Non-perturbative resummation to study hot and dense nuclear matter

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Outline

- In covariant gauge, the gluon propagator is

$$D^{ab}_{\mu\nu}(K) = -\frac{\delta_{ab}}{K^2}\left[g_{\mu\nu} - (1-\xi)\,\frac{K_\mu K_\nu}{K^2}\right]$$

- Faddeev-Popov action

$$\begin{aligned} S &= S_{YM} + S_{GF} + S_{ghost} \\ &= S_{YM} + \int d^4x \left(\bar{c}^a \partial^\mu \left(D_\mu c\right)^a - \frac{1}{2\xi}\left(\partial_\mu A^{\mu a}\right)^2\right) \end{aligned}$$

- Gribov demonstrated for the first time in 1978 that the gauge condition proposed by Faddeev and Popov is not ideal.
- Gribov considered the question of, given a certain physical configuration, how many different gauge copies of this configuration obey the particular gauge condition.
- It can be shown that the Faddeev-Popov operator has zero modes in the gauge fixing. If the gauge field is infinitesimally small, this operator will not have zero modes.

- The set of all gauge fields where the Faddeev-Popov operator has no zero modes is called the "first Gribov region" Ω.
- In the Gribov quantization, the YM partition function in Euclidean space reads

$$Z = \int_{\Omega} \mathcal{D}A(x) V(\Omega) \delta(\partial \cdot A) \det[-\partial \cdot D(A)] e^{-S_{\mathrm{YM}}}$$

The restriction of the integration to the Gribov region is realized by inserting a function $V(\Omega)$ into the partition function, where

$$V(\Omega) = \theta[1 - \sigma(0)] = \int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\beta}{2\pi i \beta} e^{\beta[1-\sigma(0)]}$$

represents the no-pole condition. Here, $1 - \sigma(P)$ is the inverse of the ghost dressing function $Z_G(P)$.

- The integration variable β is identified as the Gribov mass parameter γ_G after some redefinition.

- Gribov's gluon propagator in the Landau gauge reads

$$D_A(P) = \delta^{ab} \frac{P^2}{P^4 + \gamma_{\mathrm{G}}^4} \left(\delta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right)$$

- The ghost propagator in the Landau gauge

$$D_c(P) = \delta^{ab} \frac{1}{1 - \sigma(P)} \cdot \frac{1}{P^2},$$

- The inverse of the ghost dressing function is

$$Z_G^{-1} \equiv [1 - \sigma(P)] = \frac{N_c g^2}{128\pi^2} \left[-5 + \left(3 - \frac{\gamma_G^4}{P^4}\right) \ln\left(1 + \frac{P^4}{\gamma_G^4}\right) + \frac{\pi P^2}{\gamma_G^2} + 2\left(3 - \frac{P^4}{\gamma_G^4}\right) \frac{\gamma_G^2}{P^2} \arctan \frac{P^2}{\gamma_G^2} \right]$$

Applicability of Gribov confinement scenario

Ref: D. Zwanziger,PRD76, 125014 (2007)

- Long-distance behavior of the color-Coulomb potential $V_{\rm coul}(R) \sim \sigma_{\rm coul}\, R$, $\sigma_{\rm coul} \sim 3\sigma$ and σ being the physical string tension between a pair of external quarks.

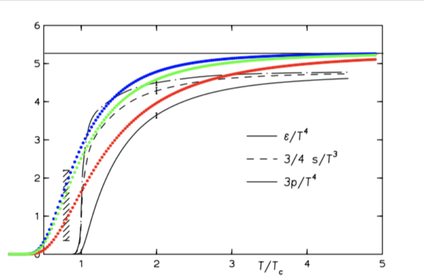
- It was also found numerically that the long-distance behavior of $V_{\rm coul}\,(R)$ is consistent with a linear increase, $\sigma_{\rm coul}\ > 0$, above the phase transition temperature, $T > T_c$, where σ vanishes.

