

# Understanding pion exchange in meson photoproduction from a Regge theory perspective

Glòria Montaña

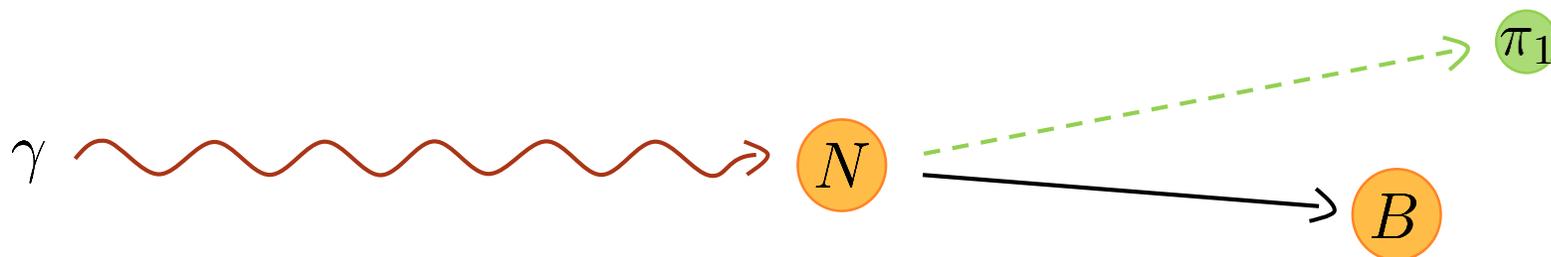
Theory Center, Thomas Jefferson National Accelerator Facility



QNP2024 – The 10<sup>th</sup> International Conference on Quarks and Nuclear Physics  
Facultat de Biologia, Universitat de Barcelona  
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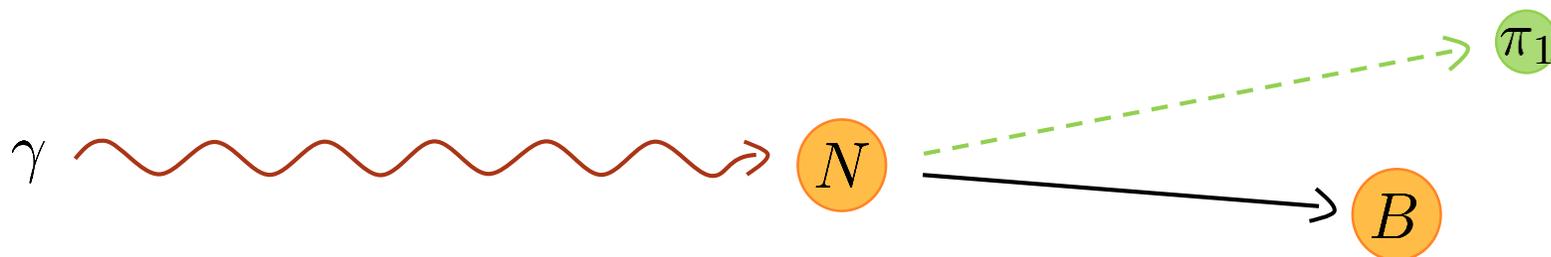
## Search for exotic hybrid mesons in photoproduction with GlueX at JLab

- Peripheral meson photoproduction:



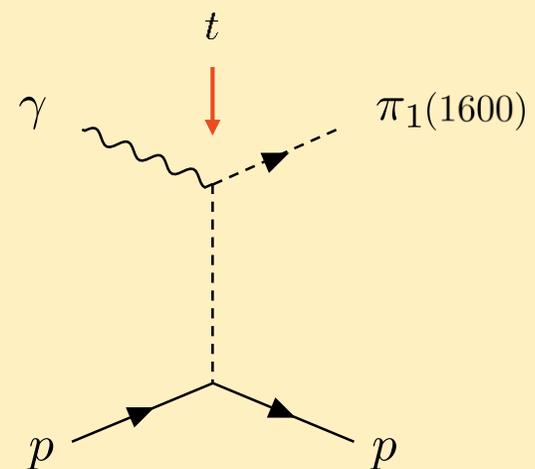
## Search for exotic hybrid mesons in photoproduction with GlueX at JLab

- Peripheral meson photoproduction:



### Production mechanism:

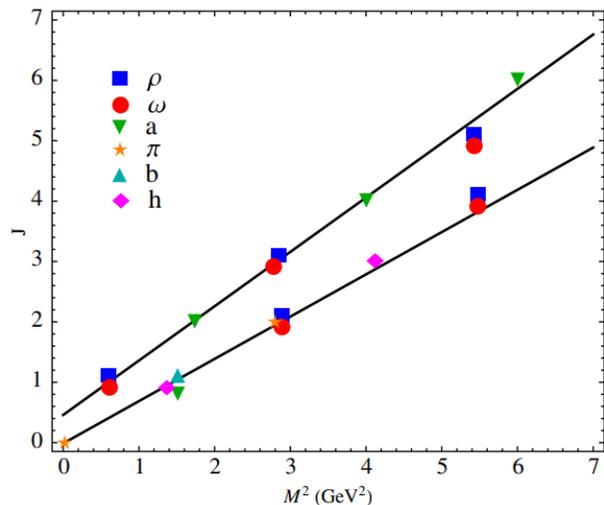
exchange of Regge trajectories in the  $t$ -channel



## Regge trajectories

- Families with same quantum numbers but different spin  $J$  (even or odd parity).
- Almost straight lines (Chew-Frautschi plot)
- In standard Regge theory parameterized by:

$$\alpha(t) = \alpha' t + \alpha_0$$



[V.Mathieu et al., *Phys.Rev.D* 98 (2018) 1, 014041]

