Radiative corrections to one- and two-meson tau decays for precise new physics tests

Pablo Roig Garcés
Cinvestav, Mexico City, Mexico

INTRODUCTION

• The $\tau$ is the only lepton massive enough to decay into hadrons, thus offering a clean environment to learn about hadronization of QCD currents

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
INTRODUCTION

• The $\tau$ is the only lepton massive enough to decay into hadrons, thus offering a clean environment to learn about hadronization of QCD currents & make stringent new physics tests. [Pich, '14, Prog.Part.Nucl.Phys. 75 (2014) 41-85]

RadCors for semileptonic tau decays and NP tests

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INTRODUCTION

- The $\tau$ is the only lepton massive enough to decay into hadrons, thus offering a clean environment to learn about hadronization of QCD currents & make stringent new physics tests. Before our last paper...

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<tr>
<td>$\pi^-$</td>
<td>0.5%</td>
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<td>LFU, NP</td>
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Short-Distance corrections: Sirlin '78; Marciano-Sirlin '93
Rev.Mod.Phys. 50 (1978) 573 & 905 (erratum)

RadCors for semileptonic tau decays and NP tests

Decker and Fikemeier '95, Arroyo-Ureña et al '21
Cirigliano et al '01, Flores-Talpa et al '06, Miranda and Roig '20
Antonelli et al '13, Flores-Baéz and Morones-Ibarra '13

Pablo Roig (Cinvestav, Mexico City)
The $\tau$ is the only lepton massive enough to decay into hadrons, thus offering a clean environment to learn about hadronization of QCD currents & make stringent new physics tests. Updated (Escribano-Miranda-Roig *Phys.Rev.D* 109 (2024) 5, 053003 = This work)

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RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
INTRODUCTION

• Electromagnetic radiative corrections require the inclusion of diagrams with both virtual (loops) & real photons (ISR & FSR).

• I will illustrate this with the RadCors to the one $\pi$ (or K) tau decays. The two-meson cases can be studied similarly. *They are just more complicated...*
1. RadCors to one-meson tau decays (Motivation)

\[ \frac{R_{\tau/P}}{P} \equiv \frac{\Gamma(\tau \to P\nu_{\tau}[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} \]

Tests LU \((g_\tau=g_\mu)\) using \(P=\pi,K\)

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
1. RadCors to one-meson tau decays (Motivation)

\[ R_{\tau/P} \equiv \frac{\Gamma(\tau \rightarrow PV_{\tau}[\gamma])}{\Gamma(P \rightarrow \mu\nu_{\mu}[\gamma])} \]

Tests LU \( (g_{\tau}=g_{\mu}) \) using \( P=\pi, K \)

I will consider both decays in turn, \( R_{\tau/P} \), results & applications. Among the conclusions, I will show results for 2 mesons.
1. RadCors to one-meson tau decays (Motivation)

- **Lepton Universality** (LU) as a basic tenet of the Standard Model (SM).
- A few **anomalies** observed in semileptonic B meson decays*.
- Lower energy observables currently provide the most precise test of LU**.

- We aim to test **muon-tau lepton universality** through the ratio \( (P = \pi, K)***: 

\[
R_{\tau/P} = \frac{\Gamma(\tau \to P\nu\tau[\gamma])}{\Gamma(P \to \mu\nu\mu[\gamma])} = \left| \frac{g_\tau}{g_\mu} \right|^2 \frac{R_{\tau/P}^{(0)}}{R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P})} 
\]

- \( g_\tau = g_\mu \) according to LU.
- \( R_{\tau/P}^{(0)} \) is the LO result 

\[
R_{\tau/P}^{(0)} = \frac{1}{2} \frac{M_\tau^3}{m_\tau^2 m_P} \frac{1 - m_P^2/M_\tau^2}{(1 - m_\mu^2/m_P^2)^2} 
\]

- \( \delta R_{\tau/P} \) encodes the radiative corrections.

- \( \delta R_{\tau/P} \) was calculated by Decker & Finkemeier (DF'95) ^:

\[
\delta R_{\tau/\pi} = (0.16 \pm 0.14)\% \quad \text{and} \quad \delta R_{\tau/K} = (0.90 \pm 0.22)\%. 
\]

- Important **phenomenological and theoretical reasons** to address the analysis again.