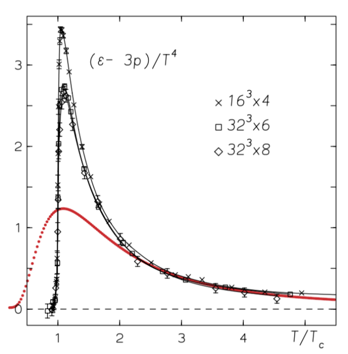
- Investigation of the temperature dependence of $\sigma_{\rm coul}$ revealed that in the deconfined phase, the Coulomb string tension increases with T, which is consistent with a magnetic mass $\sigma^{1/2}_{\rm coul}(T) \sim g_s^2(T)\,T$.

- Thus, from the numerical evidence one can say that the Gribov parameter is nonzero in the deconfined phase also.

Gluon Thermodynamics

- Gluon thermodynamics with Gribov term was calculated for the first time (in our knowledge) in 2005 in PRL 94, 182301 (2005) by D Zwanziger considering Coulomb gauge.
- The unknown Gribov parameter was determined by matching the lattice trace anomaly at high temperature





Asymptotic expression of γ_G

- K Fukushima & N Su [PRD88 (2013) 076008] used the Gribov modified gluon and ghost propagator and calculated the gluon thermodynamics in Landau gauge.
- The Gribov mass parameter was determined by the variational principle, leading to the following gap equation:

$$\sum\!\!\!\!\!\!\!\int_P \frac{1}{P^4+\gamma_{\mathrm{G}}^4} = \frac{d}{(d-1)N_{\mathrm{c}}g^2}$$

- Asymptotic solutions of the Gribov parameter is obtained as

$$\begin{aligned}\gamma_G &= \frac{3}{4}\frac{N_c}{4\sqrt{2}\pi}g^2T \qquad T\to\infty \\ &= \Lambda\exp\left(\frac{5}{12}-\frac{32\pi^2}{3N_{\mathrm{c}}g^2}\right) \qquad T\to 0.\end{aligned}$$

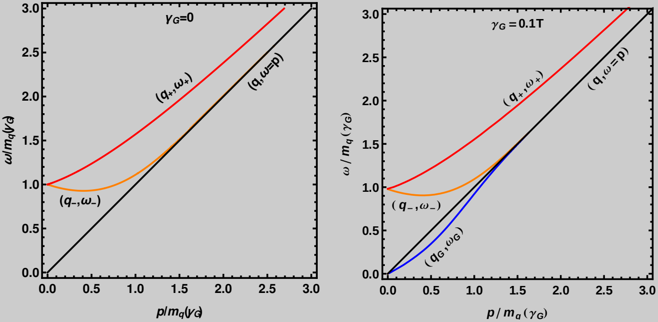
- The running coupling used $\alpha_s\,(T/T_c) \equiv \frac{g^2(T/T_c)}{4\pi} = \frac{6\pi}{11N_{\mathrm{c}}\ln[c(T/T_c)]}$

Gribov term as magnetic scale resummation in HTL

- Using the high temperature asymptotically form of γ_G, one can calculate quark self using Gribov-gluon propagator as

$$\Sigma(P) = (ig)^2 C_F \sum\!\!\!\!\!\!\!\int_{\{K\}} \gamma_\mu S_f(K)\gamma_\nu D^{\mu\nu}(P-K)$$

- Dispersion relation: PRL**114**, 161601(2015), N. Su & K. Tywoniuk



Covariant kinetic theory with Gribov resummation

- Equilibrium energy-momentum tensor

$$T^{\mu\nu}_{(0)} = \underbrace{\frac{g}{(2\pi)^4}\int d^4p\, 2\Theta\left(p^0\right)(2\pi)\,\delta\left(p^2+\frac{\gamma_G^4}{p^2}\right)}_{\int dp} p^\mu p^\nu\, f_0 + B_0(T)\, g^{\mu\nu}$$

- Gluon propagators have poles at $E_\pm = \sqrt{\boldsymbol{p}^2 \pm i\gamma_G^2}$
- The equilibrium pressure and energy density:

$$P_{eq} = \frac{g}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, \frac{\boldsymbol{p}^2}{6}\left(\frac{f_0^+}{E_+}+\frac{f_0^-}{E_-}\right) - B_0,$$

$$\varepsilon_{eq} = \frac{g}{(2\pi)^3}\int \mathrm{d}^3\boldsymbol{p}\, \frac{1}{2}\left(f_0^+ E_+ + f_0^- E_-\right) + B_0,$$

