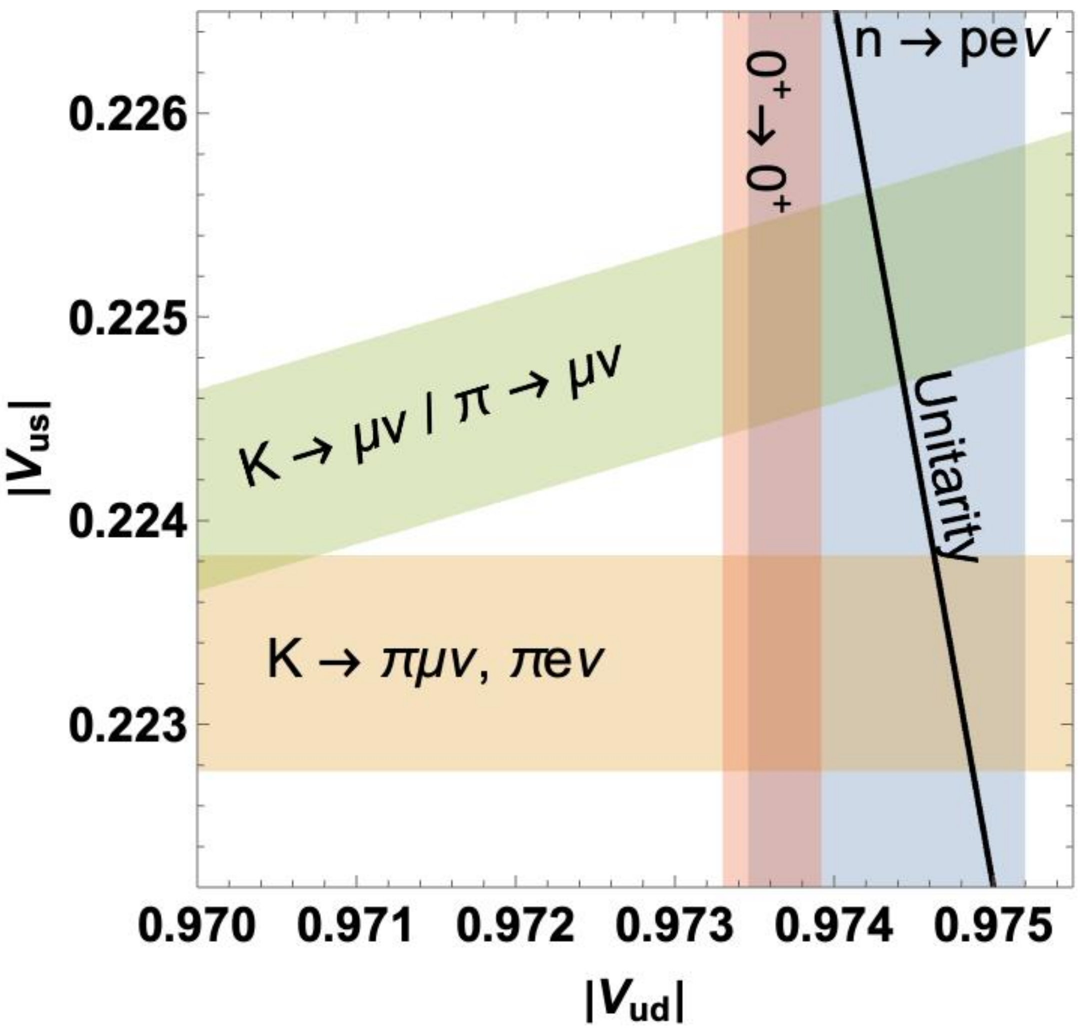
Precision Tests of the Standard Model with Cabibbo Unitarity and Nuclear β-Decays



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Collaborators:

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Michael Gennari

Mehdi Drissi

Petr Navratil

Neutron beta decay review: MG, Seng, Universe **2023**, 9(9), 422, arXiv:**2307.01145**

Nuclear beta decay review: MG, Seng, Ann.Rev.Part.Nucl.Sci. 74 (2024) 23-47, arXiv:**2311.00044**

QNP 2024, Barcelona 8-12 July, 2024

Outline

Beta decay, radiative corrections and the Standard Model

Cabibbo anomaly and BSM

Radiative corrections to β-decays: overall setup

Fermi function and nuclear radii

Isospin breaking correction

γW-box: Dispersion Theory, lattice QCD and EFT

Nuclear structure correction

Summary & Outlook

What to work on to win a Nobel prize?

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Beta decay has been an excellent choice for a century!

1896- Becquerel discovers spontaneous radioactivity of uranium, identified β with the electron

1898- Curie-Sklodowska, Curie discover polonium and radium

1903

1899- Rutherford systematized α, β, γ rays, identified α with He-4

1908

1934- F.&I. Joliot-Curie discovered β^+ decay with β^+ - positron

1935

1956- Lee & Yang proposed parity non conservation in β-decay, confirmed by Wu experiment

1957

1961- Glashow proposed electroweak unification

1967- Weinberg & Salam implemented Higgs mechanism

1973- Neutral weak current discovered at CERN

1979

1973- Kobayashi, Maskawa: 3-flavor quark mixing matrix

2008

That was the bright side…

Niepce de Saint-Victor: observed radioactivity in 1857
cited in Becquerel-father's book

Cox, McIlwraith, Kurrelmeier (1928); Chase (1929-30)
"Apparent evidence of polarization in a beam of beta rays"

1930: Pauli postulated existence of neutrinos
1934: Fermi formulated the contact theory of beta decay

1938: Klein predicted $M_W \sim \sqrt{4\pi\alpha\sqrt{2}/G_F} \sim 100\, GeV$

1957: Wu's experiment was crucial to prove Lee-Yang's
conjecture, but Chien-Shiung Wu was not awarded the NP

1963- Cabibbo: proposed 2-flavor quark mixing
to reconcile μ, β, K decay rates

Precision Era: V-A + Radiative Corrections

V - A theory (Sudarshan&Marshak and Gell-Mann&Feynman 1957); S-PS not excluded

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Muon lifetime $\tau_\mu = 2196980.3(2.2) ps$ —> Fermi constant $G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

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Kinoshita, Sirlin, Behrends, …

UV cut-off

1-loop RC to spectrum: $$\Delta P^0 d^3 p = \frac{\alpha}{2\pi} P^0 d^3 p \left[6 \ln \frac{\Lambda}{M_p} + \text{finite} \right]$$

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Is weak interaction universal for leptons and hadrons?

1967: Sirlin applied current algebra:
general UV behavior of β decay rate at 1-loop $\frac{\alpha}{2\pi} P^0 d^3p \; 3[1+2\bar{Q}] \ln(\Lambda/M)$

$\bar{Q}$: average charge of fields involved: $1 + 2\bar{Q}_{\mu,\nu_\mu} = 0$ but $1 + 2\bar{Q}_{n,p} = 2$

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Eventually, massive W-boson renders RC to beta decay UV-finite

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Cabibbo: strength shared between 2 generations

$$G_V^{\Delta S=0} = \cos\theta_C G_\mu$$

$$G_V^{\Delta S=1} = \sin\theta_C G_\mu$$

Cabibbo unitarity: $\cos^2\theta_C + \sin^2\theta_C = 1$

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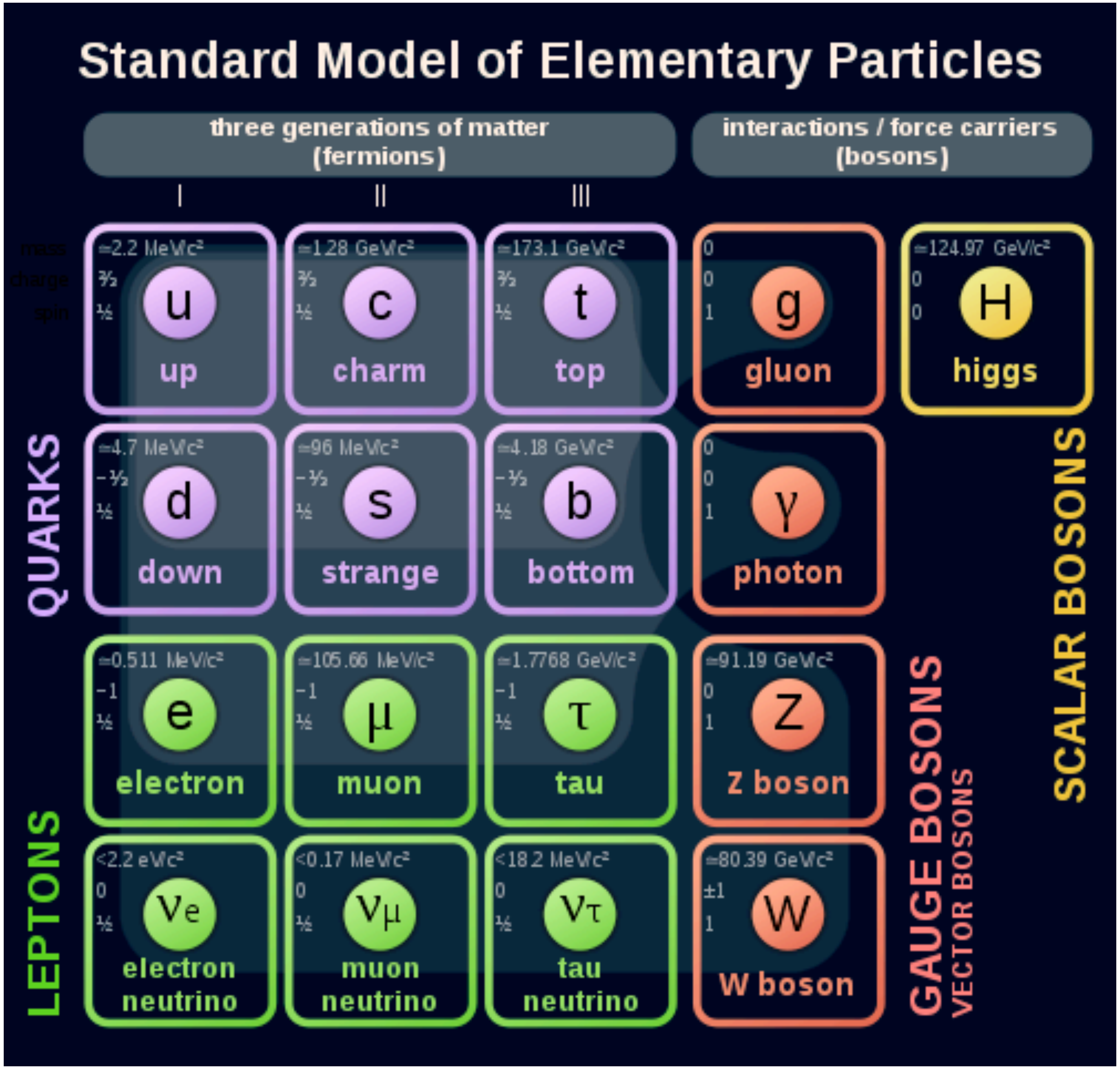
Kobayashi & Maskawa: 3 flavors + CP violation — CKM matrix V

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - completeness of the SM: $VV^\dagger = \mathbf{1}$

Top row unitarity constraint: $V_{ud}\ ^2 + V_{us}\ ^2 + V_{ub}\ ^2 = 1$

Detailed understanding of β decays largely shaped the Standard Model



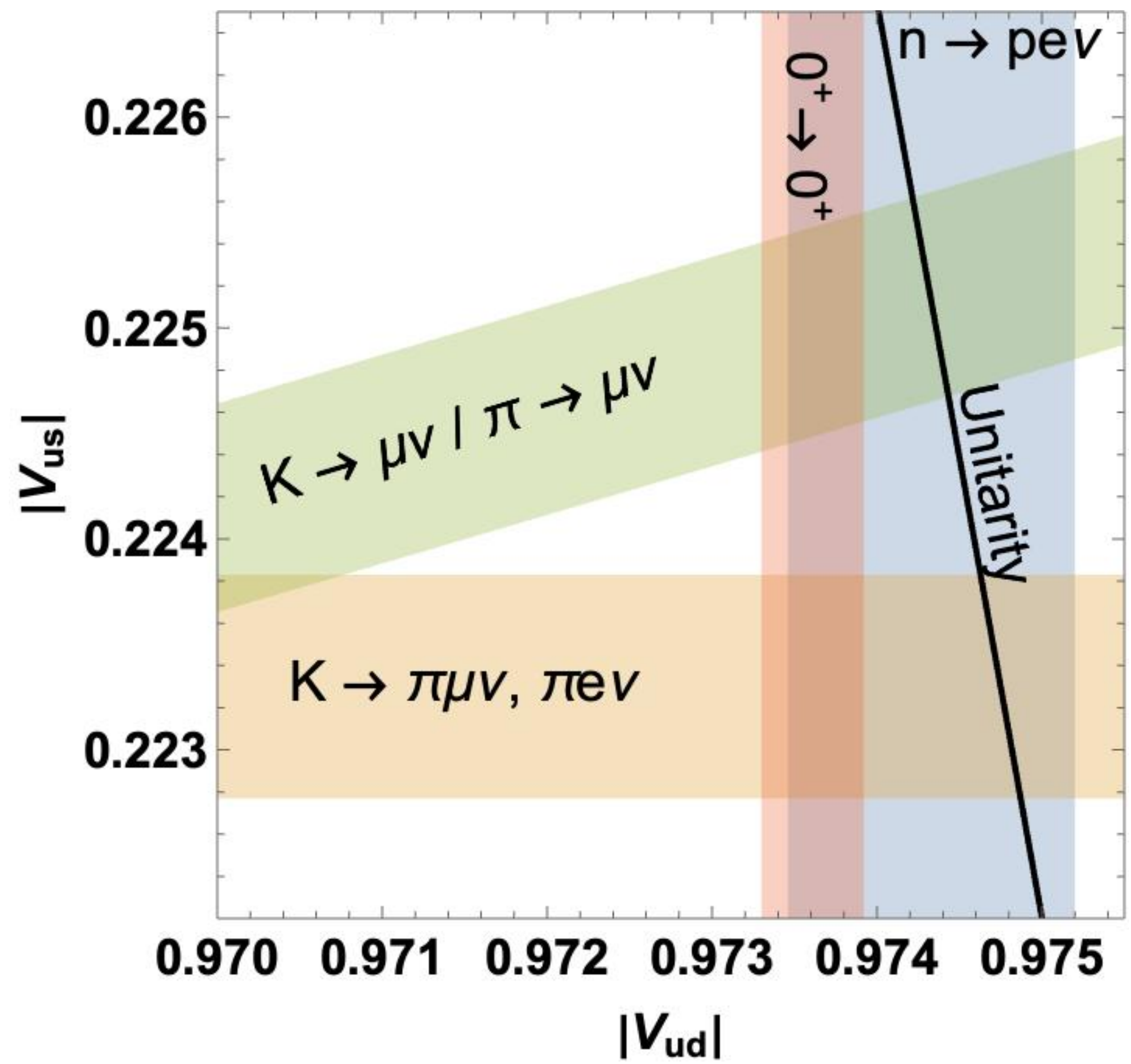
Cabibbo Angle Anomaly:
Status and BSM interpretation

Status of Cabibbo unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 $\sim 10^{-5}$

V_{ud} and V_{us} determinations
inconsistent with the SM



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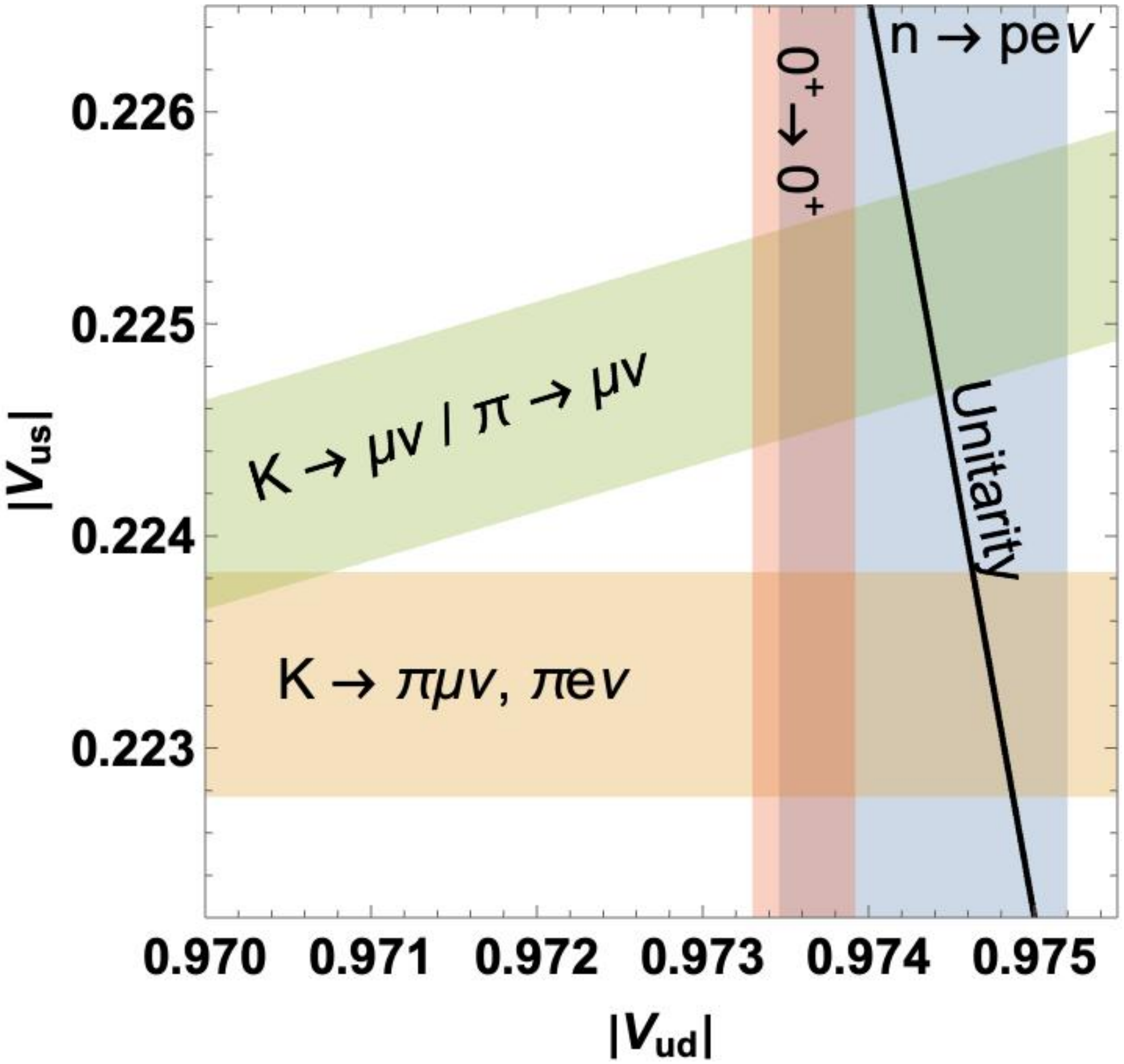

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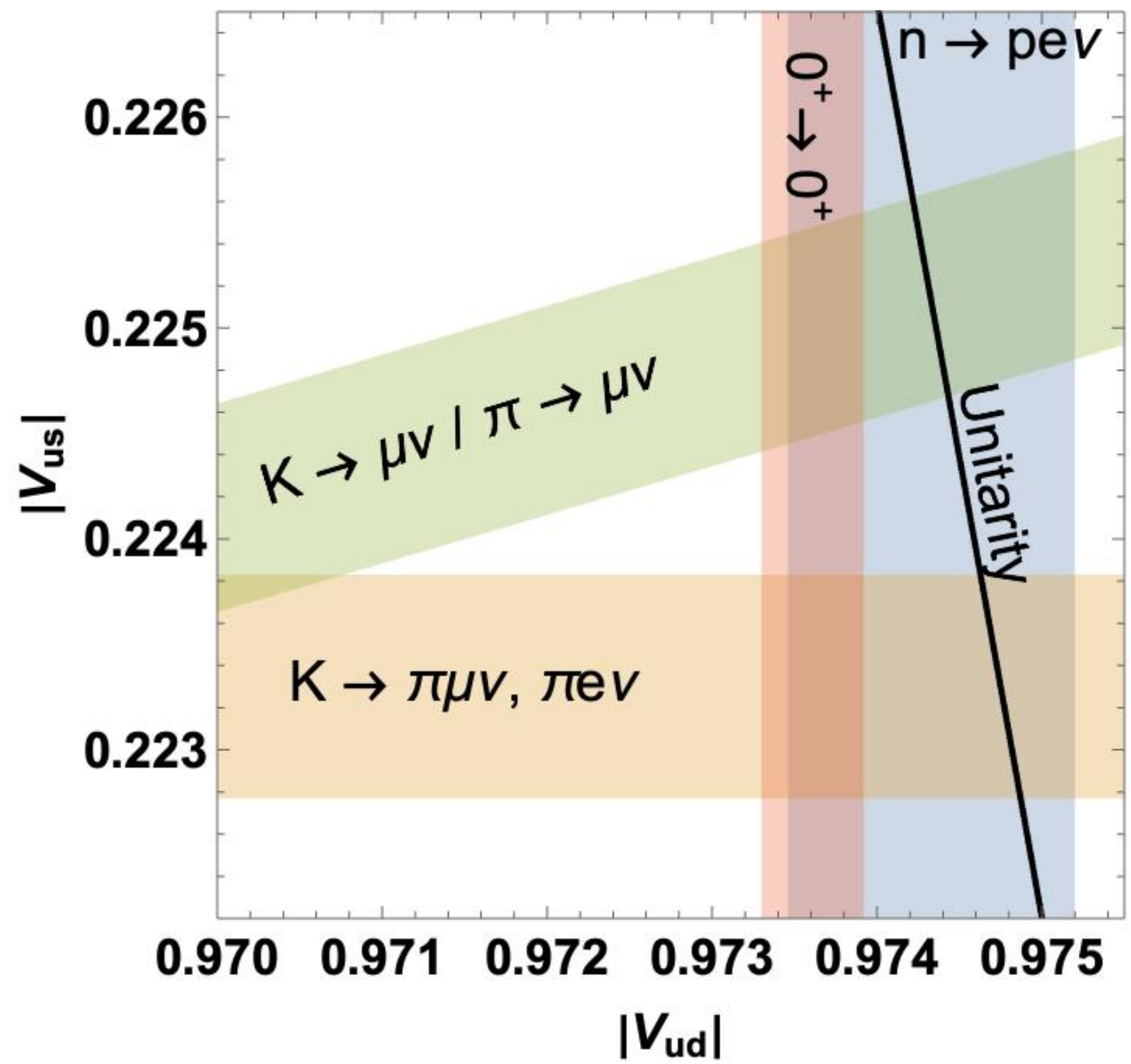
At variance with kaon decays + Cabibbo unitarity