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* Albrecht et al. '21  
** Bryman et al. '21  
*** Marciano & Sirlin '93  
^ Decker & Finkemeier '95

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
1. RadCors to one-meson tau decays (Motivation)

✓ Phenomenological disagreement in LU tests:

✓ Using $\frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])}$ and DF'95*, HFLAV** reported:

✓ $|g_\tau/g_\mu|_\pi = 0.9958 \pm 0.0026$ (at 1.6\(\sigma\) of LU)
✓ $|g_\tau/g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9\(\sigma\) of LU)

✓ Using $\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma])}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])}$, HFLAV** reported:

✓ $|g_\tau/g_\mu| = 1.0010 \pm 0.0014$ (at 0.7\(\sigma\) of LU)

✓ Using $\frac{\Gamma(W \to \tau\nu_\tau)}{\Gamma(W \to \mu\nu_\mu)}$, CMS and ATLAS*** and reported:

✓ $|g_\tau/g_\mu| = 0.995 \pm 0.006$ (at 0.8\(\sigma\) of LU)

---


RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
1. RadCors to one-meson tau decays (Motivation)

- Phenomenological disagreement in LU tests:
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- Theoretical issues within DF’95*:
  - Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
  - A cutoff to regulate the loop integrals (separating long- and short-distance corrections)
  - Unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).

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RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
1. RadCors to one-meson tau decays (Motivation)

- Phenomenological disagreement in LU tests:
  - Using $\Gamma(\tau \to P\nu_\tau[\gamma]) / \Gamma(P \to \mu\nu_\mu[\gamma])$, and DF'95*, HFLAV** reported:
    - $|g_\tau / g_\mu|_n = 0.9958 \pm 0.0026$ (at 1.6σ of LU)
    - $|g_\tau / g_\mu|_K = 0.9879 \pm 0.0063$ (at 1.9σ of LU)
  - Using $\Gamma(\tau \to e\bar{\nu}_e\nu_\tau[\gamma]) / \Gamma(\mu \to e\bar{\nu}_e\nu_\mu[\gamma])$, HFLAV** reported:
    - $|g_\tau / g_\mu| = 1.0010 \pm 0.0014$ (at 0.7σ of LU)
  - Using $\Gamma(W \to \tau\bar{\nu}_\tau) / \Gamma(W \to \mu\bar{\nu}_\mu)$, CMS and ATLAS*** and reported:
    - $|g_\tau / g_\mu| = 0.995 \pm 0.006$ (at 0.8σ of LU)

- Theoretical issues within DF'95*:
  - Hadronic form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.
  - A cutoff to regulate the loop integrals (separating long- and short-distance corrections).

- By-products of the project:
  - Radiative corrections in $\Gamma(\tau \to P\nu_\tau[\gamma])$.
  - CKM unitarity test via $\Gamma(\tau \to K\nu_\tau[\gamma])$ or via the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$.
  - Constraints on possible non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$.

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* Decker & Finkemeier '95
** HFLAV'21
*** CMS'21, ATLAS'21

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RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
2. RadCors to $P_{[^2\gamma]}$ decays ($P=\pi, K$)

- Calculated unambiguously within the Standard Model (Chiral Perturbation Theory, ChPT*).
- Notation by Marciano & Sirlin** and numbers by Cirigliano and Rosell*** (D=d,s for $\pi, K$ and $F_\pi = 92.2$ MeV):

$$\Gamma(P \to \mu\nu_\mu[\gamma]) = \frac{G_F^2 |V_{ud}|^2 F_0^2}{4\pi} m_P m_\mu \left(1 - \frac{m_\mu}{m_P} \right)^2 S_{EW} \left\{ 1 + \frac{\alpha}{\pi} F \left( \frac{m_\mu^2}{m_P^2} \right) \right\} \times \left\{ 1 - \frac{\alpha}{\pi} \left[ \frac{3}{2} \log \frac{m_\mu}{m_P} + c_1^{(P)} + \frac{m_\mu^2}{m_P^2} c_2^{(P)} \log \frac{m_\mu^2}{m_P^2} + c_3^{(P)} + c_4^{(P)} (m_\mu/m_P) - \frac{m_\mu^2}{m_P^2} c_2^{(P)} \log \frac{m_\mu^2}{m_P^2} \right] \right\}$$

- The only model-dependence is the determination of the counterterms in $c_1^{(P)}$ and $c_3^{(P)}$:

- Large-$N_C$ expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies†.