- the thermodynamic consistency is maintained only if

$$\frac{dB_0}{dT} + \frac{g}{(2\pi)^3}\gamma_G\frac{d\gamma_G}{dT}\int \mathrm{d}^3\boldsymbol{p}\, \frac{i}{2}\left(\frac{f_0^+}{E_+}-\frac{f_0^-}{E_-}\right) = 0.$$

Out-of-equilibrium and the Boltzmann equation

For the general non-equilibrium case, one can consider the energy momentum tensor $T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Delta T^{\mu\nu}$. In terms of the distribution function, we can write

$$T^{\mu\nu} = \int dp\, p^{\mu} p^{\nu} f + B(T)\, g^{\mu\nu}$$

- The energy-momentum conservation leads to

$$\partial^{\nu} B + \frac{\partial^{\nu}\Gamma_G}{\Gamma_G} \int dp\, p^2 f + \int dp\, p^{\nu} \left[p^{\mu}\partial_{\mu} f + \frac{1}{\Gamma_G} \left(\partial_{\mu}\Gamma_G\right) \partial^{\mu}_{(p)} \left(p^2 f\right) \right] = 0$$

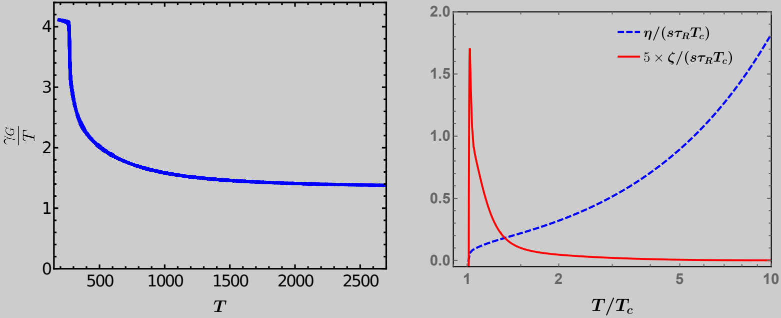
- In non-equilibrium case

$$p^{\mu}\partial_{\mu} f + \frac{p^2}{\Gamma_G} \left(\partial_{\mu}\Gamma_G\right) \partial^{\mu}_{(p)} f = C[f]$$

- Within RTA, the dissipative quantities can be written in terms of δf as,

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp\, p^\alpha p^\beta \delta f, \quad \Pi = -\frac{\Delta_{\alpha\beta}}{3}\int dp\, p^\alpha p^\beta \delta f,$$

- $\Delta^{\mu\nu}_{\alpha\beta} \to \frac{1}{2}(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha - \frac{2}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta})$ is the symmetric traceless projector orthogonal to u^μ



Ref: A. Jaiswal, NH, PLB 811(2020) 135936.

HQ diffusion coefficient within Gribov quantization

The momentum of the heavy quark evolves according to the Langevin equations as

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i, \quad \left\langle \xi_i(t)\xi_j\left(t'\right)\right\rangle = \kappa\delta_{ij}\delta\left(t - t'\right)$$

Now the diffusion constant in space, D_s, can be found by starting a particle at $x = 0$ at $t = 0$ and finding the mean-squared position at a later time,

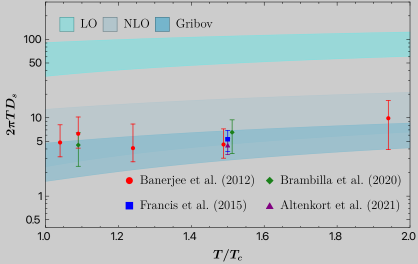
$$\langle x_i(t)x_j(t)\rangle = 2Dt\delta_{ij} \quad \rightarrow \quad 6D_s t = \left\langle x^2(t)\right\rangle .$$

The relation between position and momentum $x_i(t) = \int_0^t dt' \frac{p_i(t')}{M}$, we have

$$6D_s t = \int_0^t dt_1 \int_0^t dt_2 \frac{1}{M^2} \langle p(t_1)\, p(t_2)\rangle = \frac{6Tt}{M\eta_D} \qquad \Rightarrow D_s = \frac{T}{M\eta_D} = \frac{2T^2}{\kappa}.$$

