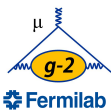




Results from the Muon $g-2$ experiment at Fermilab

Anna Driutti
 University and INFN Pisa
 on behalf of the Muon $g-2$ Collaboration



QNP2024 - The 10th International Conference on Quarks and Nuclear Physics

The g-factor and the muon anomaly

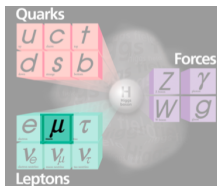
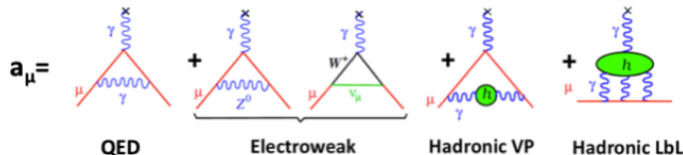
- **Muon:** elementary particle with spin-1/2 and magnetic moment proportional to spin through the **g-factor**:

$$\vec{\mu} = g \frac{q}{2m_\mu} \vec{S}$$

- At first order (Dirac theory for $s = 1/2$ particles) $g = 2$ but with higher order corrections (vacuum effects) $g > 2$:

$$\underbrace{g_\mu = 2}_{\text{Dirac}} (1 + a_\mu) \Rightarrow \boxed{a_\mu = \frac{g - 2}{2}} \quad \text{muon anomaly}$$

→ a_μ can be calculated with the SM (all particles contribute):



The g-factor and the muon anomaly

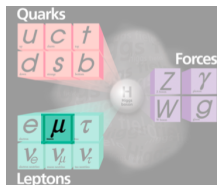
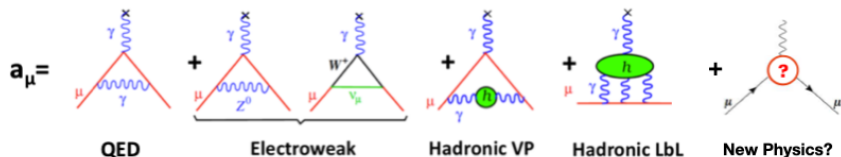
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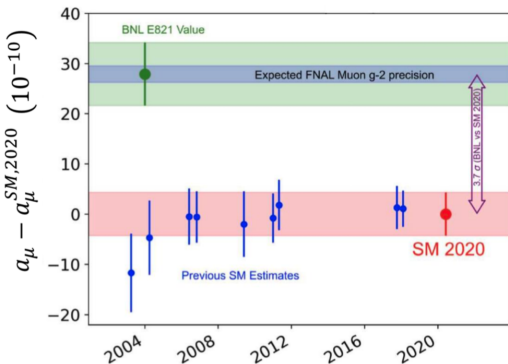
→ a_{μ} can be calculated with the SM (all particles contribute):



→ If new particles contribute the SM will disagree with measurement:
precise test of the SM and look for new physics

Experimental measurement vs. SM calculation (before 2021)

- Long-standing $> 3\sigma$ discrepancy



- **E821 (BNL) experimental value:**

$$a_\mu^{E821,BNL} = 116592080(63) \times 10^{-11}$$

[Phys. Rev. D, **73** (2006) 072003]

- **SM value** re-evaluated in 2020 by Muon g-2 Theory Initiative:

$$a_\mu^{SM,2020} = 116591810(43) \times 10^{-11}$$

[Phys. Rept. **887**, 1 (2020)]

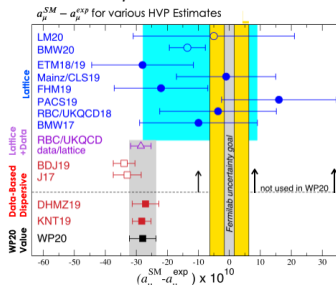
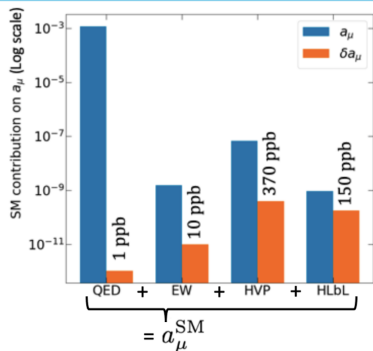
- In the meantime: **FNAL Exp.** was constructed and began collecting data in 2018, continuing operations until 2023 aiming to **improve uncertainty** with **140 ppb goal**

[E821, BNL uncertainty: 540 ppb; SM, 2020 uncertainty: 370 ppb]

SM calculation of the muon anomaly

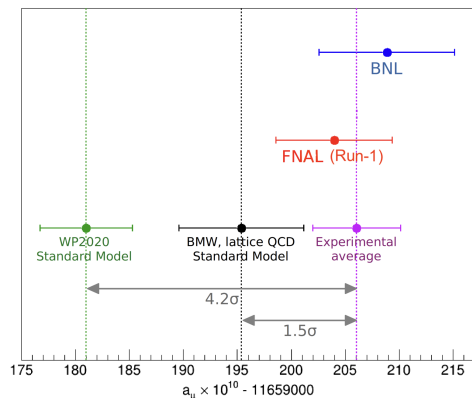
- Calculation is continuously updated
- Largest contribution but lowest uncertainty from QED
- EW terms are also well known
- Uncertainty dominated by strong-interaction contributions (HVP and HLbL)
- **Hadronic Vacuum Polarization LO-term:** virtual loops with hadrons calculated with two approaches
 - **data-driven:** experimental data (e^+e^-) plus dispersion theory (used by WP20)
 - direct calculation with **lattice QCD**
- (-) Proposal: from the shape of the $\mu - e$ elastic scattering cross section *vs.* space-like squared momentum transfer **MUonE Experiment**

References at <https://muon-gm2-theory.illinois.edu>



Experimental measurement vs. SM calculation (2021)

- In April 2021 were published:



- a new measurement from **FNAL Muon g – 2 Exp. Run-1 data** that confirmed result from BNL:

$$a_\mu(\text{FNAL}) = 116592040(54) \cdot 10^{-11} \text{ (460 ppb)}$$

$$a_\mu(\text{BNL}) = 116592089(63) \cdot 10^{-11} \text{ (540 ppb)}$$

$$a_\mu(\text{Exp}) = 116592061(41) \cdot 10^{-11} \text{ (350 ppb)}$$

[Phys. Rev. Lett. **126**, no.14, 141801 (2021)]

- a new theoretical calculation $a_\mu(\text{BMW, HVP – LO})$ based on Lattice QCD in tension with $a_\mu(\text{WP, HVP – LO})$ calculation based on e^+e^- data

[Nature **593** (2021) 51-55]

- In this talk: review of the **FNAL Run-1** measurement and I will present you the latest **FNAL Run-2/3 result** (announced on Aug 10, 2023)

[Phys. Rev. Lett. **131**, 161802 (2023)]

