Precision Tests of the Standard Model with Cabibbo Unitarity and Nuclear $\beta$-Decays

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Outline

Beta decay, radiative corrections and the Standard Model

Cabibbo anomaly and BSM

Radiative corrections to $\beta$-decays: overall setup

Fermi function and nuclear radii

Isospin breaking correction

$\gamma W$-box: Dispersion Theory, lattice QCD and EFT

Nuclear structure correction

Summary & Outlook
What to work on to win a Nobel prize?
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Beta decay has been an excellent choice for a century!

1896- Becquerel discovers spontaneous radioactivity of uranium, identified \( \beta \) with the electron

1898- Curie-Sklodowska, Curie discover polonium and radium

1899- Rutherford systematized \( \alpha, \beta, \gamma \) rays, identified \( \alpha \) with He-4

1934- F.&I. Joliot-Curie discovered \( \beta^+ \) decay with \( \beta^+ \) - positron

1956- Lee & Yang proposed parity non conservation in \( \beta \)-decay, confirmed by Wu experiment

1961- Glashow proposed electroweak unification

1967- Weinberg & Salam implemented Higgs mechanism

1973- Neutral weak current discovered at CERN

1973- Kobayashi, Maskawa: 3-flavor quark mixing matrix
Niepce de Saint-Victor: observed radioactivity in 1857 cited in Becquerel-father’s book

Cox, McIlwraith, Kurrelmeier (1928); Chase (1929-30) “Apparent evidence of polarization in a beam of beta rays”

1930: Pauli postulated existence of neutrinos
1934: Fermi formulated the contact theory of beta decay

1938: Klein predicted \( M_W \sim \sqrt{4\pi\alpha \sqrt{2/G_F}} \sim 100 \text{ GeV} \)

1957: Wu’s experiment was crucial to prove Lee-Yang’s conjecture, but Chien-Shiung Wu was not awarded the NP

1963: Cabibbo: proposed 2-flavor quark mixing to reconcile \( \mu, \beta, \) K decay rates
Precision Era: V-A + Radiative Corrections

V - A theory (Sudarshan&Marshak and Gell-Mann&Feynman 1957); S-PS not excluded
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Radiative corrections to muon decay: important evidence for V-A theory
RC to muon decay - UV finite for V-A but divergent for S-PS

Muon lifetime $\tau_\mu = 2196980.3(2.2)\,ps \rightarrow$ Fermi constant $G_\mu = 1.1663788(7) \times 10^{-5}GeV^{-2}$
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Kinoshita, Sirlin, Behrends, …

1-loop RC to spectrum: $\Delta P^0 d^3p = \frac{\alpha}{2\pi} P^0 d^3p \left[ 6 \ln \frac{\Lambda}{M_p} + \text{finite} \right]$
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Is weak interaction universal for leptons and hadrons?

1967: Sirlin applied current algebra:

general UV behavior of $\beta$ decay rate at 1-loop

$$\frac{\alpha}{2\pi} P^0 d^3p 3[1 + 2\tilde{Q}] \ln(\Lambda/M)$$

$\tilde{Q}$ : average charge of fields involved: $1 + 2\tilde{Q}_{\mu,\nu} = 0$ but $1 + 2\tilde{Q}_{n,p} = 2$

Finiteness of RC to muon decay was accidental!
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Finiteness of RC to muon decay was accidental!

Eventually, massive W-boson renders RC to beta decay UV-finite
In SM the same coupling of W-boson to leptons and hadrons, $G_V = G_\mu$

Before RC were included: $G_V \sim 0.98 G_\mu$
Precision, Universality and CKM unitarity

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Large $\log(M_Z/M_p)$ in RC for neutron $\rightarrow G_V \sim 0.95 G_\mu$

Kaon and hyperon decays? ($\Delta S = 1$) — even smaller coupling!
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Cabibbo: strength shared between 2 generations

$$G_V^{\Delta S=0} = \cos \theta_C G_\mu$$
$$G_V^{\Delta S=1} = \sin \theta_C G_\mu$$

Cabibbo unitarity: $\cos^2 \theta_C + \sin^2 \theta_C = 1$
Precision, Universality and CKM unitarity

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Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix \( V \)

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

CKM unitarity - completeness of the SM: \( VV^\dagger = 1 \)

Top row unitarity constraint: \( V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \)
Detailed understanding of $\beta$ decays largely shaped the Standard Model.
Cabibbo Angle Anomaly: Status and BSM interpretation
Status of Cabibbo unitarity

\[ V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9985(6)V_{ud}(4)V_{us} \]

\[ \sim 0.95 \quad \sim 0.05 \quad \sim 10^{-5} \]

\( V_{ud} \) and \( V_{us} \) determinations inconsistent with the SM
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Superallowed nuclear $\beta$: \[ V_{ud} = 0.9737 (3) \]
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Superallowed nuclear \[ \beta \] : \[ V_{ud} = 0.9737 \ (3) \]

At variance with kaon decays + Cabibbo unitarity

\[ K \to \pi \ell \nu : \quad V_{us} = 0.2233(5) \]

Unitarity \[ V_{ud} = \sqrt{1 - V_{us}^2} = 0.9747(1) \]
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\[ \frac{K \rightarrow \mu \nu}{\pi \rightarrow \mu \nu} : \quad V_{us}/V_{ud} = 0.2311(5) \]

Unitarity \[ V_{ud} = [1 + V_{us}/V_{ud}^2]^{-1/2} = 0.9743(1) \]
Status of Cabibbo unitarity

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PDG \([S = 2.5] : \quad V_{us} = 0.2243(8)\)

\text{Unitarity} \rightarrow \quad V_{ud} = 0.9745(2)\)
CAA summary - 3 anomalies!

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, $V_{ud}$

2 quantities to determine: $V_{us}$, $V_{ud}$

3 ways to test unitarity

$\Delta_{CKM}^{(1)} = |V_{ud}|^2 + |V_{us}^{K\ell 3}|^2 - 1 = -0.00176(56) -3.1\sigma$

$\Delta_{CKM}^{(2)} = |V_{ud}|^2 \left[ 1 + \left( \frac{|V_{us}|^{K\mu 2}}{|V_{ud}|} \right)^2 \right] - 1 = -0.00098(58) -1.7\sigma$

$\Delta_{CKM}^{(3)} = |V_{us}^{K\ell 3}|^2 \left[ \frac{1}{\left( \frac{|V_{us}/V_{ud}|^{K\mu 2}}{1} \right)^2 + 1} \right] - 1 = -0.0164(63) -2.6\sigma$

Can it be a signal of BSM?
In SM, $W$ couples only to LH chiral fermion states.

New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}-K_{\mu 2}$ difference.