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* Weinberg '79
** Marciano & Sirlin '93
*** Cirigliano & Rosell '07
* Gasser & Leutwyler '84 '85
† Ecker et al. '89
†† Kinoshita '59

Pablo Roig (Cinvestav, Mexico City)
3. RadCors to $\tau \rightarrow P\nu_\tau[\gamma]$ decays ($P=\pi, K$)

- Calculated within an effective approach encoding the hadronization:

- Large-$N_C$ expansion of QCD: ChPT is enlarged by including the lightest multiplets of spin-one resonances such that the relevant Green functions are well-behaved at high energies.*

- We follow a similar notation to $P \rightarrow \mu\nu_\mu[\gamma]$ ($D=d,s$ for $\pi,K$ and $F_\pi = 92.2$ MeV):

\[
\Gamma(\tau \rightarrow P\nu_\tau[\gamma]) = \frac{G_F^2 |V_{ud}|^2 F_2^2}{8\pi} M^4_\tau \left(1 - \frac{m^2_P}{M^2_\tau}\right)^2 S_{EW} \left\{1 + \frac{\alpha}{\pi} G(m^2_P/M^2_\tau)\right\} \times \left\{1 - \frac{3\alpha}{2\pi} \log \frac{m_\rho}{M_\tau} + \delta_{\tau P}|_{rSD} + \delta_{\tau P}|_{vSD}\right\}
\]

- Real-photon structure-dependent (rSD) contributions from Guo & Roig'10^

- Virtual-photon structure-dependent (vSD) contributions not calculated in the literature.

References:
- Kinoshita’59
- Guo & Roig’10
- Phys. Rev. Lett. 2 (1959) 477
- Pablo Roig (Cinvestav, Mexico City)
3. RadCors to $\tau \rightarrow P\nu_\tau[\gamma]$ decays ($P=\pi$, $K$)

Virtual-photon structure-dependent contribution (vSD):

\[
i\mathcal{M}[\tau \rightarrow P\nu_\tau]_{\text{SD}} = G_F V_{u\mu} e^2 \int \frac{d^4k}{(2\pi)^4} \frac{\ell^{\mu\nu}}{k^2[(p_\tau+k)^2-M_\gamma^2]} [ie_{\mu\nu}\lambda_\rho k^\rho F_\nu^P(W^2, k^2) + F_A^P(W^2, k^2)\lambda_{1\mu\nu} + 2B(k^2)\lambda_{2\mu\nu}]
\]

\[
\ell^{\mu\nu} = \bar{u}(q)\gamma^\mu(1-\gamma_5)[(p_\tau+k) + M_\gamma]\gamma^\nu u(p_\tau)
\]

\[
\lambda_{1\mu\nu} = \left[\frac{(p+k)^2 + k^2 - m_P^2}{p^2 - m_P^2}\right] g_{\mu\nu} - 2k_\mu p_\nu
\]

\[
\lambda_{2\mu\nu} = k^2 g_{\mu\nu} - \frac{k^2 p_\mu p_\nu}{(p+k)^2 - m_P^2}
\]

Form factors from Guo & Roig'10 and Guevara et al.'13:

\[
F_\nu^P(W^2, k^2) = \frac{-N_C M_V^4}{24\pi^2 F_P(k^2 - M_V^2)(W^2 - M_\gamma^2)}
\]

\[
F_A^P(W^2, k^2) = \frac{F_P M_V^2 - 2M_\gamma^2 - k^2}{2(M_\gamma^2 - W^2)(M_A^2 - W^2)}
\]

\[
B(k^2) = \frac{F_P}{M_\gamma^2 - k^2}
\]

Well-behaved two- and three-point Green functions.

Chiral and U(3) limits.

$M_V$ and $M_A$ vector- and axial-vector resonance mass: $M_V=M_p$ and $M_A=M_{a1}$ ($\pi$ case); $M_V=M_K^*$ and $M_A=M_{\eta_1}$ ($K$ case).