Experimental technique

1. Inject polarized muons into a magnetic storage ring
2. Muons circulate around the ring at the cyclotron frequency:

$$\vec{\omega}_C = \frac{q}{\gamma m_\mu} \vec{B}$$

3. Muon spin precession frequency (Larmor) is given by:

$$\vec{\omega}_S = \frac{q}{\gamma m_\mu} \vec{B} (1 + \gamma a_\mu)$$

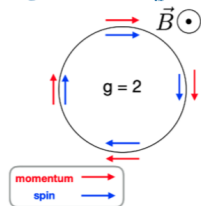
4. Muon anomaly is related to **anomalous precession frequency**:

$$\vec{\omega}_a \cong \vec{\omega}_S - \vec{\omega}_C \cong a_\mu \frac{q}{m_\mu} \vec{B}$$

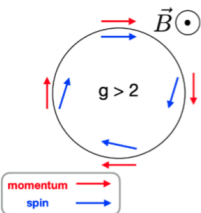
5. Measure B and ω_a to extract the anomaly



$$\text{If } g = 2 \Rightarrow \vec{\omega}_a = 0$$



$$g \neq 2 \Rightarrow \vec{\omega}_a \cong a_\mu \frac{e}{m} \vec{B}$$



Final formula

Muon anomaly is determined with:

$$a_\mu = \underbrace{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}}_{\text{ratio of frequencies } (R_\mu) \text{ measured by us}} \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}_{\text{fundamental factors (combined uncertainty 25 ppb)}}$$

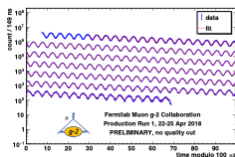
ratio of frequencies (R_μ)
measured by us

fundamental factors
(combined uncertainty 25 ppb):

ω_a : muon anomalous precession frequency

Extract from decay positron time spectra

$$N(t) = N_0 e^{-t/\tau_\mu} [1 + A \cos(\omega_a t + \phi)]$$



$\tilde{\omega}'_p(\mathbf{T}_r)$: magnetic field B in terms of (shielded) proton precession frequency (proton NMR $\hbar\omega_p = 2\mu_p B$) and weighted by the muon distribution

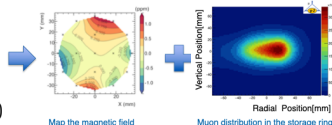
(shielded = measured in spherical water sample at $T_r = 34.7^\circ\text{C}$)

$\mu'_p(T_r)/\mu_e(H)$ from [Metrologia **13**, 179 (1977)]

$\mu_e(H)/\mu_e$ from [Rev. Mod. Phys. **88** 035009 (2016)]

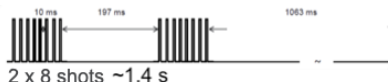
m_μ/m_e from [2018 CODATA (Web Version 8.1)]

$g_e/2$ from [Phys. Rev. Lett. **130**, 071801 (2023), Prog. Theor. Exp. Phys. 2022, 083C01 (2022), and 2023 update]

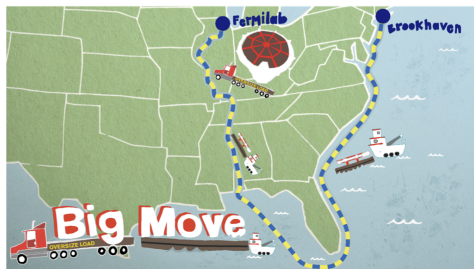


Production of the muon beam

- **Recycler Ring:** 8 GeV protons from Booster are divided in 4 bunches
- **Target Station:** p -bunches are collided with target and π^+ with 3.1 GeV/ c ($\pm 10\%$) are collected
- **Beam Transport and Delivery Ring:** magnetic lenses select μ^+ from $\pi^+ \rightarrow \mu^+ \nu_\mu$ then μ^+ are separated from p and π^+ in circular ring
- **Muon Campus:** polarized μ^+ are ready to be injected into the storage ring

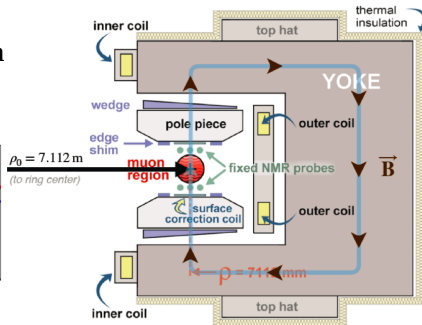
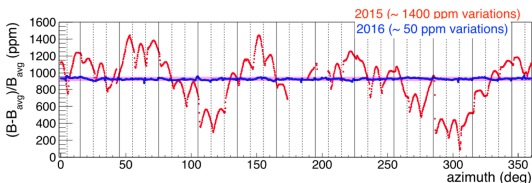
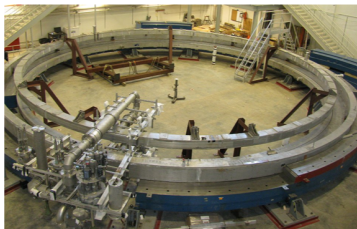


The storage ring journey: from BNL to FNAL in Summer 2013



Storage ring magnet

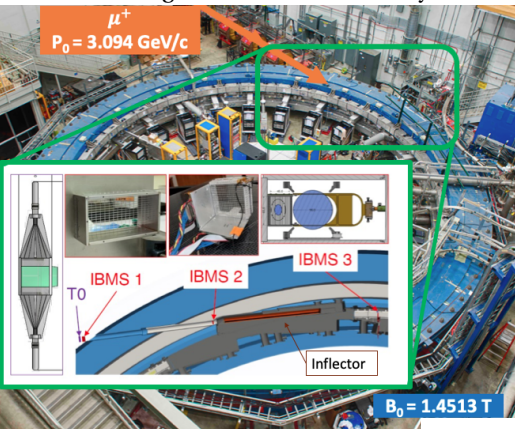
- Three superconducting coils provide 1.45 T vertical magnetic field
- Vacuum chambers surrounded by a cryosystem and C-shaped **yokes** to allow the decay positrons to reach the detectors.
- Achieved 50 ppm on field uniformity thanks to low-carbon steel **poles, edge shims, steel wedges, surface correction coil**



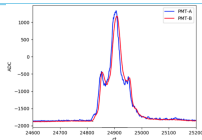
final field ~ 3 times more uniform than at BNL

Injection of the muons into the ring

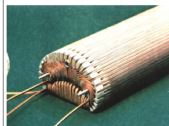
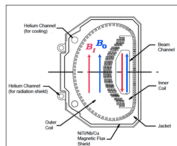
- Beam enters the ring through a 2.2 m-long 10 cm hole in the iron yoke



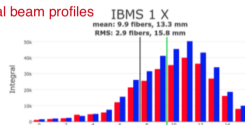
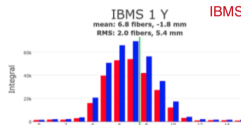
- **T0 Counter** (thin scintillator read out by PMTs) to measure **beam time profile**



- **Inflector magnet** provides nearly field free region for muons to enter the storage region

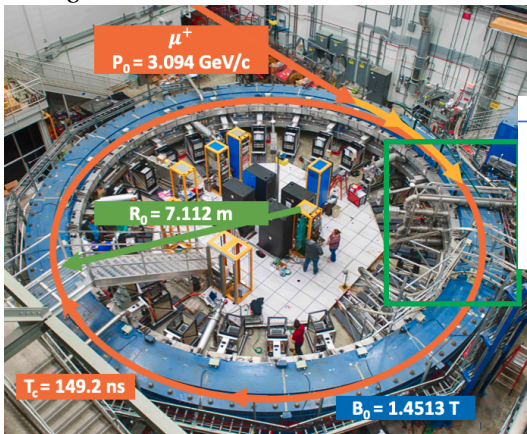


- **Inflector Beam Monitoring System** (scintillator fiber grids) to measure **beam spatial profile**



