

# Spin density matrix elements in polarized photoproduction of resonances

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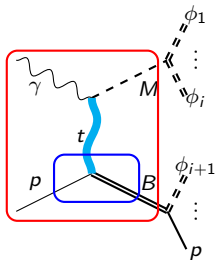
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## Goal: Understanding the photoproduction of light hybrid meson

- $\pi_1(1600)$  is a  $5\pi$  resonance  
( $\pi_1 \rightarrow b_1\pi \rightarrow (\omega\pi)\pi \rightarrow ((3\pi)\pi)\pi$ )
- $\pi_1(1600)$  production cross section is larger when produced with  $\Delta$  compared to proton [1].
- Single  $\pi$ -photoproduction is well understood (Gloria's talk, Tue)
- $\pi\Delta$  and  $b_1\Delta$  production
  - $\pi$  exchange process is to be understood (gauge invariance).
  - What is the physics behind the photoproduction of  $\pi(b_1)\Delta$ ?
  - Understand the lower vertex and absorption processes.
  - To what extent do the two vertices influence the SDMEs of the resonances? ( $\Delta$ -SDMEs from  $\pi\Delta$  vs  $b_1\Delta$ )
- Available observables:  $\frac{d\sigma}{dt}$ ,  $\Sigma$ ,  $\rho^{0,1,2}$ .
- SDMEs can be used to model the production amplitude.



The  $\vec{\gamma}p \rightarrow \pi\Delta \rightarrow \pi(\rho\pi)$  amplitude is given by,

$$A_{\lambda_\gamma, \lambda_1, \lambda_2}(\Omega) = \sum_{\lambda_\Delta} T_{\lambda_\gamma, \lambda_1, \lambda_\Delta}(s, t) D_{\lambda_\Delta, \lambda_2}^{3/2*}(\Omega) \quad (1)$$

The intensity is given by,

$$I(\Omega, \Phi, P_\gamma) = \frac{\kappa}{2} \sum_{\lambda_\gamma^{(\prime)}, \lambda_1, \lambda_2} A_{\lambda_\gamma, \lambda_1, \lambda_2}(\Omega) \hat{\rho}_{\lambda_\gamma, \lambda_\gamma'} A_{\lambda_\gamma', \lambda_1, \lambda_2}^*(\Omega) \quad (2)$$

where  $\kappa$  is the phase space factor.  $\Omega = (\theta, \phi)$  are the  $\Delta$ -decay angles.

$$I(\Omega, \Phi) = 2N \left\{ \rho_{33}^0 \sin^2 \theta + \rho_{11}^0 \left( \frac{1}{3} + \cos^2 \theta \right) - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{31}^0 \sin 2\theta \cos \phi - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{3-1}^0 \sin^2 \theta \cos 2\phi \right. \\ \left. - P_\gamma \cos 2\Phi \left[ \rho_{33}^1 \sin^2 \theta + \rho_{11}^1 \left( \frac{1}{3} + \cos^2 \theta \right) - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{31}^1 \sin 2\theta \cos \phi - \frac{2}{\sqrt{3}} \operatorname{Re} \rho_{3-1}^1 \sin^2 \theta \cos 2\phi \right] \right. \\ \left. - P_\gamma \sin 2\Phi \left[ \frac{2}{\sqrt{3}} \operatorname{Im} \rho_{31}^2 \sin 2\theta \sin \phi + \frac{2}{\sqrt{3}} \operatorname{Im} \rho_{3-1}^2 \sin^2 \theta \sin 2\phi \right] \right\}. \quad (3)$$

where the SDMEs are defined as,

$$\rho_{\lambda_\Delta \lambda_\Delta'}^0 = \frac{1}{2N} \sum_{\lambda_\gamma \lambda_1} T_{\lambda_\gamma \lambda_1 \lambda_\Delta} T_{\lambda_\gamma \lambda_1 \lambda_\Delta'}^*, \quad \rho_{\lambda_\Delta \lambda_\Delta'}^1 = \frac{1}{2N} \sum_{\lambda_\gamma \lambda_1} T_{\lambda_\gamma \lambda_1 \lambda_\Delta} T_{-\lambda_\gamma \lambda_1 \lambda_\Delta'}^* \quad (4)$$

$$\rho_{\lambda_\Delta \lambda_\Delta'}^2 = \frac{-i}{2N} \sum_{\lambda_\gamma \lambda_1} \lambda_\gamma T_{\lambda_\gamma \lambda_1 \lambda_\Delta} T_{-\lambda_\gamma \lambda_1 \lambda_\Delta'}^* \quad (5)$$

