RUHR UNIVERSITÄT BOCHUM RUB

Extracting properties of the Tcc state from lattice QCD

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July 08-12, 2024

QNP 2024

Based on *PRD* 105, 014024(2022); *PRL* 131, 131903 (2023); *PRD* 109 (2024), L071506

and 2407.04649 this Monday

in collaboration with

M. Abolnikov, X. Dong, M. Du, E. Epelbaum, A. Filin, A. Gasparyan, F.-K. Guo, C. Hanhart,
L. Meng, A. Nefediev, J. Nieves and Q. Wang

T_{cc} : an ideal case for studying exotic properties

Aaij et al [LHCb] Nature Physics (2022)
Nature Comm.(2022)

see also Talks by Juan Nieves and Gabriele Romolini

— first exotic doubly charm state: $cc\bar{u}\bar{d}$

$I = 0,\ J^P = 1^+$

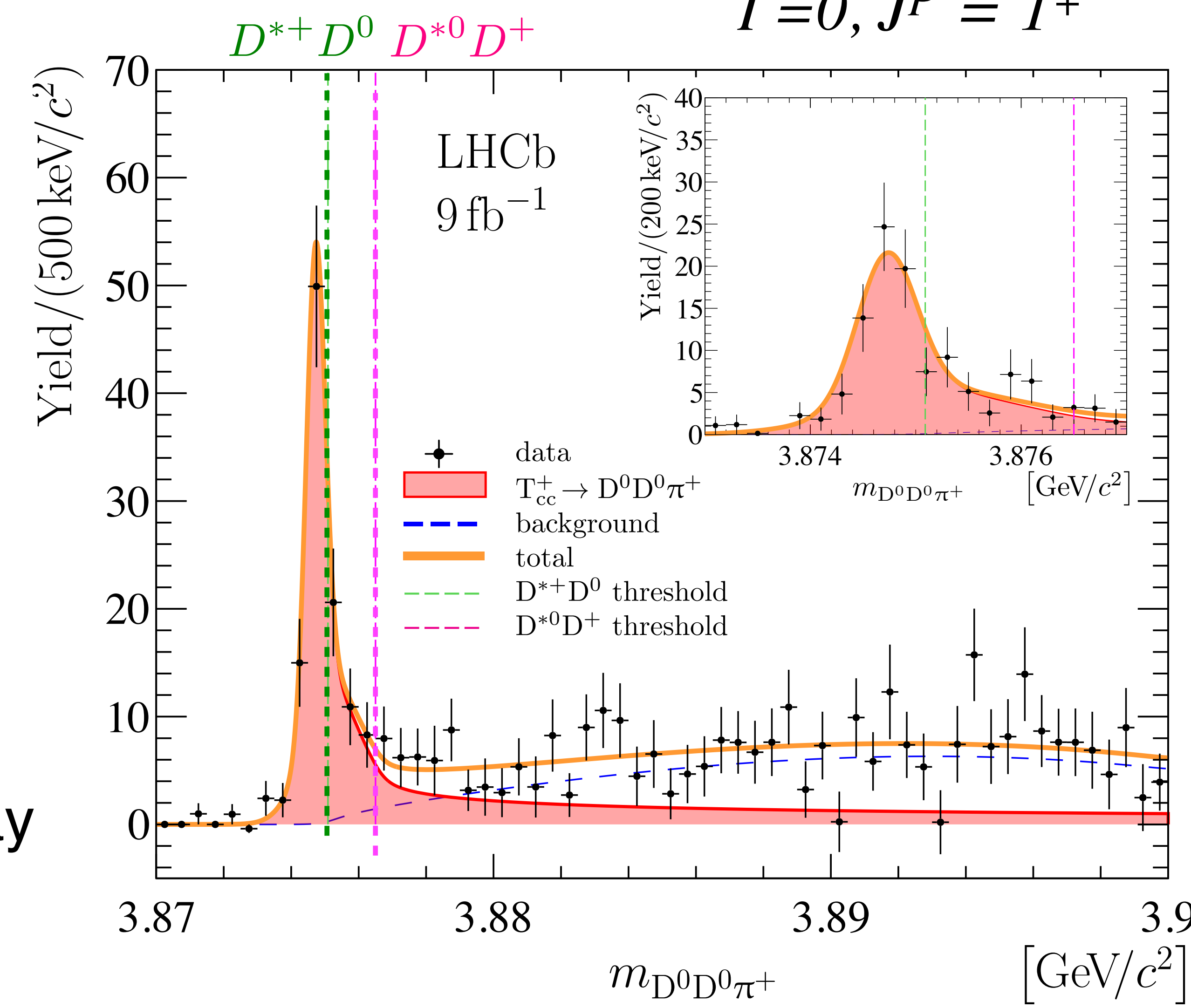
— Expansion in χEFT : $\chi = \dfrac{\sqrt{2\mu\Delta_M}}{\Lambda_\chi} < 0.1$

$$\Delta_M = m(D^+D^{*0}) - m(D^0D^{*+})$$

— No admixture of inelastic channels

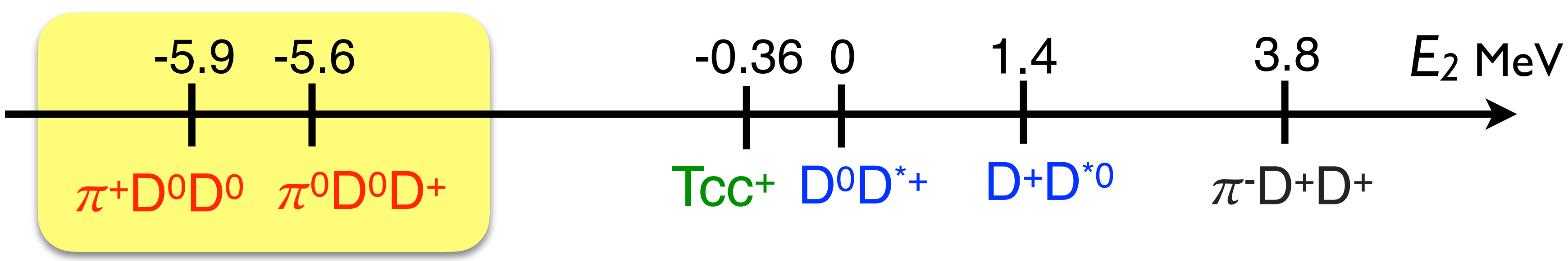
— Width: almost entirely from the only strong decay

$$T_{cc}^+ \to D^0 D^{*+} \to D^0 D^0 \pi^+ / D^0 D^+ \pi^0$$



$$\delta m_{BW} = -273 \pm 61 \pm 5^{+11}_{-14}\ \text{keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$



- Plenty of theoretical studies; in particular, the Tcc width is addressed in

Meng et al (2021), Fleming et al (2021), Ling et al (2022), Feijoo et al. (2021), Yan et al. (2022), Albaladejo (2022), Dai et al. (2023),…

T_{cc} on lattice

- HAL QCD Collaboration at mπ = 146 MeV: Lyu et al, *PRL* 131 161901 (2023)

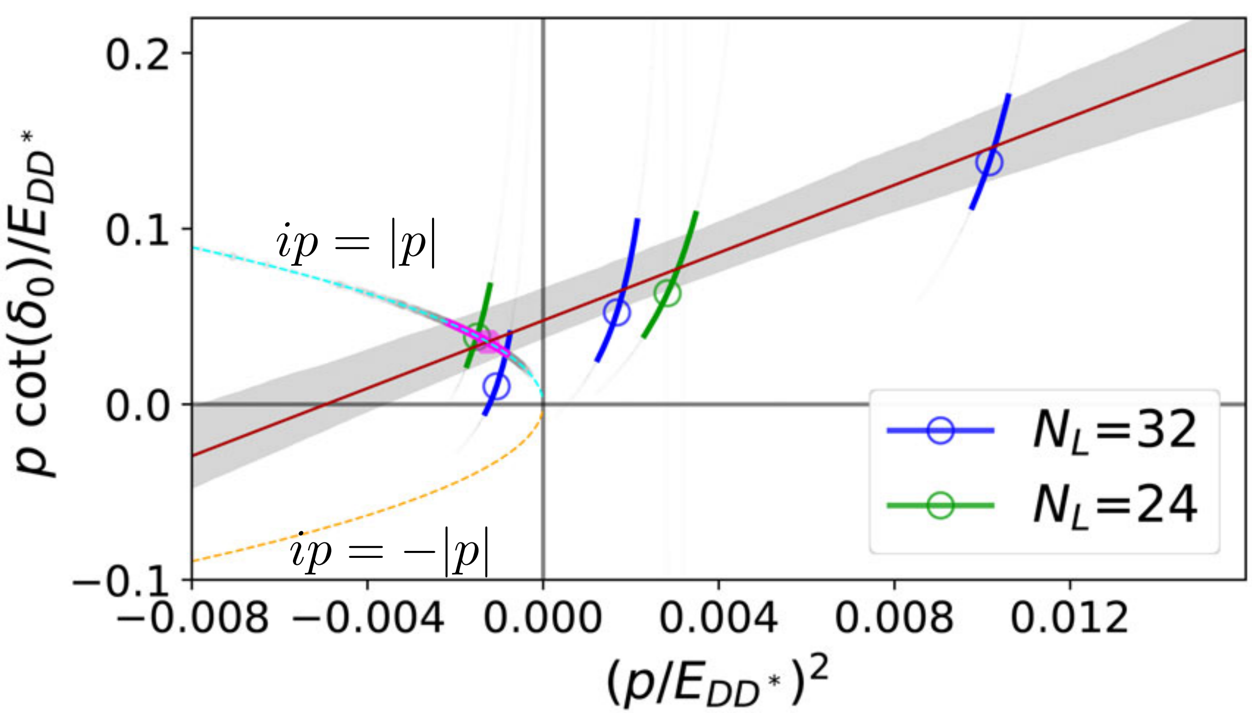
— calculate the DD* scattering potential ⇒ phase shifts above the two-body threshold

$$E_{\text{pole}} = -59\,{}^{+53\,+2}_{-99\,-67}\ \text{keV}$$ DD* virtual state

- Lüscher method based analyses of FV energy levels

— DD* spectra at mπ = 280 MeV Padmanath and Prelovsek, PRL 129, 032002 (2022)

see also Collins et al *PRD* 109 (2024) 9, 094509



— DD* phase shifts parameterised using the ERE:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2}rp^2 + \mathcal{O}(p^4)$$

$$E_{\text{pole}} = -9.9^{+3.6}_{-7.2}\ \text{MeV}$$ DD* virtual state

— DD*-D*D* coupled channel parameterization at mπ = 391 MeV T. Whyte, D. Wilson, and C. Thomas 2405.15741v1 (2024)

$$E_{\text{pole}} = (-62 \pm 34)\ \text{MeV}$$ DD* virtual state

$$E_{\text{pole}} = (-49 \pm 35 + i(11 \pm 13)/2)\ \text{MeV}$$ D*D* resonance

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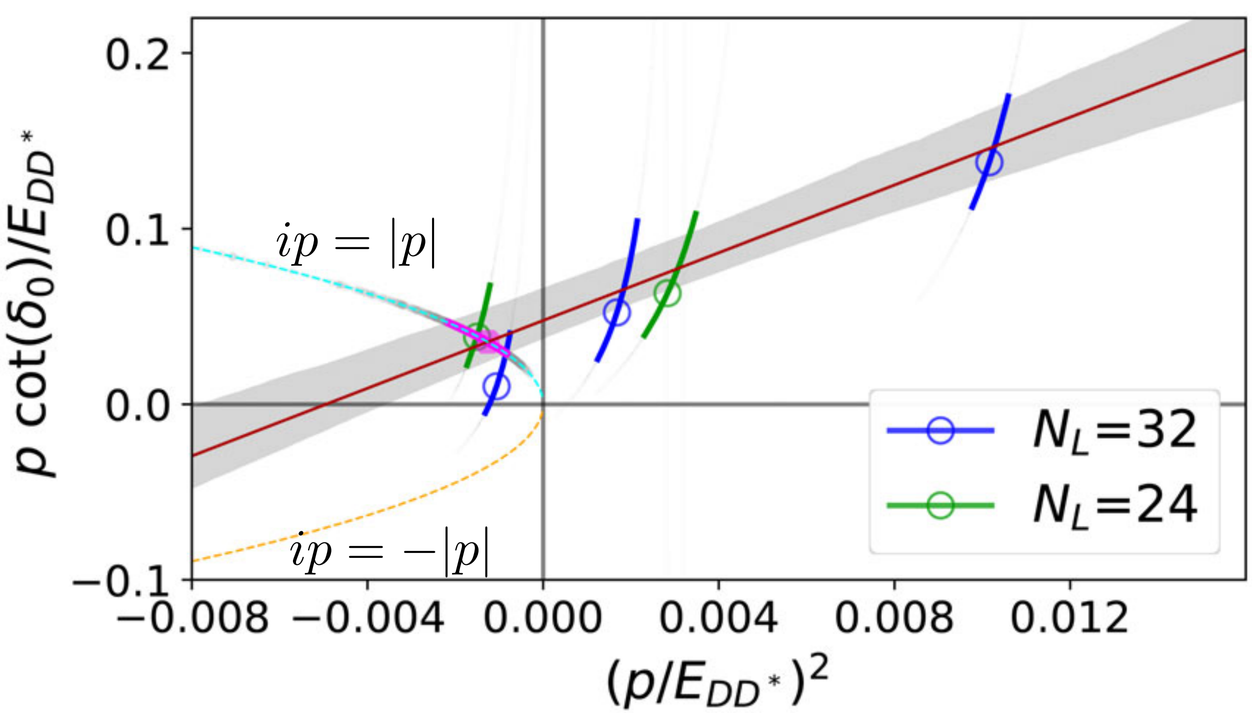

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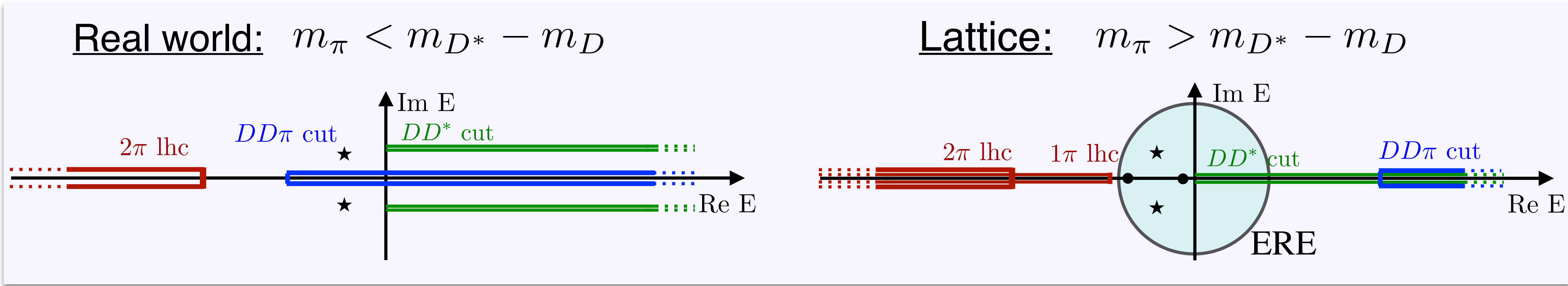
Main assumption for Lüscher method: no nearby left-hand cuts!

