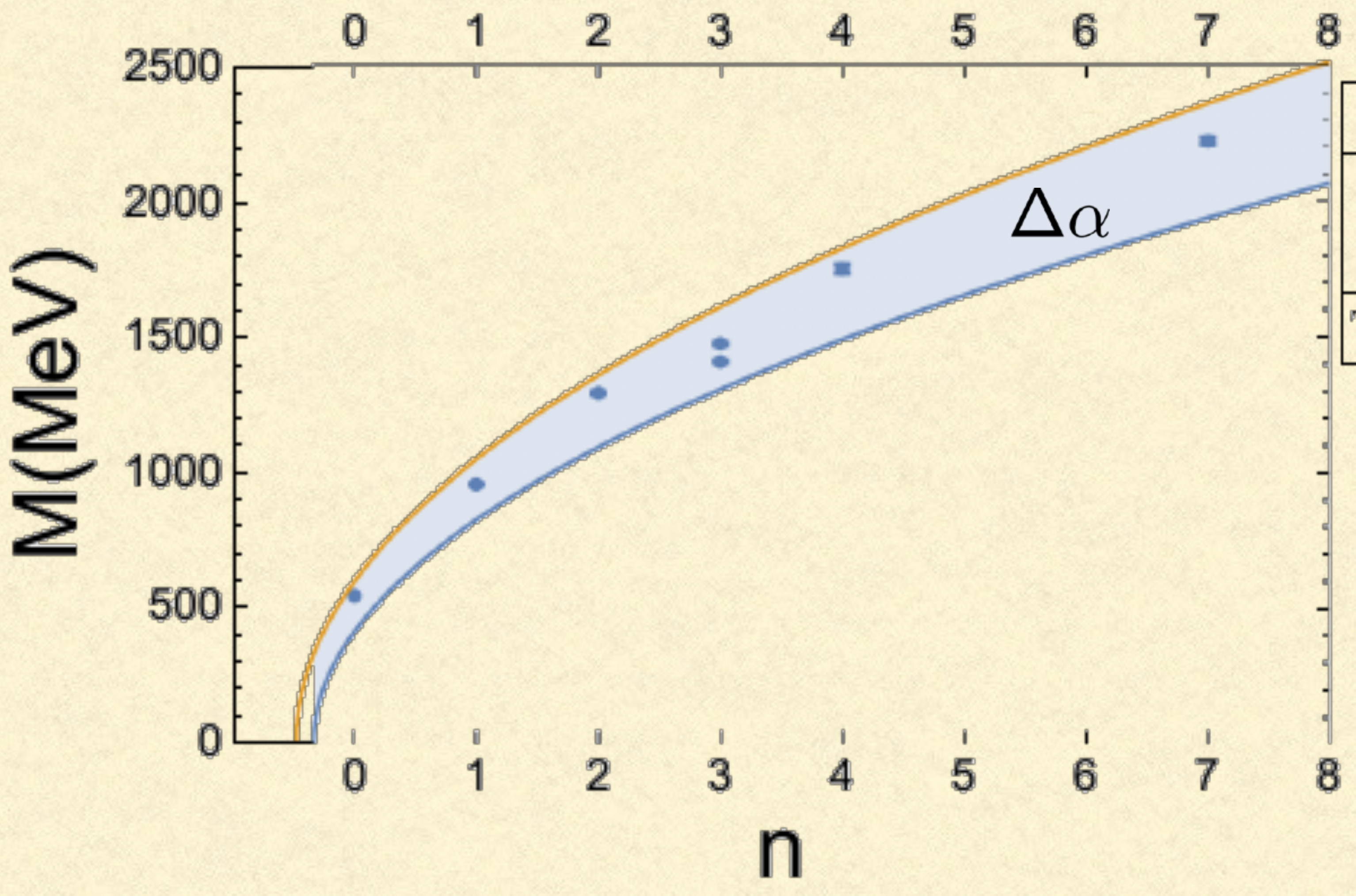
We do not change the parameters! $M_5^2 = -4$

M.R. and V. Vento, PRD 104 (2021) 3, 034016



	η	η'	$\eta(1295)$	$\eta(1405)$ - $\eta(1475)$	$\eta(1760)$	$\eta(????)$	$\eta(????)$	$\eta(2225)$
PDG	547.862 ± 0.017	957.78 ± 0.06	1295 ± 4	1408.8 ± 2.0 1475 ± 4	1751 ± 15			2221 ± 12
This work	513 ± 92	943 ± 111	1231 ± 133	1463 ± 151	1663 ± 168	1842 ± 183	2005 ± 198	2155 ± 210

The GSW model predicts this 2 new states

VECTORS IN GRAVITON SOFT-WALL ADS/QCD

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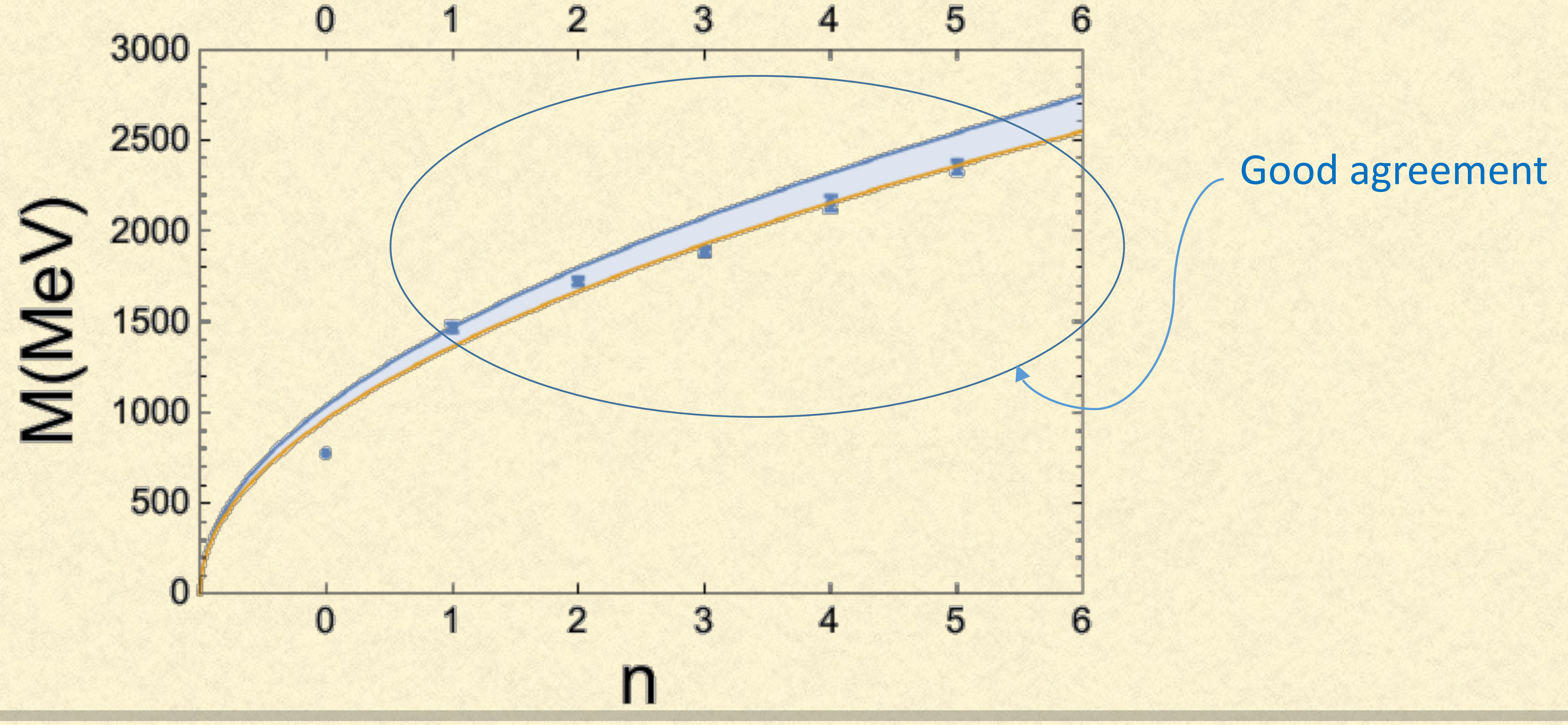
This is the action for the ρ field:

