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Revealing nucleus properties by exploring ultra-central symmetric heavy-ion collisions

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Dense & Strange Hadronic Matter Group, Technical University of Munich, Germany

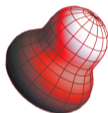
and S. M. A. Tabatabaee

School of particles and accelerators, IPM, Tehran, Iran

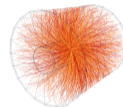
Based on: [arXiv:2406.13863]

Barcelona, Spain

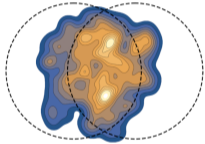
July 11th, 2024

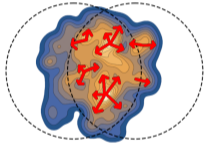


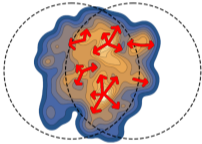
Low energy nuclear physics



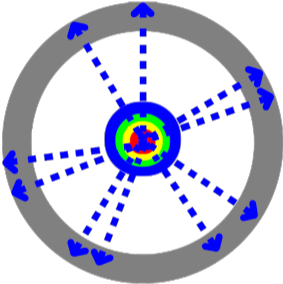
High energy heavy-ion collision

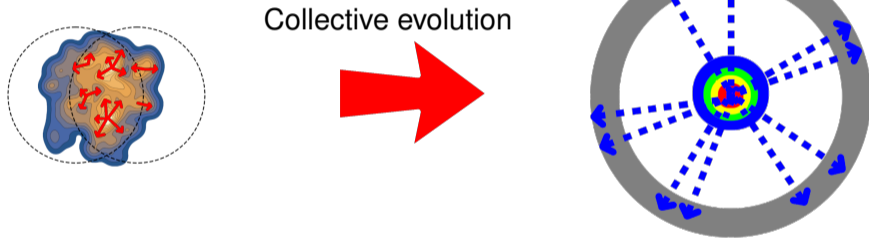






Collective evolution



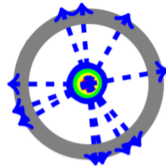
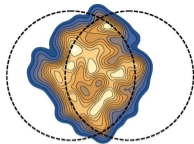


Initial spatial anisotropy \longrightarrow *Final momentum anisotropy*

[Jean-Yves Ollitrault, Phys. Rev. D 46, 229, 1992]

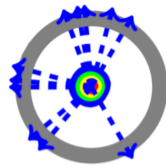
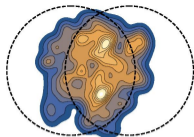
Fluctuation

- ▶ Nucleon position fluctuation:



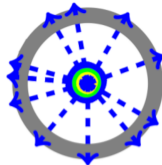
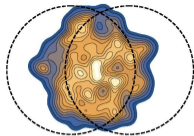
Fluctuation

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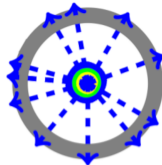
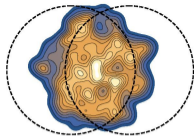
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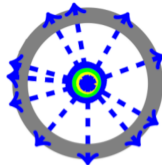
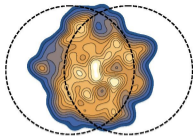
Fluctuation

- ▶ Nucleon position fluctuation:



Fluctuation

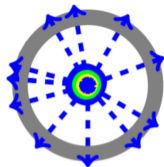
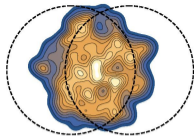
- ▶ Nucleon position fluctuation:



- ▶ Constituent weight fluctuation:

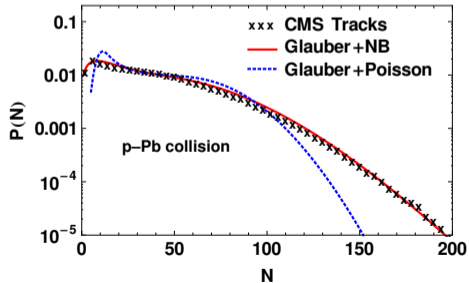
Fluctuation

- ▶ Nucleon position fluctuation:



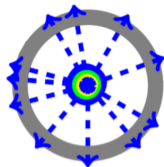
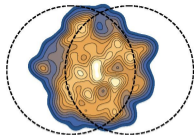
- ▶ Constituent weight fluctuation:

[P. Bozek et. al., PRC 88, 014903 (2013)]



Fluctuation

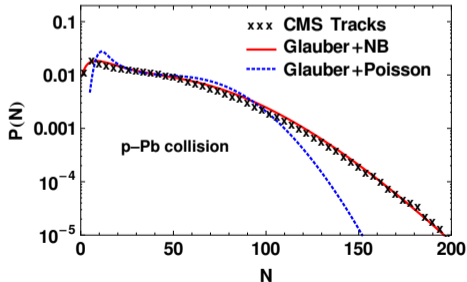
- ▶ Nucleon position fluctuation:



- ▶ Constituent weight fluctuation:

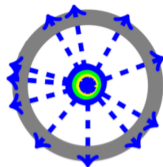
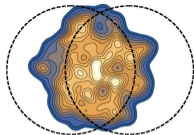
$$p_{\text{fluc}}(\gamma) = \frac{k^k}{\Gamma(k)} \gamma^{k-1} e^{-k\gamma}, \quad k = 1/\sigma_{\text{fluc}}^2.$$

[P. Bozek et. al., PRC 88, 014903 (2013)]



Fluctuation

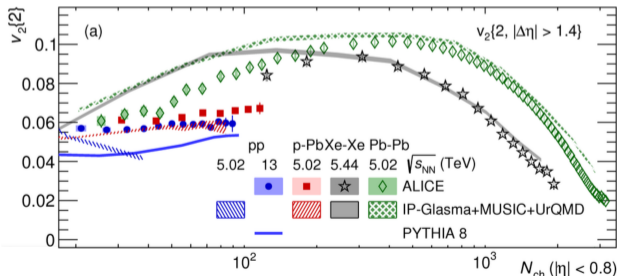
- ▶ Nucleon position fluctuation:



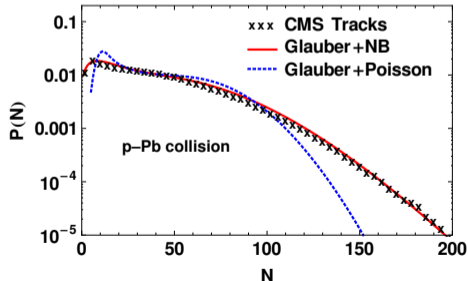
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[ALICE collaboration, PRL, 123, 142301 (2019)]

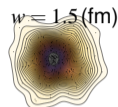
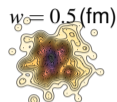
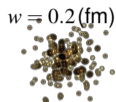
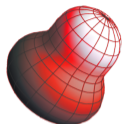


[P. Bozek et. al., PRC 88, 014903 (2013)]



$$* v_2^2\{2\} \equiv \langle v_2^2 \rangle \approx 2\sigma_v^2 + \bar{v}_2^2$$

Initial State



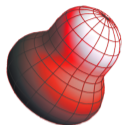
- ▶ $\rho_{\text{WS}}(\vec{x}_i)$: Woods-Saxon distribution,

$$\rho_{\text{WS}}(\vec{x}) = \frac{\rho_0}{1 + \exp [(|\vec{x}| - R(\theta, \phi))/a_0]},$$

$$R(\theta, \phi) = R_0(1 + \beta_2 Y_2^0(\theta, \phi) + \dots)$$

- ▶ $C_2(\vec{x}_1, \vec{x}_3)$: two-body correlation,

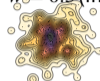
$$C_2(\vec{x}_1, \vec{x}_2) = \begin{cases} 0 & |\vec{x}_1 - \vec{x}_2| \leq d_{\min} \\ 1 & |\vec{x}_1 - \vec{x}_2| > d_{\min} \end{cases}$$



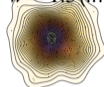
$w = 0.2$ (fm)



$w = 0.5$ (fm)



$w = 1.5$ (fm)



- ▶ $\rho_{WS}(\vec{x}_i)$: Woods-Saxon distribution,

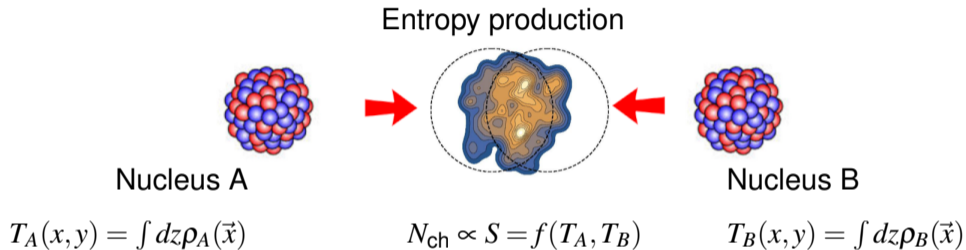
$$\rho_{WS}(\vec{x}) = \frac{\rho_0}{1 + \exp [(|\vec{x}| - R(\theta, \phi)) / a_0]},$$

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Parameter	Description
A	mass number
R_0	nuclear radius
a_0	skin thickness
β_2	quadrupole deformation
σ_{fluc}	Nucleon fluctuation
w	Nucleon width parameter
d_{\min}	Minimum inter-nucleon distance



Constituents interact and produce a medium.

Functionality of $f(T_A, T_B)$

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▶ **IP-Glasma**: (q is in the range 0.54 to 1.71) [1,2]:

$$f^{3/2}(T_A, T_B) \sim (T_A T_B)^{3q/4}$$

▶ **CGC model** [3]:

$$f^{3/2}(T_A, T_B) \propto \frac{T_A T_B}{(T_A + T_B)^{5/2}} [2T_A^2 + 7T_A T_B + 2T_B^2]$$

▶ **T_RENTo model** (p is a real parameter) [4] :

$$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

[1] Schenke, Tribedy, Venugopalan, PRL. 108, 252301 (2012), Schenke, Tribedy, Venugopalan, PRC 86, 034908 (2012)

[2] Nijs, van der Schee, (2023), arXiv:2304.06191 [nucl-th]

[3] Borghini, Borrell, Feld, Roch, Schlichting, PRC 107 (2023) 3, 034905

[4] Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901

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$$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

T_R ENTo, CGC model, (and IP-Glasma) are scale-invariant: $f(cT_A, cT_B) = cf(T_A, T_B)$.

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- ▶ **MC-Glauber model** ($\alpha \sim 0.1$) [5]:

$$f(T_A, T_B) = (T_A + T_B) + \alpha T_A T_B$$

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[5] B. Alver, M. Baker, C. Loizides, and P. Steinberg, (2008), arXiv:0805.4411 [nucl-ex]

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$$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

- ▶ **MC-Glauber model** ($\alpha \sim 0.1$) [5]:

$$f(T_A, T_B) = (T_A + T_B) + \alpha T_A T_B$$

- ▶ **Generalized T_RENTo model** (p, q are real parameters) [2]:

$$f^{3/2}(T_A, T_B) \propto E_{\text{ref}}^{2-2q} \left(\frac{T_A^p + T_B^p}{2} \right)^{q/p}$$

T_RENTo, CGC model, (and IP-Glasma) are scale-invariant: $f(cT_A, cT_B) = cf(T_A, T_B)$.

