Parton propagation in a strong color field: toward precision

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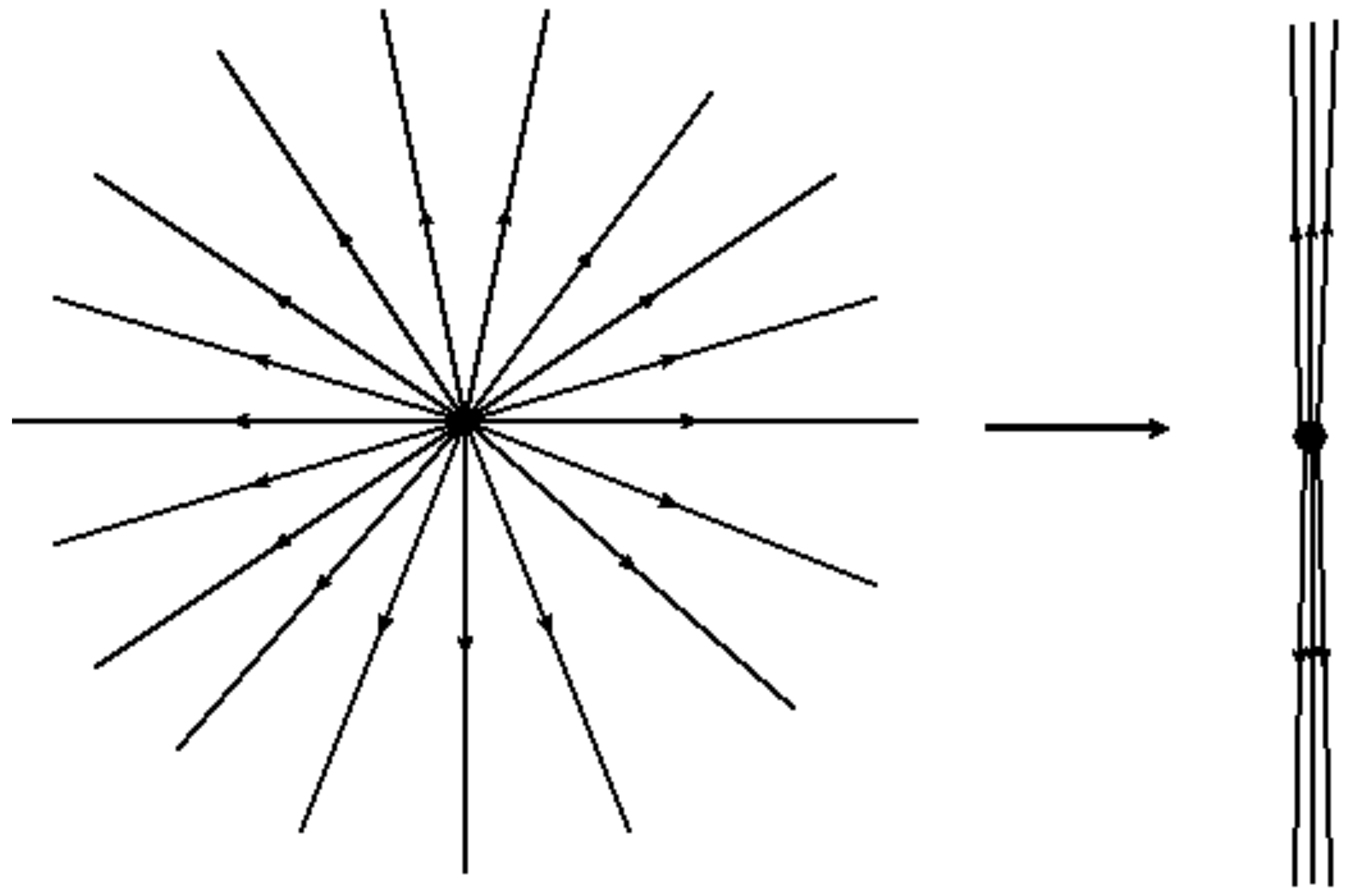
How does a Fermion propagate in a strong field?

strong EM fields

- High energy heavy ion collisions (RHIC, LHC)
- neutron stars
- ……



boost $\mathbf{F}^{+\mathbf{i}} \sim \gamma$

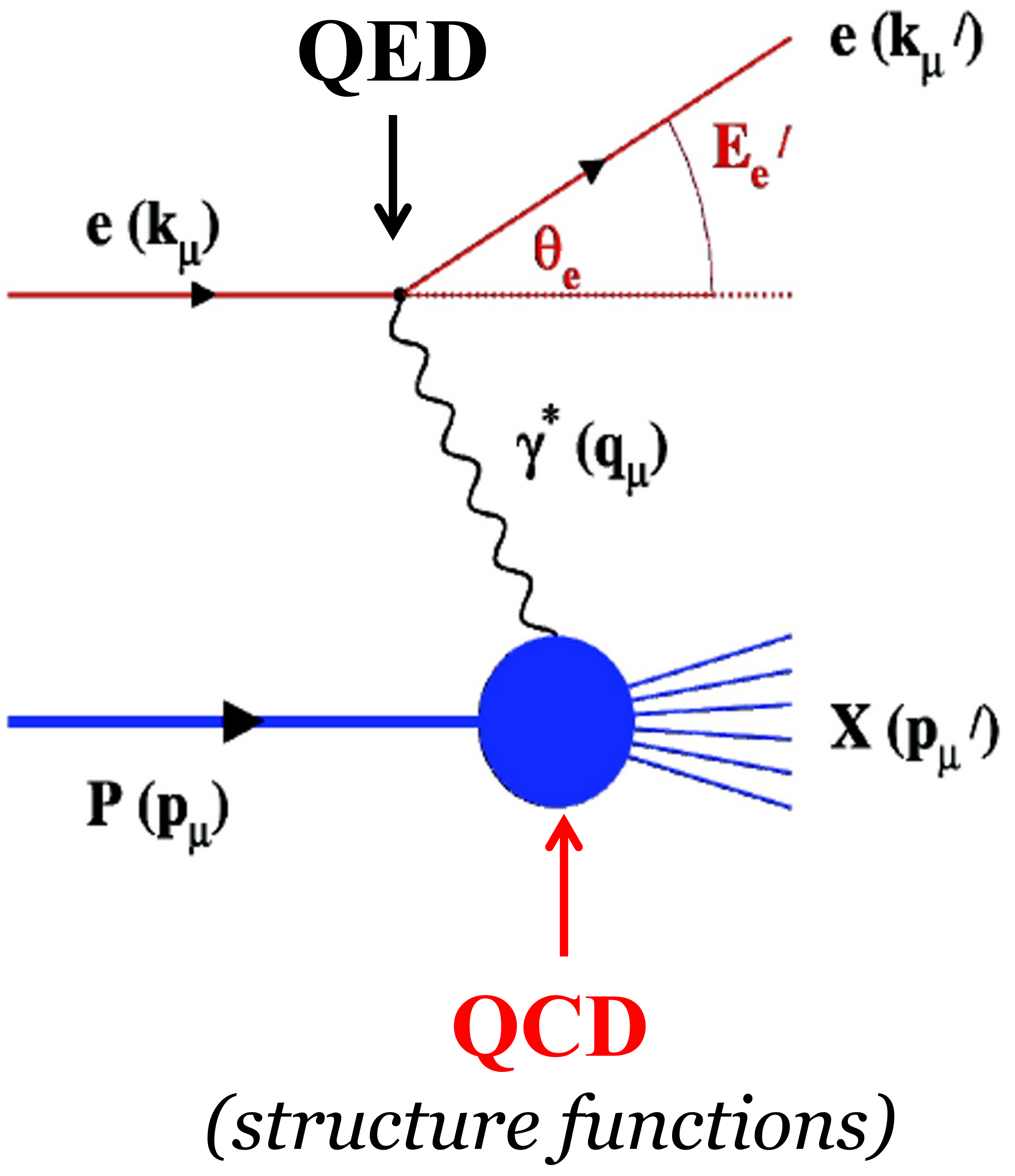
strong color fields:

- head-on high energy heavy ion collisions (RHIC, LHC)
- Deep Inelastic Scattering (DIS) at high energy (EIC)

How to generate strong color fields?

Deep Inelastic Scattering (DIS)

probing hadron structure



Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

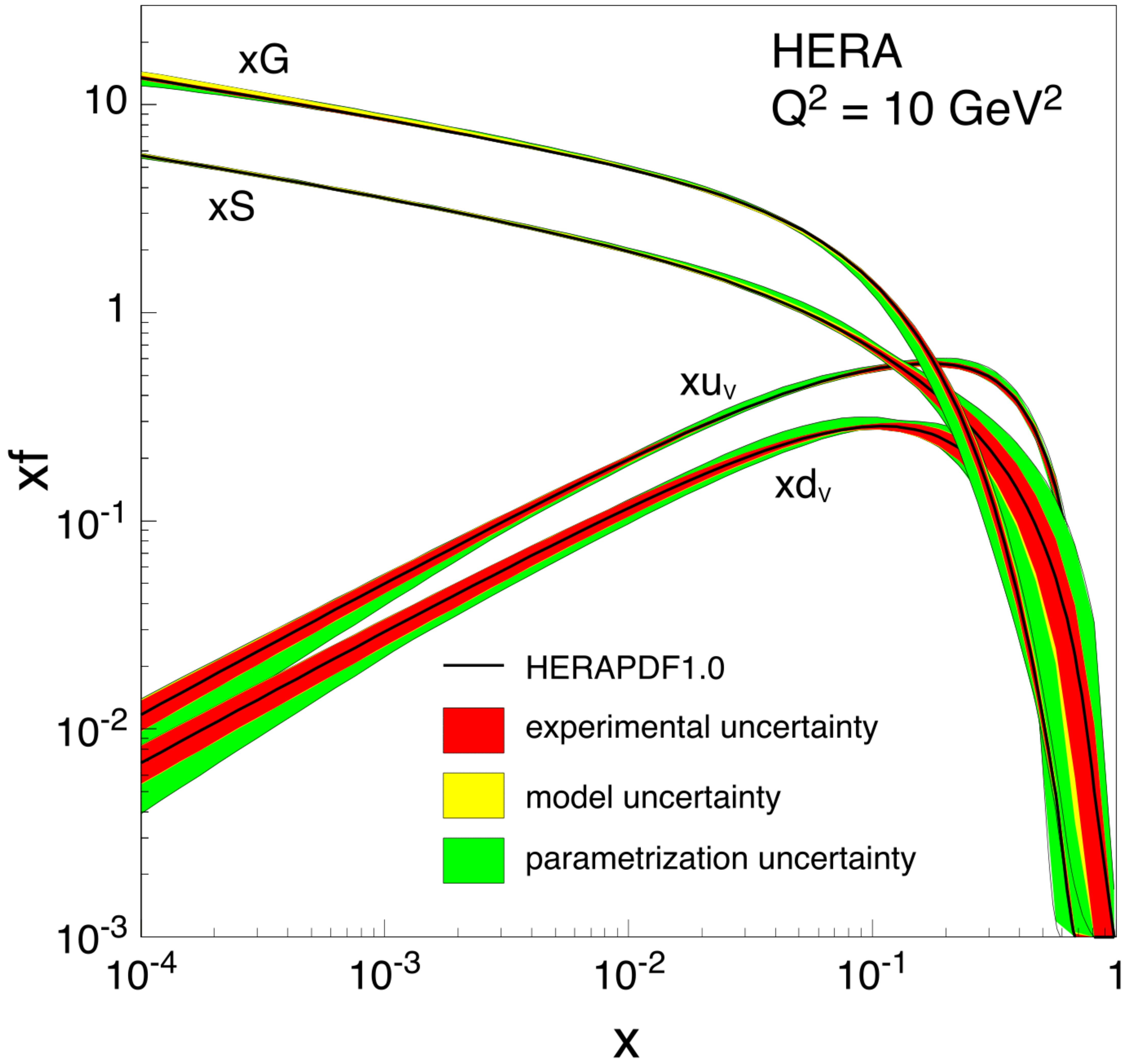
Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

Rise of the partons



$$x = \frac{p^+}{P^+}$$ ***x*** ***is the fraction of hadron energy carried by a parton***

QCD in the Regge-Gribov limit

recall $x_{Bj} \equiv \frac{Q^2}{s}$ $\quad$ $s \to \infty\,,\ Q^2$ fixed : $x_{Bj} \to 0$

