Amplitude-level Effects in Bhabha Scattering from Dark Boson Exchange

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Outline

• Introduction

• $A'$ search in 10-100 MeV mass range

• $Z'$ search in 10-100 MeV mass range

• QED predictions for the Beam Normal SSA in Bhabha scattering

• Summary
Jlab 12 GeV CW Electron Accelerator
# Capability With a Future Positron Injector

<table>
<thead>
<tr>
<th>Machine Parameter</th>
<th>Electrons</th>
<th>Positrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall Multiplicity</td>
<td>4</td>
<td>1 or 2</td>
</tr>
<tr>
<td>Energy (ABC/D)</td>
<td>11/12 GeV</td>
<td>11/12 GeV</td>
</tr>
<tr>
<td>Beam Repetition</td>
<td>249.5/499 MHz</td>
<td>249.5/499 MHz</td>
</tr>
<tr>
<td>Duty Factor</td>
<td>100% cw</td>
<td>100% cw</td>
</tr>
<tr>
<td>Unpolarized Intensity</td>
<td>170 µA</td>
<td>&gt; 1 µA</td>
</tr>
<tr>
<td>Polarized Intensity</td>
<td>170 µA</td>
<td>&gt; 50 nA</td>
</tr>
<tr>
<td>Beam Polarization</td>
<td>&gt; 85%</td>
<td>&gt; 60%</td>
</tr>
<tr>
<td>Fast/Slow Helicity Reversal</td>
<td>1920 Hz/Yes</td>
<td>1920 Hz/Yes</td>
</tr>
</tbody>
</table>

See talk by Joe Grames at [https://indico.jlab.org/event/819/](https://indico.jlab.org/event/819/) from the March 2024 PWG Workshop. There were also many talks on future experiments and related theory calculations.
The Developing Physics Program with Positron Beams

Hadronic Physics

• **Deeply Virtual Compton Scattering:** Large, and therefore easily measureable, interference effects are predicted for $e^- + N \rightarrow e^- + N + \gamma$ which will be invaluable for constraining Generalized Parton Distributions (GPDs).

• **2-photon exchange:** A wide range of unpolarized and polarized precision measurements of several % effects from the interference of 1- and 2-photon exchange will challenge theory to make electron scattering an even more precise tool for studies of EM form factors, charge radii, Pardon Distribution Functions, etc.

Beyond the Standard Model

The ability of positrons to annihilate with electrons opens the door for the production of new particles

• **Dark photon search:** A sensitive search in $e^+ + e^- \rightarrow \gamma (A')$ using the missing mass technique.
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The ability of positrons to annihilate with electrons opens the door for the production of new particles

- **Dark photon search:** A sensitive search in $e^+ + e^- \rightarrow \gamma (A')$ using the missing mass technique.

But I think there are also several exciting opportunities for Bhabha scattering, $e^+ e^- \rightarrow e^+ e^-$. Jlab’s high luminosity, plus its expertise in spin manipulation, suggest to me that measurements of unprecedented precision will be possible in a mass region which has been relatively unexplored by e+e- colliders.
Bhabha scattering is a purely leptonic reaction with very different behavior than Moller scattering.

The $e^+$ and $e^-$ are of course not identical, and there is an s-channel annihilation diagram.

In the Standard Model (SM), the exchanged boson is a $\gamma$ and a $Z^0$.

Going Beyond the SM (BSM), considering neutral bosons only, there is also potentially an $A'$ or $Z'$. 
A’ search in Bhabha Scattering
Excluded A’ Phase Space (visible decays)

The mixing between the photon and dark photon is parameterized as $\varepsilon$.

The coupling of the dark photon to the electron is $\varepsilon^* e$.

There is a region of phase space, relevant to the Jlab positron program, which has proven resistant in visible searches (and to a much lesser extent for invisible decays).

The phase space in the red dashed box seems potentially excludable in a Jlab positron program, at least with amplitude-based searches I describe.

**Figure 4.** Constraints on visible $A'$ decays considered in this study from (red) electron beam dumps, (cyan) proton beam dumps, (green) $e^+e^-$ colliders, (blue) $p p$ collisions, (magenta) meson decays, and (yellow) electron on fixed target experiments. The constraint derived from $(g - 2)_e$ is shown in grey [90, 91].