M.R. and V. Vento, PRD 104 (2021) 3, 034016
M.R. and V. Vento, JPG 47 (2020), 12, 125003

$$\bar{S} = -\frac{1}{2} \int d^5x \sqrt{-g}\, e^{-k^2 z^2} \left[\frac{1}{2} g^{MP} g^{QN} F_{MN} F_{PQ} \right]$$

We do not change the parameters! $M_5^2 = 0$

M.R. and V. Vento, PRD 104 (2021) 3, 034016



VECTORS IN GRAVITON SOFT-WALL ADS/QCD

We propose to modify the metric to properly describe the glueball dynamics:

M. R. and V. Vento EPJA 54 (2018)
M. R. and V. Vento JPG 47 (2020), 5, 055104
M. R. and V. Vento JPG 47 (2020), 12, 125003
M. R. and V. Vento, PRD 104 (2021) 3,034016

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right) \qquad \varphi(z) = k^2 z^2$$

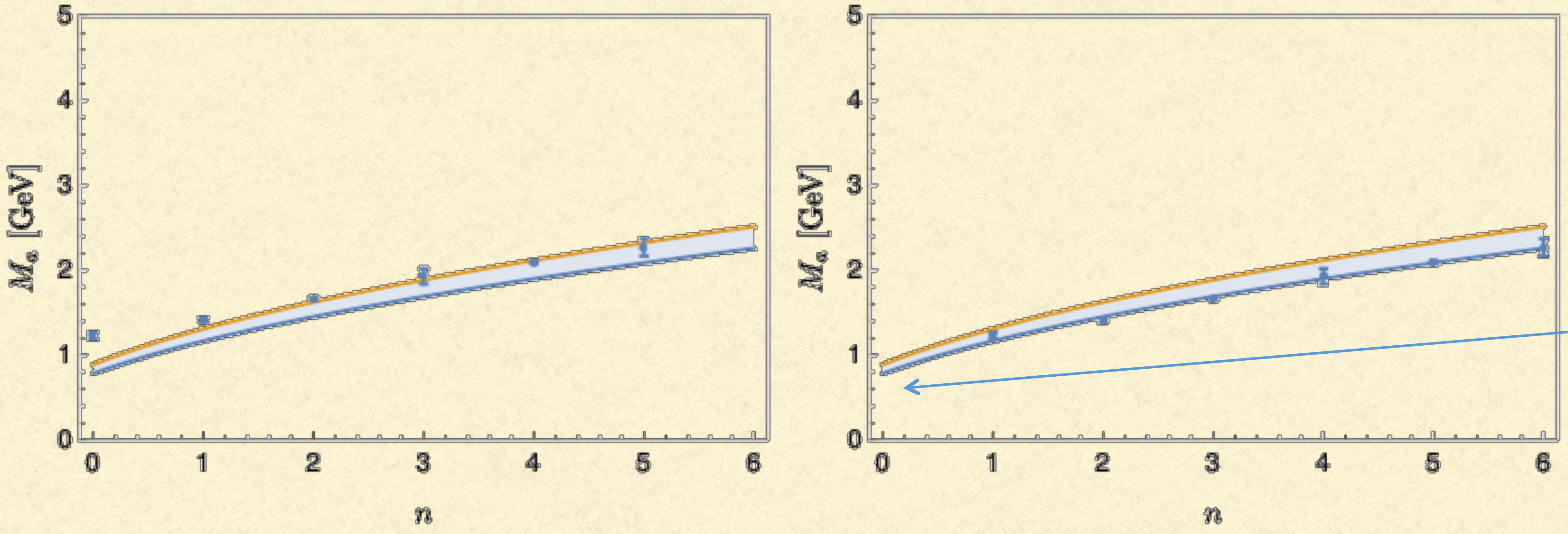
This is the action for the axial $\mathbf{a}_1$ field: $\bar{S} = -\frac{1}{2}\int d^5x \sqrt{-g}\, e^{-k^2z^2-\varphi_n}\left[\frac{1}{2} g^{MP} g^{QN} F_{MN} F^{PQ} + M_5^2 R^2 g^{PM} A_P A_M e^{\alpha k^2 z^2}\right]$

M.R. and V. Vento, PRD 104 (2021) 3, 034016
M.R. and V. Vento, JPG 47 (2020), 12, 125003

We do not change the parameters! $\qquad M_5^2 = -1$

M.R. and V. Vento, PRD 104 (2021) 3, 034016

GOOD AGREEMENT!



Here we guess the existence of an unknown lightest grand state...

	$a_1(1260)$	$a_1(1420)$	$a_1(1640)$	$a_1(1930)$	$a_1(2095)$	$a_1(2270)$
PDG & Av	1230 ± 40	1411^{+15}_{-13}	1655 ± 16	1930^{+19}_{-70}	2096^{+17}_{-121}	2270^{+55}_{-40}
This work	833 ± 53	1235 ± 72	1535 ± 87	1785 ± 100	2005 ± 111	2202 ± 122

A MODEL FOR THE PION

We need modifications to implement chiral symmetry breaking. Possible but no time to discuss all details, see M.R., F. A. Ceccopieri and V. Vento, EPJC 82 (2022) 7, 626

GSWL1: $m_q = 45$ MeV $\gamma_\pi = -0.6$

GSWL2: $m_q = 52$ MeV $\gamma_\pi = -0.17$

new parameters to take into account chiral symmetry breaking

SPECTRUM

	π^0		$\pi(1300)$			$\pi(1800)$
PDG	134.9768 ± 0.0005		1300 ± 100			1819 ± 10
SW [26]	0		1080	1527		1870
Ref. [8]	135	943 ± 111	1231 ± 133	1463 ± 151	1663 ± 168	1842 ± 183
GSWL1	140		1199 ± 41			1800 ± 6
GSWL2	140		1019 ± 27			1793 ± 16
Ref. [22]	140		1520			2120