$K \to \pi \ell \nu$: $\quad V_{us} = 0.2233(5)$

$$\text{Unitarity} \to \quad V_{ud} = \sqrt{1 - |V_{us}|^2} = 0.9747(1)$$



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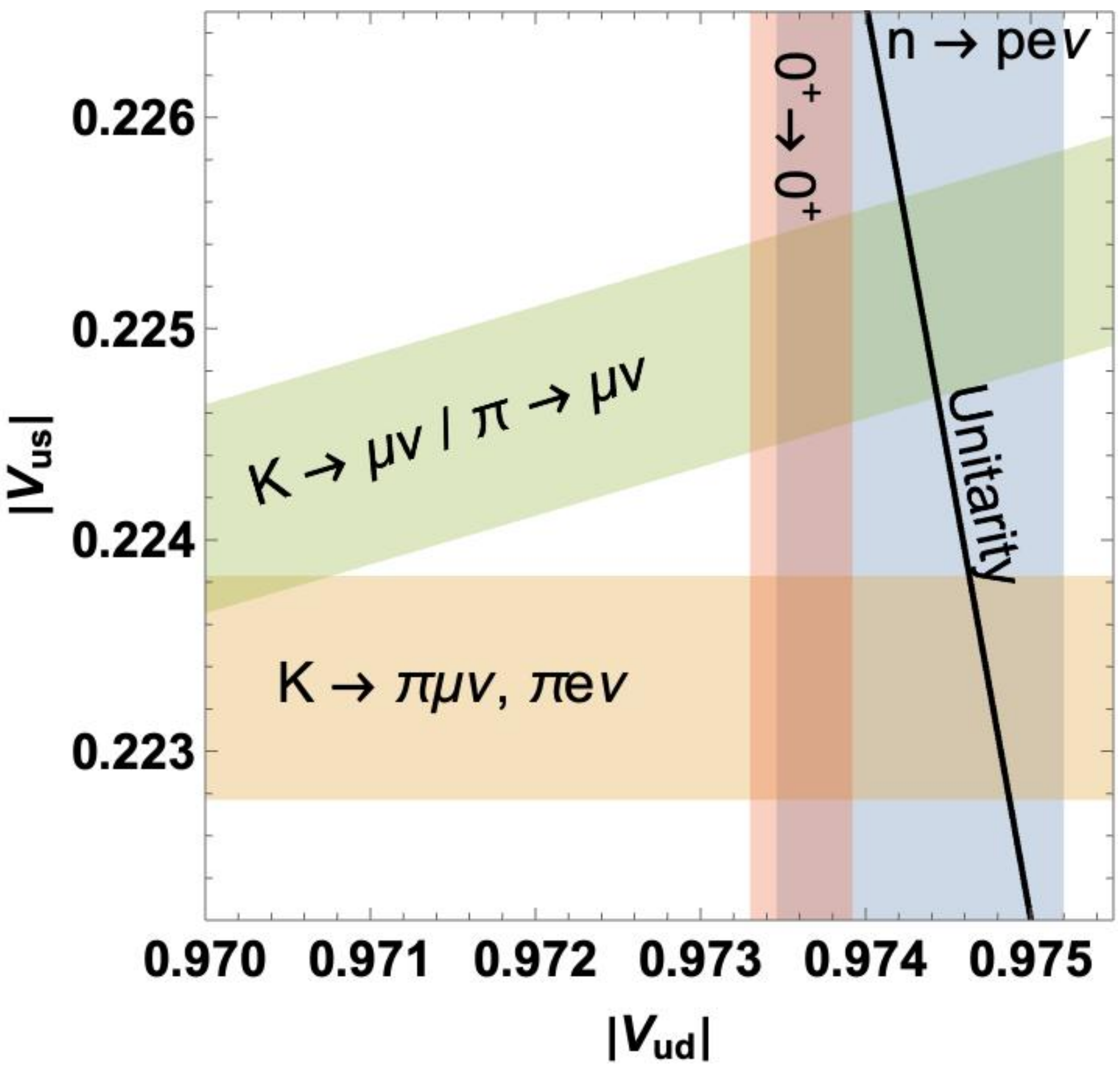
$\frac{K \to \mu\nu}{\pi \to \mu\nu}$: $V_{us}/V_{ud} = 0.2311(5)$

Unitarity $\to$ $V_{ud} = [1 + |V_{us}/V_{ud}|^2]^{-1/2} = 0.9743(1)$



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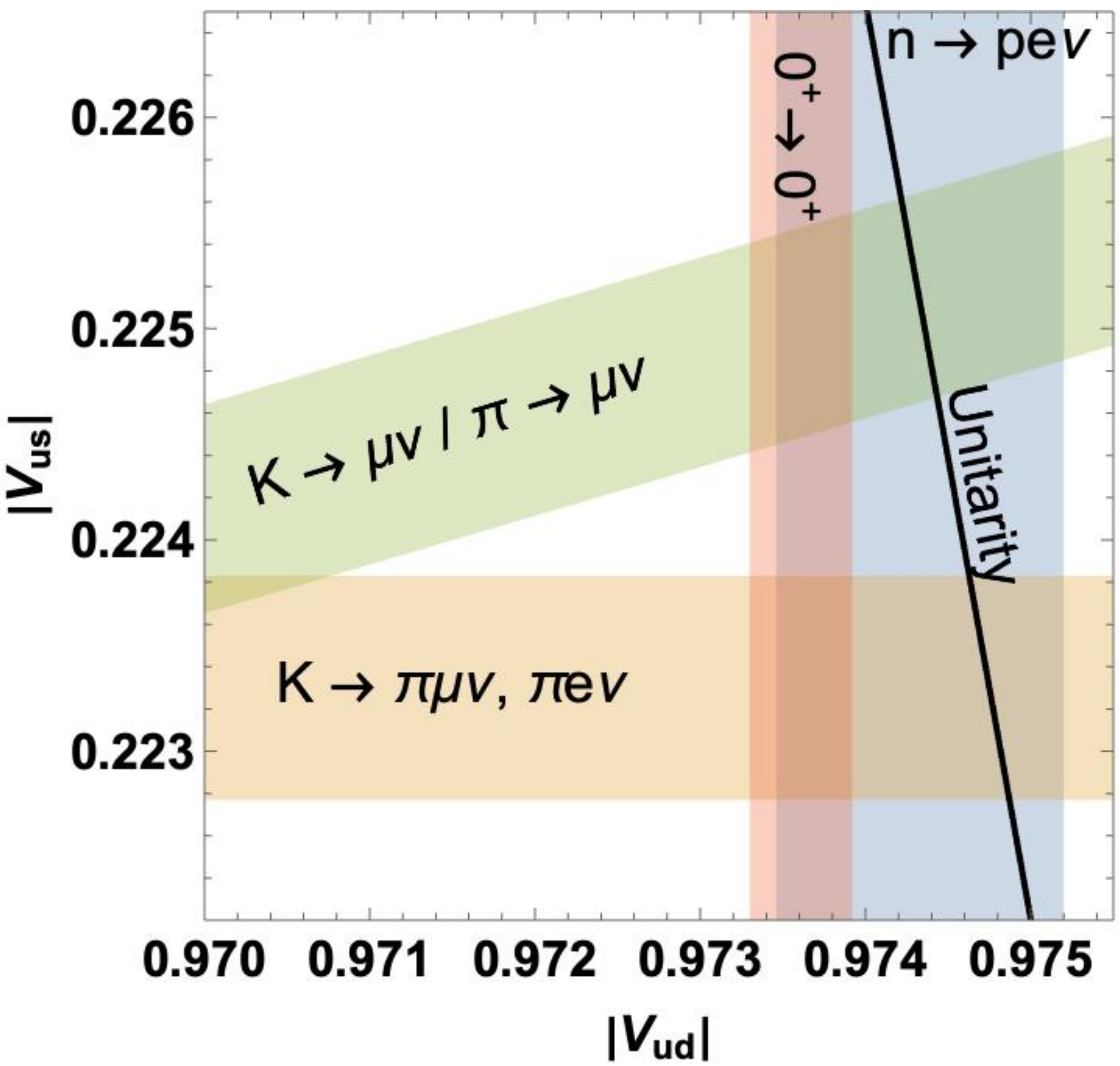

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PDG $[S = 2.5]$: $\quad |V_{us}| = 0.2243(8)$

Unitarity $\to$ $|V_{ud}| = 0.9745(2)$

CAA summary - 3 anomalies!

3 observables: $\lvert V_{us}\rvert^{K\ell 3}$, $\lvert V_{us}/V_{ud}\rvert^{K\mu 2}$, V_{ud} 2 quantities to determine: V_{us}, V_{ud}	▶	**3 ways to test unitarity**

$$\Delta^{(1)}_{\mathrm{CKM}} = |V_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\mathrm{CKM}} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$$\Delta^{(3)}_{\mathrm{CKM}} = |V^{K_{\ell 3}}_{us}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

Can it be a signal of BSM?

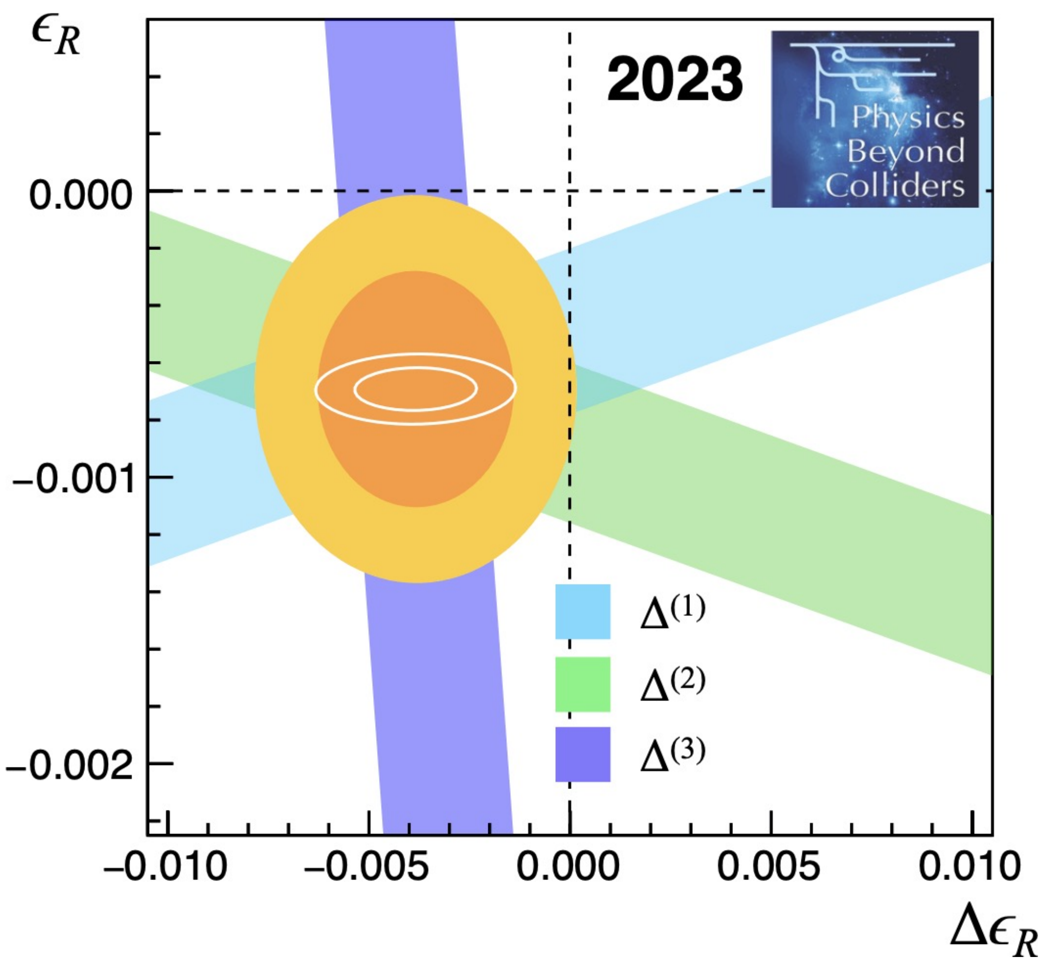
CAA in presence of RH currents

Cirigliano et al.
PLB 838 (2023)

- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\text{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\text{CKM}} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\text{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1 + \Delta^{(2)}_{\text{CKM}}}{1 + \Delta^{(3)}_{\text{CKM}}} \right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$



From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ

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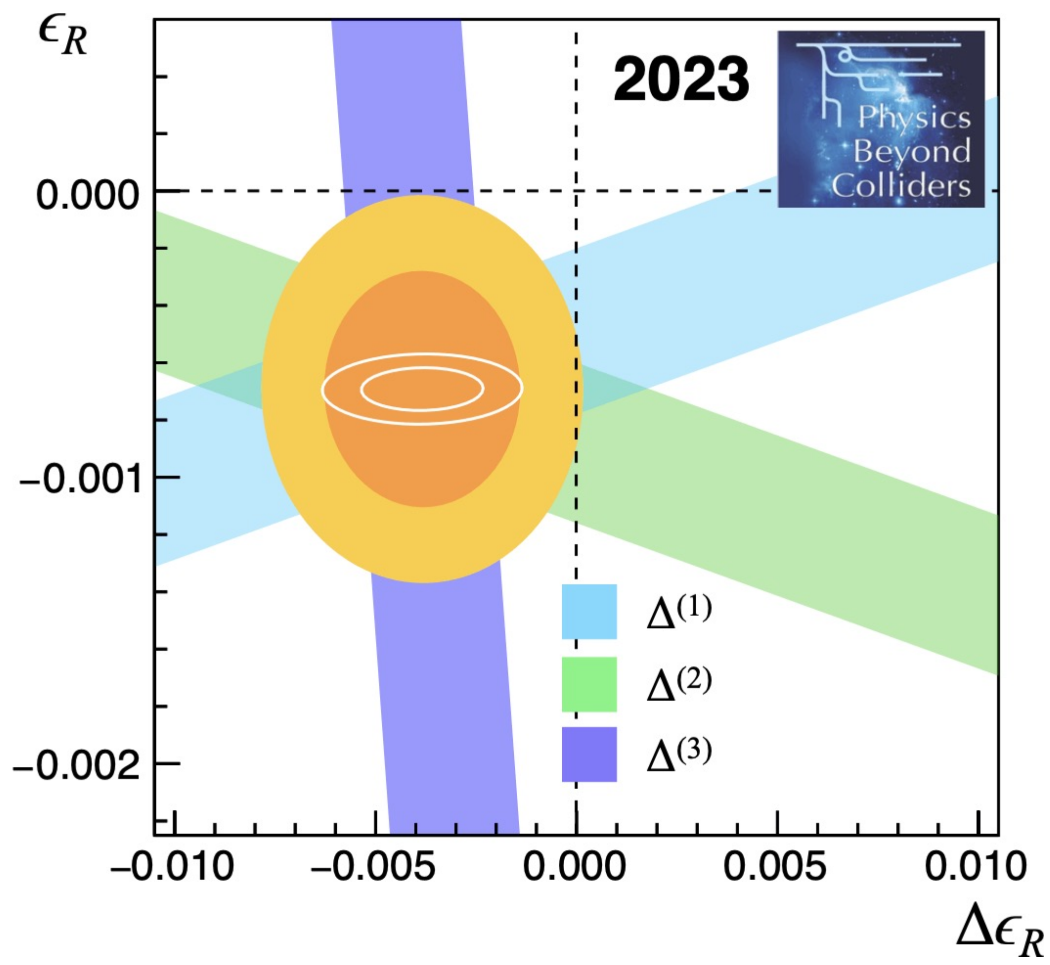


Are all SM corrections under control?

The path from kaon decays to V_{us}

V_{us} / V_{ud} from $K\mu 2 = K \to \mu\nu$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2}\left(1 - \frac{1}{2}\delta_{EM} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm{}_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm{}_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking

$f_K/f_\pi \to f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

LQCD+EM ($N_f = 2 + 1 + 1$):

$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$

Di Carlo et al, 2019

LQCD ($N_f = 2 + 1 + 1$):

$f_K/f_\pi = 1.1978(22)$

FLAG 2021 average

$$V_{us}/V_{ud} = 0.23108(23)_{\text{exp}}(42)_{\text{lat}}(16)_{\text{IB}}$$
$$(51)_{\text{tot}} = 0.22\ \%$$

V_{us} from $K\ell 3 = K \to \pi e \nu, \pi\mu\nu$

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)

$$K_{\ell 3}: \quad V_{us} = 0.22330(35)_{\rm exp}(39)_{\rm lat}(8)_{\rm IB} \; (53)_{\rm tot} = 0.24\,\%$$

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:
- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\rm EM}$ Form-factor correction for long-distance EM effects

$f_+(0)$ LQCD ($N_f = 2+1$): $f_+(0) = 0.9677(27)$

LQCD ($N_f = 2+1+1$): $f_+(0) = 0.9698(17)$

FLAG 2021 averages

Long-distance EM RC $\Delta_{K\ell}^{EM}$: new approach (ChPT + Current Algebra + LQCD)

Uncertainty reduced by 1 o.o.m. — under control

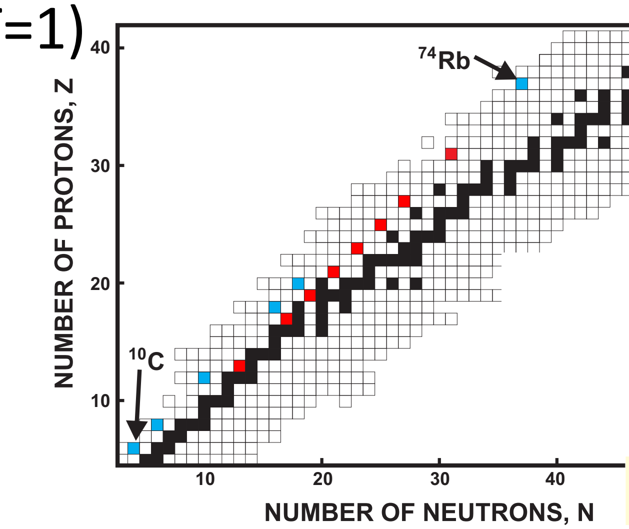
Seng, Galviz, Meißner 1910.13208
Seng, Galviz, MG, Meißner 2103.04843
Seng, Galviz, MG, Meißner 2203.05217
Feng, MG, Jin, Ma, Seng 2003.09798
Ma, Feng, MG, Jin, Seng 2102.12048

	$\delta_{\rm EM}^{K\ell}$ [10^{-3}]	ChPT [10^{-3}]
$K^0 e$	$11.6(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{e^2p^4}$	$9.9(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ e$	$2.1(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(1)_{e^2p^4}$	$1.0(1.9)_{e^2p^4}(1.6)_{\rm LEC}$
$K^0 \mu$	$15.4(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$14.0(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ \mu$	$0.5(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$0.2(1.9)_{e^2p^4}(1.6)_{\rm LEC}$

The path from nuclear beta decays to V_{ud}

V_{ud} from superallowed $0^+ - 0^+$ nuclear decays

1. Transitions within $J^P=0^+$ isotriplets (T=1)
2. Elementary process: p—>ne$^+\nu$
3. Only conserved vector current
4. 15 measured to better than 0.2%
5. Internal consistency as a check
6. SU(2) good —> corrections ~small



${}^{10}_{6}\mathrm{C} \to {}^{10}_{5}\mathrm{B}$
${}^{14}_{8}\mathrm{O} \to {}^{14}_{7}\mathrm{N}$
${}^{18}_{10}\mathrm{Ne} \to {}^{18}_{9}\mathrm{F}$
${}^{22}_{12}\mathrm{Mg} \to {}^{22}_{11}\mathrm{Na}$
${}^{26}_{14}\mathrm{Si} \to {}^{26}_{13}\mathrm{Al}$
${}^{30}_{16}\mathrm{S} \to {}^{30}_{15}\mathrm{P}$
${}^{34}_{18}\mathrm{Ar} \to {}^{34}_{17}\mathrm{Cl}$
${}^{38}_{20}\mathrm{Ca} \to {}^{38}_{19}\mathrm{K}$
${}^{42}_{22}\mathrm{Ti} \to {}^{42}_{21}\mathrm{Sc}$
${}^{46}_{24}\mathrm{Cr} \to {}^{46}_{23}\mathrm{V}$
${}^{50}_{26}\mathrm{Fe} \to {}^{50}_{25}\mathrm{Mn}$
${}^{54}_{28}\mathrm{Ni} \to {}^{54}_{27}\mathrm{Co}$

${}^{26m}_{13}\mathrm{Al} \to {}^{26}_{12}\mathrm{Mg}$
${}^{34}_{17}\mathrm{Cl} \to {}^{34}_{16}\mathrm{S}$
${}^{38m}_{19}\mathrm{K} \to {}^{38}_{18}\mathrm{Ar}$
${}^{42}_{21}\mathrm{Sc} \to {}^{42}_{20}\mathrm{Ca}$
${}^{46}_{23}\mathrm{V} \to {}^{46}_{22}\mathrm{Ti}$
${}^{50}_{25}\mathrm{Mn} \to {}^{50}_{24}\mathrm{Cr}$
${}^{54}_{27}\mathrm{Co} \to {}^{54}_{26}\mathrm{Fe}$
${}^{62}_{31}\mathrm{Ga} \to {}^{62}_{30}\mathrm{Zn}$
${}^{66}_{33}\mathrm{As} \to {}^{66}_{32}\mathrm{Ge}$
${}^{70}_{35}\mathrm{Br} \to {}^{70}_{34}\mathrm{Se}$
${}^{74}_{37}\mathrm{Rb} \to {}^{74}_{36}\mathrm{Kr}$

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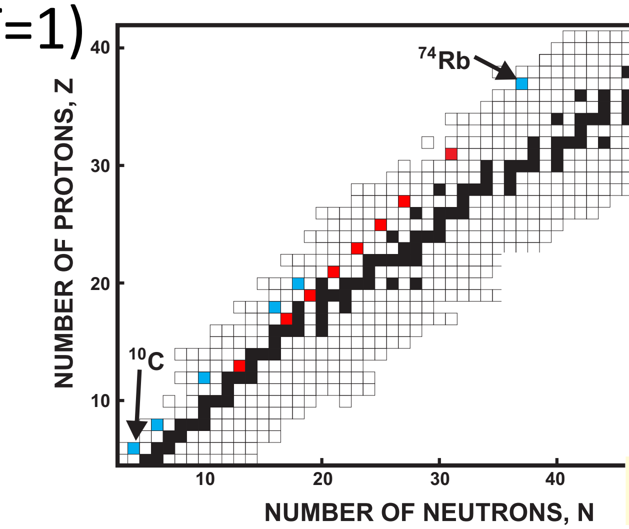
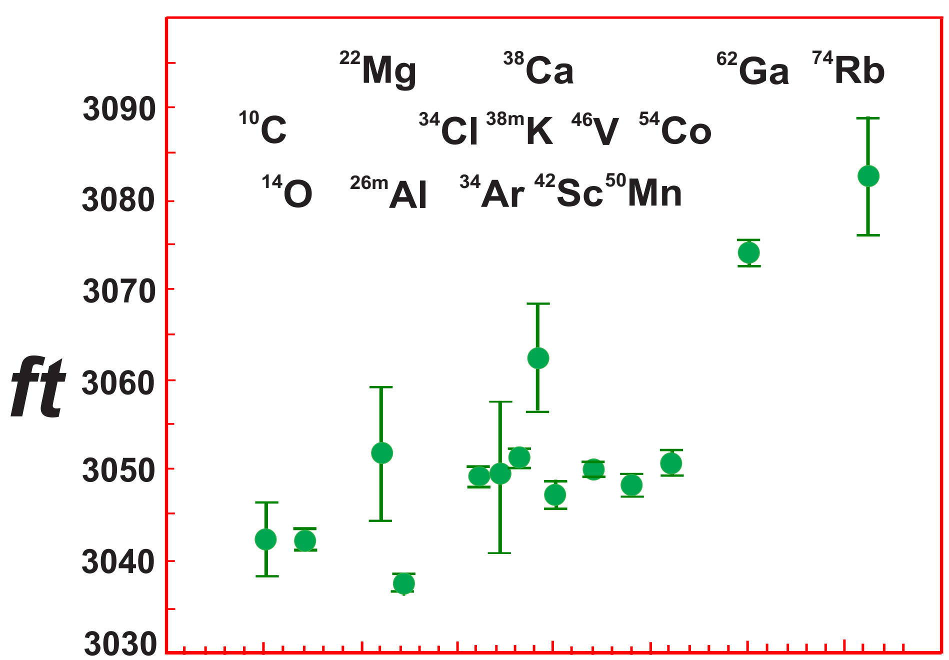


$^{10}_{6}\mathrm{C} \to ^{10}_{5}\mathrm{B}$	$^{26m}_{13}\mathrm{Al} \to ^{26}_{12}\mathrm{Mg}$
$^{14}_{8}\mathrm{O} \to ^{14}_{7}\mathrm{N}$	$^{34}_{17}\mathrm{Cl} \to ^{34}_{16}\mathrm{S}$
$^{18}_{10}\mathrm{Ne} \to ^{18}_{9}\mathrm{F}$	$^{38m}_{19}\mathrm{K} \to ^{38}_{18}\mathrm{Ar}$
$^{22}_{12}\mathrm{Mg} \to ^{22}_{11}\mathrm{Na}$	$^{42}_{21}\mathrm{Sc} \to ^{42}_{20}\mathrm{Ca}$
$^{26}_{14}\mathrm{Si} \to ^{26}_{13}\mathrm{Al}$	$^{46}_{23}\mathrm{V} \to ^{46}_{22}\mathrm{Ti}$
$^{30}_{16}\mathrm{S} \to ^{30}_{15}\mathrm{P}$	$^{50}_{25}\mathrm{Mn} \to ^{50}_{24}\mathrm{Cr}$
$^{34}_{18}\mathrm{Ar} \to ^{34}_{17}\mathrm{Cl}$	$^{54}_{27}\mathrm{Co} \to ^{54}_{26}\mathrm{Fe}$
$^{38}_{20}\mathrm{Ca} \to ^{38}_{19}\mathrm{K}$	$^{62}_{31}\mathrm{Ga} \to ^{62}_{30}\mathrm{Zn}$
$^{42}_{22}\mathrm{Ti} \to ^{42}_{21}\mathrm{Sc}$	$^{66}_{33}\mathrm{As} \to ^{66}_{32}\mathrm{Ge}$
$^{46}_{24}\mathrm{Cr} \to ^{46}_{23}\mathrm{V}$	$^{70}_{35}\mathrm{Br} \to ^{70}_{34}\mathrm{Se}$
$^{50}_{26}\mathrm{Fe} \to ^{50}_{25}\mathrm{Mn}$	$^{74}_{37}\mathrm{Rb} \to ^{74}_{36}\mathrm{Kr}$
$^{54}_{28}\mathrm{Ni} \to ^{54}_{27}\mathrm{Co}$	

Exp.: **f** - phase space (Q value)

t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within ~2% but not exactly!

Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
   (proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_{\mathrm{R}}^{\mathrm{V}}) = \mathrm{ft}(1 + \delta_{\mathrm{R}}')(1 - \delta_{\mathrm{C}} + \delta_{\mathrm{NS}})(1 + \Delta_{\mathrm{R}}^{\mathrm{V}})$$

~ Measured QED Isospin-breaking Nuclear structure Universal RC

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

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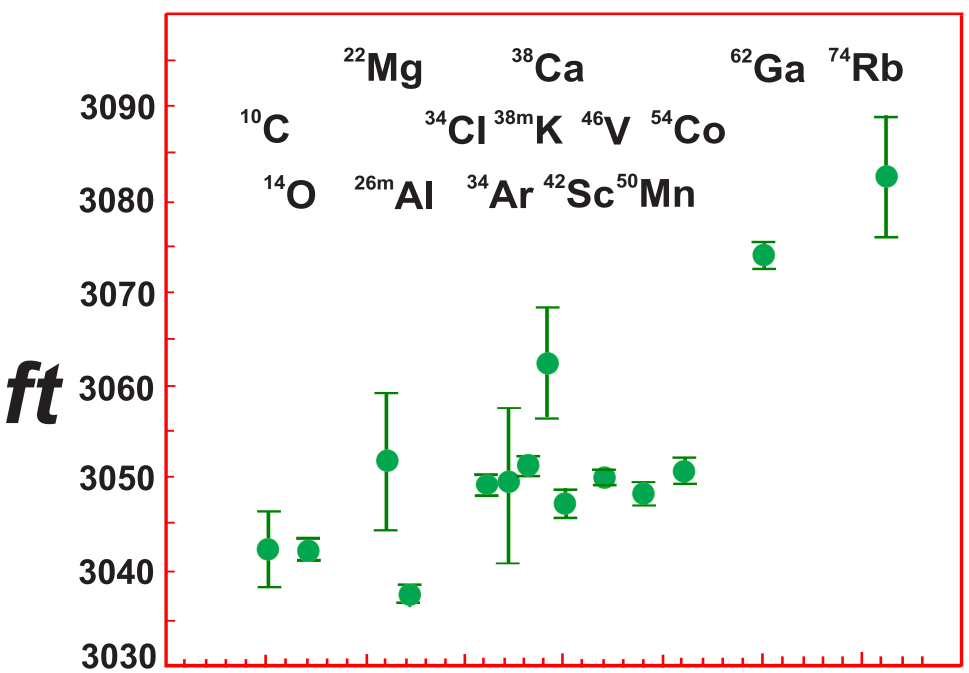
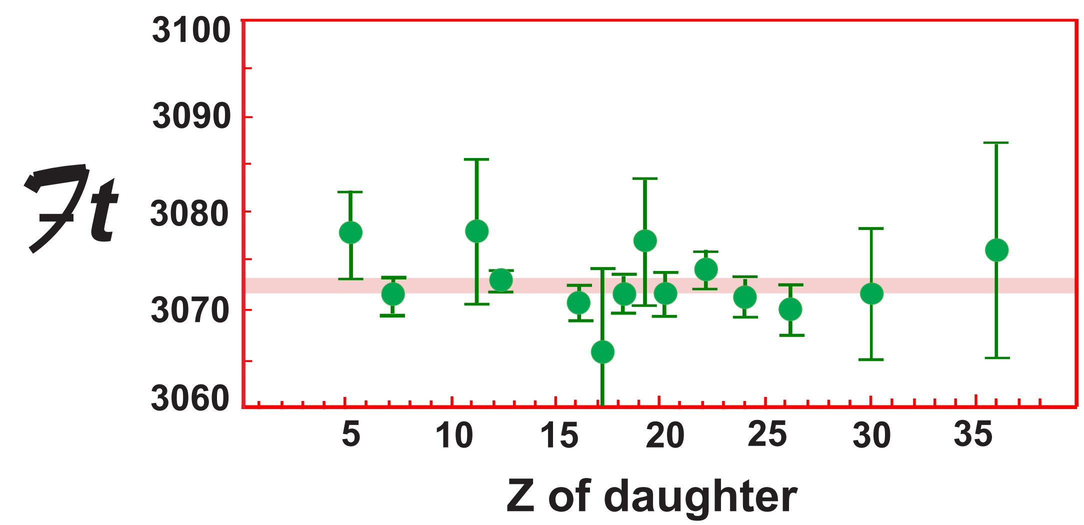
~ Measured QED Isospin-breaking Nuclear structure Universal RC





Average of 14 decays Hardy, Towner 1972 - 2020

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

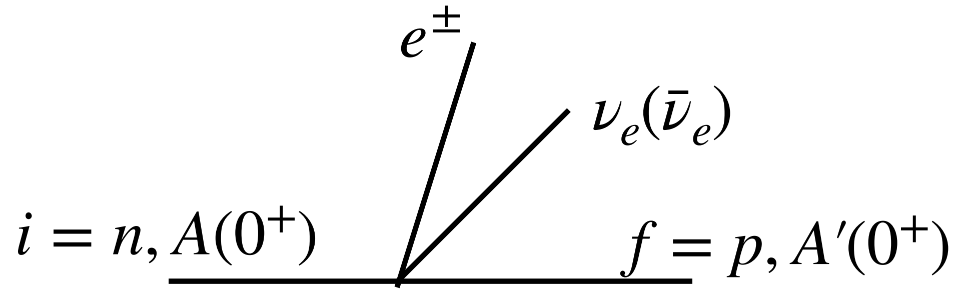
$$V_{ud}^{\ 2} = \frac{2984.43 s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$V_{ud}^{0^+-0^+} = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

Radiative Corrections to beta decay: Overall Setup

RC to beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale

$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale

$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

Universal

Nuclear scale

$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent (QCD)

Decay Q-value (endpoint energy)

$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass

$m_e \approx 0.5\,\mathrm{MeV}$

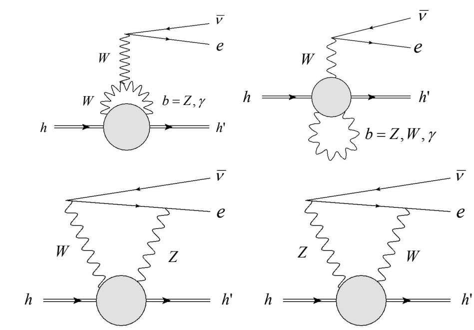
Nuclear structure independent (QED)

RC to beta decay: separating scales

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function (Dirac-Coulomb problem) + Sirlin function (soft Bremsstrahlung)

UV: large EW logs + pQCD corrections

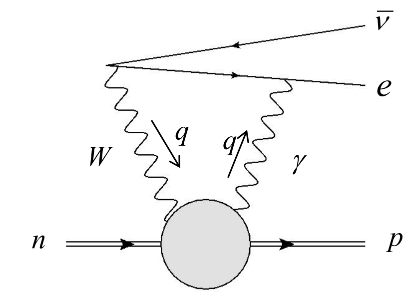


Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM

γW**-box: sensitive to all scales**

New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear

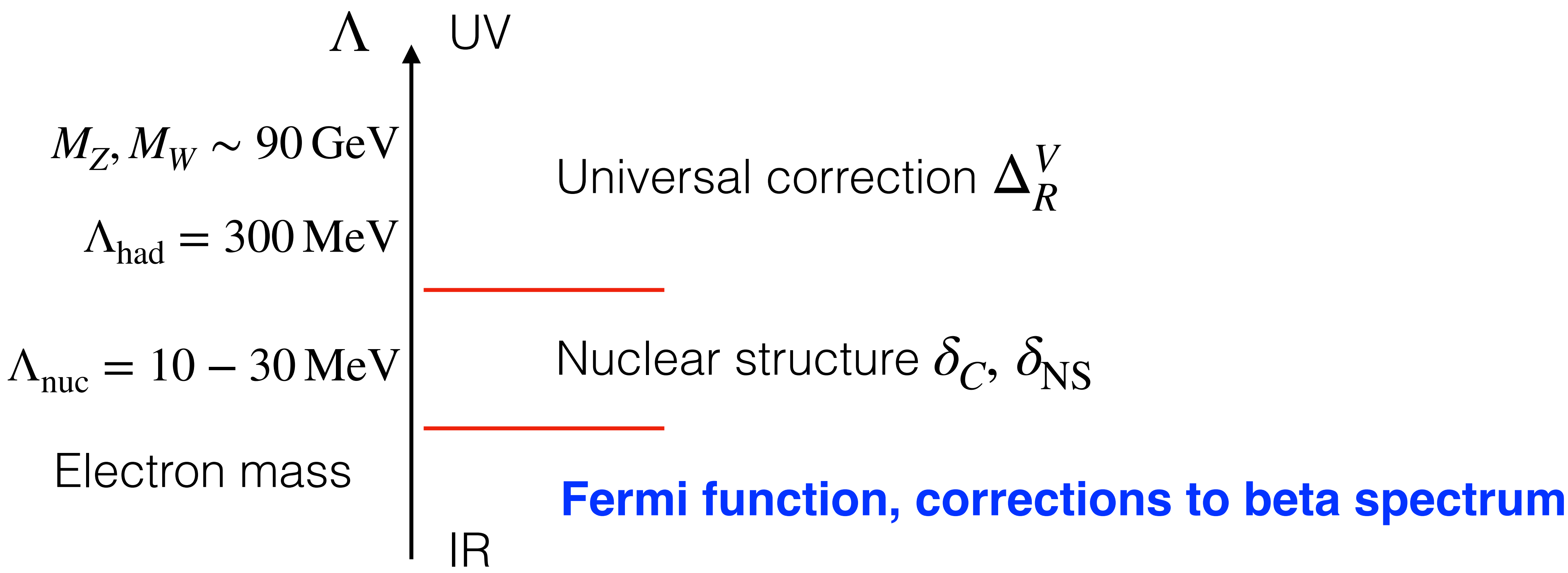


UV-sensitive γW-box on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = \; V_{ud}\;^2 \left[1 + \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_W} + \tilde{a}_g \right\} + \delta_{\rm QED}^{HO} + 2\,\Box_{\gamma W} \right]$$

Nuclear structure: $\delta_{\rm NS} = 2(\,\Box_{\gamma W}^{\rm Nucl} - \Box_{\gamma W}^{\rm free\,n}\,)$

All non-enhanced terms $\sim \alpha/2\pi \sim 10^{-3}$ — only need to ~10%



Long-Range QED Corrections to Beta Spectrum and ft-values

QED: Corrections to Decay Spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \;\; \vec{p}_e \;\; E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

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Unperturbed beta spectrum

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Unperturbed beta spectrum

Fermi function: e^+ in Coulomb field of daughter nucleus

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Shape factor: spatial distribution of decay

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Unperturbed beta spectrum

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Atomic screening and overlap corrections

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Coulomb distortion numerically large: escapes the usual scaling α/π
Fermi function $F_0 \sim Z\alpha\pi/\beta$ (coherent effect, Sommerfeld and π^2 enhancement)

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Since Fermi fn is of order 1 —> even small corrections should be assessed.

A myriad of corrections introduced/estimated by different people in past 9 decades!

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\, L_0(Z,W)\, U(Z,W)\, D_{\mathrm{FS}}(Z,W,\beta_2)\, R(W,W_0)\, R_N(W,W_0,M) \times Q(Z,W)\, S(Z,W)\, X(Z,W)\, r(Z,W)\, C(Z,W)\, D_C(Z,W,\beta_2)\, pW(W_0-W)^2\, dW$$

QED: Corrections to Decay Spectrum

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \; \vec{p}_e \;\; E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

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$$\times \; Q(Z,W) \; S(Z,W) \; X(Z,W) \; r(Z,W) \; C(Z,W) \; D_C(Z,W,\beta_2) \; pW(W_0 - W)^2 \; dW$$

Unified method of calculation (matching between them is well-defined)
numerical solution of Dirac equation with inputs from nuclear theory and experiment

Nuclear Structure Inputs in ft

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e \;\; \vec{p}_e \;\; E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

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QED

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

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QED

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

Shape factor: **nuclear weak CC transition FF** $F_{CW}(q^2)$

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QED

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Slope of the charge FF at origin: nuclear charge radius
Not all radii are known —> have to be guessed (theory)

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Charged-current weak transition form factors: only accessible with the decay itself (tough);
Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)
Typical result: very similar to charge FF

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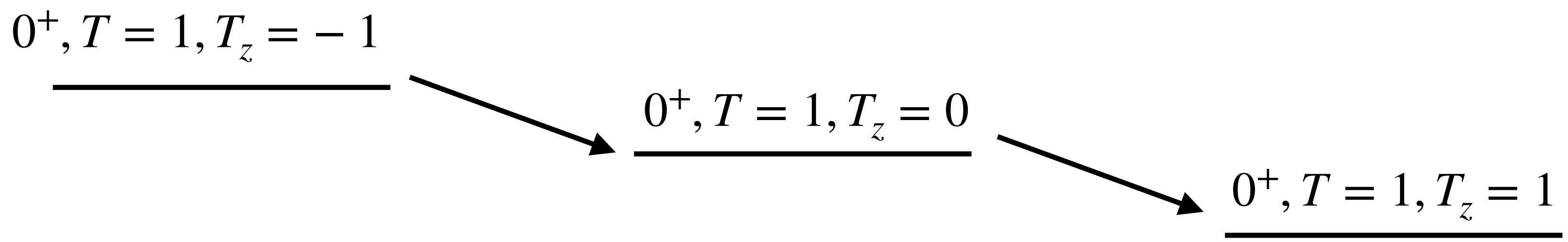
New development:
use isospin symmetry and known charge radii to predict the weak transition radius!

Isospin symmetry + Charge Radii in 0^+ isotriplet

CY Seng, 2212.02681



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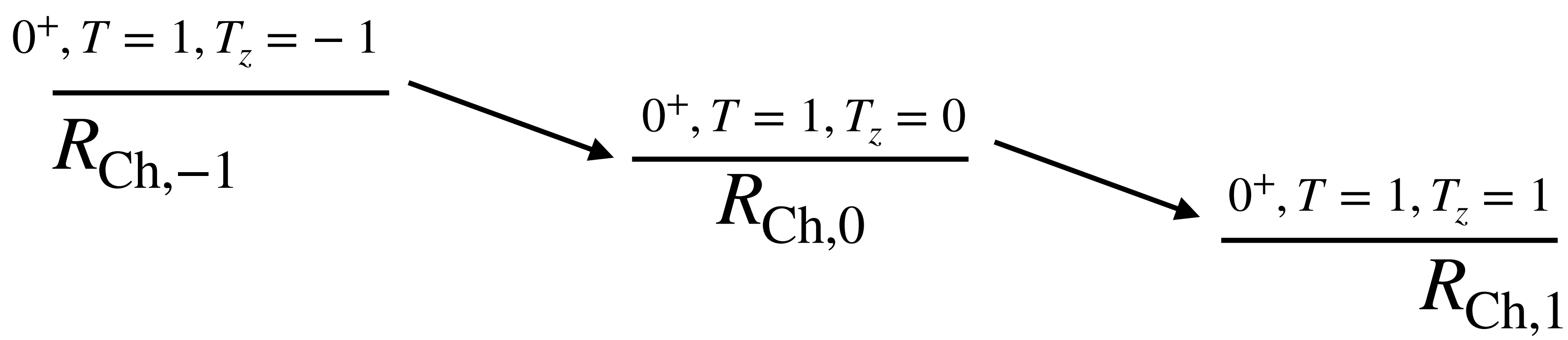

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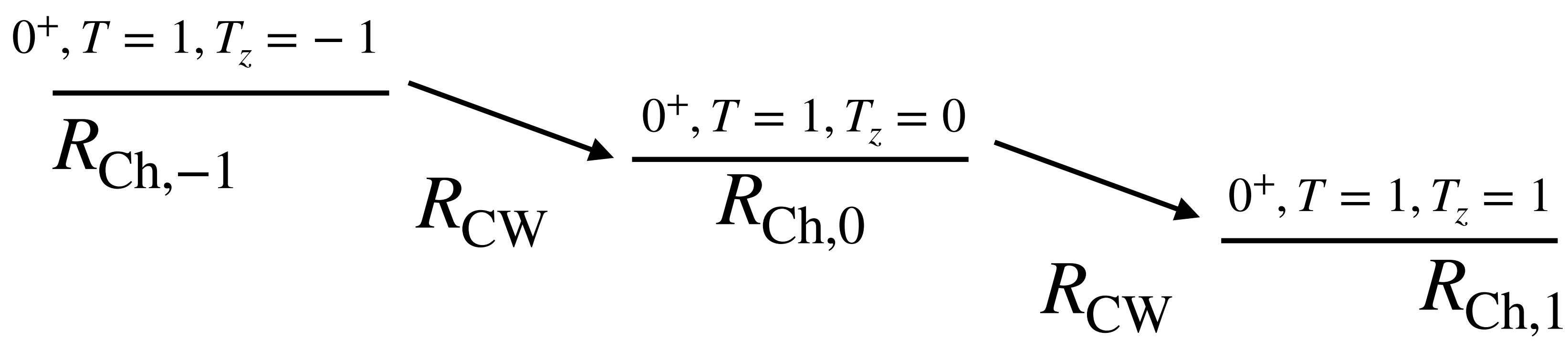

CY Seng, 2212.02681

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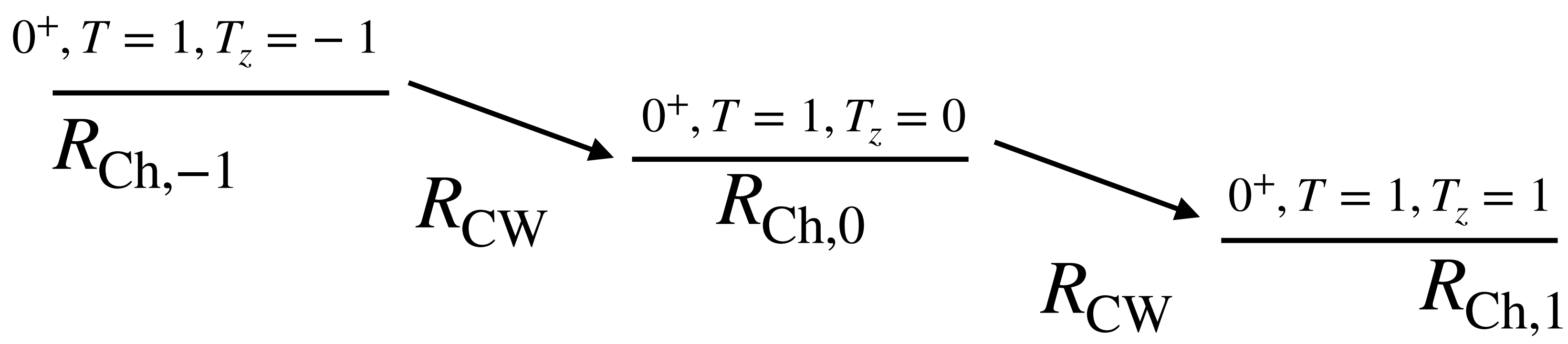


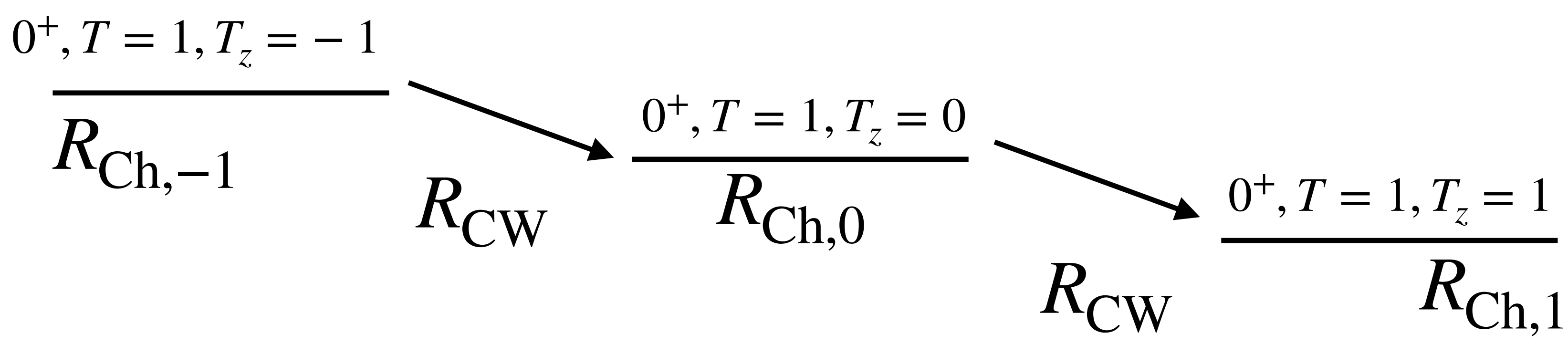
How is R_{CW} related to R_{Ch,T_z}?
Charged-Current weak current: pure isovector
Electromagnetic current isovector + isoscalar

Remove isoscalar part:
Relate weak <—> charge radii

$$
\begin{aligned}
R^2_{\mathrm{CW}} &= R^2_{\mathrm{Ch},1} + Z_0(R^2_{\mathrm{Ch},0} - R^2_{\mathrm{Ch},1}) \\
&= R^2_{\mathrm{Ch},1} + \frac{Z_{-1}}{2}(R^2_{\mathrm{Ch},-1} - R^2_{\mathrm{Ch},1})
\end{aligned}
$$

