A Comprehensive Study of Double Pion Photoproduction: Regge Approach

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Background and Motivation

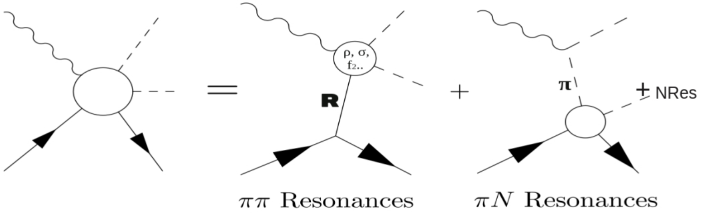
- **Importance of Two-Pion Photoproduction:**
  - Crucial for understanding light meson resonances, especially given challenges in obtaining free pion targets.
  - Recent discoveries of exotic heavy states have sparked renewed interest, advancing hadron spectroscopy.
- **Leveraging New Data:**
  - High-precision data from CLAS12 and GlueX provide new insights, refining production mechanisms and enhancing model accuracy.
- **Theoretical Insights and Challenges:**
  - Simple pomeron-based models explain the $\rho(770)$ resonance, s-channel helicity conservation (SCHC), and cross-section behavior at small momentum transfer ($|t| \lesssim 0.4, \text{GeV}^2$).
  - They fail at larger momentum transfers, where additional light meson resonances become significant.
  - This highlights the necessity for more detailed models capable of explaining complex interference patterns and resonance contributions.

Model Description

Process: $\gamma(q,\lambda_q) + p(p_1,\lambda_1) \rightarrow \pi^+(k_1) + \pi^-(k_2) + p(p_2,\lambda_2)$

$2 \rightarrow 3$ Dynamics

Our approach builds upon established dynamics within $2 \rightarrow 2$ subchannels, by extending the on-shell πN Deck mechanism to an off-shell framework



- Direct implementation of $\pi\pi$ resonances within our model
- Embedding of πN resonances in the Deck mechanism

Kinematics

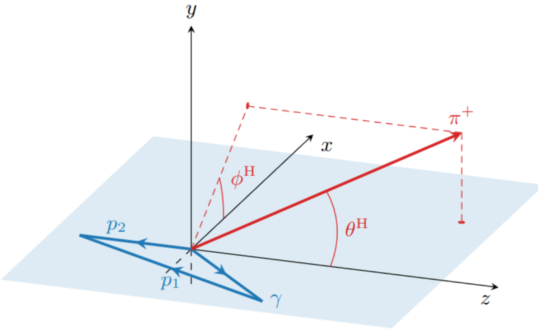
Helicity frame: the recoiling proton ($\vec{p_2}$) defines the negative z-axis, and $\Omega^{\mathrm{H}} = (\theta^{\mathrm{H}}, \phi^{\mathrm{H}})$ define the angles of the π^+.

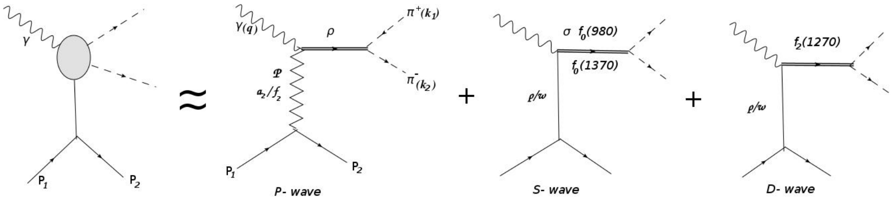
We will use the following kinematic invariants:

$$s = (p_1 + q)^2 = (p_2 + k_1 + k_2)^2,$$

$$t = (p_1 - p_2)^2 = (k_1 + k_2 - q)^2,$$

$$s_{12} = (k_1 + k_2)^2 = (p_1 - p_2 + q)^2.$$





- Effective Lagrangian: One-particle exchange model.
- Regge Propagator:

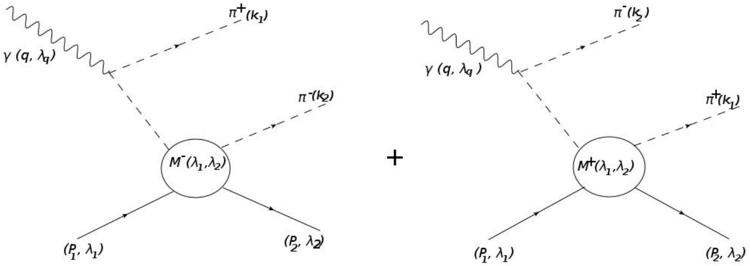
$$R^{\mathrm{E}}(s,t) = \frac{1}{s_0}\frac{\alpha^{E}(t)}{\alpha^{\mathrm{E}}(0)}\frac{1+\tau^{\mathrm{E}}e^{-i\pi\alpha^{\mathrm{E}}(t)}}{\sin\pi\alpha^{\mathrm{E}}(t)}\left(\frac{s}{s_0}\right)^{\alpha^{\mathrm{E}}(t)-1},$$

- Include the resonance decay vertex by using an energy-dependent width Breit-Wigner [Phys. Rev. D 98, 030001 (2018)].

The partial wave amplitude:

$$\mathcal{M}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{12},\Omega_H) = \sum_{lm}\mathcal{M}^{lm}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{12},\Omega_H)Y_{lm}(\Omega_H)$$

Non Resonant Production: Deck Mechanism



$$\mathcal{M}^{\text{Deck,GI}}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12},\Omega_H) = \sqrt{4\pi\alpha} \times \Bigg[\left(\frac{\epsilon(q,\lambda_q)\cdot k_1}{q\cdot k_1} - \frac{\epsilon(q,\lambda_q)\cdot(p_1+p_2)}{q\cdot(p_1+p_2)}\right)\beta(t_{\pi_1})\mathcal{M}^-_{\lambda_1\lambda_2}(s_2,t) - \left(\frac{\epsilon(q,\lambda_q)\cdot k_2}{q\cdot k_2} - \frac{\epsilon(q,\lambda_q)\cdot(p_1+p_2)}{q\cdot(p_1+p_2)}\right)\beta(t_{\pi_2})\mathcal{M}^+_{\lambda_1\lambda_2}(s_1,t)\Bigg].$$

$$F_{bkg}(s_{12}) \equiv [(s_{12}^{\text{th}} - s_{12})(s_{12}^{\text{max}} - s_{12})],$$

where

$$s_{12}^{\text{th}} = 4m_\pi^2$$
$$s_{12}^{\text{max}} = s + m_p^2 - \frac{1}{2m_p^2}\Big[(s + m_p^2)(2m_p^2 - t) - \lambda^{1/2}(s, m_p^2, 0)\lambda^{1/2}(t, m_p^2, m_p^2)\Big].$$

Thus

$$\mathcal{M}_P^{\text{nres}} = \frac{1}{s} R_{f_2}(s,t) F_{bkg}(s_{12}) \overline{u}(p_2, \lambda_2)\gamma^\mu u(p_1, \lambda_1) w_\mu(\lambda_\gamma),$$
$$\mathcal{M}_S^{\text{nres}} = \frac{1}{s} g_s^{\text{nr}} R(s,t) F_{bkg}(s_{12}) \overline{u}(p_2, \lambda_2)\gamma^\mu u(p_1, \lambda_1) v_\mu(\lambda_\gamma).$$

Parameterization of Helicity Structure

- Parameterizing helicity-dependent couplings $a^{\mathsf{E},\mathcal{R}}_{\lambda_\gamma M}(t)$ at photon-nucleon vertices for $J = 0, 1, 2$ partial waves in two-pion photoproduction. For example, the P-wave vertex:

$$\mathcal{T}^{\alpha}_{\lambda_\gamma M} = a^{\mathsf{E},\mathcal{R}}_{\lambda_\gamma M}(t) \left[q^\alpha \epsilon^\sigma_{\lambda_\gamma}(q) - q^\sigma \epsilon^\alpha_{\lambda_\gamma}(q) \right] \epsilon^*_{M\sigma}(k).$$

  These parameters were allowed to be complex. Upon fitting, these couplings capture the helicity structure and ensure gauge invariance.

- A total of 30 free parameters:
  - $S-$wave Contributions:
    - 2 for each scalar resonance i.e. $f_0(500), f_0(980), f_0(1370)$,
    - 2 for the background.
  - $P-$wave Contribution:
    - 6 for ρ production via f_2 exchange,
    - 6 for the backgroud.
  - $D-$wave Contribution: - 10 for the tensor meson $f_2(1270)$.