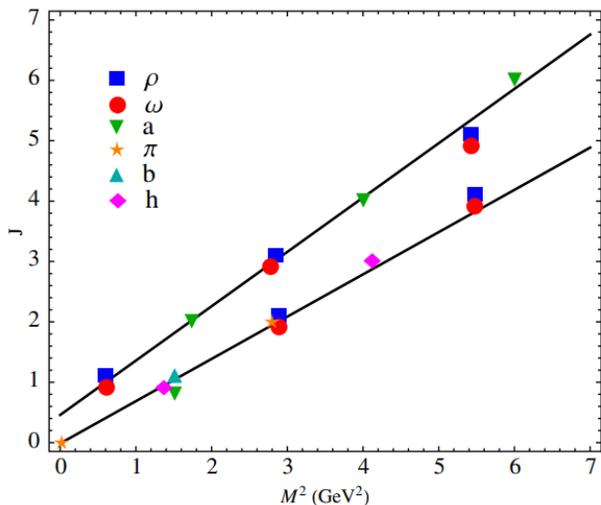
$$\alpha_{\pi}(t) = 0.7(t - m_{\pi}^2) = 0.7t - 0.014$$

$$\alpha_{\rho}(t) = 0.9(t - m_{\rho}^2) + 1 = 0.9t + 0.466$$

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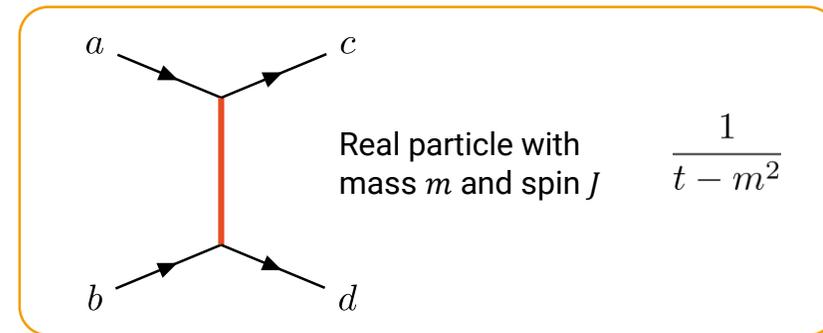
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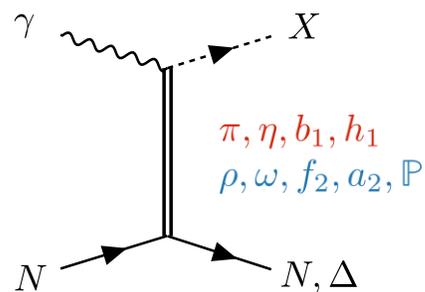
$$\mathcal{P}_{\text{Regge}} = \frac{\pi\alpha'}{\sin(\pi\alpha(t))} \frac{1 + \eta e^{-i\pi\alpha(t)}}{2} \left(\frac{s}{s_0}\right)^{\alpha(t)} \frac{1}{\Gamma(1 + \alpha(t))}$$

Annotations for the formula:

- $\frac{\pi\alpha'}{\sin(\pi\alpha(t))}$ : poles for integer  $\alpha(t)$
- $\frac{1 + \eta e^{-i\pi\alpha(t)}}{2}$ : signature factor
- $\left(\frac{s}{s_0}\right)^{\alpha(t)}$ : asymptotic behavior
- $\frac{1}{\Gamma(1 + \alpha(t))}$ : cancel non-physical poles

# Polarized photoproduction at high energies

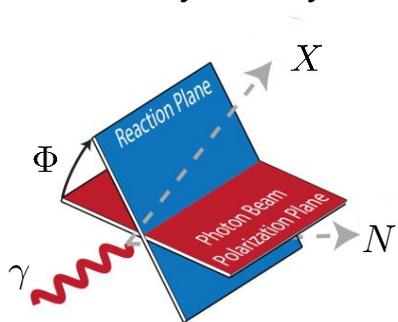
- Linear photon beam polarization used to filter out the “naturality” of exchanged particle.



→ **Unnatural** ( $P(-1)^J = -1$ ) parity:  $0^-, 1^+, 2^-, 3^+, \dots$

→ **Natural** ( $P(-1)^J = 1$ ) parity:  $0^+, 1^-, 2^+, 3^-, \dots$

- Beam asymmetry

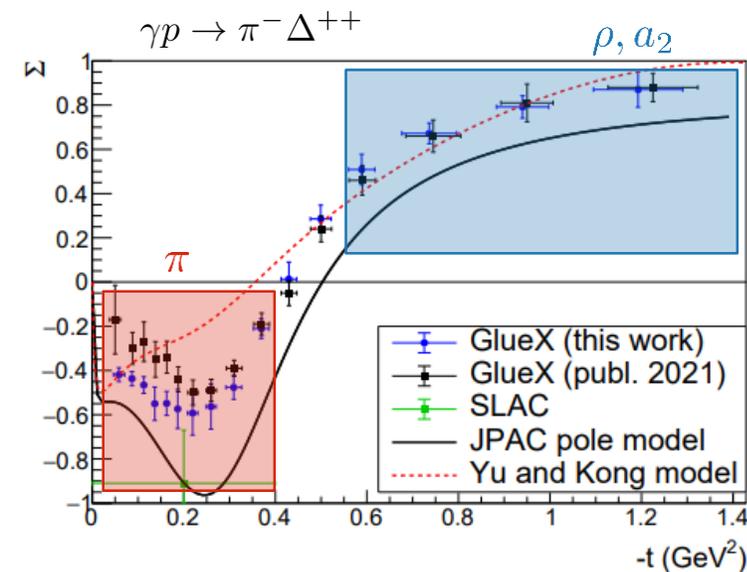


$$\Sigma = \frac{d\sigma_{\perp} - d\sigma_{\parallel}}{d\sigma_{\perp} + d\sigma_{\parallel}} \approx \frac{d\sigma_N - d\sigma_U}{d\sigma_N + d\sigma_U}$$

$\Phi = \frac{\pi}{2}$        $\Phi = 0$

Unnatural exchange
Natural exchange

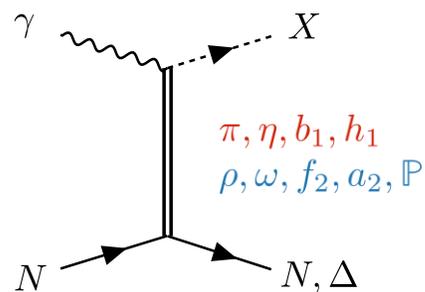
[V.Mathieu et al., *Phys.Rev.D* 92 (2015) 7]



[GlueX Collaboration, arXiv:2406.12829]  
 JPAC Model: [Nys et al. PLB 779, 77 (2018)]

