Radiative corrections to one- and two-meson tau decays for precise new physics tests

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Cinvestav

Based on Miranda-**Roig** *Phys.Rev.D* 102 (2020) 114017, Arroyo Ureña-Hernández Tomé-López Castro-**Roig**-Rosell *Phys.Rev.D* 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173, Escribano-Miranda-**Roig** *Phys.Rev.D* 109 (2024) 5, 053003

INTRODUCTION

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Nucl.Phys.B 438 (1995) 17-53

Phys.Rev.D 104 (2021) 9, L091502
JHEP 02 (2022) 173

Phys.Lett.B 513 (2001) 361-370

JHEP 08 (2002) 002

Phys.Rev.D 74 (2006) 071301

Phys.Rev.D 102 (2020) 114017

JHEP 10 (2013) 070

Phys.Rev.D 88 (2013) 7, 073009

H^-	Precision $[\mathcal{B}_H]$ PDG 2022	Rad. Corr.	Application
π^-	0.5%	✓ ✦ (blue)	LFU, NP
K^-	1.4%	✓ ✦ (blue)	V_{us}, LFU, NP
$\pi^-\pi^0$	0.4%	✓ ✦ (red)	ρ, ρ', $\cdots$, $(g-2)_\mu$, NP
K^-K^0	2.3%	✗	ρ', $\cdots$, NP
$\bar{K}^0\pi^-$	1.7%	✓ ✦ (green)	K^*, V_{us}, $C\!\!\!/P$, NP
$K^-\pi^0$	3.5%	✓ ✦ (green)	K^*, V_{us}, NP
$K^-\eta$	5.2%	✗	K^*, NP
$\pi^-\pi^+\pi^-$	0.5%	✗	a_1
$\pi^-2\pi^0$	1.1%	✗	a_1

✦ (blue) Decker and Fikemeier '95, Arroyo-Ureña et al '21
✦ (red) Cirigliano et al '01, Flores-Tlalpa et al '06, Miranda and Roig '20
✦ (green) Antonelli et al '13, Flores-Baéz and Morones-Ibarra '13

Short-Distance corrections: Sirlin '78; Marciano-Sirlin '93
Rev.Mod.Phys. 50 (1978) 573 & 905 (erratum)
Phys.Rev.Lett. 71 (1993) 3629-3632

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K^-K^0	2.3%	✘	✓	ρ', $\cdots$, NP
$\bar{K}^0\pi^-$	1.7%	✓	✓	K^*, V_{us}, CP̸, NP
$K^-\pi^0$	3.5%	✓	✓	K^*, V_{us}, NP
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This work

Escribano-Miranda-Roig
Phys.Rev.D 109 (2024) 5, 053003

INTRODUCTION

- Electromagnetic radiative corrections require the inclusion of diagrams with both virtual (loops) & real photons (ISR & FSR).
- I will illustrate this with the RadCors to the one π (or K) tau decays. The two-meson cases can be studied similarly. *They are just more complicated*...

1. RadCors to one-meson tau decays (Motivation)

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$$

Tests LU ($g_\tau = g_\mu$) using $P=\pi, K$

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I will consider both decays in turn, $R_{\tau/\pi}$, results & applications. Among the conclusions, I will show results for 2 mesons.

1. RadCors to one-meson tau decays (Motivation)

- ✓ Lepton Universality (LU) as a basic tenet of the Standard Model (SM).
  - ✓ A few anomalies observed in semileptonic B meson decays*.
  - ✓ Lower energy observables currently provide the most precise test of LU**.
- ✓ We aim to test muon-tau lepton universality through the ratio (P = π, K)***:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R_{\tau/P}^{(0)} \left(1 + \delta R_{\tau/P}\right)$$

  - ✓ $g_\tau = g_\mu$ according to LU.
  - ✓ $R_{\tau/P}^{(0)}$ is the LO result $R_{\tau/P}^{(0)} = \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}$.
  - ✓ $\delta R_{\tau/P}$ encodes the radiative corrections.
- ✓ $\delta R_{\tau/P}$ was calculated by Decker & Finkemeier (DF'95) ^ :
  - ✓ $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$.
- ✓ Important phenomenological and theoretical reasons to address the analysis again.