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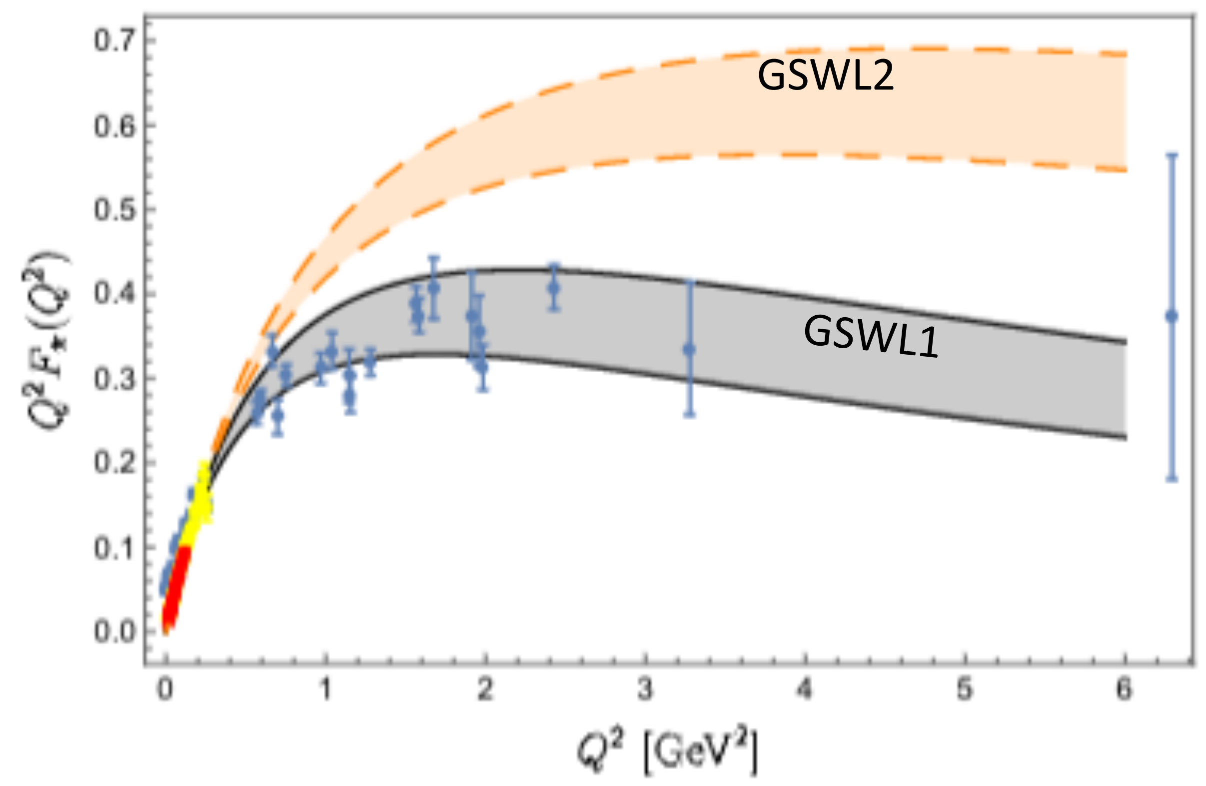
FORM FACTOR AND PION RADIUS



	Ref. [26]	Ref. [57]	Ref. [58]	GSWL1	GSWL2	Experiment [55]
$\sqrt{\langle r^2 \rangle}$ [fm]	0.524	0.673-0.684	0.644	0.67 ± 0.03	0.70 ± 0.05	0.67 ± 0.01

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GSWL2: $m_q = 52 \text{ MeV}$ $\gamma_\pi = -0.17$

DECAY CONSTANT

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d\mathbf{k}_{\perp 1}}{16\pi^3} \psi_{2/h}(x, \mathbf{k}_{\perp 1})$$

Pion w.f.
(GSW x longitudinal dyn.)

	Data [34]	GSWL1	GSWL2
f_π [MeV]	91.92 ± 3.54	126 ± 6	104 ± 7

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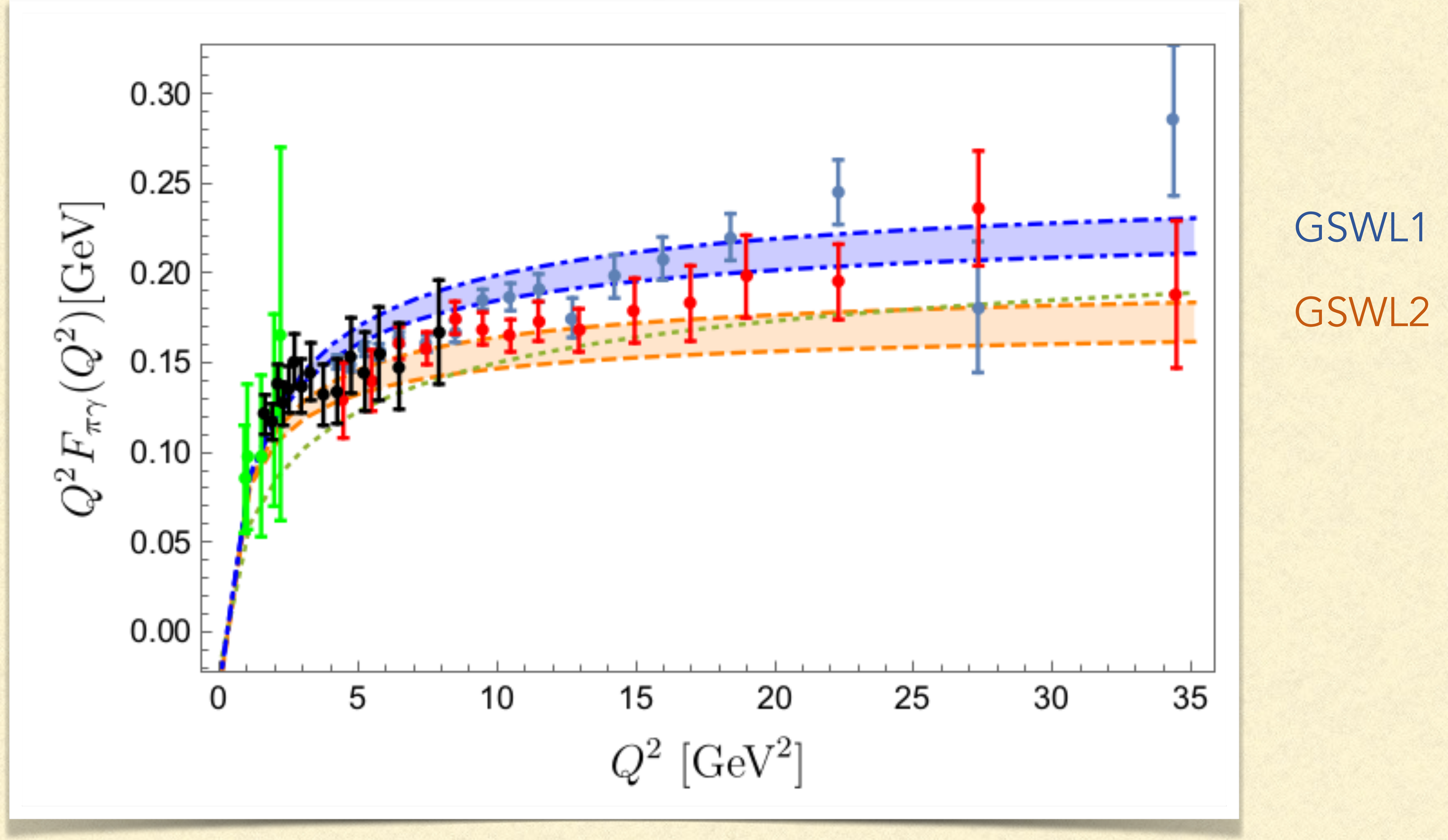
THE PION TRANSITION FORM FACTOR