## Large- $s$ scattering:

- Large- $s \rightarrow$  exponential fall-off
- Residual (polynomial) behavior [2]

$$\frac{d\sigma}{dt} = \beta^R(t) s^{2\alpha_{\text{eff}} - 2} \quad (6)$$

- Residual interaction can be modeled using Regge theory.
- Two distinct trajectories (small- $t$  and large- $t$ ).
- Similar cross-section as single pion photoproduction at large- $t$

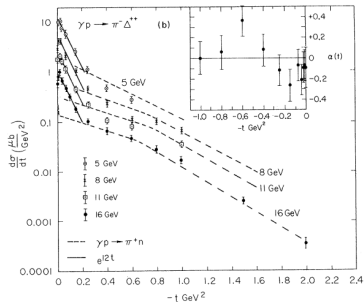


Figure:  $\pi\Delta$  photoproduction cross section as reported in Ref. [3].

GlueX operates at  $E_\gamma \sim 9$  GeV

- Simple model; exchange of  $\pi$ ,  $\rho$ ,  $b_1$ ,  $a_2$
- Upper and lower vertices factorize
- The phases of the amplitudes are fixed by the Regge theory.
- Poor man's absorption (PMA) model for  $\pi$ -exchange [4].

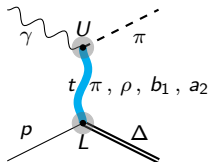


Figure: The  $t$ -channel photoproduction process of  $\pi\Delta$ .  $U$  and  $L$  are the upper and lower vertices.

General form of the helicity amplitude is [5]

$$T_{\lambda_\gamma, \lambda_1, \lambda_\Delta}(s, t) = \sum_x \left[ \xi_{\lambda_\gamma, \lambda_1, \lambda_\Delta} T_{\lambda_\gamma, \lambda_1, \lambda_\Delta}^x(s, t) \right]; \quad x \in \{\pi, \rho, b_1, a_2\} \quad (7)$$

$$T_{\lambda_\gamma, \lambda_1, \lambda_\Delta}^x(s, t) = \sqrt{-t}^{|\lambda_\gamma|} \sqrt{-t}^{|\lambda_1 - \lambda_\Delta|} \hat{\beta}_{\lambda_\gamma}^{x, U}(t) \hat{\beta}_{\lambda_1, \lambda_\Delta}^{x, L}(t) \mathcal{P}_R^x(s, t) S_x(t) \quad (8)$$

- Like exchanges take the same form of the vertex (exchange degeneracy):

$$\hat{\beta}_{\lambda_1, \lambda_\Delta}^{\pi, L}(t) = \hat{\beta}_{\lambda_1, \lambda_\Delta}^{b_1, L}(t); \quad \hat{\beta}_{\lambda_1, \lambda_\Delta}^{\rho, L}(t) = \hat{\beta}_{\lambda_1, \lambda_\Delta}^{a_2, L}(t)$$

- Overall (exponential + polynomial) suppression factors for each exchange (also removes wrong signature poles)
- Residues are polynomials in  $t$ , coupling constants fitted+fixed.
- (Irving & Worden model [2] has constant residues, all fixed phenomenologically.)

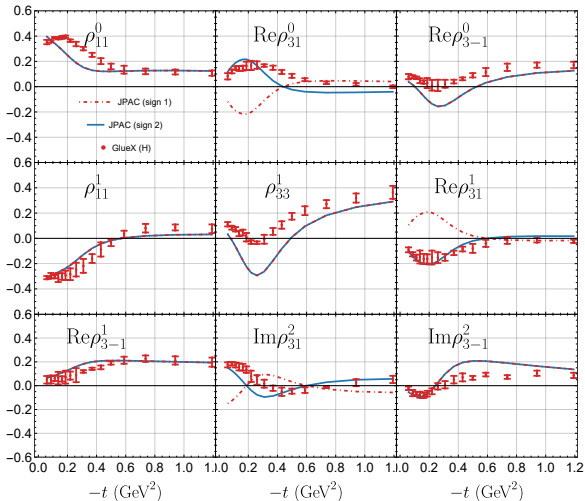


Figure: SDMEs in the helicity frame compared to the GlueX data [8].