RadCors for semileptonic tau decays and NP tests 

Pablo Roig (Cinvestav, Mexico City)
3. RadCors to $\tau \rightarrow P\nu_\tau[\gamma]$ decays ($P=$π, K)

Virtual-photon structure-dependent contribution (vSD):

$$iM[\tau \rightarrow P\nu_\tau]|_{SD} = G_{F}V_{ud}e^{2}\int \frac{d^{d}k}{(2\pi)^{d}} \frac{\ell^{\mu\nu}}{(p_{r}+k)^{2} - M_{Z}^{2}} \left[ i\epsilon_{\mu\nu\lambda\rho} k^{\lambda} p^{\rho} F^{P}_{1}(W^{2}, k^{2}) + F^{P}_{A}(W^{2}, k^{2}) \lambda_{1\mu\nu} + 2B(k^{2}) \lambda_{2\mu\nu} \right]$$

Form factors from Guo & Roig'10 and Guevara et al.'13:

$$F^{P}_{1}(W^{2}, k^{2}) = \frac{-N_{C}M_{V}^{4}}{24\pi^{2}F_{P}(k^{2} - M_{V}^{2})(W^{2} - M_{V}^{2})}$$

$$F^{P}_{A}(W^{2}, k^{2}) = \frac{F_{P}}{2} \frac{M_{A}^{2} - 2M_{V}^{2} - k^{2}}{(M_{V}^{2} - k^{2})(M_{A}^{2} - W^{2})}$$

Well-behaved two- and three-point Green functions.

Chiral and U(3) limits.

$M_{V}$ and $M_{A}$ vector- and axial-vector resonance mass: $M_{V} = M_{p}$ and $M_{A} = M_{a1}$ ($\pi$ case); $M_{V} = M_{K^{*}}$ and $M_{A} = M_{11}$ ($K$ case).

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

1. Structure-independent contribution (point-like approximation): SI.

- We confirm the results by DF‘95*.

$$\delta R_{\tau/P}\big|_{SI} = \frac{\alpha}{2\pi} \left\{ \frac{3}{2} \log\frac{M_{\tau}^2 m_{P}\mu^2}{m_{\mu}^4} + \frac{3}{2} + g\left(\frac{m_{P}^2}{M_{\tau}^2}\right) - f\left(\frac{m_{\mu}^2}{m_{P}^2}\right) \right\}$$

$$f(x) = 2 \left(\frac{1 + x}{1 - x} \log x - 2\right) \log(1 - x) - \frac{x(8 - 5x)}{2(1 - x)^2} \log x + 4 \frac{1 + x}{1 - x} \text{Li}_2(x) - \frac{x}{1 - x} \left(\frac{3}{2} + \frac{4}{3} \pi^2\right)$$

$$g(x) = 2 \left(\frac{1 + x}{1 - x} \log x - 2\right) \log(1 - x) - \frac{x(2 - 5x)}{2(1 - x)^2} \log x + 4 \frac{1 + x}{1 - x} \text{Li}_2(x) + \frac{x}{1 - x} \left(\frac{3}{2} - \frac{4}{3} \pi^2\right)$$

- $\delta R_{\tau/\pi}\big|_{SI} = 1.05\%$ and $\delta R_{\tau/K}\big|_{SI} = 1.67\%$

2. Real-photon structure-dependent contribution: rSD.

- $\delta_{P\mu}\big|_{rSD}$ from Cirigliano & IR’07**: $\delta_{\tau\mu}\big|_{rSD} = -1.3 \cdot 10^{-8}$ and $\delta_{K\mu}\big|_{rSD} = -1.7 \cdot 10^{-5}$.

- $\delta_{\tau P}\big|_{rSD}$ from Guo & Roig‘10***: $\delta_{\tau\pi}\big|_{rSD} = 0.15\%$ and $\delta_{\tau K}\big|_{rSD} = (0.18 \pm 0.05)\%$.

- $\delta R_{\tau/\pi}\big|_{rSD} = 0.15\%$ and $\delta R_{\tau/K}\big|_{rSD} = (0.18 \pm 0.15)\%$

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* Decker & Finkemeier‘95

** Cirigliano & Rosell ‘07

*** Guo & Roig‘10


RadCors for semileptonic tau decays and NP tests

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4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tauP} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- $\delta_{P\mu}|_{vSD}$ from Cirigliano & Rosell '07*: $\delta_{\mu\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.

- $\delta_{\tauP}|_{vSD}$, new calculation: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tauK}|_{vSD} = (-0.45 \pm 0.57)\%$.