$$\langle \gamma(P-q)|J^\mu|\pi(P)\rangle = ie^2 F_{\gamma\pi}(Q^2)\varepsilon^{\mu\nu\rho\sigma}P_\nu \varepsilon_\rho q_\sigma$$



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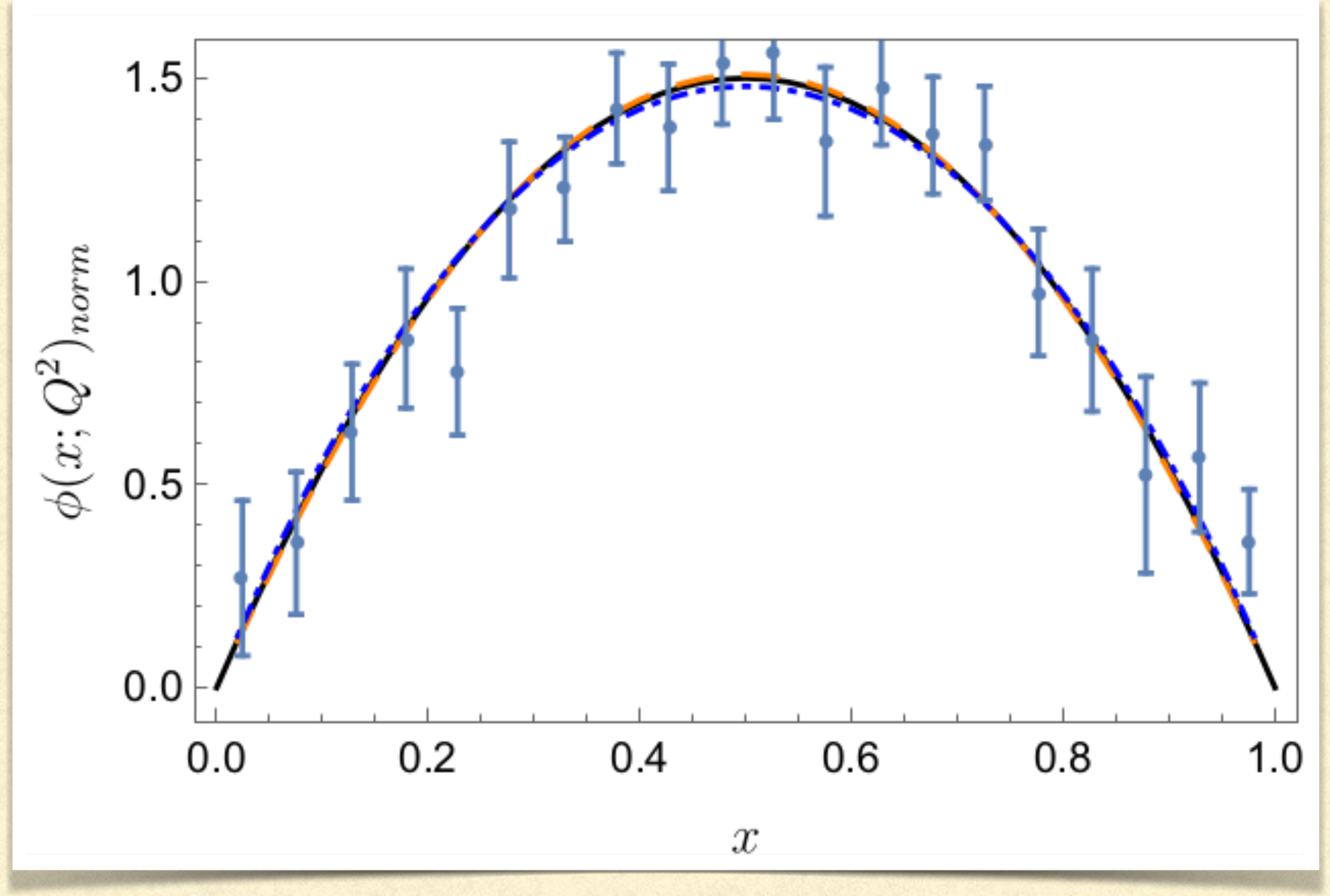
GSWL1: $m_q = 45$ MeV $\quad \gamma_\pi = -0.6$

GSWL2: $m_q = 52$ MeV $\quad \gamma_\pi = -0.17$

DISTRIBUTION AMPLITUDE

$$\phi(x;Q) = \int_0^{Q^2} \frac{d^2\mathbf{k}_{\perp 1}}{16\pi^3} \psi_{2/\pi}(x, \mathbf{k}_{\perp 1})$$

Pion w.f.
(GSW x longitudinal dyn.)



$$\int dx\ \phi(x;Q^2)_{norm} = 1$$

A MODEL FOR THE PION

We need modifications to implement chiral symmetry breaking. Possible but no time to discuss all details, see **M.R., F. A. Ceccopieri and V. Vento, EPJC 82 (2022) 7, 626**