Large factors ~Z multiply small differences

Isospin symmetry + Charge Radii in 0^+ isotriplet



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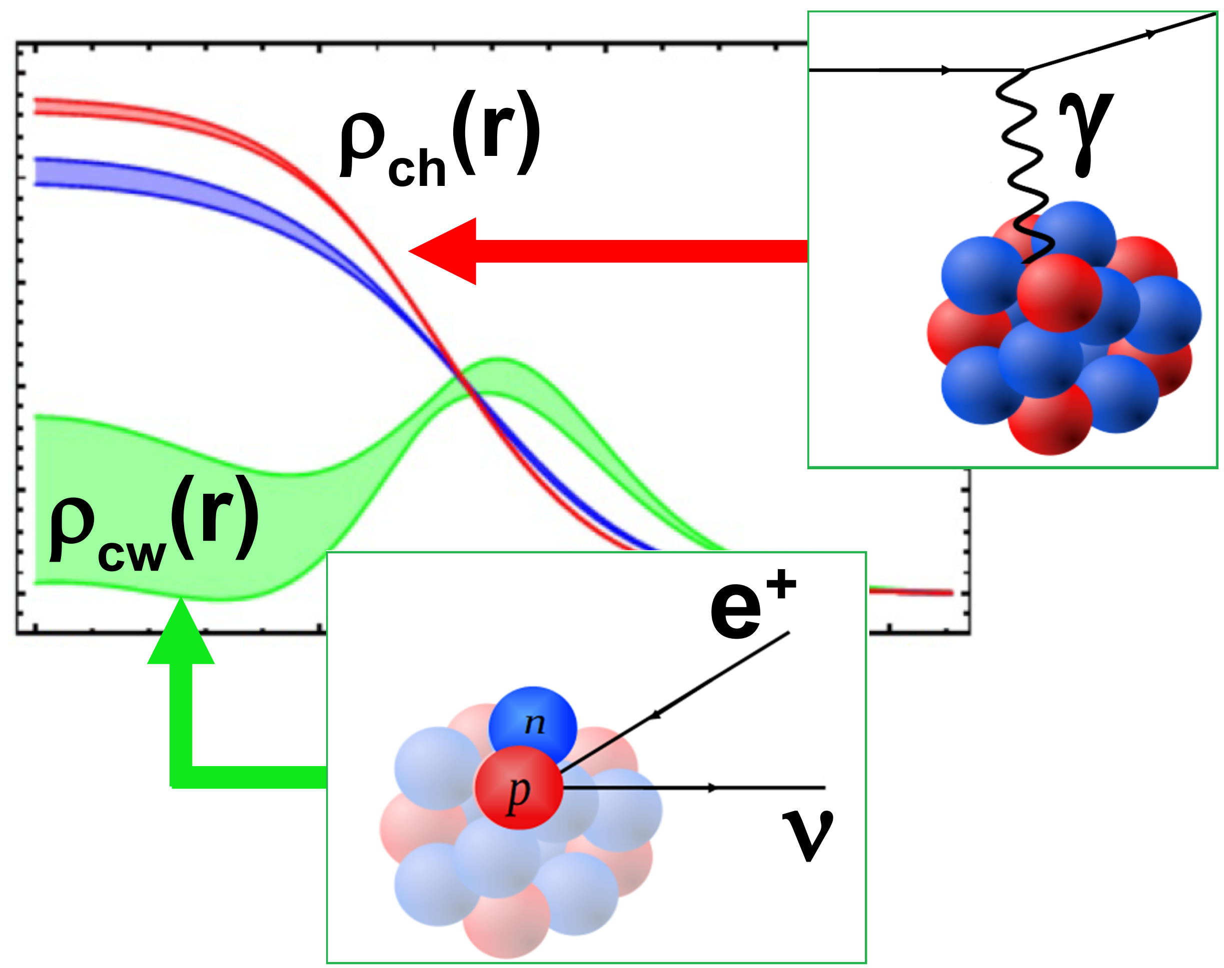

Large factors ~Z multiply small differences

Photon probes the entire nuclear charge
Only the outer protons can decay: all neutron states in the core occupied

Isospin symmetry + Charge Radii in $T=1, O^+$isotriplet

A	$\langle r^2_{\mathrm{ch},-1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},0}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{cw}}\rangle^{1/2}$ (fm)
10	$^{10}_{6}$C	$^{10}_{5}$B(ex)	$^{10}_{4}$Be: 2.3550(170)[a]	N/A
14	$^{14}_{8}$O	$^{14}_{7}$N(ex)	$^{14}_{6}$C: 2.5025(87)[a]	N/A
18	$^{18}_{10}$Ne: 2.9714(76)[a]	$^{18}_{9}$F(ex)	$^{18}_{8}$O: 2.7726(56)[a]	3.661(72)
22	$^{22}_{12}$Mg: 3.0691(89)[b]	$^{22}_{11}$Na(ex)	$^{22}_{10}$Ne: 2.9525(40)[a]	3.596(99)
26	$^{26}_{14}$Si	$^{26m}_{13}$Al: 3.130(15)[f]	$^{26}_{12}$Mg: 3.0337(18)[a]	4.11(15)
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)[a]	N/A
34	$^{34}_{18}$Ar: 3.3654(40)[a]	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)[a]	3.954(68)
38	$^{38}_{20}$Ca: 3.467(1)[c]	$^{38m}_{19}$K: 3.437(4)[d]	$^{38}_{18}$Ar: 3.4028(19)[a]	3.999(35)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)[a]	$^{42}_{20}$Ca: 3.5081(21)[a]	4.64(39)
46	$^{46}_{24}$Cr	$^{46}_{23}$V	$^{46}_{22}$Ti: 3.6070(22)[a]	N/A
50	$^{50}_{26}$Fe	$^{50}_{25}$Mn: 3.7120(196)[a]	$^{50}_{24}$Cr: 3.6588(65)[a]	4.82(39)
54	$^{54}_{28}$Ni: 3.738(4)[e]	$^{54}_{27}$Co	$^{54}_{26}$Fe: 3.6933(19)[a]	4.28(11)
62	$^{62}_{32}$Ge	$^{62}_{31}$Ga	$^{62}_{30}$Zn: 3.9031(69)[b]	N/A
66	$^{66}_{34}$Se	$^{66}_{33}$As	$^{66}_{32}$Ge	N/A
70	$^{70}_{36}$Kr	$^{70}_{35}$Br	$^{70}_{34}$Se	N/A
74	$^{74}_{38}$Sr	$^{74}_{37}$Rb: 4.1935(172)[b]	$^{74}_{36}$Kr: 4.1870(41)[a]	4.42(62)

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from R_{ch}
Shape factor—> Fermi Fn —> ft

Isospin symmetry + Charge Radii in $T=1, O^{+}$ isotriplet

A	$\langle r^2_{\mathrm{ch},-1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},0}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{cw}}\rangle^{1/2}$ (fm)
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50	${}^{50}_{26}$Fe	${}^{50}_{25}$Mn: 3.7120(196)[a]	${}^{50}_{24}$Cr: 3.6588(65)[a]	4.82(39)
54	${}^{54}_{28}$Ni: 3.738(4)[e]	${}^{54}_{27}$Co	${}^{54}_{26}$Fe: 3.6933(19)[a]	4.28(11)
62	${}^{62}_{32}$Ge	${}^{62}_{31}$Ga	${}^{62}_{30}$Zn: 3.9031(69)[b]	N/A
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74	${}^{74}_{38}$Sr	${}^{74}_{37}$Rb: 4.1935(172)[b]	${}^{74}_{36}$Kr: 4.1870(41)[a]	4.42(62)

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from R_{ch}
Shape factor—> Fermi Fn —> ft

New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%

Transition	$(ft)_{\mathrm{HT}}$ (s)	$(ft)_{\mathrm{new}}(s)$
^{18}Ne→^{18}F	2912 ± 79	2912 ± 80
^{22}Mg→^{22}Na	3051.1 ± 6.9	3050.4 ± 6.8
^{26}Si→^{26m}Al	3052.2 ± 5.6	3050.7 ± 5.6
^{34}Ar→^{34}Cl	3058.0 ± 2.8	3057.1 ± 2.8
^{38}Ca→^{38m}K	3062.8 ± 6.0	3062.2 ± 5.9
^{42}Ti→^{42}Sc	3090 ± 88	3085 ± 86
^{50}Fe→^{50}Mn	3099 ± 71	3098 ± 72
^{54}Ni→^{54}Co	3062 ± 50	3063 ± 49
^{26m}Al→^{26}Mg	3037.61 ± 0.67	3036.5 ± 1.0
^{34}Cl→^{34}S	$3049.43^{+0.95}_{-0.88}$	3048.0 ± 1.1
^{38m}K→^{38}Ar	3051.45 ± 0.92	3050.5 ± 1.1
^{42}Sc→^{42}Ca	3047.7 ± 1.2	3045.0 ± 2.7
^{50}Mn→^{50}Cr	3048.4 ± 1.2	3046.1 ± 3.6
^{54}Co→^{54}Fe	$3050.8^{+1.4}_{-1.1}$	$3051.3^{+1.7}_{-1.4}$
^{74}Rb→^{74}Kr	3082.8 ± 6.5	3086 ± 11

Isospin symmetry + Charge Radii in $T = 1, O^{+}$isotriplet

A	$\langle r^2_{\mathrm{ch},-1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},0}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{cw}}\rangle^{1/2}$ (fm)
10	${}^{10}_{6}\mathrm{C}$	${}^{10}_{5}\mathrm{B(ex)}$	${}^{10}_{4}\mathrm{Be}$: $2.3550(170)^a$	N/A
14	${}^{14}_{8}\mathrm{O}$	${}^{14}_{7}\mathrm{N(ex)}$	${}^{14}_{6}\mathrm{C}$: $2.5025(87)^a$	N/A
18	${}^{18}_{10}\mathrm{Ne}$: $2.9714(76)^a$	${}^{18}_{9}\mathrm{F(ex)}$	${}^{18}_{8}\mathrm{O}$: $2.7726(56)^a$	3.661(72)
22	${}^{22}_{12}\mathrm{Mg}$: $3.0691(89)^b$	${}^{22}_{11}\mathrm{Na(ex)}$	${}^{22}_{10}\mathrm{Ne}$: $2.9525(40)^a$	3.596(99)
26	${}^{26}_{14}\mathrm{Si}$	${}^{26m}_{13}\mathrm{Al}$: $3.130(15)^f$	${}^{26}_{12}\mathrm{Mg}$: $3.0337(18)^a$	4.11(15)
30	${}^{30}_{16}\mathrm{S}$	${}^{30}_{15}\mathrm{P(ex)}$	${}^{30}_{14}\mathrm{Si}$: $3.1336(40)^a$	N/A
34	${}^{34}_{18}\mathrm{Ar}$: $3.3654(40)^a$	${}^{34}_{17}\mathrm{Cl}$	${}^{34}_{16}\mathrm{S}$: $3.2847(21)^a$	3.954(68)
38	${}^{38}_{20}\mathrm{Ca}$: $3.467(1)^c$	${}^{38m}_{19}\mathrm{K}$: $3.437(4)^d$	${}^{38}_{18}\mathrm{Ar}$: $3.4028(19)^a$	3.999(35)
42	${}^{42}_{22}\mathrm{Ti}$	${}^{42}_{21}\mathrm{Sc}$: $3.5702(238)^a$	${}^{42}_{20}\mathrm{Ca}$: $3.5081(21)^a$	4.64(39)
46	${}^{46}_{24}\mathrm{Cr}$	${}^{46}_{23}\mathrm{V}$	${}^{46}_{22}\mathrm{Ti}$: $3.6070(22)^a$	N/A
50	${}^{50}_{26}\mathrm{Fe}$	${}^{50}_{25}\mathrm{Mn}$: $3.7120(196)^a$	${}^{50}_{24}\mathrm{Cr}$: $3.6588(65)^a$	4.82(39)
54	${}^{54}_{28}\mathrm{Ni}$: $3.738(4)^e$	${}^{54}_{27}\mathrm{Co}$	${}^{54}_{26}\mathrm{Fe}$: $3.6933(19)^a$	4.28(11)
62	${}^{62}_{32}\mathrm{Ge}$	${}^{62}_{31}\mathrm{Ga}$	${}^{62}_{30}\mathrm{Zn}$: $3.9031(69)^b$	N/A
66	${}^{66}_{34}\mathrm{Se}$	${}^{66}_{33}\mathrm{As}$	${}^{66}_{32}\mathrm{Ge}$	N/A
70	${}^{70}_{36}\mathrm{Kr}$	${}^{70}_{35}\mathrm{Br}$	${}^{70}_{34}\mathrm{Se}$	N/A
74	${}^{74}_{38}\mathrm{Sr}$	${}^{74}_{37}\mathrm{Rb}$: $4.1935(172)^b$	${}^{74}_{36}\mathrm{Kr}$: $4.1870(41)^a$	4.42(62)

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from R_{ch}
Shape factor—> Fermi Fn —> ft

Transition	$(ft)_{\mathrm{HT}}$ (s)	$(ft)_{\mathrm{new}}(s)$
${}^{18}\mathrm{Ne}\to{}^{18}\mathrm{F}$	2912 ± 79	2912 ± 80
${}^{22}\mathrm{Mg}\to{}^{22}\mathrm{Na}$	3051.1 ± 6.9	3050.4 ± 6.8
${}^{26}\mathrm{Si}\to{}^{26m}\mathrm{Al}$	3052.2 ± 5.6	3050.7 ± 5.6
${}^{34}\mathrm{Ar}\to{}^{34}\mathrm{Cl}$	3058.0 ± 2.8	3057.1 ± 2.8
${}^{38}\mathrm{Ca}\to{}^{38m}\mathrm{K}$	3062.8 ± 6.0	3062.2 ± 5.9
${}^{42}\mathrm{Ti}\to{}^{42}\mathrm{Sc}$	3090 ± 88	3085 ± 86
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New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%

More -and more precise- charge radii necessary!
Working closely with exp. (PSI, FRIB, ISOLDE, TRIUMF)

Isospin symmetry + Charge Radii in $T = 1, O^{+}$isotriplet

Above treatment assumes isospin symmetry — but we know that it is slightly broken!
Why isospin symmetry assumption is good enough?

Shape factor and finite size effects are ~small corrections to Fermi function
1-2% ISB effect on top of a RC may be assumed negligible (but needs to be tested)

Test requires that all 3 nuclear radii in the isotriplet are known;
Currently only the case for A=38 system

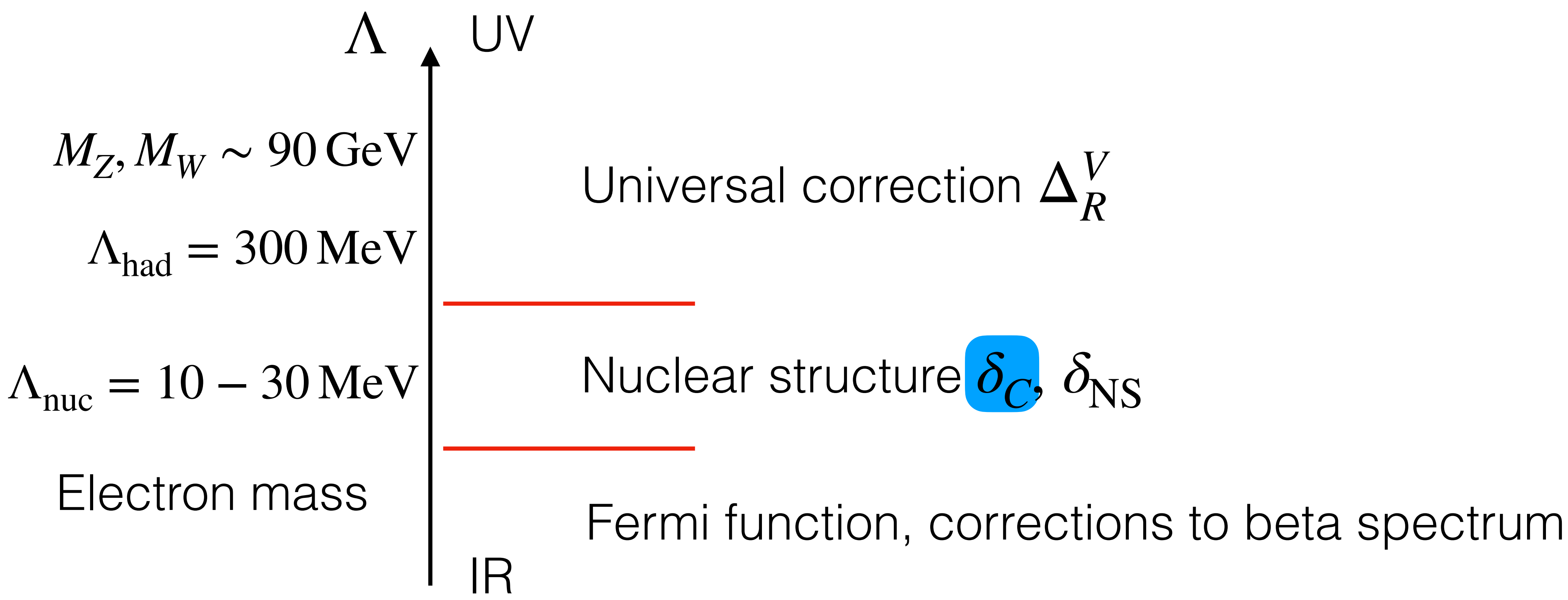
26	$^{26}_{14}$Si	$^{26m}_{13}$Al: 3.130(15)[f]	$^{26}_{12}$Mg: 3.0337(18)[a]	4.11(15)
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)[a]	N/A
34	$^{34}_{18}$Ar: 3.3654(40)[a]	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)[a]	3.954(68)
38	$^{38}_{20}$Ca: 3.467(1)[c]	$^{38m}_{19}$K: 3.437(4)[d]	$^{38}_{18}$Ar: 3.4028(19)[a]	3.999(35)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)[a]	$^{42}_{20}$Ca: 3.5081(21)[a]	4.64(39)
46	^{46}Cr	^{46}V	^{46}Ti: 3.6070(22)[a]	N/A

ISB-sensitive combination

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2 = 0 \quad \text{if isospin symmetry exact}$$

$$\frac{1}{2}\left(20 \times 3.467(1)^2 + 18 \times 3.4028(19)^2\right) - 19 \times 3.437(4)^2 = -0.00(12)(14)(52)$$

Improvement of K-38m radius necessary! (Plans at TRIUMF on IS K-38m, K-37?)



Isospin breaking in nuclear WF: δ_C

Tree-level effect — ISB “large”

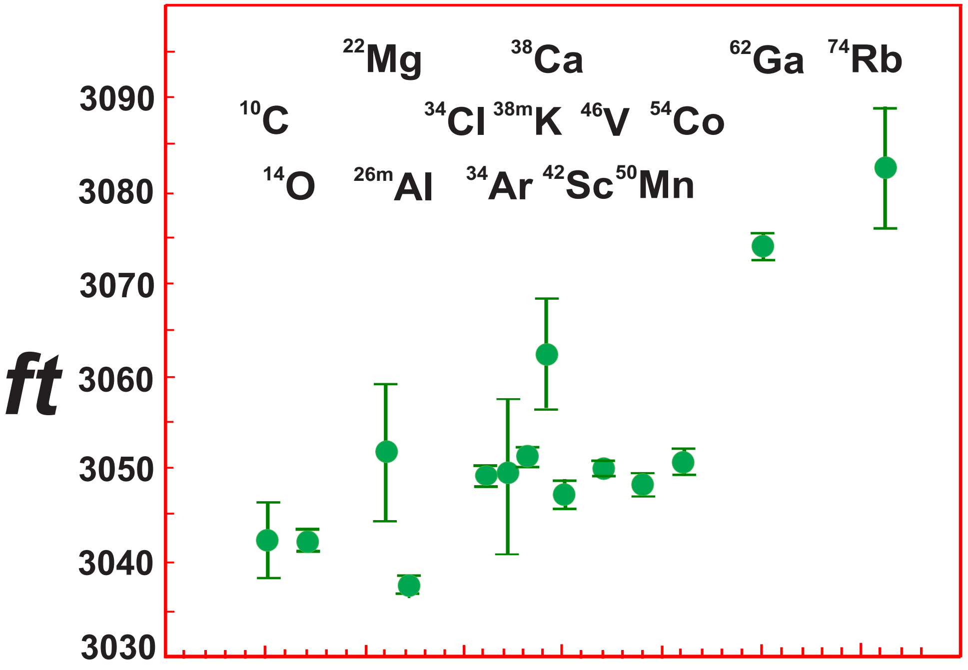
Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f \; \tau^+ \; i \rangle$$

τ^+ — Isospin operator

$i\rangle, \;\; f\rangle$ — members of T=1 isotriplet



Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f \quad \tau^+ \quad i \rangle$$

τ^+ — Isospin operator

$i\rangle, \quad f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states
(e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

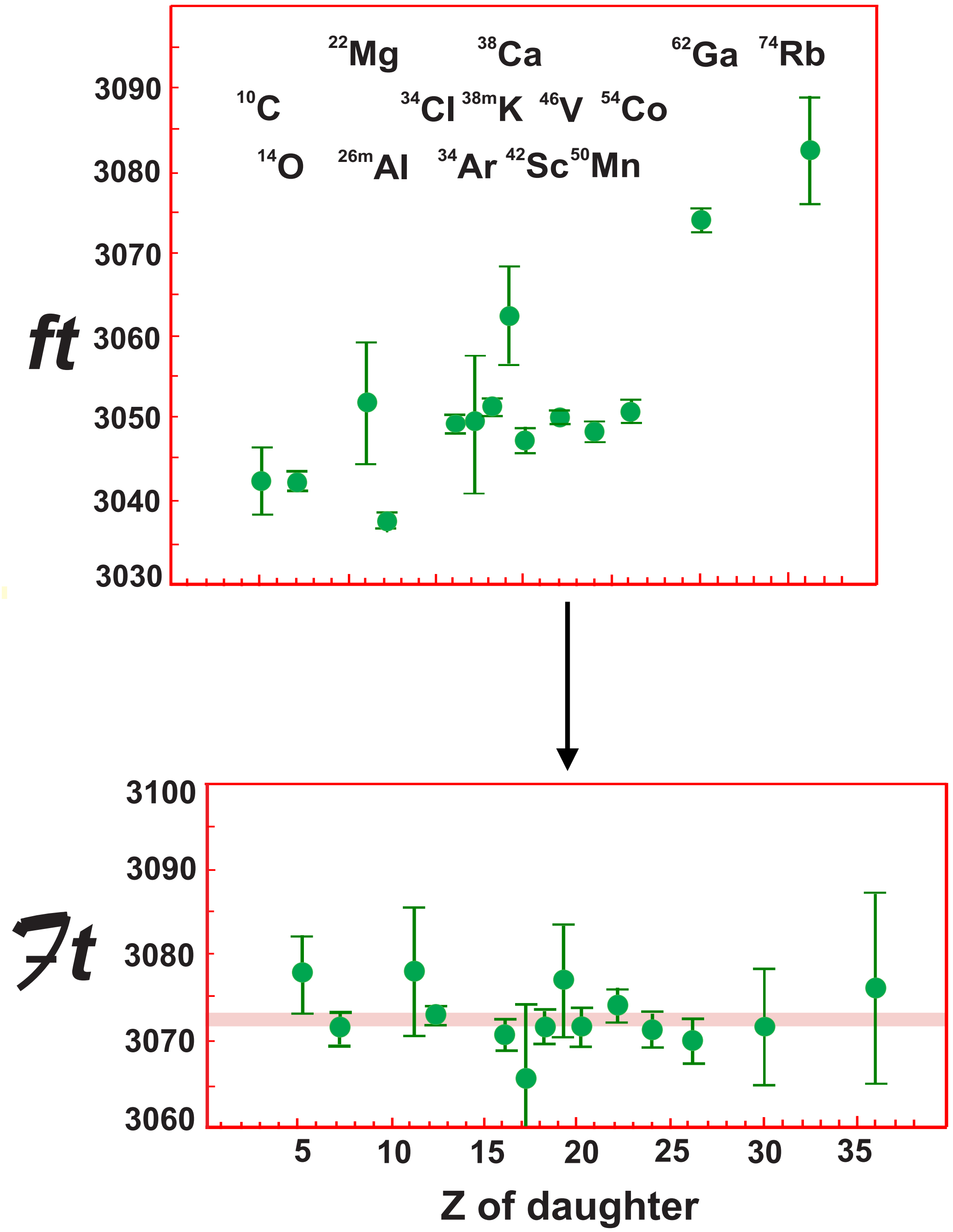
$$M_F^{\ 2} = M_0^{\ 2}(1 - \delta_C)$$

MacDonald 1958

ISB correction almost singlehandedly aligns ft-values!