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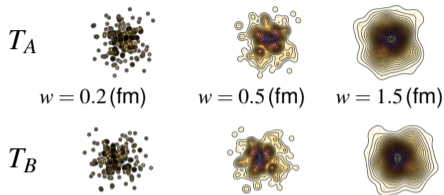
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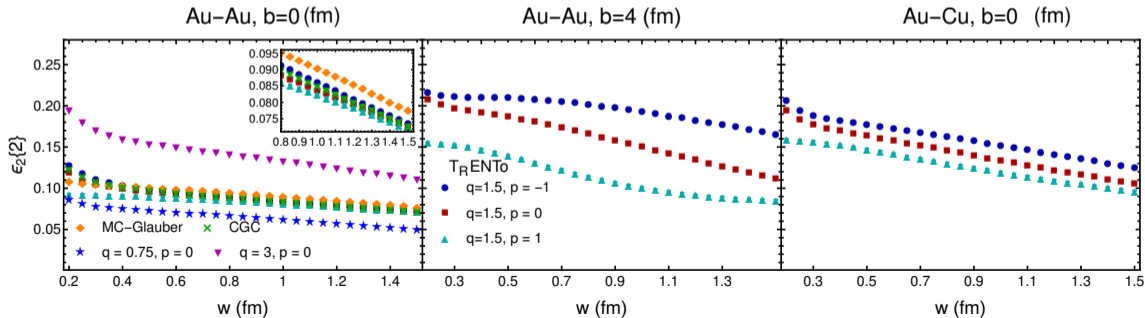
[5] B. Alver, M. Baker, C. Loizides, and P. Steinberg, (2008), arXiv:0805.4411 [nucl-ex]

Ultra-central Symmetric Collisions



- ▶ Same behavior for other initial state moments.
- ▶ From data, $w \approx 0.7$ (fm) is favored.

$$\epsilon_2^2\{2\} \equiv \langle \epsilon_2^2 \rangle$$

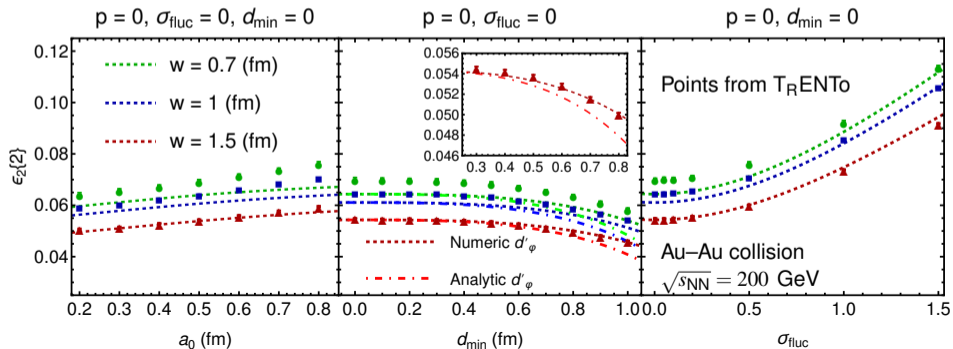


Ellipticity in ultra-central symmetric collisions

from an analogy with statistical mechanics

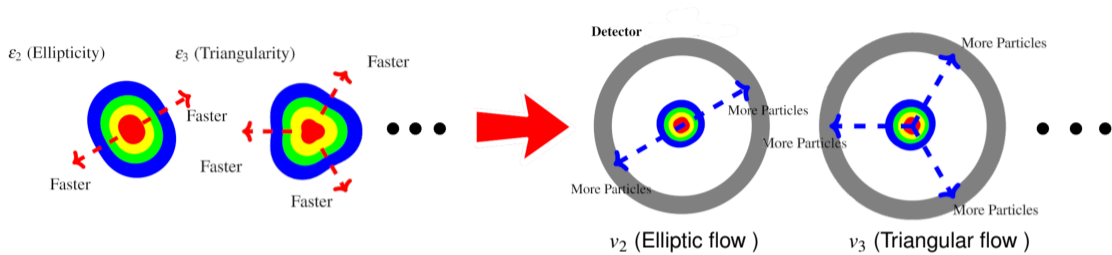
- ▶ Set of events \leftrightarrow set of ensembles
- ▶ Short range correlations \rightarrow cluster expansion method

$$\epsilon_2^2\{2\} \approx \frac{5}{7A} \frac{1}{(1 + (w/R_0)^2)^2} \left[1 + \alpha_\epsilon \left(\frac{a_0}{R_0} \right)^2 - \frac{A}{2} \left(\frac{d_{\min}}{R_0} \right)^3 + \sigma_{\text{fluc}}^2 + b_{2,\epsilon} \beta_2^2 + \dots \right],$$

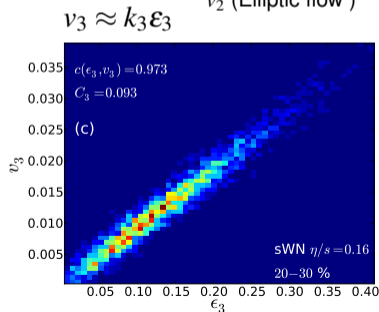
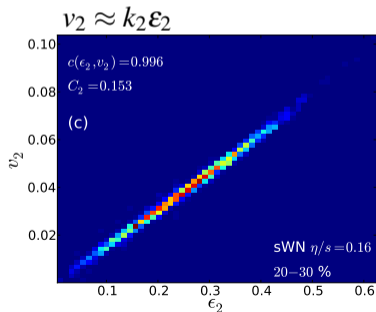
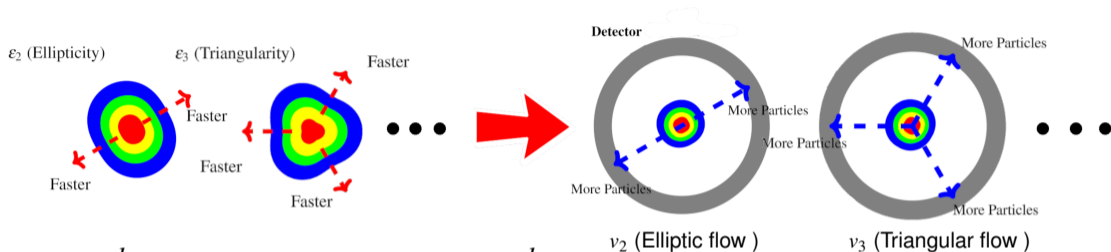