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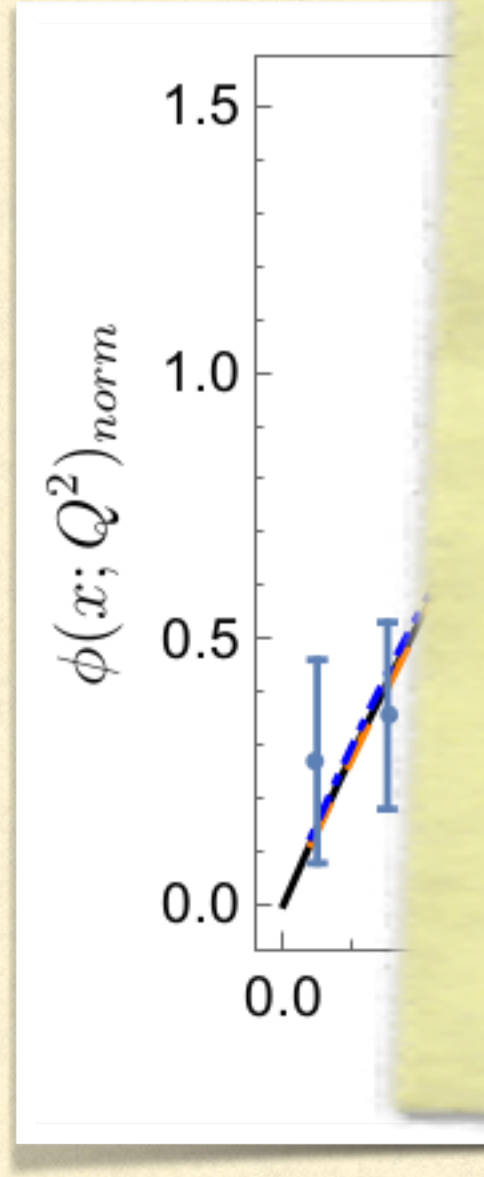
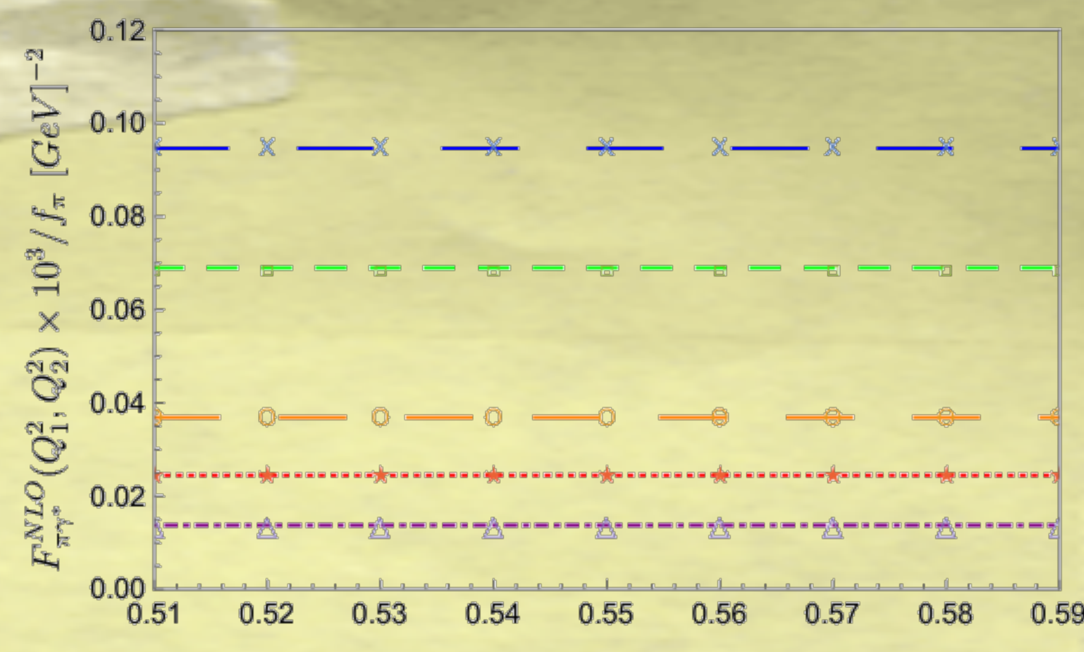
GSWL2: $m_q = 52$ MeV $\qquad \gamma_\pi = -0.17$

DISTRIBUTION AMPLITU

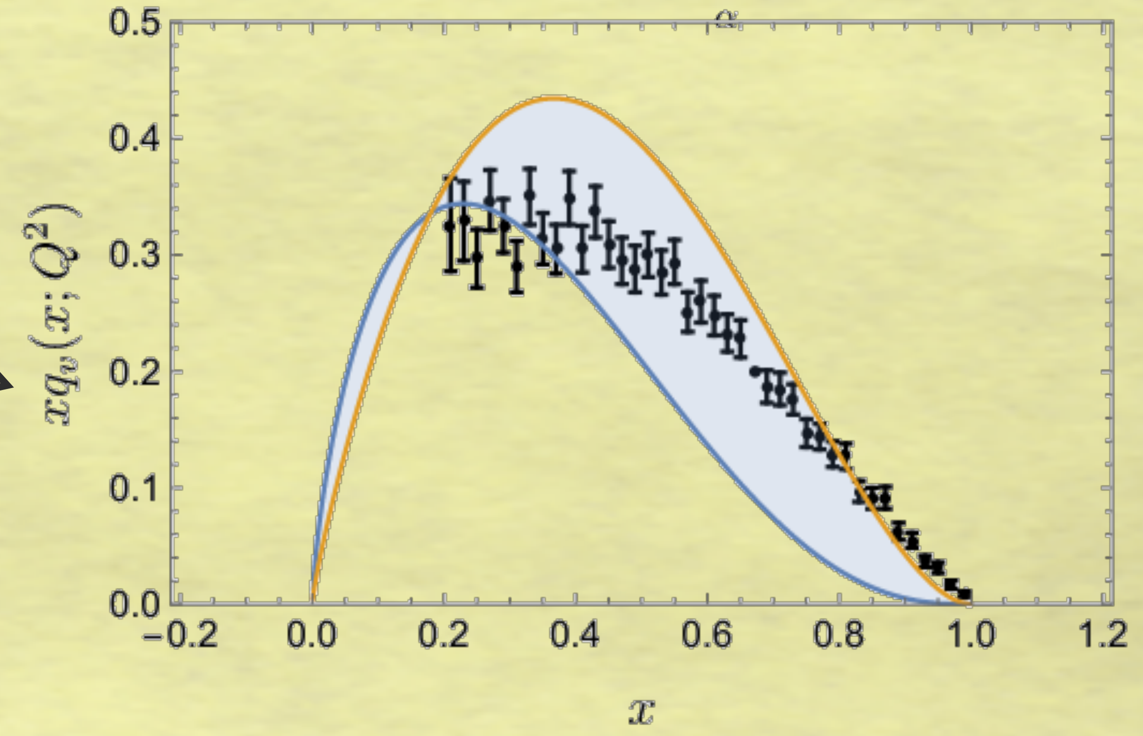


We also computed:

- TFF with 2 virtual photons



- Moments of DA
- PDF
(more investigations are needed)



-Effective form factors:
relevant quantities for Double Parton Scattering (Comparison with lattice)

THE GSW MODEL AND HYBRID SPECTRA

M. R. and V. Vento, PRD 109 (2024) 11,11403

The actions, in the gravity sector, are still those for scalars (S) and vectors (V):

$$\tilde{I} = \int d^5x \sqrt{g}\, e^{-\varphi(z)-\varphi_n(z)} \left[g^{MN}\partial_M \mathcal{S}\partial_N \mathcal{S} + e^{\alpha\varphi(z)} M_5^2 \mathcal{S}^2\right]$$

$$\bar{S} = -\frac{1}{2}\int d^5x\sqrt{-g}\,e^{-k^2z^2-\varphi_n}\left[\frac{1}{2}g^{MP}g^{QN}F_{MN}F^{PQ} + M_5^2R^2g^{PM}A_PA_M e^{\alpha k^2z^2}\right]$$

The states we considered (also including anomalous dimension to take into account χ-symmetry breaking):

$0^{-+}\quad S = \bar{\Psi}\gamma^i\lambda^a\Psi B_i^a \quad (\Delta = 5, \Delta_p = 0, p = 0, M_5^2R^2 = 5)$