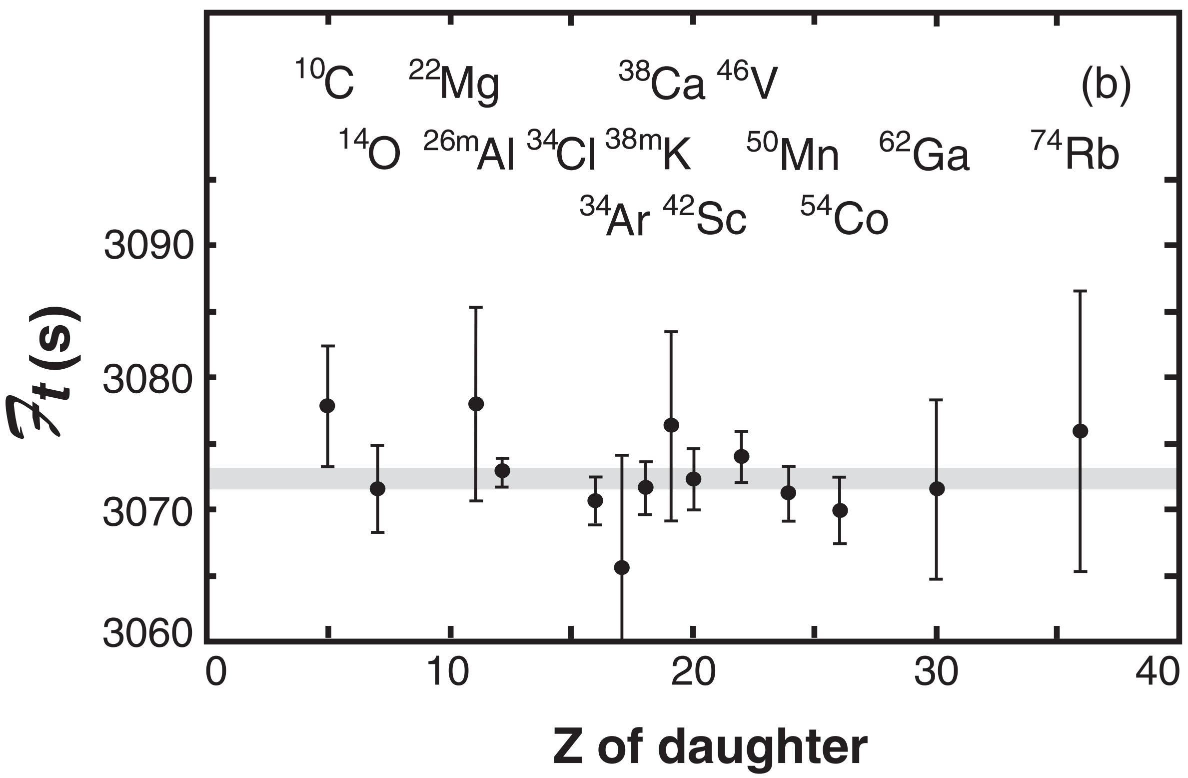
$$\delta_C \sim 0.17\% - 1.6\% \,!$$

Crucial for V_{ud} extraction



J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

Nuclear Corrections vs. scalar BSM

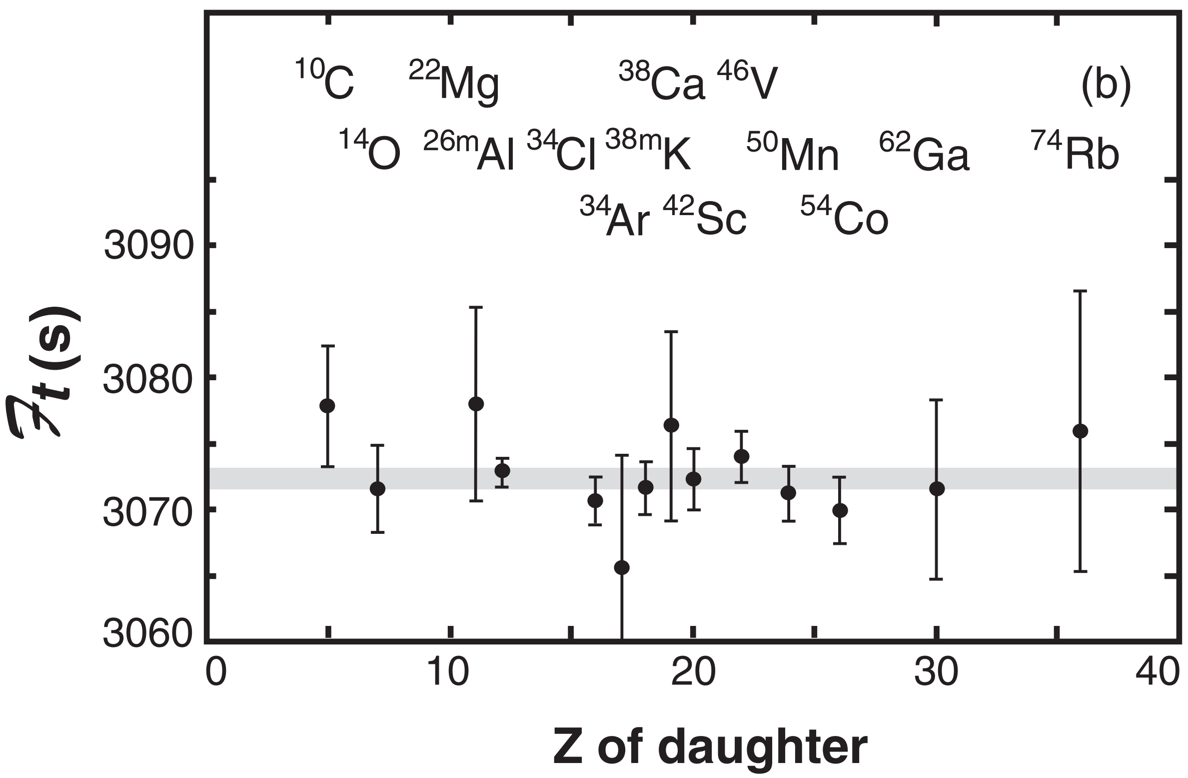


Once all corrections are included:
CVC —> Ft constant

Fit to 14 transitions:
Ft constant within 0.02%

Hardy, Towner 2020

Nuclear Corrections vs. scalar BSM



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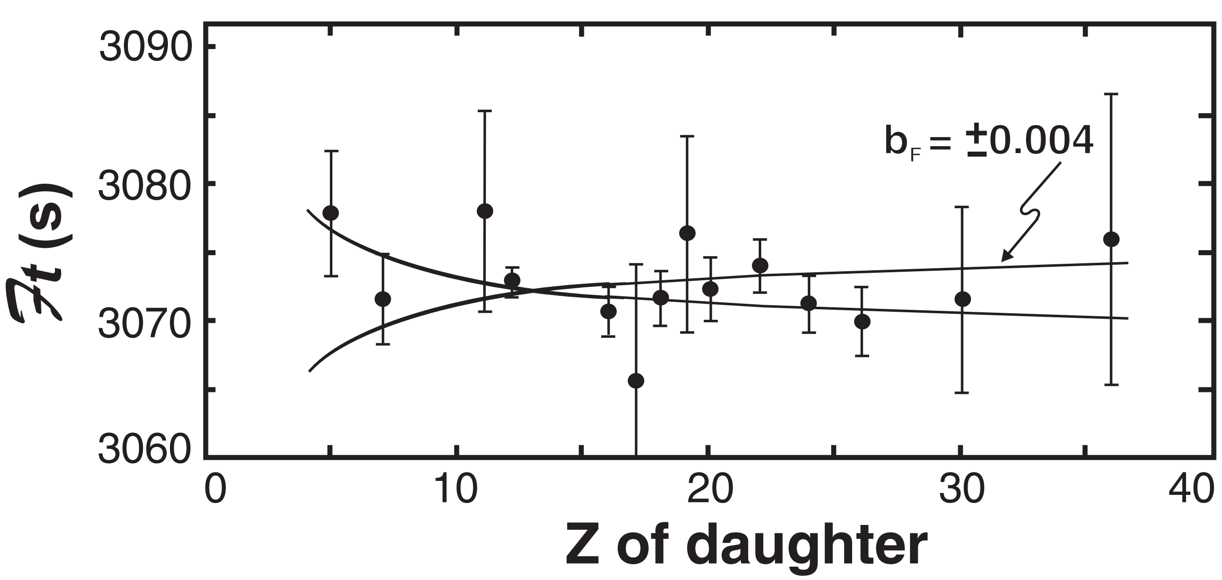


If BSM scalar currents present: Fierz interference b_F

$$\mathcal{F}t^{SM} \to \mathcal{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

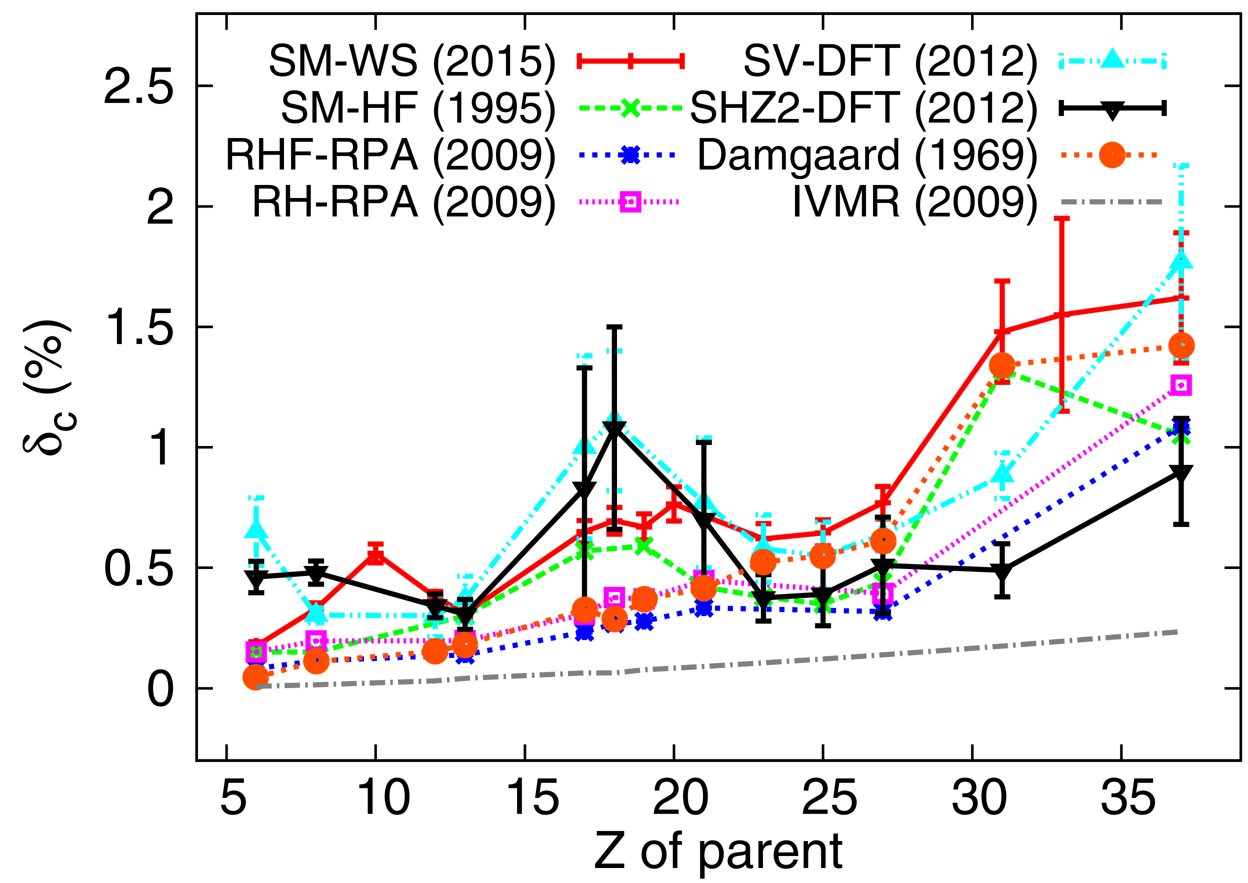
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model dependence of δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

	SM-WS	SM-HF	RPA PKO1	RPA DD-ME2	RPA PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Constraints on δ_C from nuclear radii

ISB-sensitive combinations of radii can be constructed **Seng, MG 2208.03037; 2304.03800; 2212.02681**

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

$\Delta M_B^{(1)} = 0$ used for ft-value in isospin limit

$$\Delta M_A^{(1)} \equiv -\langle r_{\mathrm{CW}}^2 \rangle + \left(\frac{N_1}{2}\langle r_{n,1}^2 \rangle - \frac{Z_1}{2}\langle r_{p,1}^2 \rangle\right)$$

Neutron radius: measurable with PV e- scattering!

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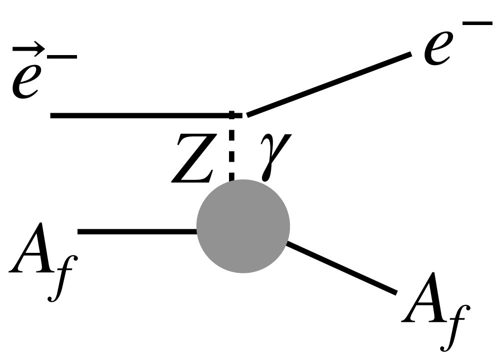
Neutron radius: measurable with PV e[-] scattering!



$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$$R_{\rm NW} \approx R_n$$

Z-boson couples to neutrons, photon - to protons;

PV asymmetry at low Q^2 sensitive to the difference $\langle r_{n,1}^2\rangle - \langle r_{p,1}^2\rangle$ - neutron skin

Extensive studies in neutron rich nuclei (PREX, CREX) —> input to physics of neutron stars

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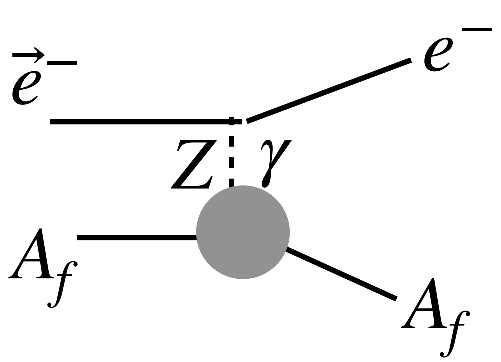
Neutron radius: measurable with PV e$^-$ scattering!



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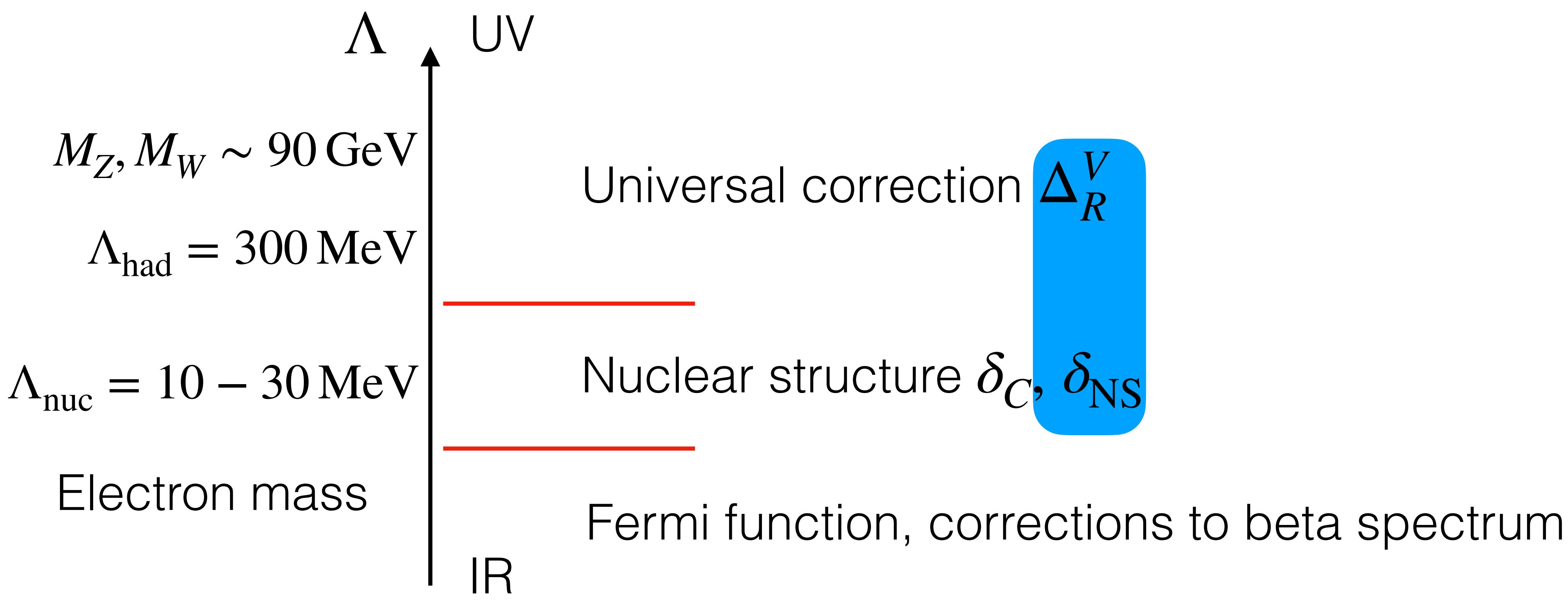
Upcoming exp. program at Mainz (MREX)
Neutron skins of stable daughters (e.g. Mg-26, Ca-42, Fe-54)
PV asymmetry on C-12 for a sub-% measurement of R_n
Unexpected connections via neutron skins:
ISB for precision tests vs. EoS of neutron-rich matter

N. Cargioli, M. Cadeddu, MG,
J. Piekarewicz, X. Roca Maza,
H. Spiesberger — *in preparation*

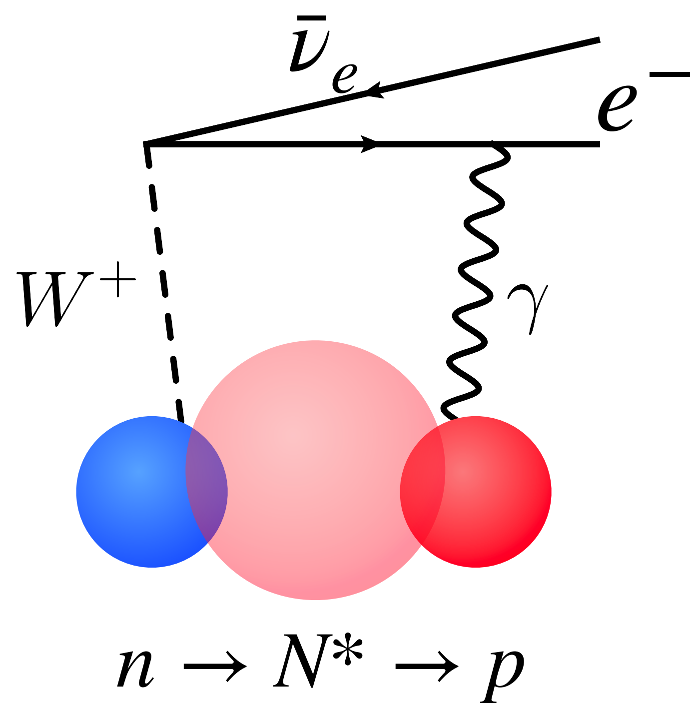


Unified Formalism for Δ_R^V and δ_{NS}

Dispersion Theory of the γW-box

Universal RC from dispersion relations

UV large log — model independent (Parton model + pQCD)
Sensitivity to nonperturbative QCD: inclusive hadron spectrum



Model dependence: interference γW structure functions

$$\mathrm{Im} T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$$

After some algebra

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi} \int_0^\infty dQ^2 \frac{M_W^2}{M_W^2 + Q^2} \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 2\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^2} \frac{F_{3,-}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi} \int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'} \frac{\nu' + 3\sqrt{\nu'^2 + Q^2}}{(\nu' + \sqrt{\nu'^2 + Q^2})^3} \frac{F_{3,+}(\nu', Q^2)}{M f_+(0)} + \mathcal{O}(E_e^3)$$

Structure functions are measurable or may be related to data

Input into dispersion integral - $\nu/\bar{\nu}$ data

Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

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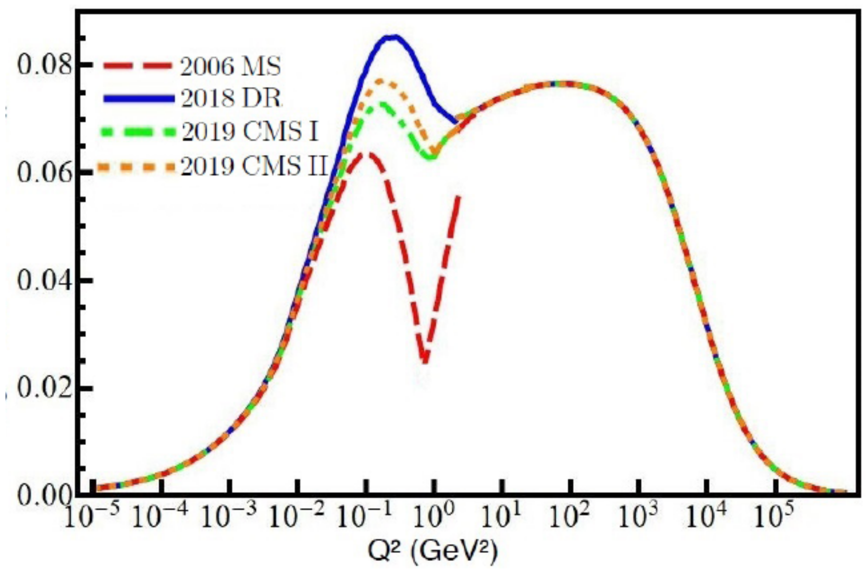
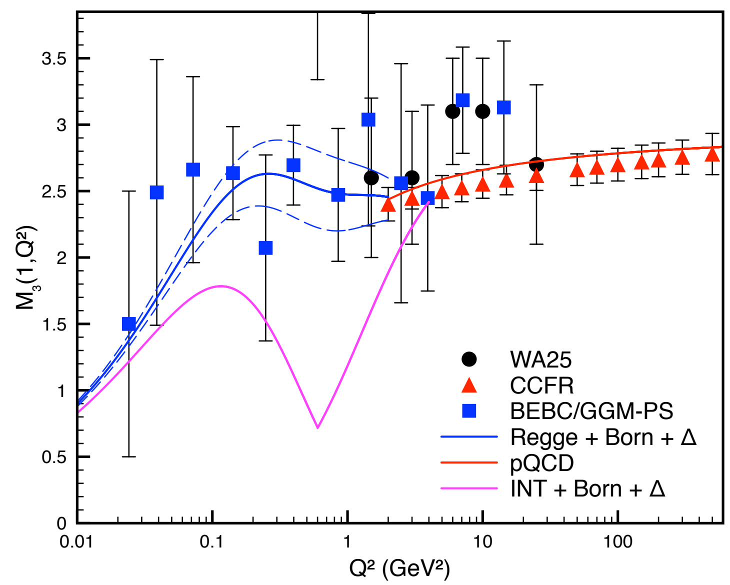
Free neutron γW box



Neutrino scattering data



Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $V_{ud} = 0.97420(10)_{Ft}(18)_{RC}$

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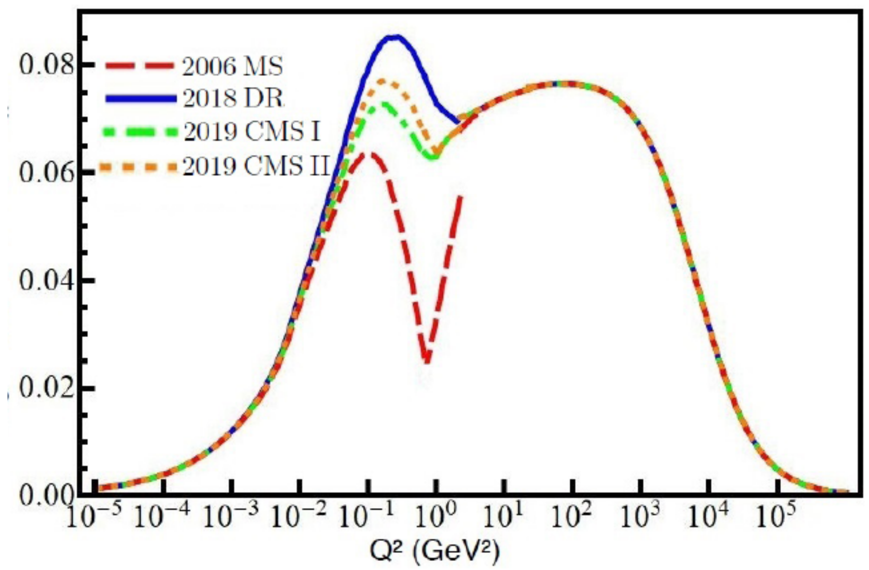

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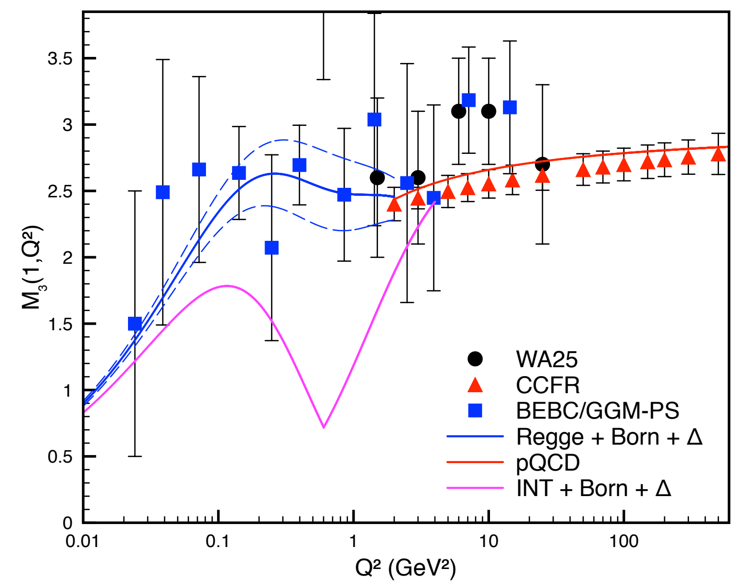