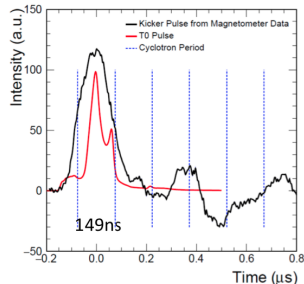
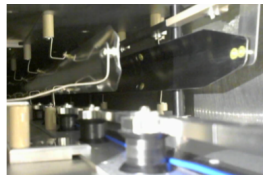
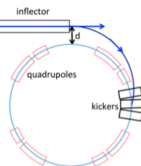
Muon storage

- Injected beam is 77 mm off from storage region center

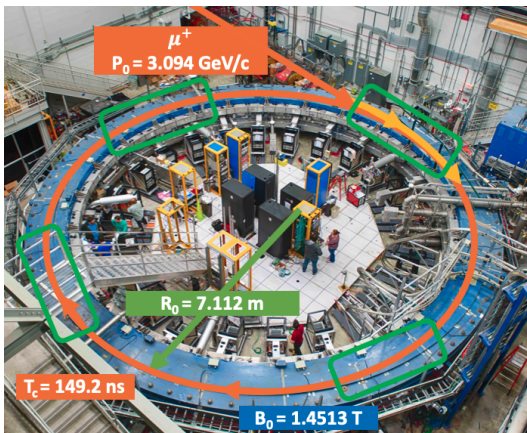


Kicker Magnets

- 3 pulsed magnets deflect beam $\sim 10 \text{ mrad}$ onto the closed storage orbit in less than 150 ns

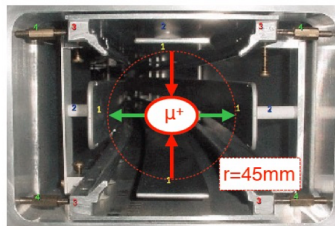


Vertical focusing



Electrostatic Quadrupoles

- 4 sets of quads provide vertical beam focusing

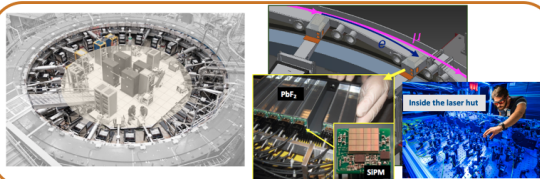


- E -field component cancels out (at first order) when muons at *magic momentum*:

$$\vec{\omega}_a \cong -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \underbrace{\frac{1}{\gamma^2 - 1}} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

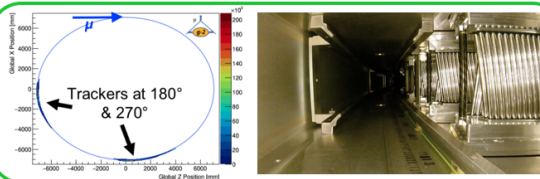
~ 0 if $\gamma = 29.3$ i.e., $p_\mu = 3.094 \text{ GeV}/c$

Detectors and field probes



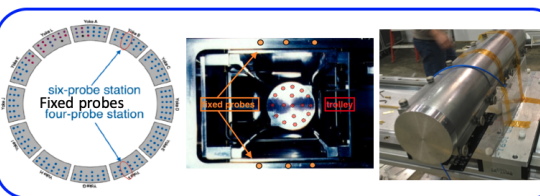
24 Calos around the ring

- Each made of 6×9 PbF_2 crystals read out by large-area SiPMs
- 1296 channels individually calibrated by 405nm-laser system



2 in-vacuum straw trackers

- Each with 8 modules consisting of 128 gas filled straws



2 types of field probes

- 378 fixed NMR probes above and below storage region
 - measure B-field 24/7
- Trolley with 17-probe NMR
 - 2D profile of B over the entire azimuth when beam is OFF

Master formula for Muon g-2 analysis

$$R_\mu = \frac{\overbrace{f_{clock} \cdot \omega_a^{meas}}^{\omega_a} \cdot \overbrace{(1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}^{\text{beam dynamics corrections}}}{\underbrace{f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi)}_{\tilde{\omega}'_p(T_r)} \cdot \underbrace{(1 + B_k + B_q)}_{\text{field corrections}}}$$

f_{clock} : blinded clock
 ω_a^{meas} : measured precession frequency

f_{calib} : absolute magnetic field calibration
 $\omega'_p(x, y, \phi)$: field maps
 $M(x, y, \phi)$: muon beam distribution

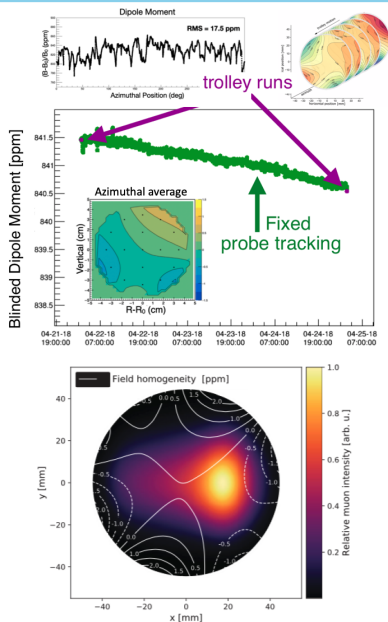
C_e : electric field correction
 C_p : pitch correction
 C_{ml} : muon loss correction
 C_{pa} : phase-acceptance correction
 C_{dd} : differential-decay correction

B_k : transient field from eddy current in kicker
 B_q : transient field from quad vibration

Measuring the magnetic field seen by the muons

$$R_{\mu} = \frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}{f_{calib} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)}$$

- ω_p' is proportional to the magnetic field and it is mapped every 3 days using 17 NMR probes on a trolley
- During data taking fixed NMR probes located above and below the storage region monitor the field
- Fixed probes to interpolate the field between trolley runs
- Field maps are weighted by beam distribution (extrapolated from the decay e^+ trajectory measured by the trackers and simulations)

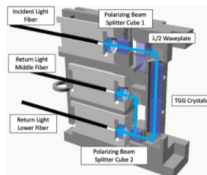


Magnetic field corrections

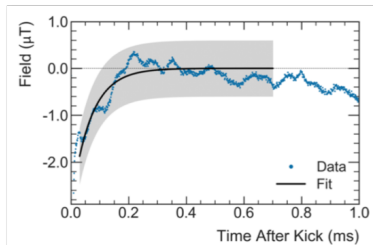
$$R\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa} + C_{dd})}{f_{calib} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Kicker transient field

- due to eddy currents produced by kicker pulses
- measured using Faraday magnetometers

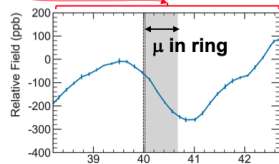
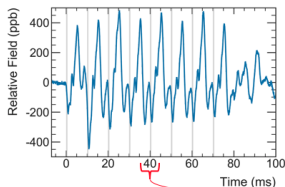
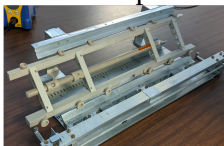