Sign 1:  $\beta_{\frac{1}{2}, \frac{3}{2}}^{\times, L}$  and  $\beta_{\frac{1}{2}, -\frac{1}{2}}^{\times, L}$  as given in [5] Sign 2: Signs of  $\beta_{\frac{1}{2}, \frac{3}{2}}^{\times, L}$  and  $\beta_{\frac{1}{2}, -\frac{1}{2}}^{\times, L}$  flipped

## Reflectivity basis

- Reflectivity operation involves  $180^\circ$  rotation about the “y-axis” + parity inversion  $\Rightarrow$  inversion of the “y-axis” [6, 7].
- The amplitude in the reflectivity basis can be defined as (valid for  $\gamma p \rightarrow \pi \Delta$ ):

$$T_{\lambda_1, \lambda_\Delta}^{(\epsilon)}(s, t) = \frac{1}{2} (T_{1, \lambda_1, \lambda_\Delta}(s, t) + \epsilon T_{-1, \lambda_1, \lambda_\Delta}(s, t)) \quad (9)$$

- The SDMEs take the form,

$$\rho_{\lambda_\Delta \lambda'_\Delta}^0 = \frac{1}{N} \sum_{\lambda_1} \left[ T_{\lambda_1, \lambda_\Delta}^{(+)} T_{\lambda_1, \lambda'_\Delta}^{(+)*} + T_{\lambda_1, \lambda_\Delta}^{(-)} T_{\lambda_1, \lambda'_\Delta}^{(-)*} \right] \quad (10)$$

$$\rho_{\lambda_\Delta \lambda'_\Delta}^1 = \frac{1}{N} \sum_{\lambda_1} \left[ T_{\lambda_1, \lambda_\Delta}^{(+)} T_{\lambda_1, \lambda'_\Delta}^{(+)*} - T_{\lambda_1, \lambda_\Delta}^{(-)} T_{\lambda_1, \lambda'_\Delta}^{(-)*} \right] \quad (11)$$

- The  $\epsilon = (-)+$  amplitudes are dominated by (un)natural parity meson exchange.

Natural and Unnatural amplitudes

$$N_{-1} = T_{\frac{1}{2} \frac{3}{2}}^{(+)} \quad N_0 = T_{\frac{1}{2} \frac{1}{2}}^{(+)} \quad N_1 = T_{\frac{1}{2} -\frac{1}{2}}^{(+)} \quad N_2 = T_{\frac{1}{2} -\frac{3}{2}}^{(+)} \quad (12)$$

$$U_{-1} = T_{\frac{1}{2} \frac{3}{2}}^{(-)} \quad U_0 = T_{\frac{1}{2} \frac{1}{2}}^{(-)} \quad U_1 = T_{\frac{1}{2} -\frac{1}{2}}^{(-)} \quad U_2 = T_{\frac{1}{2} -\frac{3}{2}}^{(-)} \quad (13)$$

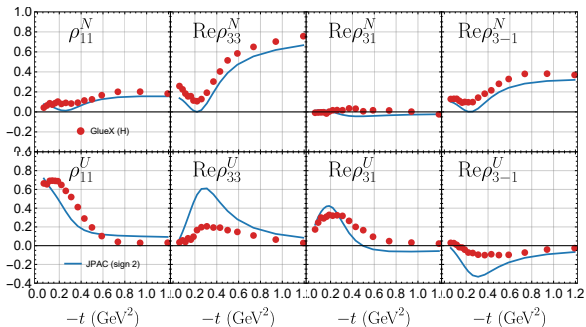


Figure: Natural and unnatural combinations of SDMEs in the helicity frame.

The non-zero value of  $\rho_{33}^N$  at small- $t$  indicates the presence of absorption corrections.