Fitting Angular Moments

- Parameters are fitted to experimental angular moments from CLAS data.
- The fit procedure involves analyzing data primarily at the highest energy bin ($3.6 - 3.8$ GeV) and evaluating the model at 3.7 GeV photon energy.
- Fits are performed for angular moments $< Y_L^M >$ where $L = 0, 1, 2$ and $M = 0...L$, where

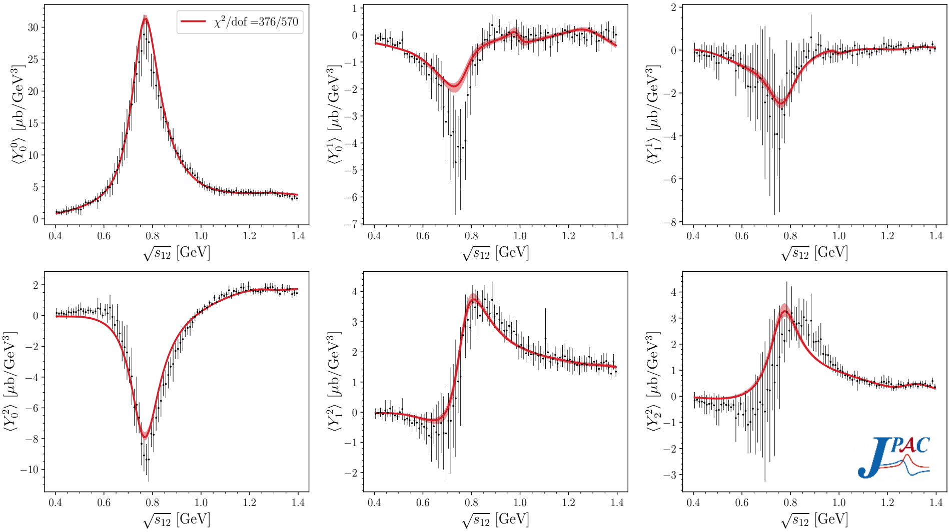
$$< Y_L^M >= \sqrt{4\pi} \int d\Omega^H \frac{d\sigma}{dt dm_{12} d\Omega^H} \text{Re}\, Y_{LM}(\Omega^H)$$

[Phys.Rev.D 80 (2009) 072005]

- Each t-bin is fitted separately (600 data points per fit) where statistical uncertainties are determined using a bootstrap method ensuring reliable parameter estimates.

Fitting Results

$E_\gamma = 3.7$ GeV and $t = -0.45$ GeV^2



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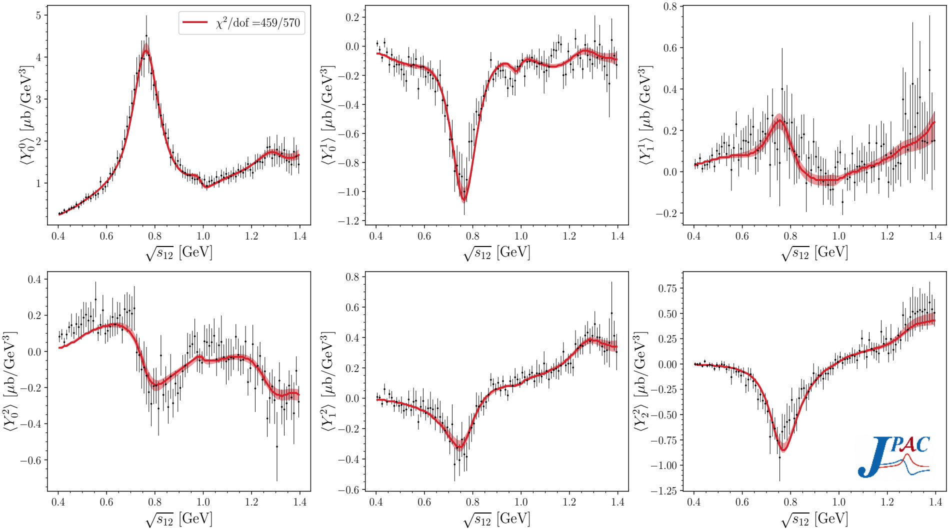