(a)



Quads transient field

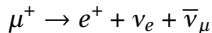
- due to mechanical vibrations from pulsing the quads
- mapped using special NMR probes



Measuring ω_a

$$R_\mu = \left(\frac{f_{\text{clock}} \cdot \omega_a^{\text{meas}} \cdot (1 + C_e + C_p + C_{mi} + C_{pa} + C_{dd})}{f_{\text{calib}} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

- Polarized muon decay:

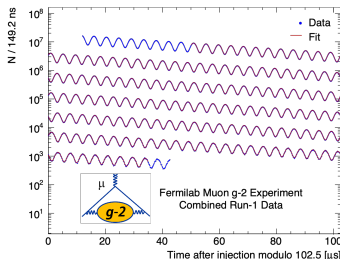
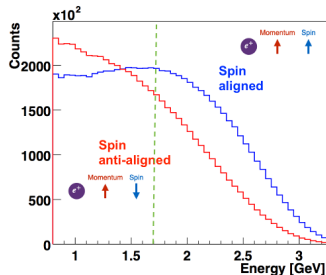


- High energy e^+ are preferentially emitted in direction of μ^+ spin (parity violation of the weak decay)
- Energy spectrum modulates at the ω_a frequency
- Counting the number of e^+ with $E_{e^+} > E_{\text{threshold}}$ as a function of time (wiggle plot) leads to ω_a :

$$N(t) = \underbrace{N_0}_{\text{normalization}} e^{-t/\tau} \left[1 + \underbrace{A}_{\text{g-2 asymmetry}} \cos(\underbrace{\omega_a t + \varphi}_{\text{g-2 phase}}) \right]$$

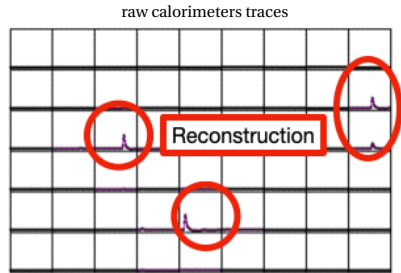
muon lab-frame lifetime
g-2 phase

E_{e^+} and t are measured by the calorimeters with a blinding factor applied to the digitization rate

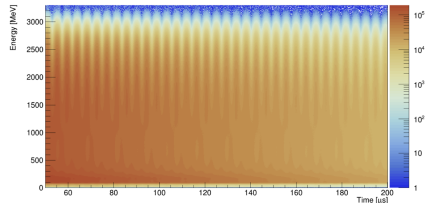


Wiggle plot

- Calorimeters data is reconstructed into energies and times
 - > 2 (Run-1) or 3 (Run-2/3) independent reconstruction routines

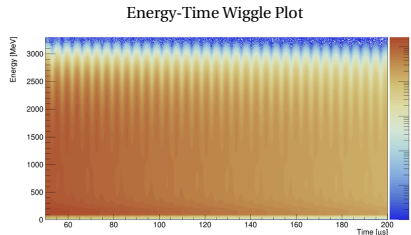


Energy-Time Wiggle Plot

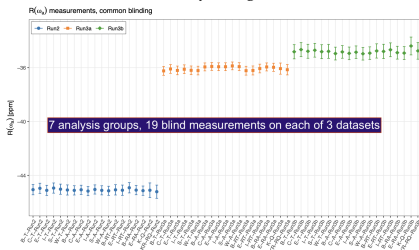


Wiggle plot

- Calorimeters data is reconstructed into energies and times
 - 2 (Run-1) or 3 (Run-2/3) independent reconstruction routines
- Different and software independently blind analysis techniques:
 - **Threshold (T) Method**
 - only positrons above energy threshold
 - **Asymmetry-Weighted (A) Method:**
 - positrons divided into energy bins and weighted by g-2 asymmetry
 - **Ratio (R) Method**
 - muon lifetime exponential decay removed before fitting
 - **Ratio Asymmetry-Weighted (RA) Method**
 - **Integrated Charge (Q) Method:**
 - sum of raw calorimeter traces (unique method independent of reconstruction)

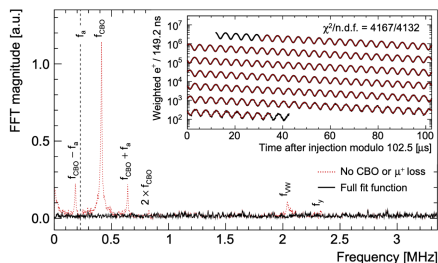


Different Analysis (e.g., Run-2/3)



Fitting procedure

- Fit \rightarrow Residuals \rightarrow Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_y, \omega_{VW} \text{ vertical oscillations}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue = fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

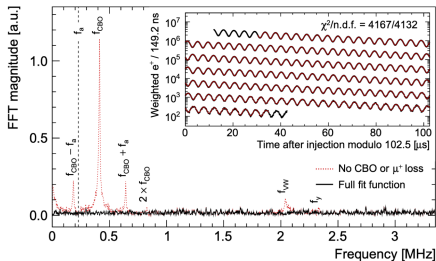
$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_{\theta} e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_{\theta} t + \phi \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_{\gamma}(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_{\phi} \cos(\omega_{CBO}(t) + \phi_{\phi}) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$\omega_{\gamma}, \omega_{VW}$ vertical oscillations

$$N_{\gamma}(t) = 1 + A_{\gamma} \cos(\omega_{\gamma}(t) + \phi_{\gamma}) e^{-\frac{t}{\tau_{\gamma}}}$$

Red = free parameters
Blue = fixed parameters

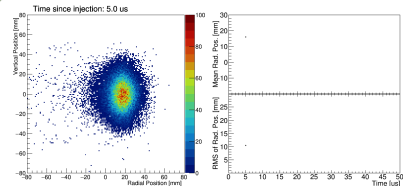
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_{\gamma}(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

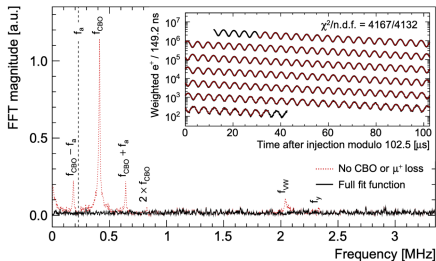
$$\omega_{VW}(t) = \omega_c - 2\omega_{\gamma}(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_0 t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

ω_y, ω_{VW} vertical oscillations

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue = fixed parameters

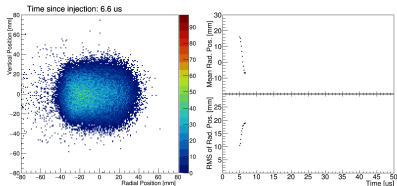
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

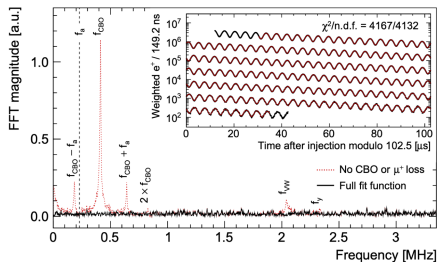
$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_0 t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