Define $\epsilon_R = \text{admixture of RH currents in non-strange sector}$

$$\epsilon_R + \Delta \epsilon_R = \text{admixture of RH currents in strange sector}$$

$$\Delta_{\text{CKM}}^{(1)} = 2\epsilon_R + 2\Delta \epsilon_R V_{us}^2$$

$$\Delta_{\text{CKM}}^{(2)} = 2\epsilon_R - 2\Delta \epsilon_R V_{us}^2$$

$$\Delta_{\text{CKM}}^{(3)} = 2\epsilon_R + 2\Delta \epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left( \frac{1 + \Delta_{\text{CKM}}^{(2)}}{1 + \Delta_{\text{CKM}}^{(3)}} \right)^{1/2} = \frac{V_{us}}{V_{ud}} K_{\ell 2}/\pi_{\ell 2} = 1 - 2\Delta \epsilon_R$$

From current fit:

$$\epsilon_R = -0.69(27) \times 10^{-3} \ (2.5\sigma)$$

$$\Delta \epsilon_R = -3.9(1.6) \times 10^{-3} \ (2.4\sigma)$$

$$\epsilon_R = \Delta \epsilon_R = 0 \ \text{excluded at} \ 3.1\sigma$$
CAA in presence of RH currents

- In SM, $W$ couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}-K_{\mu 2}$ difference
- Define $\epsilon_R = \text{admixture of RH currents in non-strange sector}$
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- $\epsilon_R = \Delta\epsilon_R = 0$ excluded at $3.1\sigma$

Are all SM corrections under control?
The path from kaon decays to $V_{us}$
$V_{us}/V_{ud}$ from $K\mu 2 = K \to \mu\nu$

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_\pi} = \left( \frac{\Gamma_{K\mu 2(\gamma)} m_{\pi \pm}}{\Gamma_{\pi\mu 2(\gamma)} m_{K \pm}} \right)^{1/2} \frac{1 - m_{\mu}^2/m_{\pi \pm}^2}{1 - m_{\mu}^2/m_{K \pm}^2} \left( 1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

**Inputs from experiment:**

*From $K^\pm$ BR fit:*

- $\text{BR}(K^\pm_{\mu 2(\gamma)}) = 0.6358(11)$
- $\tau_{K \pm} = 12.384(15) \text{ ns}$

*From PDG:*

- $\text{BR}(\pi^\pm_{\mu 2(\gamma)}) = 0.9999$
- $\tau_{\pi \pm} = 26.033(5) \text{ ns}$

**Inputs from theory:**

- $\delta_{EM}$: Long-distance EM corrections
- $\delta_{SU(2)}$: Strong isospin breaking
- $f_K/f_\pi$: Ratio of decay constants
  - Cancellation of lattice-scale uncertainties from ratio
  - NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K \pm}/f_{\pi \pm}$

**LQCD+EM ($N_f = 2 + 1 + 1$):**

$$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$$

*Di Carlo et al, 2019*

**LQCD ($N_f = 2 + 1 + 1$):**

$$f_K/f_\pi = 1.1978(22)$$

*FLAG 2021 average*

$$V_{us}/V_{ud} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{IB}}^{(51)_{\text{tot}}} = 0.22\%$$
\[ V_{us} \text{ from } K\ell 3 = K \rightarrow \pi \nu, \pi \mu \nu \]

\[ \Gamma(K\ell 3(\gamma)) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f^0_{\ell 3}(0)|^2 I_{K\ell}(\lambda_{K\ell}) \left( 1 + 2\Delta_{K}^{SU(2)} + 2\Delta_{K\ell}^{EM} \right) \]

with \( K \in \{ K^+, K^0 \} \); \( \ell \in \{ e, \mu \} \), and:
- \( C_K^2 \): 1/2 for \( K^+ \), 1 for \( K^0 \)
- \( S_{EW} \): Universal SD EW correction (1.0232)

\[ K\ell 3 : \quad V_{us} = 0.22330(35)_{\exp}(39)_{\text{lat}}(8)_{\text{IB}} \]
\[ (53)_{\text{tot}} = 0.24 \% \]

### Inputs from experiment:
- \( \Gamma(K\ell 3(\gamma)) \): Rates with well-determined treatment of radiative decays:
  - Branching ratios
  - Kaon lifetimes
- \( I_{K\ell}(\lambda_{K\ell}) \): Integral of form factor over phase space; \( \lambda \)s parameterize evolution in \( t \)

\[ f_+(0) \]
- LQCD \((N_f = 2 + 1)\): \( f_+(0) = 0.9677(27) \)
- LQCD \((N_f = 2 + 1 + 1)\): \( f_+(0) = 0.9698(17) \)

### Inputs from theory:
- \( f^0_{\ell 3}(0) \): Hadronic matrix element (form factor) at zero momentum transfer \((t = 0)\)
- \( \Delta_{K}^{SU(2)} \): Form-factor correction for \( SU(2) \) breaking
- \( \Delta_{K\ell}^{EM} \): Form-factor correction for long-distance EM effects

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**Long-distance EM RC \( \Delta_{K\ell}^{EM} \): new approach (ChPT + Current Algebra + LQCD)**

Uncertainty reduced by 1 o.o.m. — under control

Seng, Galviz, Meißner 1910.13208
Seng, Galviz, MG, Meißner 2103.04843
Seng, Galviz, MG, Meißner 2203.05217
Feng, MG, Jin, Ma, Seng 2003.09798
Ma, Feng, MG, Jin, Seng 2102.12048
The path from nuclear beta decays to $V_{ud}$
$V_{ud}$ from superallowed $0^+ \rightarrow 0^+$ nuclear decays

1. Transitions within $J^p=0^+$ isotriplets ($T=1$)
2. Elementary process: $p \rightarrow n e^+ \nu$
3. Only conserved vector current
4. 15 measured to better than 0.2%
5. Internal consistency as a check
6. SU(2) good $\rightarrow$ corrections $\sim$ small
\(V_{ud}\) from superallowed \(0^+ - 0^+\) nuclear decays

1. Transitions within \(J^p=0^+\) isotriplets (\(T=1\))
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Exp.: \(f\) - phase space (Q value)
\(t\) - partial half-life (\(t_{1/2}\), branching ratio)

ft values: same within \(\sim 2\%\) but not exactly!
Reason: SU(2) slightly broken
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)
To obtain $V_{ud}$, absorb all decay-specific corrections into universal $F_t$

$$f_t(1 + RC + ISB) = \mathcal{F}t(1 + \Delta^V_R) = \mathcal{F}t(1 + \delta'_R)(1 - \delta_C + \delta_{NS})(1 + \Delta^V_R)$$

\[\sim\text{Measured}\quad\text{QED}\quad\text{Isospin-breaking}\quad\text{Nuclear structure}\quad\text{Universal RC}\]
To obtain $V_{ud} \rightarrow$ absorb all decay-specific corrections into universal $Ft$

$$ft(1 + RC + ISB) = \mathcal{F}t(1 + \Delta_V^R) = ft(1 + \delta_R')(1 - \delta_C + \delta_{NS})(1 + \Delta_V^R)$$

~ Measured  QED  Isospin-breaking  Nuclear structure  Universal RC

**Average of 14 decays**  
Hardy, Towner 1972 - 2020

Pre-2018: $\overline{Ft} = 3072.1 \pm 0.7 \text{ s}$

PDG 2022: $\overline{Ft} = 3072 \pm 2 \text{ s}$

$$V_{ud}^2 = \frac{2984.43 \text{s}}{\mathcal{F}t(1+\Delta_V^R)}$$

$$V_{ud}^{0^+-0^+} = 0.9737 (1)_{exp, nucl} (3)_{NS} (1)_{RC}[3]_{total}$$
Radiative Corrections to beta decay: 
Overall Setup
RC to beta decay: overall setup

Tree-level amplitude

\[ i = n, A(0^+) \quad f = p, A'(0^+) \]

Electron carries away energy \( E < \) Q-value of a decay

\[ e^\pm, \nu_e(\bar{\nu}_e) \sim V_{ud} \]