# Polarized photoproduction at high energies

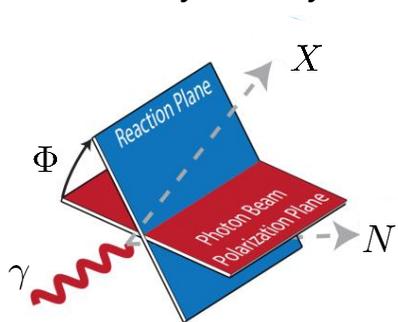
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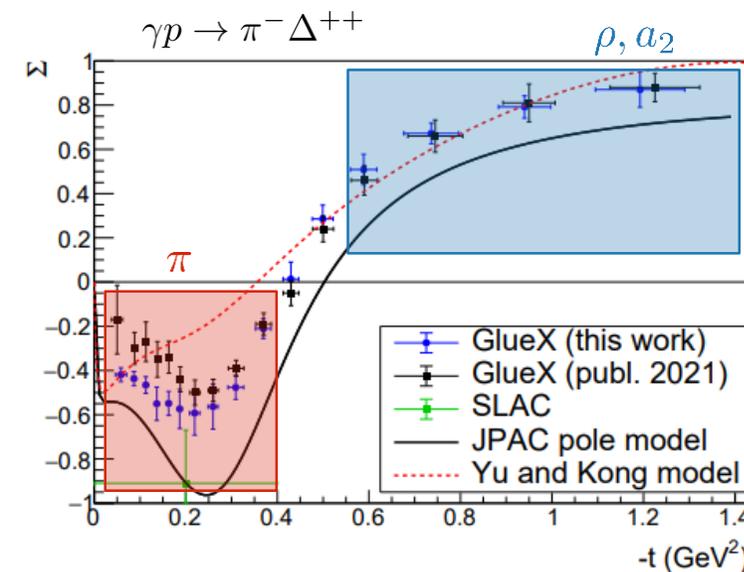
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$$\Sigma = \frac{d\sigma_{\perp} - d\sigma_{\parallel}}{d\sigma_{\perp} + d\sigma_{\parallel}} \approx \frac{d\sigma_N - d\sigma_U}{d\sigma_N + d\sigma_U}$$

$\Phi = \frac{\pi}{2}$        $\Phi = 0$       Unnatural exchange      Natural exchange

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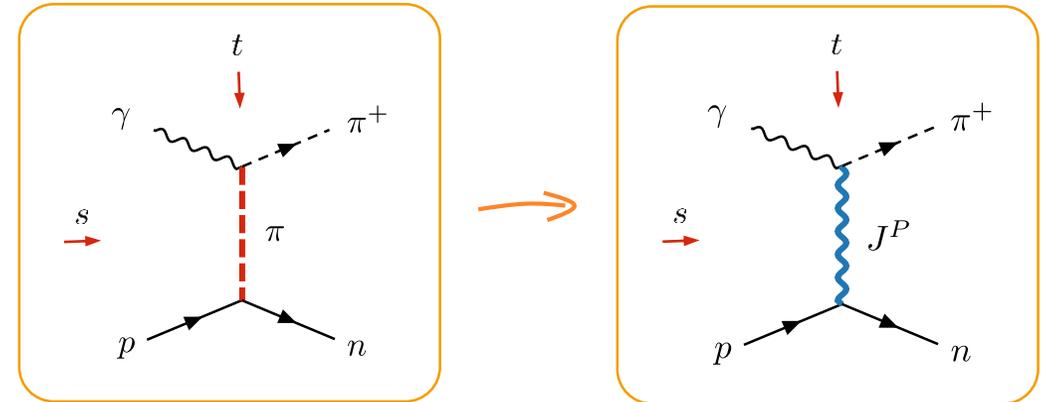
→ **Focus of this talk: pion exchange mechanism in pion photoproduction**

## Reggeization of pion exchange

- The exchanged pion is expected to reggeize.
- In the Regge-pole approximation:

$$\frac{1}{t - m_\pi^2} \longrightarrow \mathcal{P}_\pi^{\text{Regge}} = \frac{\pi \alpha'_\pi}{\sin \pi \alpha_\pi(t)} \frac{1 + e^{-i\pi \alpha_\pi(t)}}{2} \frac{1}{\Gamma(\alpha_\pi(t) + 1)} \left(\frac{s}{s_0}\right)^{\alpha_\pi(t)}$$

Pion trajectory:  $\alpha_\pi(t) = \alpha'_\pi(t - m_\pi^2)$  with  $\alpha'_\pi = 0.7$

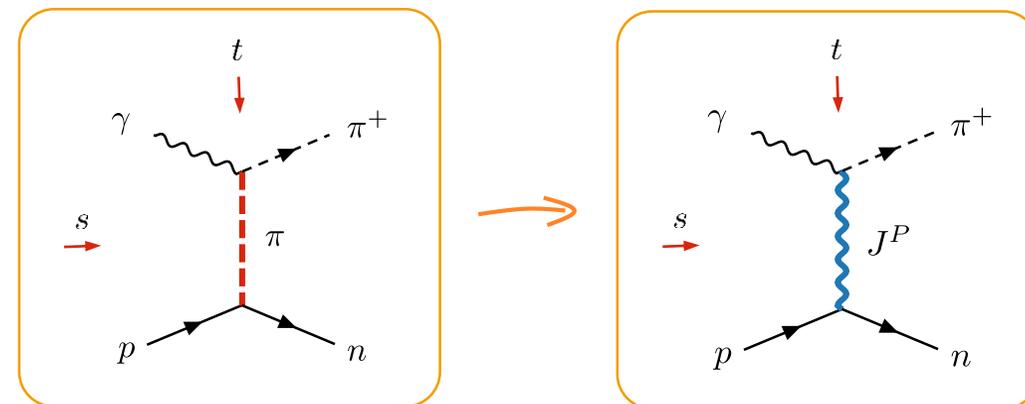


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Our approach:

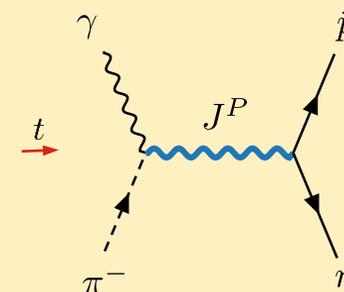
- Explicit exchange of particles with spin  $J$  in the crossed channel ( $t$ -channel).

$$\gamma(k) + \bar{\pi}(-p_\pi) \rightarrow \bar{N}(-p_i) + N(p_f)$$

- Sum the tower of exchanges.

$$A_{\lambda_\gamma \lambda_i \lambda_f}(s, t) = \sum_J (2J + 1) a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) d_{\lambda_\gamma \lambda_i - \lambda_f}^J(\theta_t)$$

- Analytical continuation to  $s$ -channel physical region.