ω_y, ω_{VW} vertical oscillations

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue = fixed parameters

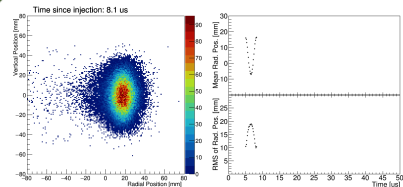
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

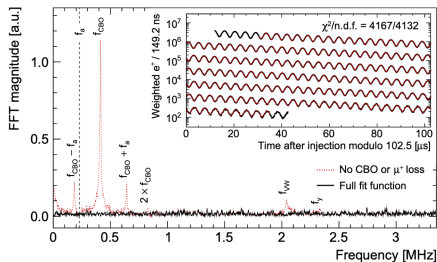
$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_B e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_B t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_Y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$\omega_{CBO}, \omega_{2CBO}$ radial oscillations

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_Y(t) = 1 + A_Y \cos(\omega_Y(t) + \phi_Y) e^{-\frac{t}{\tau_Y}}$$

ω_Y, ω_{VW} vertical oscillations

Red = free parameters
Blue = fixed parameters

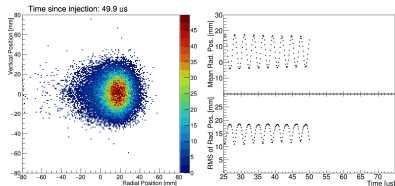
$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_Y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

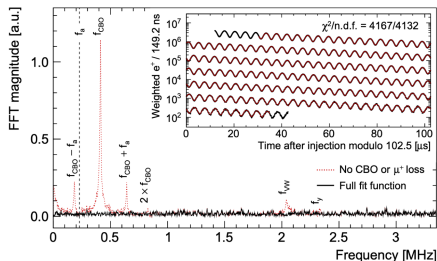
$$\omega_{VW}(t) = \omega_c - 2\omega_Y(t)$$

The muon beam oscillates and breathes:



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



Additional term to account for muons that hit the collimators and are lost:

$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_0 t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_y, \omega_{VW} \text{ vertical oscillations}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

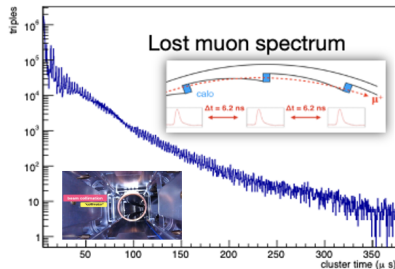
Red = free parameters
Blue = fixed parameters

$$J(t) = 1 - k_{LM} \int_0^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

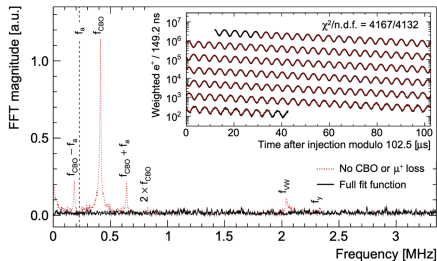
$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$



Fitting procedure

- Fit → Residuals → Fast Fourier Transform (FFT)
- Analyses of FFT fit residuals shows that simple 5-parameter model is inadequate
- Flat FFT of residuals using a (typical) 22-parameter fit function that includes beam dynamics effects



$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi + \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}} \quad \omega_{CBO}, \omega_{2CBO} \text{ radial oscillations}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{\tau_{2CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}} \quad \omega_y, \omega_{VW} \text{ vertical oscillation}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt \quad \text{Lost muons}$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

Red = free parameters
Blue = fixed parameters

+ beam dynamics corrections:

$$\omega_a = \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml} + C_{pa}) + C_{dd} \text{ (in Run-2/3)}$$

Electric Field, Pitch, Muon Loss, Phase Acceptance

Correction for Effects on Spin Precession

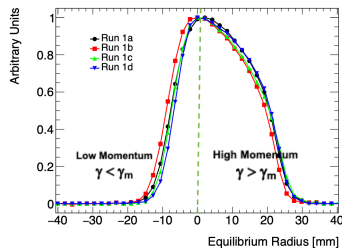
$$R_\mu = \left(f_{clock} \cdot \omega_a^{meas} \cdot (1 + \boxed{C_e} + \boxed{C_p} + C_{ml} + C_{pa} + C_{dd}) \right) / \left(f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q) \right)$$

Non-simplified spin-motion is described by:

$$\vec{\omega}_a = -\frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} - a_\mu \frac{\gamma}{\gamma + 1} (\vec{B} \cdot \vec{\beta}) \vec{\beta} \right]$$

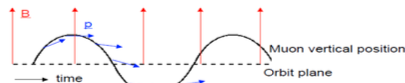
Electric Field

- due to momentum spread around p_{magic}
- measured using momentum distribution provided by the calorimeters in terms of equilibrium radius

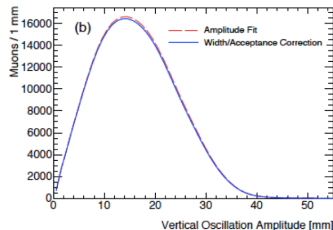


Pitch

- due to vertical beam oscillation



- measured using the beam vertical amplitude from the trackers, calorimeter data, and simulations



Corrections for Phase-Changing Effects

$$\begin{aligned} \cos(\omega_a t + \phi(t)) &= \cos(\omega_a t + \phi_0 + \phi' t + \dots) \\ &= \cos((\omega_a + \phi')t + \phi_0 + \dots) \end{aligned}$$

$$R_\mu = \left(\frac{f_{clock} \cdot \omega_a^{meas} \cdot (1 + C_e + C_p + C_{ml}) + C_{pa} + C_{dd}}{f_{calib} \cdot \omega'_p(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Muon losses

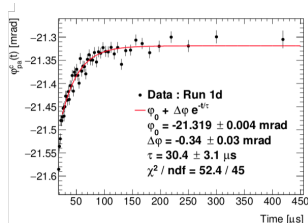
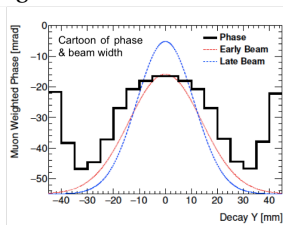
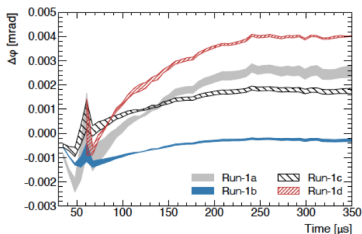
- cause a phase shift because muon-phase and muon loss rate are momentum-dependent
- measured using data-driven technique

Differential Decay

- correction to account for high momentum muons having a longer lifetime

Phase acceptance

- phase changes due to early to late variations of the beam
- measured using tracker data and simulations

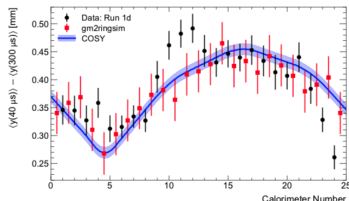
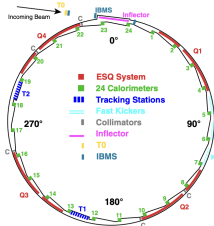
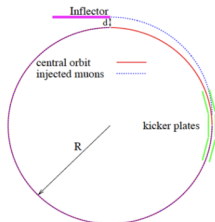
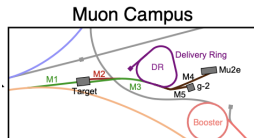


Simulation Tools

- Beam dynamics from simulations:
 - for beam dynamics corrections
 - to propagate the muon distribution around the ring
- Main simulation tools:
 - MARS
 - G4BEAMLINE
 - BMAD
 - COSY
 - GM2RINGSIM
- simulation tools are cross-checked against benchmarks and against each other.