$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\%$ and $\delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$

4. Calculation of $R_{\tau/P} = R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) = R_{\tau/P}^{(0)} (1 + \delta_{\tau P} - \delta_{P\mu})$

3. Virtual-photon structure-dependent contribution: vSD.

- $\delta_{P\mu}|_{vSD}$ from Cirigliano & Rosell '07: $\delta_{P\mu}|_{vSD} = (0.54 \pm 0.12)\%$ and $\delta_{K\mu}|_{vSD} = (0.43 \pm 0.12)\%$.

- $\delta_{\tau P}|_{vSD}$, new calculation: $\delta_{\tau\pi}|_{vSD} = (-0.48 \pm 0.56)\%$ and $\delta_{\tau K}|_{vSD} = (-0.45 \pm 0.57)\%$.

$$\delta R_{\tau/\pi}|_{vSD} = (-1.02 \pm 0.57)\% \text{ and } \delta R_{\tau/K}|_{vSD} = (-0.88 \pm 0.58)\%$$

- Uncertainties dominated by $\delta_{\tau P}|_{vSD}$:
  - P decays within ChPT [counterterms can be determined by matching ChPT with the resonance effective approach at higher energies], whereas $\tau$ decays within resonance effective approach [no matching to determine the counterterms].
  - Estimation of the model-dependence by comparing our results with a less general scenario where only well-behaved two-point Green functions and a reduced resonance Lagrangian is used: $\pm0.22\%$ and $\pm0.24\%$ for the pion and the kaon case.
  - Estimation of the counterterms by considering the running between 0.5 and 1.0 GeV: $\pm0.52\%$ (similar procedure in Marciano & Sirlin'93). Conservative estimate, since vSD counterterms affecting in P decays imply similar corrections to our estimation of the vSD counterterms in $\tau$ decays.

JHEP 10 (2007) 005
5. Results

<table>
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<th>Contribution</th>
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<th>$\delta R_{\tau/K}$</th>
<th>Ref.</th>
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<tr>
<td>SI</td>
<td>+1.05%</td>
<td>+1.67%</td>
<td>*</td>
</tr>
<tr>
<td>rSD</td>
<td>+0.15%</td>
<td>+(0.18 ± 0.05)%</td>
<td>**</td>
</tr>
<tr>
<td>vSD</td>
<td>-(1.02 ± 0.57)%</td>
<td>-(0.88 ± 0.58)%</td>
<td>new</td>
</tr>
<tr>
<td>Total</td>
<td>+(0.18 ± 0.57)%</td>
<td>+(0.97 ± 0.58)%</td>
<td>new</td>
</tr>
</tbody>
</table>

Errors are not reported if they are lower than 0.01%.

✓ Central values agree remarkably with DF'95, merely a coincidence: $\delta R_{\tau/\pi} = (0.16 ± 0.14)\%$ and $\delta R_{\tau/K} = (0.90 ± 0.22)\%$, but in that work:

✓ problematic hadronization: form factors are different for real- and virtual-photon corrections, do not satisfy the correct QCD short-distance behavior, violate unitarity, analicity and the chiral limit at leading non-trivial orders.

✓ a cutoff to regulate the loop integrals, splitting unphysically long- and short-distance regimes.

✓ unrealistic uncertainties (purely $O(e^2p^2)$ ChPT size).


RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
6. Application I: Radiative corrections in $\Gamma(\tau \to P\nu_{\tau}[\gamma])$

$$
\Gamma(\tau \to P\nu_{\tau}[\gamma]) = \frac{G_F^2 |V_{ud}|^2 F_P^2}{8\pi} M_{\tau}^3 \left( 1 - \frac{m_p^2}{M_{\tau}^2} \right)^2 S_{\text{EW}} (1 + \delta_{\tau P})
$$

✓ $\delta_{\tau P}$ includes SI and SD radiative corrections.

$$
\delta_{\tau P} = \frac{\alpha}{2\pi} \left( g \frac{m_p^2}{M_{\tau}^2} + \frac{19}{4} - \frac{2\pi^2}{3} - 3 \log \frac{m_p}{M_{\tau}} \right) + \delta_{\tau P}|_{\text{SD}} + \delta_{\tau P}|_{\nu_{\text{SD}}} = \left\{ \begin{array}{c} \delta_{\tau \pi} = (-0.24 \pm 0.56)\% \\ \delta_{\tau K} = (-0.15 \pm 0.57)\% \end{array} \right.
$$
6. Application II: lepton universality test