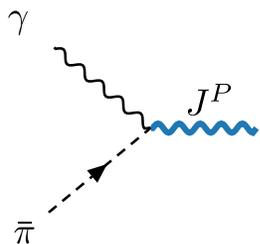


$$J = 0, 2, 4, \dots$$

$$P = -1$$

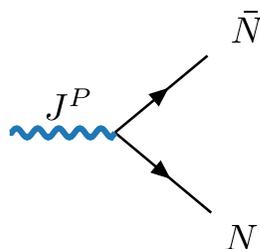
## 1.1 Exchange of arbitrary spin $J^P = (\text{even})^-$

Build amplitudes for the exchange of arbitrary spin:



$$1^- \otimes 0^- = 1^+ \quad \left\{ \begin{array}{ll} L = 1 & J = 0 \\ L = \{J - 1, J + 1\} & J \geq 2 \end{array} \right\} \quad \text{one } L \text{ vs two } L\text{'s}$$

$$V_{\lambda_\gamma}^J(M) = 2\sqrt{2}g_{\gamma\pi} \left[ k^{\nu_1} \dots k^{\nu_J} \epsilon_\mu(k, \lambda_\gamma) p_\pi^\mu - (k \cdot p_\pi) k^{\nu_1} \dots k^{\nu_{J-1}} \epsilon^{\nu_J}(k, \lambda_\gamma) \right] \epsilon_{\nu_1 \dots \nu_J}^*(M) \rightarrow \text{Gauge invariant by construction}$$

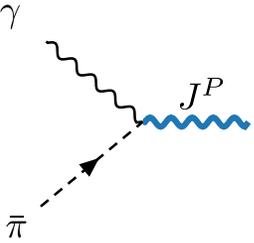


$$\frac{1}{2}^+ \otimes \frac{1}{2}^- = 0^- \oplus 1^- \rightarrow L = J$$

$$V_{\lambda_i \lambda_f}^J(M) = g_{N\bar{N}} P^{\nu_1} \dots P^{\nu_J} \epsilon_{\nu_1 \dots \nu_J}(M) \bar{u}(p_f, \lambda_f) \gamma_5 v(-p_i, \lambda_i) \quad (P^\nu = p_i^\nu + p_f^\nu)$$

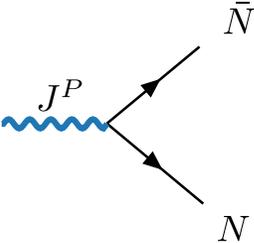
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$V_{\lambda_i \lambda_f}^J(M) = g_{N\bar{N}} P^{\nu_1} \dots P^{\nu_J} \epsilon_{\nu_1 \dots \nu_J}(M) \bar{u}(p_f, \lambda_f) \gamma_5 v(-p_i, \lambda_i) \quad (P^\nu = p_i^\nu + p_f^\nu)$

$A_{\lambda_\gamma \lambda_i \lambda_f}^J(s, t) = \sum_M \frac{V_{\lambda_\gamma}(J, M) V_{\lambda_i \lambda_f}(J, M)}{J - \alpha_\pi(t)} = a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) d_{\lambda_\gamma \lambda_i - \lambda_f}^J(\theta_t)$

↓ pole in  $J$ 
↓ spin -  $J$  amplitude ( $J > 0$ ):
 $J > 0$

$a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) \equiv \frac{2e_\pi g_J t}{J - \alpha_\pi(t)} (2\lambda_i \delta_{\lambda_i \lambda_f}) c_J^2 \sqrt{\frac{J+1}{J}} (-2p_t k_t)^J$

$(e_\pi g_J = g_{\gamma\pi} g_{N\bar{N}})$

## 1.2 Analytical continuation to $J = 0$

To include the pion, we must extend definition of the spin- $J$  amplitude to  $J = 0$ . The general expression diverges:

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- Wigner d-functions can be expressed in terms of Jacobi polynomials:

$$d_{\lambda_\gamma 0}^J(\theta_t) = \sqrt{\frac{J+1}{2J}} d_{\lambda_\gamma 0}^1(\theta_t) P_{J-1}^{11}(z_t)$$

- We impose definite signature:  $P_n^{ab}(-z) = (-1)^n P_n^{ba}(z)$  ,  $P_{J-1}^{11}(z_t) \rightarrow \frac{1}{2}(P_{J-1}^{11}(z_t) - P_{J-1}^{11}(-z_t))$
- Analytical continuation using the hyperbolic function:  $P_{J-1}^{11}(z_t) = J {}_2\tilde{F}_1\left(1 - J, J + 2; 2; \frac{1 - z_t}{2}\right)$

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The contribution to the amplitude from  $J = 0$  is finite!

$$\begin{aligned} A_{\lambda_\gamma \lambda_i \lambda_f}^{J=0}(s, t) &= a_{\lambda_\gamma \lambda_i \lambda_f}^{J \rightarrow 0}(t) d_{\lambda_\gamma \lambda_i - \lambda_f}^{J \rightarrow 0}(\theta_t) = \frac{e_\pi g_{J \rightarrow 0} t}{\alpha_\pi(t)} (2\lambda_i \lambda_\gamma \delta_{\lambda_i \lambda_f}) \sqrt{1-z_t^2} \frac{1}{2J} [P_{J-1}^{11}(z_t) - P_{J-1}^{11}(-z_t)] \Big|_{J=0} \\ &= \frac{2e_\pi g_0 t}{-\alpha_\pi(t)} (2\lambda_i \lambda_\gamma \delta_{\lambda_i \lambda_f}) \frac{z_t}{\sqrt{1-z_t^2}} \\ &\approx -i2e_\pi g_{\pi NN} \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) \frac{t}{m_\pi^2 - t} \end{aligned}$$

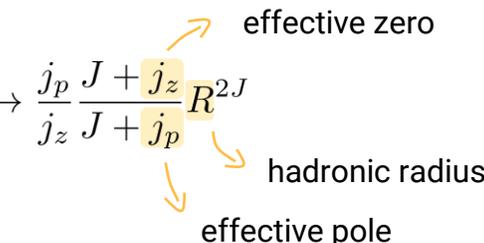


This term projects back into all partial waves!

$$(g_{J \rightarrow 0} = \alpha' g_{\pi NN})$$

## 2. Spin summation

$$\begin{aligned}
 A_{\lambda_\gamma \lambda_i \lambda_f}^{\text{Regge } \pi}(s, t) &= \sum_{J=0,2,\dots} (2J+1) a_{\lambda_\gamma \lambda_i \lambda_f}^J(t) d_{\lambda_\gamma, \lambda_i - \lambda_f}^J(\theta_t) \\
 &= -2e_\pi \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) t \sin \theta_t \sum_{J=0,1,2,\dots} (-2)^J g_J c_J^2 \frac{(2J+1)(J+1)}{2J(J-\alpha_\pi(t))} (p_t k_t)^J \frac{1}{2} \left[ P_{J-1}^{|\lambda_\gamma - \lambda'|, |\lambda_\gamma + \lambda'|}(z_t) - P_{J-1}^{|\lambda_\gamma + \lambda'|, |\lambda_\gamma - \lambda'|}(-z_t) \right]
 \end{aligned}$$

- For high spins, the coupling reflects the internal structure of the hadronic radii:  $g_J = \alpha' g_{\pi NN} h_J(r_t r_b)^J$
- The kinematical factor has alternating poles and zeros for  $J < 0$ . We approximate:  $c_J^2 h_J(2r_t r_b)^J \rightarrow \frac{j_p}{j_z} \frac{J + j_z}{J + j_p} R^{2J}$ 

- Use the generating function of the Jacobi polynomials to perform the spin summation.