Radiative corrections to tree-level amplitude

\[ \sim \frac{\alpha}{2\pi} \approx 10^{-3} \]

Precision goal for \( V_{ud} \) extraction

\[ 1 \times 10^{-4} \]

Electron carries away energy \( E < \) Q-value of a decay

\[ \frac{\alpha}{2\pi} \left( \frac{E}{\Lambda}, \ln \frac{E}{\Lambda}, \ldots \right) \]

Energy scales \( \Lambda \)

Decay Q-value (endpoint energy)

\[ Q_{if} = M_i - M_f = 1 - 10 \text{ MeV} \]

Electron mass

\[ m_e \approx 0.5 \text{ MeV} \]

Weak boson scale

\( M_Z, M_W \approx 90 \text{ GeV} \)

Hadronic scale

\( \Lambda_{\text{had}} = 300 \text{ MeV} \)

Universal

Nuclear scale

\( \Lambda_{\text{nuc}} = 10 - 30 \text{ MeV} \)

Nuclear structure dependent (QCD)

Nucleus-specific

Nuclear structure independent (QED)
RC to beta decay: separating scales

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function (Dirac-Coulomb problem) + Sirlin function (soft Bremsstrahlung)

UV: large EW logs + pQCD corrections

Inner RC: energy- and model-independent

UV structure of SM

$\gamma W$-box: sensitive to all scales

New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear

UV-sensitive $\gamma W$-box on free neutron $\Delta^V_{R}$: Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = V_{ud}^2 \left[ 1 + \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_W} + \tilde{a}_g \right\} + \delta^{HO \text{QED}} + 2 \Box_{\gamma W} \right]$$

Nuclear structure: $\delta_{NS} = 2(\Box_{\gamma W}^{\text{Nucl}} - \Box_{\gamma W}^{\text{free n}})$

All non-enhanced terms $\sim \alpha/2\pi \sim 10^{-3}$ — only need to $\sim 10\%$
Long-Range QED Corrections to Beta Spectrum and ft-values

\[ \Lambda = 300 \text{ MeV} \]

\[ \Lambda_{\text{had}} = 300 \text{ MeV} \]

\[ \Lambda_{\text{nuc}} = 10 - 30 \text{ MeV} \]

Electron mass

\[ M_Z, M_W \sim 90 \text{ GeV} \]

Universal correction \( \Delta^V_R \)

Nuclear structure \( \delta_C, \delta_{\text{NS}} \)

\[ \text{Fermi function, corrections to beta spectrum} \]
QED: Corrections to Decay Spectrum

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, \vec{p}_e \, E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e) \]
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Unperturbed beta spectrum
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Unperturbed beta spectrum

Fermi function: e\(^+\) in Coulomb field of daughter nucleus
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Shape factor: spatial distribution of decay
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Unperturbed beta spectrum

Fermi function: e$^+$ in Coulomb field of daughter nucleus

Shape factor: spatial distribution of decay

Atomic screening and overlap corrections
QED: Corrections to Decay Spectrum

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \vec{p}_e E_e (E_0 - E_e)^2 \]

- Unperturbed beta spectrum
- Fermi function: \( e^+ \) in Coulomb field of daughter nucleus
- Shape factor: spatial distribution of decay
- Atomic screening and overlap corrections
- Recoil correction

\[ F(E_e)C(E_e)Q(E_e)R(E_e)r(E_e) \]
QED: Corrections to Decay Spectrum

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \overrightarrow{p}_e E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e) \]

- Unperturbed beta spectrum
- Fermi function: \( e^+ \) in Coulomb field of daughter nucleus
- Shape factor: spatial distribution of decay
- Atomic screening and overlap corrections
- Recoil correction
- Coulomb distortion numerically large: escapes the usual scaling \( \alpha/\pi \)
- Fermi function \( F_0 \sim Z\alpha\pi/\beta \) (coherent effect, Sommerfeld and \( \pi^2 \) enhancement)
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Since Fermi fn is of order 1 \( \rightarrow \) even small corrections should be assessed.

A myriad of corrections introduced/estimated by different people in past 9 decades!

\[ N(W) dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z, W) L_0(Z, W) U(Z, W) D_{FS}(Z, W, \beta_2) R(W, W_0) R_N(W, W_0, M) \times Q(Z, W) S(Z, W) X(Z, W) r(Z, W) C(Z, W) D_C(Z, W, \beta_2) pW(W_0 - W)^2 dW \]
QED: Corrections to Decay Spectrum

\[ f = m_e^{-5} \int \frac{E_0}{m_e} dE_e \vec{p}_e E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e) \]

Unperturbed beta spectrum

Fermi function: e+ in Coulomb field of daughter nucleus

Recoil correction

Shape factor: spatial distribution of decay

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Unified method of calculation (matching between them is well-defined)

Numerical solution of Dirac equation with inputs from nuclear theory and experiment
Nuclear Structure Inputs in ft

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \vec{p}_e \ E_e(E_0 - E_e)^2 F(E_e)C(E_e)Q(E_e)R(E_e)r(E_e) \]
Nuclear Structure Inputs in ft

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, \vec{p}_e \, E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e) \]
Fermi Fn: daughter nuclear charge form factor $F_{Ch}(q^2)$

\[
f = m_e^{-5} \int_{m_e}^{E_0} dE_e \quad \vec{p}_e \quad E_e(E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)
\]
Nuclear Structure Inputs in $ft$

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \ \vec{p}_e \ \frac{E_e(E_0 - E_e)^2}{E_e} F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

Shape factor: **nuclear weak CC transition FF** $F_{CW}(q^2)$
Nuclear Structure Inputs in $ft$

\[
f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, \vec{p}_e \, E_e(E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(e) r(E_e)\]

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

Shape factor: **nuclear weak CC transition FF** $F_{CW}(q^2)$

Charge form factors: combination of e-scattering, X-ray/laser/optical atom spectroscopy
Slope of the charge FF at origin: nuclear charge radius
Not all radii are known —> have to be guessed (theory)
Nuclear Structure Inputs in ft

\[ f = m_e^{-5} \int_{m_e}^{E_0} dE_e \, \vec{p}_e \, E_e(E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e) \]

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Charged-current weak transition form factors: only accessible with the decay itself (tough);
Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)
Typical result: very similar to charge FF
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Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)

Typical result: very similar to charge FF

New development:

use isospin symmetry and known charge radii to predict the weak transition radius!
Isospin symmetry + Charge Radii in $0^+$ isotriplet

0$^+$, $T = 1, T_z = -1$

0$^+$, $T = 1, T_z = 0$

0$^+$, $T = 1, T_z = 1$

CY Seng, 2212.02681
Isospin symmetry + Charge Radii in $0^+$ isotriplet

$0^+, T = 1, T_z = -1$
$R_{Ch,-1}$

$0^+, T = 1, T_z = 0$
$R_{Ch,0}$

$0^+, T = 1, T_z = 1$
$R_{Ch,1}$

CY Seng, 2212.02681
Isospin symmetry + Charge Radii in $0^+$ isotriplet

$0^+, T = 1, T_z = -1$

$R_{Ch,-1}$

$R_{CW}$

$0^+, T = 1, T_z = 0$

$R_{Ch,0}$

$R_{CW}$

$0^+, T = 1, T_z = 1$

$R_{Ch,1}$

CY Seng, 2212.02681
How is $R_{CW}$ related to $R_{Ch,Tz}$?