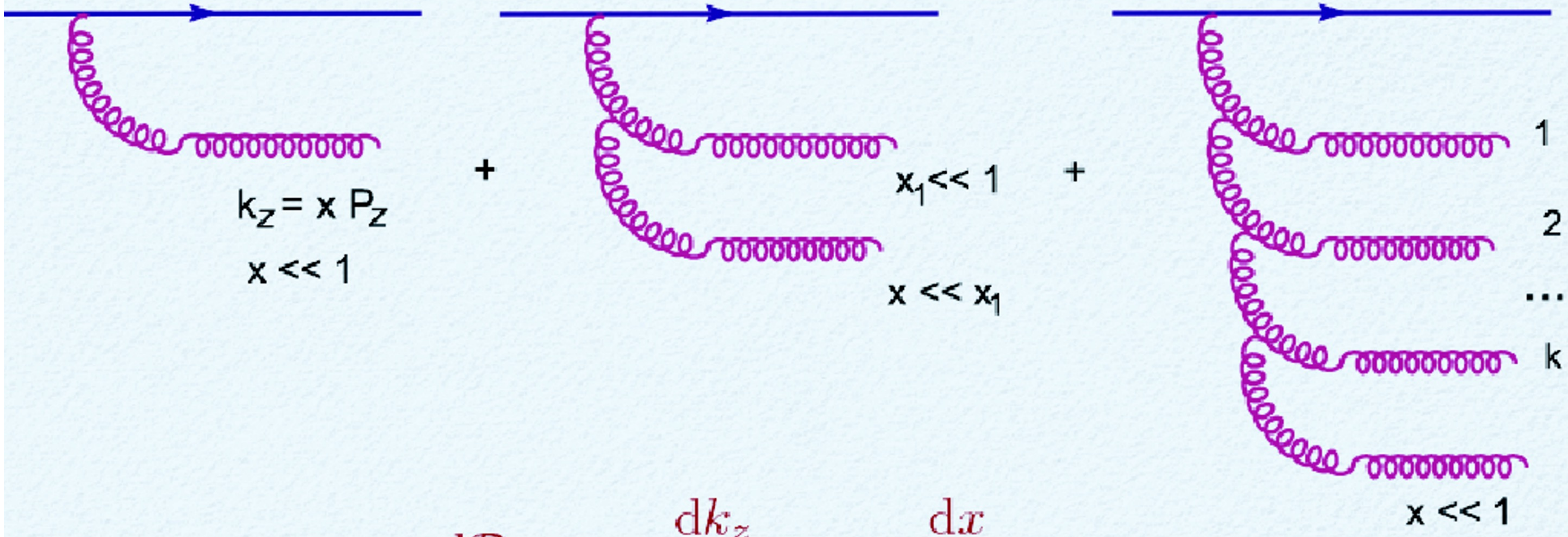
Regge

Gribov

gluon radiation at small x :pQCD

The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons

$$\mathbf{P_{gg}(x) \sim \frac{1}{x} \;\; for \;\; x \to 0}$$



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

The 'price' of an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

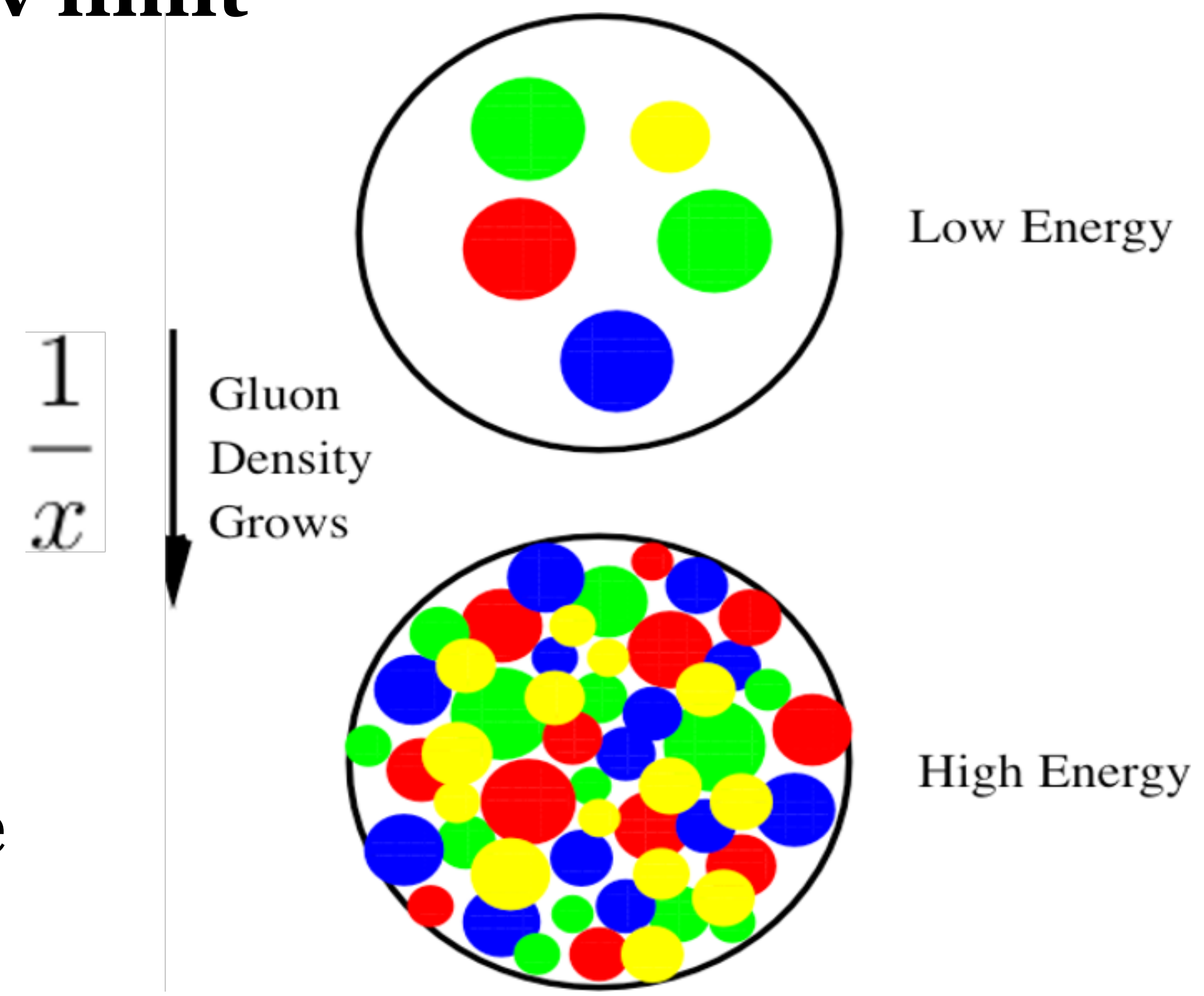
number of gluons grows fast

$$n \sim e^{\alpha_s ln 1/x}$$

Resolving the nucleus/hadron: Regge-Gribov limit

Gribov-Levin-Ryskin

Q^2 fixed and $\sqrt{S} \to \infty \quad (x \equiv \frac{Q^2}{S} \to 0)$



gluons are radiated into fixed resolved area

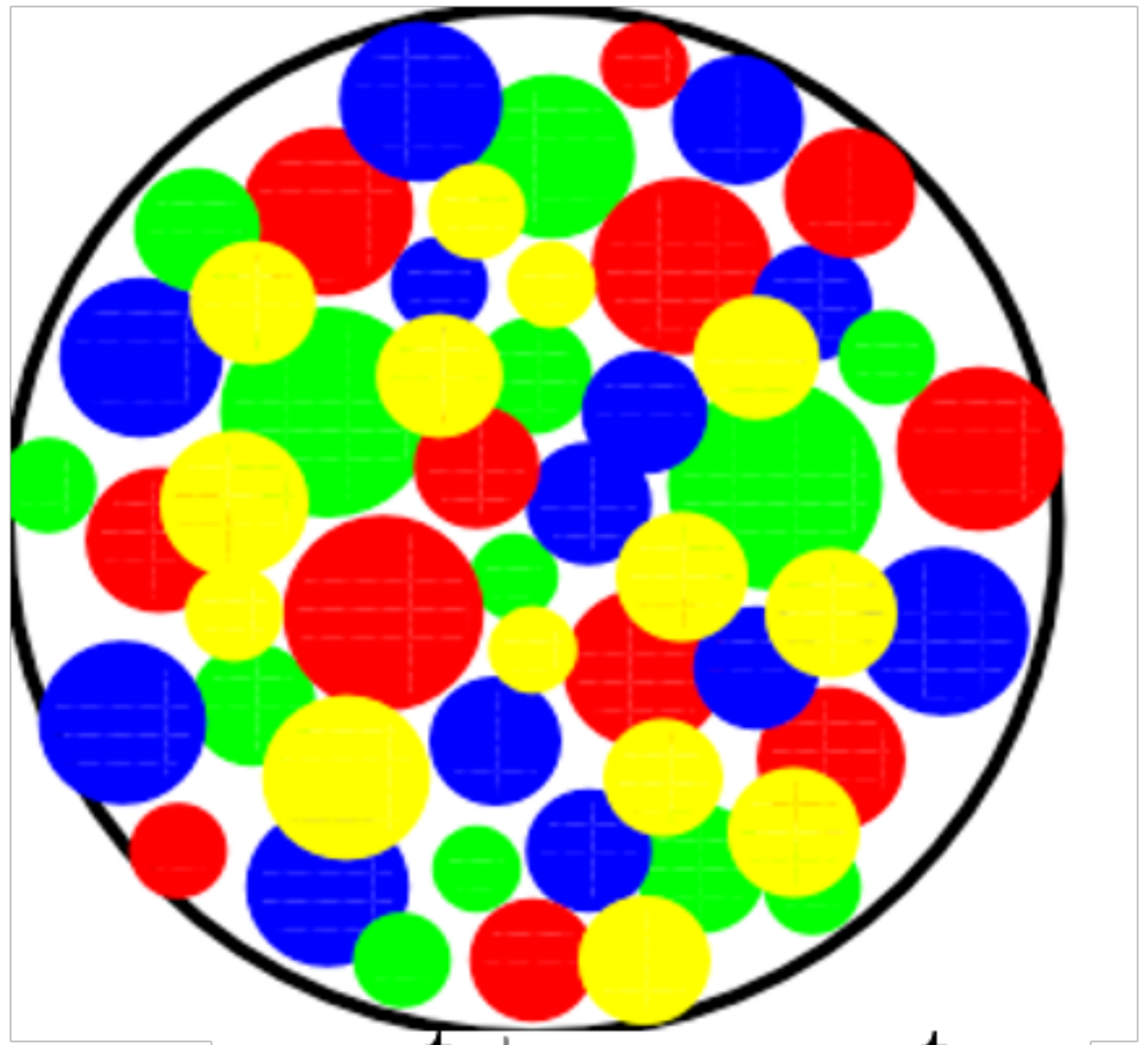
number of gluons increases due to increased longitudinal phase space

hadron/nucleus becomes a dense state of gluons (CGC) $\frac{\alpha_s}{Q^2} \frac{xG(x, Q^2)}{S_\perp} \sim 1 \quad Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$

strong color fields

possible universal properties of QCD observables ?