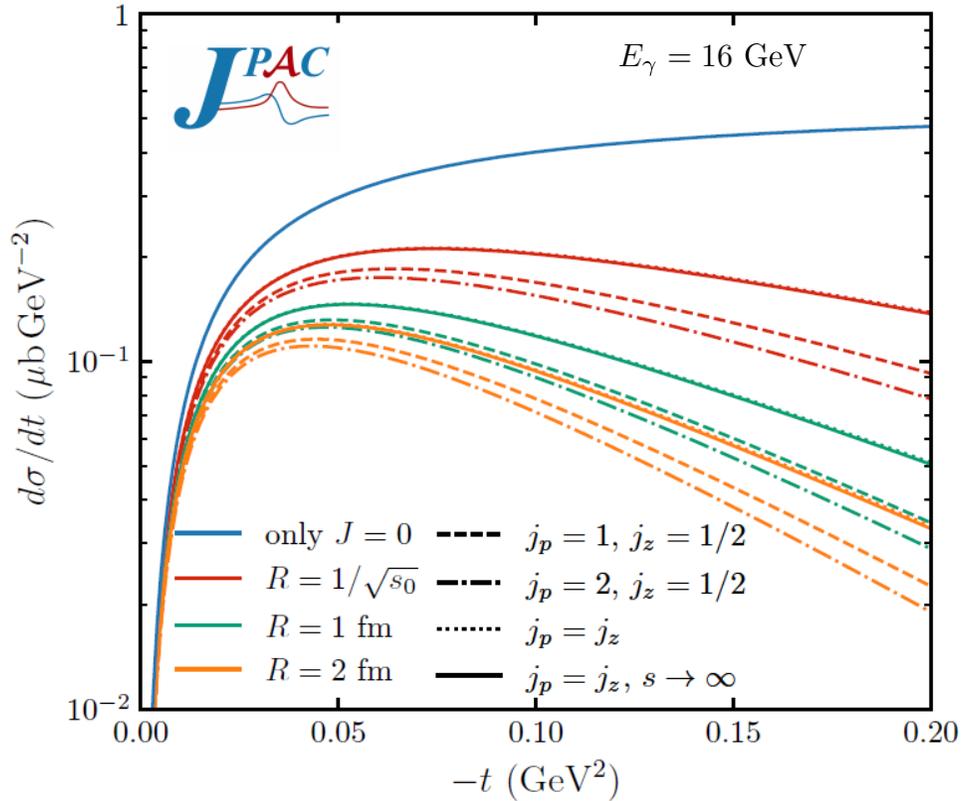
**Option 1:** solve numerically an integral equation with parameters  $R, j_p, j_z$ .

**Option 2:** consider the dominant contribution of the Regge pole, in the limit of large  $s$ .

## Results

- We define  $A_{\lambda_\gamma \lambda_i \lambda_f}^{\text{Regge } \pi}(s, t) = -i2e_\pi g_{\pi NN} \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) t \times \mathcal{A}(s, t)$

[G.Montana et al. (in preparation)]



Only  $J = 0$  contribution:  $\mathcal{A}^{J=0} = \mathcal{P}_\pi = -\frac{\alpha'}{\alpha(t)} = \frac{1}{m_\pi^2 - t}$ .

Option 1:

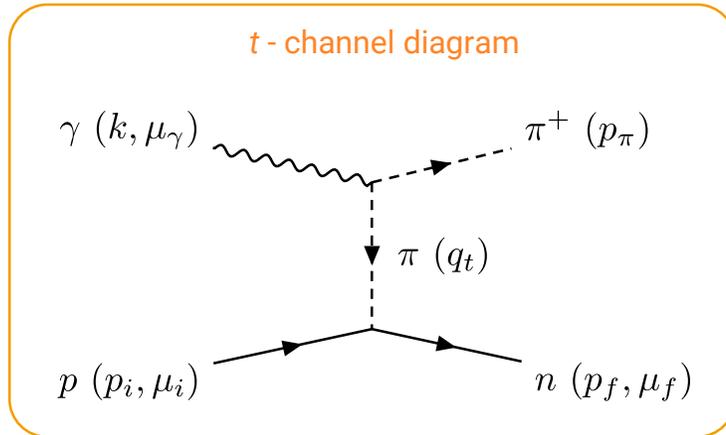
$$\begin{aligned} \mathcal{A}_{\pi}^{j_p, j_z}(s, t) = & \mathcal{P}_\pi - \left\{ \frac{i\alpha' \kappa \sin \theta_t}{2} \int_0^1 dy \left[ \frac{1}{-\alpha} + y^{-\alpha} \frac{j_p(\alpha + j_z)(\alpha + 1)(2\alpha + 1)}{\alpha j_z(\alpha + j_p)} \right. \right. \\ & \left. \left. + y^{j_p} \frac{(j_z - j_p)(1 - j_p)(1 - 2j_p)}{j_z(j_p + \alpha)} \right] \right. \\ & \times 2 \left[ \frac{1}{G(z_t, y)} \frac{1}{(1 + G(z_t, y))^2 - (\kappa y)^2} - (z_t \rightarrow -z_t) \right] \\ & \left. + \frac{4j_p}{j_z} \left[ \frac{1}{G(z_t, \kappa)} \frac{1}{1 + G(z_t, \kappa))^2 - \kappa^2} - (z_t \rightarrow -z_t) \right] \right\} \end{aligned}$$

Option 2:

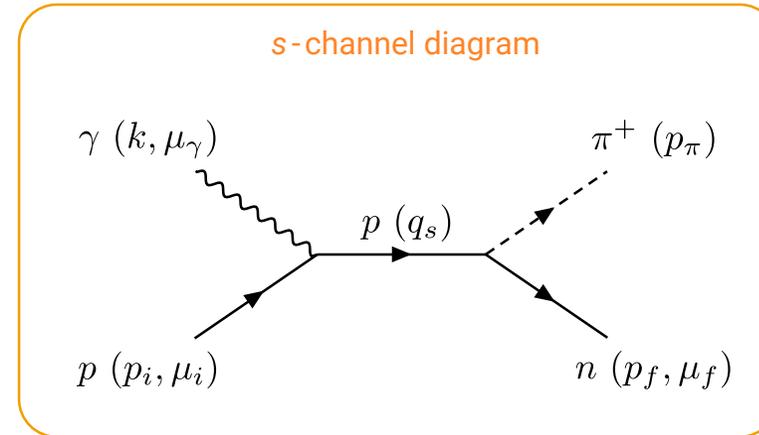
$$\mathcal{A}^{s \rightarrow \infty}(s, t) = \mathcal{P}_\pi^{s \rightarrow \infty} = -\frac{\alpha' 2\sqrt{\pi} \Gamma(\alpha + 3/2)}{\sin \pi \alpha \Gamma(\alpha + 1)} \frac{1 + e^{-i\pi\alpha}}{2} (sR^2)^\alpha$$

vs  $\mathcal{P}_\pi^{\text{Regge}} = -\frac{\pi\alpha'}{\sin \pi\alpha} \frac{1}{\Gamma(\alpha + 1)} \frac{1 + e^{-i\pi\alpha}}{2} \left(\frac{s}{s_0}\right)^\alpha$  ( $s_0 = 1 \text{ GeV}^2$ )

# Born diagrams



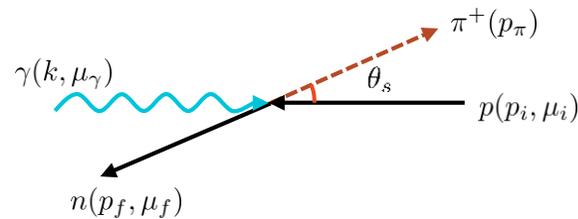
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- Separate electric and magnetic contributions:

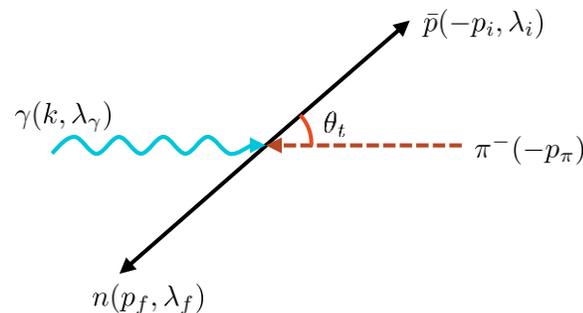
**s - channel CM frame**

$$\gamma + p \rightarrow \pi^+ + n$$



**t - channel CM frame**

$$\gamma + \pi^- \rightarrow \bar{p} + n$$



$$A_{\mu\gamma\mu_i\mu_f} = A_{\mu\gamma\mu_i\mu_f}^e + A_{\mu\gamma\mu_i\mu_f}^m$$

$$A_{\mu\gamma\mu_i\mu_f}^e = 2\sqrt{2}eg_{\pi NN} \left[ \frac{(\epsilon(k, \mu_\gamma) \cdot p_\pi)}{t - m_\pi^2} + \frac{(\epsilon(k, \mu_\gamma) \cdot p_i)}{s - m_p^2} \right] \bar{u}(p_f, \mu_f) \gamma_5 u(p_i, \mu_i)$$

dominant  $\curvearrowright$  ~~vanishes~~  $\curvearrowright$

$$A_{\lambda_\gamma\lambda_i\lambda_f} = A_{\lambda_\gamma\lambda_i\lambda_f}^e + A_{\lambda_\gamma\lambda_i\lambda_f}^m$$

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~~vanishes~~  $\curvearrowright$   $\curvearrowright$  dominant

## Minimal gauge invariant decomposition of the Born amplitude

- Necessary and sufficient Lorentz structure to ensure gauge invariance of pion diagram:

$$F^{\mu\nu} P_\mu p_{\pi,\nu} = (\epsilon(k, \lambda_\gamma) \cdot P) (k \cdot p_\pi) - (\epsilon(k, \lambda_\gamma) \cdot p_\pi) (k \cdot P) \quad \text{with} \quad F^{\mu\nu} = \epsilon^\mu k^\nu - \epsilon^\nu k^\mu$$

→ “minimal gauge invariant” pion exchange amplitude:

$$A_{\lambda_\gamma \lambda_i \lambda_f}^{\pi\text{-m.g.i.}} = 2\sqrt{2} e_\pi g_{\pi NN} \left( \frac{\epsilon(k, \lambda_\gamma) \cdot p_\pi}{t - m_\pi^2} + \frac{\epsilon(k, \lambda_\gamma) \cdot P}{s - u} \right) \bar{u}(p_f, \lambda_f) \gamma_5 v(-p_i, \lambda_i)$$

↘ restores gauge invariance of the bare pion exchange

- We can rewrite the electric Born term:

(*t*-channel CM frame)

$$A_{\lambda_\gamma \lambda_i \lambda_f}^e = 2\sqrt{2} e g_{\pi NN} \left[ \left( \frac{\epsilon(k, \lambda_\gamma) \cdot p_\pi}{t - m_\pi^2} + \frac{\epsilon(k, \lambda_\gamma) \cdot P}{s - u} \right) + \frac{1}{2} \left( \frac{\epsilon(k, \lambda_\gamma) \cdot p_\pi}{s - m_N^2} + \frac{\epsilon(k, \lambda_\gamma) \cdot P}{s - u} \frac{t - m_\pi^2}{s - m_N^2} \right) \right] \bar{u}(p_f, \lambda_f) \gamma_5 v(-p_i, \lambda_i) \approx -i 2e_\pi g_{\pi NN} \lambda_\gamma (2\lambda_i \delta_{\lambda_i \lambda_f}) \frac{t}{m_\pi^2 - t} = A_{\lambda_\gamma \lambda_i \lambda_f}^{J=0}(s, t)$$

↘ subleading at large *s*

- Gauge invariance relates the pion and nucleon exchanges.
- The nucleon exchange diagram originates the pion pole in the *t*-channel CM frame.

## Magnetic term

- The magnetic term is gauge invariant by itself.

$$A_{\mu\gamma\mu_i\mu_f}^m = \sqrt{2}g_{\pi NN} \left[ \frac{e_{N_i}}{s - M^2} + \frac{e_{N_f}}{u - M^2} \right] \bar{u}(p_f, \mu_f) \gamma_5 \not{k} \not{\epsilon}_{\mu\gamma} u(p_i, \mu_i)$$

$$\approx \mu_\gamma 2g_{\pi NN} (e_{N_i} - e_{N_f}) \delta_{\mu\gamma\mu_i} \delta_{-\mu_i\mu_f}$$

- At  $t \sim 0$  the electric term of the amplitude vanishes.
- The magnetic term has small dependence in  $t$ .
- Size of the cross section agrees with the data at  $t \sim 0$ .