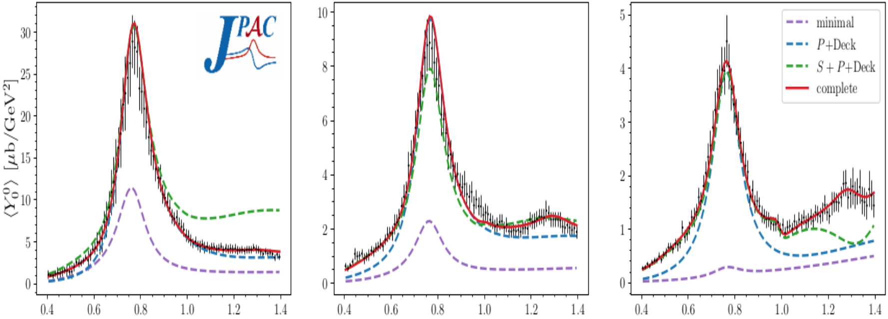
Decoding Angular Moments: The Production Puzzle

Our model interprets angular moments via physically motivated parameterizations of production amplitudes. We analyze four model decompositions to understand their contributions:

- Minimal Model: Includes Pomeron-induced resonant $\rho(770)$ production and Deck mechanism.
- $P+$ Deck: Incorporates the full $\rho(770)$ production amplitude and its corresponding background.
- $S + P+$ Deck: Extends to include a comprehensive set of related resonances and their backgrounds.
- Complete Model: Represents our fully developed model.

Insights from Y_0^0 Analysis

$t = -0.45\ GeV^2$ (left), $t = -0.65\ GeV^2$ (center), $t = -0.95\ GeV^2$ (right)

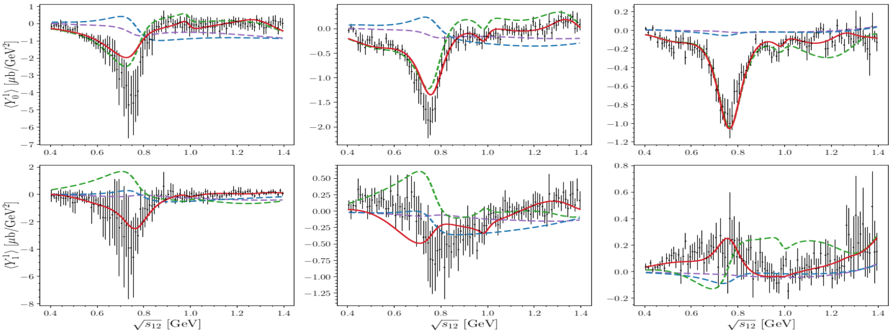


Key observations:

- Minimal model underestimates Y_0^0 across all t-bins
- $P+$ Deck captures Y_0^0 near $\rho(770)$ peak but doesn't match lineshape
- Improved fit with $S + P+$ Deck, particularly for $\sqrt{s_{12}} < m_\rho$
- Enhanced accuracy including D-wave for $\sqrt{s_{12}} > m_\rho$

Analysis Highlights for $L = 1$ Moments

$t = -0.45\ GeV^2$ (left), $t = -0.65\ GeV^2$ (center), $t = -0.95\ GeV^2$ (right)



Key observations:

- Minimal and $P+$ Deck models fail across all t-bins for Y_0^1 and Y_1^1.
- Including S-wave improves fit for Y_0^1 but still falls short.
- Including all resonances is essential to capture observed data features.

Summary

- Developed a theoretical framework integrating resonance effects and their impact on angular moments across t.
- Demonstrated the Deck Model's adaptability in off-shell pion exchange, which is crucial for understanding resonance structures.
- Validated the model with experimental data, effectively capturing intricate t-dependence.
- Proposed future studies on resonances like $f_0(980)$ and $f_2(1270)$ at higher energies due to their increasing prominence.
- Emphasized the necessity of additional amplitudes beyond Pomeron exchange, such as S-wave and D-wave contributions, to accurately reflect experimental data.

Future Work: ρ Meson SDMEs

Importance of Computing SDMEs

- Spin Density Matrix Elements (SDMEs) describe the polarization state of the studied meson (ρ, Δ^{++},..).
- Validate our model by comparing theoretical predictions with experimental data.
- Understand vector meson production and decay dynamics.
- Explore effects of different production mechanisms.
- Identification of potential areas for further theoretical development.