$0^{++}\quad S' = \varepsilon_{ijk}\bar{\Psi}\sigma^{ij}\lambda^a\Psi B^{ka} \quad (\Delta = 5, \Delta_p = 1, p = 0, M_5^2R^2 = 12),$

$1^{--}\quad V_i = \bar{\Psi}\gamma_5\lambda^a\Psi B_i^a \quad (\Delta = 5, \Delta_p = 0, p = 1, M_5^2R^2 = 8),$

$1^{+-}\quad A_i' = \varepsilon_{ijk}\bar{\Psi}\gamma_5\gamma^j\lambda^a\Psi B^{ka} \quad (\Delta = 5, \Delta_p = 1, p = 1, M_5^2R^2 = 15),$

$1^{++}\quad A_i = \varepsilon_{ijk}\bar{\Psi}\gamma^j\lambda^a\Psi E^{ka} \quad (\Delta = 5, \Delta_p = 1, p = 1, M_5^2R^2 = 15)$

$0^{+-}\Sigma = \bar{\Psi}\gamma_5\gamma^i\lambda^a\Psi B_i^a \quad (\Delta = 5, \Delta_p = 1, p = 0, M_5^2R^2 = 12)$

$0^{--}\Sigma' = \bar{\Psi}\gamma_5\lambda^a\Psi E_i^a \quad (\Delta = 5, \Delta_p = 0, p = 0, M_5^2R^2 = 5)$

$1^{-+}W_i = \varepsilon_{ijk}\bar{\Psi}\gamma^j\lambda^a\Psi B^{ka} \quad (\Delta = 5, \Delta_p = 0, p = 1, M_5^2R^2 = 8)$

$1^{-+}W_i' = \bar{\Psi}\gamma_0\lambda^a\Psi E_i^a \quad (\Delta = 5, \Delta_p = 0, p = 1, M_5^2R^2 = 8).$

	LQCD(renormalon) [40]	QCD(anysotropic) [38]	LQCD (m_π = 396 MeV) [39]	GSWm2
0^{-+}	-	-	2.1	2.074 ± 0.028
0^{++}	1.98	-	>2.4	2.694 ± 0.021
1^{--}	0.87	-	2.3	2.149 ± 0.017
1^{+-}	1.25	-	>2.4	2.747 ± 0.013
1^{++}	-	-	>2.4	2.747 ± 0.013
0^{+-}	-	-	>2.4	2.694 ± 0.021
0^{--}	-	-	-	2.074 ± 0.028
1^{-+}	2.15	2.013	2.0	2.149 ± 0.017

- Good agreement with lattice data (no free parameters)
- The model can be considered suitable to identify which observed states could be interpreted as hybrids
- The approach has been extended also to describe hybrids in the heavy quark sector

THE GSW MODEL AND HYBRID SPECTRA

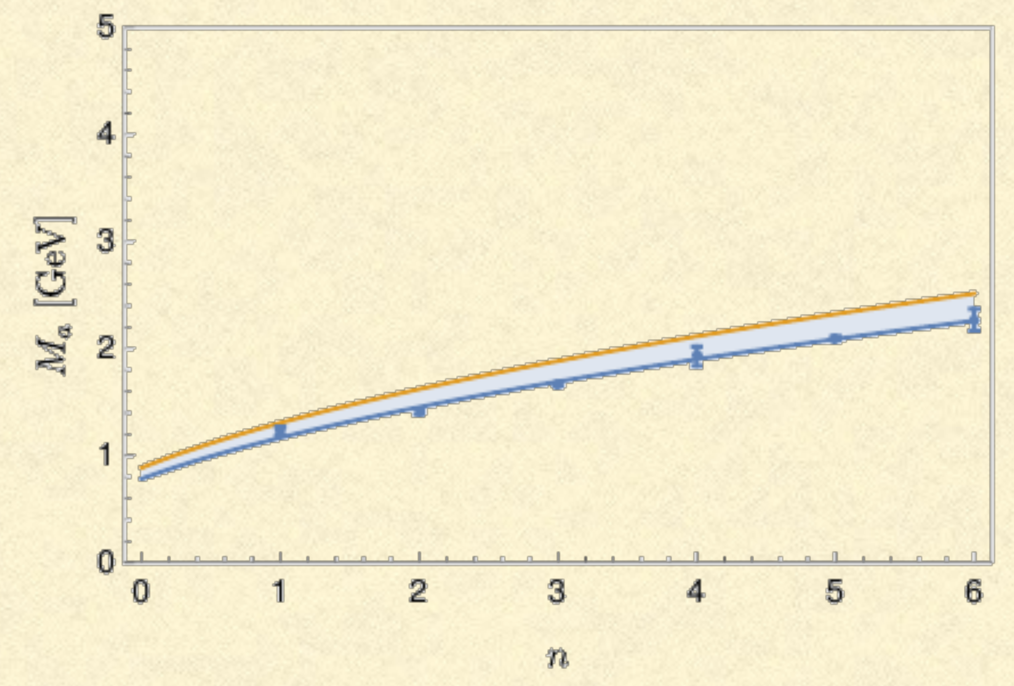
M. R. and V. Vento, PRD 109 (2024) 11,11403

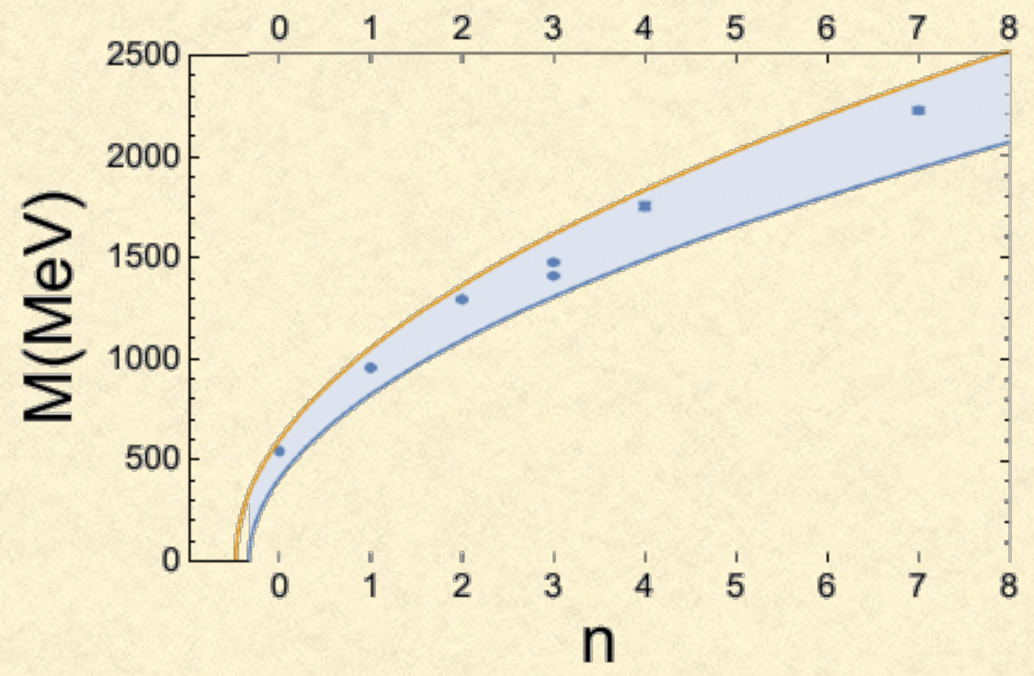
For example, we can use the model to try to identify