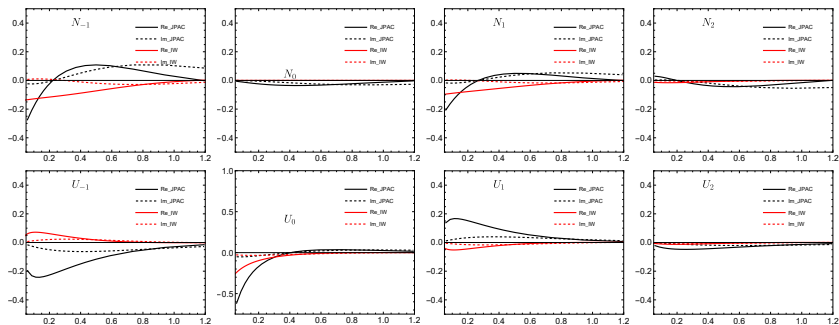
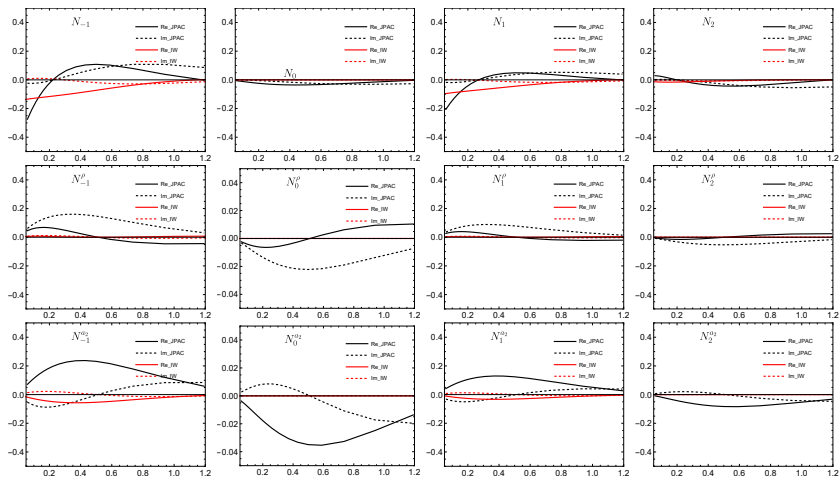


Figure: Natural and Unnatural amplitudes from the JPAC model compared with IWM.



**Figure:** The natural exchange amplitudes and their  $\rho$  and  $a_2$  components from the JPAC model compared with IWM.

- A simplistic model that assumes Regge exchange and factorization
- Explains the general features the SDMEs of  $\pi\Delta$  photoproduction; needs fine tuning to match the data.
- Diagonal elements of  $\rho^{0,1}$  can be interpreted as sum/difference of production probabilities
- $\Delta$  is produced dominantly in the helicity  $\pm 1/2$  ( $\pm 3/2$ ) configuration at small- $t$  (large- $t$ )
- (Some of the) relative phases of helicity amplitudes can be fixed from SDMEs
- All amplitudes except the natural spin-nonflip amplitude experience absorption corrections
- Absorption is evident in the SDMEs and the BSA
- $\pi$ -exchange dominates small- $t$  region;  $a_2$  exchange dominates the large- $t$  region.
- Work in progress!

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Thank you!



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An Upper Limit on the Photoproduction Cross Section of the Spin-Exotic  $\pi_1(1600)$ .  
7 2024.



A. C. Irving and R. P. Worden.

Regge Phenomenology.  
*Phys. Rept.*, 34:117–231, 1977.



A. Boyarski, Robert E. Diebold, Stanley D. Ecklund, G. E. Fischer, Y. Murata, Burton Richter, and W. S. C. Williams.

PHOTOPRODUCTION OF  $\pi^- \Delta^{++}$  (1236) FROM HYDROGEN FROM 5-GeV TO 16-GeV.  
*Phys. Rev. Lett.*, 22:148–151, 1969.



P. K. Williams.

Extrapolation model for  $\pi\pi$  scattering.  
*Phys. Rev. D*, 1:1312–1318, 1970.



J. Nys, V. Mathieu, C. Fernández-Ramírez, A. Jackura, M. Mikhasenko, A. Pilloni, N. Sherrill, J. Ryckebusch, A. P. Szczepaniak, and G. Fox.

Features of  $\pi\Delta$  Photoproduction at High Energies.  
*Phys. Lett. B*, 779:77–81, 2018.



S. U. Chung and T. L. Trueman.

Positivity Conditions on the Spin Density Matrix: A Simple Parametrization.  
*Phys. Rev. D*, 11:633, 1975.



V. Mathieu, M. Albaladejo, C. Fernández-Ramírez, A. W. Jackura, M. Mikhasenko, A. Pilloni, and A. P. Szczepaniak.

Moments of angular distribution and beam asymmetries in  $\eta\pi^0$  photoproduction at GlueX.  
*Phys. Rev. D*, 100(5):054017, 2019.