Analytic structure of the DD* scattering amplitude

– Cut structure depends on the (light-quark or) pion mass



3-body DDπ cut

Left-hand cuts

$\Rightarrow$ Prominent role for the T_{cc} width

M. Du et al *PRD* 105, 014024(2022)

$\Rightarrow$ Constraints on the ERE applicability range

M. Du et al, *PRL* 131, 131903 (2023)

$\Rightarrow$ Invalidate Lüscher's QC at least below lhc

Raposo and Hansen (2023), L. Meng et al (2023), Green et al (2021), …

– The leading nearby cut is always associated with the one-pion exchange

$\Rightarrow$ Theoretical framework has to include it!

χEFT approach at low energies

Weinberg (1992)

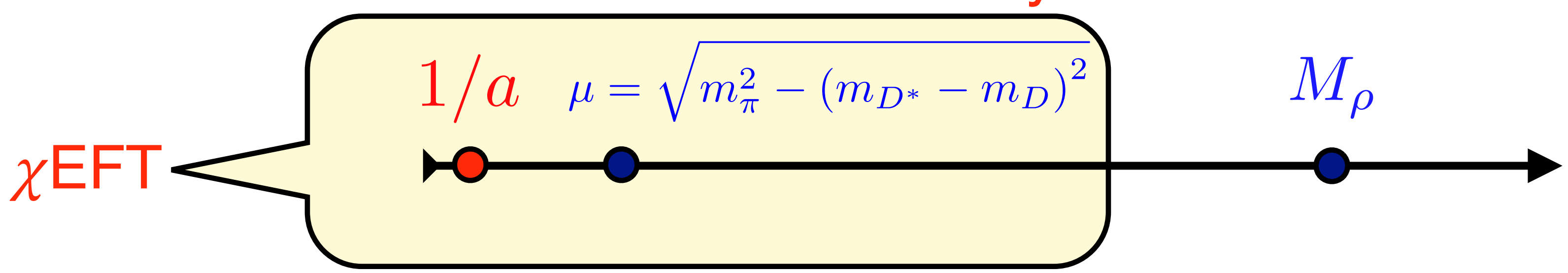
our works 2010-till now

see also AlFiky et al 2006

Fleming et al 2007

- DD* potential to a given order in χ=Q/Λ_h

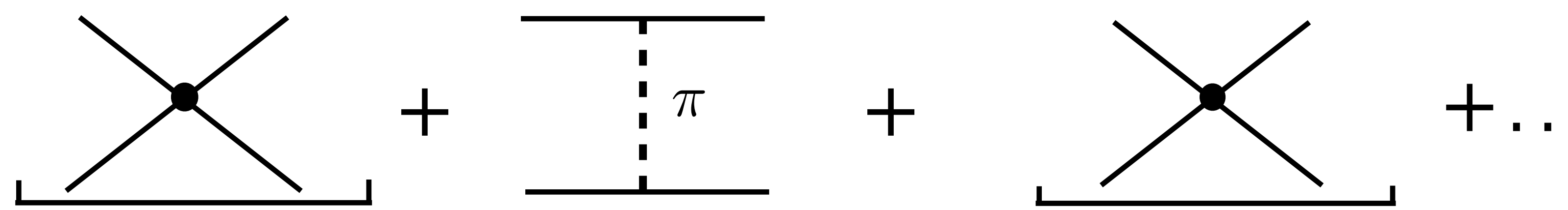
Hierarchy of scales



— Keep track of small scales ~ $1/a$ due to shallow states ⇒ **resummation**

— Range effects can also come from D^0D^{*+} — D^+D^{*0} coupled channels

$$V = V^{(0)}_{\rm OPE} + V^{(0)}_{\rm cont} + V^{(2)}_{\rm cont} + \ldots =$$



$C^{(0)}$ Long range: OPE $C^{(2)}(p^2+p'^2) + D^{(2)}(\xi^2-1)$

$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

— Incorporates the relevant cuts

— Extention to incorporate DD*-D*D* coupled channels is straightforward

- Amplitudes are solutions of the integral equations

$$T_{\alpha\beta} = V^{\rm eff}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\rm eff}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

G - Green functions

→ Consistent with Unitarity and analyticity

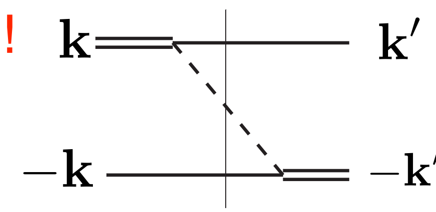
3-body DDπ cut

- OPE potential:

$$V_{\mathrm{DD^*\to DD^*}}(\mathbf{k},\mathbf{k}',E) \propto \frac{g_c^2}{(4\pi f_\pi)^2}\,\tau_1\cdot\tau_2\,\frac{(\epsilon_1\cdot\vec{q})\,(\epsilon_2'^{\,*}\cdot\vec{q})}{2E_\pi(\mathbf{k}-\mathbf{k}')}\left(\frac{1}{D_{DD\pi}(\mathbf{k},\mathbf{k}',E)}+\frac{1}{D_{D^*D^*\pi}(\mathbf{k},\mathbf{k}',E)}\right)$$

$$D_{DD\pi}(k,k',E) = E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E \quad \Rightarrow$$

goes on shell!



$D_{DD\pi}(k,k',E) \to i\pi\delta(E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E) \to \mathrm{Im\ part}$

3-body cut condition

- For each $E \geq E_{\mathrm{thr}} \equiv 2m+m_\pi$, there are real values of k and k$'$ such that $D_{DD\pi}(k,k',E)=0$

  3-body branch point is (k=k$'$=0): $E \equiv 2m+m_\pi$

3-body DDπ cut

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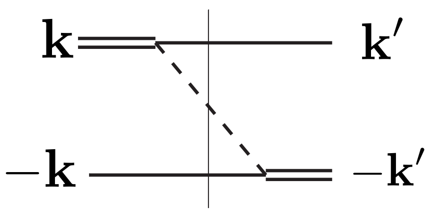


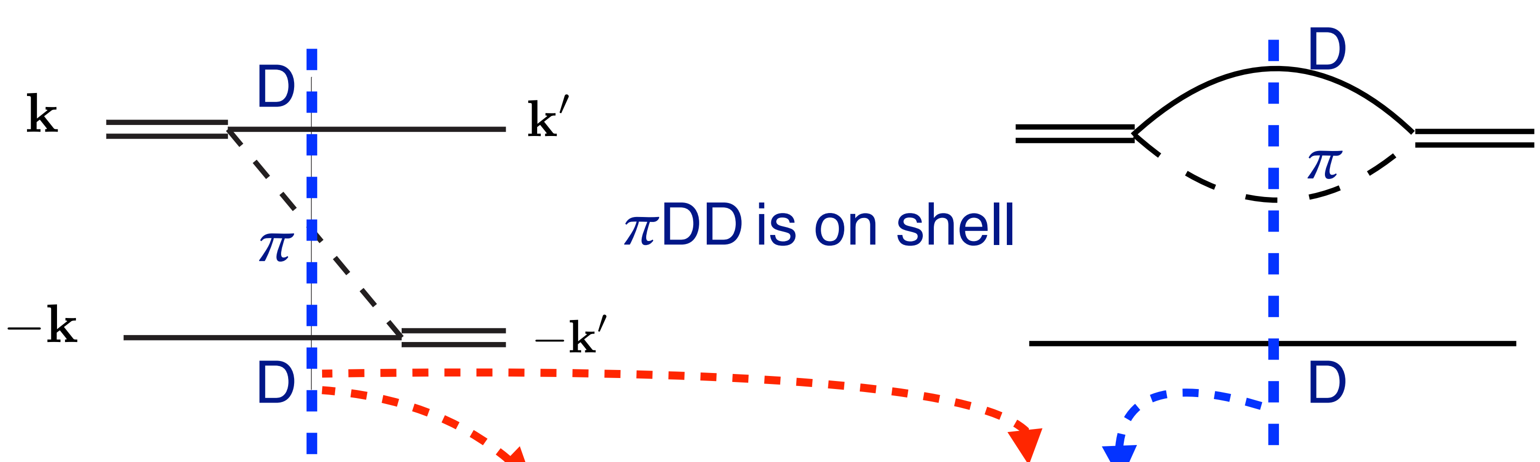
$$D_{DD\pi}(k,k',E) \to i\pi\delta(E_D(k)+E_D(k')+E_\pi(\mathbf{k}-\mathbf{k}')-E) \to \mathrm{Im\ part}$$

3-body cut condition

☛ For each $E \geq E_{\mathrm{thr}} \equiv 2m + m_\pi$, there are real values of k and k′ such that $D_{DD\pi}(k,k',E) = 0$

3-body branch point is (k=k′=0): $E \equiv 2m + m_\pi$

- 3-body cut stems from one-pion exchange (OPE) and self energies in the Green funct.



$$T_{\alpha\beta} = V^{\mathrm{eff}}_{\alpha\beta} - \sum_\gamma \int \frac{d^3q}{(2\pi)^3} V^{\mathrm{eff}}_{\alpha\gamma} G_\gamma T_{\gamma\beta}$$

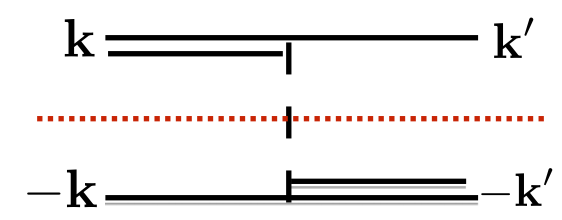
Bose statistics for DD requires both cont's to appear together

Full analogy to the X(3872)

VB et al. PRD84 (2011)

Left-hand cut

— Leading singularity is from the on shell one-pion exchange



PWD of the static potential:

$$V_{l=0}(k,k') \propto \int dz \frac{1}{(\mathbf{k}-\mathbf{k}')^2+\mu^2} = \frac{1}{2kk'} \log \frac{(k+k')^2+\mu^2}{(k-k')^2+\mu^2} \xrightarrow[k=k'=p]{\text{on shell}} \frac{1}{2p^2} \log \frac{4p^2+\mu^2}{\mu^2}$$

⇒ left-hand cut (lhc) branch point is at

$$(p_{\rm lhc}^{1\pi})^2 = -\frac{\mu^2}{4}$$

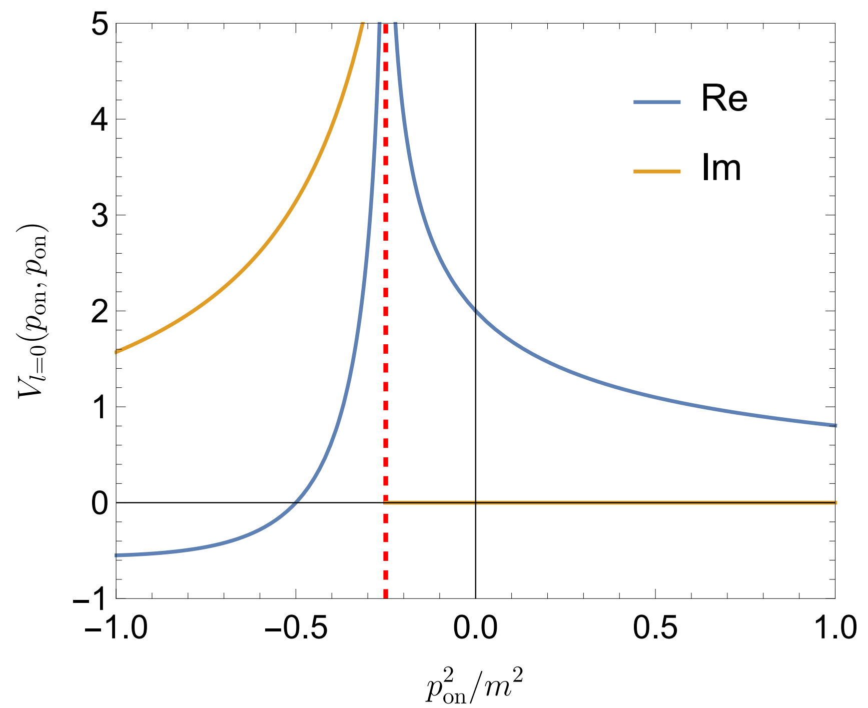
For DD*

$$\mu^2 = m_\pi^2 - \Delta M^2$$

$$\Delta M = m_{D^*} - m_D$$

Sc. amplitude is complex for E below the lhc
⇒ Lüscher's method breaks down

Meng et al, *PRD* 109 (2024), Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021)



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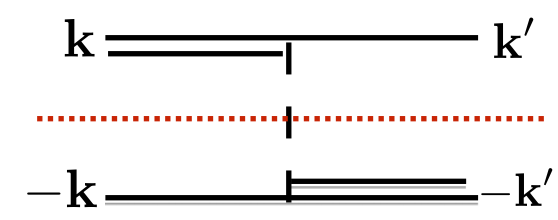