A’ Signal Proportionality in terms of $e$ and $\varepsilon$: examples using dark Bremsstrahlung

For incoherent production and decay:

- Yield for $A'$ production $\sim |Z F(q) e^3 \varepsilon|^2$
- Yield for $A'$ decay $\sim BR_{A' \rightarrow e^+e^-} |e\varepsilon|^2$
A’ Signal Proportionality in terms of e and ε: examples using dark Bremsstrahlung

For incoherent production and decay:

• Yield for A’ production ~ |Z F(q) e^3 ε |^2
• Yield for A’ decay ~ BR_{A’→e+e-} |eε|^2

Visible decay scenario: the net yield for detecting A’ → e+e- is

\sim BR_{A’→e+e-} Z^2 F^2(q) \alpha^4 \epsilon^4

(note upper limits on ε improve glacially slowly with increasing FOM, 1/FOM^{1/8}. A detached vertex search allows a huge improvement.)
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All decays scenario: signal yield for indirectly detecting an A’ by MM^2_x in e + p → e + p (X)

~ F^2(q) α^3 ε^2

(upper limits on ε improve less slowly, 1/FOM^{1/4})
A’ Signal Proportionality in terms of e and ε: examples using dark Bremsstrahlung

For incoherent production and decay:

- Yield for A’ production \( \sim |Z F(q) e^3 \epsilon|^2 \)
- Yield for A’ decay \( \sim BR_{A' \rightarrow e^+e^-} |e\epsilon|^2 \)

**Visible decay scenario:** the net yield for detecting \( A' \rightarrow e^+e^- \) is
\[
\sim BR_{A' \rightarrow e^+e^-} Z^2 F^2(q) \alpha^4 \epsilon^4
\]
(note upper limits on \( \epsilon \) improve glacially slowly with increasing FOM, \( 1/FOM^{1/8} \). A detached vertex search allows a huge improvement.)

**All decays scenario:** signal yield for indirectly detecting an A’ by MM_\( x^2 \)
in \( e + p \rightarrow e + p (X) \)
\[
\sim F^2(q) \alpha^3 \epsilon^2
\]
(upper limits on \( \epsilon \) improve less slowly, \( 1/FOM^{1/4} \))

Because \( \alpha \) and \( \epsilon \) are small numbers, one would like to design an experiment with small exponents and low backgrounds.
Alternate Strategy if a Positron Beam is Available

Assume the total amplitude is the sum of a large SM and small BSM amplitude: \( A_{\text{tot}} = A_{\text{EM}} + A_{\text{small}} \)

The yield is hand-wavingly proportional to \( A_{\text{tot}}^2 = (A_{\text{EM}} + A_{\text{small}})^2 \)

\[
= A_{\text{EM}}^2 + 2A_{\text{EM}}A_{\text{small}} + A_{\text{small}}^2
\]

Instead of looking for real dark photons as a bump proportional to \( A_{\text{small}}^2 \), can we search sensitively for resonant signatures of virtual \( A' \) exchange in the interference term, with relative magnitude

\[
2A_{\text{EM}}A_{\text{small}} / A_{\text{EM}}^2 = 2A_{\text{small}} / A_{\text{EM}}
\]

The answer is, “Yes!”
A’ Signal Proportionality in terms of e and ε: assuming a positron beam and virtual A’ exchange

**All decays scenario:** the amplitude of the interference term for a virtual A’ in \( e^+ + e^- \rightarrow e^+ + e^- \) is

\[ \sim \alpha \varepsilon^2 \]

This is not the whole story, but it’s extremely promising.
A’ Signal Proportionality in terms of e and ε: assuming a positron beam and virtual A’ exchange

All decays scenario: the amplitude of the interference term for a virtual A’ in $e^+ + e^- \rightarrow e^+ + e^-$ is

$$\sim \alpha \varepsilon^2$$

This is not the whole story, but it’s extremely promising.

If one allows for Initial State Radiation (which may be the only practical way to cover a large mass range without 100 separate beam energies) this becomes

$$\sim \alpha^{3/2} \varepsilon^2$$

which is still very promising.

Total disclosure: the experimental FOM for an A’ search in a bump hunt goes like the Signal/sqrt(Background), so experimental resolution also matters a lot, while backgrounds matter to a lesser extent.
I used H.A. Olsen and P. Osland, “Polarized Bhabha and Moller scattering in left-right-asymmetric theories”, PRD 25, 2895-2910 (1982). This paper includes $\gamma$ and $Z$ contributions, as well as a $Z'$. The latter I extended to an $A'$ by giving it purely vector couplings and a mass in range accessible to Jlab.

But it does not include radiation which will be important for designing realistic experiments.