	$\pi(1800)$	$\eta(2010)^*$	$\pi(2070)^*$	$\eta(2100)^*$	$\eta(2190)^*$	$\eta(2320)^*$	$\pi(2360)^*$
0^{-+}	1800^{+9}_{-10}	2010^{+35}_{-60}	2070 ± 35	2050^{+105}_{-50}	2190 ± 50	2320 ± 15	2360 ± 25
	$f_0(1710)$	$a_0(2020)^*$	$f_0(2060)^*$	$X(2540)^*$			
0^{++}	1740^{+8}_{-7}	2025 ± 30	2060 ± 10	2540^{+52}_{-28}			
	$\omega(1420)$	$\rho(1450)$	$\omega(1650)$	$\phi(1680)$	$\rho(1700)$		
1^{--}	1410 ± 60	1465 ± 20	1670 ± 30	1680 ± 20	1720 ± 20		
	$\omega(1960)^*$	$\phi(2170)$	$\omega(2205)^*$	$\rho(2270)^*$	$\omega(2290)^*$	$\omega(2330)^*$	
1^{--}	1960 ± 25	2162 ± 7	2205 ± 30	2270 ± 45	2290 ± 20	2330 ± 30	
	$h_1(1170)$	$h_1(1415)$	$h_1(1595)$	$b_1(1960)^*$	$h_1(1965)^*$	$h_1(2215)^*$	$b_1(2240)^*$
1^{+-}	1166 ± 8	1416 ± 8	1594^{+25}_{-75}	1960 ± 35	1965 ± 45	2215 ± 40	2240 ± 35
	$a_1(1930)^*$	$f_1(1970)^*$	$a_1(2095)^*$	$a_1(2270)^*$	$f_1(2310)^*$		
1^{++}	1930^{+30}_{-70}	1971 ± 15	2096 ± 138	2270^{+55}_{-40}	2310 ± 60		

$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2074	2536
$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2694	3179
$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2149	2647
$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2149	2647
$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2746	3251
$GSW^{n=0}_{m2}$	$GSW^{n=1}_{m2}$
2746	3251