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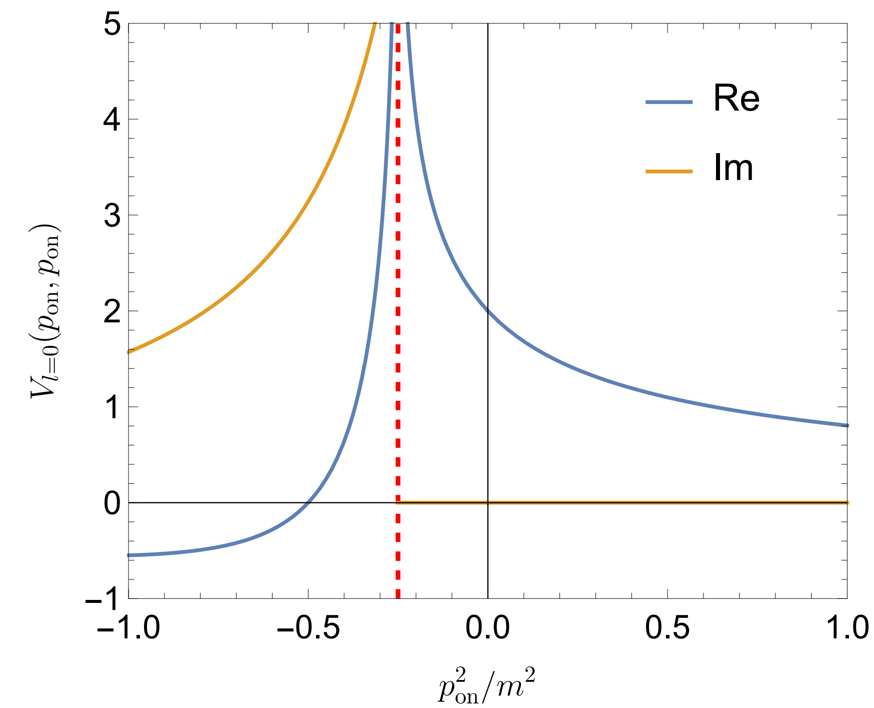
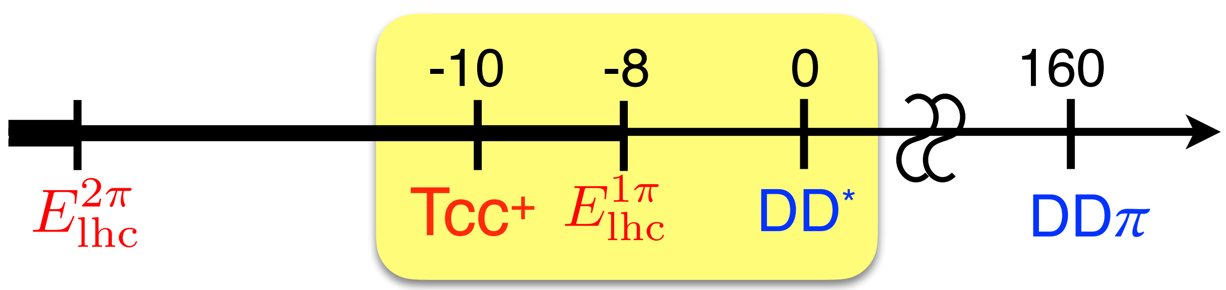


Sc. amplitude is complex for E below the lhc
⇒ Lüscher's method breaks down

Meng et al, *PRD* 109 (2024), Raposo and Hansen 2311.18793 (2023), Green et al, *PRL* 127 (2021)

At mπ = 280 MeV

$$E_{\rm lhc}^{1\pi} = \frac{(p_{\rm lhc}^{1\pi})^2}{2\mu_{DD^*}} = -8\,\text{MeV} \Rightarrow$$ $E_{\rm lhc}^{1\pi}$ sets the range of convergence of the ERE: $E \ll |E_{\rm lhc}^{1\pi}|$



⇒ ERE is not applicable

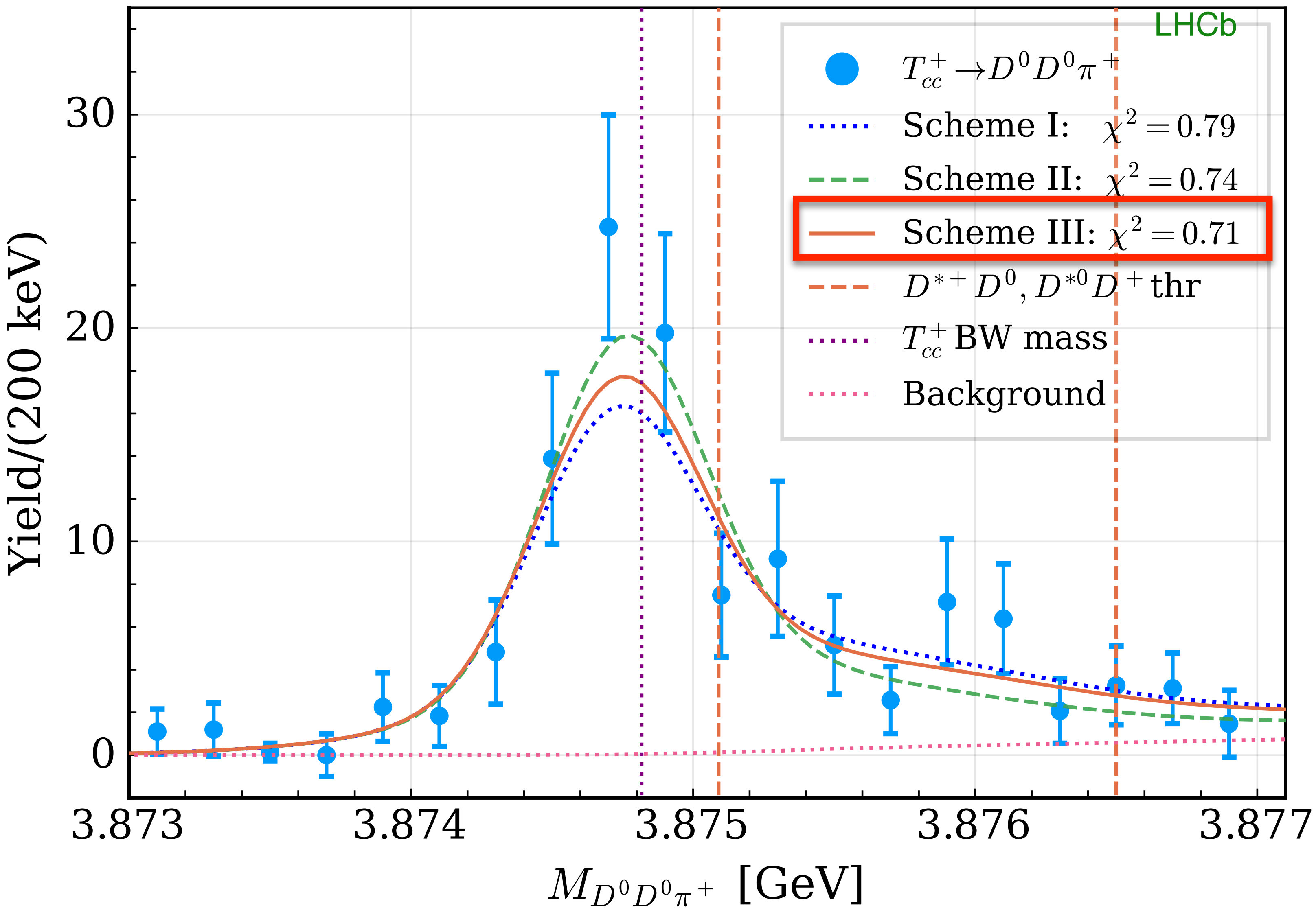
M. Du et al, *PRL* 131, 131903 (2023)

Applications

App I: LO χEFT analysis of $D^0D^0\pi^+$ data by LHCb

M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

with resolution



The pole

III
full 3-body unitarity:
OPE + dynamical D^* width
$-356^{+39}_{-38} - i(28 \pm 1)$
0.71

— Coupled $D^0D^{*+} - D^+D^{*0}$ scattering

— 1 parameter + overal normalization

Re part of the T_{cc} pole: Inconclusive about the role of 3-body effects with current exp. precision

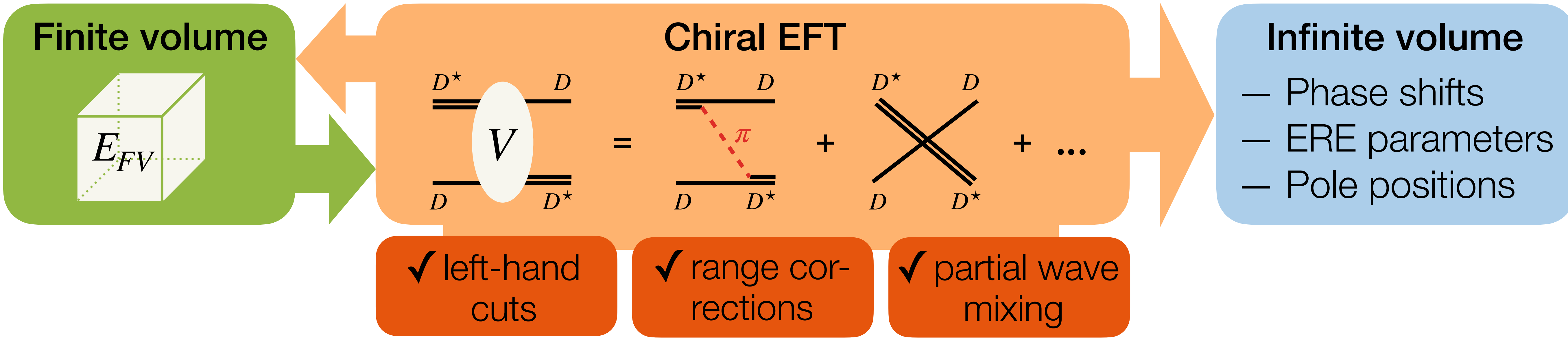
Can be reanalysed if more precise data emerge

Im part of the T_{cc} pole: Controlled by 3-body effects

$$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV}$$

App II: χEFT as an alternative to Lüscher

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)



— Construct regularized effective potential truncated to a given order

$$V = V^{(0)}_{\mathrm{OPE}} + V^{(0)}_{\mathrm{cont}} + V^{(2)}_{\mathrm{cont}} + \dots$$

$$V^{(0)+(2)}_{\mathrm{cont}}[{}^3S_1] = \left(C^{(0)}_{{}^3S_1} + C^{(2)}_{{}^3S_1}(p^2 + p'^2)\right)(\vec{\epsilon}\cdot\vec{\epsilon}'^*)$$

$$V^{(2)}_{\mathrm{cont}}[{}^3P_0] = C^{(2)}_{{}^3P_0}(\vec{p}'\cdot\vec{\epsilon}'^*)(\vec{p}\cdot\vec{\epsilon})$$

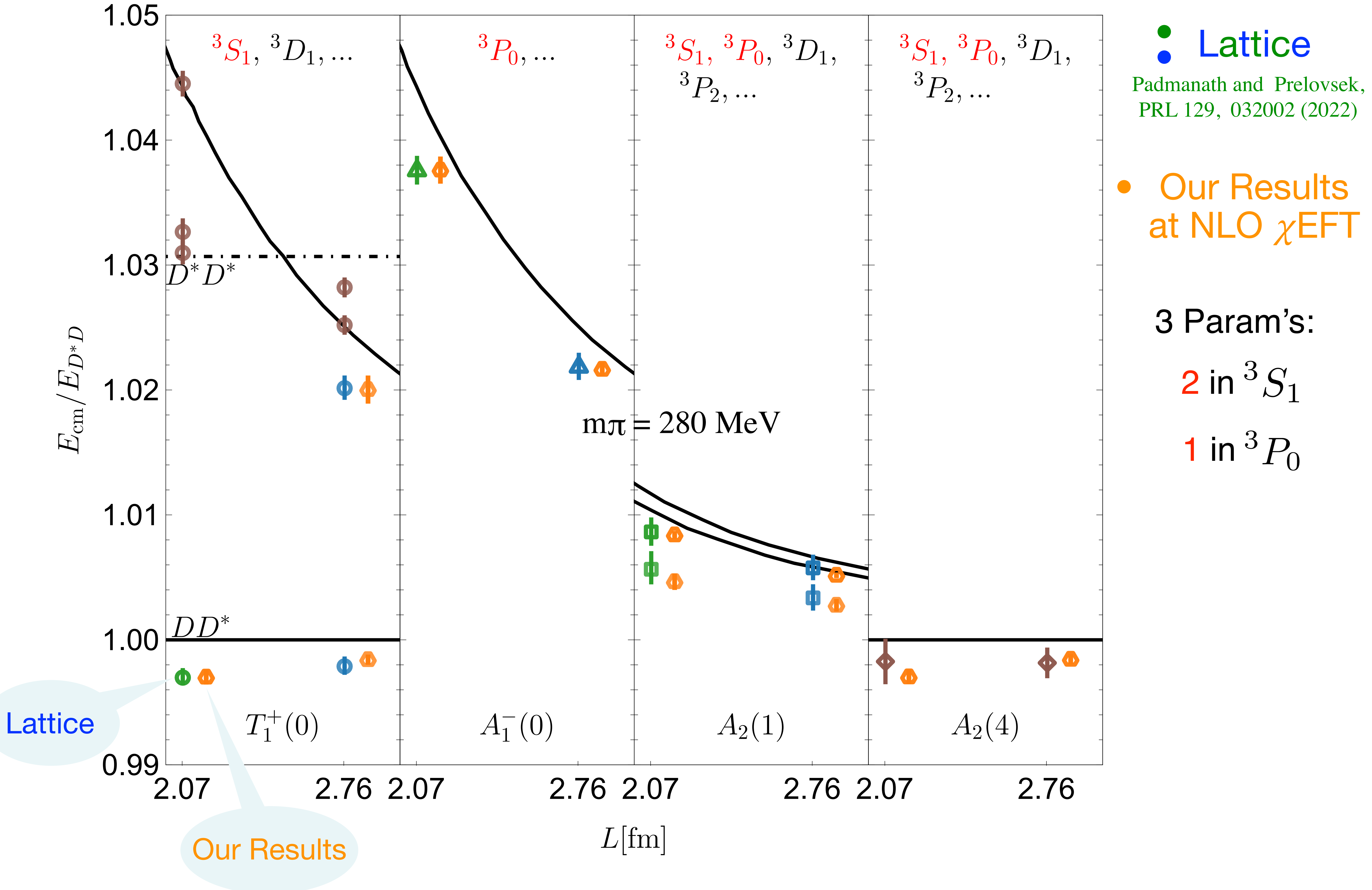
— Calculate E_{FV} in each irrep as a solution of the eigenvalue problem

$$\det\left[\mathbb{G}^{-1}(E) - \mathbb{V}(E)\right] = 0 \qquad \mathbb{G}_{\mathbf{n},\mathbf{n}'} = \mathcal{J}\frac{\delta_{\mathbf{n}',\mathbf{n}}}{L^3}\frac{1}{4E_D(\tilde{p}_{\mathbf{n}})E_{D^*}(\tilde{p}_{\mathbf{n}})}\frac{1}{E - E_D(\tilde{p}_{\mathbf{n}}) - E_{D^*}(\tilde{p}_{\mathbf{n}})}$$