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Shift upwards by 3σ + reduction of uncertainty by factor 2

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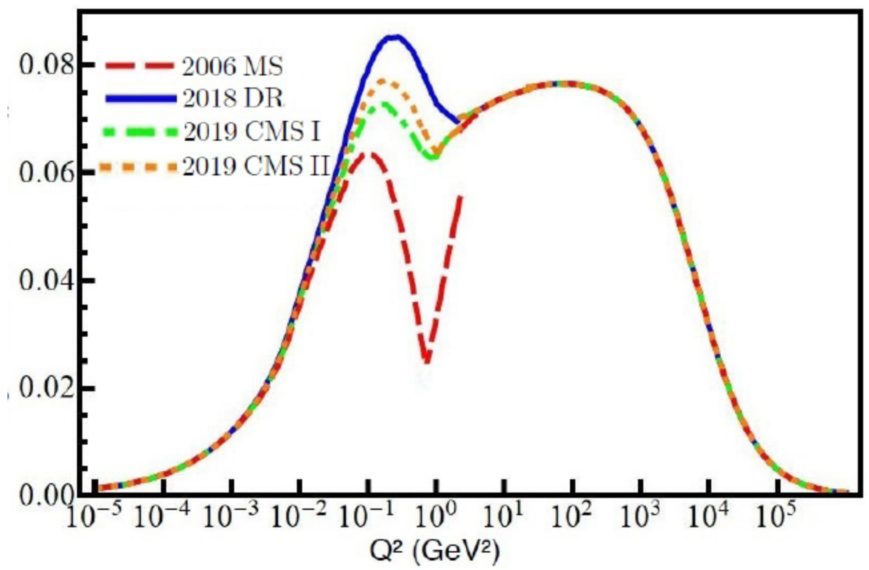

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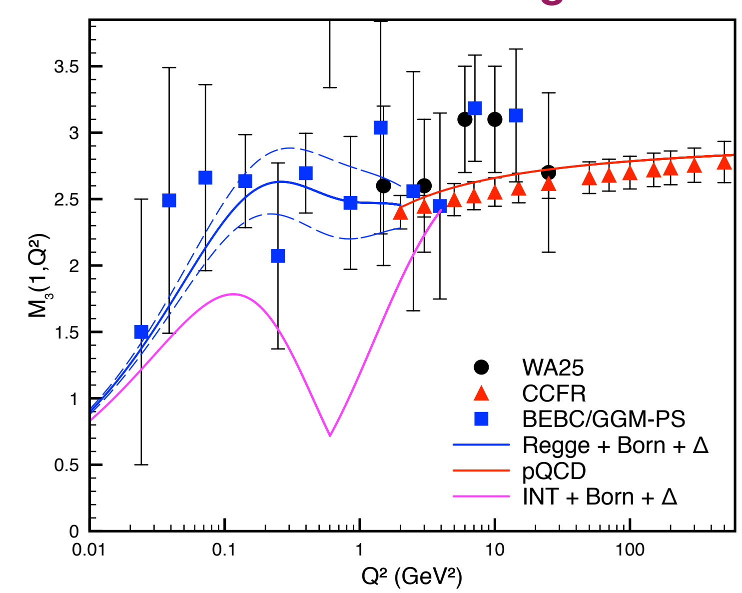

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Shift upwards by 3σ + reduction of uncertainty by factor 2

Confirmed by lattice QCD:

LQCD on pion + pheno: $\Delta_R^V = 0.02477(24)_{\mathrm{LQCD}^\pi+\mathrm{pheno}}$

Seng, MG, Feng, Jin, 2003.11264
Yoo et all, 2305.03198

LQCD on neutron: $\Delta_R^V = 0.02439(19)_{\mathrm{LQCD}^n}$

Ma, Feng, MG et al 2308.16755

EFT: scale separation for free n

Cirigliano et al, 2306.03138

Effective Field Theory: explicit separation of scales + RGE running between

SM —> LEFT (no H,t,Z,W) —> ChPT —> NR QED

Formal consistency built in, RGE, transparent error estimation (naturalness)

Precision limited by matching (LEC) and HO — relies on inputs (e.g. γW-box from DR)

To improve: need to go to higher order — new LECs, still tractable?

At present: order $O(\alpha, \alpha\alpha_s, \alpha^2)$ — realistic to go beyond?

$$\frac{\mathrm{d}\Gamma_n}{\mathrm{d}E_e} = \frac{G_F^2\,|V_{ud}|^2}{(2\pi)^5}\,(1+3\lambda^2)\;p_e E_e (E_0 - E_e)^2\,[g_V(\mu_\chi)]^2\,F_{NR}(\beta)\Big(1+\delta_{\mathrm{RC}}(E_e,\mu_\chi)\Big)\Big(1+\delta_{\mathrm{recoil}}(E_e)\Big)$$

$\lambda = g_A/g_V$ Extract from Experiment

vector coupling

$\pi^2, 1/\beta$ Enhanced

$\mathcal{O}(\alpha)$ [no logs]

$\mathcal{O}(m_e/m_N)$

Total RC: $1 + \Delta_{\mathrm{TOT}} = 1.07761(27)\,\%$

Total RC from DR: $1 + \Delta_{\mathrm{TOT}} = 1.07735(27)\,\%$

Good agreement within errors!

Nuclear-Structure RC δ_{NS}

History of δ_{NS}: γW-box on nuclei

Jaus, Rasche 1990

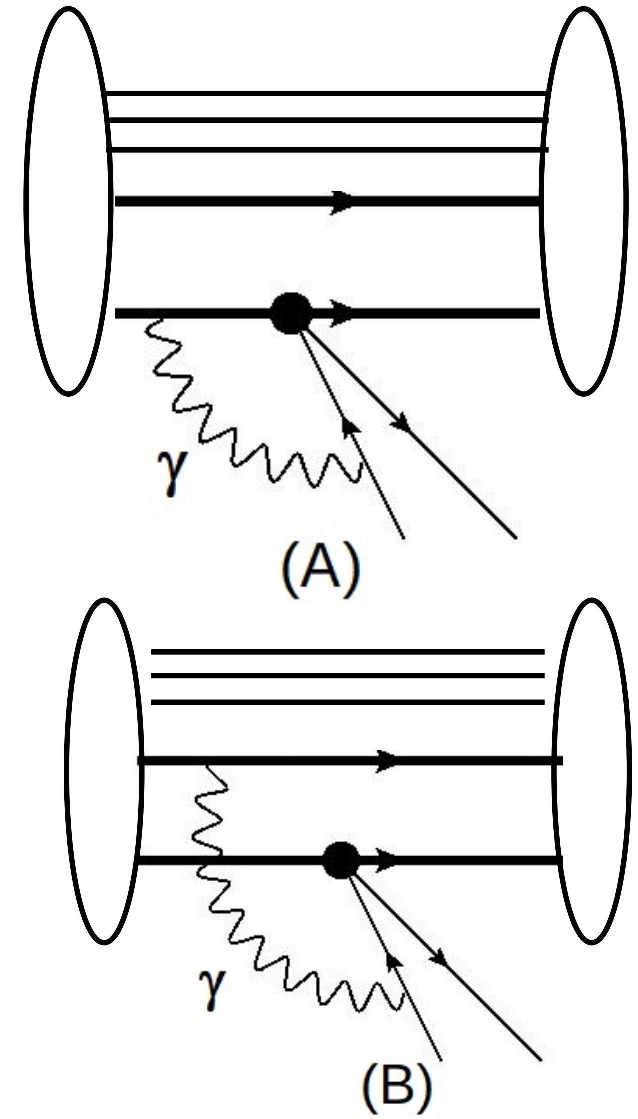
γ and W on same nucleon —> already in Δ_R^V: drop!

Towner 1994

Nucleons are bound — free-nucleon RC modified: δ_{NS}^A

Jaus, Rasche 1990; Hardy, Towner 1992-2020

γ and W on distinct nucleons —> only in nuclei: δ_{NS}^B



Implementation:
Nuclear shell model with “semi-empirical” Woods-Saxon potential
One-body nucleon currents only (axial + magnetic)
No nuclear Green’s function between the em and weak vertices
Parameters fixed to reproduce selected properties within each isotriplet
Predictive power questionable, but tailored to the task
Systematic uncertainty unclear and hard to quantify

δ_{NS} from dispersion relations

Same formulas for free neutron and nuclei;

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) \;=\; \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)}+\mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) \;=\; \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)}+\mathcal{O}(E_e^3)$$

NS correction reflects extraction of the free box
DR: a framework to control this subtraction!

$$\delta_{\mathrm{NS}} = 2[\,\Box_{\gamma W}^{\mathrm{VA,\,nucl}} - \Box_{\gamma W}^{\mathrm{VA,\,free\,n}}\,]$$

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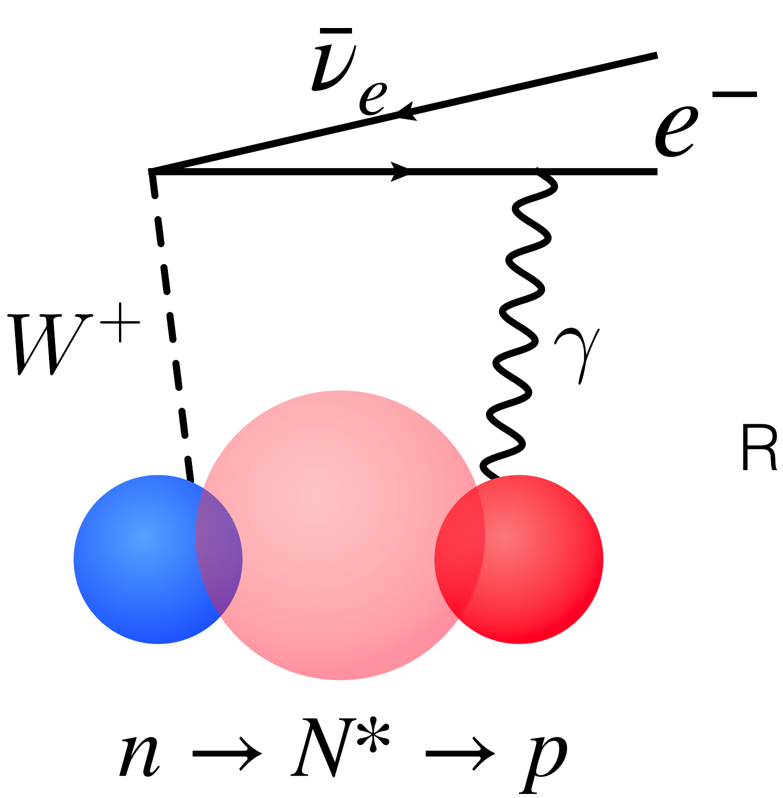
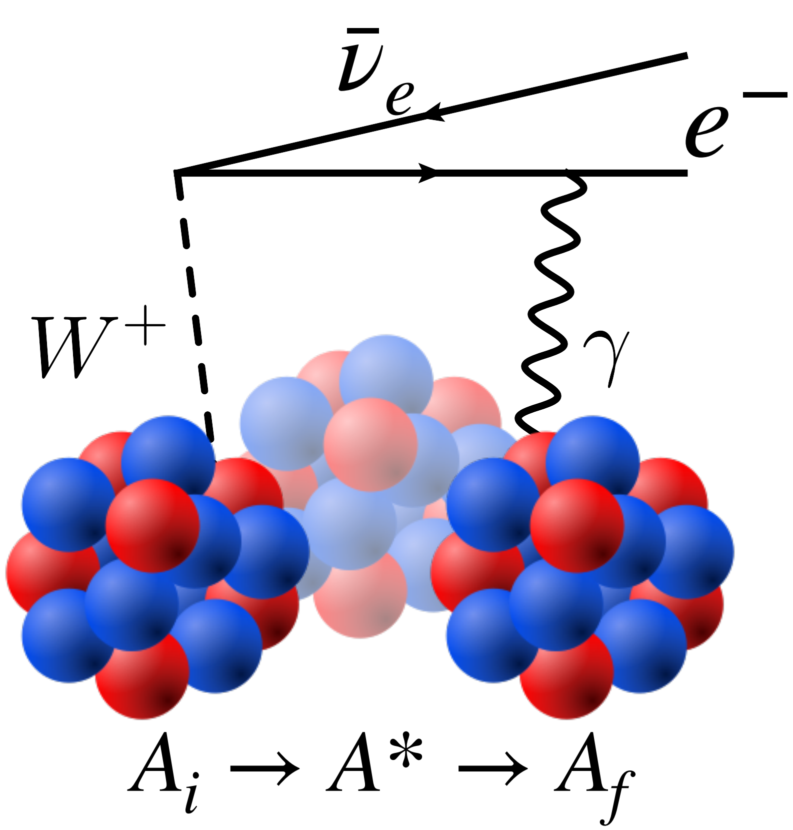


Differences due to:

Richer excitation spectrum in nuclei

Different quantum numbers
(spin, isospin)



δ_{NS} in ab-initio nuclear theory

M. Gennari, M. Drissi, MG, P. Navratil, C.-Y. Seng, arXiv: **2405.19281**

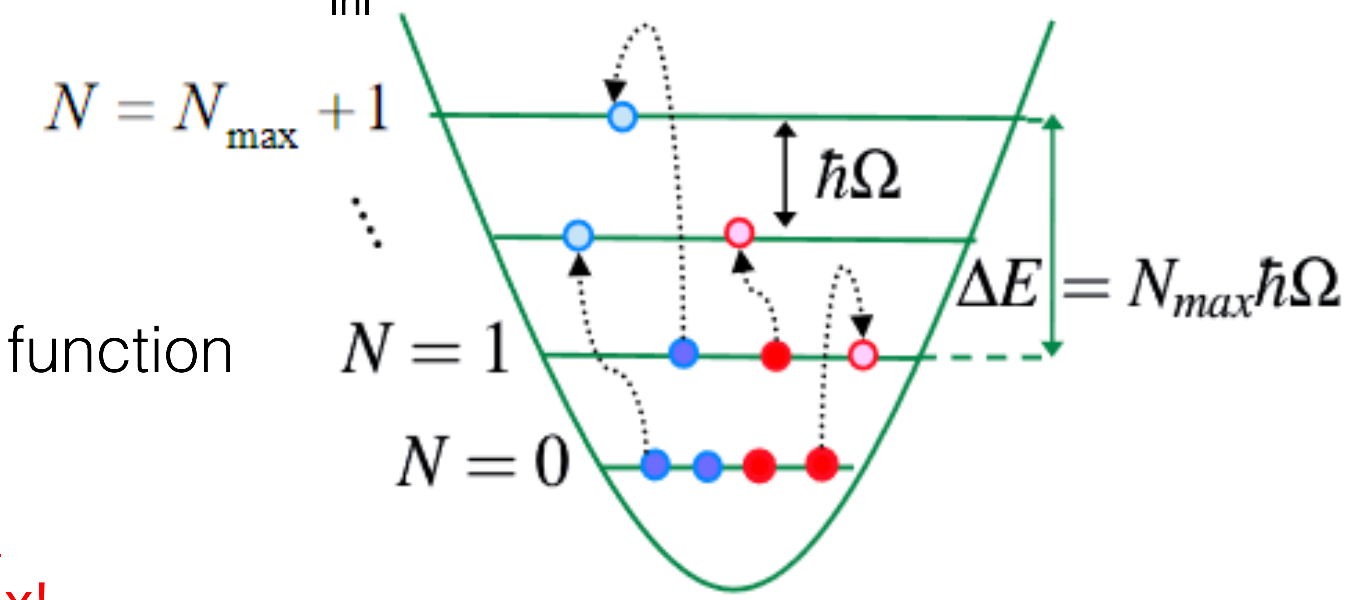
Low-momentum part of the loop: account for nucleon d.o.f. only

First case study: $^{10}C \rightarrow {}^{10}B$ in No-Core Shell Model (NCSM)

Many-body problem in HO basis with separation Ω and up to $N = N_{max} + N_{Pauli}$

- Nuclear interactions from Chiral EFT:
  - NN-N^{4}LO+3N$_{lnl}$
  - NN-N^{4}LO+3N*$_{lnl}$

Entem, Machleidt and Nosyk, 2017 PRC;
Gysbers et al., 2019 Nature;
Kravvaris, Navrátil, Quaglioni, Hebborn and Hupin, 2023 PLB



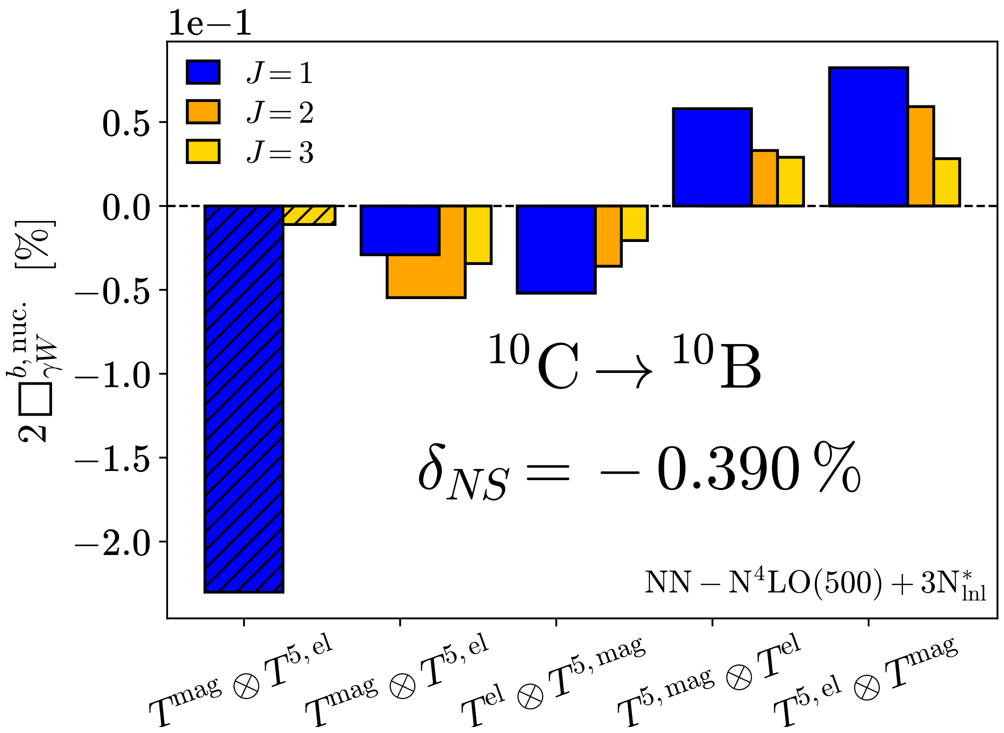
Evaluate the m.e. of nuclear Green's function

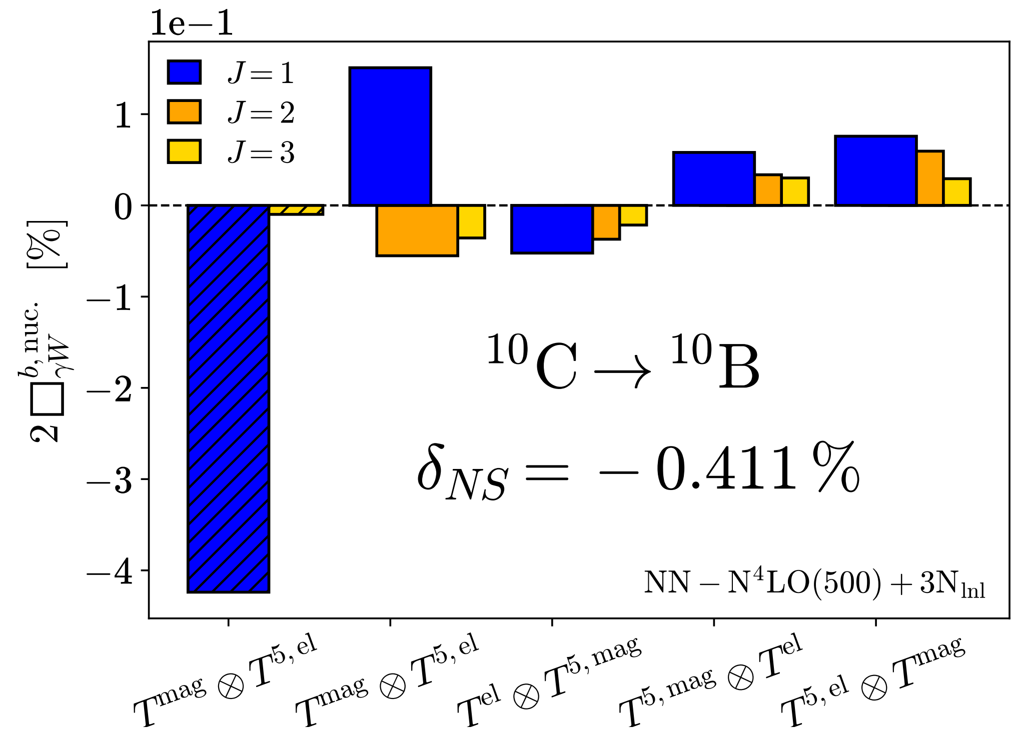
$$G(z) \equiv \frac{1}{z - H_0}$$

Difficulty: Inverting a large matrix!