Eqn (1) of Olsen and Osland gives the xsect and asymmetries for all the standard combinations of $e^+$ or $e^-$ longitudinal or transverse polarization.

Simplifying and dumbing down the notation a bit:

$$\sigma(\theta,\phi) = \sigma_0 \left\{ 1 + A_{LL} P_+^{\text{para}} P_-^{\text{para}} + A_{LU}(P_-^{\text{para}} - P_+^{\text{para}}) + P_+^{\text{perp}} P_-^{\text{perp}} \left[ A_{TT} \cos(2\phi) + A_{TT}' \sin(2\phi) \right] \right\}$$

If I drop the predominantly PV terms, it looks just like the Moller polarimetry equations:

$$\sigma(\theta,\phi) = \sigma_0 \left\{ 1 + A_{LL} P_+^{\text{para}} P_-^{\text{para}} + P_+^{\text{perp}} P_-^{\text{perp}} A_{TT} \cos(2\phi) \right\}$$

Let’s look at the $\sigma_0$ term on the next slide.
Bhabha vs Moller Comparison: $\sigma/\sigma_{QED}$ Up to $E_{cm} = 140$ GeV/c^2

This is just one observable from the Olsen and Osland paper of the dramatic difference between Bhabha and Moller scattering.

When a resonance is present, the effects on the $\sigma$ can be 10-100x larger than in Moller scattering.

For me, this motivates the idea of searching for an $A'$ using Bhabha scattering.
Yields: Purely Vector Coupling, $\varepsilon = 1E-4$, $M_{A'} = 57.5$ MeV/c$^2$

On this plotting scale, the $A'$ effects are invisibly small.

Note the flattening of the Bhabha xsect at backward angles, where t-channel photon exchange no longer dominates.
Yields: Purely Vector Coupling, $\varepsilon = 1E-4$, $M_{A'} = 57.5$ MeV/c$^2$

On this plotting scale, the $A'$ effects are invisibly small.

Note the flattening of the Bhabha xsect at backward angles, where t-channel photon exchange no longer dominates.

Taking the difference wrt the SM, off-resonance effects are tiny, comparable to Z0 exchange.

As naively expected, $A'$ effects are largest at backward angles.
Yield Signal ($\epsilon = 1E-4, M_{A'} = 57.5$ MeV/c$^2$)

Having established that $A'$ effects are less diluted at large $\theta_{CM}$, now scanning $\sqrt{s}$ at 150deg CM:

This is potentially measureable (about 0.5 ppm in the wings).

Naively, the beam energy would have to be within ~50 MeV of resonance to see a deviation from the SM.

In practice, if the beam energy is above the resonance, Initial State Radiation (ISR) will allow probing a broader range of lower $E_{cm}$.
light Z’ search via the PV asymmetry
$A_{PV}: g_A = g_V = 1, \varepsilon = 1E^{-4}, M_{A'} = 57.5 \text{ MeV}/c^2$

The tree-level SM $A_{PV}$ in Bhabha is very small. Even several hundred MeV in beam energy off resonance, the effect on $A_{PV}$ is dramatic.

Of course, one would need a year or so to achieve 10 ppb sensitivity.
$A_{PV} : g_A = g_V = 1, \varepsilon = 1E-4, M_{A'} = 57.5 \text{ MeV}/c^2$

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Of course, one would need a year or so to achieve 10 ppb sensitivity.

Near resonance, the asymmetry approaches $O(1)$ ppm. This is 100x the SM value, and requires 1/10,000 less time to observe.

Best guess is 6-12 beam energies will be needed.
QED Predictions for the Beam Normal SSA
BNSSA in Bhabha Scattering

This observable is not available in the Olsen and Osland paper. It is small at high energy.

To 4th order in the EM coupling constant, $e$, their calculation should be valid from $\sqrt{s} = up to 2*m_\mu$ threshold.
I do not yet have a formalism that includes an $A'$, so I will only show the QED prediction.

The BNSSA is the interference between 1-photon and the imaginary part of 2-photon exchange amplitudes:

$$P = 2i \text{tr}\{M_2^* \sigma \cdot s \text{ Im } M_4\}/\text{tr}\{M_2^*M_2\}$$
Bhabha Compared to Moeller – Angular Distributions

Non-relativistic in CM

BNSSA in ppm vs ThetaCM for Beam E = 1 MeV

Asymmetry (ppm)

Theta CM (deg)
Bhabha Compared to Moeller – Angular Distributions

Non-relativistic in CM

Relativistic in CM
Between the non-relativistic and relativistic regimes, there’s dramatic evolution of the shape of the Bhabha angular distribution. In Moller scattering, evolution of the shape of the angular distribution is more subtle. In Bhabha, the magnitude near $\theta_{\text{CM}} \sim 122$ degrees evolves from nearly a maximum to a stable zero crossing.
What Might a Dark Matter Contribution to the BNSSA in Bhabha Look Like?