Derivation of SDMEs for Vector Mesons

SDMEs of the decaying vector meson, are related to the photon spin density matrix as:

$$\rho(V) = T\rho(\gamma)T^{\dagger},$$

where

$$\rho(\gamma) = \frac{1}{2}I + \vec{P_\gamma} \cdot \vec{\sigma}\,,$$

for linear polarisation:

$$\vec{P_\gamma} = P_\gamma(-\cos 2\Phi, -\sin 2\Phi, 0)\,,$$

Φ is the angle between the production plane and the polarisation vector of the the photon and $0 \lesssim P_\gamma \lesssim 1$. Thus:

$$\rho(V) = \rho_0 + \sum_{i=1}^{3} P_\gamma^\alpha \rho^\alpha\,,$$

Derivation of SDMEs for Vector Mesons

ρ^α are hermitian matrices and their trace is 1. They are given by:

$$\rho^0_{\lambda\lambda'} = \frac{1}{2N} \sum_{\lambda\lambda_{N'}\lambda_N} T_{\lambda\lambda_{N'},\lambda_\gamma\lambda_N} T^*_{\lambda'\lambda_{N'},\lambda_\gamma\lambda_N},$$

$$\rho^1_{\lambda\lambda'} = \frac{1}{2N} \sum_{\lambda\lambda'_N\lambda_N} T_{\lambda\lambda_{N'},-\lambda_\gamma\lambda_N} T^*_{\lambda'\lambda_{N'},\lambda_\gamma\lambda_N},$$

$$\rho^2_{\lambda\lambda'} = \frac{i}{2N} \sum_{\lambda\lambda_{N'}\lambda_N} \lambda_\gamma T_{\lambda\lambda_{N'},-\lambda_\gamma\lambda_N} T^*_{\lambda'\lambda_{N'},\lambda_\gamma\lambda_N},$$

$$\rho^3_{\lambda\lambda'} = \frac{1}{2N} \sum_{\lambda\lambda_{N'}\lambda_N} \lambda_\gamma T_{\lambda\lambda_{N'},-\lambda_\gamma\lambda_N} T^*_{\lambda'\lambda_{N'},\lambda_\gamma\lambda_N}.$$

They satisfy the symmetry properties:

$$\rho^\alpha_{\lambda\lambda'} = (-1)^{\lambda-\lambda'} \rho^\alpha_{-\lambda-\lambda'} \quad , \quad \text{for } \alpha = 0, 1$$

,

$$\rho^\alpha_{\lambda\lambda'} = -(-1)^{\lambda-\lambda'} \rho^\alpha_{-\lambda-\lambda'} \quad , \quad \text{for } \alpha = 2, 3$$

.

The density matrix is related to the decay angular distribution:

$$W(\cos\theta, \phi) = M\rho(V)M^{\dagger} \,,$$

Then the ρ meson decay distribution is:

$$W(\cos\theta, \phi, \Phi) = W_0(\cos\theta, \phi) - P_\gamma \cos 2\Phi W_1(\cos\theta, \phi) - P_\gamma \sin 2\Phi W_2(\cos\theta, \phi) \,,$$

where

$$W_0(\cos\theta, \phi) = \frac{3}{4\pi}\left[\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00} - 1)\cos^2\theta - \sqrt{2}\,\text{Re}\,\rho^0_{10}\sin 2\theta\cos\phi - \rho^0_{1-1}\sin^2\theta\cos 2\phi\right],$$

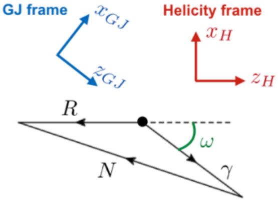
$$W_1(\cos\theta, \phi) = \frac{3}{4\pi}\left[\rho^1_{11}\sin^2\phi + \rho^1_{00}\cos^2\theta - \sqrt{2}\,\rho^1_{10}\sin 2\theta\cos\phi - \rho^1_{1-1}\sin^2\theta\cos 2\phi\right]$$

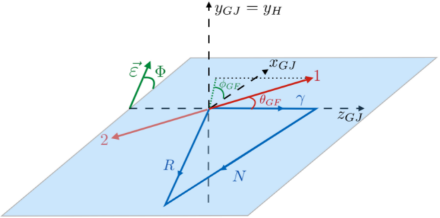
$$W_2(\cos\theta, \phi) = \frac{3}{4\pi}\left[\sqrt{2}\,\text{Im}\,\rho^2_{10}\sin 2\theta\sin\phi + \text{Im}\,\rho^2_{1-1}\sin^2\theta\sin 2\phi\right].$$