Collective expansion

Linear Response approximation



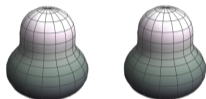
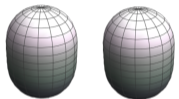
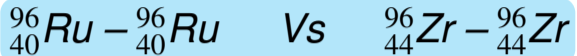
Linear Response approximation

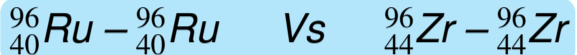


$$v_n \approx k_n \epsilon_n$$

for $n = 2, 3$

Deformed nuclei and isobar ratio





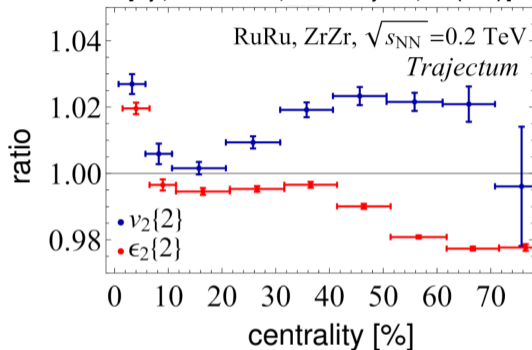
$$\frac{v_2^{\text{Ru}}\{2\}}{v_2^{\text{Zr}}\{2\}} \approx \frac{\varepsilon_2^{\text{Ru}}\{2\}}{\varepsilon_2^{\text{Zr}}\{2\}} \approx 1 + \lambda_\beta \left[(\beta_2^{\text{Ru}})^2 - (\beta_2^{\text{Zr}})^2 \right] + \lambda_a \left[a_0^{\text{Ru}} - a_0^{\text{Zr}} \right] / a_0 + \lambda_R \left[R_0^{\text{Ru}} - R_0^{\text{Zr}} \right] / R_0$$

- ▶ The dependence on deformation, $1 + \lambda_\beta \left[(\beta_2^{\text{Ru}})^2 - (\beta_2^{\text{Zr}})^2 \right]$, has been studied in [Zhang, Jia, PRL, 128, 022301].

Deformed nuclei and isobar ratio



[Nijs, van der Schee, SciPost Phys. 15, 041 (2023)]



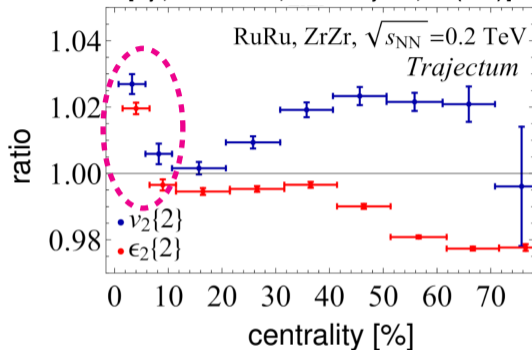
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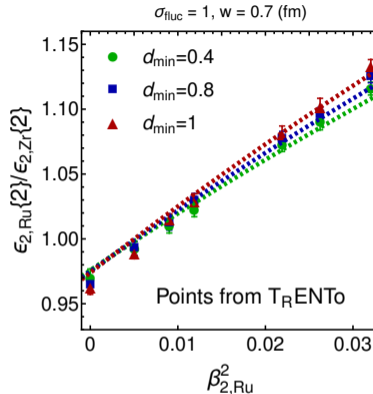
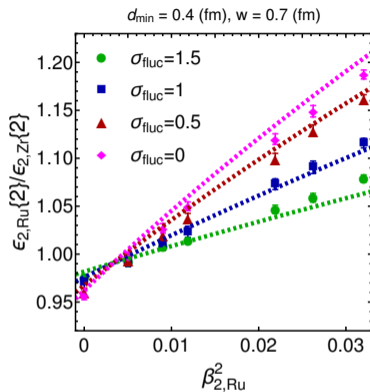
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$$\frac{v_2^{\text{Ru}}\{2\}}{v_2^{\text{Zr}}\{2\}} \approx \frac{\epsilon_2^{\text{Ru}}\{2\}}{\epsilon_2^{\text{Zr}}\{2\}} \approx 1 + \lambda_\beta \left[(\beta_2^{\text{Ru}})^2 - (\beta_2^{\text{Zr}})^2 \right] + \lambda_a \left[a_0^{\text{Ru}} - a_0^{\text{Zr}} \right] / a_0 + \lambda_R \left[R_0^{\text{Ru}} - R_0^{\text{Zr}} \right] / R_0$$

- The dependence on deformation, $1 + \lambda_\beta \left[(\beta_2^{\text{Ru}})^2 - (\beta_2^{\text{Zr}})^2 \right]$, has been studied in [Zhang, Jia, PRL, 128, 022301].