* Albrecht et al.'21 *Prog.Part.Nucl.Phys.* 120 (2021) 103885
** Bryman et al.'21 *Ann.Rev.Nucl.Part.Sci.* 72 (2022) 69-91

*** Marciano & Sirlin'93 *Phys.Rev.Lett.* 71 (1993) 3629-3632
^ Decker & Finkemeier'95 *Nucl.Phys.B* 438 (1995) 17-53

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- ✓ Phenomenological disagreement in LU tests:
  - ✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6σ of LU)
    - ✓ $|g_\tau/g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9σ of LU)
  - ✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:
    - ✓ $|g_\tau/g_\mu| = 1.0010 \pm 0.0014$ (at 0.7σ of LU)
  - ✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:
    - ✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)

* Decker & Finkemeier'95 *Nucl.Phys.B* 438 (1995) 17-53
** HFLAV'21 *Eur.Phys.J.C* 81 (2021) 3, 226
*** CMS'21, ATLAS'21 *Phys.Rev.D* 105 (2022) 7, 072008 *Nature Phys.* 17 (2021) 7, 813-818

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Nature Phys. 17 (2021) 7, 813-818

✓ Theoretical issues within DF'95*:

✓ Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.

✓ A cutoff to regulate the loop integrals (separating long- and short-distance corrections)

✓ Unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

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✓ By-products of the project:

✓ Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.

✓ CKM unitarity test via $\Gamma(\tau \to K\nu_\tau[\gamma])$ or via the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$.

✓ Constraints on possible non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$^.

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*** CMS'21, ATLAS'21 — *Phys.Rev.D* 105 (2022) 7, 072008; *Nature Phys.* 17 (2021) 7, 813-818

^ Cirigliano et al.'10 '19, '22 — *Nucl.Phys.B* 830 (2010) 95-115; *Phys.Rev.Lett.* 122 (2019) 22, 221801; *JHEP* 04 (2022) 152
^ González-Alonso & Martin-Camalich '16 — *JHEP* 12 (2016) 052
^ Gonzàlez-Solís et al. '20 — *Phys.Lett.B* 804 (2020) 135371

2. RadCors to $P_{l2[\gamma]}$ decays ($P=\pi$, K)

- ✓ Calculated unambigously within the Standard Model (Chiral Perturbation Theory, ChPT*).
- ✓ Notation by Marciano & Sirlin** and numbers by Cirigliano Rosell*** (D=d,s for π,K and $F_\pi \approx 92.2$ MeV):

LO result — short-distance EW correction ≈ 1.0232** — structure independent (SI) contributions (point-like approximation)^

$$\Gamma(P\to\mu\nu_\mu[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{4\pi}m_P\,m_\mu^2\left(1-\frac{m_\mu^2}{m_P^2}\right)^2 S_{\rm EW}\left\{1+\frac{\alpha}{\pi}F(m_\mu^2/m_P^2)\right\}\times$$
$$\left\{1-\frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{m_\rho}{m_P}+c_1^{(P)}+\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}+c_3^{(P)}+c_4^{(P)}(m_\mu/m_P)\right)-\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}\right]\right\}$$

structure-dependent (SD) contributions

Physica A 96 (1979) 1-2, 327-340

Annals Phys. 158 (1984) 142

Nucl.Phys.B 250 (1985) 465-516

- ✓ The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:
- ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

Phys.Rev.Lett. 71 (1993) 3629-3632

Nucl.Phys.B 321 (1989) 311-342

* Weinberg'79
* Gasser & Leutwyler'84 '85
** Marciano & Sirlin'93

*** Cirigliano & Rosell '07 *Phys.Rev.Lett.* 99 (2007) 231801 *JHEP* 10 (2007) 005
^ Kinoshita'59 *Phys.Rev.Lett.* 2 (1959) 477