Thank you

Back-Up Slides

Frames





Particle	Helicity Frame	Gottfried-Jackson Frame
$\boldsymbol{p_1}$	$\lvert\vec{p_1}\rvert(\sin\theta_1, 0, \cos\theta_1)$	$\lvert\vec{p_1}\rvert(-\sin\theta_1, 0, \cos\theta_1)$
$\boldsymbol{p_2}$	$\lvert\vec{p_2}\rvert(0, 0, -1)$	$\lvert\vec{p_2}\rvert(-\sin\theta_2, 0, \cos\theta_2)$
$\boldsymbol{q}$	$\lvert\vec{q}\rvert(-\sin\theta_q, 0, \cos\theta_q)$	$\lvert\vec{q}\rvert(0, 0, 1)$
$\boldsymbol{k_1}$	$\boldsymbol{k_1} = \lvert\vec{k_1}\rvert(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$	$\boldsymbol{k_1} = \lvert\vec{k_1}\rvert(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
$\boldsymbol{k_2}$	$\boldsymbol{k_2} = -\boldsymbol{k_1}$	$\boldsymbol{k_2} = -\boldsymbol{k_1}$

Resonant Production

Energy-dependent width Breit-Wigner function,following the parameterization provided by the Particle Data Group (PDG) [Phys. Rev. D 98, 030001 (2018)]:

$$\mathrm{BW}^{\mathrm{dep}}(s,l) = \frac{n(s)}{m_{\mathrm{BW}}^2 - s - i m_{\mathrm{BW}} \Gamma_{\mathrm{tot}}(s)}, \text{ where } n(s) = \left(\frac{q}{q_0}\right)^l F_l(q, q_0)$$

Pion-proton Scattering

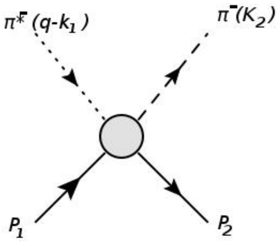


Figure: Feynman diagram for $\pi^- p \to \pi^- p$

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

$$M_\lambda^- = \bar{u}_\lambda(p_2)\left[A^-(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q-k_1+k_2)^\mu B^-(s,t,t_\pi)\right]u_\lambda(p_1),$$

where $t_\pi = (q-k_1)^2$

Pion-proton Scattering

Similarly for the positive exchanged pion:

$$M_\lambda^+ = \bar{u}_\lambda(p_2)\left[A^+(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q-k_2+k_1)^\mu B^+(s,t,t_\pi)\right]u_\lambda(p_1),$$

where $t_\pi = (q-k_2)^2$.
In the πN center of mass frame the t-channel A and B defined as follows:

$$\frac{1}{4\pi}A^\pm = \frac{\sqrt{s}+m_p}{Z_1^+Z_2^+}f_1^\pm - \frac{\sqrt{s}-m_p}{Z_1^-Z_2^-}f_2^\pm,$$
$$\frac{1}{4\pi}B^\pm = \frac{1}{Z_1^+Z_2^+}f_1^\pm - \frac{1}{Z_1^-Z_2^-}f_2^\pm.$$

Where f_1 and f_2 are called the reduced helicity amplitudes, $Z_i^\pm = \sqrt{E_i \pm m_p}$.

Pion-proton scattering

The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos\theta) - \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos\theta),$$

$$f_2 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos\theta).$$

In our model the pion virtuality appears clearly in the incoming proton energy (E_1), momentum (P_1) as well as our scattering angle ($\cos\theta$). Hence we can say that the scalar functions A and B in our case depends on the pion virtuality.

$$E_1 = \frac{s_i - t_\pi + m_p^2}{2\sqrt{s_i}},$$

$$\cos\theta = \frac{2s_i(t - 2m_p^2) + (s_i - t_\pi + m_p^2)(s_i - m_\pi^2 + m_p^2)}{\sqrt{\lambda(s_i, t_\pi, m_p^2)}\sqrt{\lambda(s_i, m_\pi^2, m_p^2)}}.$$