\[
R_\tau / P \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left| \frac{g_\tau}{g_\mu} \right|^2 \frac{1}{2} \frac{M_\tau^3}{m_P^2 m_P} \frac{(1 - m_P^2 / m_\tau^2)^2}{(1 - m_\mu^2 / m_P^2)^2} (1 + \delta R_\tau / P)
\]

\[
\left| \frac{g_\tau}{g_\mu} \right|_{\pi} = 0.9964 \pm 0.0028_{\text{th}} \pm 0.0025_{\text{exp}} = 0.9964 \pm 0.0038
\]

\[
\left| \frac{g_\tau}{g_\mu} \right|_{K} = 0.9857 \pm 0.0028_{\text{th}} \pm 0.0072_{\text{exp}} = 0.9857 \pm 0.0078
\]

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
6. Application II: lepton universality test

\[ R_{\tau/P} \equiv \frac{\Gamma(\tau \rightarrow P \nu_\tau [\gamma])}{\Gamma(P \rightarrow \mu \nu_\mu [\gamma])} = \frac{|g_\tau|}{|g_\mu|} \left[ \frac{1}{2} \frac{M_\tau^3}{m_\mu^2 m_P} \frac{(1 - m_P^2/M_\tau^2)^2}{(1 - m_\mu^2/m_P^2)^2} \right] \left( 1 + \delta R_{\tau/P} \right) \]

\[ \delta R_{\pi K} = (0.18 \pm 0.57)\% \]
\[ \delta R_{\pi K} = (0.97 \pm 0.58)\% \]

| \[ \frac{|g_\tau|}{|g_\mu|_\pi} = 0.9964 \pm 0.0028_{\text{th}} \pm 0.0025_{\text{exp}} = 0.9964 \pm 0.0038 \] |
| \[ \frac{|g_\tau|}{|g_\mu|_K} = 0.9857 \pm 0.0028_{\text{th}} \pm 0.0072_{\text{exp}} = 0.9857 \pm 0.0078 \] |

✓ π case: at 0.9σ of LU vs. 1.6σ of LU in HFLAV'21* using DF'95**

✓ K case: at 1.8σ of LU vs. 1.9σ of LU in HFLAV'21* using DF'95**

* HFLAV’21
** Decker & Finkemeier’95


RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
6. Application III: CKM unitarity test in the ratio $\frac{\Gamma(\tau \to K_{\nu\tau}[\gamma])}{\Gamma(\tau \to \pi_{\nu\tau}[\gamma])}$

\[
\frac{\Gamma(\tau \to K_{\nu\tau}[\gamma])}{\Gamma(\tau \to \pi_{\nu\tau}[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K^2}{F_\pi^2} \frac{(1 - m_K^2/M_\tau^2)^2}{(1 - m_\pi^2/M_\tau^2)^2} (1 + \delta)
\]

\[
\frac{V_{us}}{V_{ud}} = 0.2288 \pm 0.0010_{th} \pm 0.0017_{exp} = 0.2288 \pm 0.0020
\]
6. Application III: CKM unitarity test in the ratio $\Gamma(\tau \to K\nu_\tau[\gamma]) / \Gamma(\tau \to \pi\nu_\tau[\gamma])$

\[ \frac{\Gamma(\tau \to K\nu_\tau[\gamma])}{\Gamma(\tau \to \pi\nu_\tau[\gamma])} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{F_K}{F_\pi} \frac{(1-m_K^2/M_\tau^2)^2}{(1-m_\pi^2/M_\tau^2)^2} (1+\delta) \]

FLAG'20*: $F_K/F_\pi = 1.1932 \pm 0.0019$

$\delta = \delta_{rK} - \delta_{r\pi} = +(0.10 \pm 0.80)\%$

$\frac{|V_{us}|}{|V_{ud}|} = 0.2288 \pm 0.0010_{th} \pm 0.0017_{exp} = 0.2288 \pm 0.0020$

✓ 2.1σ away from CKM unitarity, considering $|V_{ud}| = 0.9737 \pm 0.0031$**.

✓ To be compared with $|V_{us}/V_{ud}| = 0.2291 \pm 0.0009$***, obtained with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in $\tau$ decays.