† Ecker et al.'89
† Cirigliano et al.'06 *Nucl.Phys.B* 753 (2006) 139-177

3. RadCors to $\tau \to P\nu_\tau[\gamma]$ decays ($P=\pi$, K)

- ✓ Calculated within an effective approach encoding the hadronization:
  - ✓ Large-N_C expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies*.
- ✓ We follow a similar notation to $P\to\mu\nu_\mu[\gamma]$ (D=d,s for π,K and $F_\pi \approx 92.2$ MeV):

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

LO result; short-distance EW correction ≈ 1.0232**; structure independent (SI) contributions (point-like approximation)***

$$\Gamma(\tau\to P\nu_\tau[\gamma]) = \frac{G_F^2|V_{uD}|^2F_P^2}{8\pi}M_\tau^3\left(1-\frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW}\left\{1+\frac{\alpha}{\pi}G(m_P^2/M_\tau^2)\right\}\times$$
$$\left\{1-\frac{3\alpha}{2\pi}\log\frac{m_\rho}{M_\tau}+\delta_{\tau P}\big|_{\rm rSD}+\delta_{\tau P}\big|_{\rm vSD}\right\}$$

real-photon structure-dependent (rSD) contributions; virtual-photon structure-dependent (vSD) contributions

- ✓ Real-photon structure-dependent (rSD) contributions from Guo & Roig'10^.
- ✓ Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

* Ecker et al.'89 *Nucl.Phys.B* 321 (1989) 311-342
* Cirigliano et al.'06 *Nucl.Phys.B* 753 (2006) 139-177

*** Kinoshita'59 *Phys.Rev.Lett.* 2 (1959) 477
^ Guo & Roig'10 *Phys.Rev.D* 82 (2010) 113016

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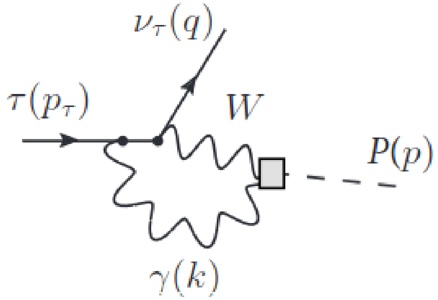
✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau\to P\nu_\tau]|_{\rm SD}=G_F V_{uD} e^2\int\frac{{\rm d}^d k}{(2\pi)^d}\frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]}\left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2)+F_A^P(W^2,k^2)\lambda_{1\mu\nu}+2B(k^2)\lambda_{2\mu\nu}\right]$$

$$\begin{aligned}\ell^{\mu\nu} &= \bar{u}(q)\gamma^\mu(1-\gamma_5)[(\not p_\tau+\not k)+M_\tau]\gamma^\nu u(p_\tau)\\ \lambda_{1\mu\nu} &= \left[(p+k)^2+k^2-m_P^2\right]g_{\mu\nu}-2k_\mu p_\nu\\ \lambda_{2\mu\nu} &= k^2 g_{\mu\nu}-\frac{k^2(p+k)_\mu p_\nu}{(p+k)^2-m_P^2}\end{aligned}$$



✓ Form factors from Guo & Roig'10 and Guevara et al.'13*:

$$\begin{aligned}F_V^P(W^2,k^2) &= \frac{-N_C M_V^4}{24\pi^2 F_P(k^2-M_V^2)(W^2-M_V^2)}\\ F_A^P(W^2,k^2) &= \frac{F_P}{2}\frac{M_A^2-2M_V^2-k^2}{(M_V^2-k^2)(M_A^2-W^2)}\\ B(k^2) &= \frac{F_P}{M_V^2-k^2}\end{aligned}$$

✓ Well-behaved two- and three-point Green functions.

✓ Chiral and U(3) limits.

✓ M_V and M_A vector- and axial-vector resonance mass: $M_V=M_\rho$ and $M_A=M_{a1}$ (π case); $M_V=M_{K^*}$ and $M_A\approx M_{f1}$ (K case).