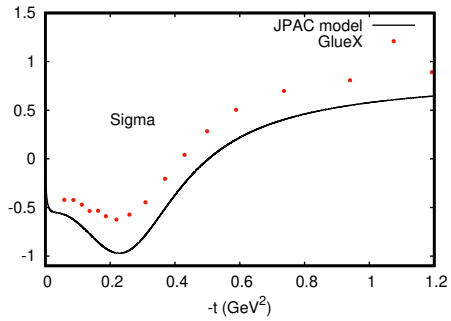
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Measurement of Spin-Density Matrix Elements in  $\Delta^{++}(1232)$  photoproduction.  
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Back up

$\hat{\rho}_{\mu_i \mu_f}^{e,if}$	Expression
$\hat{\beta}_{+1}^{\pi, \gamma \pi}(t)$	$\sqrt{2}e$
$\hat{\beta}_{+1}^{\rho, \gamma \pi}(t)$	$\frac{g_{\rho \pi \gamma}}{2m_\rho}$
$\hat{\beta}_{+1}^{b_1, \gamma \pi}(t)$	$\frac{g_{b_1 \pi \gamma}}{2m_{b_1}}$
$\hat{\beta}_{+1}^{a_2, \gamma \pi}(t)$	$\frac{g_{a_2 \pi \gamma}}{2m_{a_2}^2}$
$\hat{\beta}_{+\frac{1}{2}+\frac{3}{2}}^{\pi, N\Delta}(t)$	$\frac{g_{\pi N\Delta}(m_N+m_\Delta)}{\sqrt{2}m_\Delta}$
$\hat{\beta}_{-\frac{1}{2}+\frac{1}{2}}^{\pi, N\Delta}(t)$	$\frac{g_{\pi N\Delta}(-m_N^2+m_N m_\Delta+2m_\Delta^2+t)}{\sqrt{6}m_\Delta^2}$
$\hat{\beta}_{+\frac{1}{2}+\frac{1}{2}}^{\pi, N\Delta}(t)$	$\frac{-g_{\pi N\Delta}(-m_N^3-m_N^2 m_\Delta+m_\Delta^3+2m_\Delta t+m_N(m_\Delta^2+t))}{\sqrt{6}m_\Delta^2}$
$\hat{\beta}_{-\frac{1}{2}+\frac{3}{2}}^{\pi, N\Delta}(t)$	$\frac{-g_{\pi N\Delta}}{\sqrt{2}m_\Delta}$
$\hat{\beta}_{+\frac{1}{2}+\frac{3}{2}}^{\rho, N\Delta}(t)$	$\frac{-(2m_\Delta g_{\rho N\Delta}^{(1)}+g_{\rho N\Delta}^{(2)}(m_N-m_\Delta))}{2m_\Delta^2}$
$\hat{\beta}_{-\frac{1}{2}+\frac{1}{2}}^{\rho, N\Delta}(t)$	$\frac{-(2m_N m_\Delta g_{\rho N\Delta}^{(1)}+g_{\rho N\Delta}^{(2)}(-m_N m_\Delta+m_\Delta^2+2t)+2t g_{\rho N\Delta}^{(3)})}{2\sqrt{3}m_\Delta^3}$
$\hat{\beta}_{+\frac{1}{2}+\frac{1}{2}}^{\rho, N\Delta}(t)$	$\frac{-(2m_\Delta g_{\rho N\Delta}^{(1)}+g_{\rho N\Delta}^{(2)}(2m_N-3m_\Delta)+2g_{\rho N\Delta}^{(3)}(m_N-m_\Delta))}{2\sqrt{3}m_\Delta^3} (-t)$
$\hat{\beta}_{-\frac{1}{2}+\frac{3}{2}}^{\rho, N\Delta}(t)$	$\frac{g_{\rho N\Delta}^{(2)}}{2m_\Delta^2}$

Residues from the JPAC model.



BSA from the JPAC model compared to the GlueX data [8].



