Far-from-equilibrium attractors in kinetic theory for a mixture of quark and gluon fluids

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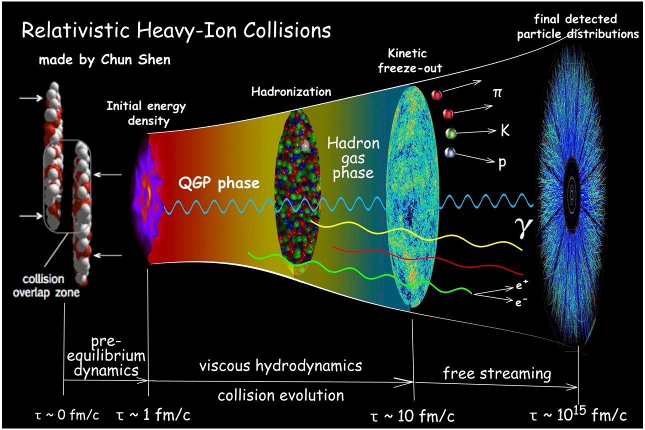
Contents

Main features of heavy-ion physics

The produced medium

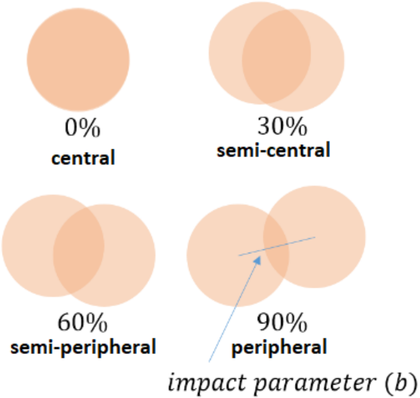


Experiments at **LHC** and **RHIC** $\longrightarrow$ a hot, dense medium is believed to be formed

- This state of matter is called **Quark-Gluon Plasma** (**QGP**)
- Our interest: connection between the pre-equilibrium and hydro phases

Some phenomenological quantities

A collision can be characterized with different values of **centrality**



[1808.01411]

Centrality quantifies the overlapping between **two colliding ions**

Some phenomenological quantities

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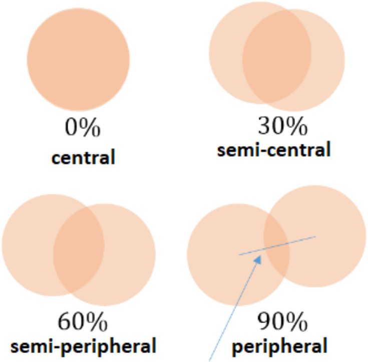

[1808.01411]

Centrality quantifies the overlapping between **two colliding ions**

We compare values of the **specific viscosity** η/s for **different fluids**

Fluid	T [K]	η [Pa · s]	η/s
H_2O	370	$2.9 \cdot 10^{-4}$	8.2
^{4}He	2.0	$2.9 \cdot 10^{-6}$	1.9
H_2O	650	$6.0 \cdot 10^{-5}$	2.0
^{4}He	5.1	$1.7 \cdot 10^{-6}$	0.7
QGP	$2 \cdot 10^{12}$	$\lesssim 5 \cdot 10^{11}$	$\lesssim 0.4$

[0712.3715]

- **QGP** has the **lowest specific viscosity** due to the **huge amount of entropy** produced in heavy-ion collisions

η/s is related to the relaxation time $\tau_{\rm eq}$

Relativistic Hydrodynamics in heavy-ion collisions (1)

Hydrodynamics leads to accurate predictions of the final **hadron spectrum**, which can be calculated according to the **Cooper-Frye formula**:

$$E\,\frac{dN}{d^3p} = \int_{\Sigma_{\rm FO}} d\Sigma_\mu\, p^\mu\, f(x;p)\,, \quad \Sigma_{\rm FO}:\ \text{3D hypersurface} \longleftrightarrow T(x) = T_{\rm FO}$$