— Adjust LEC's C's from best fits to E_{FV}: C's are independent of the volume size L

— Employ the EFT potential to calculate infinite volume amplitudes using LSE

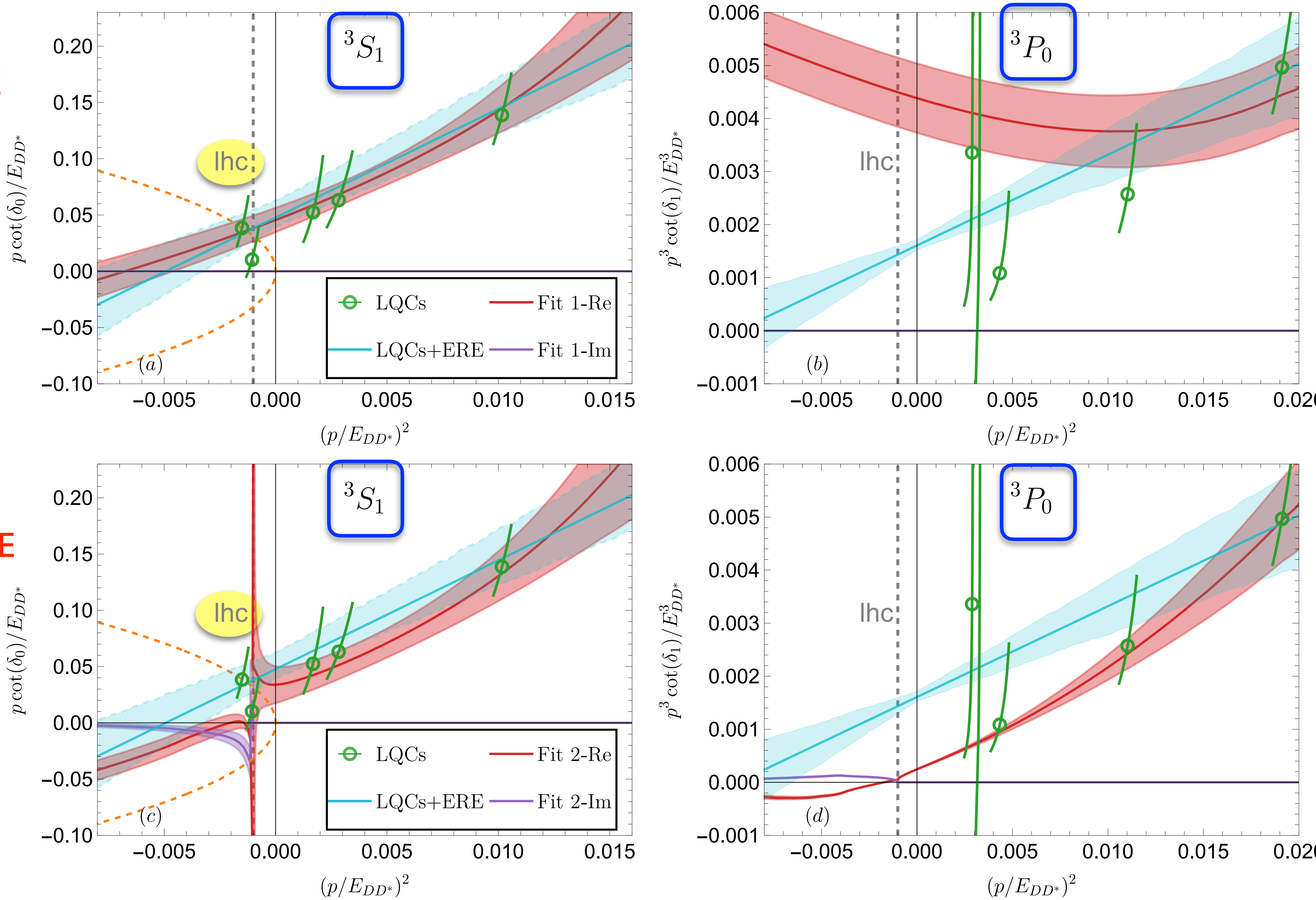
DD* Finite Volume Energy Levels at $m_\pi = 280$ MeV



Predict IFV phase shifts and Tcc pole

No OPE

With OPE



- $^3P_0$ shape controlled by OPE
- $^3S_1$ near lhc controlled by OPE

	$a_{^3S_1}$ [fm]	$r_{^3S_1}$ [fm]	$\delta m_{T_{cc}}$ [MeV]	$a_{^3P_0}$ [fm^3]	$r_{^3P_0}$ [fm^{-1}]	χ^2/dof	# of param's
LQCs+ERE fit [23]	1.04 ± 0.29	$0.96^{+0.18}_{-0.20}$	$-9.9^{+3.6}_{-7.2}$	$0.076^{+0.008}_{-0.009}$	6.9 ± 2.1	3.7/5	4
Fit 1: cont.	1.09 ± 0.35	0.75 ± 0.14	-10.6 ± 4.4	0.028 ± 0.004	-4.3 ± 0.05	5.52/6	3
Fit 2: cont.+OPE	1.46 ± 0.57	0.096 ± 0.53	$-6.6(\pm1.5) - i4.0(\pm3.7)$	0.497 ± 0.007	5.63 ± 0.19	2.95/6	3

[23] Padmanath and Prelovsek, PRL 129, 032002 (2022)

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

T_{cc} at m_π = 280 MeV is a resonance state with 85% probability

App III: Pion-mass dependence of the Tcc pole

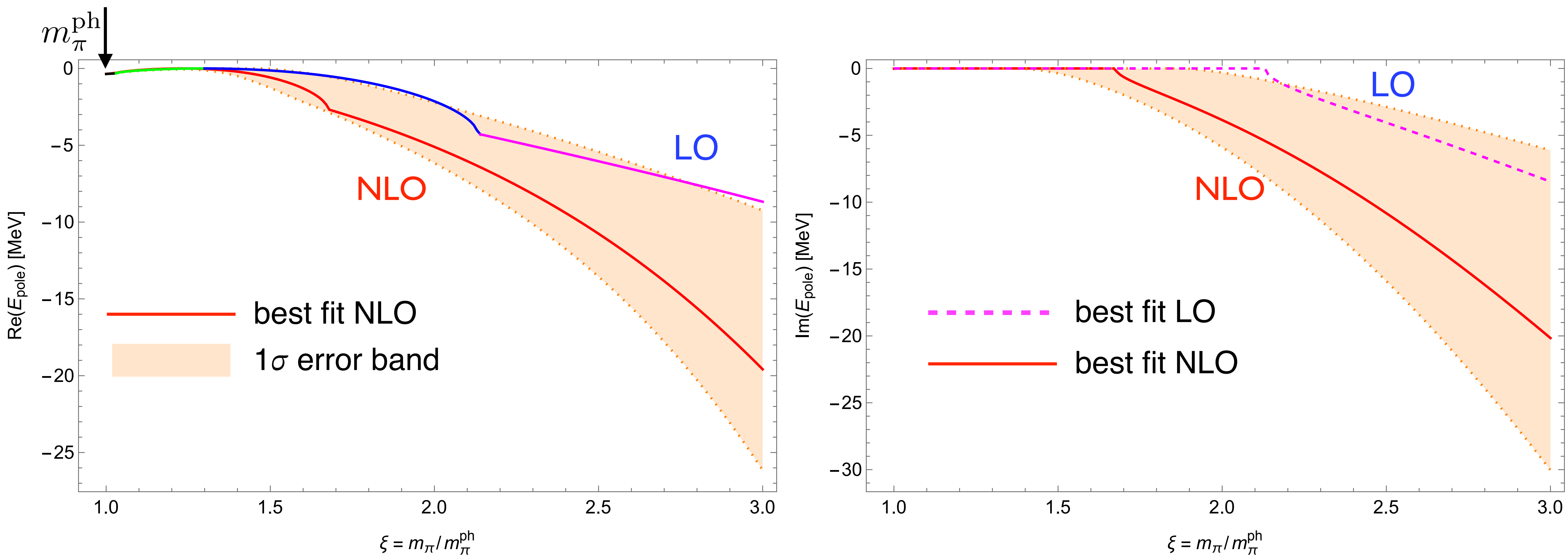
M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

- Employ knowledge of the NLO potential at $m_\pi = m_\pi^{\rm ph}$ and $m_\pi = 280$ MeV

$$V = V_{\rm OPE}^{(0)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} \qquad V_{\rm cont}^{(0)+(2)}[{}^3S_1] = C_{{}^3S_1}^{(0)} + C_{{}^3S_1}^{(2)}(p^2+p'^2) + D_{{}^3S_1}^{(2)}(\xi^2-1)$$

$$\xi = \frac{m_\pi}{m_\pi^{\rm ph}}$$

- Calculate scattering amplitude for any m_π



- Tcc pole transitions: quasi-bound → bound → virtual → resonance

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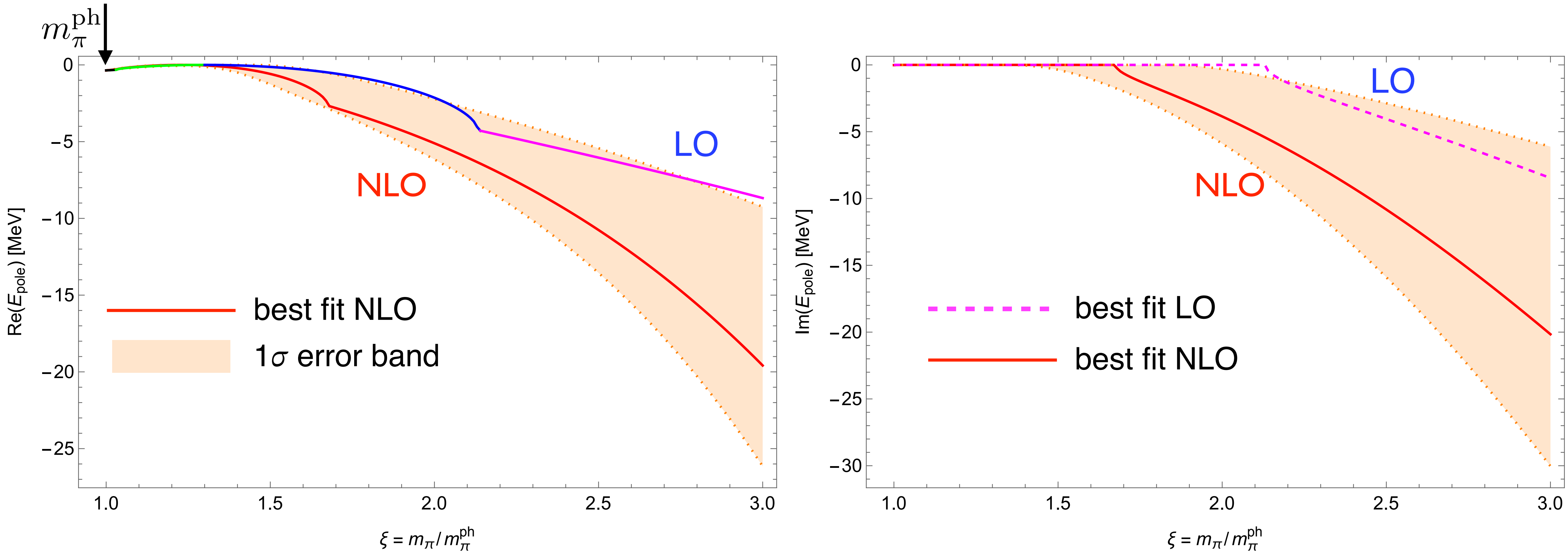


- Tcc pole transitions: quasi-bound → bound → virtual → resonance

- NLO is qualitatively consistent to LO; resonance is formed at smaller m_π

⇒ Trajectory consistent with hadronic molecule Matuschek, VB, Guo, Hanhart, EPJA 57, 101 (2021)

Truncation uncertainty of chiral expansion

Add higher-order interactions + naturalness

$$V = V_{\rm OPE}^{(0)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} + V_{\rm cont}^{(4)}$$

$$V_{\rm cont}^{(0)+(2)}[{}^3S_1] = C_{{}^3S_1}^{(0)} + C_{{}^3S_1}^{(2)}(p^2 + p'^2) + D_{{}^3S_1}^{(2)}(\xi^2 - 1)$$

Fits

$$C_2 = \frac{\alpha_2}{F_\pi^2}\frac{1}{\Lambda_\chi^2}; \quad \alpha_2 = 0.2$$

$$D_2 = \frac{\tilde{\alpha}_2}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^2; \quad \tilde{\alpha}_2 = 0.4$$