Charged-Current weak current: pure isovector
Electromagnetic current isovector + isoscalar

Remove isoscalar part:
Relate weak $\leftrightarrow$ charge radii

$$R_{CW}^2 = R_{Ch,1}^2 + Z_0(R_{Ch,0}^2 - R_{Ch,1}^2)$$

$$= R_{Ch,1}^2 + \frac{Z^{-1}}{2}(R_{Ch,-1}^2 - R_{Ch,1}^2)$$

Large factors $\sim Z$ multiply small differences
Isospin symmetry + Charge Radii in $0^+$ isotriplet

$0^+, \, T = 1, \, T_z = -1$

$R_{Ch,-1}$

$R_{CW}$

$0^+, \, T = 1, \, T_z = 0$

$R_{Ch,0}$

$R_{CW}$

$0^+, \, T = 1, \, T_z = 1$

$R_{Ch,1}$

How is $R_{CW}$ related to $R_{Ch,T_z}$?
Charged-Current weak current: pure isovector
Electromagnetic current isovector + isoscalar

Remove isoscalar part:
Relate weak $\rightarrow$ charge radii

$$R_{CW}^2 = R_{Ch,1}^2 + Z_0 (R_{Ch,0}^2 - R_{Ch,1}^2)$$

$$= R_{Ch,1}^2 + \frac{Z - 1}{2} (R_{Ch,-1}^2 - R_{Ch,1}^2)$$

Large factors $\sim Z$ multiply small differences

Photon probes the entire nuclear charge
Only the outer protons can decay: all neutron states in the core occupied
Isospin symmetry + Charge Radii in $T = 1$, $O^+$ isotriplet

Seng, 2212.02681  
MG, Seng 2311.16755

Weak radii differ significantly from $R_{ch}$
Shape factor $\rightarrow$ Fermi Fn $\rightarrow$ $f_t$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\langle r_{ch, -1}^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_{ch, 0}^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_{ch, 1}^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_{cw}^2 \rangle^{1/2}$ (fm)</th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>$^{10}_{6}$C</td>
<td>$^{10}_{5}$B(ex)</td>
<td>$^{10}_{4}$Be: 2.3550(170)$^a$</td>
<td>N/A</td>
</tr>
<tr>
<td>14</td>
<td>$^{14}_{8}$O</td>
<td>$^{14}_{7}$N(ex)</td>
<td>$^{14}_{6}$C: 2.5025(87)$^a$</td>
<td>N/A</td>
</tr>
<tr>
<td>18</td>
<td>$^{18}_{10}$Ne: 2.9714(76)$^a$</td>
<td>$^{18}_{9}$F(ex)</td>
<td>$^{18}_{8}$O: 2.7726(56)$^a$</td>
<td>3.661(72)</td>
</tr>
<tr>
<td>22</td>
<td>$^{22}_{12}$Mg: 3.0691(89)$^b$</td>
<td>$^{22}_{11}$Na(ex)</td>
<td>$^{22}_{10}$Ne: 2.9525(40)$^a$</td>
<td>3.596(99)</td>
</tr>
<tr>
<td>26</td>
<td>$^{26}_{14}$Si</td>
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<td>$^{26}_{12}$Mg: 3.0337(18)$^a$</td>
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<td>30</td>
<td>$^{30}_{16}$S</td>
<td>$^{30}_{15}$P(ex)</td>
<td>$^{30}_{14}$Si: 3.1336(40)$^a$</td>
<td>N/A</td>
</tr>
<tr>
<td>34</td>
<td>$^{34}_{18}$Ar: 3.3654(40)$^a$</td>
<td>$^{34}_{17}$Cl</td>
<td>$^{34}_{16}$S: 3.2847(21)$^a$</td>
<td>3.954(68)</td>
</tr>
<tr>
<td>38</td>
<td>$^{38}_{20}$Ca: 3.467(1)$^c$</td>
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<td>3.999(35)</td>
</tr>
<tr>
<td>42</td>
<td>$^{42}_{22}$Ti</td>
<td>$^{42}_{21}$Sc: 3.5702(238)$^a$</td>
<td>$^{42}_{20}$Ca: 3.5081(21)$^a$</td>
<td>4.64(39)</td>
</tr>
<tr>
<td>46</td>
<td>$^{46}_{24}$Cr</td>
<td>$^{46}_{23}$V</td>
<td>$^{46}_{22}$Ti: 3.6070(22)$^a$</td>
<td>N/A</td>
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<tr>
<td>50</td>
<td>$^{50}_{26}$Fe</td>
<td>$^{50}_{25}$Mn: 3.7120(196)$^a$</td>
<td>$^{50}_{24}$Cr: 3.6588(65)$^a$</td>
<td>4.82(39)</td>
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<tr>
<td>54</td>
<td>$^{54}_{28}$Ni: 3.738(4)$^e$</td>
<td>$^{54}_{27}$Co</td>
<td>$^{54}_{26}$Fe: 3.6933(19)$^a$</td>
<td>4.28(11)</td>
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<tr>
<td>62</td>
<td>$^{62}_{32}$Ge</td>
<td>$^{62}_{31}$Ga</td>
<td>$^{62}_{30}$Zn: 3.9031(69)$^b$</td>
<td>N/A</td>
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<tr>
<td>66</td>
<td>$^{66}_{34}$Se</td>
<td>$^{66}_{33}$As</td>
<td>$^{66}_{32}$Ge</td>
<td>N/A</td>
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<tr>
<td>70</td>
<td>$^{70}_{36}$Kr</td>
<td>$^{70}_{35}$Br</td>
<td>$^{70}_{34}$Se</td>
<td>N/A</td>
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<tr>
<td>74</td>
<td>$^{74}_{38}$Sr</td>
<td>$^{74}_{37}$Rb: 4.1935(172)$^b$</td>
<td>$^{74}_{36}$Kr: 4.1870(41)$^a$</td>
<td>4.42(62)</td>
</tr>
</tbody>
</table>
Isospin symmetry + Charge Radii in $T = 1, O^+$ isotriplet

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from $R_{ch}$
Shape factor $\rightarrow$ Fermi Fn $\rightarrow$ ft