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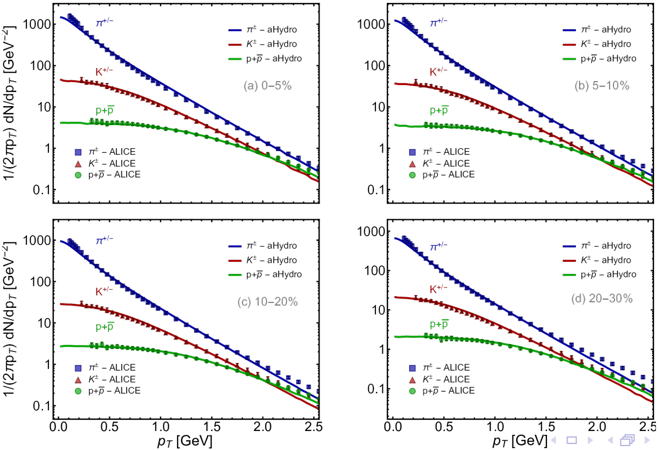


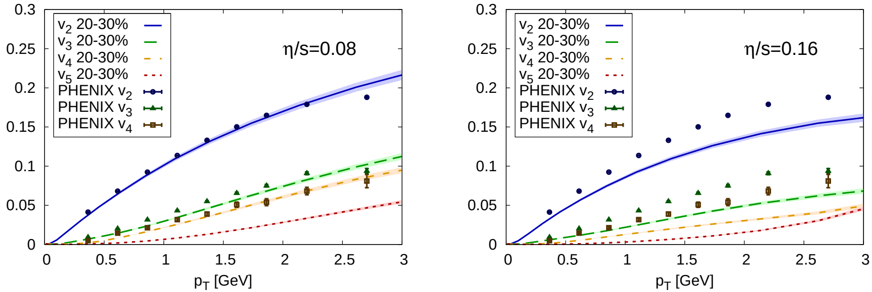
Excellent agreement with experimental data for $p_T \lesssim 2$ **GeV**

[1705.10191]

Relativistic Hydrodynamics in heavy-ion collisions (2)

Azimuthal asymmetry of the **hadron spectra** (event-by-event fluctuations)

$$\frac{dN}{d\varphi} = \frac{\overline{N}}{2\pi}\left\{1 + 2\sum_{k\geq 2} v_k \cos\left[k\left(\varphi - \psi_k\right)\right]\right\}, \quad \psi_k \to \text{azimuthal orientation}$$



[1109.6289]

Relativistic Hydrodynamics: converts **momentum anisotropy** in a fluid (out of equilibrium) into **spatial anisotropy** of the hadron distributions

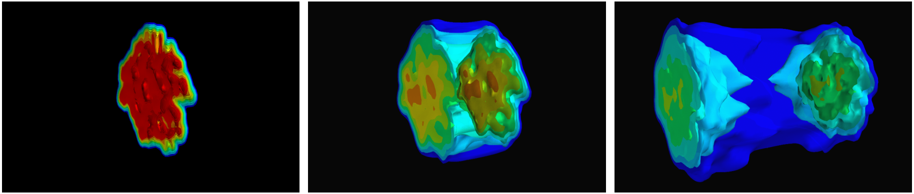
The Bjorken-flow framework

During the **early stages** after the collision, we can neglect transverse flow

$$\downarrow$$

(0+1)D longitudinal boost-invariant expansion $(v_z = z/t\,,\ \partial_\mu u^\mu = 1/\tau)$

RED: **high energy density**, BLUE: **low energy density**



made by Björn Schenke

- QGP: **short lifetime** ~ 10 fm/c and **transverse extension** $\sim$ nuclear radius

Despite its simplicity, the **Bjorken model** is still capable of describing the main **collective properties** of the system

Why hydrodynamics is so (un)reasonably effective? (1)

Shortly after its formation, the **QGP** quickly reaches a state where it exhibits **collective flow patterns**

The **hydrodynamization** process results in:

- **loss of information** about the **initial condition**
- approach to an **attractor solution** at late times ($\tau \sim 1$ fm/c), but when the system is still **out of equilibrium**
- exponential decay of the **non-hydrodynamic modes** in the one-particle distribution function

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Identification of observables with a **universal behavior**, e.g. longitudinal pressure and particle-number density