$$\rho_{\frac{1}{2}\frac{1}{2}}^0 + \rho_{\frac{1}{2}\frac{1}{2}}^1 = \frac{2}{N} (|N_0|^2 + |N_1|^2) \quad \text{Re} \left( \rho_{\frac{3}{2}\frac{1}{2}}^0 + \rho_{\frac{3}{2}\frac{1}{2}}^1 \right) = \frac{2}{N} \text{Re} (N_{-1}N_0^* - N_1N_2^*) \quad (14)$$

$$\rho_{\frac{1}{2}\frac{1}{2}}^0 - \rho_{\frac{1}{2}\frac{1}{2}}^1 = \frac{2}{N} (|U_0|^2 + |U_1|^2) \quad \text{Re} \left( \rho_{\frac{3}{2}\frac{1}{2}}^0 - \rho_{\frac{3}{2}\frac{1}{2}}^1 \right) = \frac{2}{N} \text{Re} (U_{-1}U_0^* - U_1U_2^*) \quad (15)$$

$$\rho_{\frac{3}{2}\frac{3}{2}}^0 + \rho_{\frac{3}{2}\frac{3}{2}}^1 = \frac{2}{N} (|N_{-1}|^2 + |N_2|^2) \quad \text{Re} \left( \rho_{\frac{3}{2}-\frac{1}{2}}^0 + \rho_{\frac{3}{2}-\frac{1}{2}}^1 \right) = \frac{2}{N} \text{Re} (N_0N_2^* + N_1N_{-1}^*) \quad (16)$$

$$\rho_{\frac{3}{2}\frac{3}{2}}^0 - \rho_{\frac{3}{2}\frac{3}{2}}^1 = \frac{2}{N} (|U_{-1}|^2 + |U_2|^2) \quad \text{Re} \left( \rho_{\frac{3}{2}-\frac{1}{2}}^0 - \rho_{\frac{3}{2}-\frac{1}{2}}^1 \right) = \frac{2}{N} \text{Re} (U_0U_2^* + U_1U_{-1}^*) \quad (17)$$

$$\text{Im} \rho_{\frac{3}{2}\frac{1}{2}}^2 = \frac{1}{N} \text{Re} (N_{-1}U_0^* + N_2U_1^* - N_1U_2^* - N_0U_{-1}^*) \quad (18)$$

$$\text{Im} \rho_{\frac{3}{2}-\frac{1}{2}}^2 = \frac{1}{N} \text{Re} (N_{-1}U_1^* - N_2U_0^* - U_{-1}N_1^* + U_2N_0^*) \quad (19)$$

$$N = 2 (|N_{-1}|^2 + |N_0|^2 + |N_1|^2 + |N_2|^2 + |U_{-1}|^2 + |U_0|^2 + |U_1|^2 + |U_2|^2) \quad (20)$$

$$\Sigma = \frac{2}{2N} \sum_{\sigma=-1,0,1,2} (|N_\sigma|^2 - |U_\sigma|^2) \quad (21)$$

## Absorption

- Additional corrections to Regge exchange
- Experimental evidences in the form of
  - Non-zero polarization of  $\pi N \rightarrow \pi N$  scattering
  - Forward scattering cross section of  $\pi$ -photoproduction (Gloria's talk on Tue),  $np \rightarrow pn$ , etc
  - Peripherality of  $\pi$ -exchange reactions
  - **SDMEs and BSA of  $\pi\Delta$  photoproduction** (more later)
- "Effective" way of taking care of multiple exchanges (FSI, etc).

$$T_{\lambda\gamma, \lambda_1, \lambda_\Delta}(s, t) = \sum_{\times} \left[ \xi_{\lambda\gamma, \lambda_1, \lambda_\Delta} T_{\lambda\gamma, \lambda_1, \lambda_\Delta}^{\times}(s, t) \right] ; \quad \times \in \{\pi, \rho, b_1, a_2\} \quad (22)$$

$$T_{\lambda\gamma, \lambda_1, \lambda_\Delta}^{\times}(s, t) = \sqrt{-t}^{|\lambda\gamma|} \sqrt{-t}^{|\lambda_1 - \lambda_\Delta|} \hat{\beta}_{\lambda\gamma}^{\times, U}(t) \hat{\beta}_{\lambda_1, \lambda_\Delta}^{\times, L}(t) \mathcal{P}_R^{\times}(s, t) \mathcal{S}_{\times}(t) \quad (23)$$

- The  $\sqrt{-t}$  factors arise from angular momentum and the model
- The model factors for unnatural exchange are evaluated at  $t = m_\pi^2 \rightarrow$  PMA.