- Corrections: add form factor to magnetic term

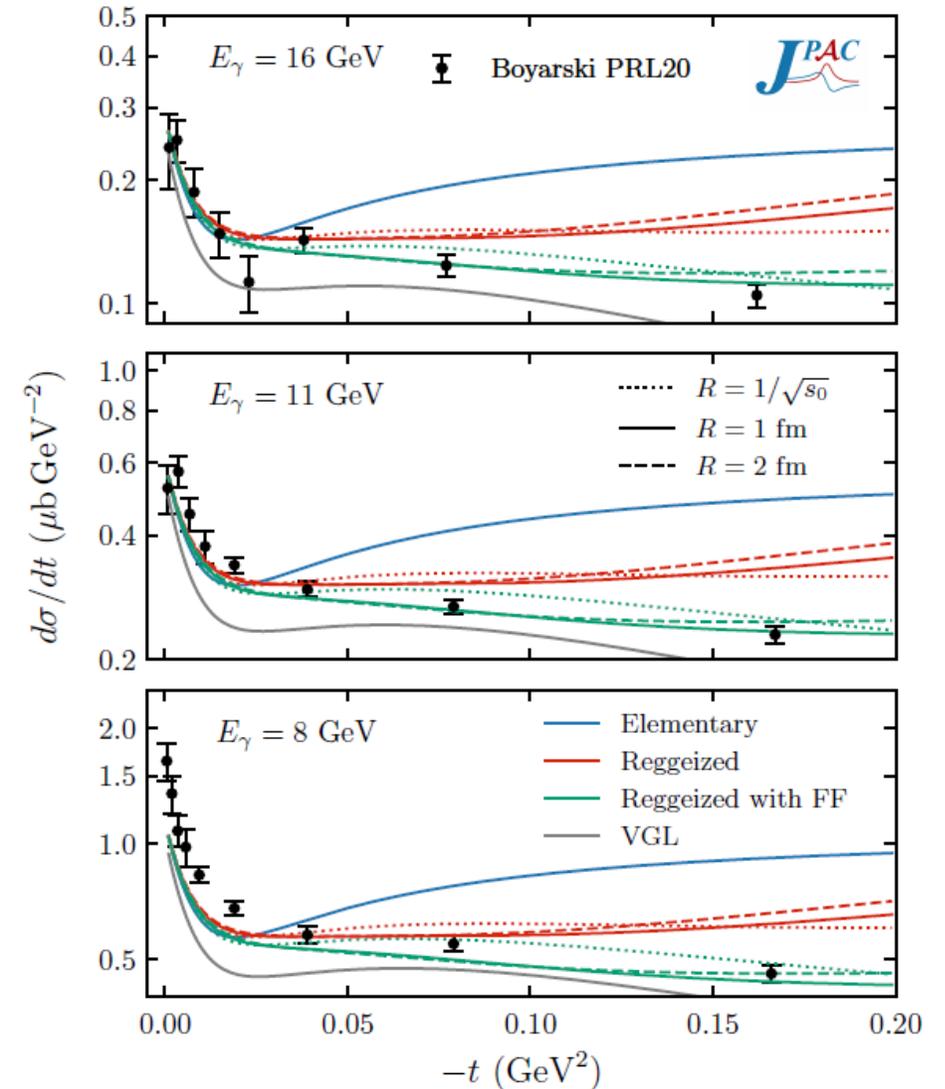
$$\beta(t) = \frac{\Lambda}{\Lambda^2 - t} \quad \text{with} \quad \Lambda \sim 1 \text{ GeV}$$

- Comparison with the VGL model

→ reggeizes by multiplying full Born amplitude by  $(t - m_\pi^2) \times \mathcal{P}_{\text{Regge}}$

[M.Guidal, J.M.Laget and M. Vanderhaeghen, Nucl.Phys.A 627 (1997) 645-678]

[G.Montana et al. (in preparation)]

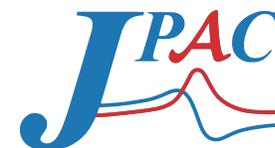


## CONCLUSIONS

- A precise comprehension of the production mechanisms is crucial for the light hybrid meson searches.
- At high energies, meson photoproduction reactions are dominated by the exchange of Regge trajectories, in particular, the pion trajectory plays a major role at low momentum transfer.
- How do we reggeize the pion in an appropriate way?
  - Current conservation requires the nucleon Born terms (gauge invariance).
  - It was not clear how to add  $t$ - and  $s$ -channel consistently without double counting:  $t$ -channel and  $s$ -channel partial wave series should independently represent the full amplitude.
  - Examination of the analytical  $J$  dependence emerging from the contraction of the vertices coupling  $\gamma\pi$  and  $N\bar{N}$  to  $J^P = (\text{even})^-$  reveals that it is analytical at  $J = 0$  and physically contains part of the ( $s$ -channel, or  $u$ -channel depending on charge) nucleon exchange.

## What's next?

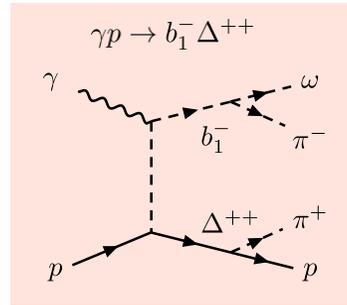
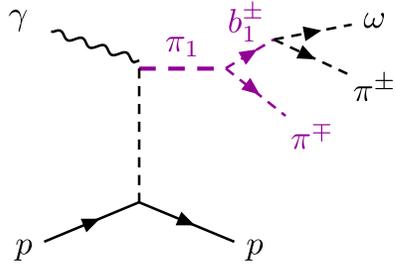
- Revisit the pion exchange in  $\gamma p \rightarrow \pi^- \Delta^{++}$  and understand  $\Delta^{++}$  SDMEs.
- Extension of the formalism to natural parity exchanges.
- Amplitudes for photoproduction of  $b_1, a_2, \pi_1$  with proton and  $\Delta^{++}$  recoils.