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Pablo Roig (Cinvestav, Mexico City)
6. Application IV: CKM unitarity test in $\Gamma(\tau \to K\nu_\tau[\gamma])$

$\Gamma(\tau \to K\nu_\tau[\gamma]) = \frac{G_F^2 F_K^2}{8\pi} |V_{us}|^2 M_\tau^3 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 S_{EW} (1 + \delta_{\tau K})$

$|V_{us}| = 0.2220 \pm 0.0008_{\text{th}} \pm 0.0016_{\text{exp}} = 0.2220 \pm 0.0018$
6. Application IV: CKM unitarity test in $\Gamma(\tau \rightarrow K\nu_{\tau}[\gamma])$


$\sqrt{2} F_K = (155.7 \pm 0.3) \text{ MeV}$

$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

$|V_{us}| = 0.2220 \pm 0.0008_{\text{th}} \pm 0.0016_{\text{exp}} = 0.2220 \pm 0.0018$

- 2.6σ away from CKM unitarity, considering $|V_{ud}| = 0.97373 \pm 0.00031^{***}$.
- To be compared with $|V_{us}| = 0.2234 \pm 0.0015^*$ or $|V_{us}| = 0.2231 \pm 0.0006^+$, obtained this last one with kaon semileptonic decays. Our error does not reach this level due to lack of statistics in $\tau$ decays.

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
6. Application V: constraining non-standard interactions in $\Gamma(\tau \to P\nu_\tau[\gamma])$


\[ \Gamma(\tau \to P\nu_\tau[\gamma]) = \frac{G_F^2 |\tilde{V}_{ud}|^2 F_P^2}{8\pi} M_\tau^3 \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{EW} \left(1 + \delta_{\tau P} + 2\Delta_{\tau P}\right) \]

Values of $\Delta_{\tau P}$ reported in the \overline{MS}-scheme and at a scale of $\mu=2 \text{ GeV}$.

\[ \Delta_{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{M_\tau(m_u + m_D)} \epsilon_P^\tau = \begin{cases} \Delta_{\tau \pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \Delta_{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{cases} \]

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
6. Application V: constraining non-standard interactions in $\Gamma(\tau \rightarrow P \nu_\tau [\gamma])$

- $|V_{ud}| = 0.97373 \pm 0.0003^*$
- $|V_{us}/V_{ud}| = 0.2288 \pm 0.0020$

**FLAG’20**:
- $\sqrt{2} F_\pi = (130.2 \pm 0.8)$ MeV
- $\sqrt{2} F_K = (155.7 \pm 0.3)$ MeV

**short-distance EW correction**
- $\approx 1.0232^{**}$
- $\delta_{\tau \pi} = (-0.24 \pm 0.56)^\%$
- $\delta_{\tau K} = (-0.15 \pm 0.57)^\%$

$\Gamma(\tau \rightarrow P \nu_\tau [\gamma]) = \frac{G_F^2 |V_{ud}|^2 F_P^2}{8\pi} \frac{M_\tau^3}{2} \left(1 - \frac{m_P^2}{M_\tau^2}\right)^2 S_{EW} \left(1 + \delta_{\tau P} + 2\Delta^P\right)$

Values of $\Delta^P$ reported in the MS-scheme and at a scale of $\mu=2$ GeV.

$\Delta^P = \frac{\epsilon^c_L - \epsilon^c_R - \epsilon^c_R - \frac{m_P^2}{M_\tau (m_u + m_D)} \epsilon^c_P}{\epsilon^c_L - \epsilon^c_R - \epsilon^c_R - \frac{m_P^2}{M_\tau (m_u + m_D)} \epsilon^c_P} = \left\{ \begin{array}{c} \delta_{\tau \pi} = -(0.15 \pm 0.72) \cdot 10^{-2} \\ \delta_{\tau K} = -(0.36 \pm 1.18) \cdot 10^{-2} \end{array} \right\}$

- To be compared with $\Delta^\pi = -(0.15 \pm 0.67)\cdot 10^{-2}$ of Cirigliano et al.’19$^*$.

- To be compared with $\Delta^\pi = -(0.12 \pm 0.68)\cdot 10^{-2}$ and $\Delta^K = -(0.41 \pm 0.93)\cdot 10^{-2}$ of González-Solís et al.’20$^†$.