Phys.Rev.D 82 (2010) 113016

Phys.Rev.D 88 (2013) 3, 033007
Phys.Rev.D 105 (2022) 7, 076007

* Guo & Roig'10
* Guevara et al.'13

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Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

- ✓ Virtual-photon structure-dependent contribution (vSD):

$$i\mathcal{M}[\tau \to P\nu_\tau]|_{\rm SD} = G_F V_{uD} e^2 \int \frac{d^dk}{(2\pi)^d} \frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\tau^2]} \left[i\epsilon_{\mu\nu\lambda\rho}k^\lambda p^\rho F_V^P(W^2,k^2) + F_A^P(W^2,k^2)\lambda_{1\mu\nu} + 2B(k^2)\lambda_{2\mu\nu}\right]$$

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\ell^{\mu\nu} &= \bar{u}(q)\gamma^\mu(1-\gamma_5)[(\not p_\tau + \not k) + M_\tau]\gamma^\nu u(p_\tau) \\
\lambda_{1\mu\nu} &= \left[(p+k)^2 + k^2 - m_P^2\right] g_{\mu\nu} - 2k_\mu p_\nu \\
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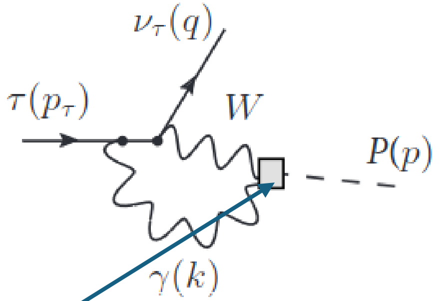

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F_A^P(W^2,k^2) &= \frac{F_P}{2}\frac{M_A^2 - 2M_V^2 - k^2}{(M_V^2 - k^2)(M_A^2 - W^2)} \\
B(k^2) &= \frac{F_P}{M_V^2 - k^2}
\end{aligned}$$

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- ✓ Chiral and U(3) limits.
- ✓ M_V and M_A vector- and axial-vector resonance mass: $M_V=M_\rho$ and $M_A=M_{a1}$ (π case); $M_V=M_{K^*}$ and $M_A \approx M_{f1}$ (K case).

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Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

1. Structure-independent contribution (point-like approximation): SI.

- ✓ We confirm the results by DF'95*.

$$\delta R_{\tau/P}\big|_{\rm SI} = \frac{\alpha}{2\pi}\left\{\frac{3}{2}\log\frac{M_\tau^2 m_P^2}{m_\mu^4} + \frac{3}{2} + g\left(\frac{m_P^2}{M_\tau^2}\right) - f\left(\frac{m_\mu^2}{m_P^2}\right)\right\}$$

$$f(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(8-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) - \frac{x}{1-x}\left(\frac{3}{2} + \frac{4}{3}\pi^2\right)$$

$$g(x) = 2\left(\frac{1+x}{1-x}\log x - 2\right)\log(1-x) - \frac{x(2-5x)}{2(1-x)^2}\log x + 4\frac{1+x}{1-x}{\rm Li}_2(x) + \frac{x}{1-x}\left(\frac{3}{2} - \frac{4}{3}\pi^2\right)$$

$\delta R_{\tau/\pi}|_{SI}$ = 1.05% and $\delta R_{\tau/K}|_{SI}$ = 1.67%

2. Real-photon structure-dependent contribution: rSD.

- ✓ $\delta_{P\mu}|_{rSD}$ from Cirigliano & IR'07**: $\delta_{\pi\mu}|_{rSD} = -1.3\cdot 10^{-8}$ and $\delta_{K\mu}|_{rSD} = -1.7\cdot 10^{-5}$.
- ✓ $\delta_{\tau P}|_{rSD}$ from Guo & Roig'10***: $\delta_{\tau\pi}|_{rSD}$ = 0.15% and $\delta_{\tau K}|_{rSD}$ = (0.18 ± 0.05)%.