- We have calculated momentum diffusion κ from $qH \to qH$ and $gH \to gH$ considering mediating gluon as Gribov gluon propagator
-

$$\kappa = \frac{1}{48M^2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^4 kk'}\int q^2 dq \int_{-1}^{1} d\cos\theta_{\boldsymbol{kq}}\delta\left(k'-k\right)q^2 \times \left[|\mathcal{M}|^2_{\text{quark}}\, n_F(k)\left[1-n_F\left(k'\right)\right]+|\mathcal{M}|^2_{\text{gluon}}\, n_B(k)\left[1+n_B\left(k'\right)\right]\right].$$



Phys.Lett.B 838 (2023) 137714

- The band shows the uncertainty arising from different schemes of lattice-measured $\alpha_s(T)$.

Screening mass

- At finite temperature, Lorentz symmetry is broken ⇒ temporal and spatial directions are, in general, unrelated.
- Correlation function in the time direction is used to define spectral functions, which gives information about the plasma's real-time properties, such as particle production rate.
- Correlation functions in spatial direction give info about
  (1) The length scale at which thermal fluctuation are correlated
  (2) The length scale at which the external charges are screened.
- Different operator gives different correlation lengths depending on their discrete and continuous global symmetry properties.
- Screening mass can show us how perturbative the medium is.
- Perturbative estimation: $m/T = 2\pi + \frac{g^2 C_F}{2\pi}\left(\frac{1}{2} + E_0\right)$

Detailed setup

- Definition:

$$C_z\left[O^a, O^b\right] = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^2\mathrm{x}_\perp \left\langle O^a\left(\tau, \mathrm{x}_\perp, z\right) O^b(0, \mathbf{0}, 0)\right\rangle$$

- In the limit of $z \to \infty, C_z\left[O^a, O^b\right] \sim \mathrm{e}^{-2\omega_0 z} = \mathrm{e}^{-mz}$,

$$\omega_n = 2\pi T\left(n + \frac{1}{2}\right), \qquad \zeta^{-1} = 2\pi T = m \to \text{Screening mass}$$

- For the correlation lengths ζ of mesonic observables, $\mathcal{O} = \bar{\psi}\Gamma F^a\psi$, where

$$\Gamma = \left\{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5\right\},$$

- Physical significance of some of these operators:
$\bar{\psi}\gamma_5 F^s\psi \propto \eta'$-meson ,
$\bar{\psi}\gamma_5 F^n\psi \propto$ pion,
$\bar{\psi}\gamma_0 F^s\psi \propto$ baryon number density,
$\bar{\psi}\gamma_0 F^n\psi \propto$ electric charge density (for $N_\mathrm{f} = 3$).

Next-to-leading order for flavour non-singlet correlators

- Infinitely many higher order graphs that need to be considered.

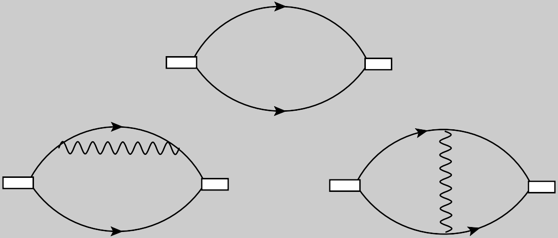


Figure: The diagrams which contribute to meson correlation function: (a) free theory correlator (b) quark self-energy graph (c) interaction of quark and antiquark through gluon exchange.

Continued....

- A convenient way of resummation of all the diagrams is offered by the effective field theory, namely NRQCD.
- Correlation lengths can be seen as (2+1)-dimensional bound states of heavy particles of mass "p_0", which is much larger than infrared scale gT, g^2T.
- Correlation function in leading order dominates only at zero Matsubara mode.

