10th International Conference on Quarks and Nuclear Physics (QNP 2024)

July 11, 2024

τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization

Phys.Lett.B 850 (2024) 138492

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IFAE, Spain

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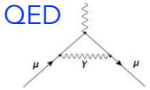
Generalitat de Catalunya
Departament de Recerca i Universitats

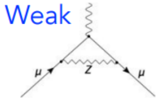
The Anomalous Magnetic Moment of the Muon

WP20 - Phys. Rept. 887 (2020) 1-166

Contributions from known particles: The Standard Model

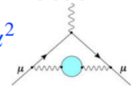
$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

QED  +… $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

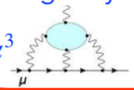
Weak  +… $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

α^2  +… $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

…Light-by-Light (HLbL)

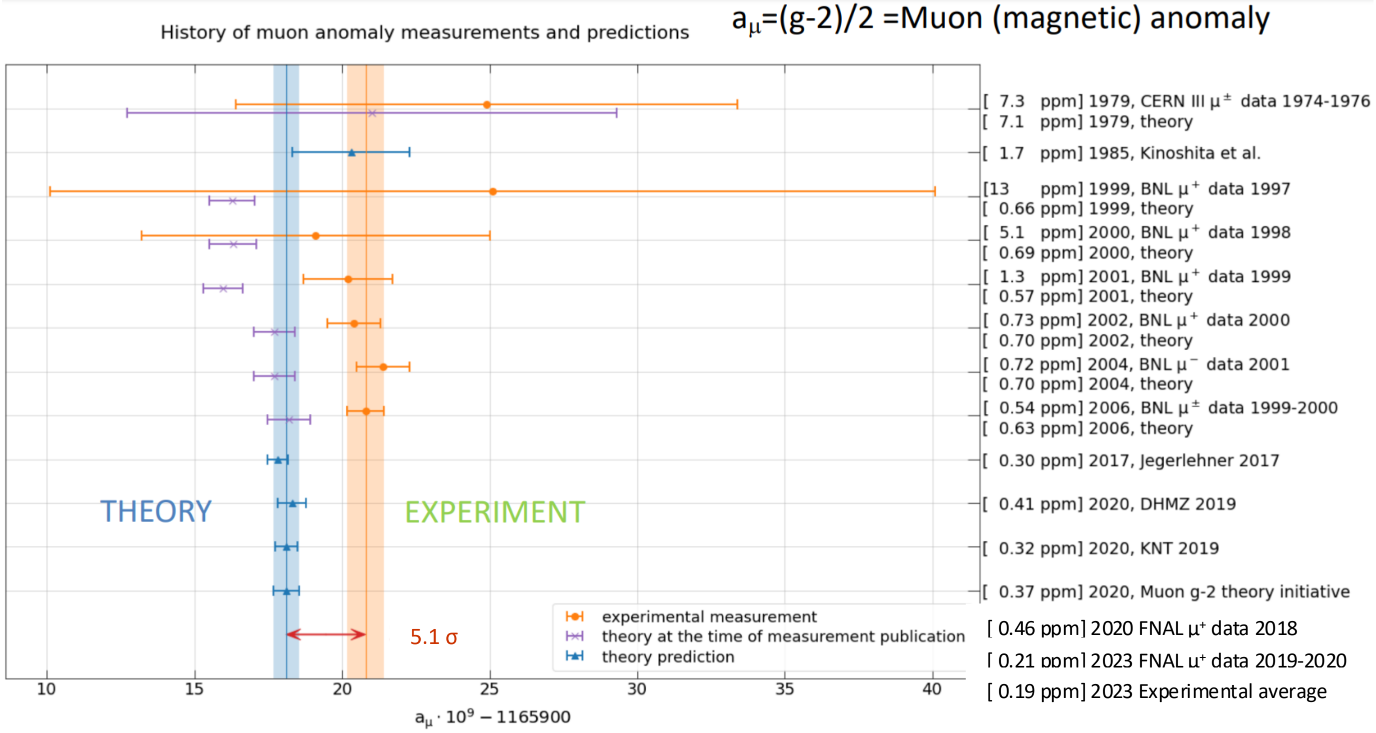
α^3  +… $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Numbers from Theory Initiative Whitepaper

Uncertainty dominated by hadronic contributions

C. Lehner. CERN EP seminar, 8 April 2021

$a_\mu = (g-2)/2$ = Muon (magnetic) anomaly



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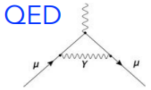

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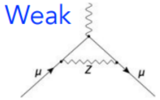

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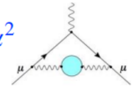

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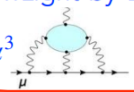

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2nd WP by the end of 2024

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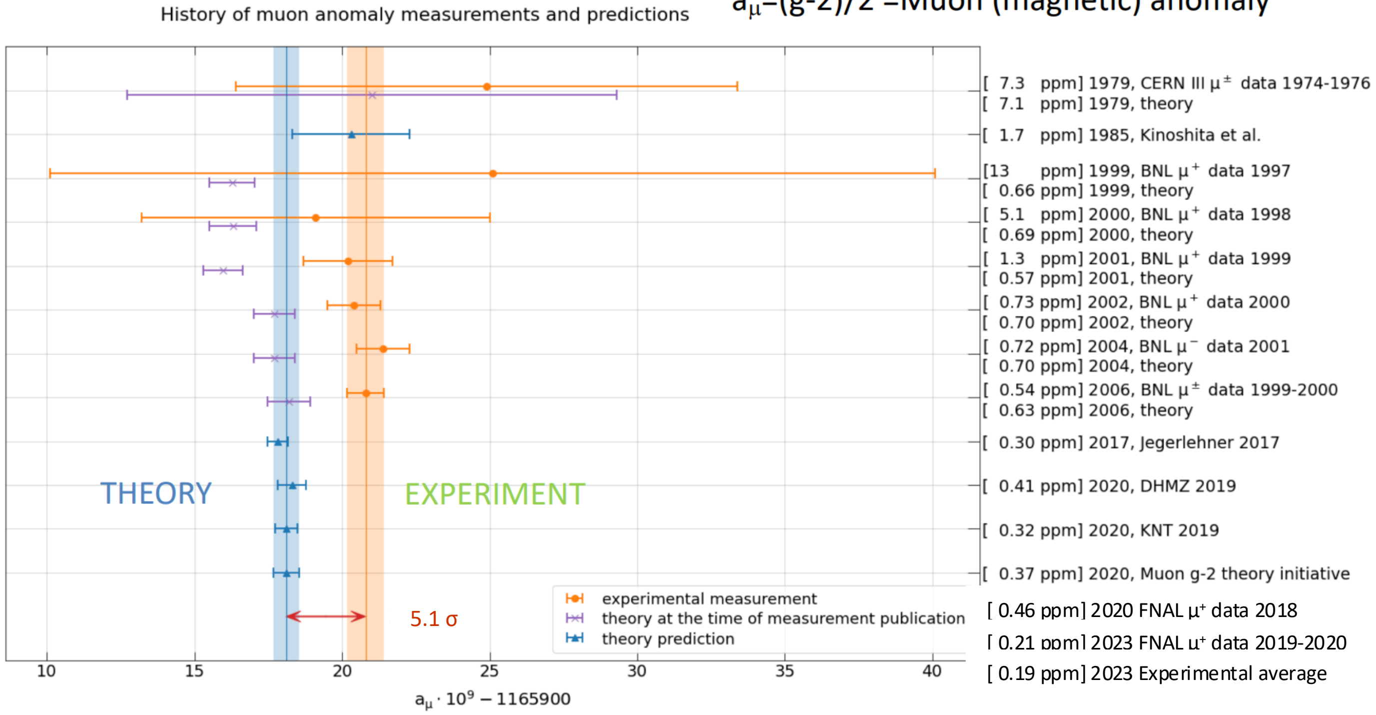

[0.14 ppm] 2025 FNAL Goal

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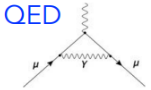
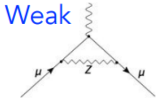
QED  +… $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak  +… $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

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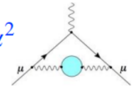
α^2  +… $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

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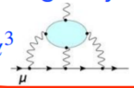
α^3  +… $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

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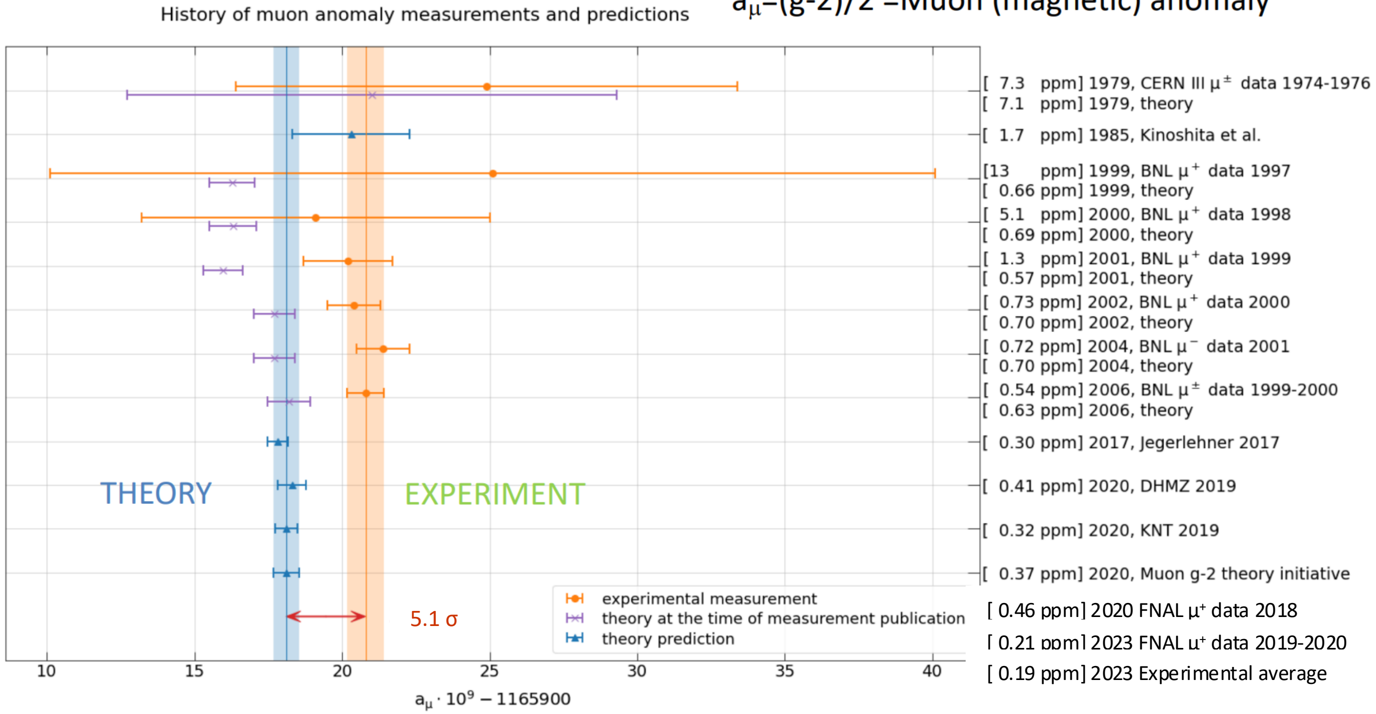

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The E34 experiment at J-PARC/Japan aim to perform ultra-precision measurements of g-2 and EDM by using a different method.

Hadronic vacuum polarization

- At low energies QCD gets strongly interacting and a perturbative calculation is not feasible.
- Luckily, analyticity and unitarity allow us to express the leading hadronic vacuum polarization (HVP) contributions via a dispersion relation in terms of experimental data:

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s),$$

Gourdin, De Rafael. Nucl.Phys.B 10 (1969) 667-674

where *K(s)* is a Kernel function ⟹ $K(s) \sim 1/s,$

$$R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{\sigma_{pt}}, \quad \sigma_{pt} = \frac{4\pi\alpha^2}{3s}$$

- An evaluation of the HVP, LO contribution can be obtained from the measurements of **$\sigma(e^+e^-\to$hadrons)** or the **$\tau \to \nu_\tau$ + hadron** decays which can be related to the isovector component of the **$e^+e^-\to$hadrons** cross section through isospin-symmetry.