Isobar ratio in ultra-central collisions



$$\frac{v_2^{Ru}\{2\}}{v_2^{Zr}\{2\}} \approx 1 + \lambda_\beta(A, R_0, a_0, \sigma_{\text{fluc}}, d_{\min}) \delta\beta^2 + \lambda_a(A, R_0, a_0, \sigma_{\text{fluc}}, d_{\min}) \frac{\delta a}{R_0} + 4 \frac{\delta R}{R_0}$$

$$\lambda_\beta(A, R_0, a_0, \sigma_{\text{fluc}}, d_{\min}) = \frac{21A/40\pi}{(1 + \sigma_{\text{fluc}}^2)(1 + 6\pi^2 a_0^2/R_0^2) - \frac{A}{2} \left(\frac{d_{\min}}{R_0}\right)^3}$$

1. More realistic short-range correlation

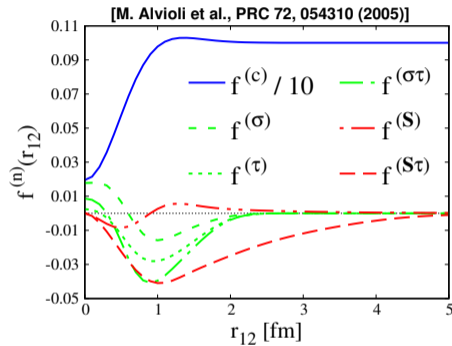
- Spin-isospin NN correlation
- Neutron skin thickness
- Sampling short-range correlations (see [1,2])

[1] M. Alvioli and M. Strikman, PRC 100, 024912 (2019)

[2] Luzum, Hippert, Ollitrault, Eur.Phys.J.A 59 (2023) 5, 110, Eur.Phys.J.A 59 (2023) 110

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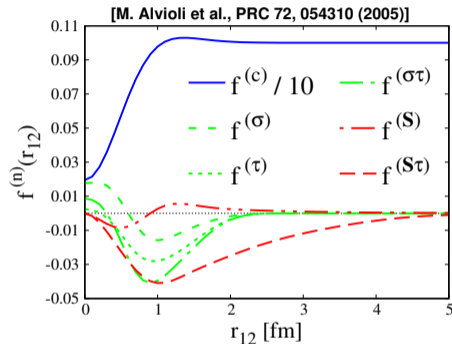
1. More realistic short-range correlation

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- Neutron skin thickness
- Sampling short-range correlations (see [1,2])

2. Calculate other flow observables

- A large set of flow observables have been developed in the past years [3],

$$v_3\{2\}, \quad v_2\{4\}, \quad \langle [p_T] \rangle, \quad \text{SC}(2,3) = \langle v_2^2 v_3^2 \rangle - \langle v_2^2 \rangle \langle v_3^2 \rangle, \quad \langle v_2^3 v_3^2 \cos(2\Phi_3 - 3\Phi_2) \rangle, \dots$$



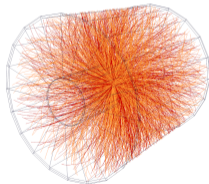
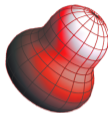
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[2] Luzum, Hippert, Ollitrault, Eur.Phys.J.A 59 (2023) 5, 110, Eur.Phys.J.A 59 (2023) 110

[3] Borghini, et al., PRC 64 (2001) 054901. - Bilandzic, et al., PRC 89 (6) (2014) 064904. - A. Bilandzic, M. Lesch, C. Mordasini, SFT, PRC 105, 024912 (2022)

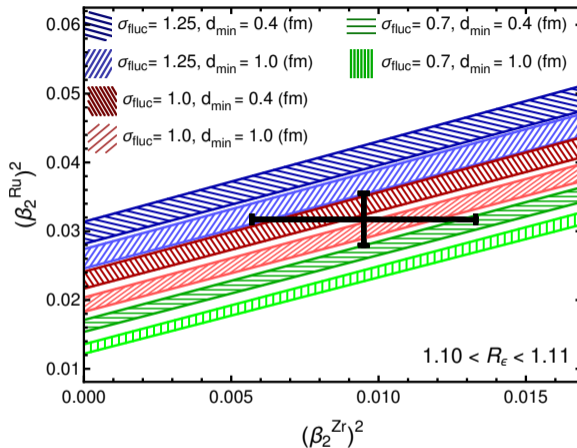
Summary

- ▶ Studying ultra-central symmetric collisions provides a great opportunity to investigate nuclear structure.
- ▶ We estimated the dependence of the elliptic flow on the initial state parameters, including nuclear structure.
- ▶ We demonstrated that the isobar ratio in ultra-central collisions is sensitive to constituent weight fluctuation and short-range correlations.



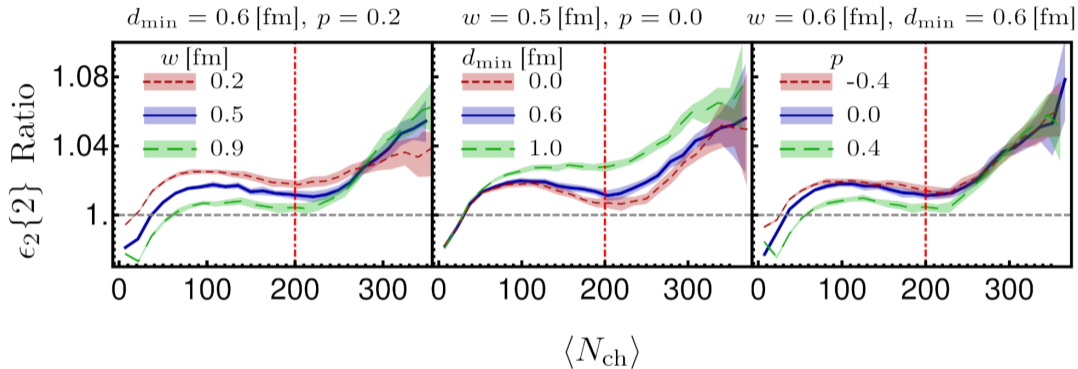
Thank You!