**Muon
Campus
Simulation**

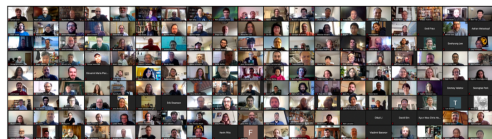
**Storage Ring
Simulation**



$$R_{\mu} = \left(\frac{f_{\text{clock}} \cdot \omega_a^{\text{meas}} \cdot (1 + C_e + C_p + C_{mi} + C_{pa} + C_{dd})}{f_{\text{calib}} \cdot \omega_p'(x, y, \phi) \otimes M(x, y, \phi) \cdot (1 + B_k + B_q)} \right)$$

Clock frequency (f_{clock}):

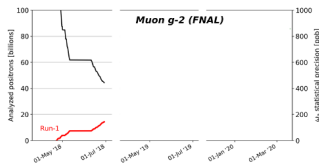
- frequency that our DAQ clock ticks
- **stable** at ppt level
- hardware-blinded to have $(40 - \varepsilon)$ MHz
 - ε kept **secret** from all collaborators
- **revealed** only when physics analysis is completed:
 - Run-1 result unblinded on Feb 25, 2021 during a virtual meeting
 - Run-2/3 result unblinded on Jul 24, 2023 during the collaboration meeting



First production run

Statistics:

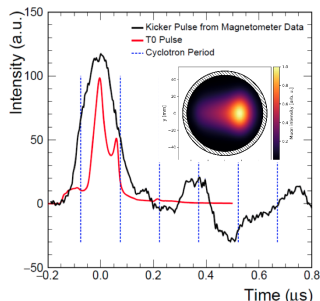
- March 26 – July 7 2018 : **Run1**
- $1.2 \times$ BNL after data quality selection



Main challenges:

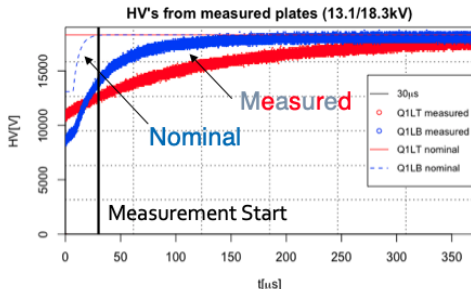
- Non-ideal kick

- low amplitude and ringing
- beam not centered in storage region



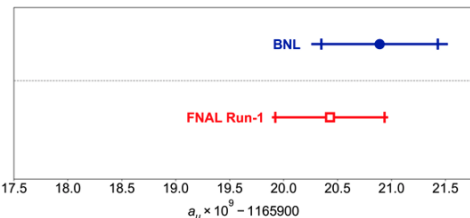
- 2 of 32 HV Quad resistors were damaged

- slow recovery time, enhanced C_{pa}



- Temperature variations larger than 1°C

Run-1 Result



- Run-1 result uncertainty is **statistics dominated**
- Major systematic uncertainties: **Phase Acceptance and Quad field transients**
- Next:** reduce as much as possible the experimental uncertainty on g-2!

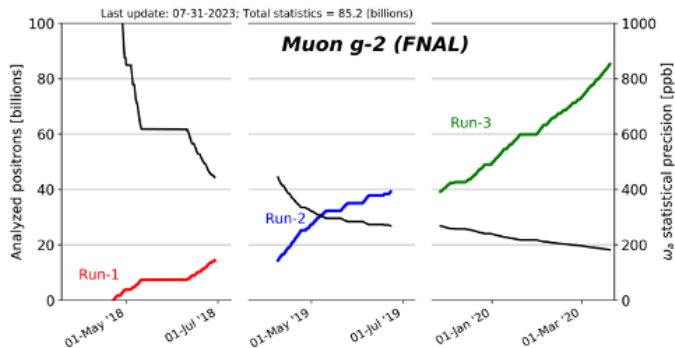
- First **FNAL** g - 2 result :

$$a_\mu = 116592040(54) \times 10^{-11} \text{ (462 ppb)}$$

- Good agreement with **BNL** g - 2

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a (statistical)	-	434
ω_a (systematic)	-	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{\text{calib}} \langle \omega_p'(x, y, \phi) \times M(x, y, \phi) \rangle$	-	56
B_k	-27	37
B_q	-17	92
$\mu_p'(34.7^\circ)/\mu_e$	-	10
m_μ/m_e	-	22
$g_e/2$	-	0
Total systematic	-	157
Total fundamental factors	-	25
Totals	544	462

Run-2 and Run-3 Statistics Improvement



Statistics:

- ~ 4.7 more data in Run-2/3 than Run-1

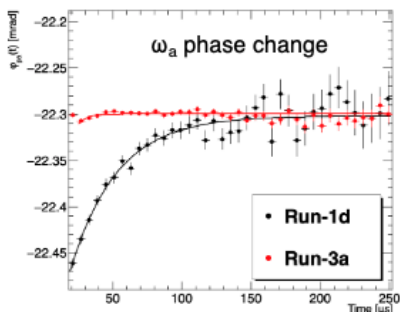
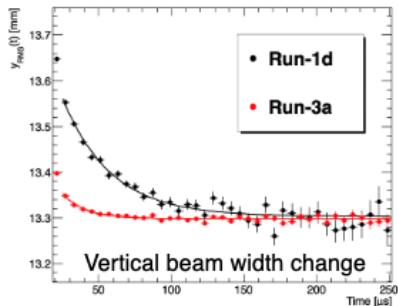
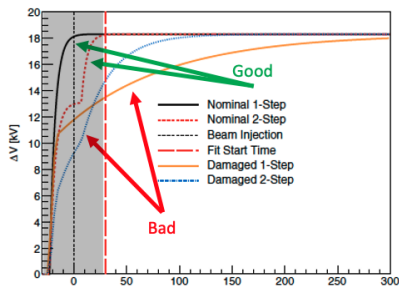
Dataset	Stat. Unc.
Run-1	434 ppb
Run-2/3	201 ppb
Run-1+Run-2/3	185 ppb

Run-2 and Run-3 Hardware Improvements

Before Run-2:

→ Replaced faulty quads HV resistors
Less beam motion and reduced C_{pa}

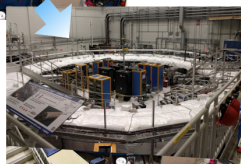
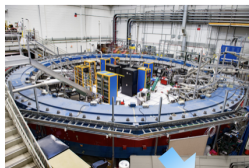
$$C_{pa} : -158 \pm 75 \text{ ppb} \rightarrow -27 \pm 13 \text{ ppb}$$