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- Since both are subject to **theoretical uncertainties**, it is a good strategy to keep using both.

Hadronic vacuum polarization

- About 73% of the contributions to the HVP and 58% of the total uncertainty correspond to the $\pi^+\pi^-(\gamma)$ final state at low energies $(4m_\pi^2 \leq s \leq 0.8\,\mathrm{GeV}^2)$.
- For the two-pion final state,

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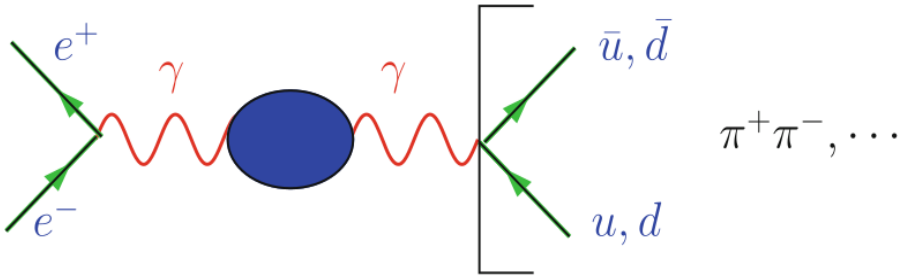

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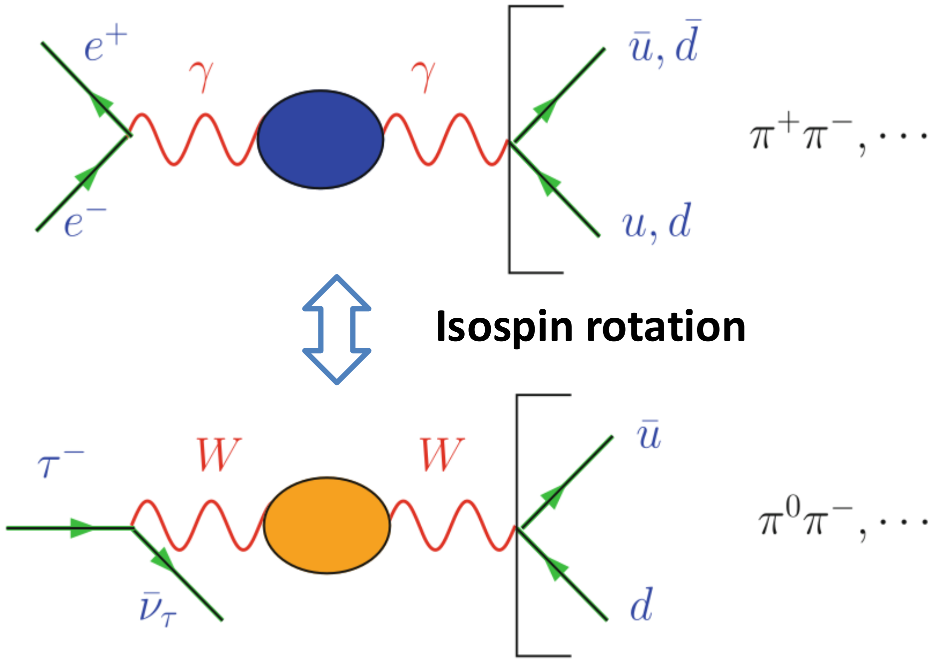
$$\sigma_{\pi^+\pi^-}(s) = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\right]\frac{R_{IB}(s)}{S_{EW}},$$

- $\frac{K_\sigma(s)}{K_\Gamma(s)}$: Kinematics
- $\frac{d\Gamma_{\pi\pi[\gamma]}}{ds}$: Measurement
- S_{EW}: Short-distance EW RadCor



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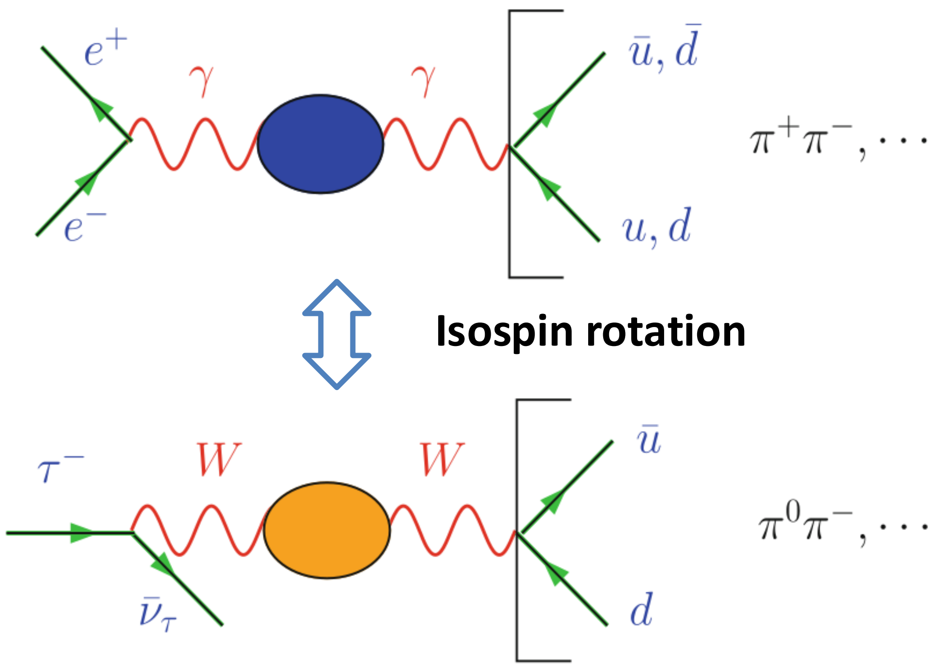

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- The ratio of neutral to charged current di-pion form factor and the long-distance em RadCor are challenging.
- $G_{EM}(s)$ receives contributions from real and virtual photons.

Cirigliano et al. Phy. Lett. B513 (2001). JHEP 08 (2002) 002

Hadronic vacuum polarization

- There is a **discrepancy** between the values of $a_\mu^{HVP,LO}[\pi\pi]$ obtained through **e^+e^-** and **τ** decays. According to Cirigliano et al. this could be a **NP effect**,

$$\frac{a_\mu^\tau - a_\mu^{ee}}{2\, a_\mu^{ee}} = \epsilon_L^{d\tau} - \epsilon_L^{de} + \epsilon_R^{d\tau} - \epsilon_R^{de} + c_T\, \hat{\epsilon}_T^{d\tau}$$

Phy. Rev. Lett. 122 (2019)
JHEP 04 (2022) 152

- There is a **solution** given by **Jegerlehner** and **Szafron** that induces an **additional correction** due to the **$\rho-\gamma$** mixing where **ρ^0** is regarded as a gauge boson.

Eur. Phys. J. C 71 (2011) 1632

- **NP effects** in $\tau^-\to\pi^-\pi^0\nu_\tau$ decays were studied using an **EFT** framework for some observables.

JHEP 11 (2018) 038

- A **global fit** using **hadronic tau decays** to set bounds on **NP** effective couplings at the **low-energy** limit of **SMEFT** was performed by Gonzàlez-Solís et al.

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Nature 593 (2021) 7857, 51-55

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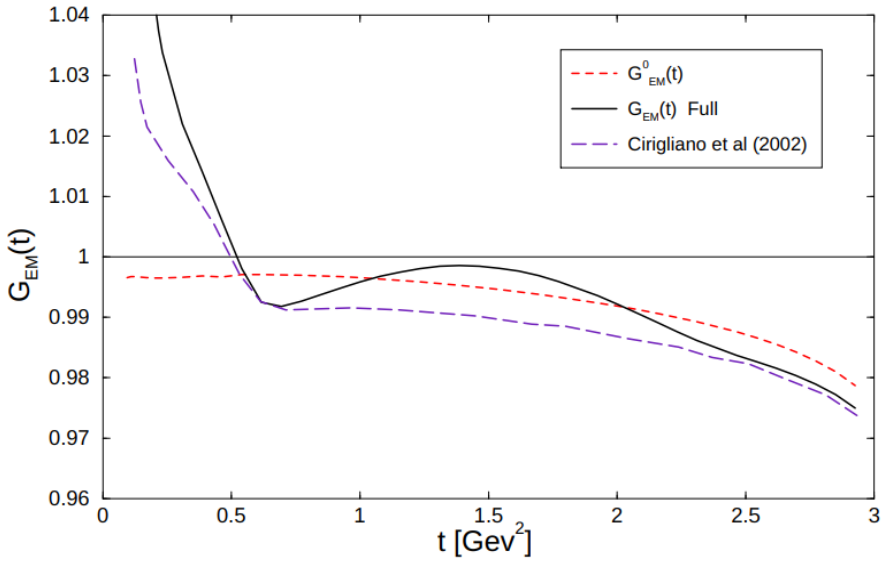
- The **recent measurement** of the e^+e^- cross section by **CMD-3** is in conflict with all previous determinations.