A very large nucleus at high energy: MV model

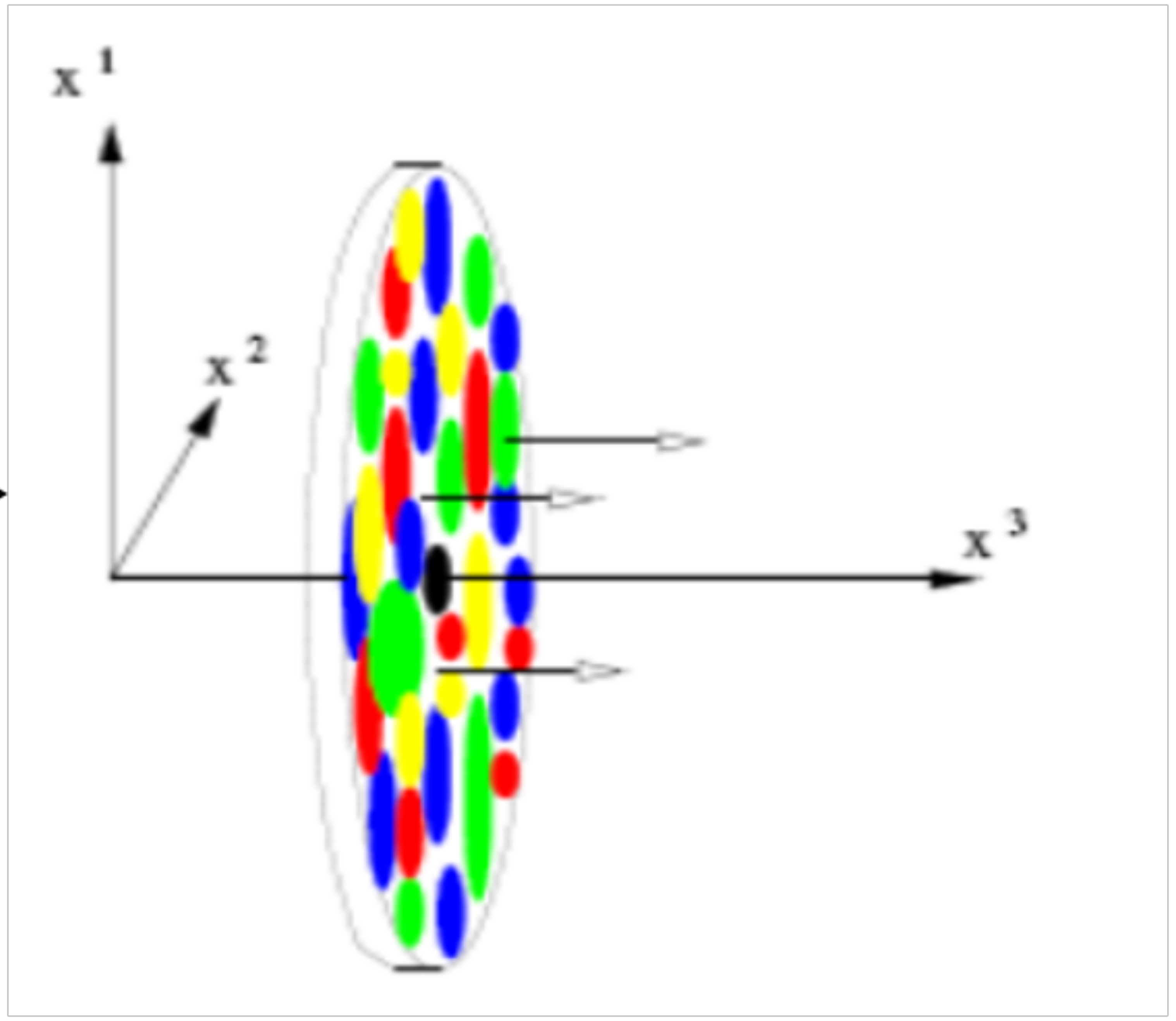


boost

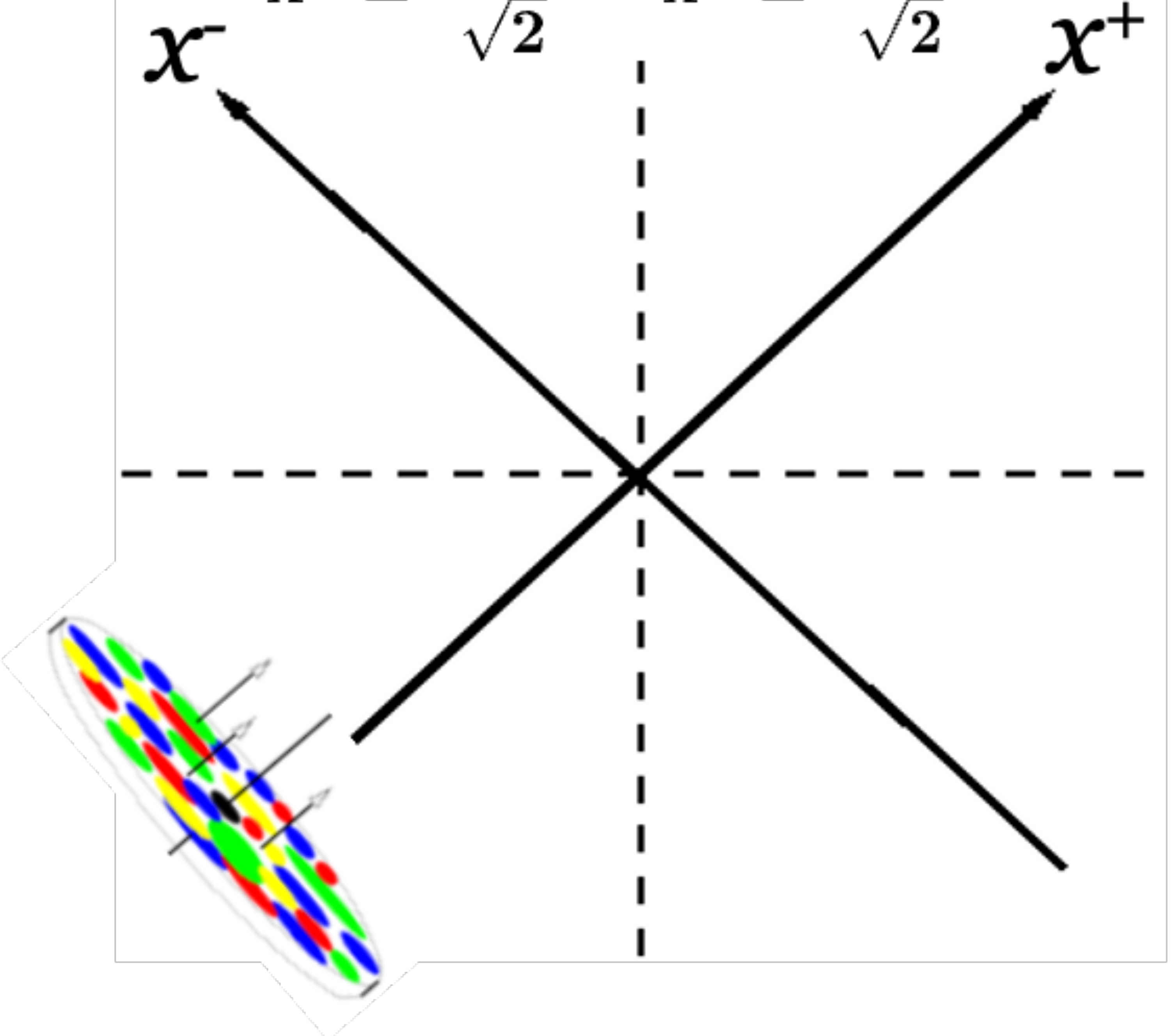
$$R \to \frac{R}{\gamma}$$

$\gamma \sim 100$ RHIC

$\gamma \sim 2500$ LHC



$$\mathbf{x^+} \equiv \frac{\mathbf{t+z}}{\sqrt{2}} \qquad \mathbf{x^-} \equiv \frac{\mathbf{t-z}}{\sqrt{2}}$$



sheet of color charge moving along x+ and sitting at x- = 0

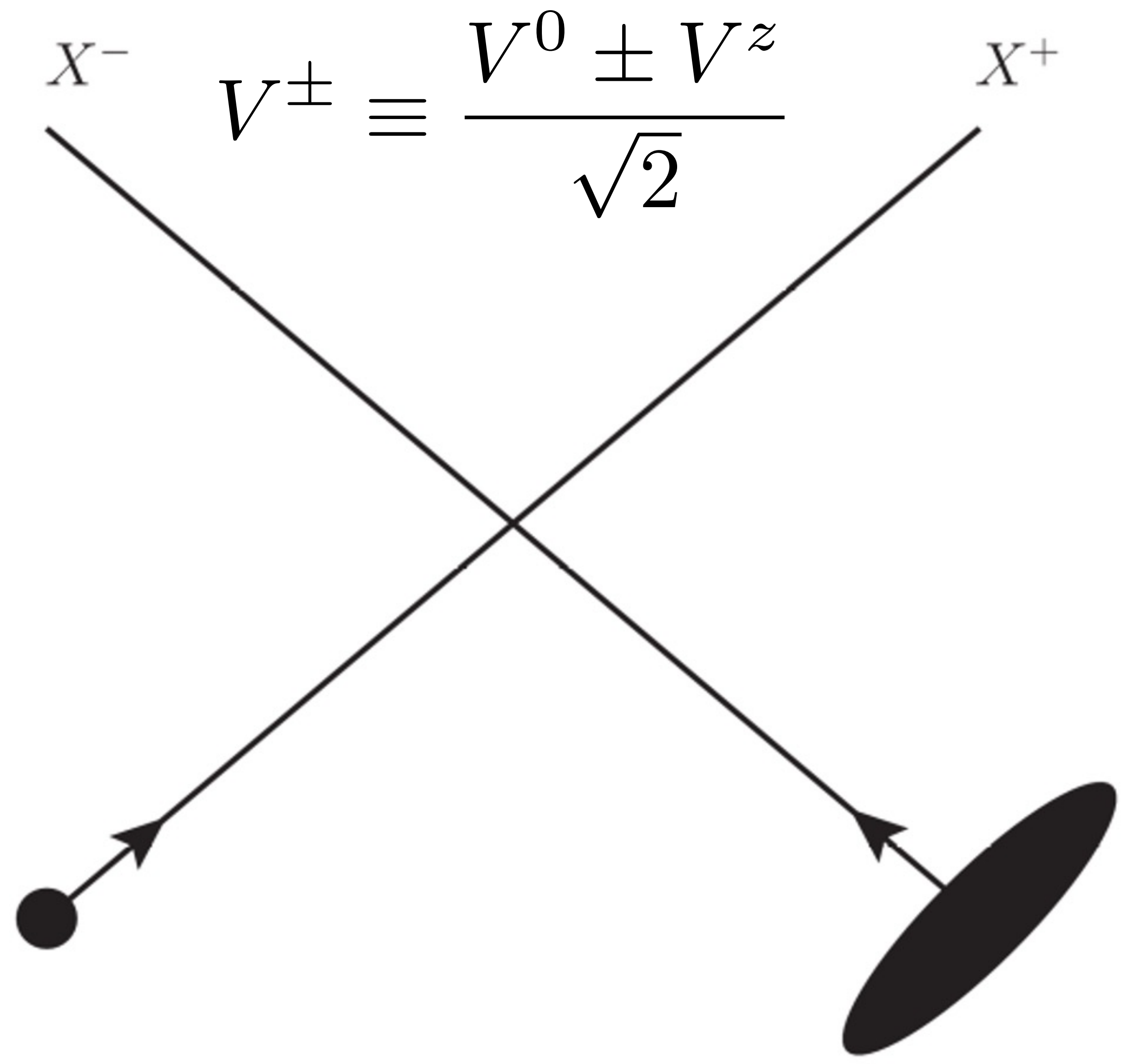
$$\mathbf{J^\mu_a(x)} \equiv \delta^{\mu +}\,\delta(\mathbf{x^-})\,\rho_\mathbf{a}(\mathbf{x_t})$$

color current *color charge*

$$\mathbf{A^a_i}(\mathbf{x^-}, \mathbf{x_t}) = \theta(\mathbf{x^-})\,\alpha^\mathbf{a}_\mathbf{i}(\mathbf{x_t})$$

with $\partial_i\,\alpha_i^a = g\,\rho^a \qquad \alpha_\mathbf{i} \sim \frac{\mathbf{1}}{\mathbf{g}}\mathbf{U}\partial_\mathbf{i}\mathbf{U}^\dagger$

Eikonal approximation



$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0 \qquad \text{(in } A^+ = 0 \text{ gauge)}$$

does not depend on x^-

solution to classical EOM:

$$A_a^-(x^+, x_t) \equiv n^- \, S_a(x^+, x_t)$$

with

$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$

$$n^2 = 2\, n^+ n^- - n_\perp^2 = 0$$

recall (eikonal limit):

$$\bar{u}(q)\gamma^\mu u(p) \rightarrow \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$$

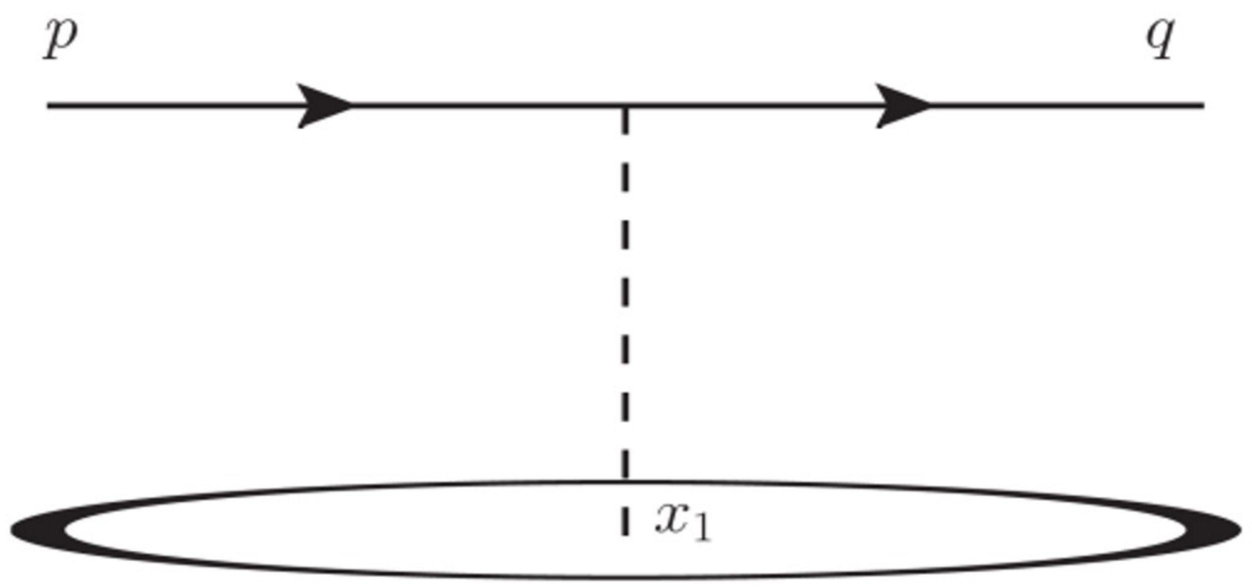
$$\bar{u}(q) \not{A}\, u(p) \rightarrow p \cdot A \sim p^+ A^-$$

scattering of a quark from background color field $A_a^-(x^+, x_t)$

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p)$$

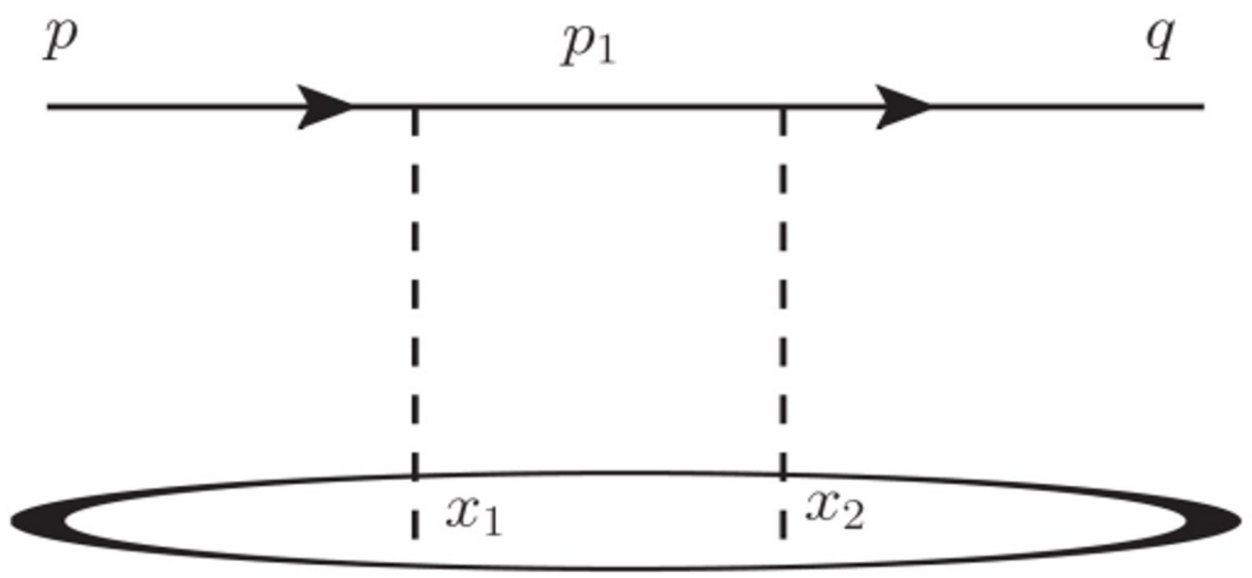
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}}$$

$$\bar{u}(q)\, \left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)$$



$$iM_2 = (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1 - p)x_1}\, e^{i(q-p_1)x_2}$$

$$\bar{u}(q)\, \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p)$$



$$\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

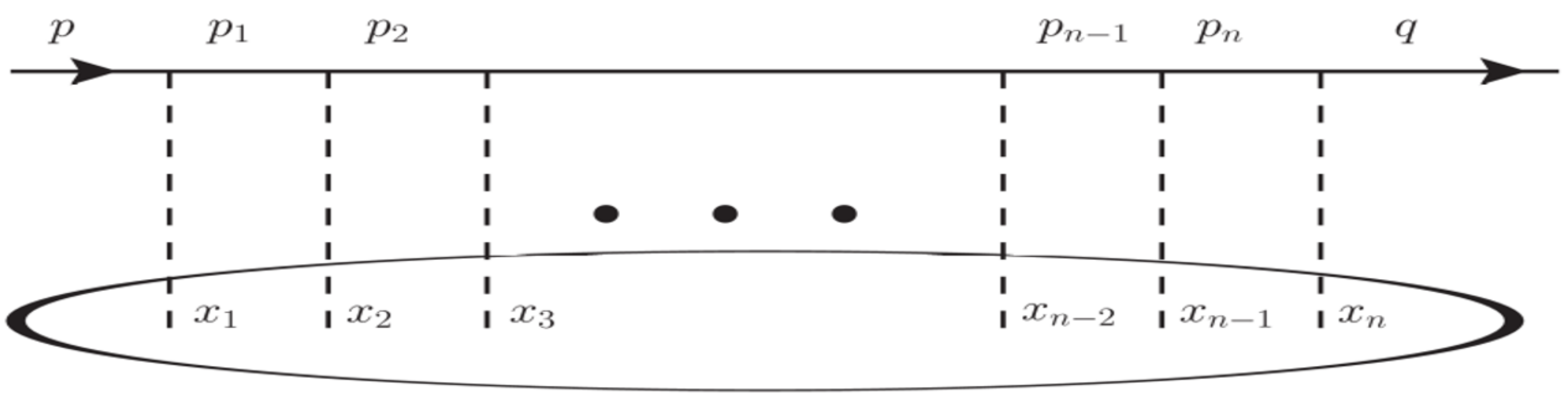
contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n\cdot p}\, \not{n} = \not{n}$

$$iM_2 = (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}}$$

$$\bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)$$

Eikonal scattering from a dense target (proton/nucleus)



$$
\begin{aligned}
i\mathcal{M}_n &= 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t} \\
&\quad \Big\{(ig)^n\,(-i)^n(i)^n\int dx_1^+\,dx_2^+\cdots dx_n^+\,\theta(x_n^+ - x_{n-1}^+)\cdots\theta(x_2^+ - x_1^+) \\
&\quad [S(x_n^+, x_t)\,S(x_{n-1}^+, x_t)\,\cdots S(x_2^+, x_t) S(x_1^+, x_t)]\Big\}\,u(p)
\end{aligned}
$$

sum all multiple scatterings $$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\,[V(x_t) - 1]\,u(p)$$