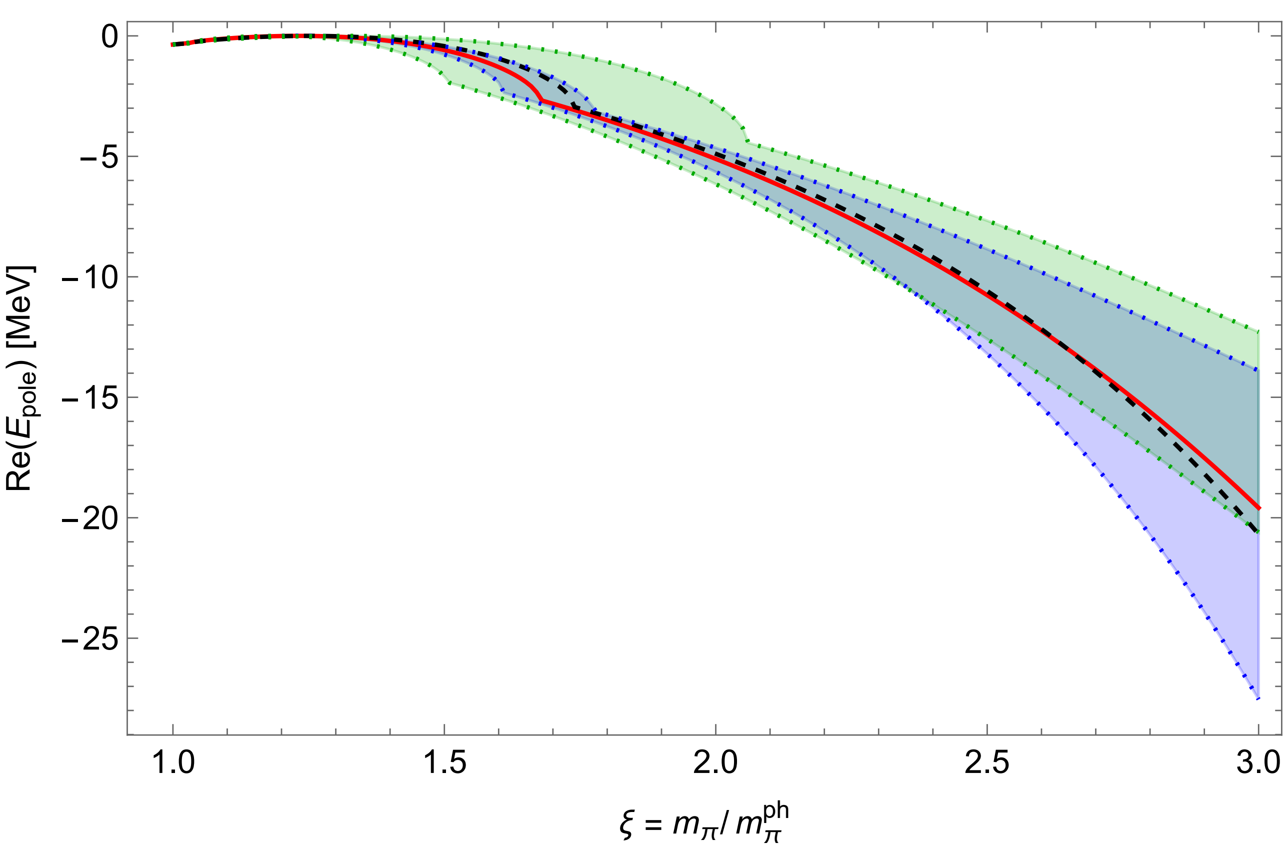
Naturalness

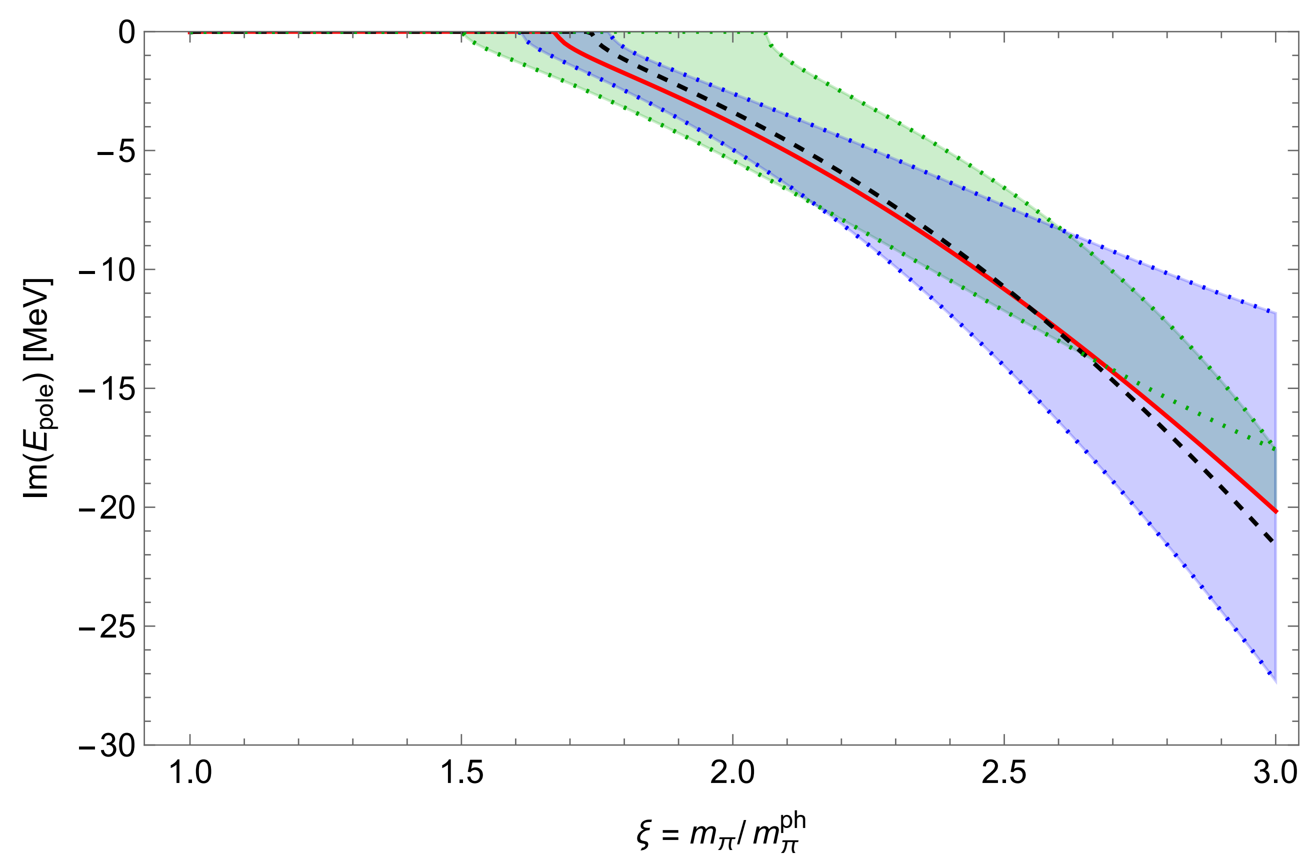
$$D_4 = \frac{\alpha_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi^2}\right)^2; \quad \alpha_4 \in [-1, 1]$$

$$\tilde{D}_4 = \frac{\tilde{\alpha}_4}{F_\pi^2}\left(\frac{m_\pi^{\rm ph}}{\Lambda_\chi}\right)^4; \quad \tilde{\alpha}_4 \in [-1, 1]$$

$$V_{\rm cont}^{(4)} = D_4(\xi^2 - 1)(p^2 + p'^2) + \tilde{D}_4(\xi^4 - 1)$$

⇒ Comparable with statistical uncertainty





T_{cc} pole vs m_π: Contact vs Pionful

M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

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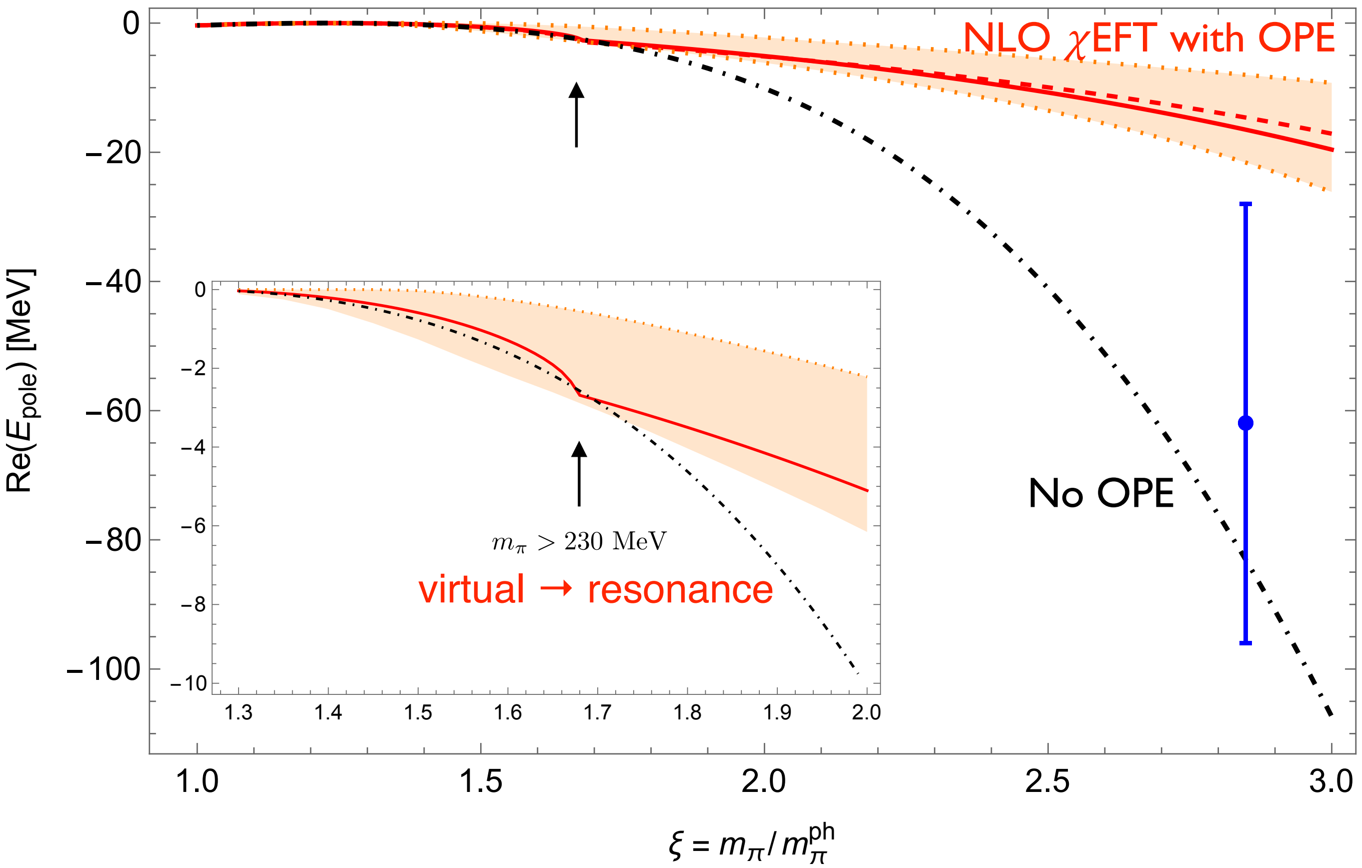
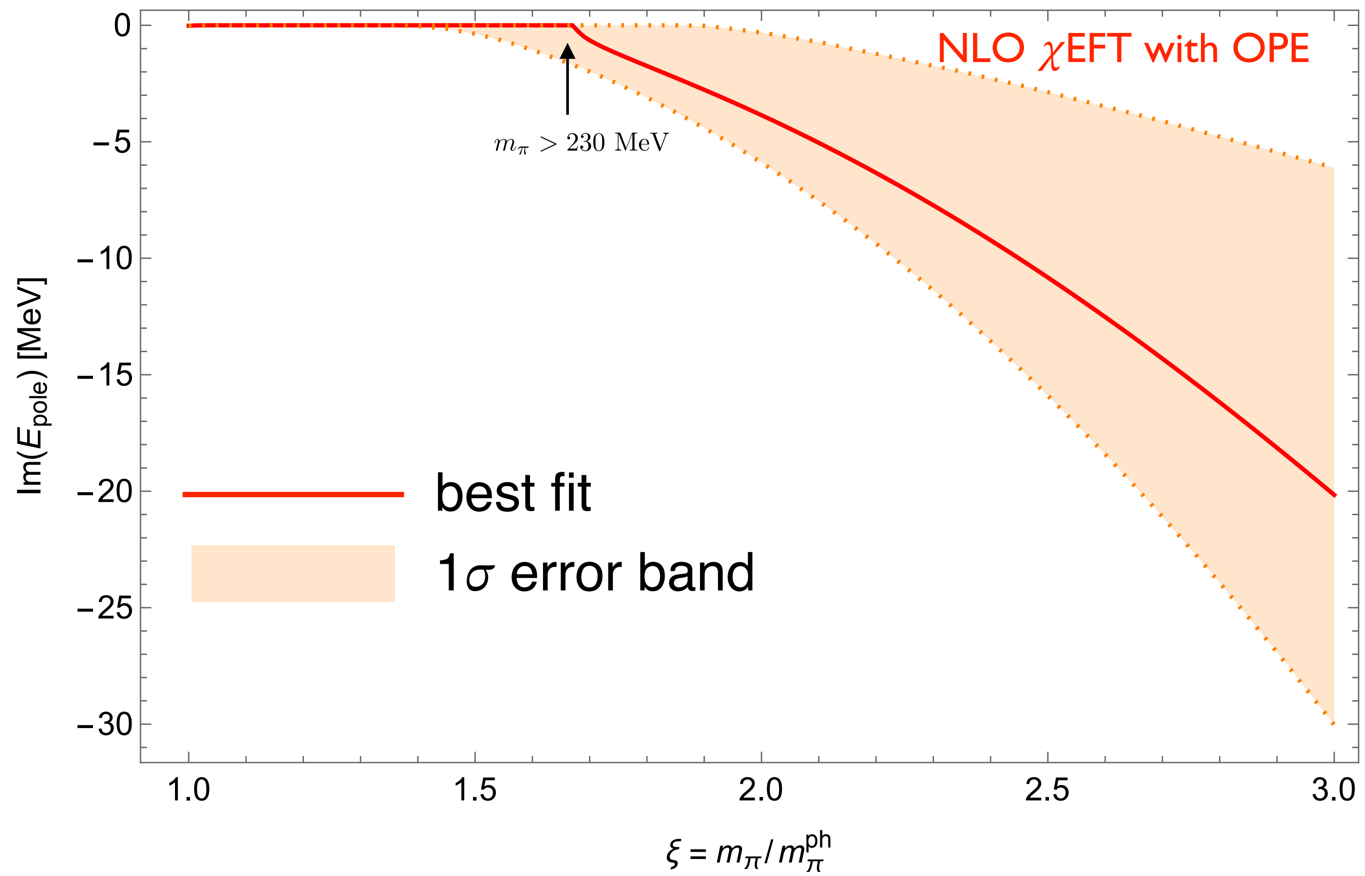