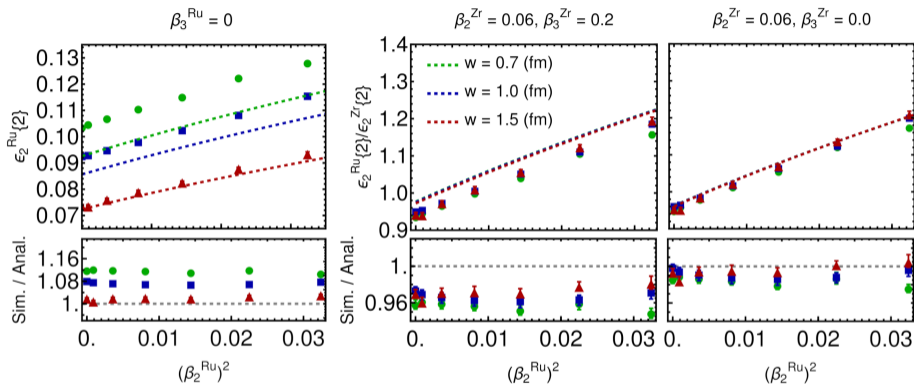
Backup Slides

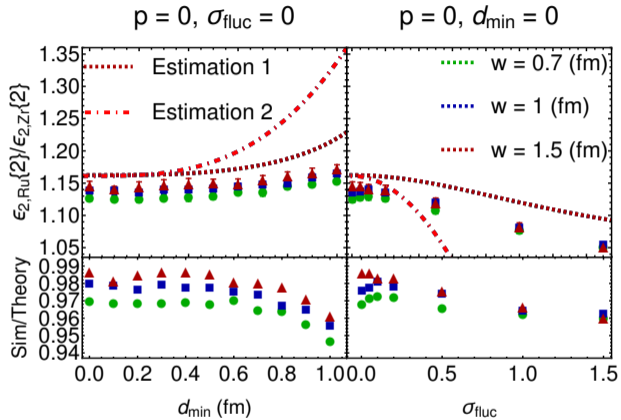


Equation $v_2^{\text{Ru}}\{2\}/v_2^{\text{Zr}}\{2\} = \text{constant}$ put a constraint on the nuclear structure parameters.

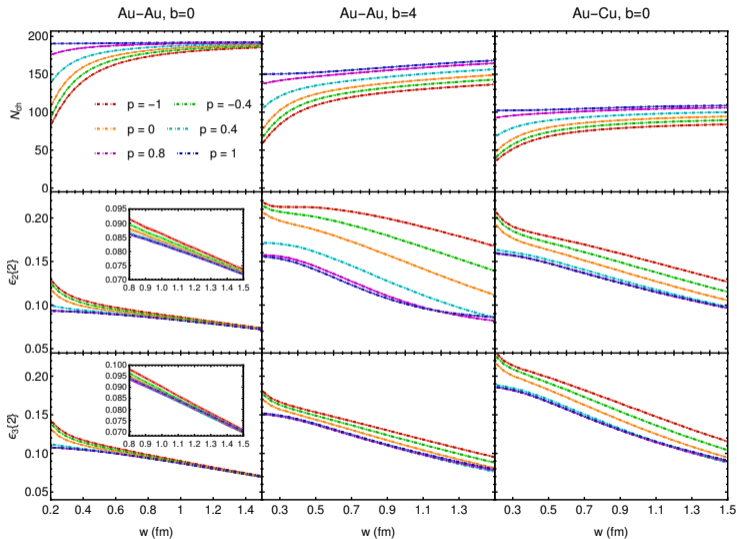
* Data point from: [Pritychenko et al, Atom.Data Nucl.Data Tabl. 107, 1-139 (2016)]



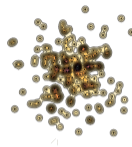




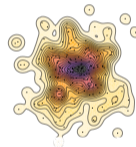
The effect of scale-invariance in ultracentral symmetric collision



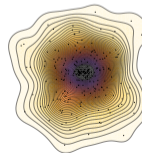
T_A



$w = 0.2$ (fm)

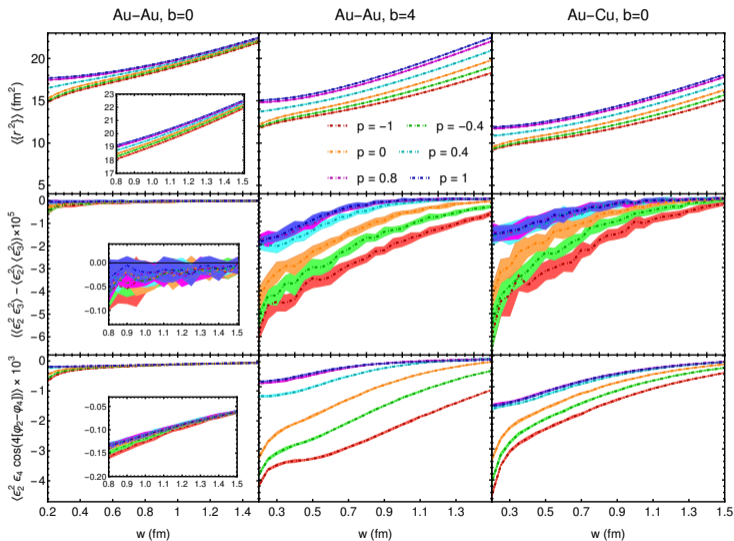


$w = 0.5$ (fm)

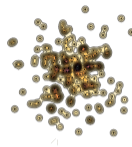


$w = 1.0$ (fm)

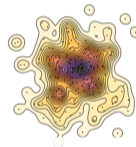
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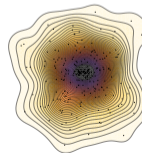
T_A



$w = 0.2$ (fm)