$\delta R_{\tau/\pi}|_{rSD}$ = 0.15% and $\delta R_{\tau/K}|_{rSD}$ = (0.18 ± 0.15)%

* Decker & Finkemeier'95 *Nucl.Phys.B* 438 (1995) 17-53

** Cirigliano & Rosell '07 *Phys.Rev.Lett.* 99 (2007) 231801 *JHEP* 10 (2007) 005

*** Guo & Roig'10 *Phys.Rev.D* 82 (2010) 113016

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Arroyo-Ureña et al., *Phys.Rev.D* 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

3. Virtual-photon structure-dependent contribution: vSD.

- ✓ $\delta_{P\mu}|_{vSD}$ from **Cirigliano & Rosell '07***: $\delta_{\pi\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.
- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$

*** Cirigliano & Rosell '07:** *Phys.Rev.Lett.* 99 (2007) 231801 *JHEP* 10 (2007) 005

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- ✓ $\delta_{\tau P}|_{vSD}$, **new calculation**: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$

- ✓ Uncertainties dominated by $\delta_{\tau P}|_{vSD}$:
  - ✓ P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas τ decays within resonance effective approach [no matching to determine the counterterms].
  - ✓ Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: ±0.22% and ±0.24% for the pion and the kaon case.
  - ✓ Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: ±0.52% (similar procedure in Marciano & Sirlin'93). **Conservative estimate**, since vSD counterterms affecting in P decays imply similar corrections to our estimation of the vSD counterterms in τ decays.

*** Cirigliano & Rosell '07:**

Phys.Rev.Lett. 99 (2007) 231801

JHEP 10 (2007) 005

5. Results

Arroyo-Ureña et al., *Phys.Rev.D* 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

Contribution	$\delta R_{\tau/\pi}$	$\delta R_{\tau/K}$	Ref.
SI	$+1.05\%$	$+1.67\%$	*
rSD	$+0.15\%$	$+(0.18 \pm 0.05)\%$	**
vSD	$-(1.02 \pm 0.57)\%$	$-(0.88 \pm 0.58)\%$	new
Total	$+(0.18 \pm 0.57)\%$	$+(0.97 \pm 0.58)\%$	new

Errors are not reported if they are lower than 0.01%.

- ✓ Central values agree remarkably with DF'95, merely a coincidence: $\delta R_{\tau/\pi} = (0.16 \pm 0.14)\%$ and $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$, **but** in that work:
  - ✓ problematic hadronization: form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
  - ✓ a cutoff to regulate the loop integrals, splitting unphysically long- and short-distance regimes.
  - ✓ unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

*** Decker & Finkemeier'95** *Nucl.Phys.B* 438 (1995) 17-53
**** Cirigliano & Rosell'07** *Phys.Rev.Lett.* 99 (2007) 231801 *JHEP* 10 (2007) 005
***** Guo & Roig'10** *Phys.Rev.D* 82 (2010) 113016

6. Application I: Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$

Arroyo-Ureña et al., *Phys.Rev.D* 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

short-distance EW correction ≈ 1.0232*

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |V_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} (1 + \delta_{\tau P})$$

✓ $\delta_{\tau P}$ includes SI and SD radiative corrections.

$$\delta_{\tau P} = \frac{\alpha}{2\pi}\left(g\left(\frac{m_P^2}{M_\tau^2}\right) + \frac{19}{4} - \frac{2\pi^2}{3} - 3\log\frac{m_\rho}{M_\tau}\right) + \delta_{\tau P}\big|_{\rm rSD} + \delta_{\tau P}\big|_{\rm vSD} = \begin{cases} \delta_{\tau\pi} = (-0.24 \pm 0.56)\% \\ \delta_{\tau K} = (-0.15 \pm 0.57)\% \end{cases}$$

Phys.Rev.Lett. 71 (1993) 3629-3632

* Marciano & Sirlin'93

6. Application II: lepton universality test

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}\left(1+\delta R_{\tau/P}\right)$$

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\text{th}} \pm 0.0025_{\text{exp}} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\text{th}} \pm 0.0072_{\text{exp}} = 0.9857 \pm 0.0078$$

6. Application II: lepton universality test

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1-m_P^2/M_\tau^2)^2}{(1-m_\mu^2/m_P^2)^2}(1+\delta R_{\tau/P})$$

PDG

$\delta R_{\tau/\pi}$ = (0.18 ± 0.57)%
$\delta R_{\tau/K}$ = (0.97 ± 0.58)%

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0028_{\rm th} \pm 0.0025_{\rm exp} = 0.9964 \pm 0.0038$$

$$\left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0028_{\rm th} \pm 0.0072_{\rm exp} = 0.9857 \pm 0.0078$$

- ✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95**
- ✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95**

* HFLAV'21 — *Eur.Phys.J.C* 81 (2021) 3, 226
** Decker & Finkemeier'95 — *Nucl.Phys.B* 438 (1995) 17-53

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

FLAG'20*:
$F_K/F_\pi = 1.1932 \pm 0.0019$

PDG

$$\delta \;=\; \delta_{\tau K} - \delta_{\tau\pi} = +(0.10 \pm 0.80)\%$$

$$\frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2}(1+\delta)$$

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2288 \pm 0.0010_{\rm th} \pm 0.0017_{\rm exp} = 0.2288 \pm 0.0020$$

- ✓ 2.1σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$**.
- ✓ To be compared with $|V_{us}/V_{ud}|=0.2291\pm0.0009$***, obtained with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

Eur.Phys.J.C 80 (2020) 2, 113 * FLAG'20
Phys.Rev.C 102 (2020) 4, 045501** Hardy & Towner'20
Phys.Rev.D 105 (2022) 1, 013005*** Seng et al.'21

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi}|V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\rm EW}\ (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\rm th} \pm 0.0016_{\rm exp} = 0.2220 \pm 0.0018$$

6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

PDG

FLAG'20*: $\sqrt{2}F_K = (155.7 \pm 0.3)$ MeV

short-distance EW correction ≈ 1.0232**

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

$$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi} |V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{\mathrm{EW}} (1 + \delta_{\tau K})$$

$$|V_{us}| = 0.2220 \pm 0.0008_{\mathrm{th}} \pm 0.0016_{\mathrm{exp}} = 0.2220 \pm 0.0018$$

- ✓ 2.6σ away from CKM unitarity, considering $|V_{ud}|=0.97373\pm0.00031$***.
- ✓ To be compared with $|V_{us}|=0.2234\pm0.0015$^ or $|V_{us}|=0.2231\pm0.0006$†, obtained this last one with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in τ decays.

EPJ C 80 (2020) 2, 113
PRL 71 (1993) 3629-3632
PRC 102 (2020) 4, 045501
EPJ C 81 (2021) 3, 226
PRD 105 (2022) 1, 013005

* FLAG'20
** Marciano & Sirlin'93
*** Hardy & Towner'20
^ HFLAV'21
† Seng et al.'21

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\rm EW} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$

Values of $\Delta^{\tau P}$ reported in the $\overline{\rm MS}$-scheme and at a scale of μ=2 GeV.

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)} \epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{cases}$$

6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

$|V_{ud}|$ = 0.97373 ± 0.00031*
$|V_{us}/V_{ud}|$ = 0.2288 ± 0.0020

FLAG'20*:
$\sqrt{2}F_\pi$ = (130.2 ± 0.8) MeV
$\sqrt{2}F_K$ = (155.7 ± 0.3) MeV

short-distance EW correction ≈ 1.0232**

$$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$$
$$\delta_{\tau K} = (-0.15 \pm 0.57)\%$$

PDG

$$\Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\widetilde{V}_{uD}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{\mathrm{EW}} \left(1 + \delta_{\tau P} + 2\Delta^{\tau P}\right)$$

Values of $\Delta^{\tau P}$ reported in the $\overline{\mathrm{MS}}$-scheme and at a scale of μ=2 GeV.

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)}\epsilon_P^\tau = \begin{cases} \Delta^{\tau\pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \Delta^{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{cases}$$

- ✓ To be compared with $\Delta^{\tau\pi} = -(0.15 \pm 0.67)\cdot 10^{-2}$ of Cirigliano et al.'19^.
- ✓ To be compared with $\Delta^{\tau\pi} = -(0.12 \pm 0.68)\cdot 10^{-2}$ and $\Delta^{\tau K} = (-0.41 \pm 0.93)\cdot 10^{-2}$ of González-Solís et al.'20†.