Although the diagram below doesn’t look 2 photon-ish at all, I believe it would satisfy the cut rules and contribute an Imaginary amplitude … but only when the measurement is above the threshold for producing an on-shell $A'$. (Higher order diagrams, such as a box diagram or loop with both $A'$ and photon exchange, would be suppressed by an additional factor of $\alpha$. These would be the major contributors to the BNSSA in Moeller scattering.)

(There may be no equivalent $e+e^- \rightarrow \gamma^* \rightarrow e+e^-$ contribution because the $\gamma^*$ is off shell.)

This seems worth a closer look, because a measurement at $\sqrt{s}$ could seemingly constrain the existence an $A'$ at all $A'$ masses $< \sqrt{s}$. Potentially has broad mass sensitivity!
A higher beam energy will reach higher $\sqrt{s}$, but the asymmetry and the cross section will be smaller.

The asymmetry is not small by the standards of the Jlab parity program.

Calculations including an $A'$ are needed.

The location of the zero crossing could in principle be sensitive to the imaginary part of an additional, small BSM amplitude.
Summary

• A positron beam facility at Jlab would enable a detailed study of Bhabha scattering in the relatively unexplored mass range of 10 to 100 MeV/c².

• Targets consisting of atomic electrons will permit practical e+e- luminosities of 10^35 to 10^36.

• Cross sections are large by Jlab standards due to a 1/s kinematic factor, the small value of s, and the lack of a form factor.

• The resulting high count rates, combined with Jlab’s expertise in spin manipulation, would enable measurements of Bhabha observables with unprecedented precision.

• A dataset with multiple beam energies for e⁺ + e⁻ → e⁺ + e⁻ (γ) would allow us to sensitively search for an A’ or Z’ by looking for the near-resonance signature of interference with the photon.

• The Beam Normal SSA may have some sensitivity to all lighter BSM physics e.g., by shifting the zero crossing. The calculations here were purely QED. New calculations are needed which include the leading order A’ contribution.

What I’ll explore next:

• The above dataset would also probe the fine structure constant α(s,t) at momentum transfer scales between that of the hydrogen atom and muon g-2, thus testing our understanding of the vacuum.

• There are mildly exotic azimuthal asymmetries due to longitudinally polarized e+ beam scattering from a transversely polarized e- target. In a purely leptonic process, these are a way to search for BSM couplings which are neither Vector nor Axial.
extras
A’ Signal Proportionality in terms of e and $\varepsilon$: assuming a positron beam and missing mass technique

All decays scenario: the yield for producing an A’ in $e^+ + e^- \rightarrow \gamma (A')$ is

$$\sim \alpha^2 \varepsilon^2$$

which has a promising sensitivity. This technique must be pursued.

But in addition to manageable backgrounds from

$e^+ + e^- \rightarrow \gamma \gamma^* \rightarrow \gamma (e^+e^-)$

$e^+ + e^- \rightarrow \gamma \ (1\gamma, \ etc)$

there may be important sources of photons from the incoming or outgoing positron beamline.
Injecting $e^+$ to CEBAF 12 GeV

Once $e^+$ source is ready, civil construction connects the LERF by a new tunnel to CEBAF. The transport line will maintain the $e^+$ polarization in plane.

$e^+$ transported in new beamline and injected to CEBAF for 12 GeV, with magnet polarities reversed

From talk by Joe Grames

(an LDRD is exploring the transport of large emittance beams at CEBAF)
Design two new injectors (e⁻ and e⁺)

Two challenging injectors have to be built

- >1 mA polarized e⁻ injector >150 MeV
- >100 kW target & cw-collection beam line

>300 kV dc-high voltage GaAs photogun generates milliAmp e⁻ beam with polarization ~90%

Cε*BAF Parameter | Status | Goal
--- | --- | ---
$p_0$ [MeV/c] | 60 | 60
$\sigma_{p/p_0}$ [%] | 0.68 | ±1
$\sigma_z$ [ps] | 3 | ≤ 4
Normalized $\epsilon_\gamma$ [mm mrad] | 140 | ≤ 40
$p_f$ [MeV/c] | 123 | 123
$I_{e^+} (P > 60\%)$ [nA] | 170 | > 50