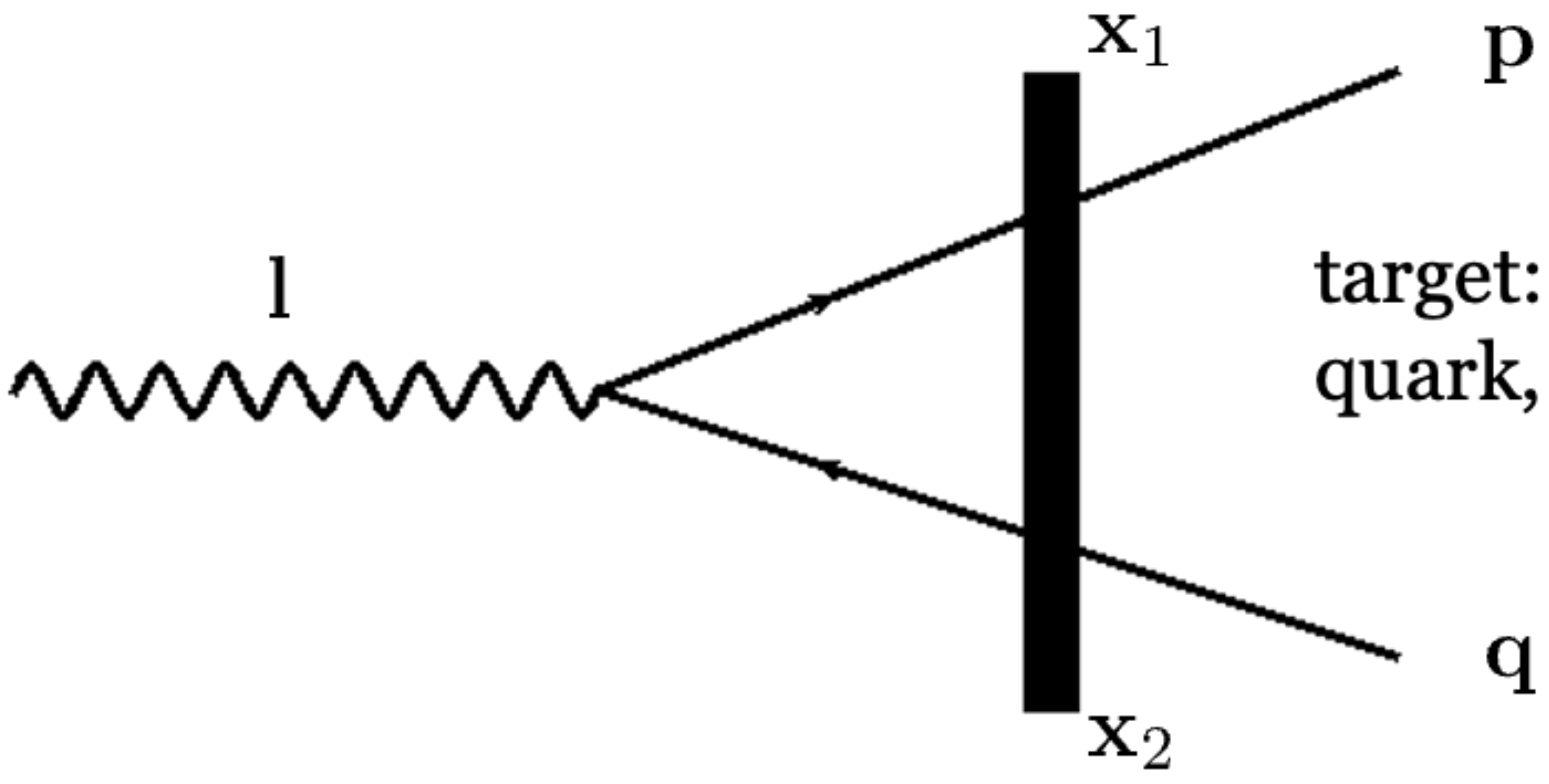
with $V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\,n^-\,S_a(x^+, x_t)\,t_a\right\}$

Wilson lines: effective degree of freedom

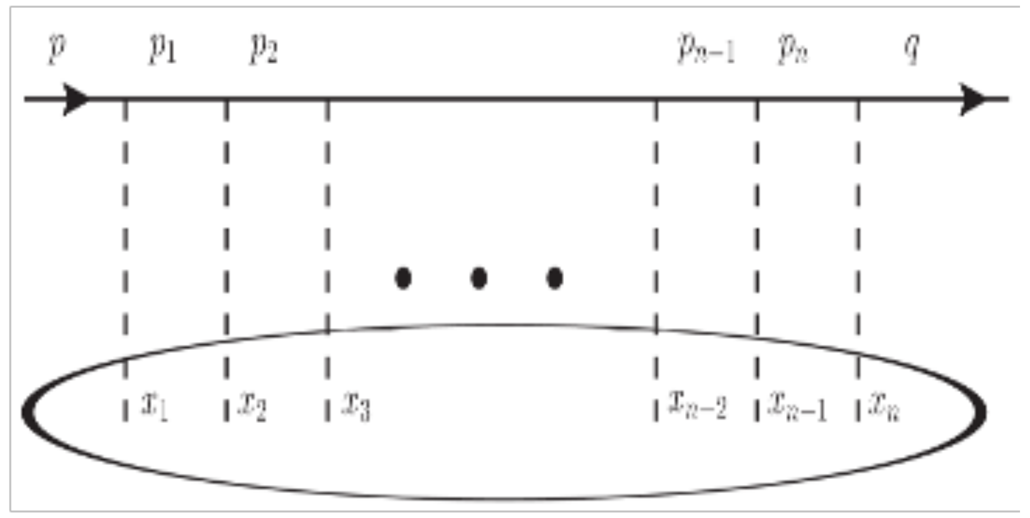
$$\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim F.T. < Tr\,V(x_t)\,V^\dagger(y_t) >$$

dipole

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1) +$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

$$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$$

quadrupole

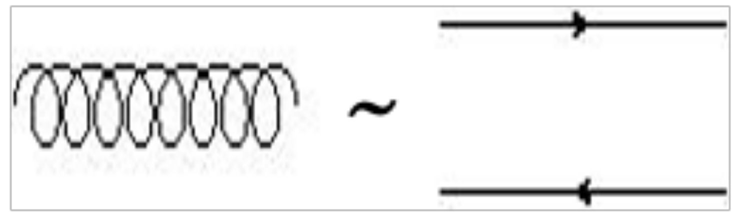
$$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$$

Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

One-loop corrections: BK-JIMWLK eq.

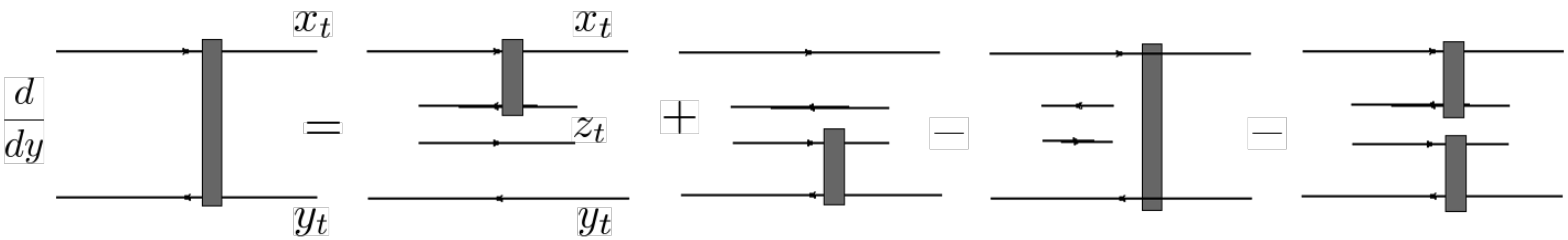
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy}T(x_t, y_t) = \frac{N_c\alpha_s}{2\pi^2}\int d^2z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2}\left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t)T(z_t, y_t)\right]$$

$T \equiv 1 - S$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma} \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2p_t dy}}{A^{1/3}\frac{d\sigma^{pp}}{d^2p_t dy}}$$

nuclear shadowing

suppression of p_t spectra

disappearance of back to back peaks

....

Toward precision

Sub-eikonal corrections: suppressed by $\sqrt{s}$

corrections to dijet production in DIS, ...

Altinoluk, Beuf, Czajka,... (2023)

....

Higher order corrections; suppressed by α_s

NLO corrections to DIS, SIDIS, DIDIS, pA

....

Beuf,...... (2017-2022)

F. Bergabo, JJM (2022 -2024)

....

toward unifying small and large x (multiple scattering)

JJM, 1708.07533, 1809.04625, 1912.08878

scattering from small x modes of the target field $\mathbf{A^- = n^-\ S}$ involves only

small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s},\ p^- = 0, p_t = 0)$$
$$S = S(p^+ \sim 0, p^-/P^- \ll 1, p_t)$$

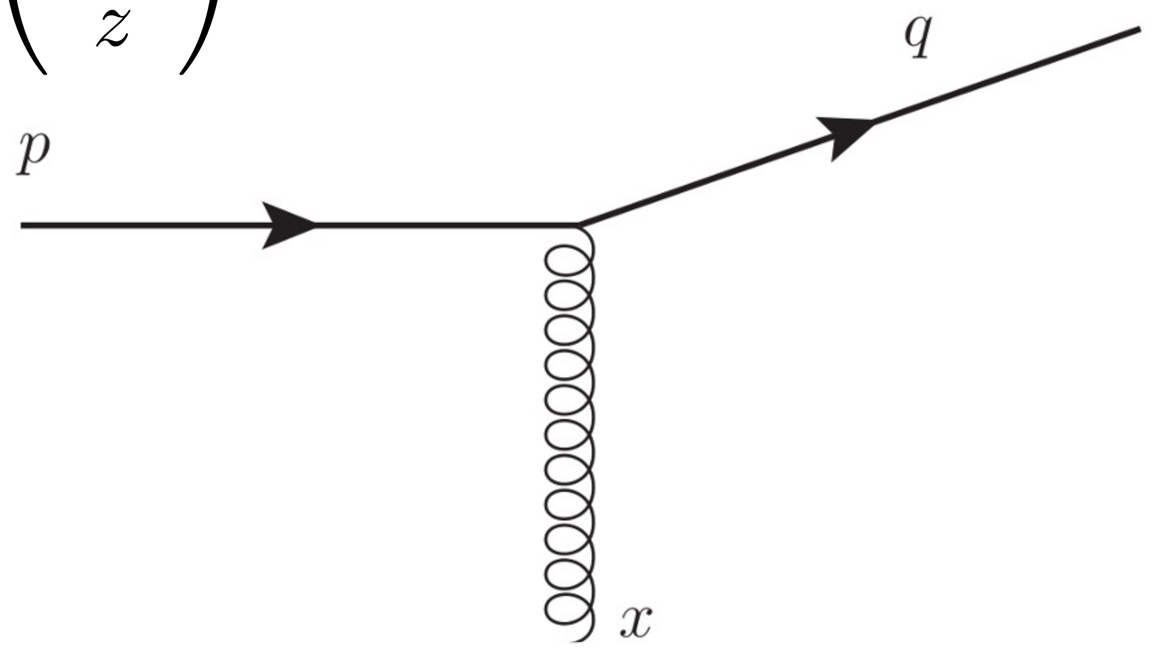
allow hard scattering by including one all x field $A^\mu_a(x^+, x^-, x_t)$
during which there is large momenta exchanged and
quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