<table>
<thead>
<tr>
<th>Transition</th>
<th>$(ft)_{HT}$ (s)</th>
<th>$(ft)_{new}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{18}$Ne$\rightarrow^{18}$F</td>
<td>2912 ± 79</td>
<td>2912 ± 80</td>
</tr>
<tr>
<td>$^{22}$Mg$\rightarrow^{22}$Na</td>
<td>3051.1 ± 6.9</td>
<td>3050.4 ± 6.8</td>
</tr>
<tr>
<td>$^{26}$Si$\rightarrow^{26m}$Al</td>
<td>3052.2 ± 5.6</td>
<td>3050.7 ± 5.6</td>
</tr>
<tr>
<td>$^{34}$Ar$\rightarrow^{34}$Cl</td>
<td>3058.0 ± 2.8</td>
<td>3057.1 ± 2.8</td>
</tr>
<tr>
<td>$^{38}$Ca$\rightarrow^{38m}$K</td>
<td>3062.8 ± 6.0</td>
<td>3062.2 ± 5.9</td>
</tr>
<tr>
<td>$^{42}$Ti$\rightarrow^{42}$Sc</td>
<td>3090 ± 88</td>
<td>3085 ± 86</td>
</tr>
<tr>
<td>$^{50}$Fe$\rightarrow^{50}$Mn</td>
<td>3099 ± 71</td>
<td>3098 ± 72</td>
</tr>
<tr>
<td>$^{54}$Ni$\rightarrow^{54}$Co</td>
<td>3062 ± 50</td>
<td>3063 ± 49</td>
</tr>
<tr>
<td>$^{26m}$Al$\rightarrow^{26}$Mg</td>
<td>3037.61 ± 0.67</td>
<td>3036.5 ± 1.0</td>
</tr>
<tr>
<td>$^{34}$Cl$\rightarrow^{34}$S</td>
<td>3049.43$^{+0.95}_{-0.88}$</td>
<td>3048.0 ± 1.1</td>
</tr>
<tr>
<td>$^{38m}$K$\rightarrow^{38}$Ar</td>
<td>3051.45 ± 0.92</td>
<td>3050.5 ± 1.1</td>
</tr>
<tr>
<td>$^{42}$Sc$\rightarrow^{42}$Ca</td>
<td>3047.7 ± 1.2</td>
<td>3045.0 ± 2.7</td>
</tr>
<tr>
<td>$^{50}$Mn$\rightarrow^{50}$Cr</td>
<td>3048.4 ± 1.2</td>
<td>3046.1 ± 3.6</td>
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<tr>
<td>$^{54}$Co$\rightarrow^{54}$Fe</td>
<td>3050.8$^{+1.4}_{-1.1}$</td>
<td>3051.3$^{+1.7}_{-1.4}$</td>
</tr>
<tr>
<td>$^{74}$Rb$\rightarrow^{74}$Kr</td>
<td>3082.8 ± 6.5</td>
<td>3086 ± 11</td>
</tr>
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New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%
Isospin symmetry + Charge Radii in $T = 1$, $O^+$ isoscal triplet

New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%

More -and more precise- charge radii necessary!
Working closely with exp. (PSI, FRIB, ISOLDE, TRIUMF)
Isospin symmetry + Charge Radii in $T = 1, O^+$isotriplet

Above treatment assumes isospin symmetry — but we know that it is slightly broken! Why isospin symmetry assumption is good enough?

Shape factor and finite size effects are ~small corrections to Fermi function
1-2% ISB effect on top of a RC may be assumed negligible (but needs to be tested)

Test requires that all 3 nuclear radii in the isotriplet are known;
Currently only the case for A=38 system

<table>
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<tr>
<td>A6</td>
<td>$^{46}_{21}$Cr</td>
<td>$^{46}_{20}$V</td>
<td>$^{46}<em>{19}$Ti, 3 $^{50}</em>{20}$V,a</td>
<td>N/A</td>
</tr>
</tbody>
</table>

ISB-sensitive combination

$$\Delta M_B^{(1)} = \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2 = 0 \quad \text{if isospin symmetry exact}$$

$$\frac{1}{2} \left( 20 \times 3.467(1)^2 + 18 \times 3.4028(19)^2 \right) - 19 \times 3.437(4)^2 = -0.00(12)(14)(52)$$

Improvement of K-38m radius necessary! (Plans at TRIUMF on IS K-38m, K-37?)
Isospin breaking in nuclear WF: $\delta_C$

Tree-level effect — ISB “large”
Isospin symmetry breaking in superallowed $\beta$-decay

Tree-level Fermi matrix element

$$M_F = \langle f \mid \tau^+ \mid i \rangle$$

$\tau^+$ — Isospin operator

$\langle \tau^+ \mid i \rangle$, $\langle f \rangle$ — members of T=1 isotriplet
Isospin symmetry breaking in superallowed $\beta$-decay

Tree-level Fermi matrix element

$$M_F = \langle f \mid \tau^+ \mid i \rangle$$

$\tau^+$ — Isospin operator

$i\rangle, \ f\rangle$ — members of $T=1$ isotriplet

If isospin symmetry were exact, $M_F \rightarrow M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states
(e.g. Coulomb, nucleon mass difference, ...)

In presence of isospin symmetry breaking (ISB):

$$M_F^2 = M_0^2 (1 - \delta_C)$$

MacDonald 1958

ISB correction almost singlehandedly aligns ft-values!

$$\delta_C \sim 0.17\% - 1.6\%$$!

Crucial for $V_{ud}$ extraction

*J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501*
Once all corrections are included:
CVC $\rightarrow$ Ft constant

Fit to 14 transitions:
Ft constant within 0.02%

Hardy, Towner 2020
Once all corrections are included: CVC → Ft constant

Fit to 14 transitions: Ft constant within 0.02%

Hardy, Towner 2020

If BSM scalar currents present: Fierz interference $b_F$

$$\mathcal{F}_t^{SM} \rightarrow \mathcal{F}_t^{SM} \left(1 + b_F \frac{m_e}{\langle E_e \rangle} \right)$$

$Q_{EC} \uparrow$ with $Z$ → effect of $b_F \downarrow$ with $Z$

Introduces nonlinearity in the Ft plot

$b_F = -0.0028(26) \sim$ consistent with 0
Nuclear model dependence of $\delta_C$

**J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501**

<table>
<thead>
<tr>
<th>RPA</th>
<th>SM-WS</th>
<th>SM-HF</th>
<th>PKO1</th>
<th>DD-ME2</th>
<th>PC-F1</th>
<th>IVMRa</th>
<th>DFT</th>
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</thead>
<tbody>
<tr>
<td>$T_z = -1$</td>
<td></td>
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<td></td>
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<tr>
<td>$^{10}$C</td>
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<td>0.150</td>
<td>0.109</td>
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<td>$^{14}$O</td>
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<td>$^{22}$Mg</td>
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<td>$^{34}$Ar</td>
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<td>0.379</td>
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<tr>
<td>$^{38}$Ca</td>
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<td>0.313</td>
<td>0.441</td>
<td>0.347</td>
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<td>$T_z = 0$</td>
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<tr>
<td>$^{26}$Al</td>
<td>0.310</td>
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<td>$^{34}$Cl</td>
<td>0.650</td>
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<td>0.234</td>
<td>0.307</td>
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<td>$^{38}$mK</td>
<td>0.670</td>
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<td>$^{42}$Sc</td>
<td>0.665</td>
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<td>0.448</td>
<td>0.345</td>
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<tr>
<td>$^{46}$V</td>
<td>0.620</td>
<td>0.600</td>
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<td>0.580</td>
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<tr>
<td>$^{50}$Mn</td>
<td>0.645</td>
<td>0.610</td>
<td></td>
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<td></td>
<td>0.550</td>
<td></td>
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<tr>
<td>$^{64}$Co</td>
<td>0.770</td>
<td>0.685</td>
<td>0.319</td>
<td>0.393</td>
<td>0.339</td>
<td>0.638</td>
<td></td>
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<tr>
<td>$^{62}$Ga</td>
<td>1.475</td>
<td>1.205</td>
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<td>0.882</td>
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<tr>
<td>$^{74}$Rb</td>
<td>1.615</td>
<td>1.405</td>
<td>1.088</td>
<td>1.258</td>
<td>0.668</td>
<td>1.770</td>
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<tr>
<td>$\chi^2/\nu$</td>
<td>1.4</td>
<td>6.4</td>
<td>4.9</td>
<td>3.7</td>
<td>6.1</td>
<td>4.3</td>
<td></td>
</tr>
</tbody>
</table>

**L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324**

![Graph showing $\delta_C$ dependence on nuclear model](image)