Lanczos continuous fraction method

Ab-initio δ_{NS}: numerical results





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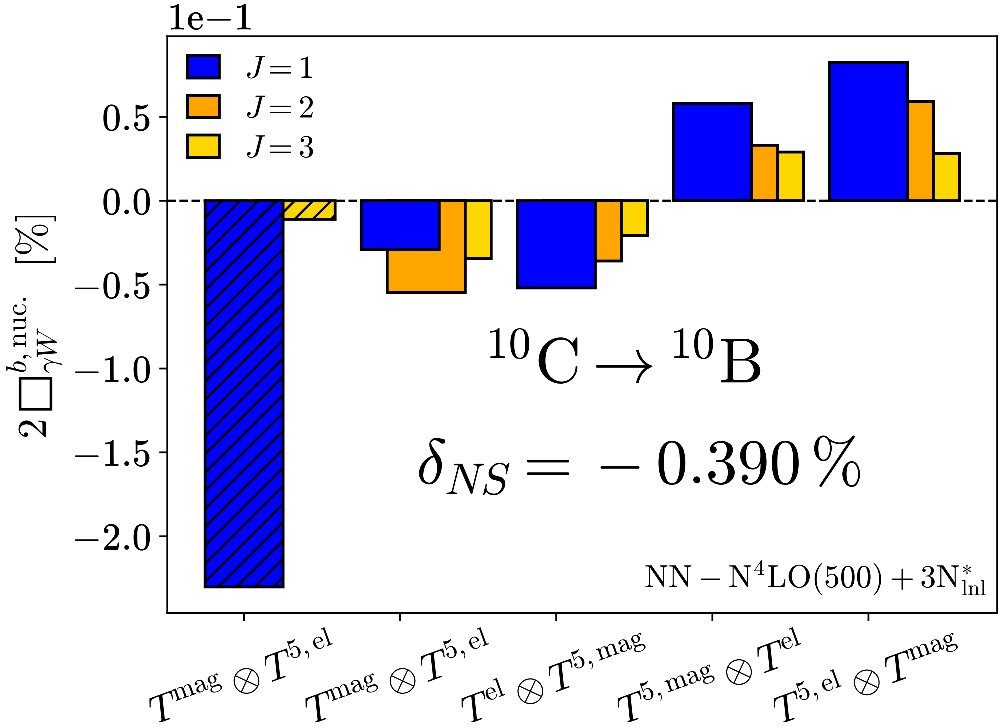
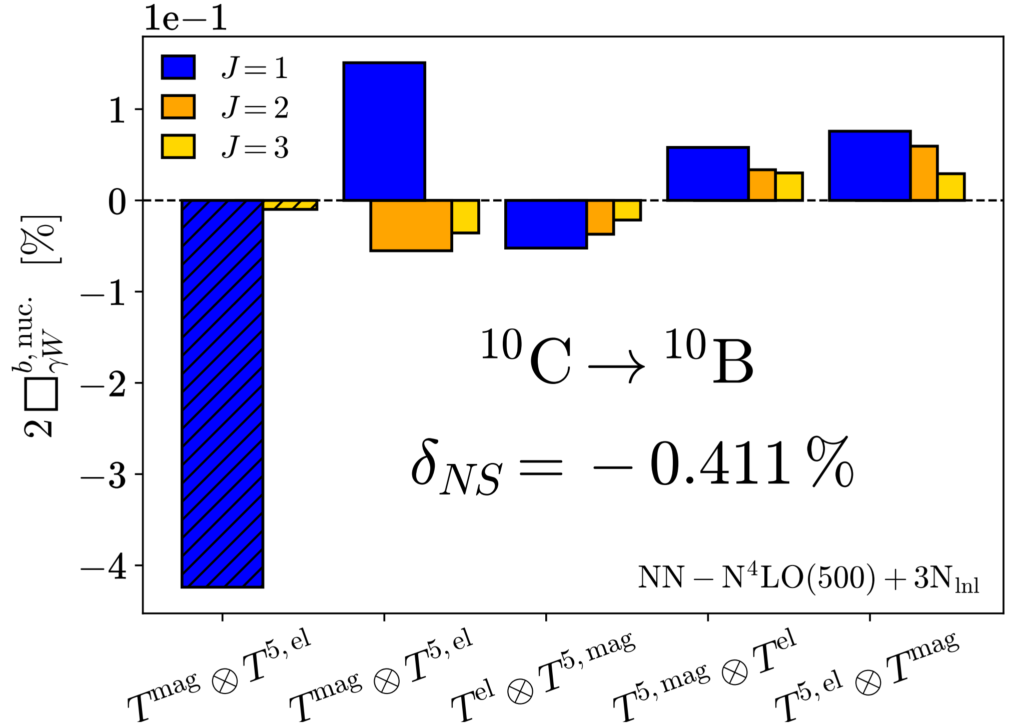
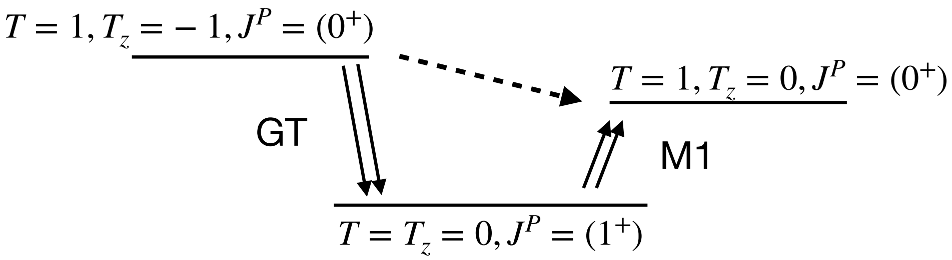


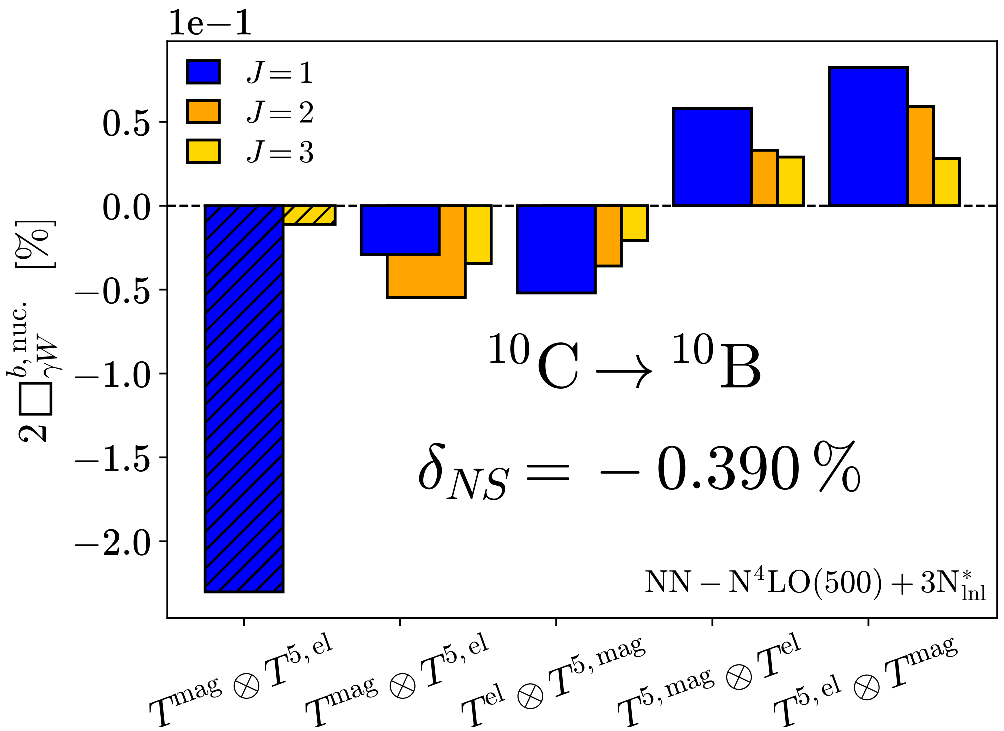


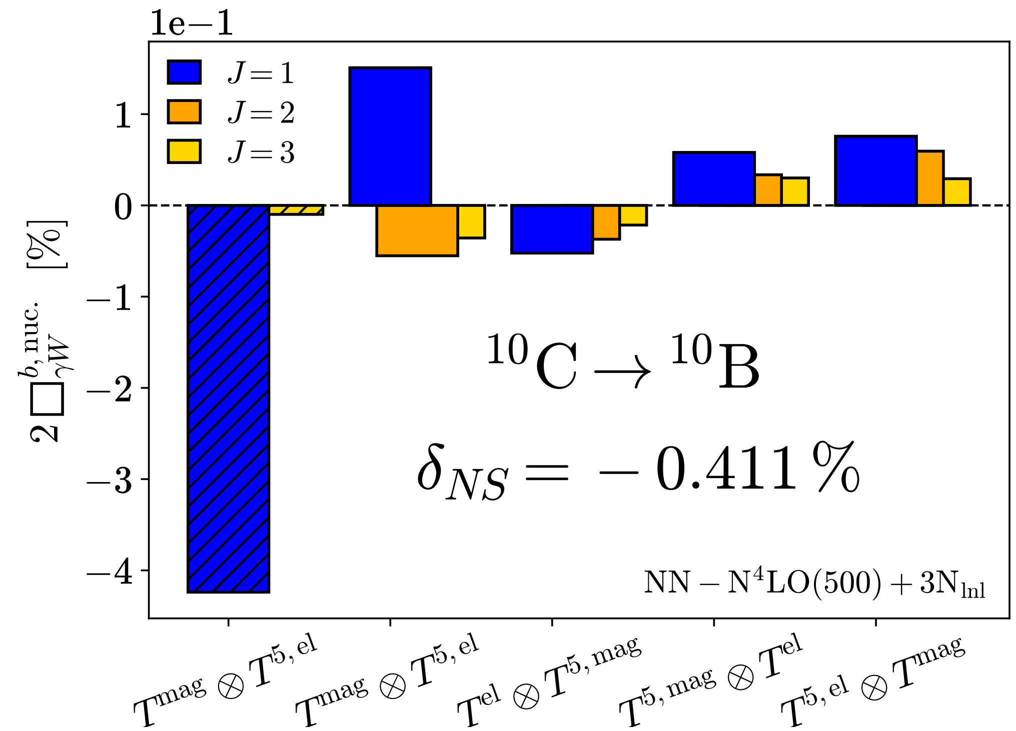
Large negative contribution: low-lying 1^+ level in ^{10}B

Large GT and M1 rates favor a two-step process



Ab-initio δ_{NS}: numerical results





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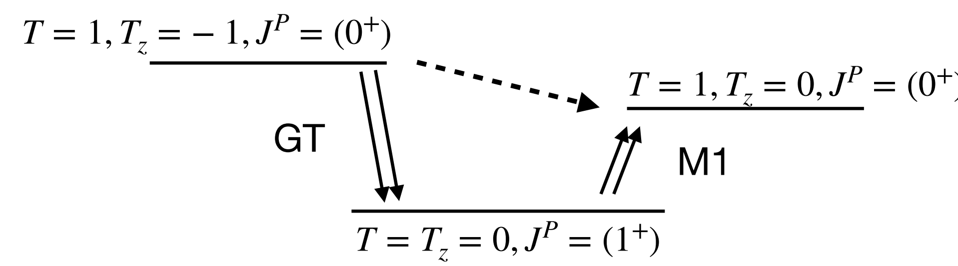
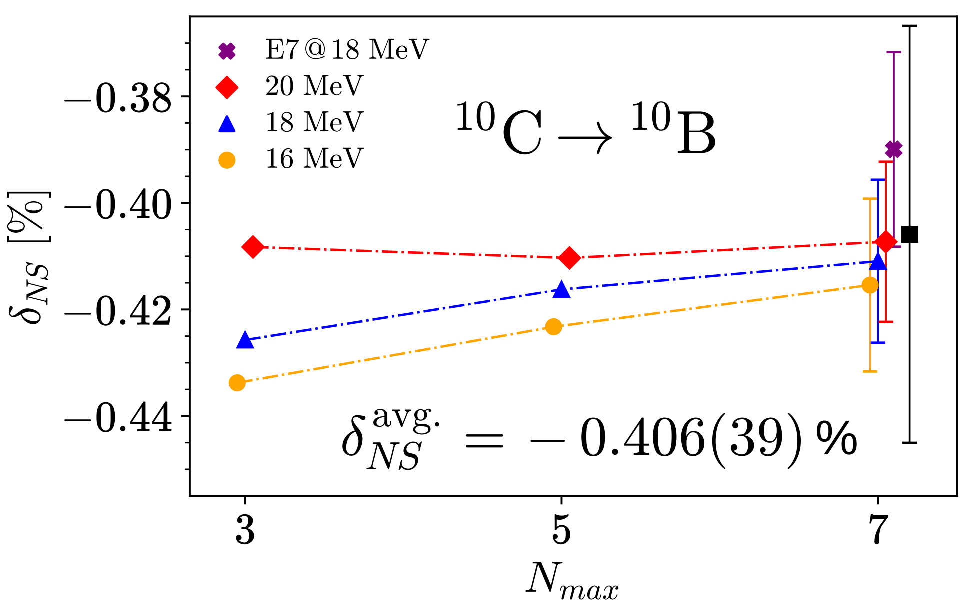


Check **Ω-independence** and **convergence w.r.t. N_{max}**



Final result for $^{10}C \to {}^{10}B$:

$$\delta_{\mathrm{NS}} = -0.406(39)\,\%$$

arXiv: **2405.19281**

Compare to Hardy-Towner (old-fashion SM)

$$\delta_{\mathrm{NS}} = -0.347(35)\,\% \qquad (2014)$$

$$\delta_{\mathrm{NS}} = -0.400(50)\,\% \qquad (2020)$$

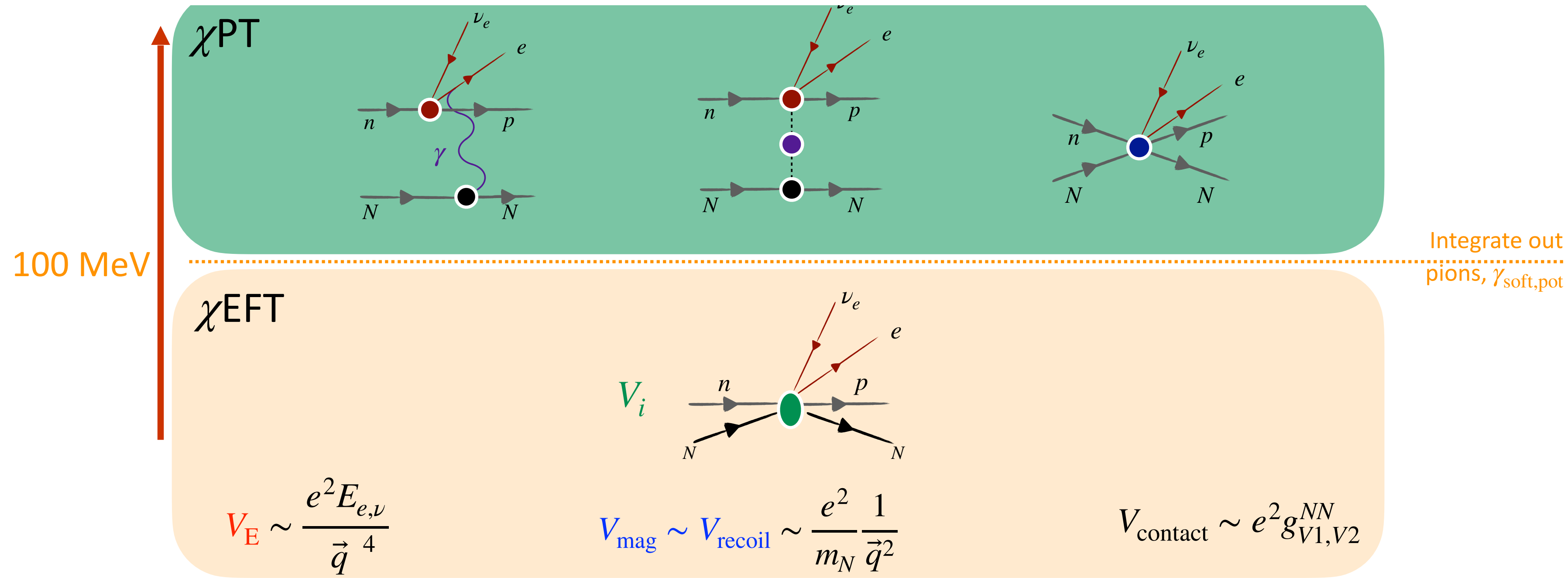
Ab-initio δ_{NS} in EFT: $^{14}O \to {}^{14}N$ with Variational Monte Carlo

V. Cirigliano et al, arXiv: **2405.18469**

$$\delta_{NS}^{(0)} = -(1.76+0.11\pm0.88)\cdot 10^{-3}$$

Uncertainty:
assuming unknown
counter term to be of
"natural size"

$$g^{NN}_{V1,V2} = 1/(4m_N F_\pi^2)$$



Compare to Hardy-Towner 2020: $\delta_{NS,B} = -1.96(50)\cdot 10^{-3}$

Promising avenue: all logs under control and are consistent

Downside: EFT non-renormalizable —> unknown counter terms external to the theory

Need extra input (dispersion theory; explicit modeling; fit to data)

Summary & Outlook

Cabibbo Angle Anomaly at 2-3σ

Nuclear uncertainties under scrutiny: δ_{NS} in ab-initio and EFT
δ_C & δ_{NS} for 15 decays from ^{10}C to ^{74}Rb — Community effort required!

Future experiments:

Neutron: UCNτ, τSPECT ($\delta\tau_n$: $0.4 \to 0.1s$); PERC, Nab (δg_A : $4 \to 1 \times 10^{-4}$)
Competitive! But: resolve existing discrepancies (e.g. "beam-bottle" lifetime)
Kaon decays: NA62, BELLE II $K\ell 3$ vs $K\mu 2$ (+ Lattice effort!)
Pion: $\pi^+ \to \pi^0 e^+ \nu$ PIONEER @ PSI (δBR: 0.3%—>0.03%)
Nuclear charge radii across superallowed isotriplets
Stable: μ-atoms @ PSI, radii of unstable nuclei @ ISOLDE, TRIUMF
Neutron skins of stable daughters with PVES @ MESA
Interplay with the nuclear EoS program: neutron skin via symmetry energy vs. ISB

Cabibbo anomaly interpretable in terms of BSM

Superallowed decays: bounds on scalar BSM from dataset consistency

International workshop on Electroweak Precision InterseCtions

EPIC 2024

September 22-27 2024, Cala Serena Beach Resort (Geremeas)

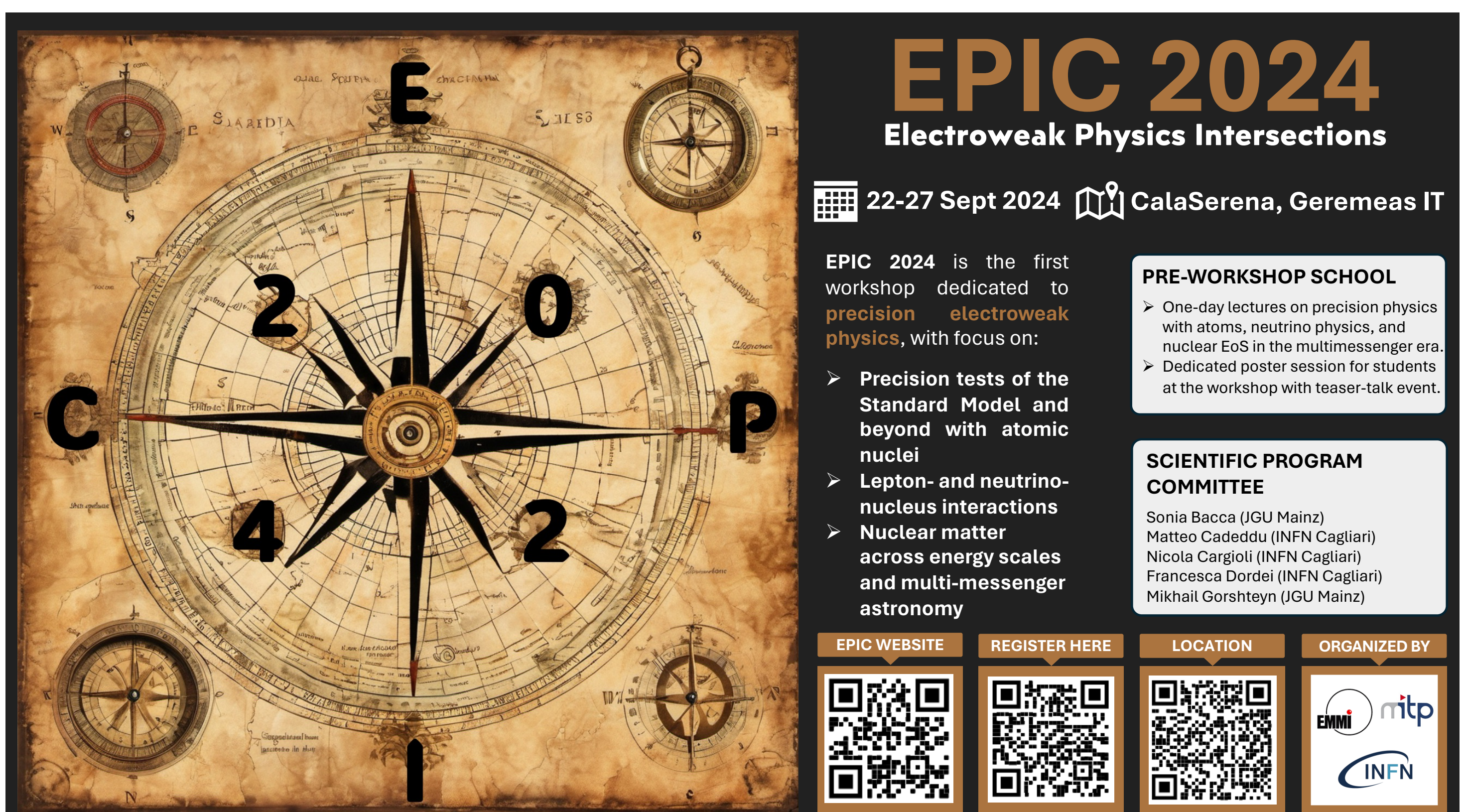
Bring together different communities:

Particle, Nuclear, Atomic, Neutrino, Astro, GW

Study existing synergies & elaborate new ones!

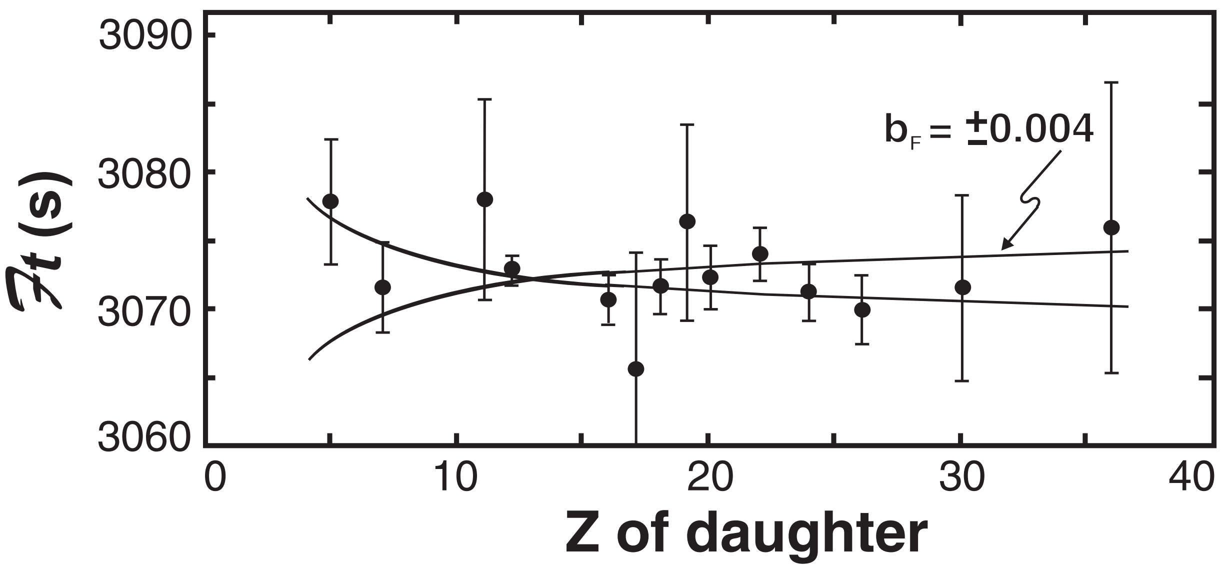
1-day pre-workshop school for PhD students

1st event this year, plan for biennial workshop series



Backup

BSM searches with superallowed beta decays



Induced scalar CC —> Fierz interference bF

$$\mathcal{F}t^{SM} \to \mathcal{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$b_F = -0.0028(26)$ ~ consistent with 0

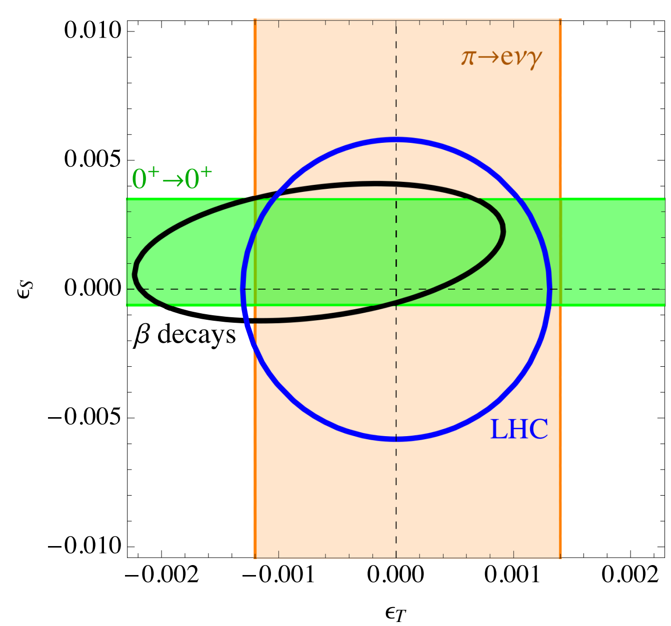
Independently of Vud and CKM unitarity: internal consistency of the data base with SM!
Like $\Delta^{(3)}_{\text{CKM}}$ does not require other experimental inputs to make a statement on (B)SM

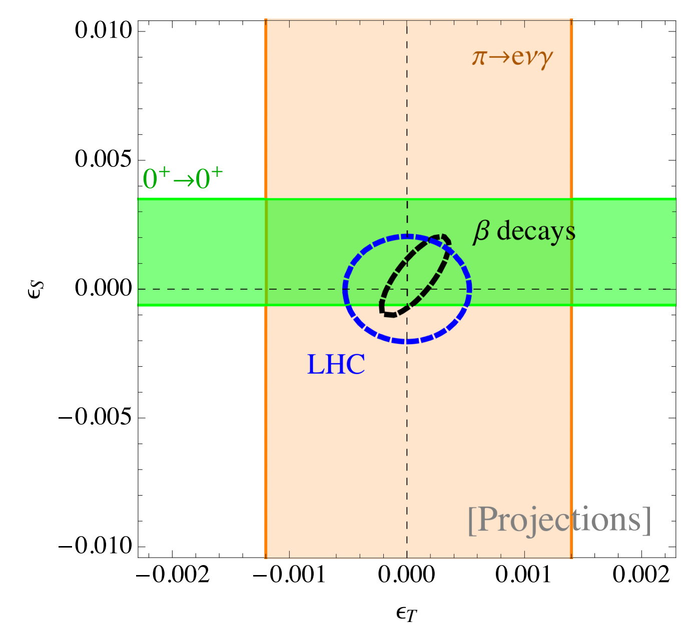
Entangled with nuclear theory uncertainties — a global effort of nuclear theory community needed

S, T interaction flips helicity:
Suppressed at high energy

Beta decay vs. LHC on S,T
Complementarity now and in the future!