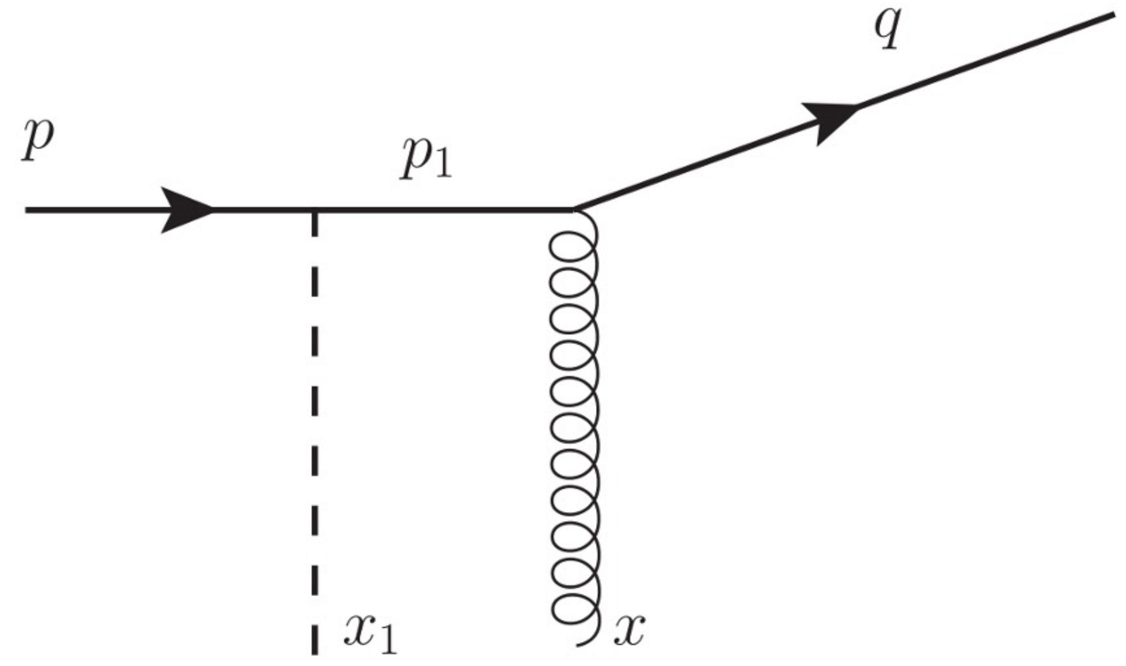
hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

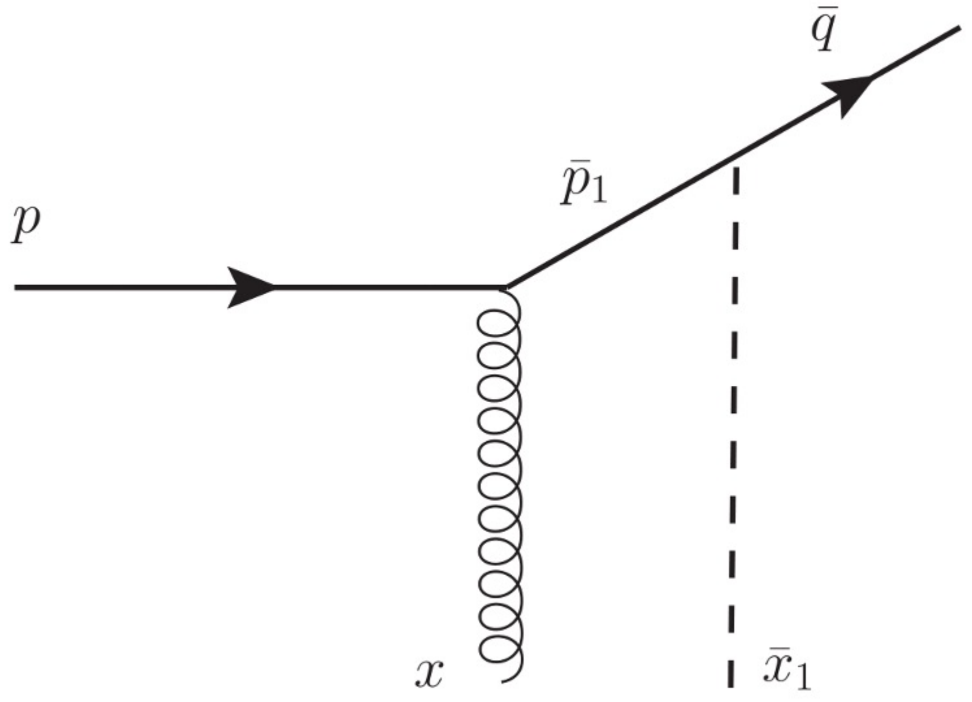
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x) \right] \, u(p)$$



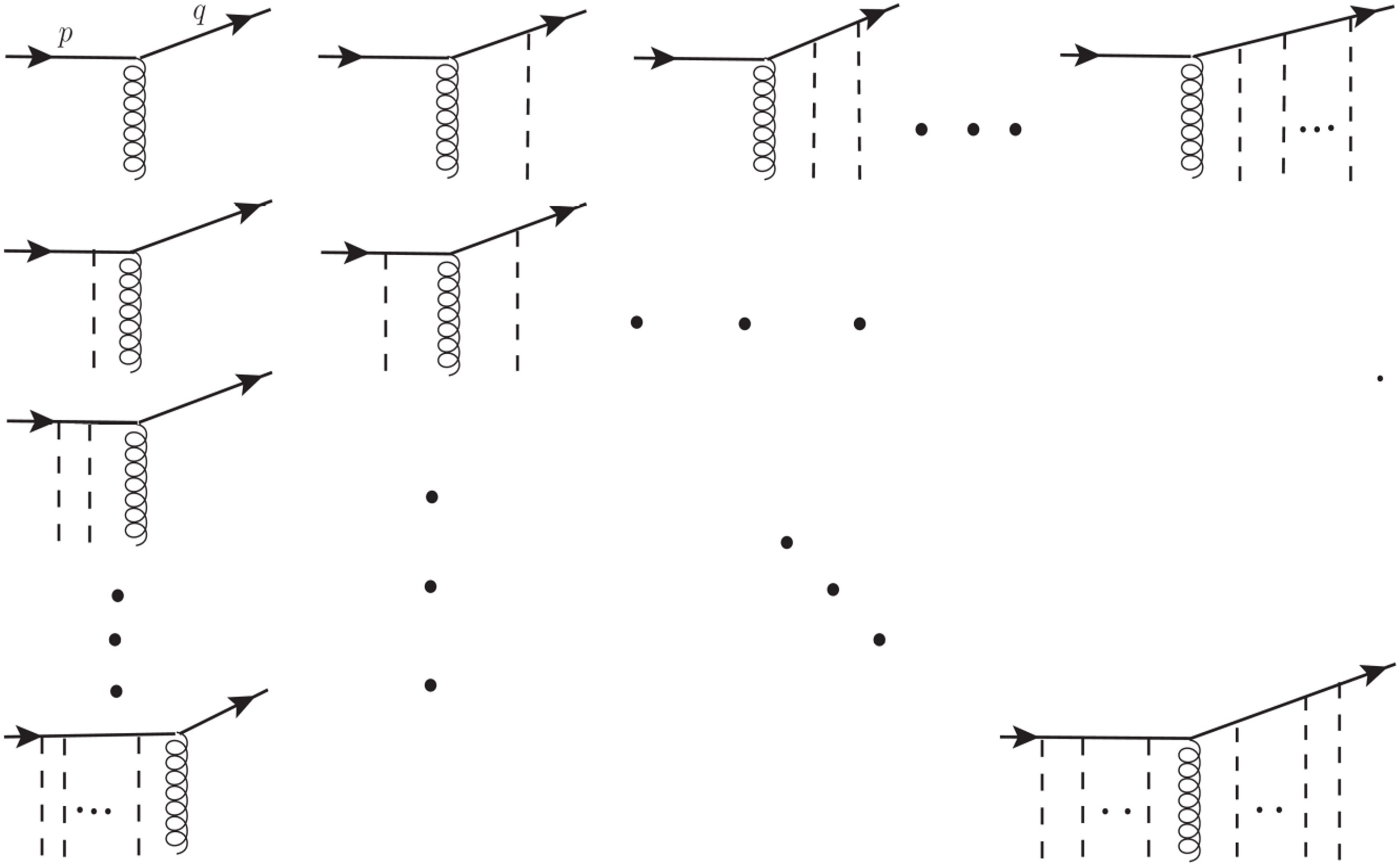
$$i\mathcal{M}_2 \quad = \quad (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \, \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] \, u(p)$$



$$i\mathcal{M}_2 \quad = \quad (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \, \bar{u}(\bar{q}) \left[\not{\bar{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x) \right] \, u(p)$$



with $\quad \bar{v}^\mu = \Lambda^\mu_\nu \, v^\nu$



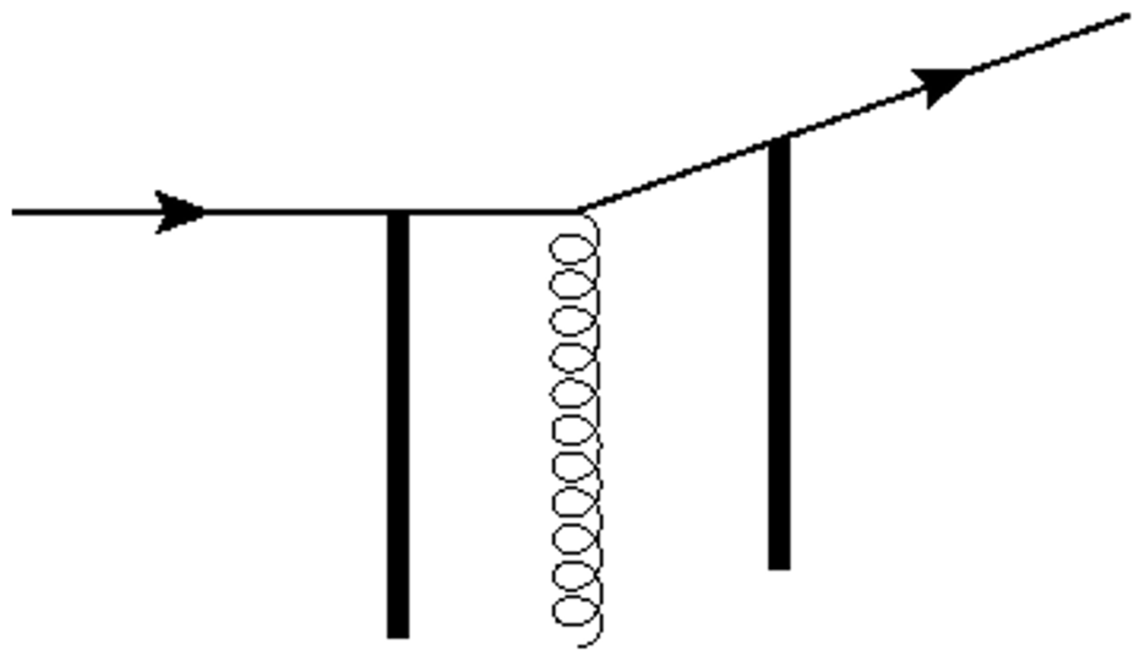
p
q

summing all the terms gives:

$$i\mathcal{M}_1 \quad = \quad \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\,\frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t}$$

$$\bar{u}(\bar{q})\,\Big[\overline{V}_{AP}(x^+,\bar{z}_t)\,\not{n}\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\big[ig\not{A}(x)\big]\,\frac{\not{k}}{2k^+}\,\not{n}\,V_{AP}(z_t,x^+)\Big]\,u(p)$$

with

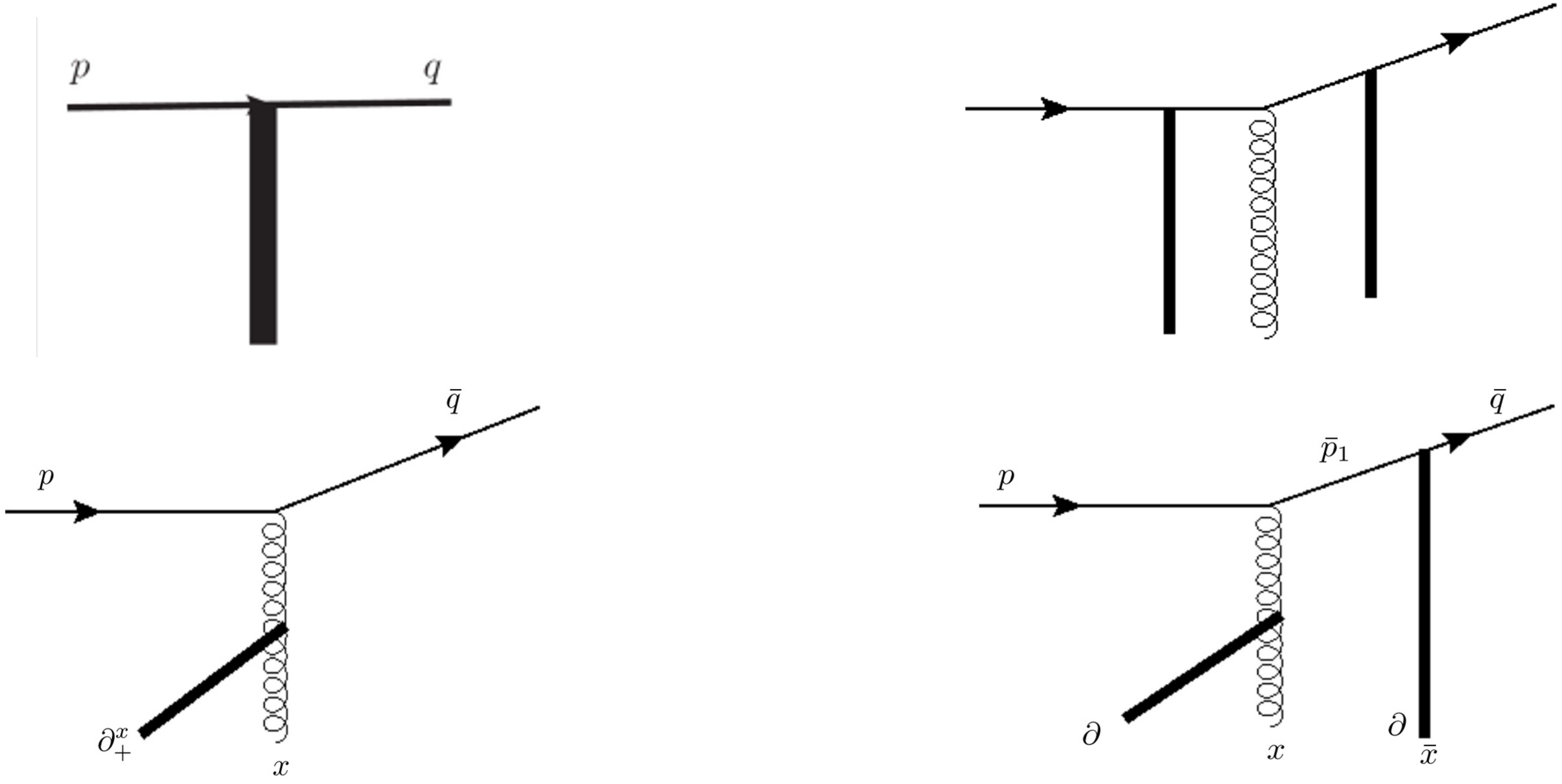


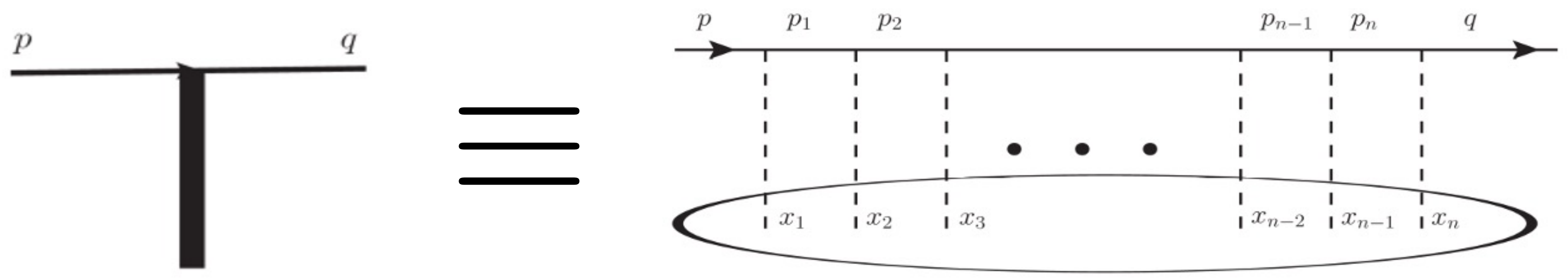
$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\,\bar{S}_a^-(\bar{z}_t,\bar{z}^+)\,t_a\right\}$$

$$V_{AP}(z_t,x^+) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z_t,z^+)\,t_a\right\}$$

this is the building block for DIS structure functions, single inclusive particle production in pA,....