HT: $\chi^2$ as criterion to prefer SM-WS; $V_{ud}$ and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio $\delta_C$ calculations (NCSM, GFMC, CC, IMSRG)

Especially interesting for light nuclei accessible to different techniques!
Constraints on $\delta_C$ from nuclear radii

ISB-sensitive combinations of radii can be constructed

$$\Delta M_B^{(1)} \equiv \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2$$

$\Delta M_B^{(1)} = 0$ used for ft-value in isospin limit

$$\Delta M_A^{(1)} \equiv - \langle r_{CW}^2 \rangle + \left( \frac{N_1}{2} \langle r_{n,1}^2 \rangle - \frac{Z_1}{2} \langle r_{p,1}^2 \rangle \right)$$

Neutron radius: measurable with PV e-$\bar{\nu}$ scattering!
Constraints on $\delta_C$ from nuclear radii

ISB-sensitive combinations of radii can be constructed

$$\Delta M_B^{(1)} \equiv \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2$$

$$\Delta M_A^{(1)} \equiv - \langle r_{CW}^2 \rangle + \left( \frac{N_1}{2} \langle r_{n,1}^2 \rangle - \frac{Z_1}{2} \langle r_{p,1}^2 \rangle \right)$$

$\Delta M_B^{(1)} = 0$ used for ft-value in isospin limit

Neutron radius: measurable with PV e⁻ scattering!

Z-boson couples to neutrons, photon - to protons;
PV asymmetry at low $Q^2$ sensitive to the difference $\langle r_{n,1}^2 \rangle - \langle r_{p,1}^2 \rangle$ - neutron skin

Extensive studies in neutron rich nuclei (PREX, CREX) $\rightarrow$ input to physics of neutron stars

$$A^{PV} = - \frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \frac{Q_W}{Z} \frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$R_{NW} \approx R_n$
Constraints on $\delta_C$ from nuclear radii

ISB-sensitive combinations of radii can be constructed

$$\Delta M_B^{(1)} \equiv \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2$$

$$\Delta M_A^{(1)} \equiv -\left< r_{CW}^2 \right> + \left( \frac{N_1}{2} \left< r_{n,1}^2 \right> - \frac{Z_1}{2} \left< r_{p,1}^2 \right> \right)$$

\[ \Delta M_B^{(1)} = 0 \text{ used for ft-value in isospin limit} \]

Neutron radius: measurable with PV $e^-$ scattering!

Z-boson couples to neutrons, photon - to protons;
PV asymmetry at low $Q^2$ sensitive to the difference $\left< r_{n,1}^2 \right> - \left< r_{p,1}^2 \right> -$ neutron skin

Extensive studies in neutron rich nuclei (PREX, CREX) --- input to physics of neutron stars

Upcoming exp. program at Mainz (MREX)
Neutron skins of stable daughters (e.g. Mg-26, Ca-42, Fe-54)
PV asymmetry on C-12 for a sub-% measurement of $R_n$
Unexpected connections via neutron skins:
ISB for precision tests vs. EoS of neutron-rich matter

\[ R_{NW} \approx R_n \]

\[ A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{Q_W}{Z} \frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)} \]

Unified Formalism for $\Delta^V_R$ and $\delta_{NS}$

Dispersion Theory of the $\gamma W$-box
Universal RC from dispersion relations

UV large log — model independent (Parton model + pQCD)
Sensitivity to nonperturbative QCD: inclusive hadron spectrum

Model dependence: interference $\gamma W$ structure functions

$$\text{Im} T^{\mu\nu}_{\gamma W} = \ldots + \frac{i e^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F^W_3(x, Q^2)$$

After some algebra

$$\square^{b,e}_{\gamma W}(E_e) = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{M^2_W}{M^2_W + Q^2} \int_{\nu_{\text{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 2\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^2} \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^2)$$

$$\square^{b,o}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^\infty dQ^2 \int_{\nu_{\text{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 3\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^3} \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^3)$$

Structure functions are measurable or may be related to data
Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC $\gamma_W$ SF (no data) $\longleftrightarrow$ Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current
Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC $\gamma W$ SF (no data) $\leftarrow\rightarrow$ Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

Marciano, Sirlin 2006: $\Delta^V_R = 0.02361(38) \rightarrow V_{ud} = 0.97420(10)_{FT}(18)_{RC}$

DR (Seng et al. 2018): $\Delta^V_R = 0.02467(22) \rightarrow V_{ud} = 0.97370(10)_{FT}(10)_{RC}$
Input into dispersion integral - $\nu/\bar{\nu}$ data

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Shift upwards by $3\sigma$ + reduction of uncertainty by factor 2
Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC $\gamma W$ SF (no data) $\longleftrightarrow$ Purely CC WW SF (inclusive neutrino data)

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Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38) \longrightarrow V_{ud} = 0.97420(10)_{\text{Fr}}(18)_{\text{RC}}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22) \longrightarrow V_{ud} = 0.97370(10)_{\text{Fr}}(10)_{\text{RC}}$

Shift upwards by 3$\sigma$ + reduction of uncertainty by factor 2

Confirmed by lattice QCD:

LQCD on pion + pheno: $\Delta_R^V = 0.02477(24)_{\text{LQCD}^\pi + \text{pheno}}$

LQCD on neutron: $\Delta_R^V = 0.02439(19)_{\text{LQCD}^n}$

Seng, MG, Feng, Jin, 2003.11264
Yoo et all, 2305.03198
Ma, Feng, MG et al 2308.16755
EFT: scale separation for free \( n \)

Effective Field Theory: explicit separation of scales + RGE running between

\[ \text{SM} \rightarrow \text{LEFT (no H,t,Z,W)} \rightarrow \text{ChPT} \rightarrow \text{NR QED} \]

Formal consistency built in, RGE, transparent error estimation (naturalness)

Precision limited by matching (LEC) and HO — relies on inputs (e.g. \( \gamma W \)-box from DR)

To improve: need to go to higher order — new LECs, still tractable?

At present: order \( O(\alpha, \alpha_s, \alpha^2) \) — realistic to go beyond?

\[
\frac{d\Gamma_n}{dE_e} = \frac{G_F^2 |V_{ud}|^2}{(2\pi)^5} \left(1 + 3\lambda^2\right) p_e E_e (E_0 - E_e)^2 \left[ g_V(\mu_\chi) \right]^2 F_{NR}(\beta) \left(1 + \delta_{RC}(E_e, \mu_\chi)\right) \left(1 + \delta_{\text{recoil}}(E_e)\right)
\]

\( \lambda = g_A/g_V \)

Extract from Experiment

\( \text{vector coupling} \)

\( \pi^2, 1/\beta \)

Enhanced

\( O(\alpha) \)

[no logs]

\( O(m_e/m_N) \)

Total RC: \( 1 + \Delta_{\text{TOT}} = 1.07761(27) \) %

Total RC from DR: \( 1 + \Delta_{\text{TOT}} = 1.07735(27) \) %

Good agreement within errors!
Nuclear-Structure RC $\delta_{NS}$
History of $\delta_{NS}$: $\gamma W$-box on nuclei

Jaus, Rasche 1990
$\gamma$ and $W$ on same nucleon $\rightarrow$ already in $\Delta^V_R$: drop!