RadCors for semileptonic tau decays and NP tests

Pablo Roig (Cinvestav, Mexico City)
7. Conclusions

✓ The observable and our result:

\[
R_{\tau/P} \equiv \frac{\frac{\Gamma(\tau \rightarrow P\nu_\tau[\gamma])}{\Gamma(P \rightarrow \mu\nu_\mu[\gamma])}}{g_\tau |g_\mu|^2 g_\mu |g_\mu|} = 1 + \delta R_{\tau/P} \rightarrow \left\{ \begin{array}{c}
\delta R_{\tau/\pi} = (0.18 \pm 0.57)\
\delta R_{\tau/K} = (0.97 \pm 0.58)
\end{array} \right. \]

✓ Framework: ChPT for \(\pi\) decays and a resonance extension of ChPT for \(\tau\) decays.

✓ Consistent with DF'95\(^*\), but with more robust assumptions and yielding a reliable uncertainty.

✓ Applications:

✓ Theoretical determination of radiative corrections in \(\Gamma(\tau \rightarrow P\nu_\tau[\gamma])\).

✓ \(|g_\tau/g_\mu|\) at 0.9\(\sigma\) (\(\pi\)) and 1.8\(\sigma\) (K) of LU, reducing HFLAV'21\(^**\) disagreement with LU.

✓ CKM unitarity in \(\Gamma(\tau \rightarrow K\nu_\tau[\gamma])/\Gamma(\tau \rightarrow \pi\nu_\tau[\gamma])\): \(|V_{us}/V_{ud}| = 0.2288 \pm 0.0020\), at 2.1\(\sigma\) from unitarity.

✓ CKM unitarity in \(\Gamma(\tau \rightarrow K\nu_\tau[\gamma])\): \(|V_{us}| = 0.2220 \pm 0.0018\), at 2.6\(\sigma\) from unitarity.

✓ Constraining non-standard interactions in \(\Gamma(\tau \rightarrow P\nu_\tau[\gamma])\): update of \(\Delta^P\).

✓ Our results have been incorporated in the very recent HFLAV'22. \(\text{Phys.Rev.D 107 (2023) 5, 052008}\)


7. Conclusions

èle The observable and our result:

\[ R_{\tau/P} = \frac{\Gamma(\tau \rightarrow P\nu_\tau[\gamma])}{\Gamma(P \rightarrow \mu\nu_\mu[\gamma])} = \left| \frac{g_{\tau}}{g_\mu} \right|^2 R_{\tau/P}^{(0)} (1 + \delta R_{\tau/P}) \rightarrow \left\{ \begin{array}{l}
\delta R_{\tau/\pi} = (0.18 \pm 0.57)\% \\
\delta R_{\tau/K} = (0.97 \pm 0.58)\%
\end{array} \right. \]

Framework: ChPT for \( \tau \) decays.

Consistent with DF.

Applications:

- Theoretical de

\[ \frac{\delta K_{-\pi}^{\pi^{-}}}{\delta EM} = -(0.009^{+0.010}_{-0.018})\% \]

\[ \frac{\delta K^{0}_{-\pi}}{\delta EM} = -(0.166^{+0.100}_{-0.157})\% \]

\[ \frac{\delta K_{-K}^{K^{0}}}{\delta EM} = -(0.030^{+0.032}_{-0.180})\% \]

\[ \frac{\delta K_{-\pi}^{-\pi}}{\delta EM} = -(0.186^{+0.114}_{-0.203})\% \]

\[ (\tau \rightarrow P\nu_\tau[\gamma]) \]

We have halved the uncertainty!

- |\( g_{\tau}/g_\mu |_P \) at 0.9\( \sigma \) (\( \pi \)) and 1.8\( \sigma \) (K) of LU, reducing HFLAV'21** disagreement with LU.

- CKM unitarity in \( \Gamma(\tau \rightarrow K\nu_\nu,\gamma)/\Gamma(\tau \rightarrow \pi\nu_\nu,\gamma) \): \( |V_{us}/V_{ud}| = 0.2288 \pm 0.0020 \), at 2.1\( \sigma \) from unitarity.

- CKM unitarity in \( \Gamma(\tau \rightarrow K\nu_\nu,\gamma) \): \( |V_{us}| = 0.2220 \pm 0.0018 \), at 2.6\( \sigma \) from unitarity.

- Constraining non-standard interactions in \( \Gamma(\tau \rightarrow P\nu_\tau[\gamma]) \): update of \( \Delta_{\nu P} \).

Our results have been incorporated in the very recent HFLAV'22. Phys.Rev.D 107 (2023) 5, 052008

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* Decker & Finkemeier'95
* HFLAV'21

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RadCors for semileptonic tau decays and NP tests

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