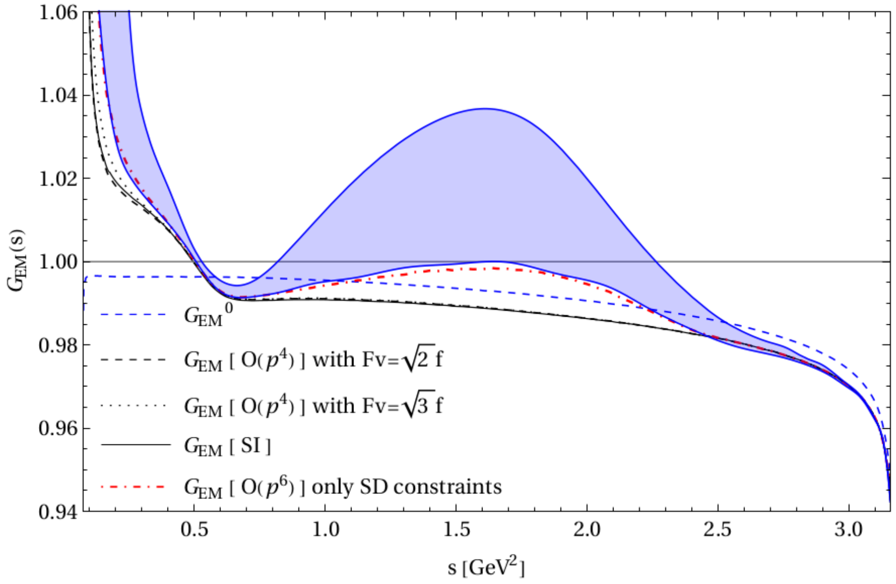
Phys.Rev.D 109 (2024) 11, 112002

Long-distance radiative corrections

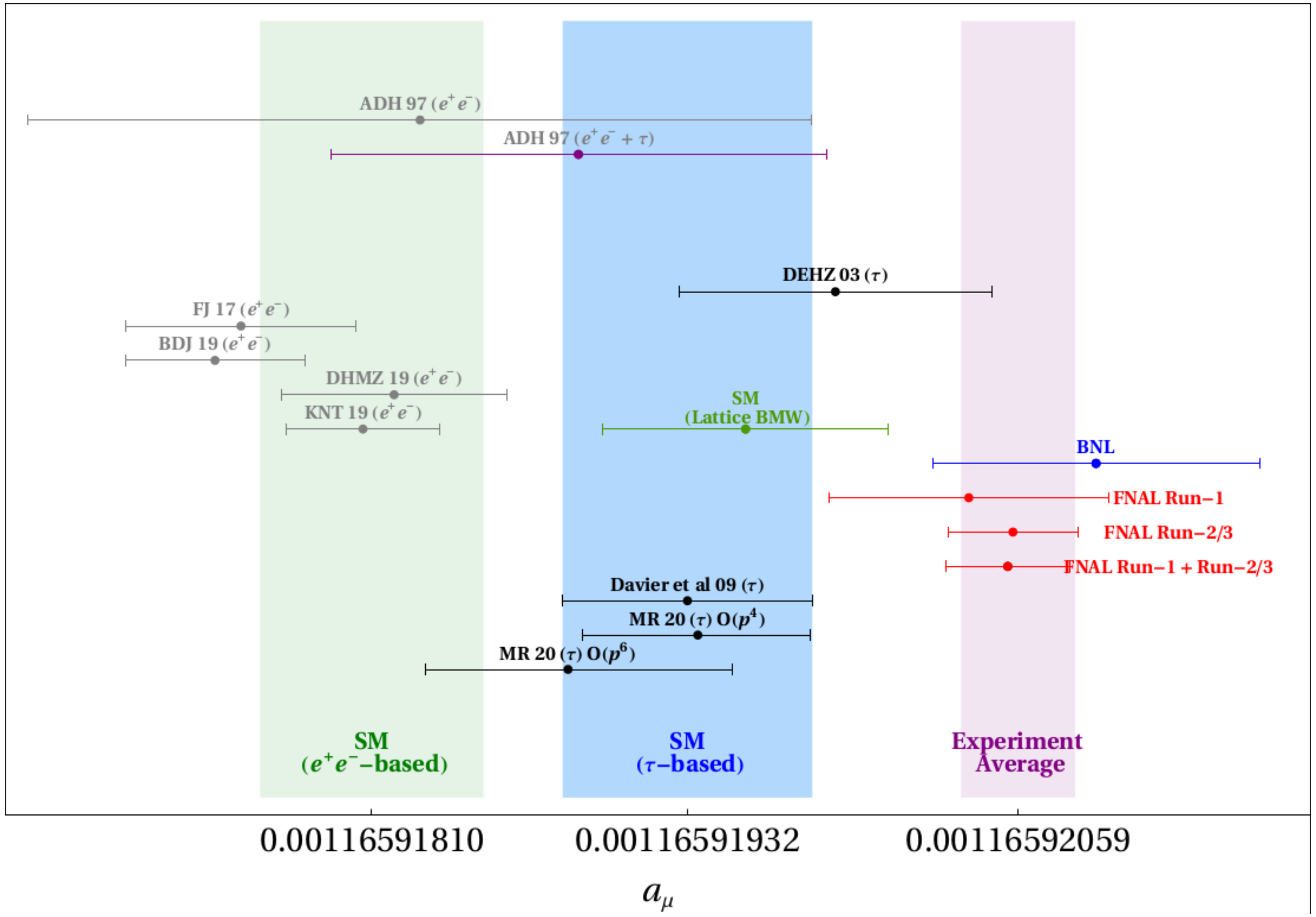
- G_{EM} was originally studied by Cirigliano et al in the frame of **RChT** at **$O(p^4)$**. JHEP 08 (2002) 002
- A recalculation was performed by Flores-Baez et al using a **VMD** model. Phys.Rev.D 74 (2006) 071301
- The two model predictions **disagree** due to the presence of diagrams involving the **ρωπ** vertex.
- We extend the **RChT** estimation including contributions up to **$O(p^6)$**. Phys. Rev. D 102 (2020) 114017



Nucl. Phys. B Proc.Suppl. 169 (2007) 250-254



Data-driven calculations of HVP



	$\Delta a_\mu^{\rm HVP,LO}[\pi\pi,\tau](\times 10^{10})$
Total	$-16.07(1.85)$

WP20 [Phys.Rept.887, arXiv:2006.04822]

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Source	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$
$S_{\rm EW}$	$-11.96(15)$	
PS	$-7.47(0)$	
FSR	$+4.56(46)$	
$G_{\rm EM}$	$-1.71\left(^{0.61}_{1.48}\right)$	$-7.61\left(^{6.50}_{4.56}\right)$
FF	$+7.13(1.48)\left(^{1.59}_{1.54}\right)\left(^{85}_{80}\right)$	
Total	$-9.45\left(^{2.51}_{2.83}\right)$	$-15.35\left(^{6.98}_{5.17}\right)$

SEW: Sirlin '78; Marciano-Sirlin '93
FF (Mixing,...) : Maltman '05; Maltman-Yorke '06, '11; Davier et al '09
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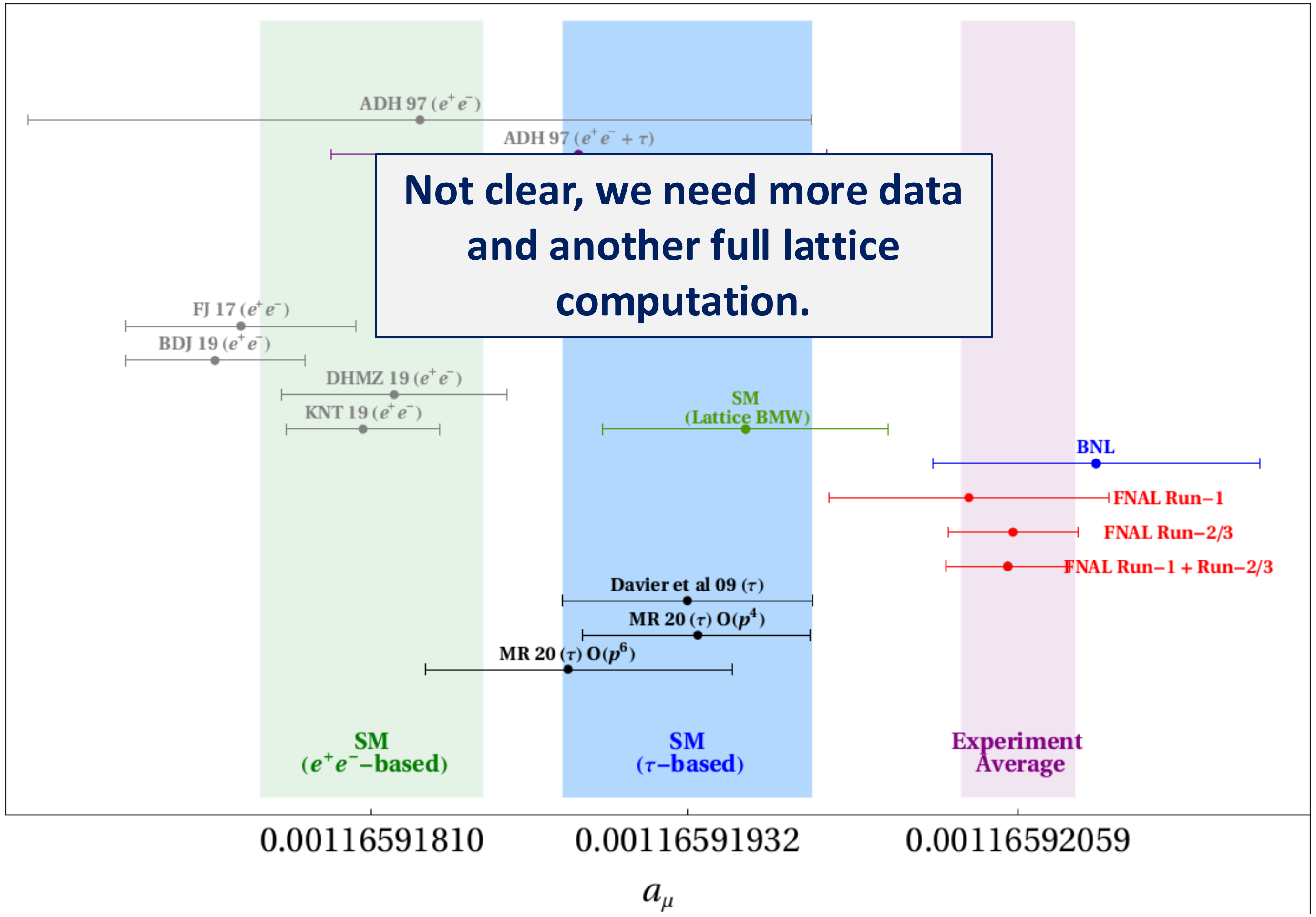

Not clear, we need more data and another full lattice computation.

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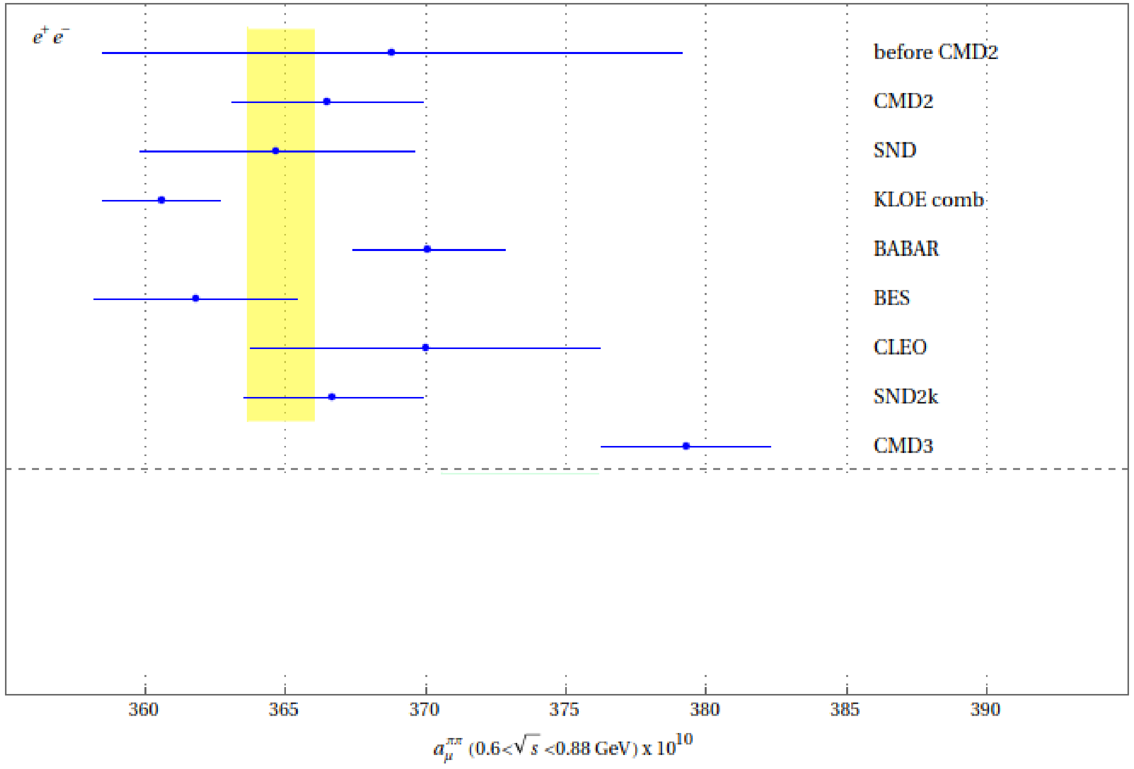
HVP, LO from data

- Comparison of results for the **HVP, LO**, evaluated between **0.6 GeV** and **0.88 GeV** .