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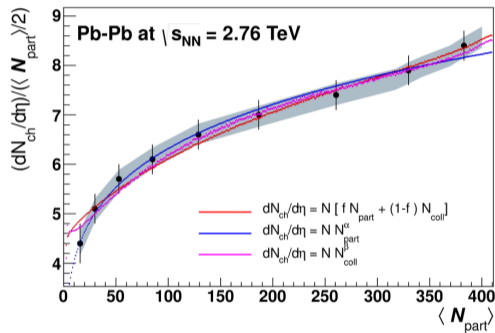
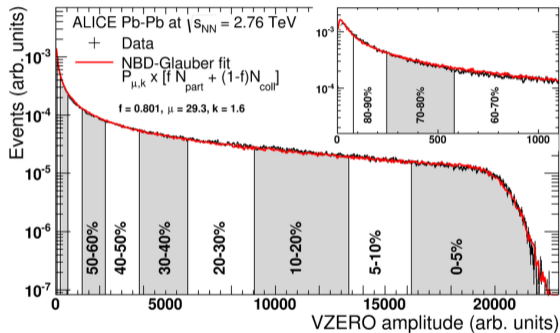


$w = 1.0$ (fm)

Charge multiplicity and function $f(N_A, N_B)$

$$N_{part} = N_A + N_B, \quad N_{col} = N_A N_B$$

[ALICE collaboration, Phys.Rev.Lett. 106 (2011) 032301], [ALICE collaboration, Phys.Rev.C 88 (2013) 4, 044909]



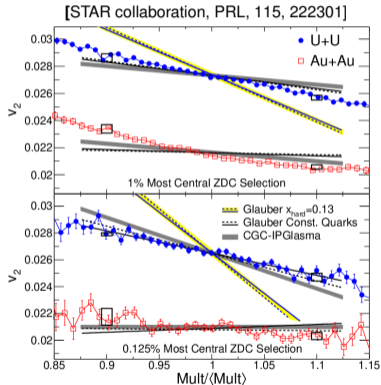
Model	Normalization	Error	Fit Par.	Error	χ^2/NDF
$f \cdot N_{part} + (1-f) \cdot N_{coll}$	2.441	0.281	$f = 0.788$	0.021	0.347
N_{part}^α	1.317	0.116	$\alpha = 1.190$	0.017	0.182
N_{coll}^β	4.102	0.297	$\beta = 0.803$	0.012	0.225

Functionality of $f(T_A, T_B)$

- ▶ $f_{\text{Glauber}}(T_A, T_B) = (T_A + T_B) + \alpha T_A T_B$
- ▶ U-U collisions disfavor binary term [Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901]

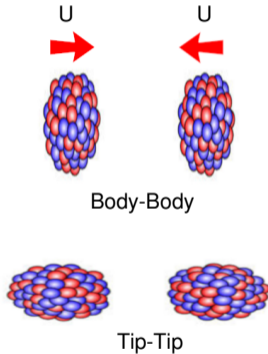
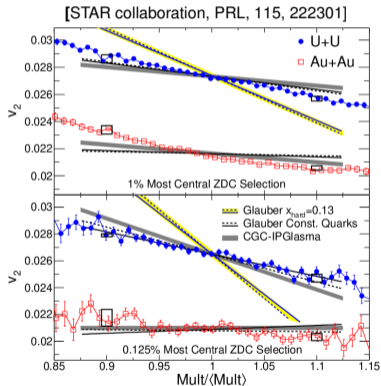
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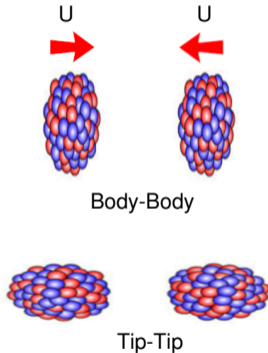
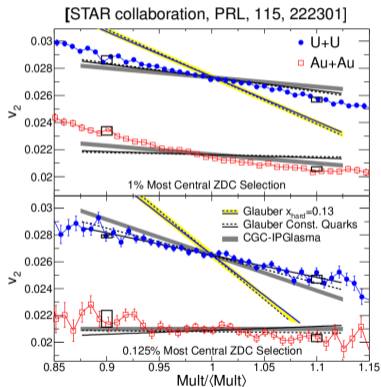
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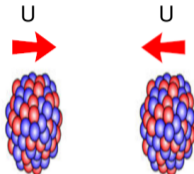
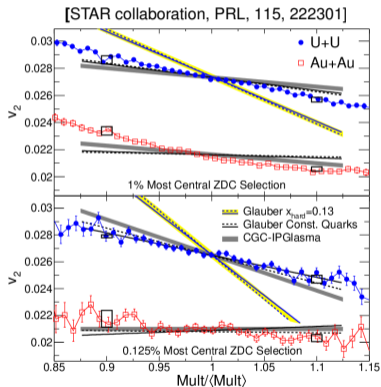


- ▶ Less binary collision
- ▶ Larger ellipticity

- ▶ More binary collision
- ▶ Smaller ellipticity

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Body-Body

- ▶ Less binary collision
- ▶ Larger ellipticity



Tip-Tip

- ▶ More binary collision
- ▶ Smaller ellipticity

$$f(cT_A, cT_B) \approx cf(T_A, T_B)$$

- ▶ T_RENTo event generator:

$$f(T_A, T_B) \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

Assuming scaling invariance, $f(cT_A, cT_B) = cf(T_A, T_B)$:

For the **ultracentral symmetric** A–A collisions with **large nucleon size**:

- ▶ The participants thickness function A and B are “similar” to each other,

$$T_A \approx T_B$$

- ▶ $f(cT_A, cT_A) = cf(T_A, T_A) \rightarrow g(cT_A) = cg(T_A) \rightarrow g(x)$ **is linear!**

For the most central symmetric A–A collisions:

observables should have small dependence on functionality $f(T_A, T_B)$.