Run-2 and Run-3 Hardware Improvements

● Before Run-2:

- Replaced faulty quads HV resistors
- Magnet covered with a thermal blanket



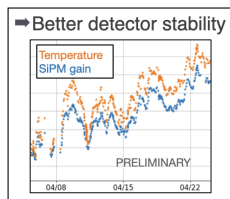
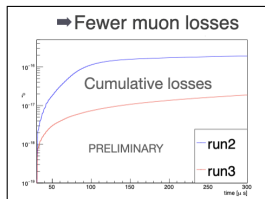
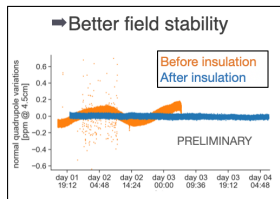
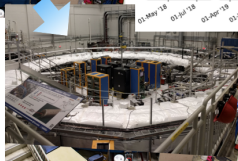
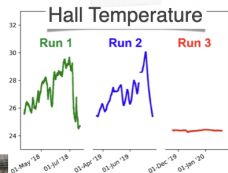
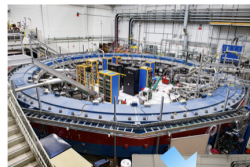
Run-2 and Run-3 Hardware Improvements

● Before Run-2:

- Replaced faulty quads HV resistors
- Magnet covered with a thermal blanket

● Before Run-3:

- Hall temperature control improved



Run-2 and Run-3 Hardware Improvements

● Before Run-2:

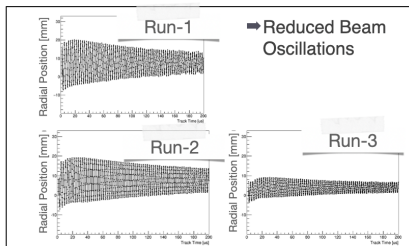
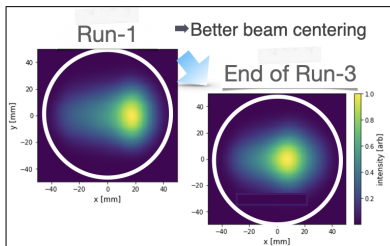
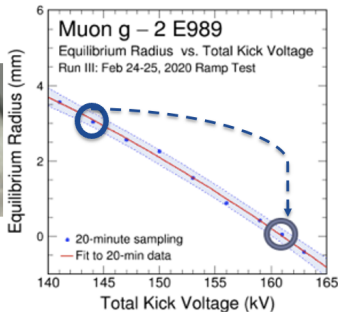
- Replaced faulty quads HV resistors
- Magnet covered with a thermal blanket

● Before Run-3:

- Hall temperature control improved

● During Run-2 and Run-3:

- Replaced kicker cables ⇒ kickers at HV design value



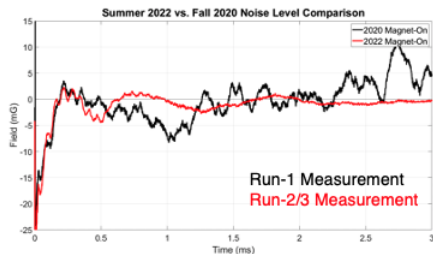
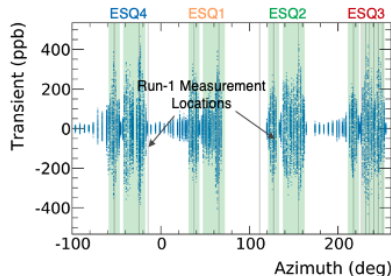
Run-2 and Run-3 Measurement Improvements

- Improved ω_a **analysis technique**:
 - added new positron reconstruction algorithms
 - improved pile-up subtraction technique
- Improved **quadrupole field transient** (B_q) uncertainty by measuring all azimuthal locations

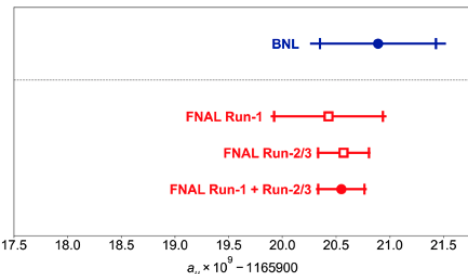
$$\delta_{B_q} : 92 \text{ ppb} \rightarrow 20 \text{ ppb}$$

- Improved **kicker field transient** (B_k) uncertainty by performing new measurements and a cross-check with a new magnetometer

$$\delta_{B_k} : 37 \text{ ppb} \rightarrow 13 \text{ ppb}$$



Run-2/3 Result



- Both Run-1 and Run-2/3 results uncertainties are **statistics dominated**
- Run-2/3 systematic uncertainty of 70 ppb is lower than our TDR goal of 100 ppb!**
- Run-1 + Run-2/3 **combination uncertainty of 203 ppb** (assuming systematics 100% correlated)

- New (2023) FNAL $g - 2$ result :**

$$a_\mu = 116592057(25) \times 10^{-11} \text{ (215 ppb)}$$

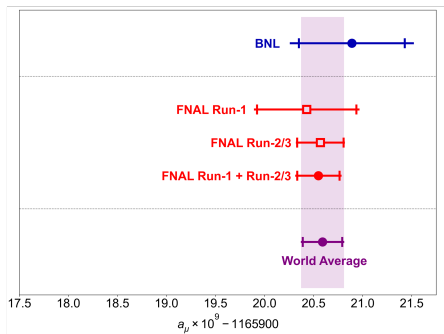
- Good agreement with **FNAL Run-1 BNL $g - 2$**

Quantity	Correction [ppb]	Uncertainty [ppb]
ω_a^m (statistical)	-	201
ω_a^m (systematic)	-	25
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{mi}	0	3
$f_{\text{calib}}(\omega'_p(\vec{r}) \times M(\vec{r}))$	-	46
B_h	-21	13
B_q	-21	20
$\mu'_p(34.7^\circ)/\mu_e$	-	11
m_μ/m_e	-	22
$g_e/2$	-	0
Total systematic	-	70
Total external parameters	-	25
Totals	622	215

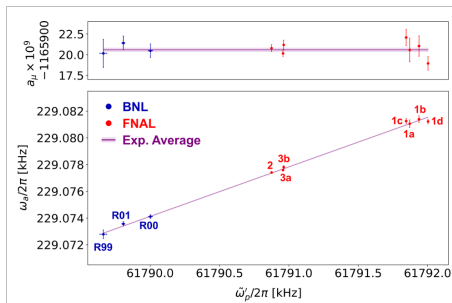
Experimental world average and field dependence

- Combined **world average** dominated by **FNAL value**:

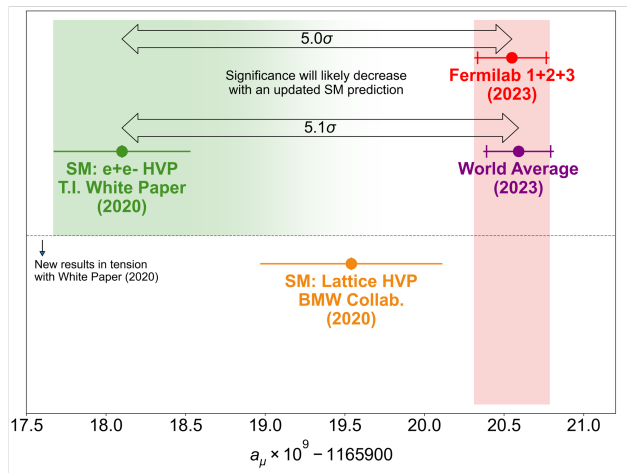
$$a_\mu(\text{Exp}) = 116592059(22) \cdot 10^{-11} \text{ (190 ppb)}$$



- Measurements were taken at different Magnetic Fields:

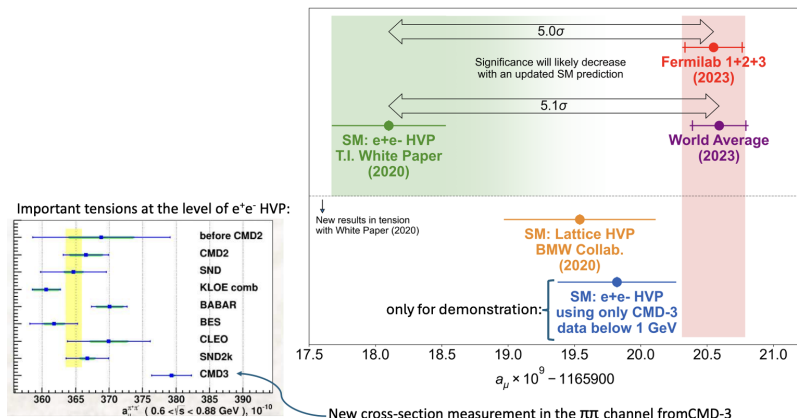


Experimental measurement vs. SM calculation



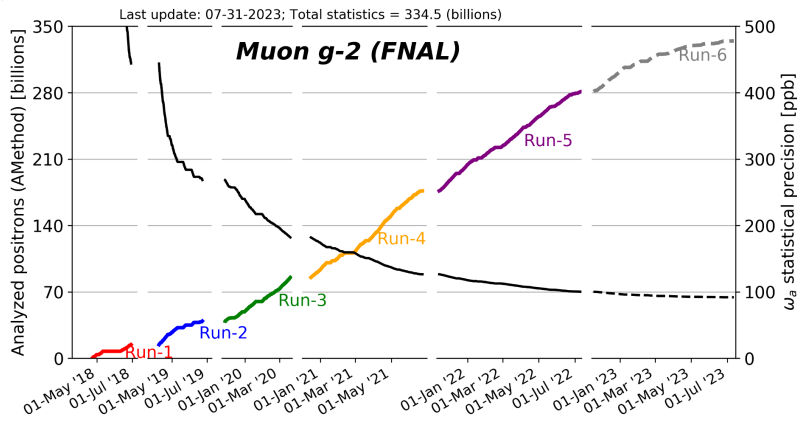
- 5.1 σ discrepancy between 2023 World average and WP (2020)
- BMW result (*i.e.*, changing in WP (2020) result the HVP term from dispersion with lattice-QCD calculation) falls in between WP (2020) and the experiment

Experimental measurement vs. SM calculation



- 5.1 σ discrepancy between 2023 World average and WP (2020)
- BMW result (*i.e.*, changing in WP (2020) result the HVP term from dispersion with lattice-QCD calculation) falls in between WP (2020) and the experiment
- New $e^+e^- \rightarrow \pi^+\pi^-$ result from CMD-3 in tensions with previous exp. results
- A clarification of the theoretical prediction is needed for the comparison.

What's next?



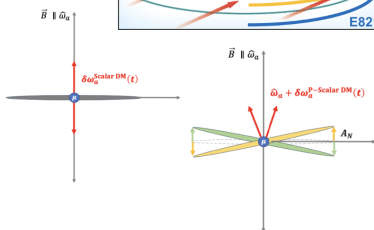
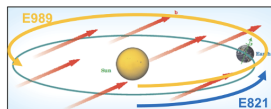
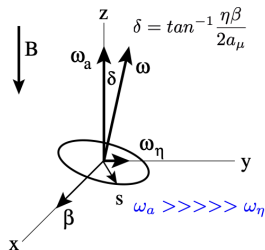
- Completed all runs (collected $> 21 \times \text{BNL}$): there is **more data still to analyze!**
- In Run-4/5/6 not only statistical improvement:
 - **improved running conditions** (quad RF in Run-5/6 reduced horizontal beam oscillations)
 - **extensive systematic measurements & Studies** in Run-6 for better understanding and modeling of beam dynamics also with new detectors (scintillating fibers) for direct beam measurements

Not only Muon g-2 measurement

EDM previous searches statistical limited ($10^{-19} e \text{ cm}$), goal to reach $10^{-21} e \text{ cm}$. Search for an up-down oscillation, out of phase with ω_a .

CPT/LV using long period of data collected we can look if the spin precession rate changes over a sidereal day (as predicted by Standard-Model Extension).

DM Muon $g - 2$ experiment enables the direct search for two (scalar and pseudoscalar) ultralight dark matter candidates that primarily interact with muons.



Summary and Conclusions

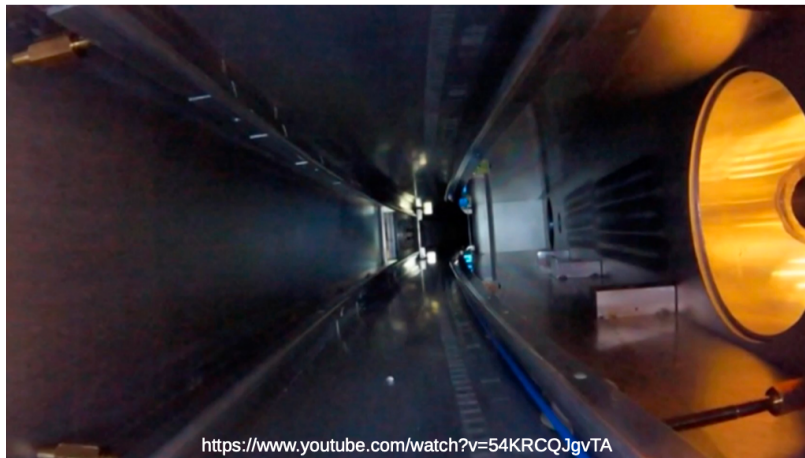
- FNAL $g - 2$ Experiment goal is to measure a_μ with a **precision of 140 ppb** (4×BNL precision)
- The result from the analysis of the Run-1/2/3 data **confirmed** result from BNL experiment
- With **Run-2 and Run-3 data** measurement achieved a factor 2 uncertainty reduction both in statistics and systematics!
- Next: analysis of **Run-4, Run-5 and Run-6** (expect to achieve the uncertainty goal), also other analysis EDM, CPT/LV and Dark Matter searches are been developed.

Thanks!

Enjoy the latest paper:
PRL **131**, 161802 (2023)
and stay tuned for the
next result!



Bonus Slide: A journey inside the storage ring:



<https://www.youtube.com/watch?v=54KRCQJgvTA>

Click here to Start the Muon g-2 trolley journey