Gonzalez-Alonso et al 1803.08732





V_{ud} from neutron decay

Neutron decay: 2 measurements needed

$$|V_{ud}|^2 = \frac{5024.7\ \mathrm{s}}{\tau_n(1+3g_A^2)(1+\Delta_R^V)}$$

RC Δ_R^V: bottleneck since 40 years

Pre-2018: $\Delta_R^V = 0.02361(38)$ *Marciano, Sirlin PRL 2006*

Post-2018: $\Delta_R^V = 0.02479(21)$ *MG, Seng Universe 2023*

Since 2018: DR+data+pQCD+EFT+LQCD

Δ_R^V uncertainty: factor 2 reduction

C-Y Seng et al., PRL 2018; PRD 2019
A. Czarnecki, B. Marciano, A. Sirlin, PRD 2018
K. Shiells et al, PRD 2021; L. Hayen PRD 2021
P-X Ma, X. Feng, MG, L-C Jin, et al *2308.16755*

Experiment: factor 3-5 uncertainties improvement; discrepancies in τ_n and g_A

3.4σ

$g_A = -1.27641(56)$ — **PERKEO-III** *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

$g_A = -1.2677(28)$ — **aSPECT** *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*

4σ

$\tau_n = 877.75(28)^{+16}_{-12}$ — **UCNτ** *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

$\tau_n = 887.7(2.3)$ — **BL1 (NIST)** *Yue et al, PRL 111 (2013) 222501*

PDG average

$$V_{ud}^{\text{free n}} = 0.9743\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

Single best measurements only

$$V_{ud}^{\text{free n}} = 0.9740\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

Future exp coming! RC under control

V_{ud} from semileptonic pion decay

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\mathrm{s}^{-1}} \qquad V_{ud}^{\pi\ell 3} = 0.9739\,(27)_{exp}\,(1)_{RC}$$

RC to semileptonic pion decay

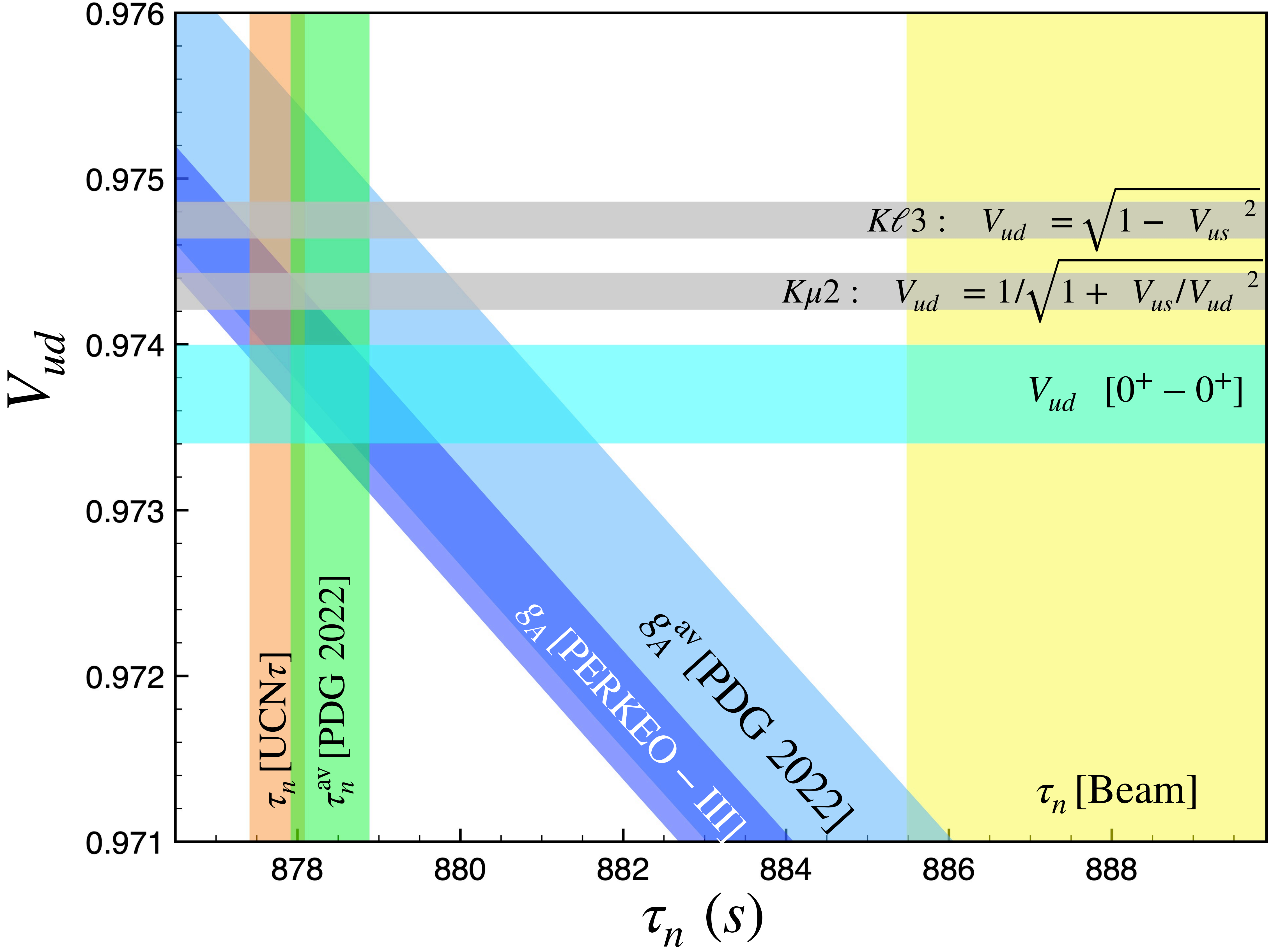
δ uncertainty: factor 3 reduction

ChPT: $\delta = -0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$ *Cirigliano et al, 2003; Passera et al, 2011*

DR + LQCD + ChPT: $\delta = 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$ *Feng et al, 2020; Yoo et al, 2023*

Future exp: 1 o.o.m. (PIONEER @ PSI)

Status of Cabibbo Unitarity



γW-box from DR + Lattice QCD input

Currently available neutrino data at low Q^2 - low quality;
Look for alternative input — compute Nachtmann moment $M_3^{(0)}$ on the lattice

First direct LQCD computation $\pi^- \to \pi^0 e^- \nu_e$ **Feng, MG, Jin, Ma, Seng 2003.09798**

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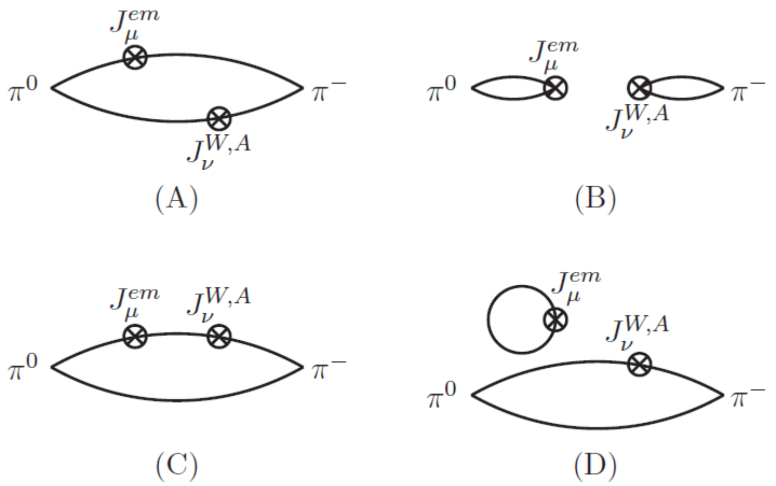
5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations
w. 2+1 flavor domain wall fermion



Quark contraction diagrams

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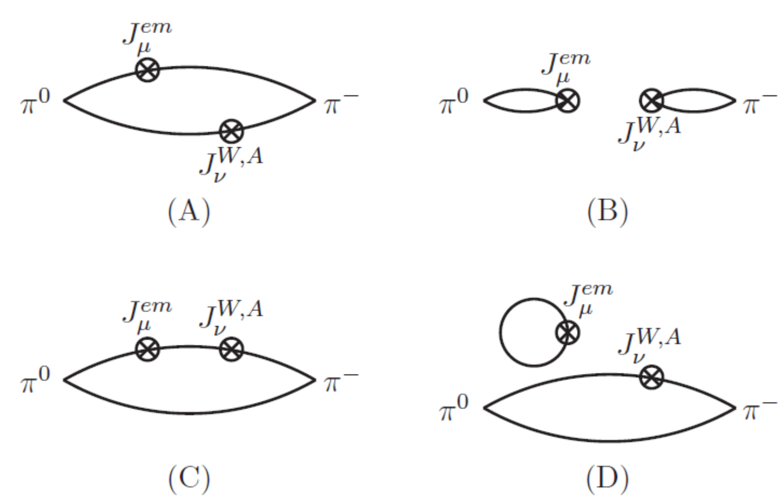
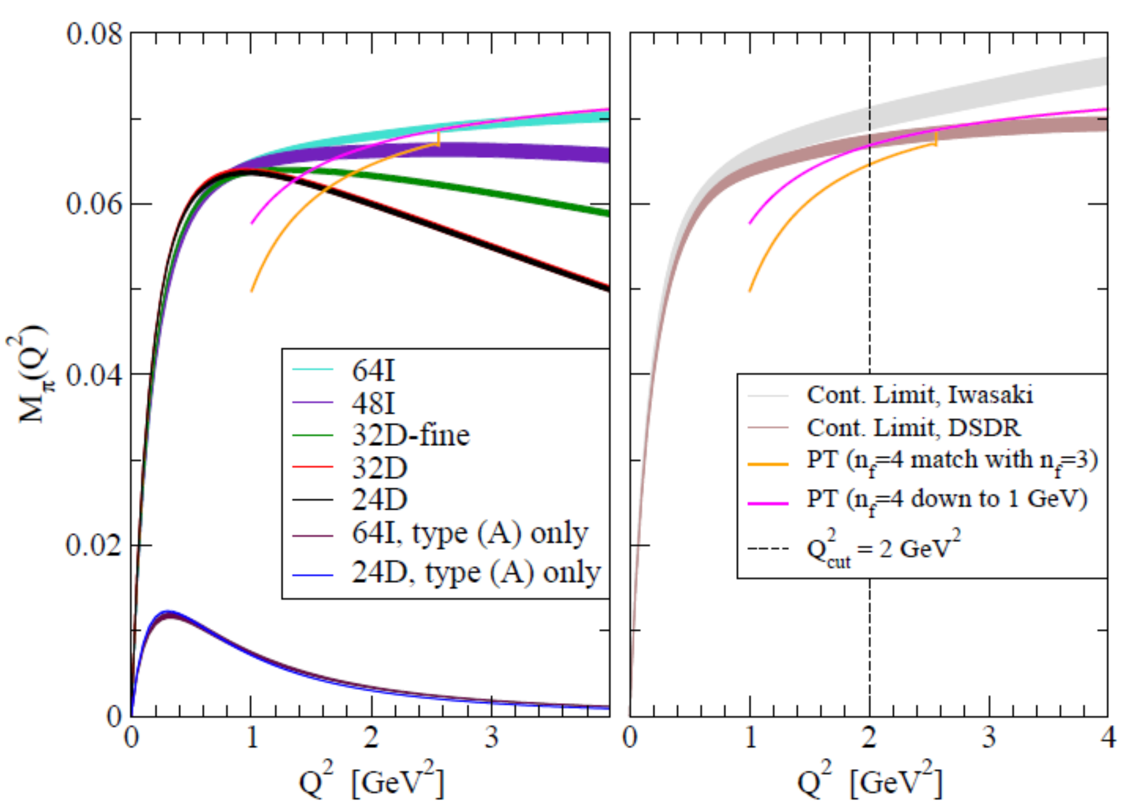


Quark contraction diagrams

5 LQCD gauge ensembles at physical pion mass
Generated by RBC and UKQCD collaborations
w. 2+1 flavor domain wall fermion

Match onto pQCD at $Q^2 \sim 2\,\mathrm{GeV}^2$



(before cont. extrapolation) (after cont. extrapolation)

$$\Box_{\gamma W}^{VA,\pi} = 2.830(11)_{\rm stat}(26)_{\rm sys}$$

Independent calculation by Los Alamos group

Yoo et all, 2305.03198

$$\Box_{\gamma W}^{VA,\pi} = 2.810(26)_{\rm stat+sys}$$

First lattice QCD calculation of γW-box

Direct impact for pion decay $\pi^+ \to \pi^0 e^+ \nu_e$

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\mathrm{s}^{-1}}$$

Previous calculation of δ — in ChPT

Cirigliano, Knecht, Neufeld and Pichl, EPJC 2003

Significant reduction of the uncertainty!

$$\delta: \quad 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}} \to 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$$

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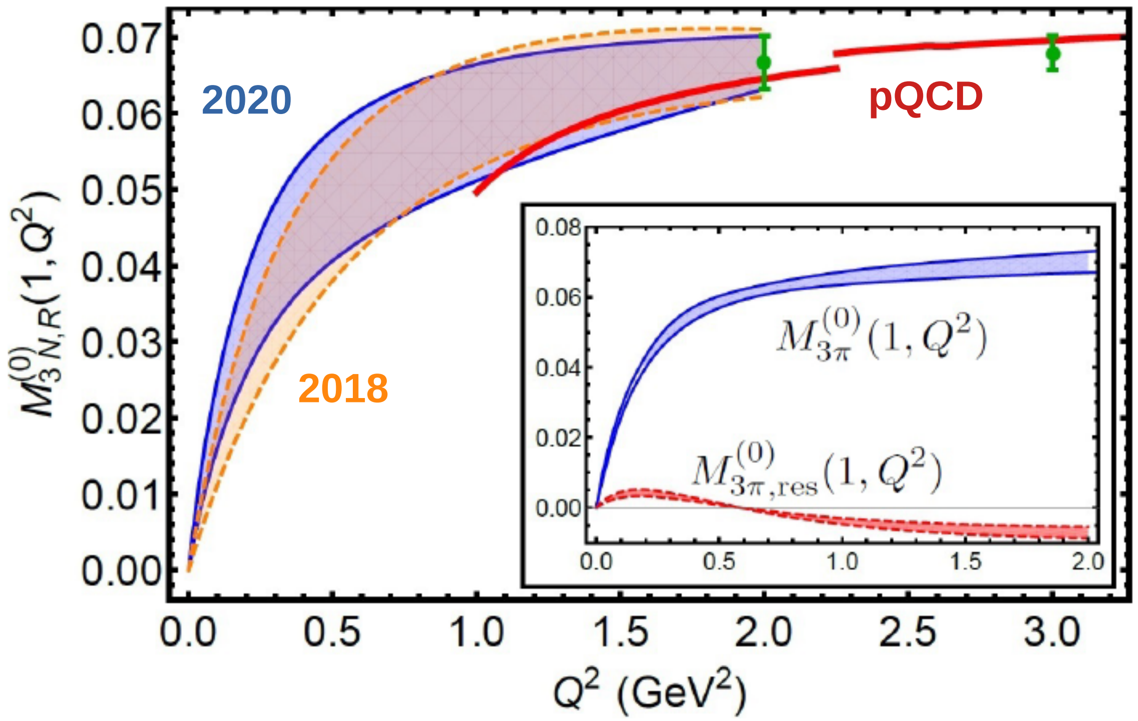
Indirectly constrains the free neutron γW-box
— requires some phenomenology
Based on Regge universality & factorization



Independent confirmation

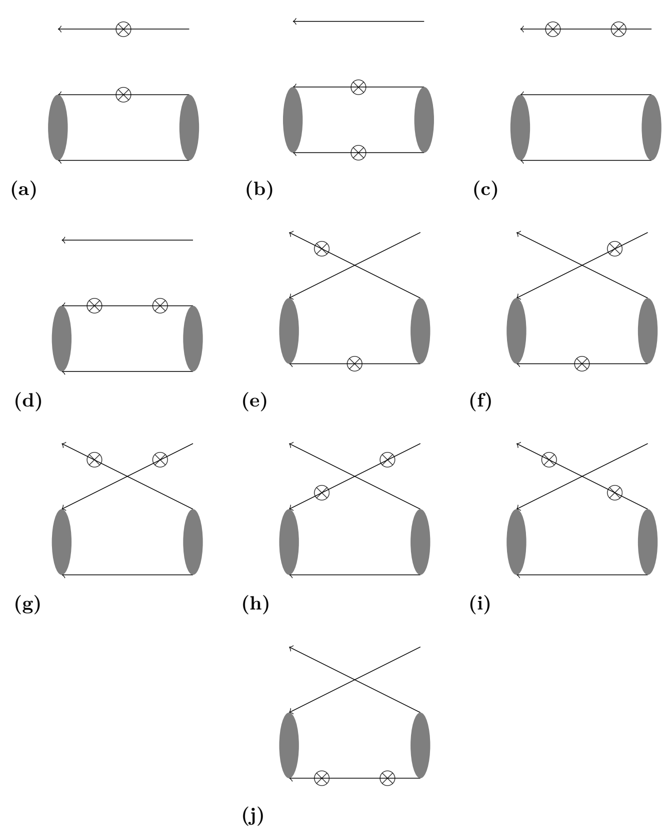
$$\Delta_R^V = 0.02467(22)_{\mathrm{DR}} \to 0.02477(24)_{\mathrm{LQCD+DR}}$$

Seng, MG, Feng, Jin, 2003.11264

First LQCD calculation of γW-box on the neutron

Much more challenging than pion:

Numerically heavier
Excited state contamination requires longer time
Large contribution from low Q $\sim g_A \mu^V$ absent for pion



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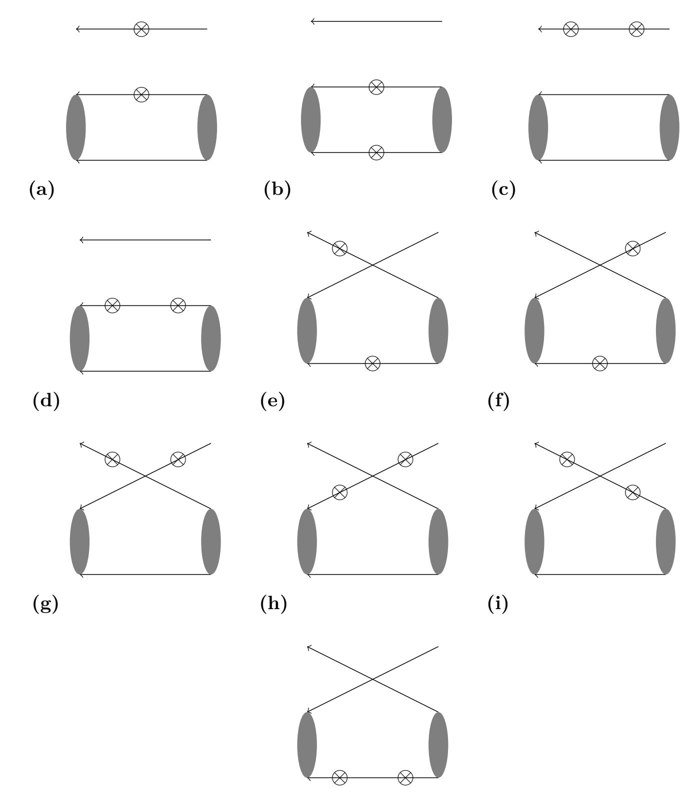

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Large contribution from low Q $\sim g_A \mu^V$ absent for pion

Split into long/short distance separated by t_s

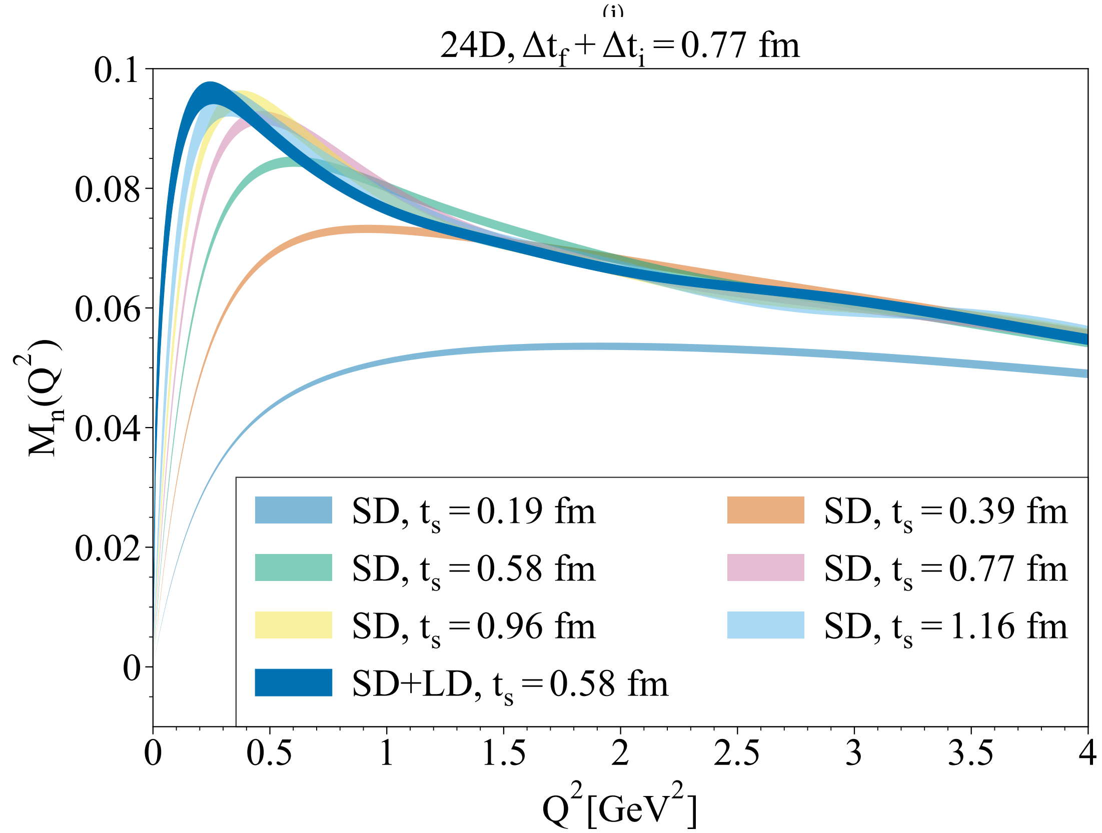
$$M_n(Q^2) = M_n^{\mathrm{SD}}(Q^2, t_s) + M_n^{\mathrm{LD}}(Q^2, t_s, t_g)$$

RBC/UKQCD 2+1 domain wall fermion

Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	N_{conf}
24D	142.6(3)	24	64	1.023(2)	207
32D-fine	143.6(9)	32	64	1.378(5)	69

$$\Delta_R^V = 0.02439(19)_{\mathrm{LQCD}} \text{ vs } 0.02467(22)_{\mathrm{DR}}$$

The result slightly lower than DR;
Finer lattice calculations underway



Ma, Feng, MG et al 2308.16755