Towner 1994
Nucleons are bound — free-nucleon RC modified: $\delta^A_{NS}$

Jaus, Rasche 1990; Hardy, Towner 1992-2020
$\gamma$ and $W$ on distinct nucleons $\rightarrow$ only in nuclei: $\delta^B_{NS}$

Implementation:
Nuclear shell model with “semi-empirical” Woods-Saxon potential
One-body nucleon currents only (axial + magnetic)
No nuclear Green’s function between the em and weak vertices
Parameters fixed to reproduce selected properties within each isotriplet
Predictive power questionable, but tailored to the task
Systematic uncertainty unclear and hard to quantify
\[ \delta_{NS} \text{ from dispersion relations} \]

Same formulas for free neutron and nuclei:

\[
\square_{\gamma W}^{b,e}(E_e) = \frac{\alpha}{\pi} \int_0^{\infty} dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_{\nu_{\text{thr}}}^{\infty} \frac{d\nu'}{\nu'} \left( \nu' + 2\sqrt{\nu'^2 + Q^2} \right) \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + O(E_e^2)
\]

\[
\square_{\gamma W}^{b,o}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^{\infty} dQ^2 \int_{\nu_{\text{thr}}}^{\infty} \frac{d\nu'}{\nu'} \left( \nu' + 3\sqrt{\nu'^2 + Q^2} \right) \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + O(E_e^3)
\]

NS correction reflects extraction of the free box

\[ \delta_{NS} = 2[ \square_{\gamma W}^{VA, \nucl} - \square_{\gamma W}^{VA, \text{free n}} ] \]
\[ \delta_{NS} \text{ from dispersion relations} \]

Same formulas for free neutron and nuclei;

\[
\square_{\gamma W}^{b,e}(E_e) = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_{\nu_{thr}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 2 \sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^2} \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + O(E_e^2)
\]

\[
\square_{\gamma W}^{b,o}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^\infty dQ^2 \int_{\nu_{thr}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 3 \sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^3} \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + O(E_e^3)
\]

NS correction reflects extraction of the free box
DR: a framework to control this subtraction!

\[ \delta_{NS} = 2[ \square_{\gamma W}^{\nu A,\text{nucl}} - \square_{\gamma W}^{\nu A,\text{free n}} ] \]

differences due to:

Richer excitation spectrum in nuclei

Different quantum numbers (spin, isospin)
\[ \delta_{NS} \text{ in ab-initio nuclear theory} \]


Low-momentum part of the loop: account for nucleon d.o.f. only
First case study: \(^{10}\text{C} \rightarrow ^{10}\text{B}\) in No-Core Shell Model (NCSM)
Many-body problem in HO basis with separation \(\Omega\) and up to \(N = N_{\text{max}} + N_{\text{Pauli}}\)

- Nuclear interactions from Chiral EFT:
  - \(\text{NN-N}^4\text{LO} + 3N_{\text{lnl}}\)
  - \(\text{NN-N}^4\text{LO} + 3N_{\text{*lnl}}^*\)

Evaluate the m.e. of nuclear Green's function

\[ G(z) \equiv \frac{1}{z - H_0} \]

Difficulty: Inverting a large matrix!

Lanczos continuous fraction method
Ab-initio $\delta_{NS}$: numerical results

Numerical results

From $\text{res}, T_3$

$\delta_{NS} = -0.390\%$

$10^C \rightarrow 10^B$

Different nuclear forces cause substantial re-distribution between different contributions, but small change to the sum.

$NN - N^4LO(500) + 3N_{\text{nli}}$

$\delta_{NS} = -0.411\%$

$10^C \rightarrow 10^B$
Ab-initio $\delta_{NS}$: numerical results

Large negative contribution: low-lying 1+ level in $^{10}$B
Large GT and M1 rates favor a two-step process

$^{10}$C $\rightarrow$ $^{10}$B

$\delta_{NS} = -0.390 \%$

$^{10}$C $\rightarrow$ $^{10}$B

$\delta_{NS} = -0.411 \%$
Ab-initio $\delta_{NS}$: numerical results

Large negative contribution: low-lying $1^+$ level in $^{10}\text{B}$
Large GT and M1 rates favor a two-step process

Check $\Omega$-independence and convergence w.r.t. $N_{\text{max}}$

Final result for $^{10}\text{C} \rightarrow ^{10}\text{B}$:

$$\delta_{NS} = -0.406(39)\%$$

arXiv: 2405.19281

Compare to Hardy-Towner (old-fashion SM)

$$\delta_{NS} = -0.347(35)\% \quad (2014)$$
$$\delta_{NS} = -0.400(50)\% \quad (2020)$$
Ab-initio $\delta_{NS}$ in EFT: $^{14}O \rightarrow ^{14}N$ with Variational Monte Carlo

V. Cirigliano et al, arXiv: 2405.18469

\[
\delta_{NS}^{(0)} = -(1.76+0.11\pm0.88) \cdot 10^{-3}
\]

Uncertainty: assuming unknown counter term to be of "natural size"

\[
g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)
\]

Compare to Hardy-Towner 2020: $\delta_{NS,B} = -(1.96(50) \cdot 10^{-3}$

Promising avenue: all logs under control and are consistent

Downside: EFT non-renormalizable $\rightarrow$ unknown counter terms external to the theory

Need extra input (dispersion theory; explicit modeling; fit to data)
Summary & Outlook

Cabibbo Angle Anomaly at 2-3σ

Nuclear uncertainties under scrutiny: $δ_{NS}$ in ab-initio and EFT
$δ_{C}$ & $δ_{NS}$ for 15 decays from $^{10}C$ to $^{74}Rb$ — Community effort required!

Future experiments:

Neutron: UCNτ, τSPECT ($δτ_n : 0.4 \rightarrow 0.1s$); PERC, Nab ($δg_A : 4 \rightarrow 1 \times 10^{-4}$)
  Competitive! But: resolve existing discrepancies (e.g. “beam-bottle” lifetime)
Kaon decays: NA62, BELLE II $K\ell 3$ vs $K\mu 2$ (+ Lattice effort!)
Pion: $\pi^+ \rightarrow \pi^0 e^+\nu$ PIONEER @ PSI ($δBR: 0.3\% \rightarrow 0.03\%$)

Nuclear charge radii across superallowed isotriplets
  Stable: μ-atoms @ PSI, radii of unstable nuclei @ ISOLDE, TRIUMF
Neutron skins of stable daughters with PVES @ MESA
  Interplay with the nuclear EoS program: neutron skin via symmetry energy vs. ISB

Cabibbo anomaly interpretable in terms of BSM

Superallowed decays: bounds on scalar BSM from dataset consistency
International workshop on **Electroweak Precision Intersections**  
**EPIC 2024**  
September 22-27 2024, Cala Serena Beach Resort (Geremeas)

Bring together different communities:  
Particle, Nuclear, Atomic, Neutrino, Astro, GW  
Study existing synergies & elaborate new ones!  
1-day pre-workshop school for PhD students  
1st event this year, plan for biennial workshop series
BSM searches with superallowed beta decays

Induced scalar CC $\rightarrow$ Fierz interference $bF$

$$\mathcal{F}_t^{SM} \rightarrow \mathcal{F}_t^{SM} \left( 1 + b_F \frac{m_e}{\langle E_e \rangle} \right)$$

$$b_F = -0.0028(26) \sim \text{consistent with 0}$$

Independently of $V_{ud}$ and CKM unitarity: internal consistency of the data base with SM!
Like $\Delta^{(3)}_{\text{CKM}}$ does not require other experimental inputs to make a statement on (B)SM

Entangled with nuclear theory uncertainties — a global effort of nuclear theory community needed

S, T interaction flips helicity:
Suppressed at high energy

Beta decay vs. LHC on S,T
Complementarity now and in the future!