Experiment	$a_\mu^{\pi^+\pi^-,LO} \cdot 10^{10}$
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

CMD-3. Phys.Rev.D 109 (2024) 11, 112002

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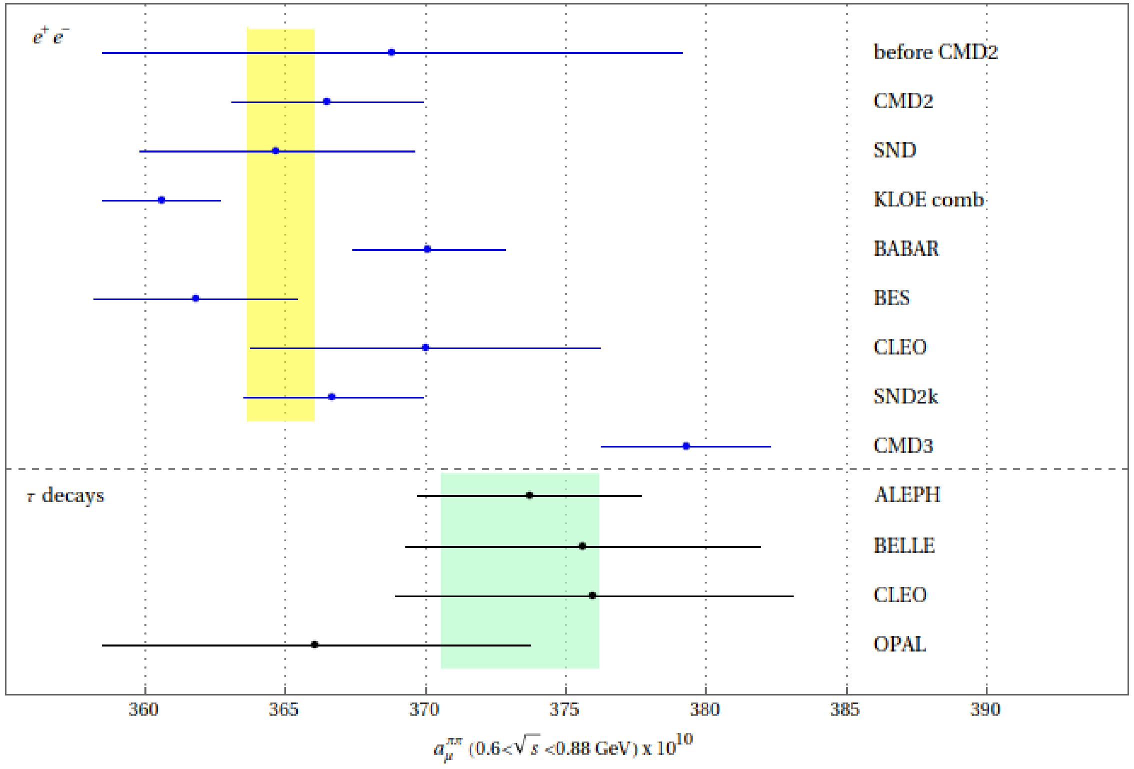

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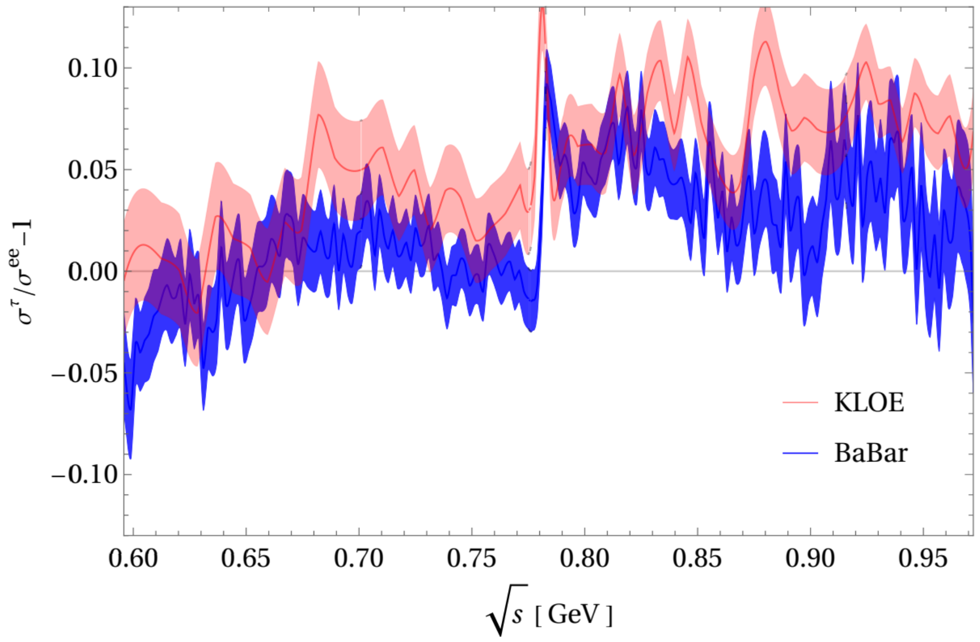
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0
ALEPH	373.7 ± 4.0
Belle	375.6 ± 6.3
CLEO	376.0 ± 7.1
OPAL	366.1 ± 7.6

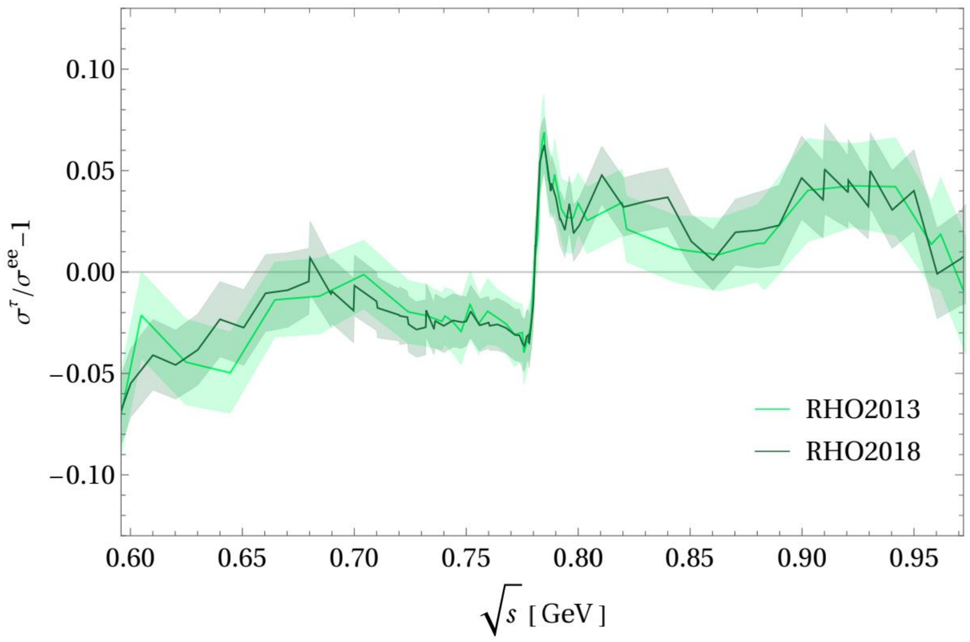
e^+e^- data

τ data

CMD-3. Phys.Rev.D 109 (2024) 11, 112002

- Large tensions among experiments: **KLOE**, **BaBar** and **CMD3**.





Comparison between the different data sets: KLOE and BaBar (left-hand) and CMD-3 (right-hand).

P. Masjuan. Phys.Lett.B 850 (2024) 138492

Euclidean windows

- **Euclidean window quantities** allow for the separation of the most challenging **short** and **long** time-distance contributions: **internal lattice cross-check**.

$$a_\mu = 4\alpha^2 \sum_t w_t \left[\Theta_{\mathrm{SD}}(t) + \Theta_{\mathrm{W}}(t) + \Theta_{\mathrm{LD}}(t)\right] G(t)$$

RBC/UKQCD 2018

- A **dispersive** result for the total **intermediate** window contribution,

$$a_\mu^{\mathrm{W}} = 229.4(1.4) \times 10^{-10}$$

which is in **4.3σ** tension with recent lattice results.

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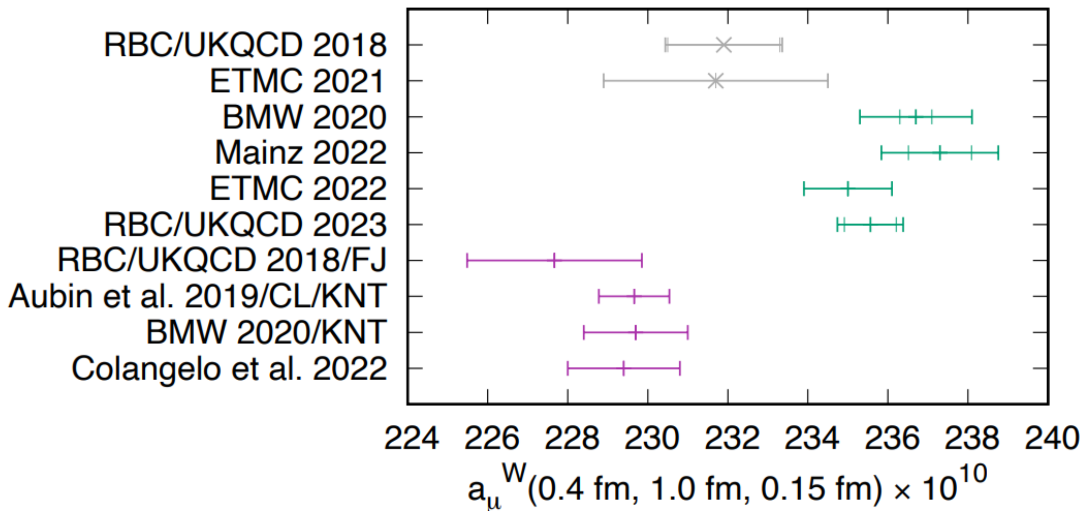
- The discrepancy between **data-driven** and **LQCD** is almost entirely due to the **light-quark connected** contribution, which is dominated by the **2π** channel ~81%.

Phys.Rev.Lett. 131 (2023) 25, 251803

Before CMD-3



RBC/UKQCD 2023

Isospin-breaking corrections

- We can **estimate** the effect of each **IB correction** through

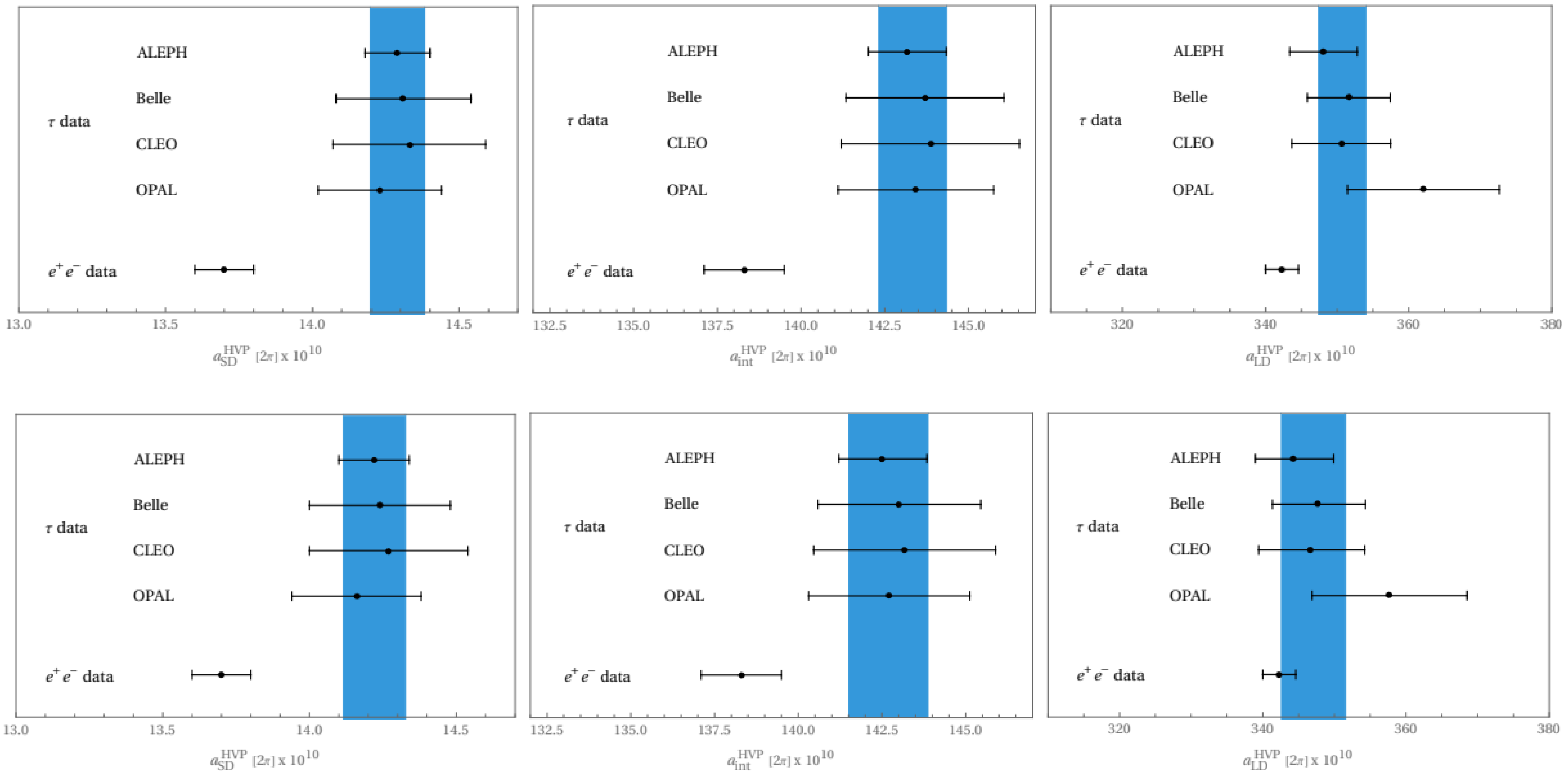
$$\Delta a_\mu^{\text{HVP, LO}}[\pi\pi,\tau] = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{m_\tau^2} ds\, K(s) \left[\frac{K_\sigma(s)}{K_\Gamma(s)} \frac{d\Gamma_{\pi\pi[\gamma]}}{ds}\right] \left(\frac{R_{\text{IB}}(s)}{S_{\text{EW}}} - 1\right),$$

Contributions to $\Delta a_\mu^{\text{HVP, LO}}$ in units of **10^{-11}** using the **dispersive** representation of the **form factor**.