full amplitude: $\mathbf{i\mathcal{M} = i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3}$

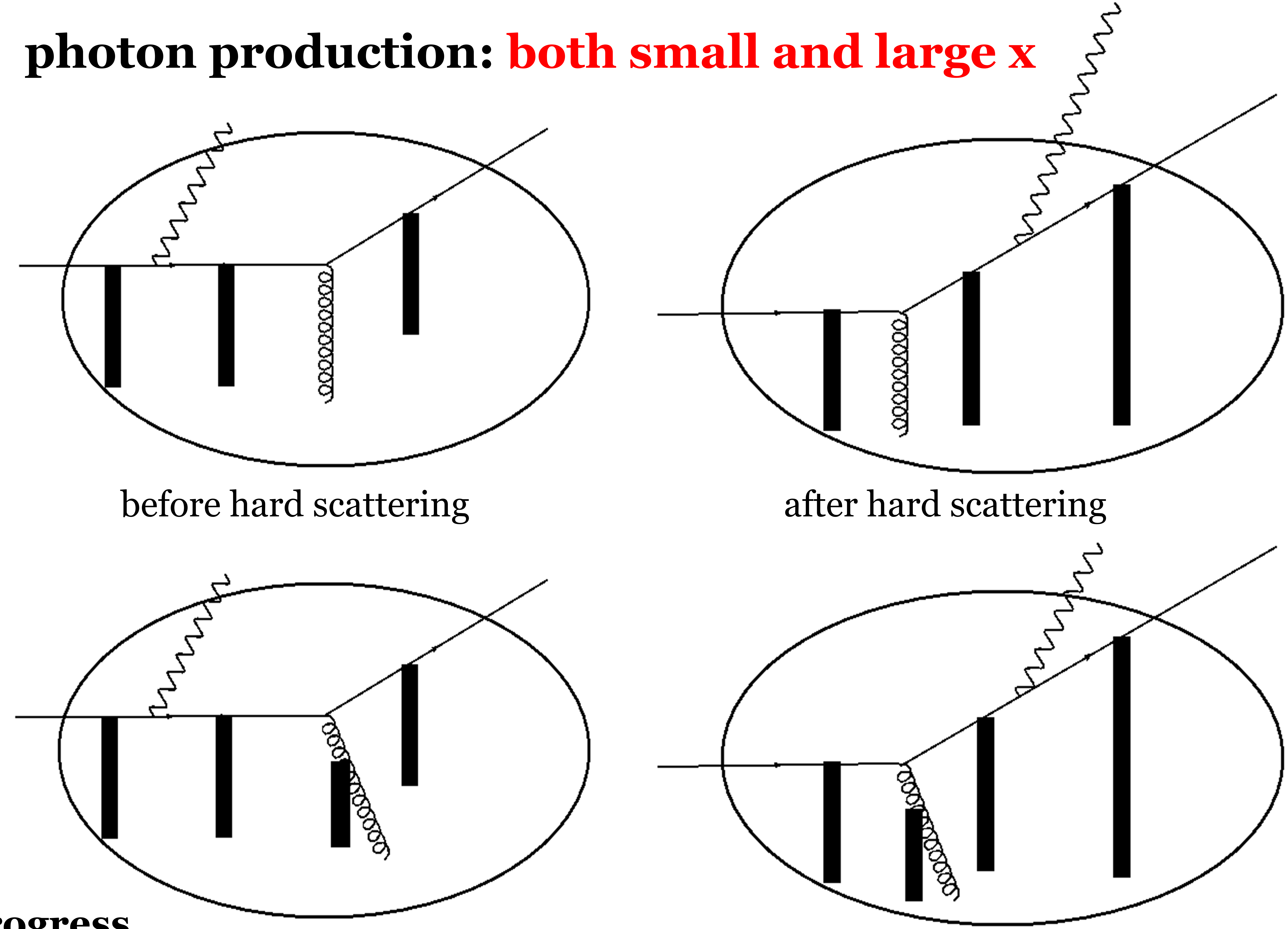




soft (eikonal) limit: $A^\mu(x) \to n^- \, S(x^+, x_t) \qquad n \cdot \bar{q} \to n \cdot p$

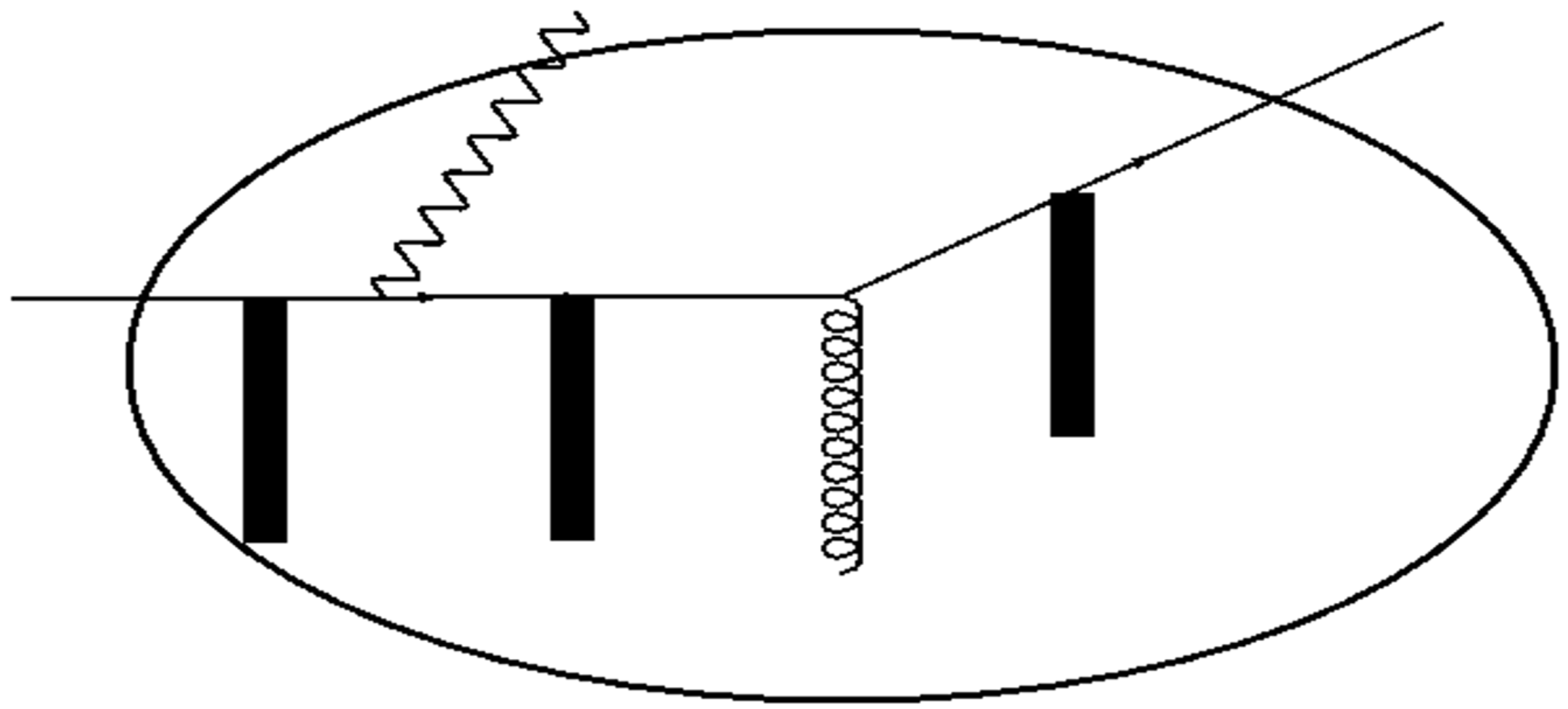
$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

photon production: both small and large x



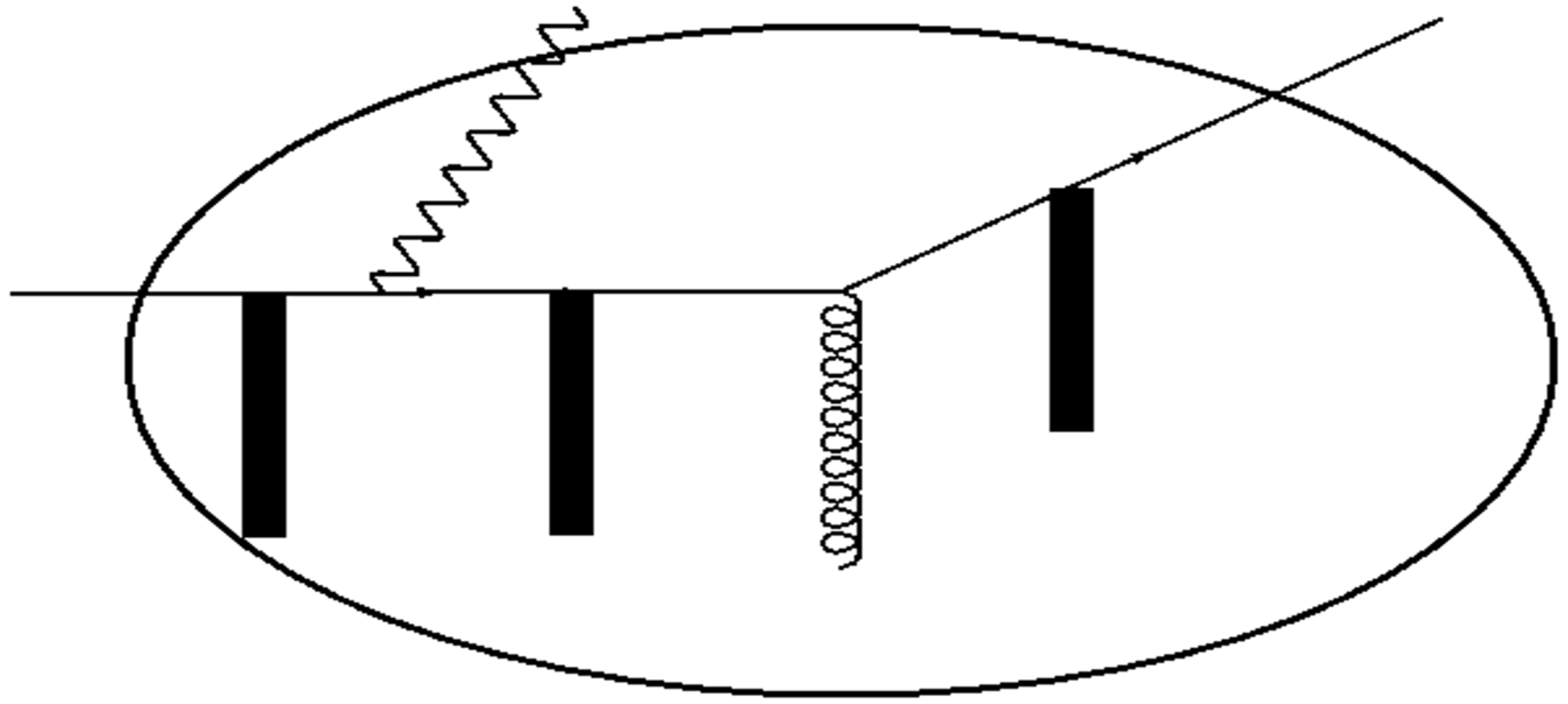
JJM, in progress

photon radiation: helicity amplitudes



$$
\begin{aligned}
&i\mathcal{M}_1(p,q,l) = \\
&eg \int \frac{d^2k_{2t}}{(2\pi)^2}\,\frac{d^2k_{3t}}{(2\pi)^2}\,\frac{d^2\bar{k}_{1t}}{(2\pi)^2} \int d^4x\, d^2y_{1t}\, d^2y_{2t}\, d^2\bar{y}_{1t}\, dz^+\, \theta(x^+ - z^+)\, e^{i(l^+ + \bar{q}^+ - p^+)x^-} \\
&e^{-i(\bar{q}_t - \bar{k}_{1t})\cdot \bar{y}_{1t}}\, e^{-i(\bar{k}_{1t} - k_{3t})\cdot x_t}\, e^{-i(k_{3t} - k_{2t})\cdot y_{2t}}\, e^{-i(l_t + k_{2t} - p_t)\cdot y_{1t}}\, \bar{u}(\bar{q})\, \overline{V}(\bar{y}_{1t}; x^+, \infty)\, \frac{\not{\bar{n}}\, \not{\bar{k}}_1}{2\bar{n}\cdot \bar{q}} \\
&\not{A}(x)\, \left[\frac{\not{k}_3}{2n\cdot (p-l)}\, V(y_{2t}; z^+, x^+)\, \frac{\not{n}\, \not{k}_2}{2n\cdot (p-l)} + i\, \frac{\delta(x^+ - z^+)}{2n\cdot (p-l)}\not{n}\right] \\
&\not{\epsilon}(l)\, \frac{\not{k}_1}{2n\cdot p}\, V(y_{1t}; -\infty, z^+)\, \not{n}\, u(p)
\end{aligned}
$$

photon production: both small and large x



$$\mathcal{N}_{1-1} = \bar{u}(\bar{q})\,\frac{\not{\bar{n}}\,\not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}}\,\not{A}(x)\,\frac{\not{k}_3\,\not{n}\,\not{k}_2\not{\epsilon}(l)\,\not{k}_1\,\not{n}}{2n\cdot p\,2n\cdot(p-l)\,2n\cdot(p-l)}\,u(p)$$

$$\mathcal{N}_{1-2} = \bar{u}(\bar{q})\,\frac{\not{\bar{n}}\,\not{\bar{k}}_1}{2\bar{n}\cdot\bar{q}}\,\not{A}(x)\,\frac{\not{n}\,\not{\epsilon}(l)\,\not{k}_1\,\not{n}}{2n\cdot p\,2n\cdot(p-l)}\,u(p)$$