Gonzalez-Alonso et al 1803.08732
**V_{ud}** from neutron decay

Neutron decay: 2 measurements needed

\[ V_{ud}^2 = \frac{5024.7 \text{ s}}{\tau_n (1 + 3 g_A^2)(1 + \Delta V_R)} \]

Pre-2018: \( \Delta V_R = 0.02361(38) \) *Marciano, Sirlin PRL 2006*
Post-2018: \( \Delta V_R = 0.02479(21) \) *MG, Seng Universe 2023*

**RC** \( \Delta V_R \): bottleneck since 40 years

Since 2018: DR+data+pQCD+EFT+LQCD
\( \Delta V_R \) uncertainty: factor 2 reduction

**Experiment:** factor 3-5 uncertainties improvement; discrepancies in \( \tau_n \) and \( g_A \)

- \( g_A = -1.27641(56) \) *PERKEO-III B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*
- \( g_A = -1.2677(28) \) *aSPECT M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*
- \( \tau_n = 877.75(28)^{+16}_{-12} \) *UCN\( \tau \) F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*
- \( \tau_n = 887.7(2.3) \) *BL1 (NIST) Yue et al, PRL 111 (2013) 222501*

**PDG average**

\[ V_{ud}^{\text{free n}} = 0.9743 (3) \tau_n (8) g_A (1)_{RC[9]} \text{total} \]

**Single best measurements only**

\[ V_{ud}^{\text{free n}} = 0.9740 (2) \tau_n (3) g_A (1)_{RC[4]} \text{total} \]

Future exp coming! RC under control
V_{ud} from semileptonic pion decay

Pion decay $\pi^+ \rightarrow \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$V_{ud}^2 = \frac{0.9799}{(1 + \delta)} \frac{\Gamma_{\pi \ell^3}}{0.3988(23) \text{ s}^{-1}}$$

$$V_{ud} = 0.9739(27)_{\exp} (1)_{RC}$$

RC to semileptonic pion decay

$\delta$ uncertainty: factor 3 reduction

ChPT: $\delta = -0.0334(10)_{\text{LEC}(3)_{HO}}$ Cirigliano et al, 2003; Passera et al, 2011

DR + LQCD + ChPT: $\delta = 0.0332(1)_{\gamma W(3)_{HO}}$ Feng et al, 2020; Yoo et al, 2023

Future exp: 1 o.o.m. (PIONEER @ PSI)
Status of Cabibbo Unitarity

\[ K\ell^3 : \quad V_{ud} = \sqrt{1 - V_{us}^2} \]

\[ K\mu^2 : \quad V_{ud} = 1/\sqrt{1 + V_{us}/V_{ud}^2} \]

\[ V_{ud} \quad [0^+ - 0^+] \]

\[ \tau_n \quad [\text{Beam}] \]

\[ \tau_n \quad [\text{UCN}] \]

\[ \tau_n^{av} \quad [\text{PDG 2022}] \]

\[ \delta\Lambda \quad [\text{PERKEO - III}] \]

\[ \delta\Lambda^{av} \quad [\text{PDG 2022}] \]
Currently available neutrino data at low $Q^2$ - low quality;
Look for alternative input — compute Nachtmann moment $M_3^{(0)}$ on the lattice

First direct LQCD computation $\pi^- \rightarrow \pi^0 e^- \nu_e$  
Feng, MG, Jin, Ma, Seng 2003.09798
Currently available neutrino data at low $Q^2$ - low quality;  
Look for alternative input — compute Nachtmann moment $M^{(0)}_3$ on the lattice

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Feng, MG, Jin, Ma, Seng 2003.09798

First lattice QCD calculation

5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations w. 2+1 flavor domain wall fermion

Quark contraction diagrams
\[ \gamma W \]-box from DR + Lattice QCD input

Currently available neutrino data at low \( Q^2 \) - low quality; Look for alternative input — compute Nachtmann moment \( M^0_3 \) on the lattice

First direct LQCD computation \( \pi^- \to \pi^0 e^- \nu_e \)

5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations w. 2+1 flavor domain wall fermion

Match onto pQCD at \( Q^2 \sim 2 \text{ GeV}^2 \)

\[ \square V_{A, \pi}^{\gamma W} = 2.830(11)_{\text{stat}}(26)_{\text{sys}} \]

Independent calculation by Los Alamos group

\[ \square V_{A, \pi}^{\gamma W} = 2.810(26)_{\text{stat+sys}} \]
First lattice QCD calculation of $\gamma W$-box

Direct impact for pion decay $\pi^+ \rightarrow \pi^0 e^+\nu_e$

Previous calculation of $\delta$ — in ChPT

Significant reduction of the uncertainty!

$$V_{ud}^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi e^3}}{0.3988(23) \text{s}^{-1}}$$

Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

$$\delta : \ 0.0334(10)_{\text{LEC(3)HO}} \rightarrow 0.0332(1)_{\gamma W(3)_{\text{HO}}}$$
First lattice QCD calculation of $\gamma_W$-box

Direct impact for pion decay $\pi^+ \rightarrow \pi^0 e^+ \nu_e$

Previous calculation of $\delta$ — in ChPT

Significant reduction of the uncertainty!

Indirectly constrains the free neutron $\gamma_W$-box — requires some phenomenology

Based on Regge universality & factorization

Independent confirmation

$$\Delta_Y^R = 0.02467(22)_{DR} \rightarrow 0.02477(24)_{LQCD+DR}$$

Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

$$V_{ud}^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi e^3}}{0.3988(23) \text{s}^{-1}}$$

$\delta : 0.0334(10)_{\text{LEC}(3)_{HO}} \rightarrow 0.0332(1)_{\gamma_W(3)_{HO}}$

Seng, MG, Feng, Jin, 2003.11264
First LQCD calculation of $\gamma W$-box on the neutron

Much more challenging than pion:

Numerically heavier
Excited state contamination requires longer time
Large contribution from low $Q \sim g_A \mu^V$ absent for pion
First LQCD calculation of $\gamma W$-box on the neutron

Much more challenging than pion:

Numerically heavier
Excited state contamination requires longer time
Large contribution from low $Q \sim g_A \mu^V$ absent for pion

Split into long/short distance separated by $t_s$

$$M_n(Q^2) = M_n^{SD}(Q^2, t_s) + M_n^{LD}(Q^2, t_s, t_g)$$

RBC/UKQCD 2+1 domain wall fermion

<table>
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<tr>
<th>Ensemble</th>
<th>$m_{\pi}$ [MeV]</th>
<th>$L$</th>
<th>$T$</th>
<th>$a^{-1}$ [GeV]</th>
<th>$N_{conf}$</th>
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<td>24D</td>
<td>142.6(3)</td>
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<td>207</td>
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<tr>
<td>32D-fine</td>
<td>143.6(9)</td>
<td>32</td>
<td>64</td>
<td>1.378(5)</td>
<td>69</td>
</tr>
</tbody>
</table>

$$\Delta_R^V = 0.02439(19)_{\text{LQCD}} \text{ vs } 0.02467(22)_{\text{DR}}$$

The result slightly lower than DR;
Finer lattice calculations underway

Ma, Feng, MG et al 2308.16755