$\Delta a_\mu^{\text{HVP,LO}}$	SD		int		LD		Total	
	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$	$\mathcal{O}(p^4)$	$\mathcal{O}(p^6)$
S_{EW}	$-0.31(0)$		$-2.96(0)$		$-7.04(0)$		$-10.31(0)$	
PS	$-0.13(0)$		$-1.39(0)$		$-5.93(0)$		$-7.45(0)$	
FSR	$+0.13(1)$		$+1.23(12)$		$+3.19(32)$		$+4.55(45)$	
G_{EM}	$+0.06\binom{0}{2}$	$-0.03\binom{9}{7}$	$+0.29\binom{6}{19}$	$-0.58\binom{93}{71}$	$-2.05\binom{0.55}{1.27}$	$-6.98\binom{5.48}{3.78}$	$-1.70\binom{0.61}{1.48}$	$-7.59\binom{6.50}{4.56}$
FF	$+0.16(5)(1)\binom{2}{1}$		$+1.90(49)\binom{29}{27}\binom{21}{20}$		$+5.04(94)\binom{1.29}{1.26}\binom{62}{59}$		$+7.10(1.48)\binom{1.59}{1.54}\binom{85}{80}$	
Total	$-0.09(6)$	$-0.18\binom{11}{9}$	$-0.93\binom{62}{64}$	$-1.80\binom{1.12}{0.93}$	$-6.79\binom{1.83}{2.13}$	$-11.72\binom{5.75}{4.15}$	$-7.81\binom{2.51}{2.83}$	$-13.70\binom{6.98}{5.17}$

In agreement with:
Phys. Rev. D 102 (2020) 114017

Windows quantities for 2π below 1.0 GeV



+ 4.3 σ
+ 3.2 σ
+ 2.1 σ

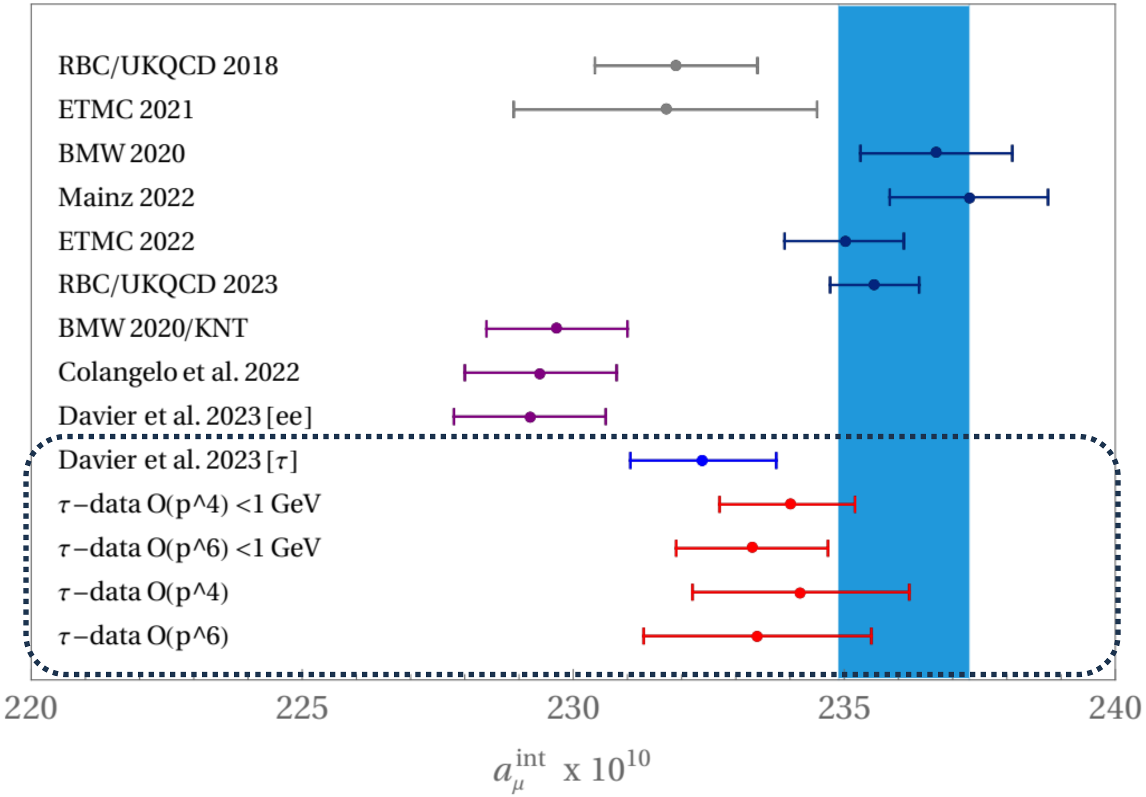
+ 3.6 σ
+ 2.6 σ
+ 0.9 σ

G. Colangelo. Phys.Lett.B 833 (2022) 137313
P. Masjuan. Phys.Lett.B 850 (2024) 138492

Hadronic vacuum polarization

- When all other contributions are added, we get the overall result for each window quantity.
- The contributions of the intermediate window using tau data are slightly closer to the lattice results (~1.5σ).

	$a_\mu^{\mathrm{HVP,LO}}$			
	SD	int	LD	Total
τ-data $\mathcal{O}(p^4) \leq 1$ GeV	69.0(5)	$234.0(^{1.2}_{1.3})$	$402.5(^{3.3}_{3.4})$	$705.5(^{5.0}_{5.2})$
τ-data $\mathcal{O}(p^6) \leq 1$ GeV	68.9(5)	233.3(1.4)	$398.5(^{4.9}_{4.2})$	$700.7(^{6.8}_{6.1})$
τ-data $\mathcal{O}(p^4)$	69.0(7)	234.2(2.0)	$402.6(^{3.8}_{3.9})$	$705.8(^{6.5}_{6.6})$
τ-data $\mathcal{O}(p^6)$	68.9(7)	233.4(2.1)	$398.5(^{5.3}_{4.6})$	$700.8(^{8.1}_{7.4})$
RBC/UKQCD 2018 [12]	–	231.9(1.5)	–	715.4(18.7)
ETMC 2021 [148]	–	231.7(2.8)	–	–
BMW 2020 [66]	–	236.7(1.4)	–	707.5(5.5)
Mainz/CLS 2022 [67]	–	237.30(1.46)	–	–
ETMC 2022 [68]	69.33(29)	235.0(1.1)	–	–
RBC/UKQCD 2023 [62]	–	235.56(82)	–	–
WP [38]	–	–	–	693.1(4.0)
BMW 2020/KNT [4, 66]	–	229.7(1.3)	–	–
Colangelo et al. 2022 [69]	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
Davier et al. 2023 $[e^+e^-]$ [147]	–	229.2(1.4)	–	694.0(4.0)
Davier et al. 2023 $[\tau]$ [125]	–	232.4(1.3)	–	–



Conclusions

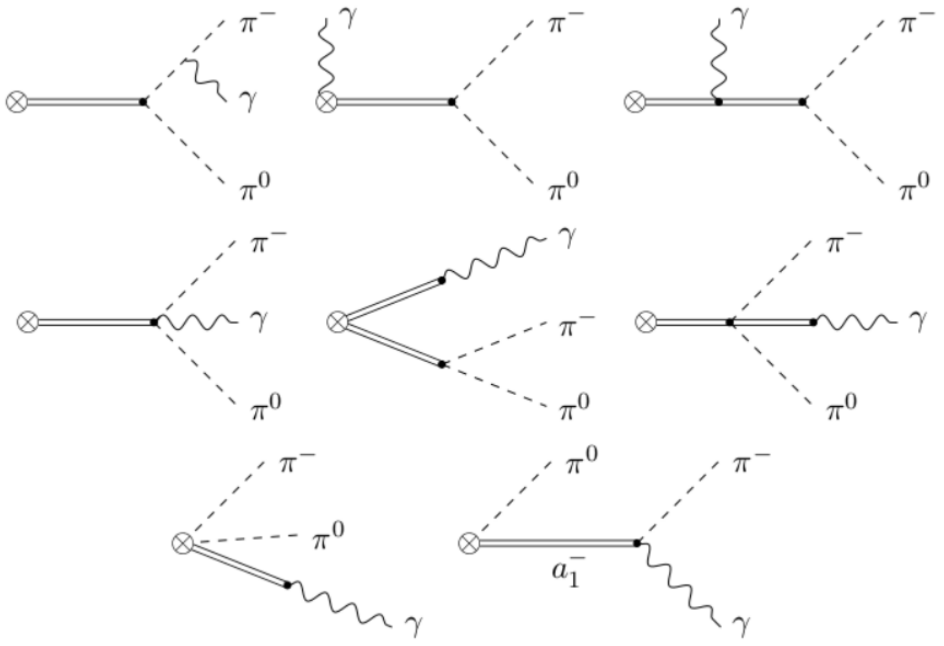
- There is a global effort in improving the hadronic contributions to a_μ. Specifically, dedicated studies to improve the HVP part from lattice, dispersion relations and improved e^+e^- data and Monte Carlos are being undertaken.
- Through the years, the tau data-driven estimation has always been approximately $[2,2.5]\sigma$ away from the experimental average.
- The most recent lattice results (Mainz/CLS, ETMC, RBC/UKQCD) agree remarkably with BMWc in the intermediate window.
- We show that tau based results are compatible with the lattice evaluations in the intermediate window, being the e^+e^- based values in tension with both of them. This difference should be further scrutinized.