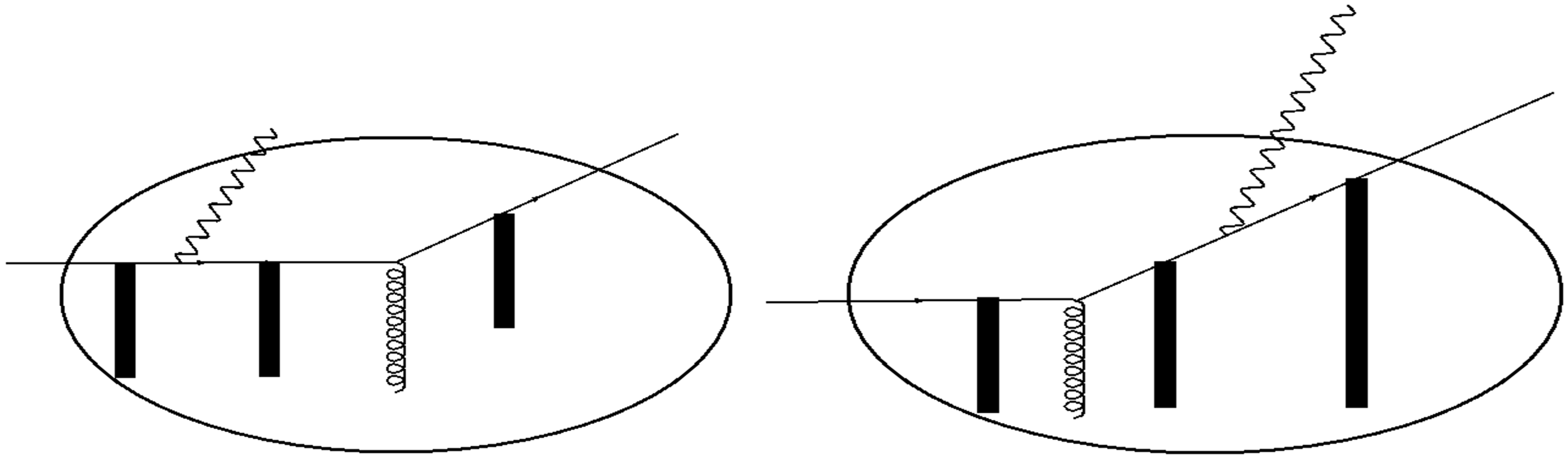
$$\mathcal{N}^{++}_{1-1} = (\mathcal{N}^{--}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\frac{[n\cdot l\,k_{2\perp}\cdot\epsilon^\star_\perp - n\cdot(p-l)\,l_\perp\cdot\epsilon^\star_\perp]}{n\cdot l\,n\cdot(p-l)}\,\langle\bar{k}^+_1|\not{A}(x)|k^+_3\rangle$$

$$\mathcal{N}^{++}_{1-2} = (\mathcal{N}^{--}_{1-2})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\langle\bar{k}^+_1|\not{A}(x)|n^+\rangle$$

$$\mathcal{N}^{+-}_{1-1} = (\mathcal{N}^{-+}_{1-1})^\star = -\sqrt{\frac{n\cdot p}{n\cdot(p-l)}}\,\frac{[n\cdot p\,l_\perp\cdot\epsilon_\perp - n\cdot l\,k_{1\perp}\cdot\epsilon_\perp]}{n\cdot p\,n\cdot l}\,\langle\bar{k}^+_1|\not{A}(x)|k^+_3\rangle$$

$$\mathcal{N}^{+-}_{1-2} = \mathcal{N}^{-+}_{1-2} = 0$$

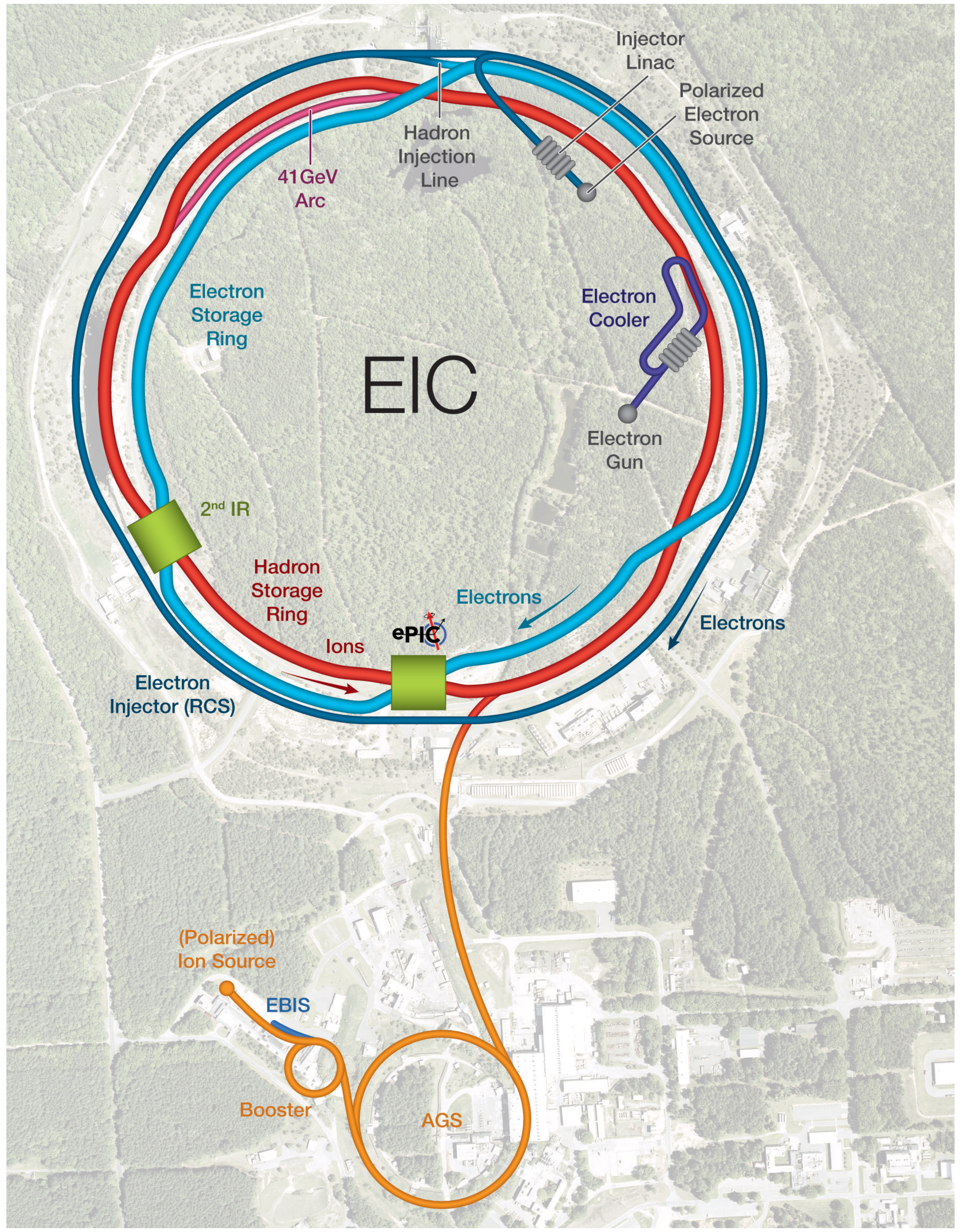
pQCD limit (large x: gluon PDF X partonic cross section):



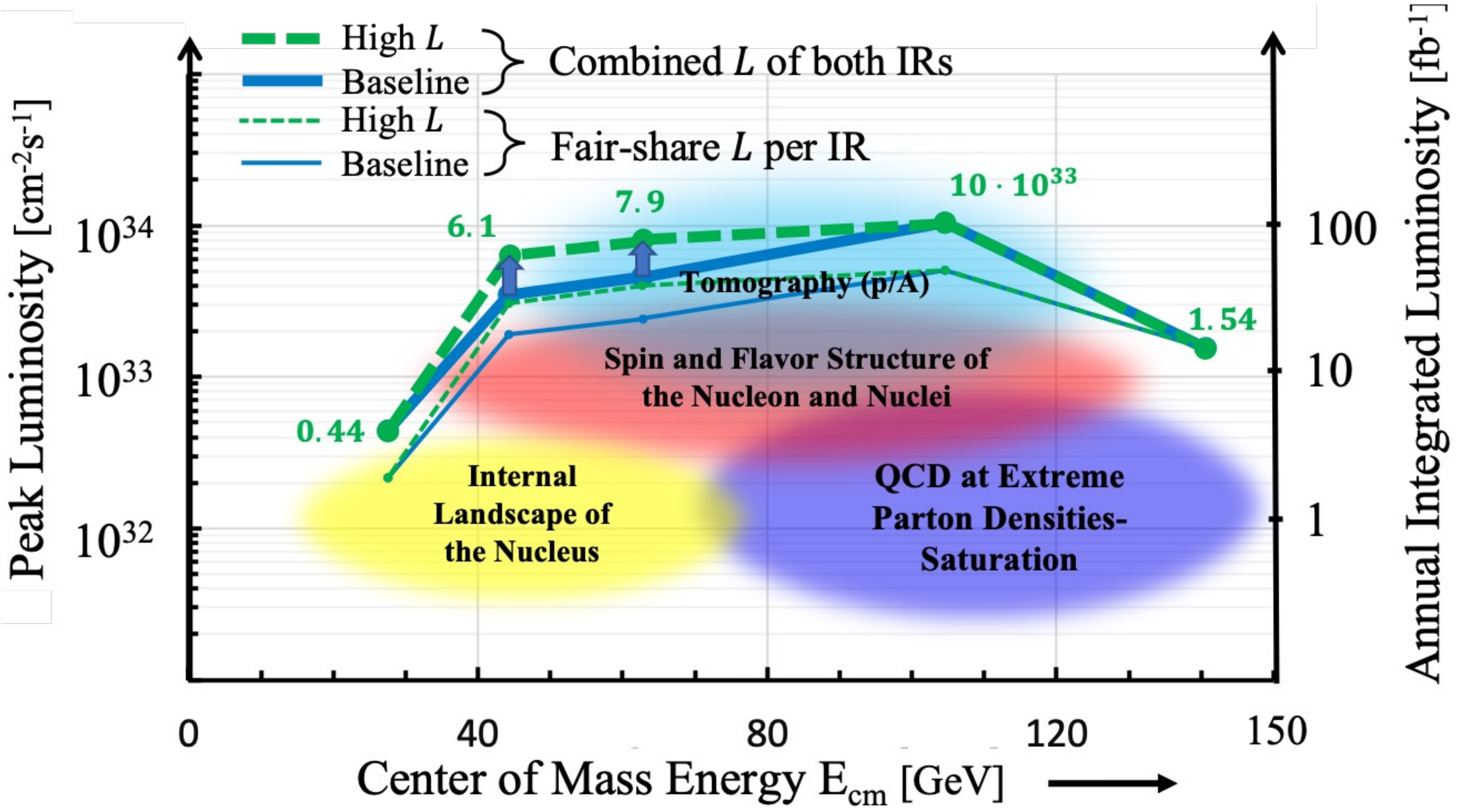
V = U = 1

Electron-Ion Collider (EIC)

EIC Accelerator Design



Center of Mass Energies:	20GeV - 140GeV
Luminosity:	10^{33} - 10^{34} $cm^{-2}s^{-1}$ / 10-100fb^{-1} / year
Highly Polarized Beams:	70%
Large Ion Species Range:	p to U
Number of Interaction Regions:	Up to 2!



slide courtesy of A. Deshpande

Summary

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,…, to be probed precisely at EIC