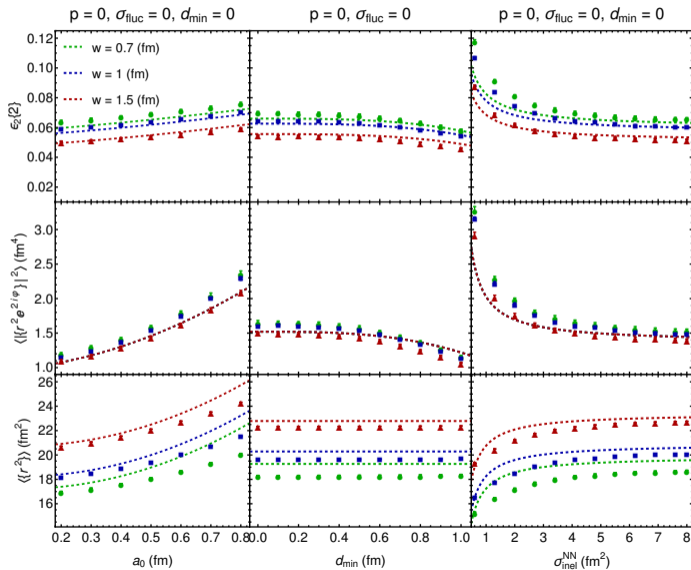
$$\langle \{r^2\} \rangle^2 = \frac{4R_0^4 \omega^2}{25} \left[\omega^2 + \alpha_r \left(\frac{a_0}{R_0} \right)^2 + s_r \left(\frac{a_0}{R_0} \right) \left(\frac{\pi R_0^2/A}{\sigma_{inel}^{NN}} \right) + b_{2,r} \beta_2^2 + b_{3,r} \beta_3^2 + \dots \right]$$

$$\left\langle \left| \{r^2 e^{2i\varphi}\} \right|^2 \right\rangle = \frac{1}{A} \frac{4R_0^4}{35} \left[1 + \alpha_\varphi \left(\frac{a_0}{R_0} \right)^2 + s_\varphi \left(\frac{a_0}{R_0} \right) \left(\frac{\pi R_0^2/A}{\sigma_{inel}^{NN}} \right) + d_\varphi \left(\frac{d_{min}}{R_0} \right)^3 + v_\varphi \frac{\sigma_{fluc}^2}{n_c} + b_{2,\varphi} \beta_2^2 + \dots \right]$$

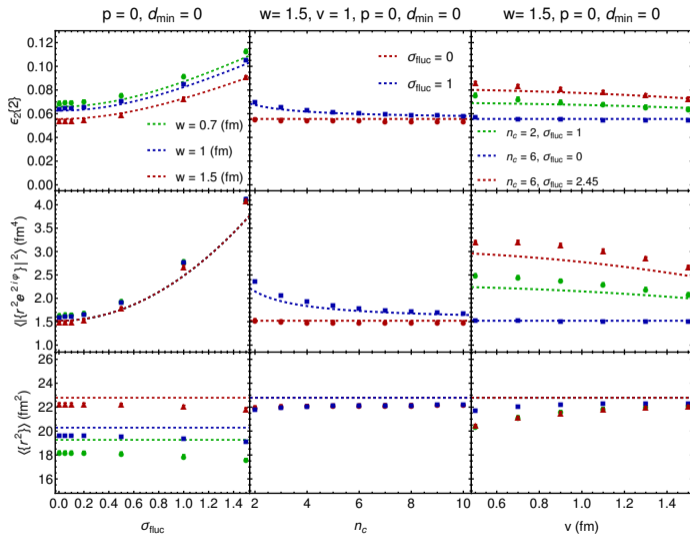
$$\varepsilon_2^2 \{2\} = \frac{5}{7} \frac{1}{A \omega^4} \left[1 + \alpha_\varepsilon \left(\frac{a_0}{R_0} \right)^2 + s_\varepsilon \left(\frac{a_0}{R_0} \right) \left(\frac{s_\perp}{\sigma_{inel}^{NN}} \right) + d_\varphi \left(\frac{d_{min}}{R_0} \right)^3 + v_\varphi \frac{\sigma_{fluc}^2}{n_c} + b_{2,\varepsilon} \beta_2^2 + b_{3,\varepsilon} \beta_3^2 + \dots \right]$$

coefficient	value
ω^2	$1 + 5(w/R_0)^2$
α_r	$14\pi^2/3$
α_φ	$6\pi^2$
s_r	-6
s_φ	$51/4$
d'_φ	$-A + \alpha_\varphi A (a_0/R_0)^2$
d_φ	$d'_\varphi - \alpha_r A (a_0/R_0)^2$
v_φ	$1 + 10(w^2 - v^2)/R_0^2 + \alpha_\varphi (a_0/R_0)^2$
$b_{2,r}$	$29/8\pi$
$b_{3,r}$	$7/2\pi$
$b_{2,\varphi}$	$21A/20\pi$
α_ε	$\alpha_\varphi - \alpha_r/\omega^2$
s_ε	$s_\varphi - s_r/\omega^2$
$b_{2,\varepsilon}$	$b_{2,\varphi} - b_{2,r}\omega^2$
$b_{3,\varepsilon}$	$-b_{3,r}/\omega^2$

Analytical result



Analytical result



Generating random sources with two-body correlation

Our analytical analysis has revealed a problem in creating sources with correlations.

To implement the minimum distance:

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To implement the minimum distance:

1. **Iterative** (used in T_RENTo [1]): Generate the n th source based on WS and compare its distance to all other $(n - 1)$ th sources. If there is any distance smaller than d_{\min} reject the n th source and generate a new one.

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2. **Shifting position** [2]: Generate A sources at once and shift positions symmetrically for pairs with distance less than d_{\min} .

[1] Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901

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2. **Shifting position** [2]: Generate A sources at once and shift positions symmetrically for pairs with distance less than d_{\min} .
3. **Direct**: Generate A sources at once and compare all mutual distances. If there is any pair with distance smaller than d_{\min} reject all A sources and generate a new set.

[1] Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901

[2] Luzum, Hippert, Ollitrault, Eur.Phys.J.A 59 (2023) 5, 110, Eur.Phys.J.A 59 (2023) 110

Comparing “Direct” method, iterative method and shifting method