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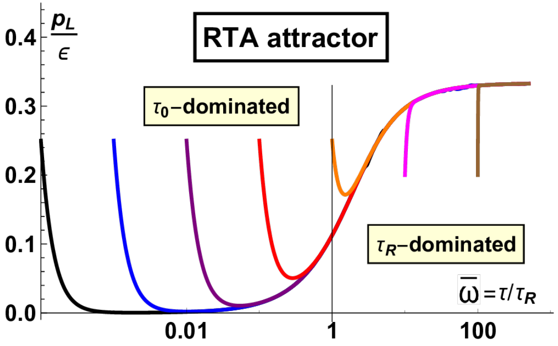
Identification of observables with a **universal behavior**, e.g. longitudinal pressure and particle-number density

- For some observables, **attractors may extend also to early times**, when the system is in the **pre-hydrodynamic regime**, due to the incredibly **fast expansion** (universality at early times $\leftrightarrow$ **no hydrodynamization**)

Why hydrodynamics is so (un)reasonably effective? (2)

Summarizing, there are two types of **attractor**:

- **late-time** (forward) attractor: $\tau > \tau_{eq}$ (**interactions**) $\longleftrightarrow$ **hydrodynamics**
- **early-time** (pullback) attractor: $\tau < \tau_{eq}$ (**free streaming**)

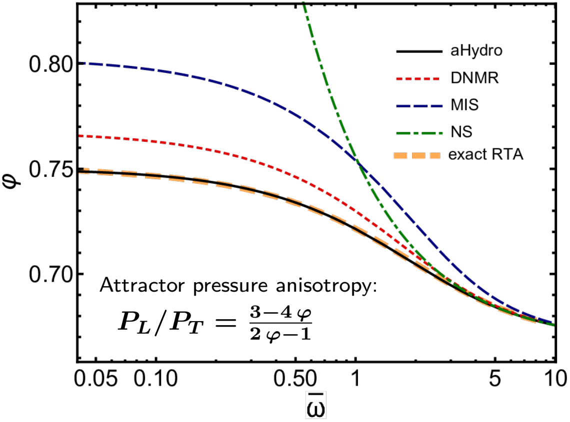


[1907.08101]

Scaled time: $\overline{\omega} \equiv \frac{\text{particle interaction rate}}{\text{fluid expansion rate}} = \tau/\tau_{eq} \longrightarrow \tau \equiv \sqrt{t^2 - z^2}$

Why hydrodynamics is so (un)reasonably effective? (3)

Attractors exist also for **different hydrodynamic schemes**, but at late times all these solutions approach the **attractor from kinetic theory** (**exact solution**)



[1709.06644]

- This fact motivates the use of **kinetic theory to study such a system**
- Kinetic theory also provides a **description of the pre-hydrodynamic stage**

The QGP as a two-component fluid

General setup for a quark-gluon mixture

Quark and **gluon** species are treated separately, in order to better describe the system relying on a **more realistic QCD-based framework**

- a **fugacity parameter** γ_q allows quarks to be **out of chemical equilibrium**
- symmetric quarks and anti-quarks (**vanishing baryon chemical potential**)

$\downarrow$

the quark species is assumed to be massless (**conformality**)

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The evolution of the distribution function of each species $(a = q, g)$ in kinetic theory is given by the **Boltzmann equation**, which reads in **RTA**:

$$\partial_\tau f_a = \frac{f_{\mathrm{eq},a} - f_a}{\tau_{\mathrm{eq},a}}\,, \quad \text{with } \tau_{\mathrm{eq},g} = \frac{5\,\eta/s}{T}$$

when the system is undergoing a **Bjorken expansion**.

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when the system is undergoing a **Bjorken expansion**.