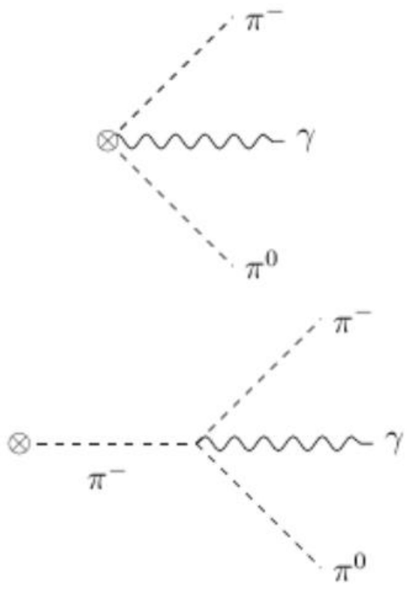
References

- V. Cirigliano, G. Ecker and H. Neufeld, "Radiative tau decay and the magnetic moment of the muon", JHEP 0208, 002 (2002). e-Print: hep-ph/0207310 [hep-ph]
- A. Miranda and P. Roig. "New τ -based evaluation of the hadronic contribution to the vacuum polarization piece of the muon anomalous magnetic moment". Phys.Rev.D 102 (2020) 114017. e-Print: 2007.11019 [hep-ph]
- G. Colangelo et al. "Data-driven evaluations of Euclidean windows to scrutinize hadronic vacuum polarization". Published in: Phys.Lett.B 833 (2022) 137313. e-Print: 2205.12963 [hep-ph]
- P. Masjuan, A. Miranda and P. Roig. "τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization". Published in: Phys.Lett.B 850 (2024) 138492. e-Print: 2305.20005 [hep-ph]
- M. Davier, A. Hoecker, A.M. Lutz, B. Malaescu, Z. Zhang. "Tensions in $e^+e^- \to \pi^+\pi^-(\gamma)$ measurements: the new landscape of data-driven hadronic vacuum polarization predictions for the muon g-2". e-Print: 2312.02053 [hep-ph]

Contributions at $O(p^4)$

- At $O(p^4)$ in χPT with resonances (RχT), the diagrams that contribute to these decays are:

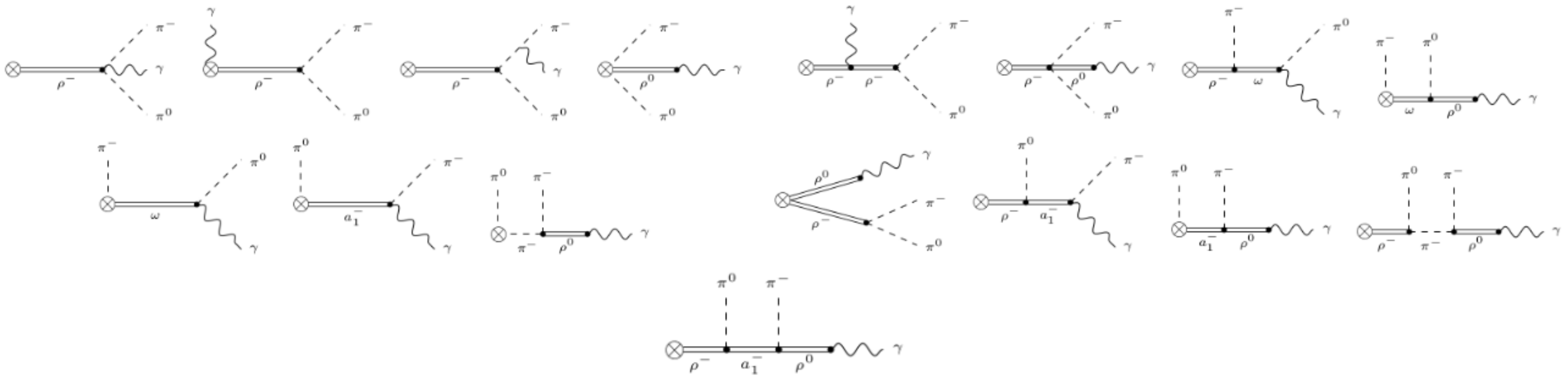




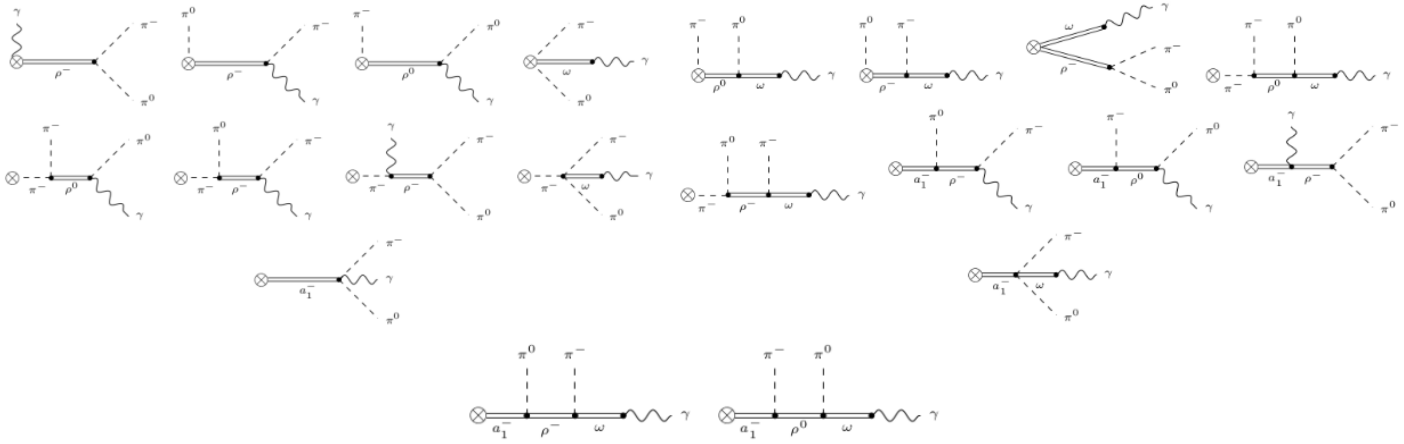
JHEP 08 (2002) 002

Contributions at $O(p^6)$

- Using the basis given by Cirigliano et al. Nucl. Phys. B753 (2006) and Kampf & Novotný, Phys. Rev. D84 (2011), we get the following contributions at $O(p^6)$:



for $V_{\mu\nu}$, and



for $A_{\mu\nu}$.

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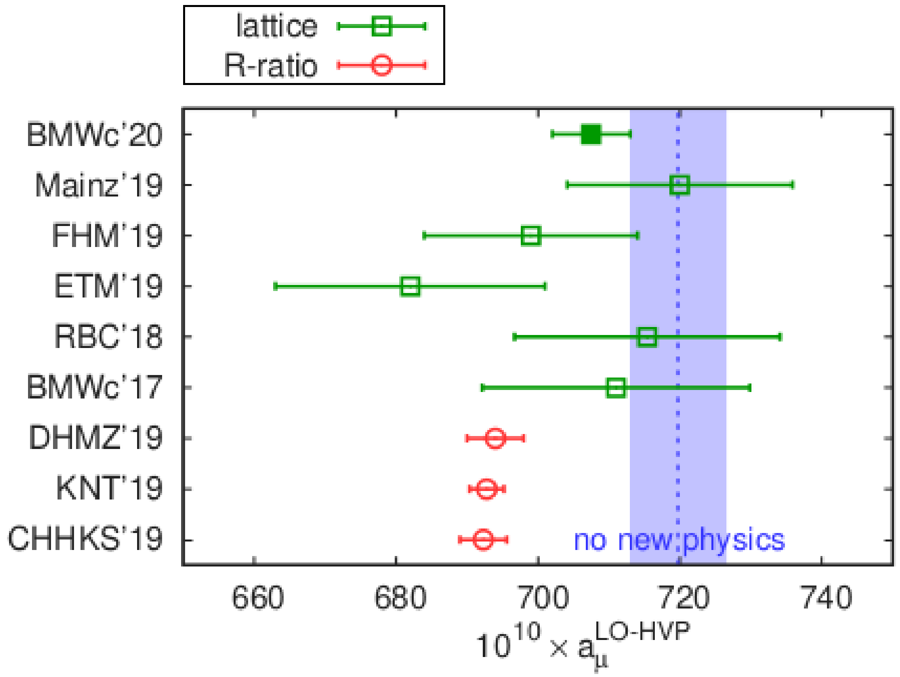
HVP, LO from lattice QCD

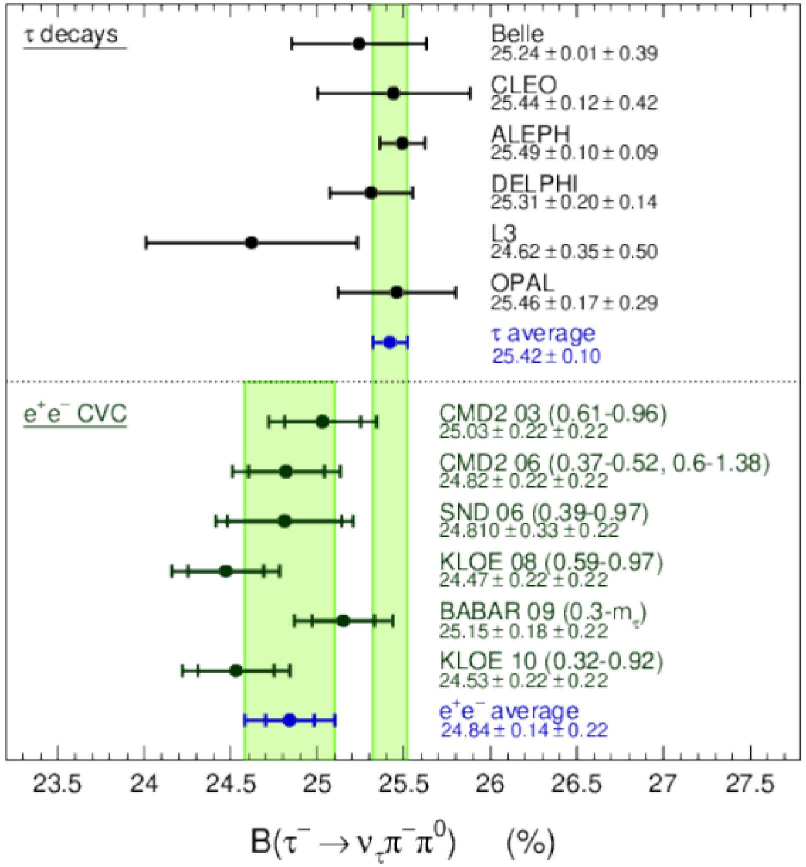
- Comparison of recent results for the leading-order, hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon:



Nature (2021)

τ vs e^+e^-

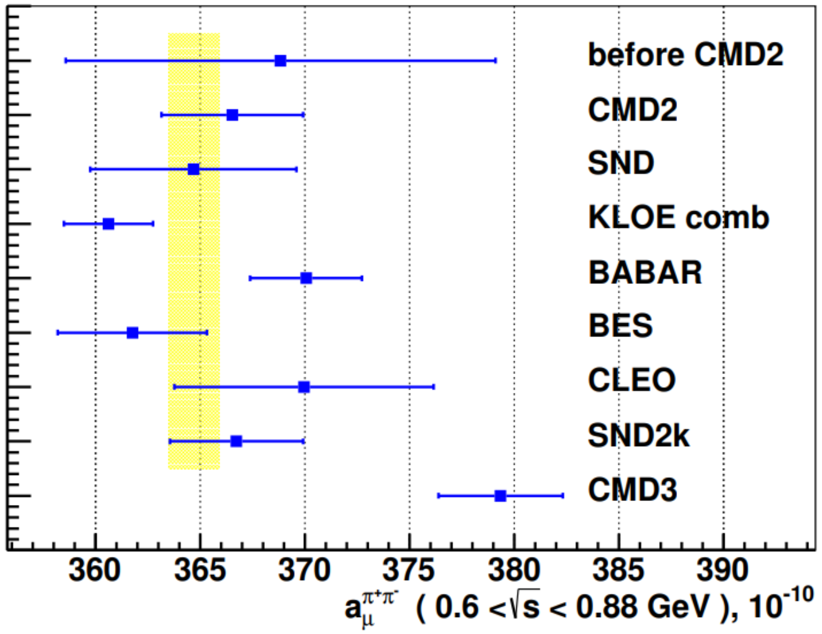
- The measured branching fractions for $\tau^- \to \pi^-\pi^0 \nu_\tau$ compared to the predictions from the $e^+ e^- \to \pi^+\pi^-$ spectral functions, applying the IB corrections.



Eur.Phys.J.C66:127-136,2010

HVP, LO from e^+e^- data

- Comparison of results for the HVP, LO, evaluated between 0.6 GeV and 0.88 GeV .