120 kW e⁻ beam irradiates water cooled spinning tungsten target,

From talk by Joe Grames
List of Measurements

Longitudinal beam polarization and unpolarized atomic e- target
6-12 beam energies, mainly searching for a bi-polar signal over the mass range 10 – 100 MeV/c²

- $A'$ search in Yield vs $\sqrt{s}$
- $Z'$ search in $A_{pv}$ vs $\sqrt{s}$

not discussed: the same dataset above can be used to address the running of $\alpha(s,t)$

Transverse beam polarization and unpolarized atomic e- target
1-2 of the above beam energies, with particular focus on the zero crossing near $\theta_{CM} \sim 120\text{deg}$

- BNSSA vs $\theta_{CM}$

Longitudinal beam polarization and transversely polarized e- target

not discussed: no calculations yet or even a concept for a run plan
There is some literature on BSM particles which have significant axial-vector couplings. It’s easy to explore that in this formalism.

In the purely vector or purely axial-vector scenario, terms proportional to $g_v^*g_a$ vanish, leaving a $g_v^2 - g_a^2$ term which switches sign.

The purely axial-vector coupling yields the mirror image of the purely vector coupling.
Yield Signal Dependence on $\epsilon$
(Purely Vector Coupling, $M_{A'} = 57.5$ MeV/c², Width = 57.5 keV)

Signal amplitudes scale like $\epsilon^2$, so $\epsilon = 1E-5$ is almost impossible, $\epsilon = 1E-4$ is challenging, and $\epsilon = 1E-3$ would be easy.

(For the $\epsilon = 1E-5$ case, the noticeable slope is from Z0 exchange, which is highly suppressed at this CM angle as well as by $1-4\sin^2\theta_w$. Hand-wavingly, I think it’s fair to say that the $A'$ signal at $\epsilon = 1E-4$ is weak interaction scale.)
$E_{cm}$ in Bhabha Scattering in Jlab Fixed Target Kinematics

At a 12 GeV CEBAF, the CM energy range will be $\sim 20-105$ MeV/$c^2$.

$$E_{cm} = \sqrt{s} = \sqrt{2m_e^2 + 2E_{\text{beam}} * m_e}$$

$$\sim \sqrt{E_{\text{beam}}}$$

Notes:

- due to the $\sqrt{s}$ factor above, it takes a roughly 100 MeV change in $E_{\text{beam}}$ to produce a 1 MeV change in $E_{cm}$.  
  (Hold that thought for later!)

- since the differential xsect contains a factor of $1/s$, and $s$ is small, 
  the xsect is large by Jlab standards, $O(1)$-$O(100)$ $\mu$B/sr at 90deg CM.
The Glitch in the Matrix: Why is This Not a Bump Hunt?

\[ M_{A'} = 57.5 \text{ MeV/c}^2, \text{ Width} = 57.5 \text{ keV} \]

\[ \frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{4s} \left[ \cos^4(\theta/2) \left| 1 + f(s) g_{GL}^2 - \frac{1 + f(t) g_{GL}^2}{\sin^2(\theta/2)} \right|^2 + \left| 1 + f(s) g_{GR}^2 - \frac{1 + f(t) g_{GR}^2}{\sin^2(\theta/2)} \right|^2 \right] \]

\[ + 2 \sin^4(\theta/2) \left| 1 + f(s) g_{GR} g_{GL} \right|^2 + \left[ \frac{2}{\sin^4(\theta/2)} \right] \left(1 + f(t) g_{GR} g_{GL} \right)^2 \right] \}

\[ \text{Xsect} \sim \left| 1 + f(s) g_{R} g_{L} \right|^2 \]

\[ \sim \left| 1 + \text{Ref}(s) g_{R} g_{L} + i \text{Im}(s) g_{R} g_{L} \right|^2 \]

\[ \sim 1 + 2\text{Ref}(s) g_{R} g_{L} + [\text{Ref}(s)^2 + \text{Im}(s)^2] (g_{R} g_{L})^2 \]

\[ \sim 1 + 2\text{Ref}(s) g_{R} g_{L} + \text{H.O.T.} \]

Real Part

Imag. Part

So at small, relevant values of \( \varepsilon \), the signal in Bhabha scattering will have the shape of \( \text{Ref}(s) \).

proportional to \( \varepsilon^2 \)

proportional to \( \varepsilon^4 \)
Evolution into a Bump with Increasing $\varepsilon$: idiot check
(Purely Vector Coupling, $M_{A'} = 57.5$ MeV/c², Width = 57.5 keV)

- $\varepsilon = 1e^{-3}$
- $\varepsilon = 0.03$
- $\varepsilon = 0.1$

The interference pattern ($A_{EM}^* A_{small}$) shape evolves into a bump with increasing $\varepsilon$. I think of the bump as representing real $A'$ production (proportional to $A_{small}^2$).
Yield Signals Compared
(ε = 1E-4 and 1E-3 on same vertical scale)

This is a reminder how much easier ε > 1E-3 would be.