- Our pionless trajectory is consistent with new lattice data at $m_\pi = 391$ MeV
  - — Virtual state extracted using Lüscher + amplitude parameterization without pions

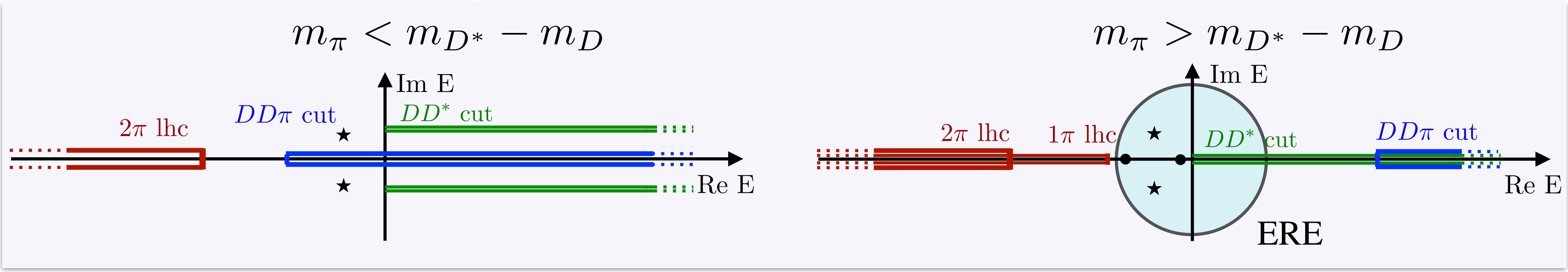
T. Whyte, D. Wilson, and C. Thomas arXiv:2405.15741v1

- But due to repulsion from the OPE, NLO χEFT yields a resonance for $m_\pi > 230$ MeV

⇒ Long-range physics significantly changes the pole trajectory

Summary

- Systematic analysis of experimental and lattice data in χEFT



- χEFT as an alternative to Lüscher in the presence of left-hand cuts

  → DD* E_{FV} mπ = 280 MeV: Lüscher QC fails around lhc, ok above DD* thr, uncertainty unclear

  → plenty of possible applications

  → for other methods see Raposo, Hansen, arXiv:2311.18793; Bubna et al. JHEP 05 (2024) 168; Hansen et al. JHEP 06 (2024) 051

- Tcc properties and pole transitions: consistent with a molecule

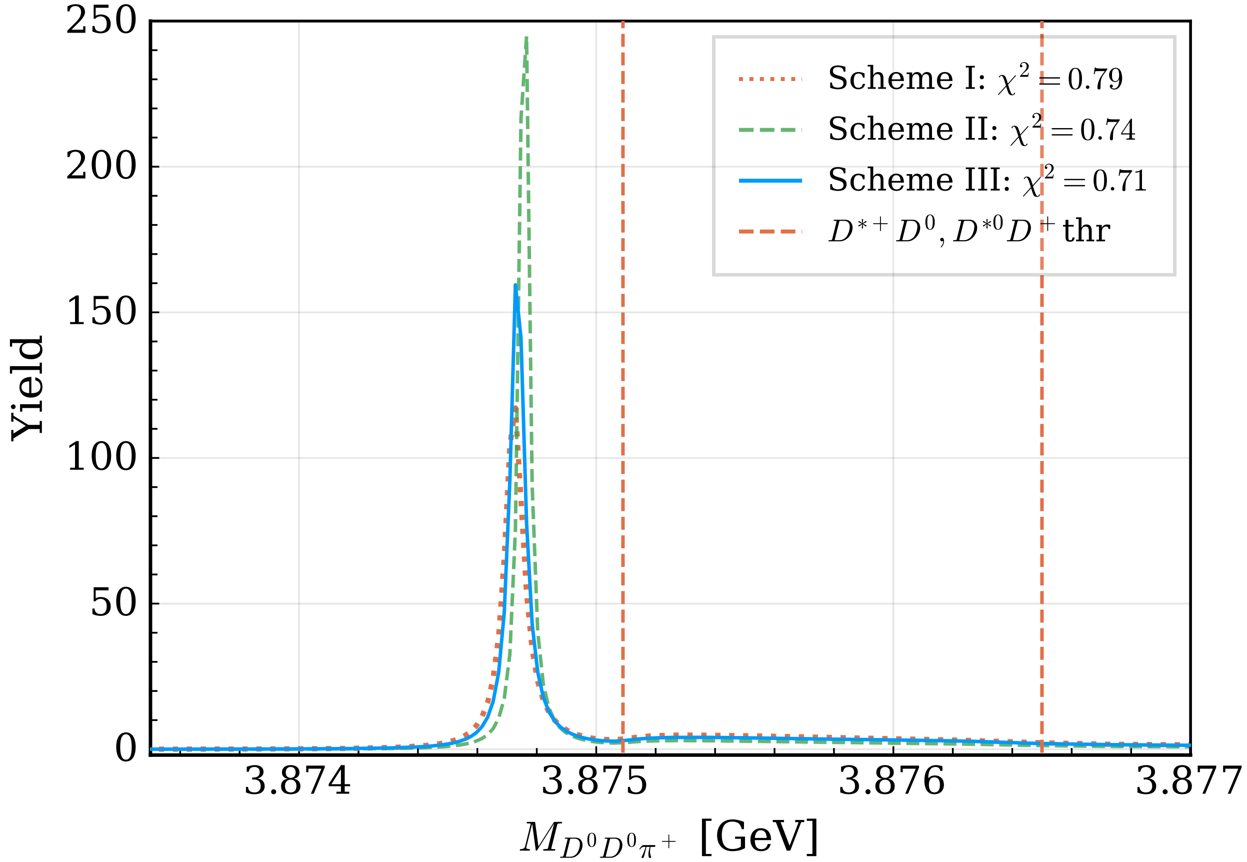
  quasi-bound → bound → virtual → resonance

Great progress in experiment and lattice together with χEFT ⇒ Precision physics

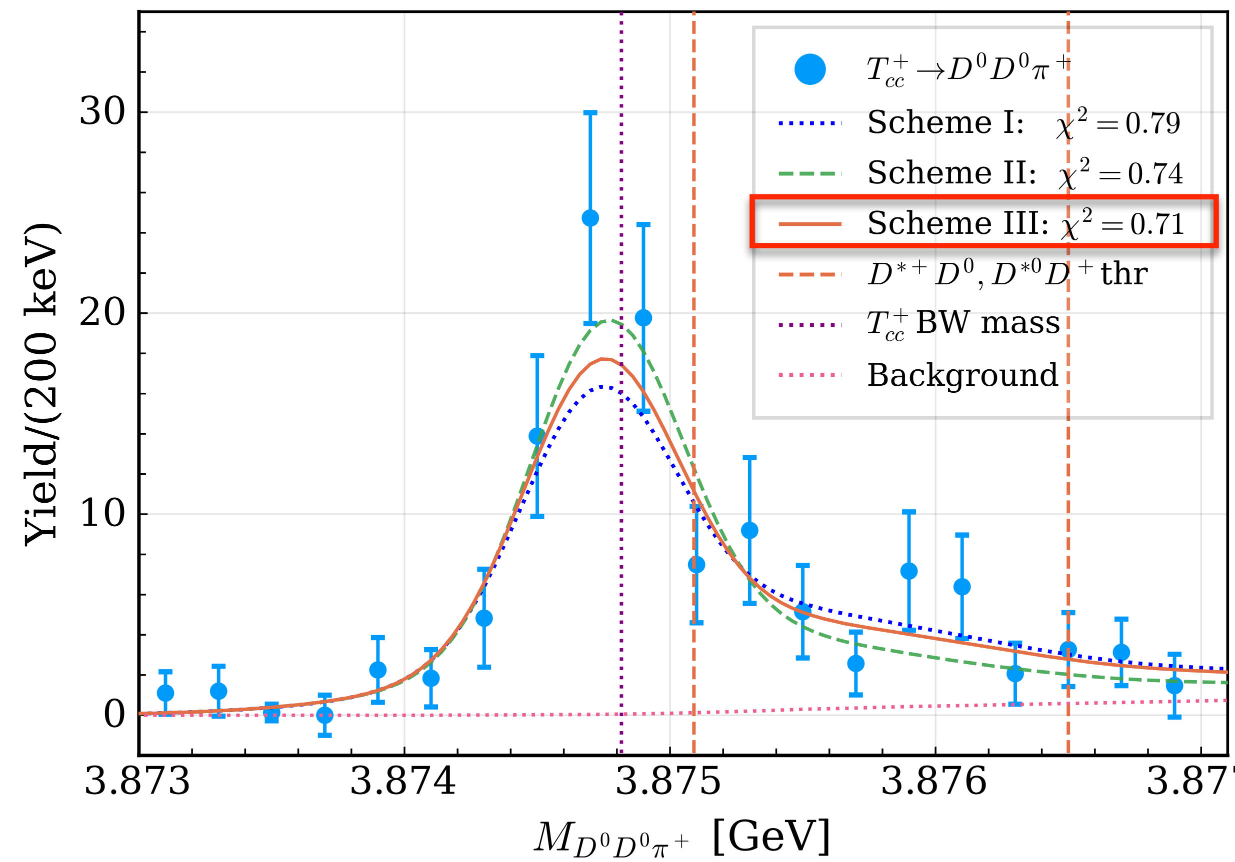
Backup

App I: Fits to the $D^0D^0\pi^+$ mass spectrum

w/o resolution



with resolution



Scheme	I	II	III
Description	2-body unitarity: No OPE, static D^* width	Incomplete 3-body unitarity: No OPE, dynamical D^* width	full 3-body unitarity: OPE + dynamical D^* width
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$
χ^2	0.79	0.74	0.71

Real part of the pole: all Fits are consistent within 1σ —more precise data are needed

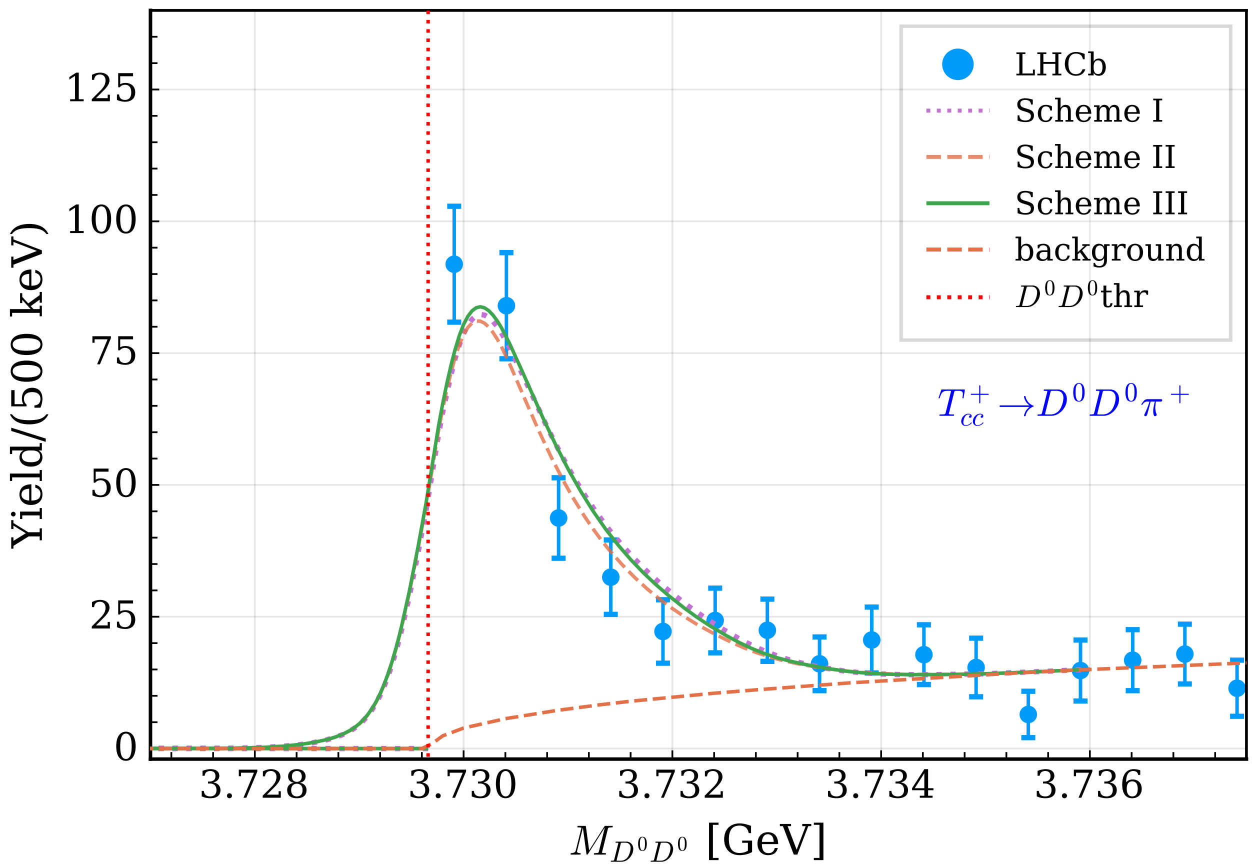
Width of Tcc+ : Accuracy requires 3-body effects