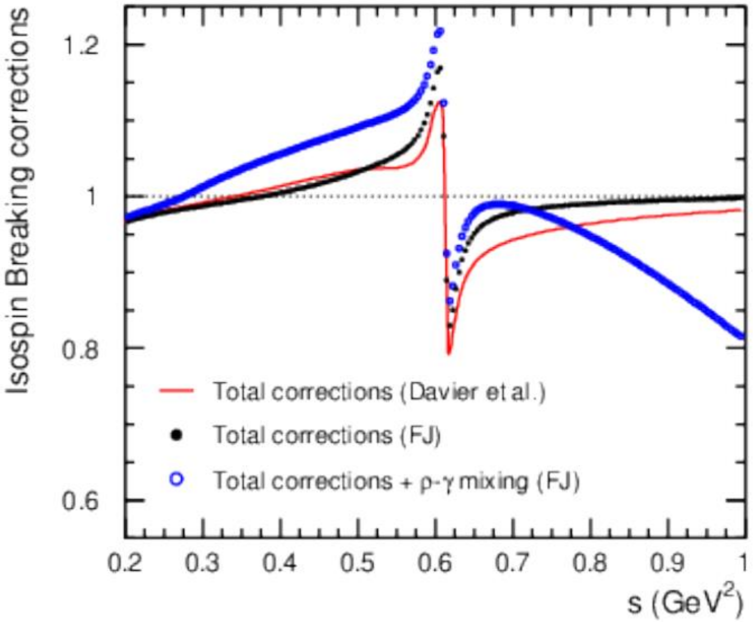


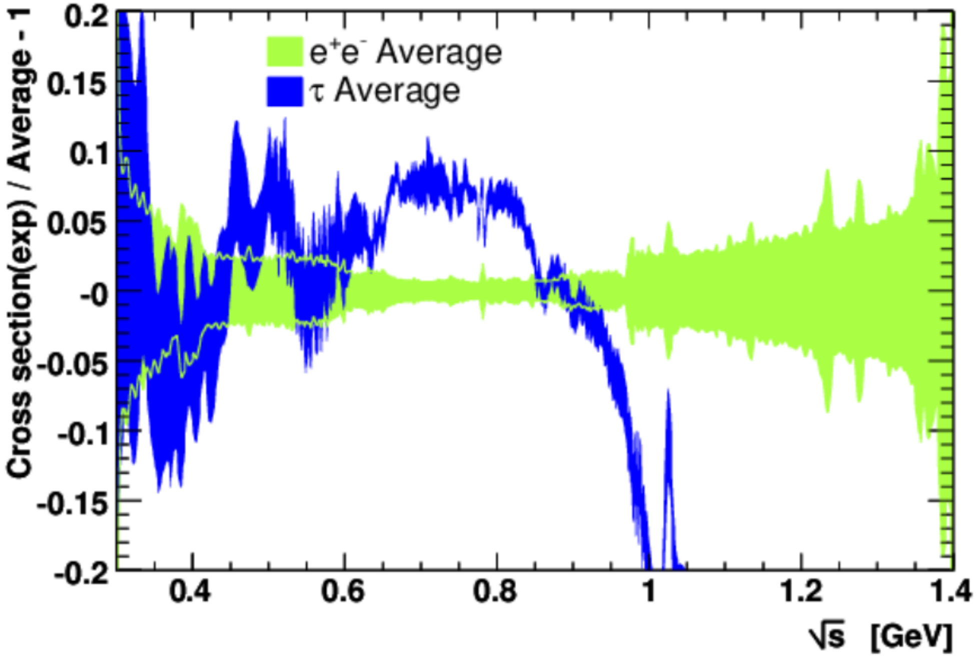
Experiment	$a_\mu^{\pi^+\pi^-,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

CMD-3. 2302.08834 [hep-ex]

ρ − γ mixing

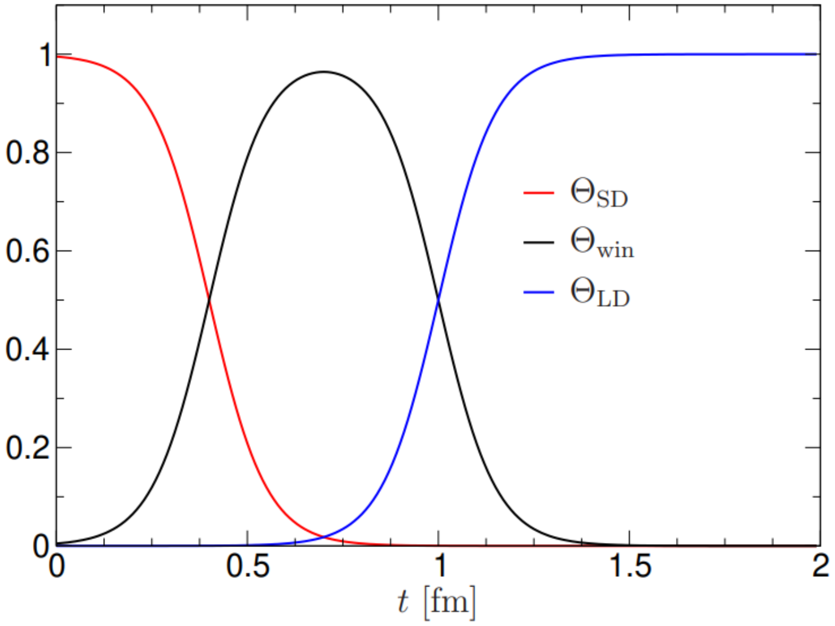
- ρ − γ mixing corrections proposed in Eur.Phys.J.C71:1632,2011.

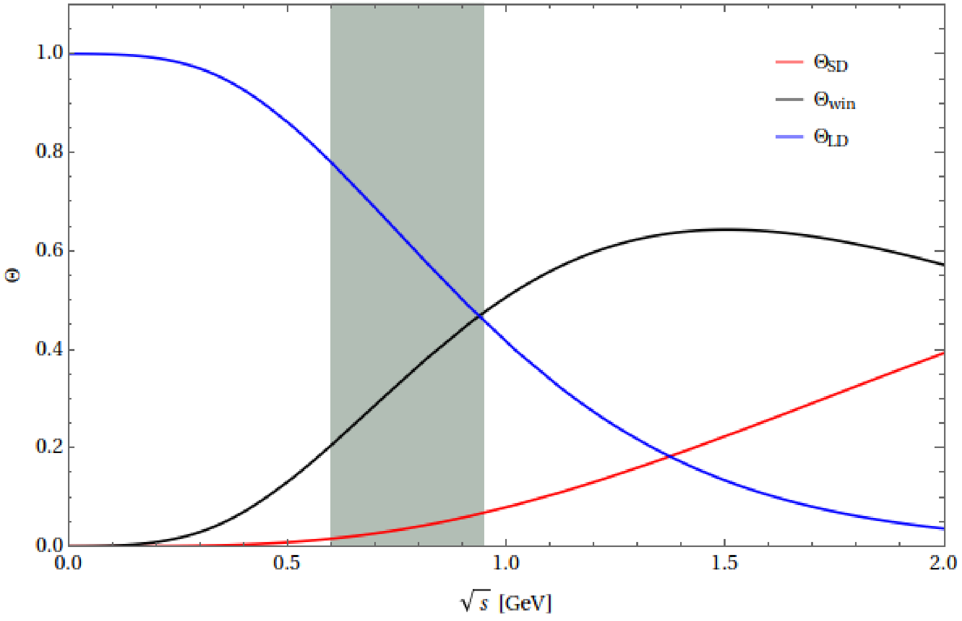




Euclidean windows

- Smoothly divide integral in several parts





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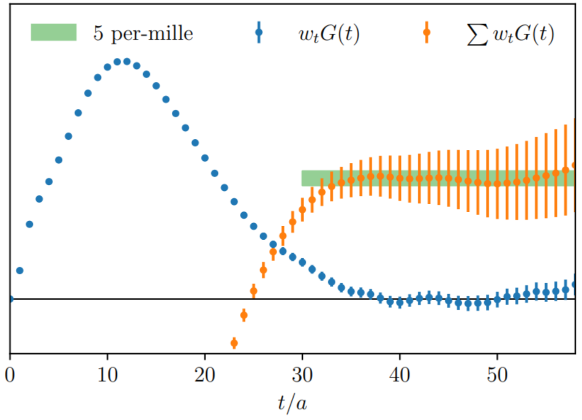
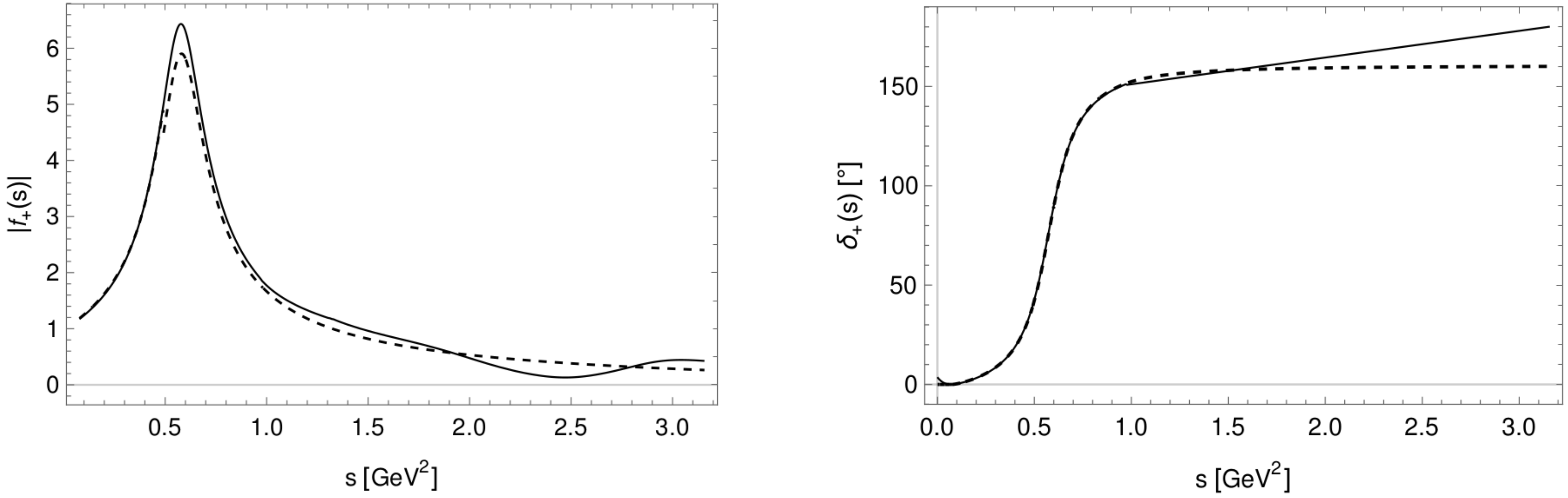
- Short-distance → cutoff effects
- Long-distance → Monte-Carlo noise
- Intermediate window: accessible with current resources
  - Precision of 0.4 - 0.6 %