PRC 102 (2020) 4, 045501
EPJ C 80 (2020) 2, 113
PRL 71 (1993) 3629-3632

* Hardy & Towner'20
** FLAG'20
*** Marciano & Sirlin'93

Phys.Rev.Lett. 122 (2019) 22, 221801
^ Cirigliano et al.'19 , '22 — *JHEP* 04 (2022) 152
† Gonzàlez-Solís et al. '20 — *Phys.Lett.B* 804 (2020) 135371

7. Conclusions

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

- ✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right) \longrightarrow \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

- ✓ Framework: ChPT for π decays and a resonance extension of ChPT for τ decays.
- ✓ Consistent with DF'95*, but with more robust assumptions and yielding a reliable uncertainty.
- ✓ Applications:
  - ✓ Theoretical determination of radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])/\Gamma(\tau\to\pi\nu_\tau[\gamma])$: $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$, at 2.1σ from unitarity.
  - ✓ CKM unitarity in $\Gamma(\tau\to K\nu_\tau[\gamma])$: $|V_{us}| = 0.2220 \pm 0.0018$, at 2.6σ from unitarity.
  - ✓ Constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$: update of $\Delta^{\tau P}$.
- ✓ Our results have been incorporated in the very recent **HFLAV'22**. *Phys.Rev.D* 107 (2023) 5, 052008

* Decker & Finkemeier'95 — *Nucl.Phys.B* 438 (1995) 17-53
** HFLAV'21 — *Eur.Phys.J.C* 81 (2021) 3, 226

7. Conclusions

Arroyo-Ureña et al., Phys.Rev.D 104 (2021) 9, L091502 & *JHEP* 02 (2022) 173

✓ The observable and **our result**:

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 R^{(0)}_{\tau/P}\left(1+\delta R_{\tau/P}\right) \longrightarrow \begin{cases} \delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\ \delta R_{\tau/K} = (0.97 \pm 0.58)\% \end{cases}$$

$$\delta_{\rm EM}^{K^-\pi^0} = -\left(0.009^{+0.010}_{-0.118}\right)\%$$
$$\delta_{\rm EM}^{\bar{K}^0\pi^-} = -\left(0.166^{+0.100}_{-0.157}\right)\%$$
$$\delta_{\rm EM}^{K^-K^0} = -\left(0.030^{+0.032}_{-0.180}\right)\%$$
$$\delta_{\rm EM}^{\pi^-\pi^0} = -\left(0.186^{+0.114}_{-0.203}\right)\%$$

$$\delta_{\rm EM}^{K^-\eta} = -\left(0.026^{+0.029}_{-0.163}\right)\% \qquad \delta_{\rm EM}^{K^-\eta'} = -\left(0.304^{+0.422}_{-0.185}\right)\%$$

Miranda-**Roig** *Phys.Rev.D* 102 (2020) 114017,
Escribano-Miranda-**Roig** *Phys.Rev.D* 109 (2024) 5, 053003

We have halved the uncertainty!

✓ Framework: ChPT f … of ChPT for τ decays.

✓ Consistent with DF' …

✓ Applications:

- ✓ Theoretical de … (τ → Pν_τ[γ]).
- ✓ $|g_\tau/g_\mu|_P$ at 0.9σ (π) and 1.8σ (K) of LU, reducing HFLAV'21** disagreement with LU.
- ✓ CKM unitarity in Γ(τ→Kν_τ[γ])/Γ(τ→πν_τ[γ]): $|V_{us}/V_{ud}|$ = 0.2288 ± 0.0020, at 2.1σ from unitarity.
- ✓ CKM unitarity in Γ(τ→Kν_τ[γ]): $|V_{us}|$ = 0.2220 ± 0.0018, at 2.6σ from unitarity.
- ✓ Constraining non-standard interactions in Γ(τ → Pν_τ[γ]): update of $\Delta^{\tau P}$.

✓ Our results have been incorporated in the very recent **HFLAV'22**. *Phys.Rev.D* 107 (2023) 5, 052008

* Decker & Finkemeier'95 — *Nucl.Phys.B* 438 (1995) 17-53
** HFLAV'21 — *Eur.Phys.J.C* 81 (2021) 3, 226