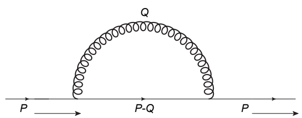
$$\mathcal{L}_E^{\psi} = \bar{\psi}\left[i\gamma_0 p_0 - ig\gamma_0 A_0 + \gamma_k D_k + \gamma_3 D_3\right]\psi$$

- The "diagonalized" on-shell effective Lagrangian for two independent light modes with a non-relativistic structure up to $\mathcal{O}(g^2)T$ is

$$\mathcal{L}_E^{\psi} = i\chi^{\dagger}\left(M - g_{\mathrm{E}}A_0 + D_t - \frac{\nabla_{\perp}^2}{2p_0}\right)\chi + i\phi^{\dagger}\left(M - g_{\mathrm{E}}A_0 - D_t - \frac{\nabla_{\perp}^2}{2p_0}\right)\phi$$

Matching conditions from QCD to NRQCD

- With Gribov propagator, quark self-energy becomes



$$\Sigma(P) = -ig^2 C_F \sum\!\!\!\!\!\!\int_Q \frac{\gamma_\mu (\not{P} - \not{Q})\gamma_\mu}{(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4}\right) + ig^2$$
$$\times \quad C_F \sum\!\!\!\!\!\!\int_Q \frac{\not{Q}(\not{P}-\not{Q})\not{Q}}{Q^2(P-Q)_f^2}\left(\frac{Q^2}{Q^4+\gamma_G^4} - \frac{\xi\, Q^2}{Q^4+\gamma_G^4}\right)$$

- With the above quark self-energy, the Euclidean dispersion relation on the QCD becomes

$$p_3 \approx i\Big[p_0 - g^2 C_F (I_1 + I_2)\Big]$$

Continued...

- $$I_1 = \frac{-1}{p_0}\int_0^\infty \frac{q^2dq}{(2\pi)^2}\left[\frac{n^+}{E_+} + \frac{n^-}{E_-}\right], \quad I_2 = \frac{1}{p_0}\left[\frac{-T^2}{24} + X\right]$$

  with $n^\pm \to$ B.E distribution function, $\tilde{n} \to$ F.D distribution function and $E_\pm = \sqrt{q^2 \pm \gamma_G^2}$ and

  $$X = \frac{\gamma_G^4}{T^2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{8EE_+E_-}\left[\left\{\frac{\tilde{n}+n^-}{i\pi - E + E_-} - \frac{\tilde{n}+n^-}{i\pi + E - E_-}\right\} - (n^- \to n^+)\right]\frac{1}{E_+ - E_-}$$

- On NRQCD$_3$ side, the pole location is simply $p_3 = iM$. Now, after doing the matching, we will get

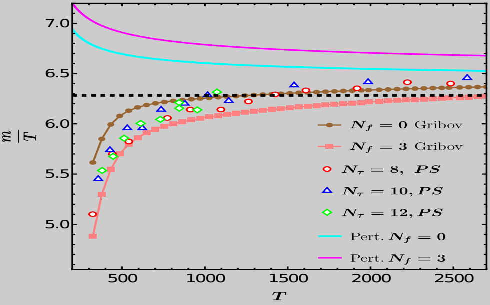
  $$M = p_0 - g^2 C_F (I_1 + I_2)$$

Screening mass

Phys.Lett.B 845 (2023) 138143

- There will be another contribution from the soft gluon exchanged and it can be obtained solving the energy eigen value of the screening states.
- EOM of the screening state becomes $\left[-\frac{\nabla_r^2}{p_0}+V(r)\right]\Psi_0 = g_{\mathrm{E}}^2\frac{C_F}{2\pi}E_0\Psi_0$
- After solving for E_0, the Screening mass can be obtained as

$$m = 2\pi T + g^2 T\frac{C_F}{2\pi}\left[E_0 - \frac{4\pi}{T}\left(I_1+I_2\right)\right].$$



Heavy quark dynamics

Phys. Rev. D109(2024), 114043

- The motion of HQs in the QCD medium can be considered as a Brownian motion and is well described by the Fokker-Planck equation

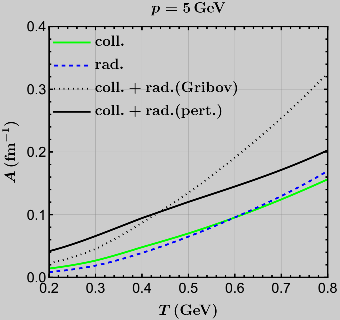
$$\frac{\partial f_{\rm HQ}}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\boldsymbol{p}) f_{\rm HQ} + \frac{\partial}{\partial p_j}\left[B_{ij}(\boldsymbol{p}) f_{\rm HQ}\right]\right],$$