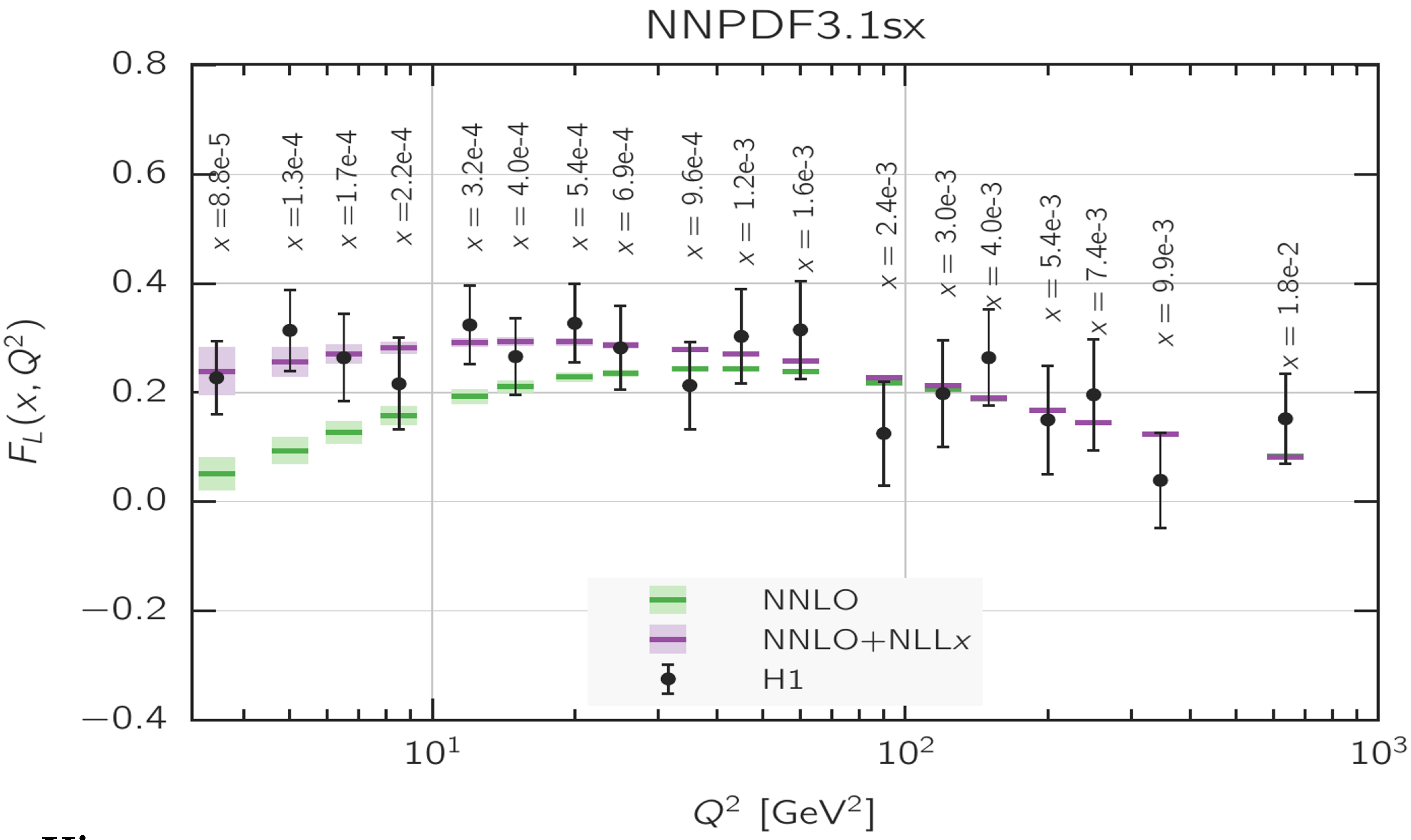
toward precision: NLO, sub-eikonal corrections, …

CGC is limited to small x (low p_t)

Need to better understand parton propagation in QCD fields

from weak to stronger to very strong fields

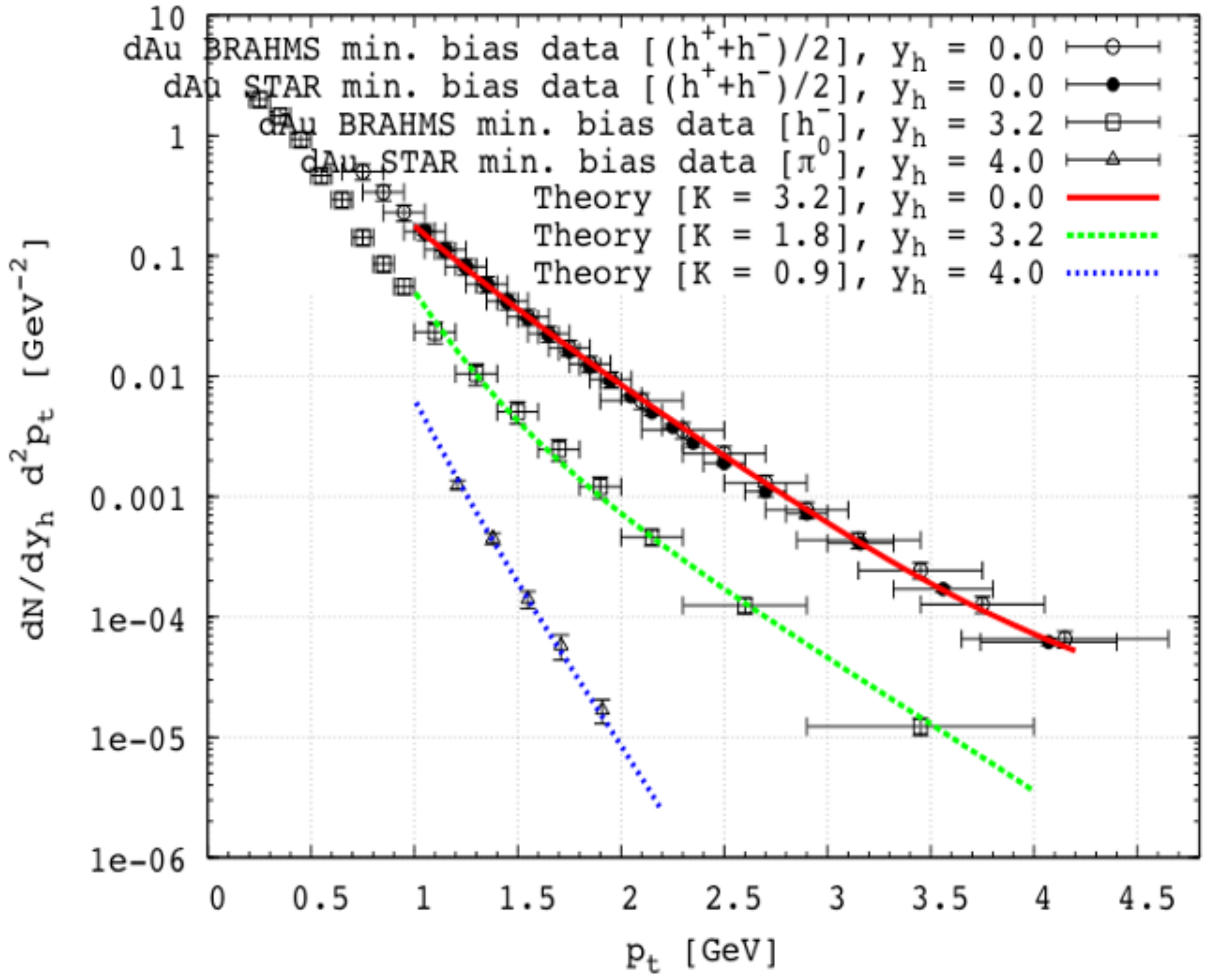
F_L at HERA

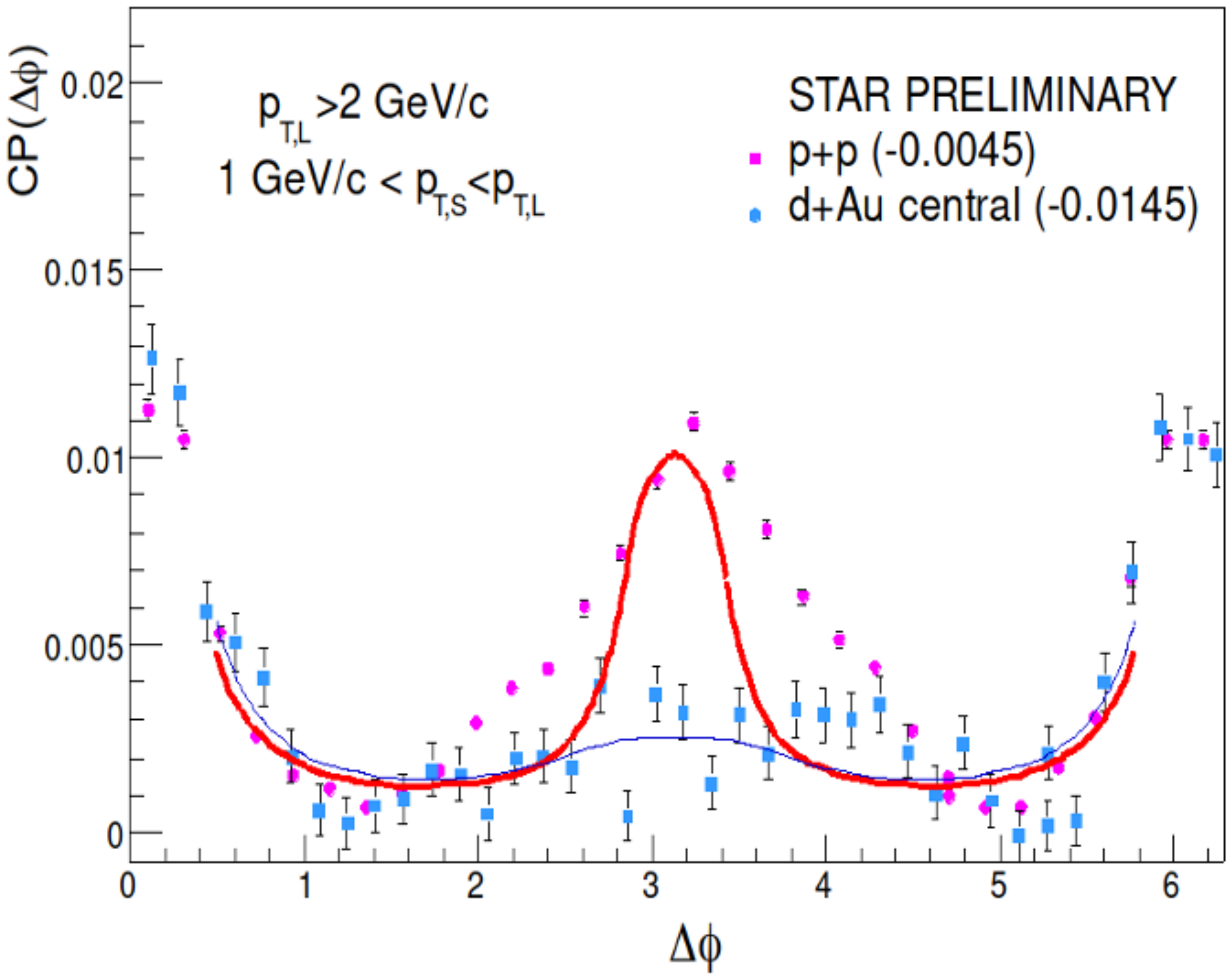


arXiv:1710.05935

CGC at RHIC

Single and double inclusive hadron production in dA collisions

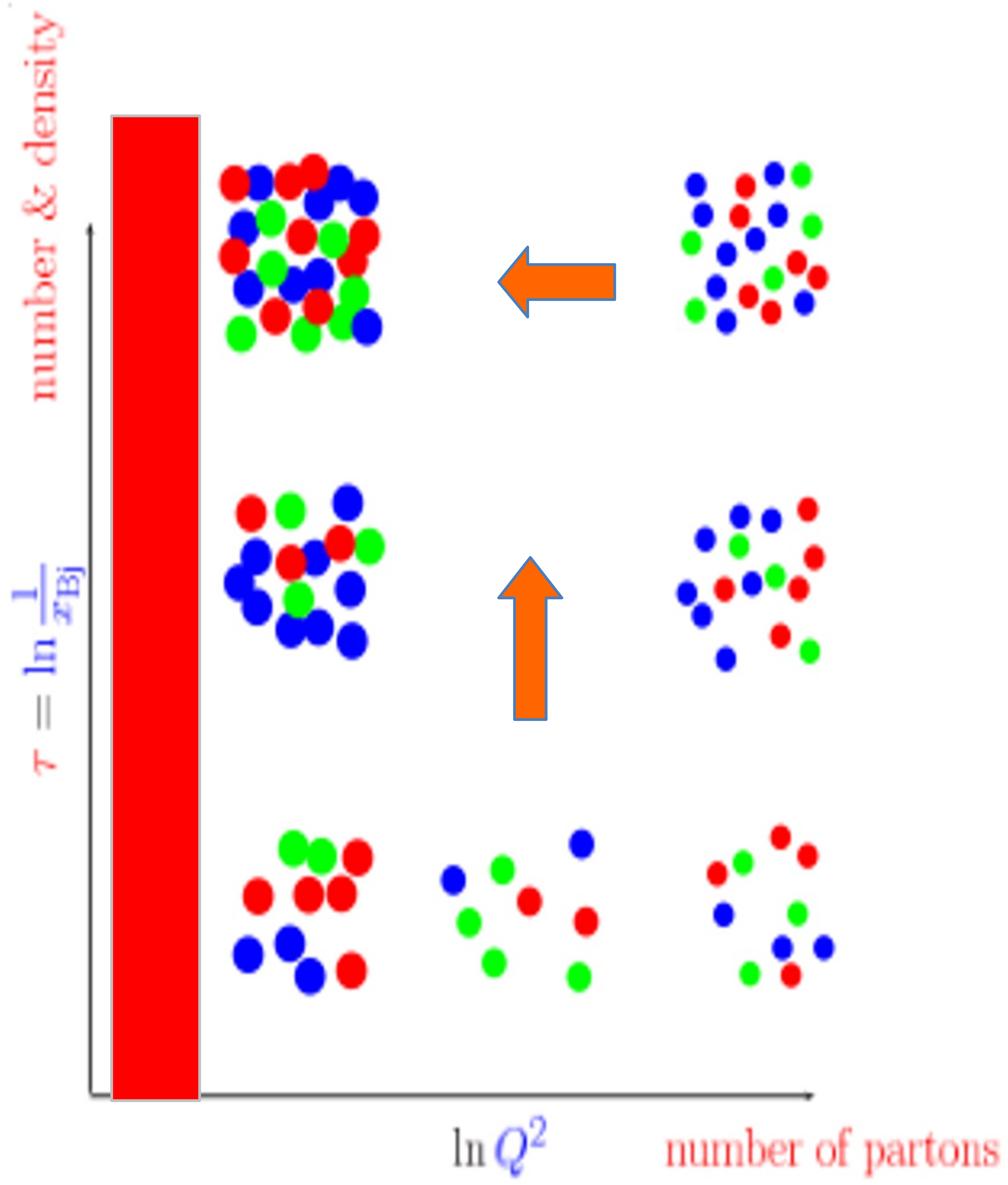




Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

QCD at small x:
many-body dynamics of universal gluonic matter (CGC)



How does this happen ?

How do correlation functions evolve ?

Are there scaling laws ?

Can CGC explain aspects of HIC ?

Initial conditions for hydro?
Thermalization ?