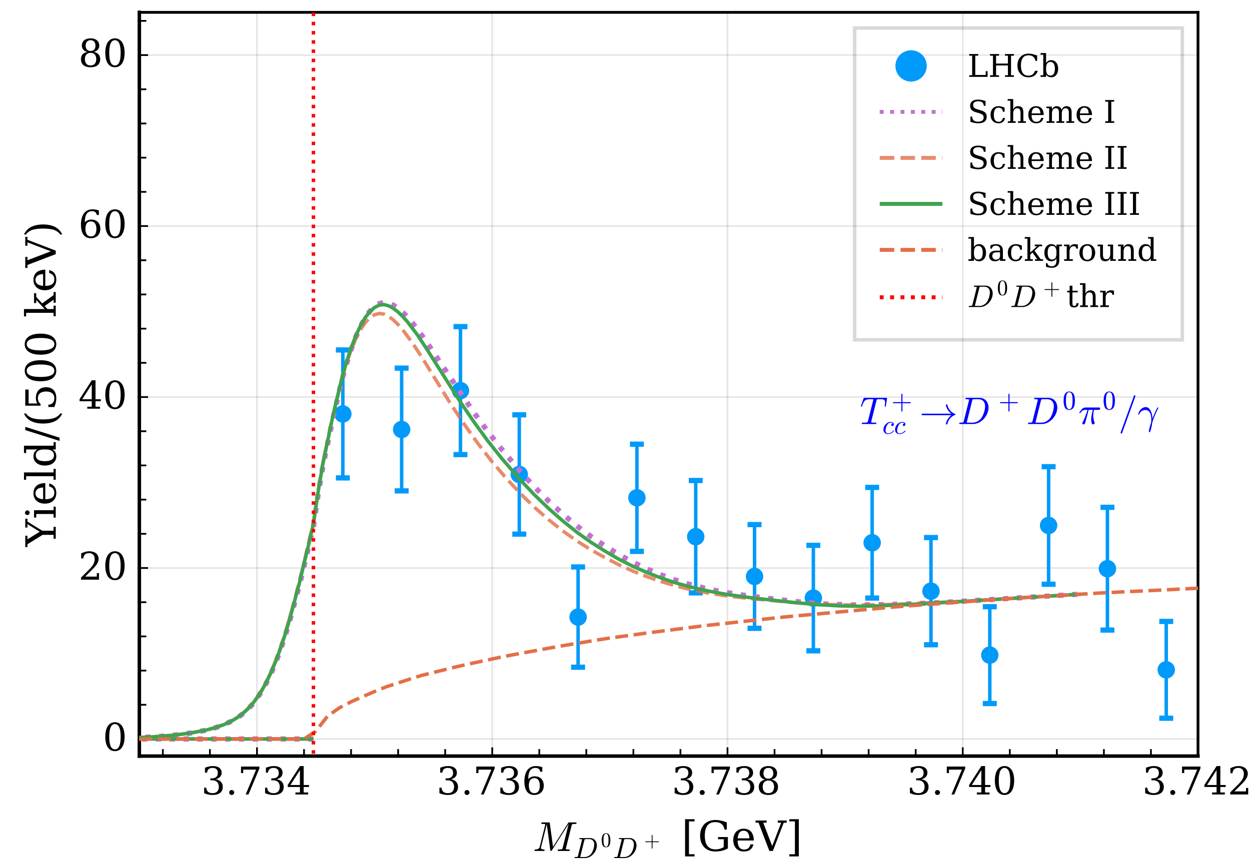
$$\Gamma_{T_{cc}}^{3-\text{body}} = 56 \pm 2\,\text{keV} \xrightarrow[\text{OPE}]{\text{remove}} 36\text{ keV} \xrightarrow[\text{dynam.width}]{\text{remove}} 74\text{ keV}$$

App I: Various predictions

D^0D^0 and D^0D^+ spectra

with resolution





Heavy quark spin partners

see also Albaladejo PLB 829 (2022) in contact EFT

$$V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+)$$

$$\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^* = -503(40)\ \text{keV}$$

⇒ (quasi)bound D*D* state ~ 0.5 MeV below the threshold

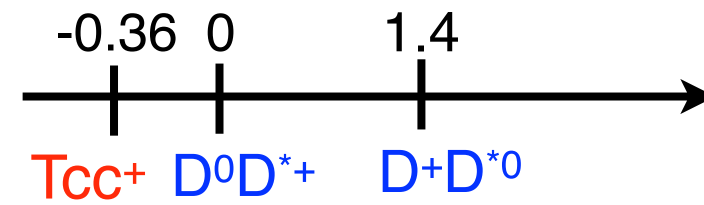
M. Du, VB, X. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRD* 105, 014024(2022)

Low-energy parameters

Du et al. PRD 105, 014024 (2022)

Scattering amplitude in the 1st (close to the pole) channel :

$$T_{D^{*+}D^0 \to D^{*+}D^0}(k) = -\frac{2\pi}{\mu_{c0}}\left(\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}(k^4)\right)^{-1}$$



$$r_0' = r_0 - \Delta r$$

Eff. range in the 1st channel

$$\Delta r = -\sqrt{\frac{\mu_2}{2\mu_1^2 \delta_2}} \simeq -3.8 \text{ fm}$$

Negative “correction” from 2nd $D^{*0}D^+$ channel caused by isospin breaking δ_2

$$\delta_2 = m_{\text{thr}2} - m_{\text{thr}1}$$

VB et al., PLB 833 (2022)

a_0 [fm]	r_0 [fm]	r_0' [fm]	X_A
$\left(-6.72^{+0.36}_{-0.45} \pm 0.27\right) - i\left(0.10^{+0.03}_{-0.03} \pm 0.03\right)$	$-2.40 \pm 0.01 \pm 0.85$	$1.38 \pm 0.01 \pm 0.85$	$0.84 \pm 0.01 \pm 0.06$

$r_0' \ll |a_0|$

– r_0' positive and is of natural size

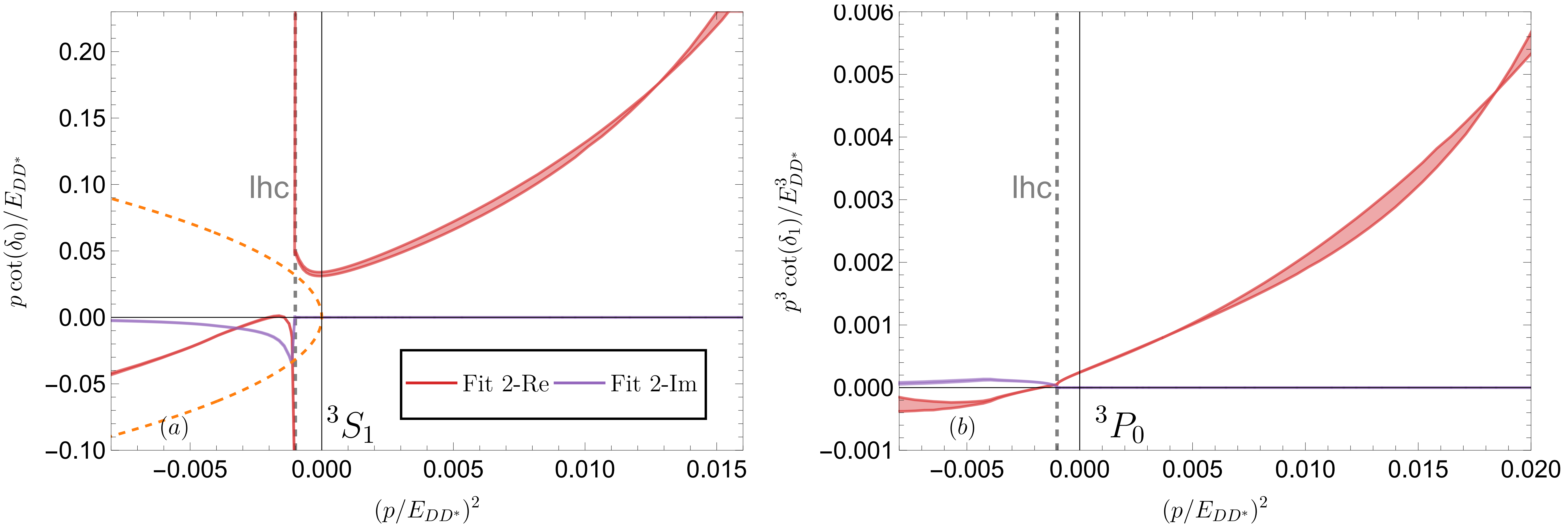
– Contrib. to r_0' from OPE is ~ 0.4 fm

T_{cc} is consistent with a pure isoscalar molecule!

App II: Residual cutoff dependence

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

- Cutoff variation from 0.7 to 1.2 GeV



$\Rightarrow$ very small

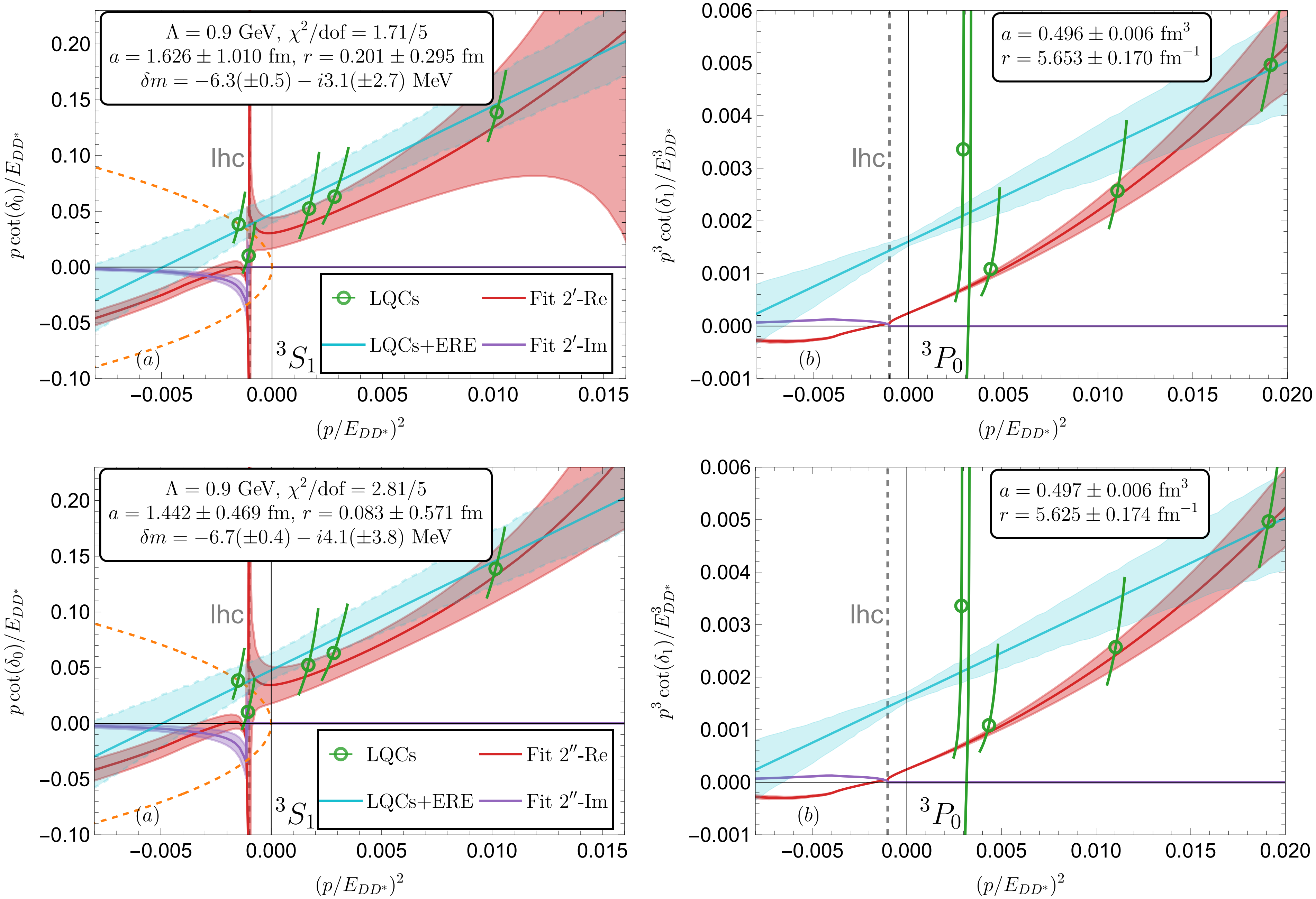
Testing chiral truncation uncertainty at $m_\pi = 280$ MeV

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

- Additional contact terms

$$V^{(2)}_{\text{cont}}[{}^3S_1 - {}^3D_1] = C^{(2)}_{SD}\, p'^2$$

$$V^{(2)}_{\text{cont}}[{}^3P_2] = C^{(2)}_{{}^3P_2}$$



⇒ Some effect of the S-D term on phase shifts at larger momenta

⇒ The impact near the threshold and on the pole is minor

The pion coupling from fits to data

Meng, VB, Filin, Epelbaum and Gasparyan *PRD letter* 109, L071506 (2024)

Two parameter fits to lattice + physical value:

- Linear extrapolation:

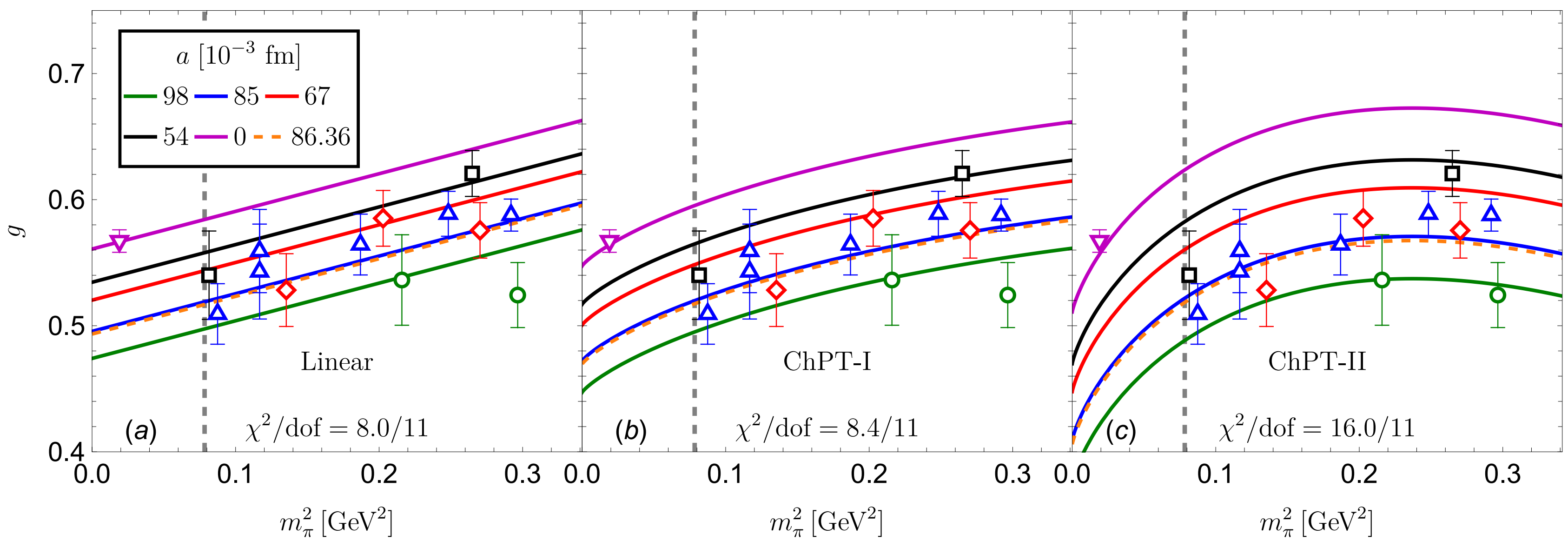
$$g(a, m_\pi) = g_0(1 + \alpha m_\pi^2 + \beta a^2)\,,$$