0.5 ppm in the wings becomes 50 ppm!
Contributions to the Bhabha Xsect: s channel

The unpolarized xsect is proportional to $\alpha^2$:

$$
\frac{d\sigma^B}{d\Omega} = \frac{\alpha^2}{4s} \left[ \cos^4(\theta/2) \left( 1 + f(s)g_L^2 - \frac{1 + f(t)g_L^2}{\sin^2(\theta/2)} \right)^2 + \left( 1 + f(s)g_R^2 - \frac{1 + f(t)g_R^2}{\sin^2(\theta/2)} \right)^2 \right] 
+ 2 \sin^4(\theta/2) \left( 1 + f(s)g_R g_L \right)^2 + \left( 2/\sin^4(\theta/2) \right) \left[ 1 + f(t)g_R g_L \right]^2, 
$$

(Polarized xsect differences can be defined from the asymmetries in Eqns 15-18.)

$f(t)$ is for spacelike Z and is purely real. These terms tend to diverge as $\theta \to 0$ deg, which will dilute any interesting $A'$ effects that we add to the s-channel.

$f(s)$ is for time-like Z, has a Real part and an Imaginary part. Generally, effects from a resonant $A'$ will be largest at backward angles (see red arrow at right, pointing to a “shelf” in the xsect).

$$
f(q^2) = \begin{cases} 
\frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2 + iM_Z \Gamma_{Z1} / Z}, & q^2 > 0 \ (q \text{ timelike}), \ i.e., \ f(s) \\
\frac{1}{4 \sin^2(2\theta_W)} \frac{q^2}{q^2 - M_Z^2}, & q^2 \leq 0 \ (q \text{ spacelike}). \ i.e., \ f(t) 
\end{cases}
$$

The unpolarized xsect is proportional to $\alpha^2$:
Contributions to the Bhabha Xsect: t channel

The unpolarized xsect is proportional to $\alpha^2$:

$$\frac{d\sigma^B_0}{d\Omega} = \frac{\alpha^2}{4s} \left[ \cos^4(\theta/2) \left\{ \left[ 1 + f(s)_{GL}^2 \right] - \frac{1 + f(t)_{GL}^2}{\sin^2(\theta/2)} \right\}^2 + \left[ 1 + f(s)_{GR}^2 \right] - \frac{1 + f(t)_{GR}^2}{\sin^2(\theta/2)} \right]^2 + 2 \sin^6(\theta/2) \left[ 1 + f(s)_{GR}^2 \right] \right\} \right],$$

$$+ 2 \sin^6(\theta/2) \left[ 1 + f(s)_{GR}^2 \right] \right\} \right],$$

$$+ \left[ 2 / \sin^4(\theta/2) \right] \left[ 1 + f(t)_{GR}^2 \right] \right\} \right],$$

(Polarized xsect differences can be defined from the asymmetries in Eqns 15-18.)

$f(t)$ is for spacelike $Z$ and is purely real. These terms tend to diverge as $\theta \to 0$ deg, which will dilute any interesting $A'$ effects in the s-channel.

$$f(q^2) = \begin{cases} 
\frac{1}{4 \sin^2(2\theta_w)} \frac{q^2}{q^2 - M_Z^2 + i M_Z F_{\text{Z}}}, & q^2 > 0 \ (q \text{ timelike}), \ i.e., \ f(s) \\
\frac{1}{4 \sin^2(2\theta_w)} \frac{q^2}{q^2 - M_Z^2}, & q^2 \leq 0 \ (q \text{ spacelike}). \ i.e., \ f(t) 
\end{cases}$$

$$\text{Xsect at 3600 MeV}$$

$$\text{xsect} \quad \text{abs}(\text{Xsect}_{LL}) \quad \text{abs}(\text{Xsect}_{TT}) \quad \text{abs}(\text{Xsect}_{LU})$$

Theta CM [deg]