- one can decompose the drag and diffusion tensors as

$$A_i = p_i A(p^2)\,, \qquad A = \langle\!\langle 1 \rangle\!\rangle - \frac{\langle\!\langle \boldsymbol{p}\cdot\boldsymbol{p}' \rangle\!\rangle}{p^2}\,.$$

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_0\left(p^2\right) + \frac{p_i p_j}{p^2} B_1\left(p^2\right)\,,$$

$$B_0 = \frac{1}{4}\left[\langle\!\langle p'^2 \rangle\!\rangle - \frac{\langle\!\langle (\boldsymbol{p}'\cdot\boldsymbol{p})^2 \rangle\!\rangle}{p^2}\right],\ B_1 = \frac{1}{2}\left[\frac{\langle\!\langle (\boldsymbol{p}'\cdot\boldsymbol{p})^2 \rangle\!\rangle}{p^2} - 2\langle\!\langle \boldsymbol{p}'\cdot\boldsymbol{p} \rangle\!\rangle + p^2\langle\!\langle 1 \rangle\!\rangle\right]$$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coll.
rad.
coll. + rad.(Gribov)
coll. + rad.(pert.)
A (fm^−1)
T (GeV)
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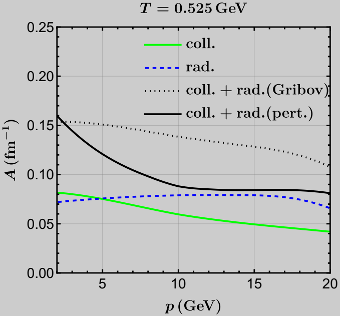
0.2
0.3
0.4
0.5
0.6
0.7
0.8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A (fm^−1)
p (GeV)
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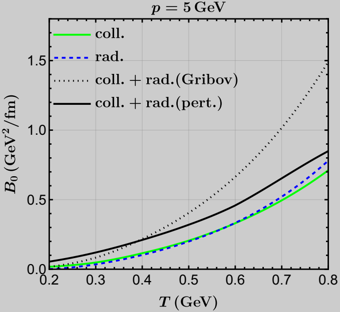
20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B0 (GeV²/fm)
T (GeV)
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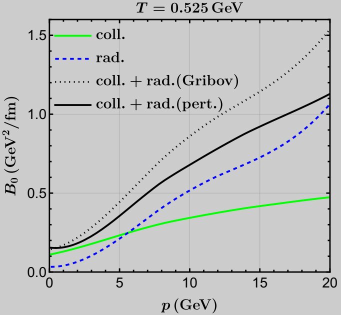
T = 0.525 GeV
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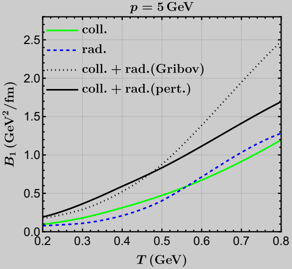
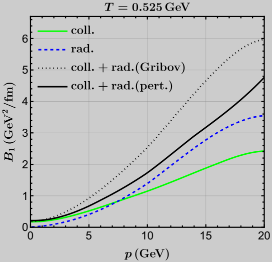
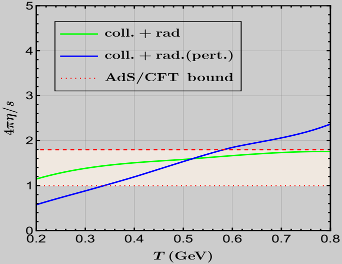




- If HQ momentum is considered in the $\hat{z}$ direction, then $B_0 = \frac{1}{4}\langle\langle k_\perp^2 \rangle\rangle = \frac{1}{4}\hat{q}$. where $\hat{q}$ is the jet quenching parameter.
- Up to the next-to-leading order $\eta/s = 1.63T^3/\hat{q}$.



Conclusion

- We have discussed the limitation of Faddeev-Popov way in non-perturbative regime and the way out by limiting calculation within first Gribov region.
- We have discussed our results for the covariant kinetic theory and transport coefficients for the Gribov Plasma.
- We have also discussed our recent results for the heavy-quark diffusion rate and QCD screening mass and heavy quark dynamics in Gribov Plasma.

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