- ChPT-I:

$$g(a, m_\pi) = g_0\left(1 - \frac{2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$

- ChPT-II:

$$g(a, m_\pi) = g_0\left(1 - \frac{1 + 2g_0^2}{(4\pi f_0)^2} m_\pi^2 \ln m_\pi^2 + \alpha m_\pi^2 + \beta a^2\right)$$



Lattice data: Becirevic and Sanfilippo Phys. Lett. B 721, 94 (2013)

	g_0	α [GeV^{-2}]	β [fm^{-2}]	g
Linear	0.561(9)	0.53(13)	−16.1(44)	0.517(15)
ChPT-I	0.547(8)	0.24(14)	−19.1(45)	0.517(15)
ChPT-II	0.511(8)	−0.59(15)	−27.6(48)	0.519(15)

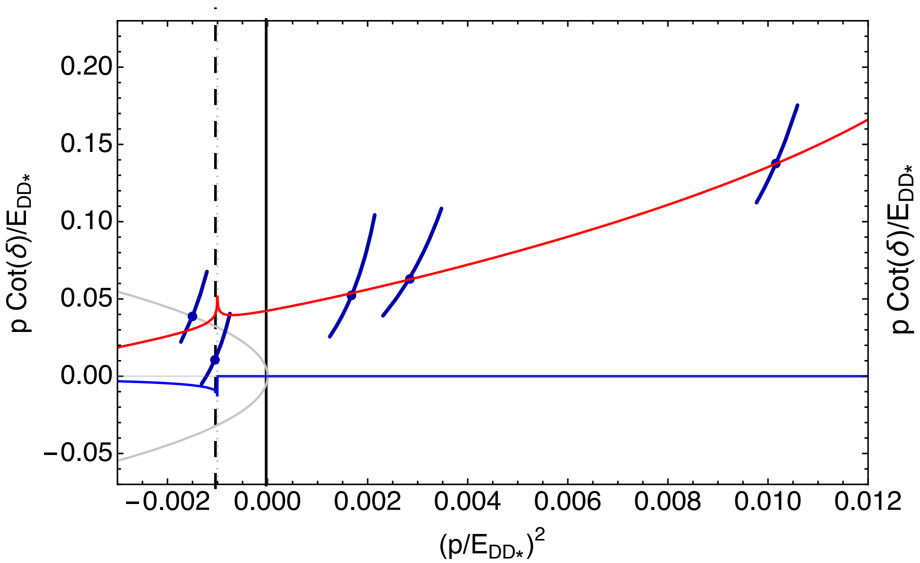
$g \equiv g(0.08636, 0.280)$

Dependence on the pion coupling

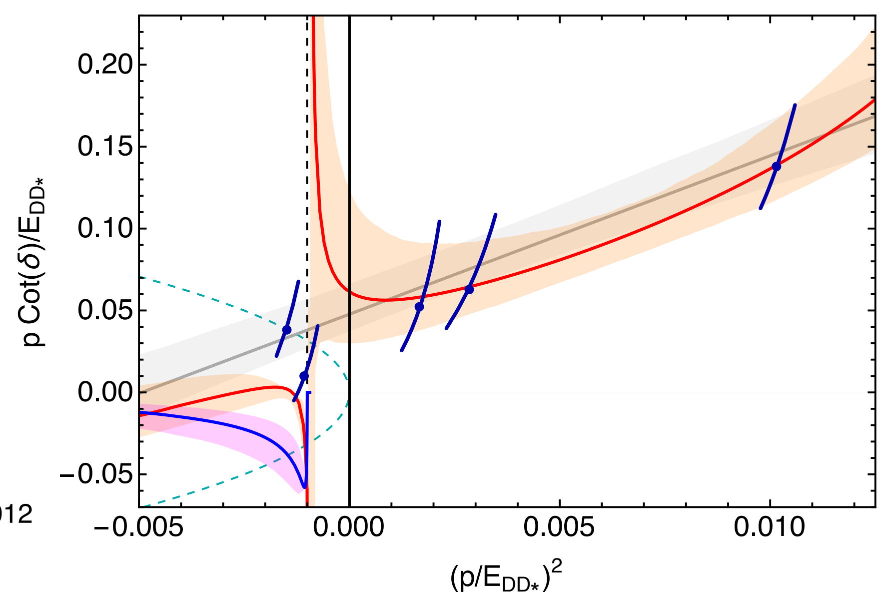
M. Du, A. Filin, VB, X. Dong, E. Epelbaum, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves and Q. Wang *PRL* 131, 131903 (2023)

- Importance of lhc is controlled by its position and strength (residue)

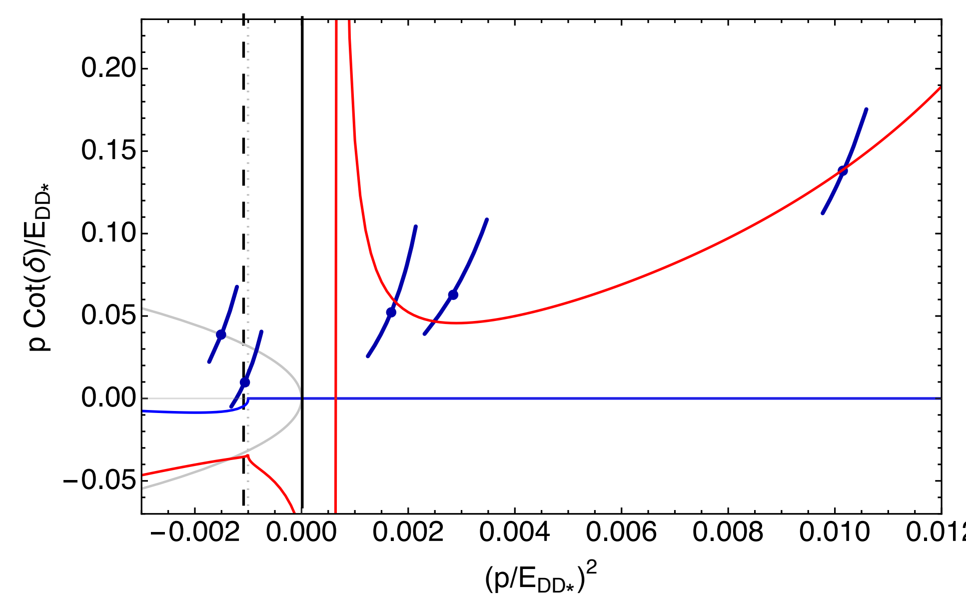
$\frac{1}{10} V^{\mathrm{OPE}}_{\mathrm{DD}^* \to \mathrm{DD}^*}(m_\pi = 280\mathrm{MeV})$



$V^{\mathrm{OPE}}_{\mathrm{DD}^* \to \mathrm{DD}^*}(m_\pi = 280\mathrm{MeV})$



$10\ V^{\mathrm{OPE}}_{\mathrm{DD}^* \to \mathrm{DD}^*}(m_\pi = 280\mathrm{MeV})$



- The smaller the coupling the closer the fit is to the ERE

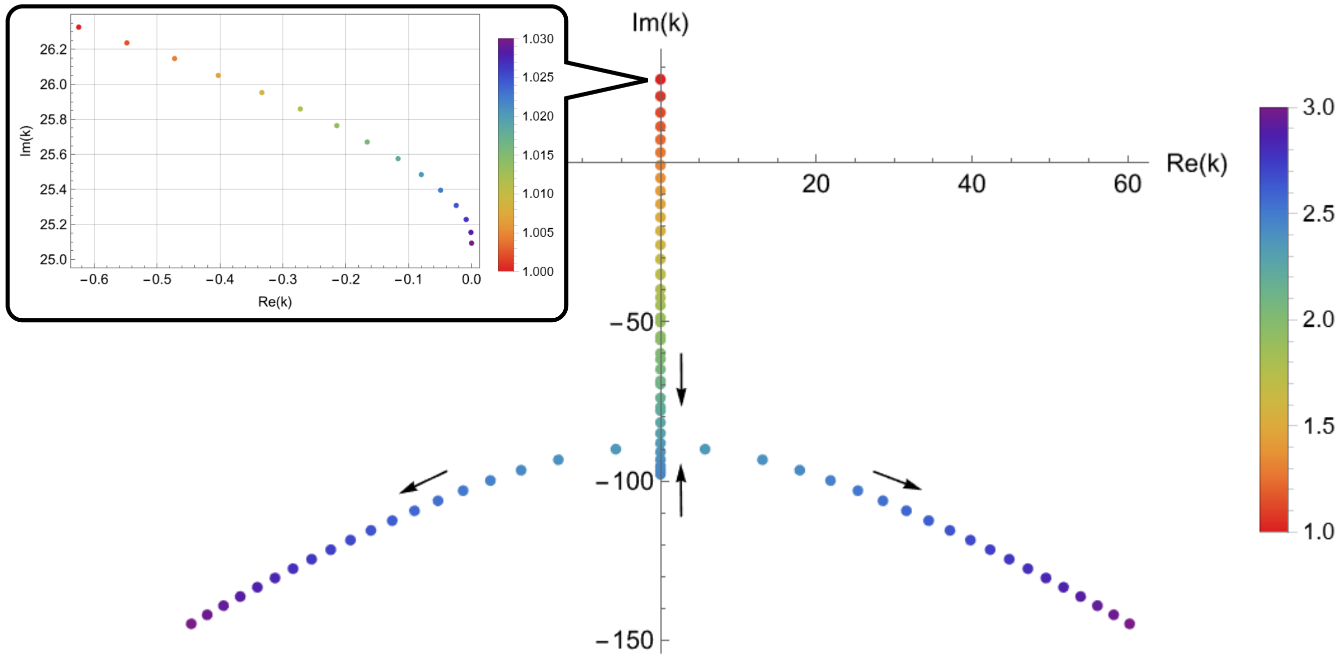
App III: Pion-mass dependence of the Tcc pole at LO

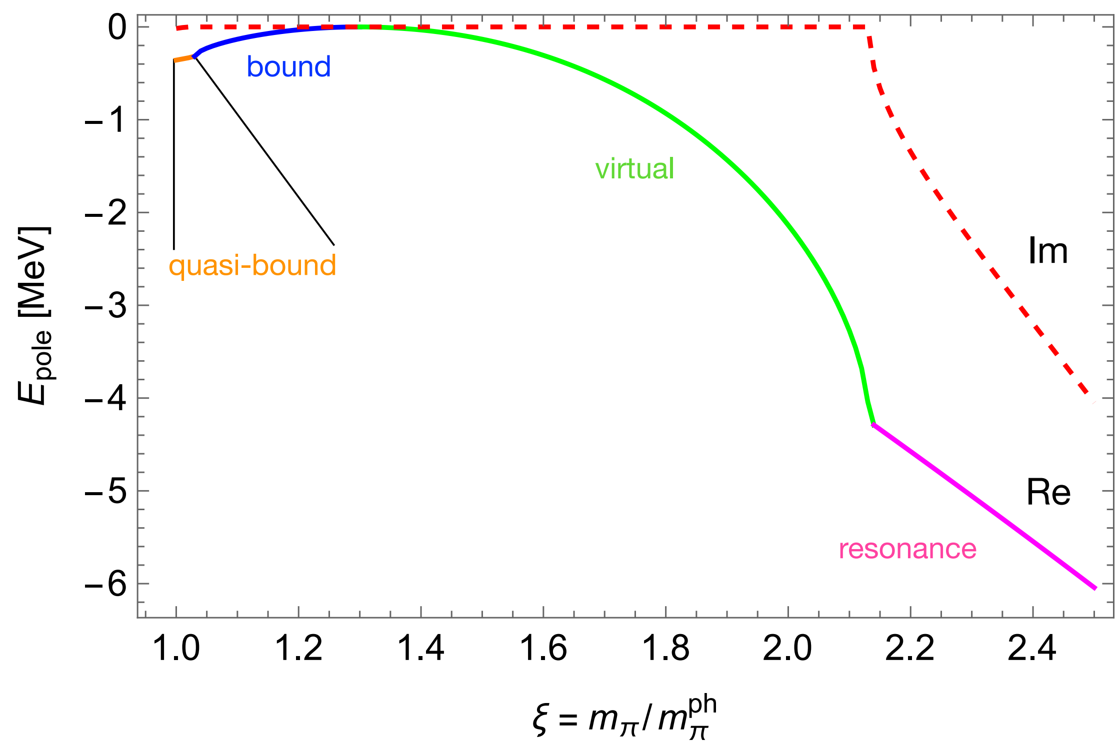
M. Abolnikov, VB, E. Epelbaum, A. Filin, C. Hanhart and L. Meng 2407.04649 [hep-ph]

LO results

$$V = V_{\rm OPE}^{(0)} + C_{^3S_1}^{(0)}$$

Fixed from the Tcc pole at physical $m_\pi^{\rm ph}$





Tcc pole transitions: quasi-bound → bound → virtual → resonance

Consistent with hadronic molecule

I. Matuschek, VB, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A 57, 101 (2021)