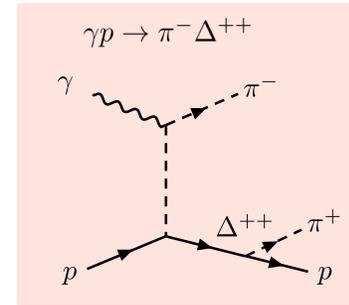
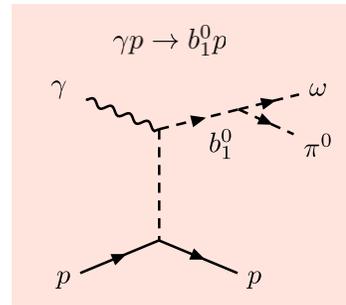
# BACKUP

# Search for exotic hybrid mesons in photoproduction with GlueX at JLab

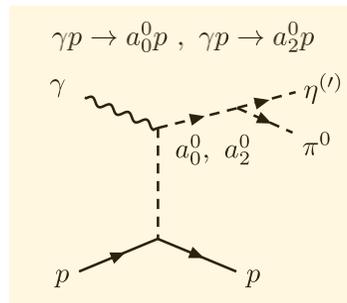
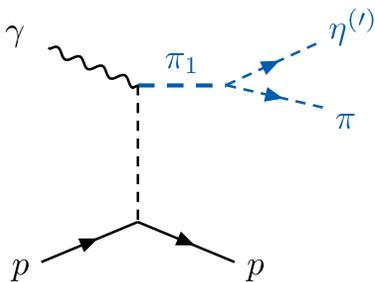
- Amplitude analyses of multi-meson final states require models for production amplitudes of several processes.



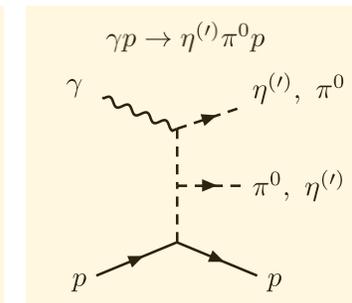
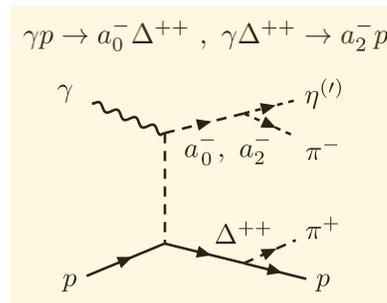
[Talk by A. Schertz, Fri PM]



[Talk by V. Shastry, Sat AM]



[Talk by M. Albrecht, Tue AM]

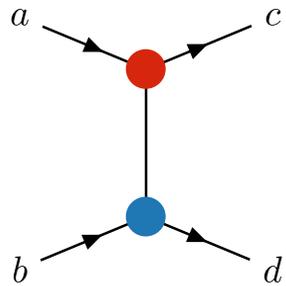


Begin by understanding the production mechanism of non-exotic mesons.

# Implications of Regge pole amplitudes

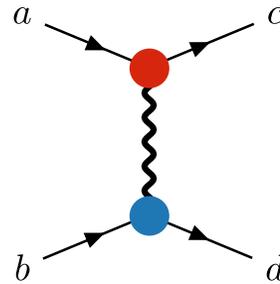
## Factorization

Amplitude for particle exchange “factorizes” (follows from unitarity).



$$A(s, t) = g_{ac} \frac{1}{t - m^2} g_{bd}$$

The reggeon residue  $\beta(t)$ :



- Contains all information about incoming and outgoing particles.
- Related to the reggeon-hadron interaction vertices.
- Satisfies factorization:  $\beta(t) = \beta_{ac}(t)\beta_{bd}(t)$

## Power law energy dependence

$$A(s, t) \sim s^{\alpha(t)}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} |A(s, t)|^2 = s^{2\alpha(t)-2}$$

Leading Regge poles (biggest  $\alpha(t)$ ) dominate asymptotically.

## Phase

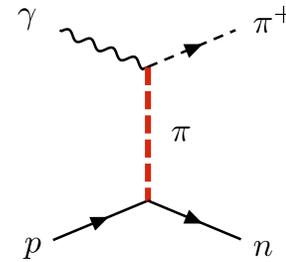
The phase comes from the signature factor:  $\frac{1 + \eta e^{-i\pi\alpha(t)}}{2}$

Exchange degeneracy (equal trajectories with opposite signatures) leads to rotating or constant phases.

- Corrections to these hypothesis, usually ~10-20%. [J.Nys et al. (JPAC), *Phys.Rev.D* 98 (2018) 3, 034020]

# Charged pion photoproduction within Born models

- Description at low energies in terms of effective Lagrangians.
- High energies: reggeization of the pion Born diagram.

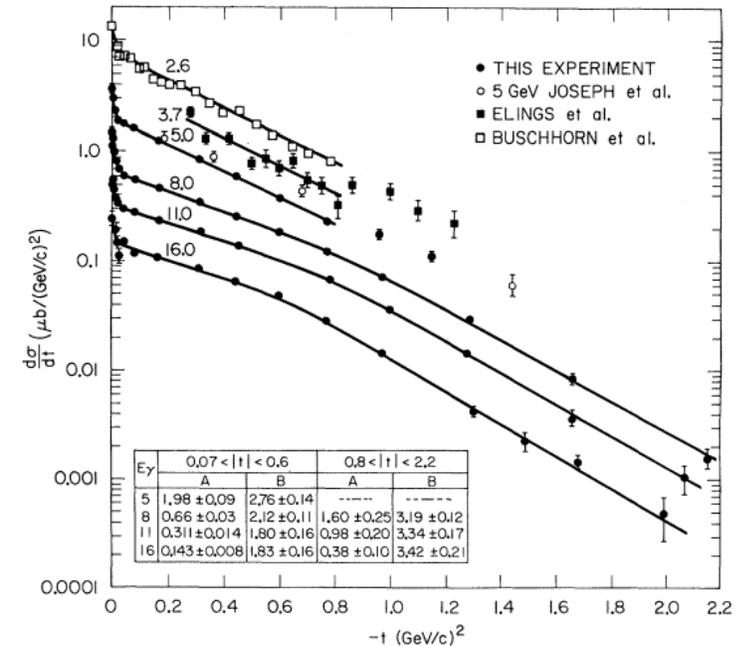


## Known issues

- What is pion exchange and how does it reggeize?
- Cannot describe forward cross-section data in  $\gamma p \rightarrow \pi^+ n$  (same for  $np \rightarrow pn$ ).

## Proposed solutions

- Existence of parity-doublet conspirator of the pion.  
[J.S.Ball, W.R. Frazer and M. Jacob, *Phys.Rev.Lett.* 20 (1968) 518]
- Regge cuts and absorption (final state interactions).  
[F. Henyey, G.L.Kane, J.Pumplin, *Phys.Rev.* 182 (1969) 1579]
- Nucleon Born terms.  
[L.Jones, *Rev.Mod.Phys.* 52 (1980) 545]



[A. Boyarski et al., *Phys.Rev.Lett.* 20 (1968) 300]