Mattia Bruno. HADRON2023, June 5th 2023

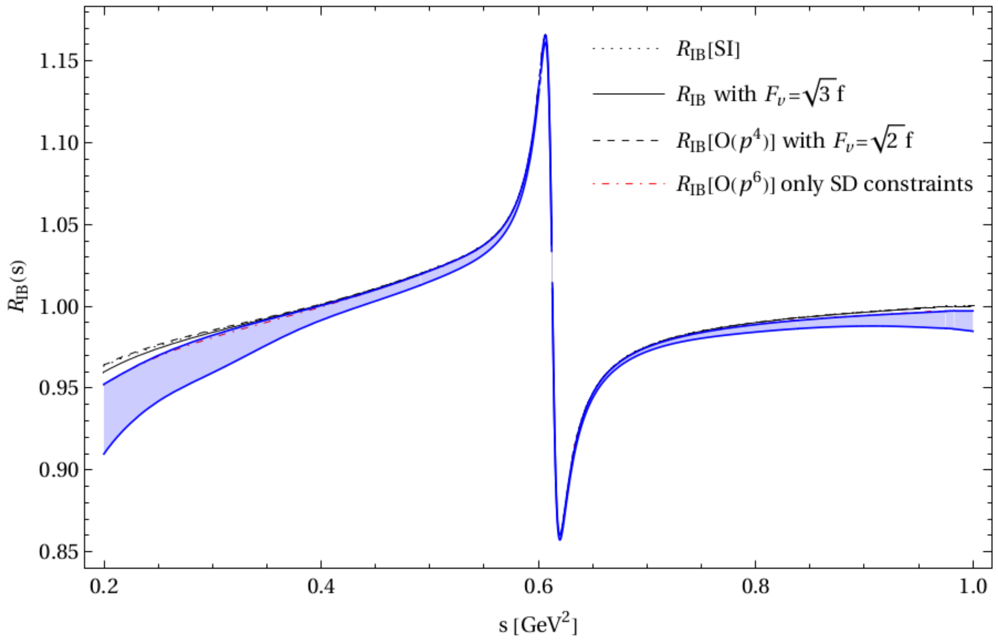


Form factor



6
5
4
3
2
1
0
$|f_+(s)|$
0.5
1.0
1.5
2.0
2.5
3.0
s [GeV^2]
150
100
50
0
$\delta_+(s)$ [°]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
s [GeV^2]

Overall IB corrections



Short-distance constraints

- Using the relations for 2-point Green functions at $O(p^4)$, we have:

$$F_V = \sqrt{2}F \quad G_V = \frac{F}{\sqrt{2}} \quad F_A = F.$$

- Using the relations for 2 and 3-point Green functions at $O(p^6)$, we have:

$$F_V = \sqrt{3}F \quad G_V = \frac{F}{\sqrt{3}} \quad F_A = \sqrt{2}F.$$

Short-distance constraints

- For the parameters contributing to the leading-order chiral LECs:

$$F_V G_V = F^2, \qquad F_V^2 - F_A^2 = F^2,$$
$$F_V^2 M_V^2 = F_A^2 M_A^2, \qquad 4 c_d c_m = F^2,$$
$$8\left(c_m^2 - d_m^2\right) = F^2, \qquad c_m = c_d = \sqrt{2} d_m = F/2.$$

- For the even-intrinsic parity sector:

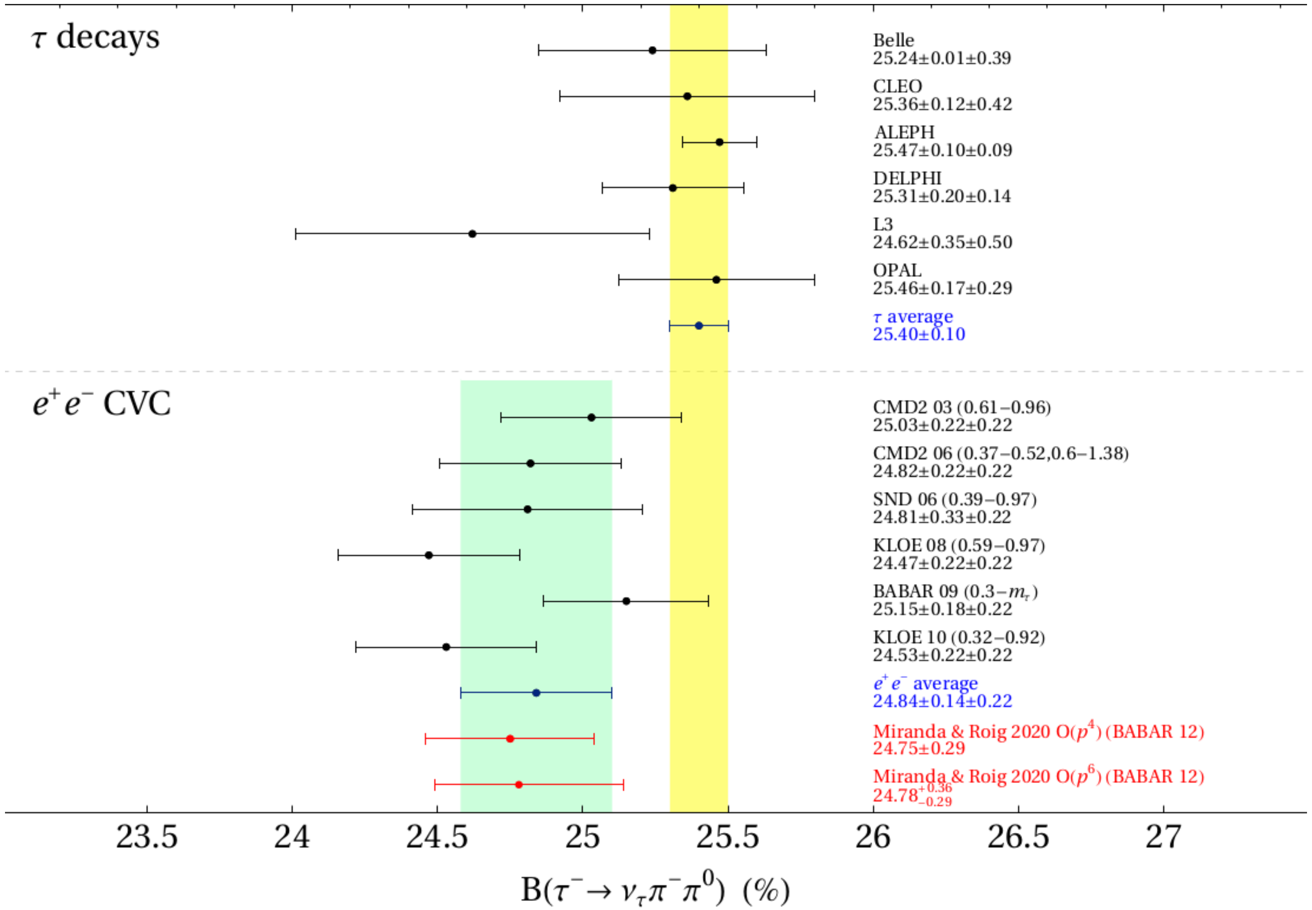
$$\lambda_{13}^P = 0, \quad \lambda_{17}^S = \lambda_{18}^S = 0,$$
$$\lambda_{17}^A = 0, \quad \lambda_{21}^V = \lambda_{22}^V = 0.$$

- The analysis of the <VAS> Green function yields:

$$\kappa_2^S = \kappa_{14}^A = 0, \quad \kappa_4^V = 2\kappa_{15}^V, \quad \kappa_6^{VA} = \frac{F^2}{32 F_A F_V},$$
$$F_V\left(2\kappa_1^{SV} + \kappa_2^{SV}\right) = 2 F_A \kappa_1^{SA} = \frac{F^2}{16\sqrt{2} c_m}.$$

CVC prediction of $B_{\pi\pi 0}$

- An important independent **cross-check** is provided by the **tau branching fraction**, another key quantity which can be directly **measured**.



$$B_{\pi\pi^0}^{\mathrm{CVC}} = B_e \int_{4m_\pi^2}^{m_\tau^2} ds\,\sigma_{\pi^+\pi^-(\gamma)}(s)\mathcal{N}(s)\frac{S_{\mathrm{EW}}}{R_{\mathrm{IB}}(s)}$$

$$\mathcal{N}(s) = \frac{3|V_{ud}|^2}{2\pi\alpha_0^2 m_\tau^2}\,s\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$$

CMD-3. Phys.Rev.D 109 (2024) 11, 112002

Short-distance constraints

- For the parameters contributing to the leading-order chiral LECs:

$$F_V G_V = F^2, \qquad F_V^2 - F_A^2 = F^2,$$
$$F_V^2 M_V^2 = F_A^2 M_A^2, \qquad 4 c_d c_m = F^2,$$
$$8\left(c_m^2 - d_m^2\right) = F^2, \qquad c_m = c_d = \sqrt{2} d_m = F/2.$$

- For the even-intrinsic parity sector:

$$\lambda_{13}^P = 0, \qquad \lambda_{17}^S = \lambda_{18}^S = 0,$$
$$\lambda_{17}^A = 0, \qquad \lambda_{21}^V = \lambda_{22}^V = 0.$$

- The analysis of the <VAS> Green function yields:

$$\kappa_2^S = \kappa_{14}^A = 0, \quad \kappa_4^V = 2\kappa_{15}^V, \quad \kappa_6^{VA} = \frac{F^2}{32 F_A F_V},$$
$$F_V\left(2\kappa_1^{SV} + \kappa_2^{SV}\right) = 2 F_A \kappa_1^{SA} = \frac{F^2}{16\sqrt{2} c_m}.$$

Short-distance constraints

- The study of the <VAP> and <SPP> Green functions yield the following restrictions on the resonance couplings:

$$\sqrt{2}\lambda_0 = -4\lambda_1^{VA} - \lambda_2^{VA} - \frac{\lambda_4^{VA}}{2} - \lambda_5^{VA} = \frac{1}{2\sqrt{2}}\left(\lambda' + \lambda''\right),$$
$$\sqrt{2}\lambda' = \lambda_2^{VA} - \lambda_3^{VA} + \frac{\lambda_4^{VA}}{2} + \lambda_5^{VA} = \frac{M_A}{2M_V},$$
$$\sqrt{2}\lambda'' = \lambda_2^{VA} - \frac{\lambda_4^{VA}}{2} - \lambda_5^{VA} = \frac{M_A^2 - 2M_V^2}{2M_V M_A},$$
$$\lambda_1^{PV} = -4\lambda_2^{PV} = -\frac{F\sqrt{M_A^2 - M_V^2}}{4\sqrt{2}d_m M_A}, \quad \lambda_1^{PA} = \frac{F\sqrt{M_A^2 - M_V^2}}{16\sqrt{2}d_m M_V}.$$

- For the odd-intrinsic parity sector:

$$\kappa_{14}^V = \frac{N_C}{256\sqrt{2}\pi^2 F_V}, \quad 2\kappa_{12}^V + \kappa_{16}^V = -\frac{N_C}{32\sqrt{2}\pi^2 F_V}, \quad \kappa_{17}^V = -\frac{N_C}{64\sqrt{2}\pi^2 F_V}, \quad \kappa_5^P = 0,$$
$$\kappa_2^{VV} = \frac{F^2 + 16\sqrt{2}d_m F_V \kappa_3^{PV}}{32F_V^2} - \frac{N_C M_V^2}{512\pi^2 F_V^2}, \quad 8\kappa_2^{VV} - \kappa_3^{VV} = \frac{F^2}{8F_V^2}.$$

Fit results

- We perform a global fit using the relations for the resonance saturation of the anomalous sector LECs:

$$\kappa_1^V = (-2.1 \pm 0.7) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_2^V = (-8.8 \pm 9.1) \cdot 10^{-3}\ \text{GeV}^{-1},$$
$$\kappa_3^V = (2.2 \pm 5.8) \cdot 10^{-3}\ \text{GeV}^{-1},$$
$$\kappa_6^V = (-2.1 \pm 0.3) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_7^V = (1.2 \pm 0.5) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_8^V = (3.1 \pm 0.9) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_9^V = (-0.1 \pm 5.9) \cdot 10^{-3}\ \text{GeV}^{-1},$$
$$\kappa_{10}^V = (-5.9 \pm 9.6) \cdot 10^{-3}\ \text{GeV}^{-1},$$
$$\kappa_{11}^V = (-3.0 \pm 0.6) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_{12}^V = (1.0 \pm 0.8) \cdot 10^{-2}\ \text{GeV}^{-1},$$
$$\kappa_{13}^V = (-5.3 \pm 1.1) \cdot 10^{-3}\ \text{GeV}^{-1},$$
$$\kappa_{18}^V = (4.7 \pm 0.8) \cdot 10^{-3}\ \text{GeV}^{-1}.$$

Phys.Rev.D 92 (2015) 025014
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- These values are in good agreement with our earlier estimation $|\kappa^V_i| < 0.025$ GeV^{-1} .