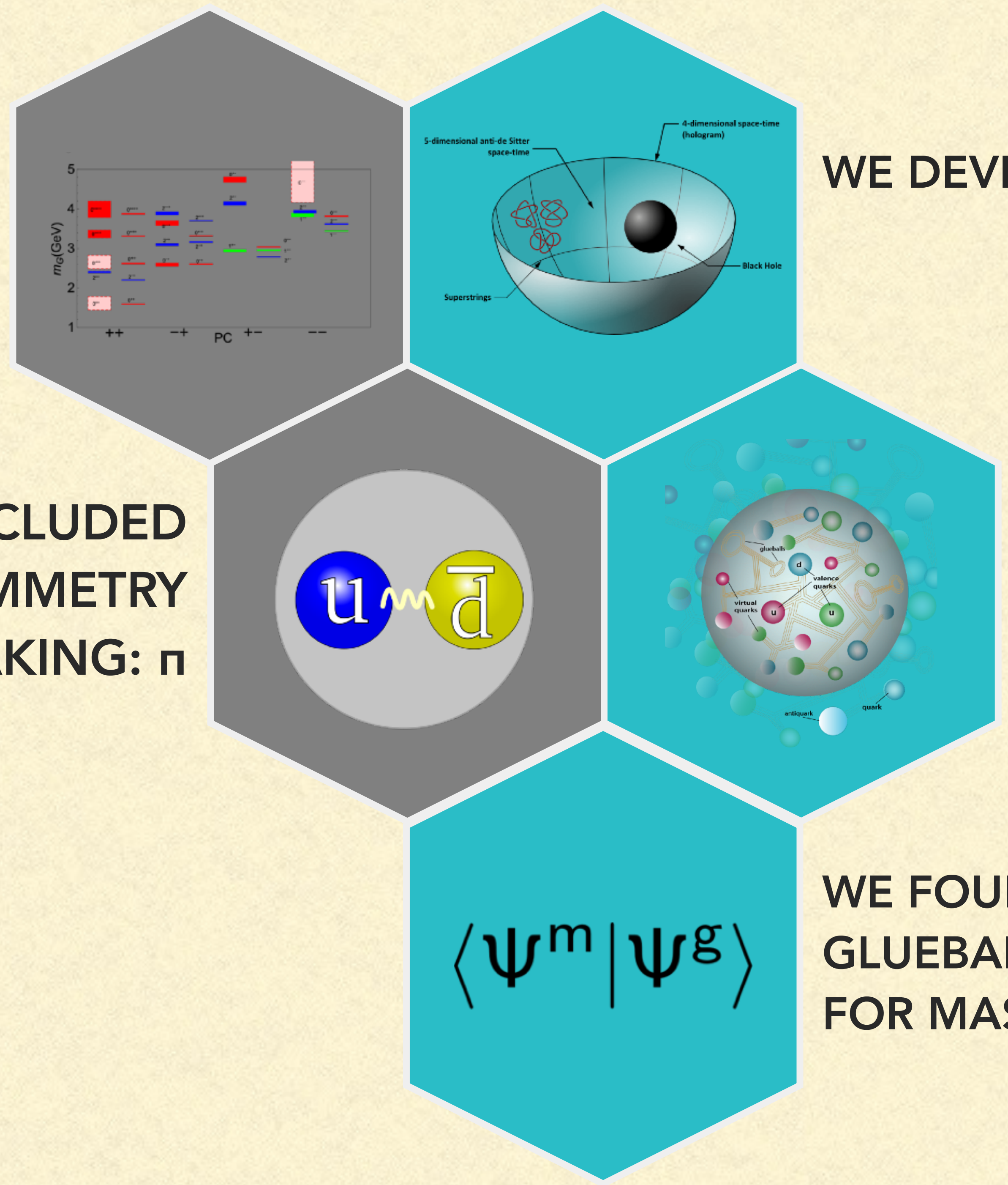
CONCLUSIONS





WE CONSIDER THE GLUEBALL & MESON SPECTRA

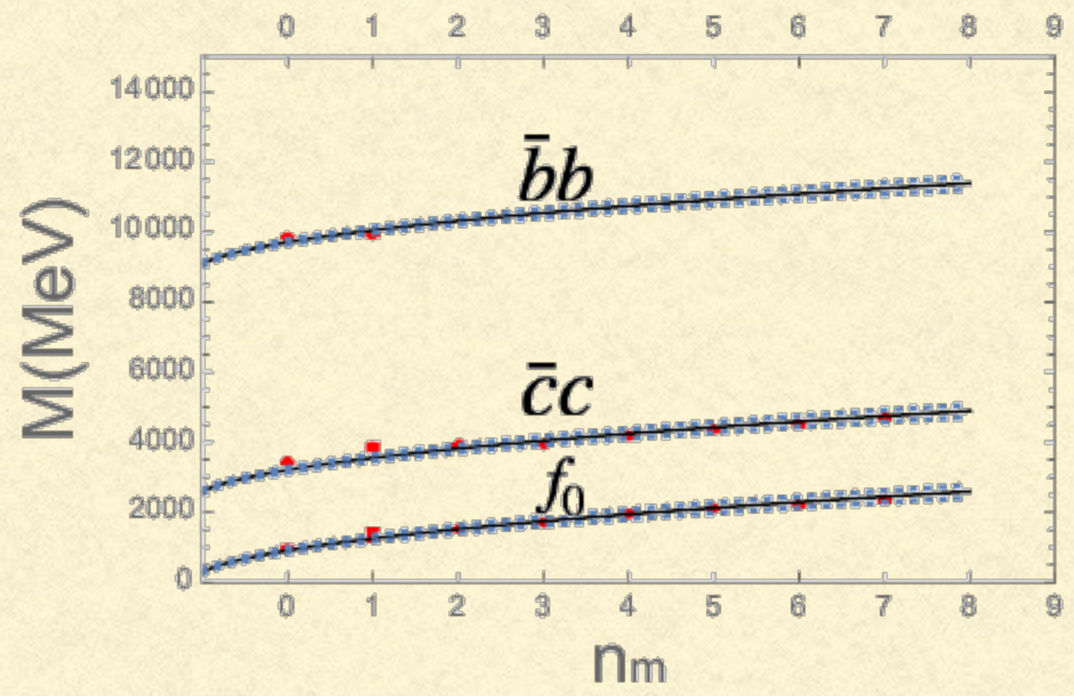
WE DEVELOPED THE GSW AdS/QCD MODEL

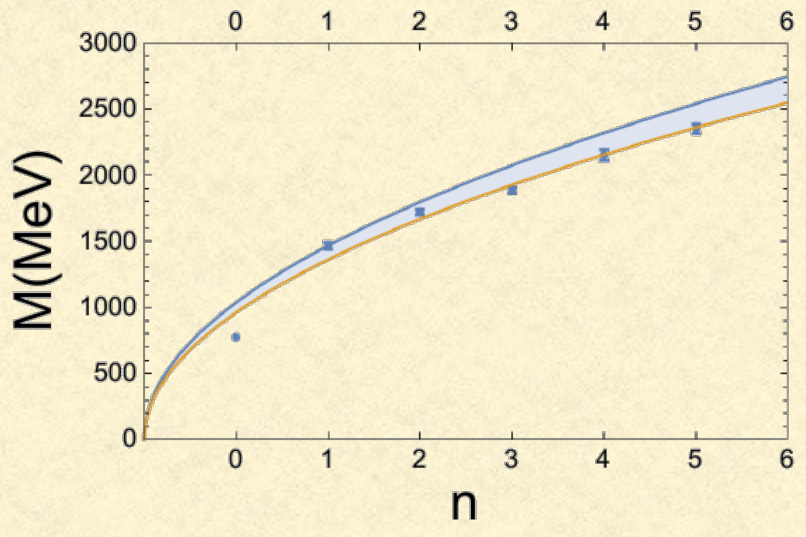


WE INCLUDED CHIRAL-SYMMETRY BREAKING: π

WE DESCRIBED QUITE WELL GLUEBALL & MESON SPECTRA WITH 2 PARAMETERS

WE FOUND THAT PURE SCALAR GLUEBALLS COULD BE FOUND FOR MASSES ABOVE 2 GeV





A GSW MODEL FOR THE PION

M.R., F. A. Ceccopieri and V. Vento, EPJC 82 (2022) 7, 626

In order to move from the eta spectrum to the pion one, the potential should modified:

$$S = \int d^5x \; e^{-\varphi_0(z)-\varphi_n(z)}\sqrt{-g}\Big[g^{MN}\partial_M\Phi(x)\partial_N\Phi(x) - 4e^{\alpha k^2 z^2}\Phi(x)^2\Big]$$

The additional dilaton, responsible for the confinement can lead to:

$$V_\pi(z) = \frac{15}{4z^2} + 2k^2 + k^4 z^2 - \frac{4}{z^2}\left[1 + (\alpha + \xi_\pi)k^2 z^2 + \frac{1}{2}(\alpha^2 + \gamma_\pi)k^4 z^4\right]$$

- Parameters used to describe: glueballs, light sclars, heavy scalars, eta, vectors.

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- Parameters used to describe: glueballs, light sclars, heavy scalars, eta, vectors.

- Two shifts of the parameters to describe the pion (included in the additional dilaton)

$$V_\pi(z) = V_\eta(z) - 4k^2 \xi_\pi - 2\gamma_\pi k^4 z^2$$

A GSW MODEL FOR THE PION

M.R., F. A. Ceccopieri and V. Vento, EPJC 82 (2022) 7, 626

In order then to include the quark masses, we follow an idea applied to the SW and other models.

- James P. Vary et al. "Heavy Quarkonium in a Holographic Basis", Phys. Lett. B, 758:118–124, 2016
- M. Burkardt, "Mesons in a collinear QCD model", Phys. Rev. D, 56:7105–7118, 1997
- James P. Vary et al. "Light-front holography with chiral symmetry breaking", Phys. Lett. B, 825:136860, 2022
- Guy F. de Teramond and Stanley J. Brodsky. "Longitudinal dynamics and chiral symmetry breaking in holographic light-front QCD". PRD, 104(11):116009, 2021

Qualitatively one can understand it by looking at the "free" hadron mass (where no dynamics is included):

$$M_0^2 = \frac{k_\perp^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}$$

Depends only on the longitudinal variable

The idea is therefore to generalize the equation of motion by including a "longitudinal" dynamics

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$$\left[-\frac{d^2}{dz^2} + V_\pi(z) + \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + V_{||}(z)\right]\bar{\Phi}(z,x) = M^2\bar{\Phi}(x,z)$$

$$V_{||}(x) = -\sigma^2\partial_x\left[x(1-x)\partial_x\right] \qquad \sigma = \frac{M_\pi^2(0) - 4m_q^2}{2m_q}$$

Used and proposed in:
J.P. Vary et al, PLB 758 (2016)
J.P. Vary et al, PLB 825 (2022)

- terms coming from the GSW model

- terms coming from the additional pure longitudinal dynamics:

- full w.f. (product of the GSW and the longitudinal ones) and mass

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M.R., F. A. Ceccopieri and V. Vento, EPJC 82 (2022) 7, 626

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- Two shifts of the parameters to describe the pion (included in the additional dilaton)

$$V_\pi(z) = V_\eta(z) - 4k^2\xi_\pi - 2\gamma_\pi k^4 z^2$$

- If we impose chiral symmetry:

$$M_\pi(0) = 0 \quad \longleftrightarrow \quad \xi_\pi = \frac{1 - 2\alpha + \sqrt{1 - 2\alpha^2 - 2\gamma_\pi}}{2}$$