Improving $\pi\pi$ dispersive amplitude analyses and resonance determination with Forward Dispersion Relations

Pablo Rabán

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▶ 1 Motivation

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▶ 2 Improving $\pi\pi$ dispersive amplitudes

- * Global fit to data
- * Results (preliminary)

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- ▶ 3 Resonance determination

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2. Extract resonances produced in the interaction in a model-independent way



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✓ Constrained fit to data CFD(2011) up to 1.4 GeV R. Garcia-Martín et al., Phys.Rev. D83, 074004 (2011)

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✓ Model-independent resonance determination: pole positions and couplings for $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_2(1270)$ and $\rho(770)$

R. García-Martín et al., Phys. Rev. Lett. 107, 072001 (2011) J.R. Peláez et al., Phys. Rev. Lett. 130, 051902 (2019)

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× Improving the precision and imposing dispersive constraints up to higher energies allows us to study the well-known $\rho(1450)$

Our objectives are:

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- * Resolve the incompatibilities and decide among the different solutions above 1.4 GeV
- * Parameterization and model-independent analyses in order to extract resonances, without imposing them in our analysis

1. Global parameterization of $t_{\ell}^{I}(s)$ (partial-waves) through fit to data (improving matching with Regge, precision, η descriptions...)



1. Global parameterization of $t'_{\ell}(s)$ (partial-waves) through fit to data (improving matching with Regge, precision, η descriptions...)



* New global G-waves (relevant for precision above 1 GeV)



2. Reconstruct the amplitudes T'(s, 0) from the global partialwave parameterizations

$$T'(s,\theta) = 32\pi \sum_{\ell} (2\ell+1) P_{\ell}(\cos\theta) t'_{\ell}(s)$$

Convenient isospin basis for the Forward Dispersion Relations

$$T^{00} = \frac{1}{3}(T^0 + 2T^2), \ T^{0+} = \frac{1}{2}(T^1 + T^2), \ T'_t = \frac{1}{6}(2T^0 + 3T^1 - 5T^2)$$

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Example:



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3. Imposing the Forward and Roy-like dispersion relations through a constrained fit to data

$$\operatorname{Re}T^{00}(s,0) = T^{00}(4m_{\pi}^{2},0) + \frac{s(s-4m_{\pi}^{2})}{\pi} \int_{4m_{\pi}^{2}}^{\infty} \frac{(2s'-4m_{\pi}^{2})\operatorname{Im}T^{00}(s',0)}{s'(s'-s)(s'-4m_{\pi}^{2})(s'+s-4m_{\pi}^{2})} ds'$$

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4. Extract resonances from the FDRs



✓ Are the Forward Dispersion Relations fulfilled?



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 $\checkmark\,$ Roy and GKPY (once-subtracted Roy eqs) for S0, P and S2 partial-waves are also satisfied by our new global parameterizations



Inelasticity data description improved \checkmark





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But... What about the $\rho(1450)$?

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* Need of a robust and general analytic continuation: continued fractions

$$C_N(s) = a_0 / \left(1 + \frac{a_1(s-s_1)}{1 + \frac{a_2(s-s_2)}{\cdots \cdot a_{N-1}(s-s_{N-1})}} \right)$$

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* In order to determine a **pole resonance**, we look for poles for different N, and we only obtain the physical poles in a stable way!

3. Continued fractions (preliminary)

* **Stable** results for different N (e.g. for the $\rho(1450)$)



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Isoscalar resonances (F ⁰⁰)						
	Μ (MeV) Γ (MeV) β					
$f_0(500)$	460^{+9}_{-7}	534^{+16}_{-7}	3.28 ^{+0.22} _{-0.18} GeV			
$f_0(980)$	986 ⁺⁷ ₋₅	54^{+10}_{-16}	$1.9^{+0.5}_{-0.3}~{ m GeV}$			
$f_0(1370)$	1238^{+38}_{-31}	550^{+58}_{-116}	$9.2^{+1.9}_{-1.1}~{ m GeV}$			
$f_0(1500)$	1524 ± 30	84^{+38}_{-36}	5.3 ± 1.4 GeV			
$f_2(1270)$	1267.7 ± 1.1	$195.2^{+2.2}_{-1.8}$	$4.48^{+0.30}_{-0.31} \text{ GeV}^{-1}$			

Isovector resonances (F ⁰⁺)					
Μ (MeV) Γ (MeV) g					
ho(770)	$757.0^{+1.6}_{-1.0}$	$153.0^{+7.6}_{-0.8}$	$6.16\substack{+0.02\\-0.15}$		
ho(1450)	1461 ± 15	290^{+32}_{-38}	2.01 ± 0.45		
$ ho_3(1690)/ ho(1700)$	1675^{+44}_{-56}	248^{+56}_{-45}	XXX		

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- * No hints of $\rho(1250)$ (present in old PDG editions and recently claimed for the same data N. Hammoud et al., Phys.Rev. D102, 054029 (2020))

Summary

★ Relevance of our parameterizations

- Simple parameterizations of $\pi\pi$ interaction and their uncertainties, up to 1.8-2 GeV
- ▶ Consistent with data and dispersion relations up to 1.6 GeV
- Being used in different experiments and works

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★ Results (preliminary)

- lmproved $\pi\pi$ amplitudes (inelasticities, Regge matching, precision,...)
- Avoid models and parameterizations for resonance extraction
- ▶ Isoscalar resonances: $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ y $f_2(1270)$
- ▶ Isovector resonances: $\rho(770)$, $\rho(1450)$ and $\rho_3(1690)/\rho(1700)$
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★ To Do

- ▶ Improve the constraints fulfillment and double check the results
- Final results for Solutions II and III (better results for Solution I so far)

Thank you for your attention