## Natural and Unnatural amplitudes

$$N_{-1} = T_{\frac{1}{2}-\frac{3}{2}}^{(+)} \quad N_0 = T_{\frac{1}{2}-\frac{1}{2}}^{(+)} \quad N_1 = T_{\frac{1}{2}-\frac{1}{2}}^{(+)} \quad N_2 = T_{\frac{1}{2}-\frac{3}{2}}^{(+)} \quad (24)$$

$$U_{-1} = T_{\frac{1}{2}-\frac{3}{2}}^{(-)} \quad U_0 = T_{\frac{1}{2}-\frac{1}{2}}^{(-)} \quad U_1 = T_{\frac{1}{2}-\frac{1}{2}}^{(-)} \quad U_2 = T_{\frac{1}{2}-\frac{3}{2}}^{(-)} \quad (25)$$

$\lambda_1 = -\frac{1}{2}$  amplitudes are related via parity.  
Eg:

$$N_0 = (\beta_1^\rho S_\rho \mathcal{P}_\rho - \beta_1^{a_2} S_{a_2} \mathcal{P}_{a_2}) \sqrt{-t} \beta_{\frac{1}{2}-\frac{1}{2}}^\rho \quad (26)$$

$$N_1 = (\beta_1^\rho S_\rho \mathcal{P}_\rho - \beta_1^{a_2} S_{a_2} \mathcal{P}_{a_2}) (-t) \beta_{\frac{1}{2}-\frac{1}{2}}^\rho \\ + \frac{1}{2} (-\beta_1^\pi S_\pi \mathcal{P}_\pi + \beta_1^{b_1} \sqrt{-t} S_{b_1} \mathcal{P}_{b_1}) (-m_\pi^2) \left(1 - \frac{t}{m_\pi^2}\right) \beta_{\frac{1}{2}-\frac{1}{2}}^\pi \quad (27)$$

$$U_0 = (-\beta_1^\pi S_\pi \mathcal{P}_\pi + \beta_1^{b_1} \sqrt{-t} S_{b_1} \mathcal{P}_{b_1}) \sqrt{-t} \beta_{\frac{1}{2}-\frac{1}{2}}^\pi \quad (28)$$

$$U_1 = \frac{1}{2} (-\beta_1^\pi S_\pi \mathcal{P}_\pi + \beta_1^{b_1} \sqrt{-t} S_{b_1} \mathcal{P}_{b_1}) (-m_\pi^2) \left(1 + \frac{t}{m_\pi^2}\right) \beta_{\frac{1}{2}-\frac{1}{2}}^\pi \quad (29)$$

- Absorption correction to natural exchanges – No pole contributions

- Negative reflectivity amplitudes are purely  $\pi$  and  $b_1$  exchanges
- Positive reflectivity amplitudes are  $\rho$  and  $a_2$  exchanges, and get absorption corrections from  $\pi$  and  $b_1$  exchanges.

The suppression factors are,

$$S_{\pi} = c_{\pi} e^{(b_U t)} (\alpha_U(t) + 2)/2 \quad (30)$$

$$S_{b_1} = c_{\pi} e^{(b_U t)} (\alpha_U(t) + 1) \quad (31)$$

$$S_{\rho} = e^{(b_N t)} (\alpha_N(t) + 1)/2 \quad (32)$$

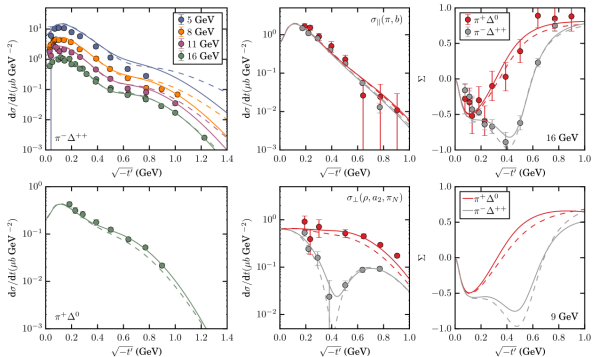
$$S_{a_2} = e^{(b_N t)} \alpha_N(t) (\alpha_N(t) + 2)/3, \quad (33)$$

## JPAC model [5]

- Residues are polynomials in  $t$
- Residues constructed phenomenologically using covariant Lagrangians (possible sign ambiguity)
- Upper vertex coupling constants fixed using decay widths
- Lower vertex is the same for all exchanges of a given naturality because of exchange degeneracy
- Lower vertex:  $\pi p \Delta$  coupling constant is fixed from the  $\Delta \rightarrow p \pi$  decay width,  $\rho p \Delta$  coupling constants fitted to the scattering data

## Irving & Worden Model (IWM) [2]

- Residues are constants
- All vertices fixed phenomenologically



Cross sections for  $\pi\Delta$  photoproduction from JPAC model compared to data (from [5]).