- f_a **relaxes to the isotropic Maxwell-Boltzmann distribution function** $f_{\mathrm{eq},a} \sim \exp\left[-(p \cdot u)/T\right]$ in a time scale given by the relaxation time τ_{eq}
- **Casimir scaling**: $\tau_{\mathrm{eq},g}/\tau_{\mathrm{eq},q} \equiv C_R = 4/9$, meaning that **gluons relax** to the equilibrium configuration **faster than quarks**

Formal solution for the Boltzmann equation

In this special case, the Boltzmann equation admits an **exact solution**:

$$f_a(\tau; w, p_T) = \underbrace{\boldsymbol{D_a(\tau, \tau_0)}\, \boldsymbol{f_{0,a}(w, p_T)}}_{\text{free}-\text{streaming term}} + \underbrace{\int_{\tau_0}^{\tau} \frac{d\tau'}{\boldsymbol{\tau_{\mathrm{eq},a}(\tau')}}\, \boldsymbol{D_a(\tau, \tau')}\, f_{\mathrm{eq},a}(\tau'; w, p_T)}_{\text{evolution term (collisions)}}$$

where we used the **damping function** to take into account the fraction of particles which have not suffered any collision in a given time interval:

$$\boldsymbol{D_a(\tau_2, \tau_1)} \equiv \exp\left[-\int_{\tau_1}^{\tau_2} \frac{d\tau'}{\tau_{\mathrm{eq},a}(\tau')}\right], \qquad w \equiv t\, p_z - z\, E \text{ (boost invariant)}$$

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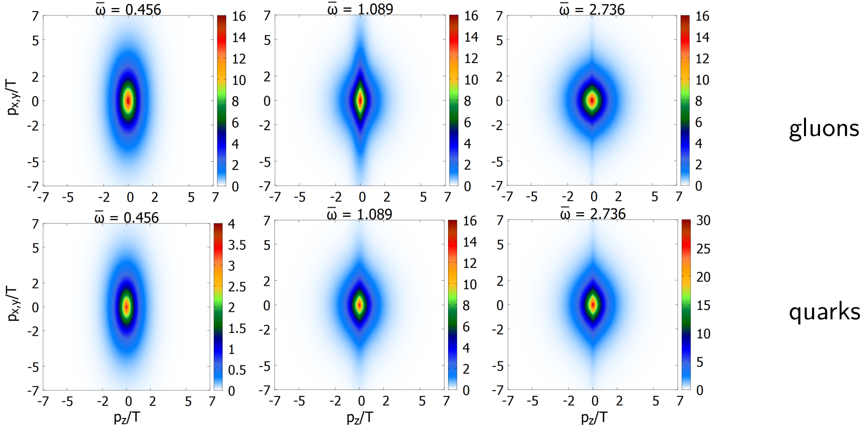
Due to the rapid expansion at **early times**, the distribution function exhibits a **high degree of anisotropy** in the fluid LRF (Strickland-Romatschke form):

$$\boldsymbol{f_{0,a}} \underset{\text{LRF}}{=} \boldsymbol{G_{a,0}} \exp\left[-\frac{\sqrt{p_T^2 + (1+\xi_0)\, p_z^2}}{\Lambda_0}\right], \text{ with } G_{a,0} \equiv \begin{cases} g_g\,, & a = g \\ 2\, g_q\, \gamma_{q,0}\,, & a = q \end{cases}$$

- **relativistic degrees of freedom**: $g_g = 2 \times 8 = 16$, $g_q = 2 \times 3 \times 3 = 18$

Evolution of the one-particle distribution function



$\alpha_0 \equiv (1+\xi_0)^{-1/2} = 0.4,\ \gamma_{q,0} = 0.1,\ \tau_0 = 0.15\,\mathrm{fm/c},\ T_0 = 600\,\mathrm{MeV},\ \eta/s = 0.2$

- At early times, the particle distributions exhibit **squeezed** (non-hydrodynamic) **modes for small p_z** (**free-streaming term**)
- At late times, the system tends to **isotropize** (**evolution term**) and the squeezed modes are exponentially suppressed, due to the damping function

How does the quark abundance change?

From the Boltzmann equation one is able to reconstruct the **time-evolution of the fugacity parameter**, working in the origin of momentum space:

$$\gamma_q(\tau) = D_q(\tau,\tau_0)\,\gamma_{q,0} + \left[\gamma_{q,\mathrm{eq}} - D_q(\tau,\tau_0)\right], \quad \text{with } \gamma_{q,\mathrm{eq}} = 1$$

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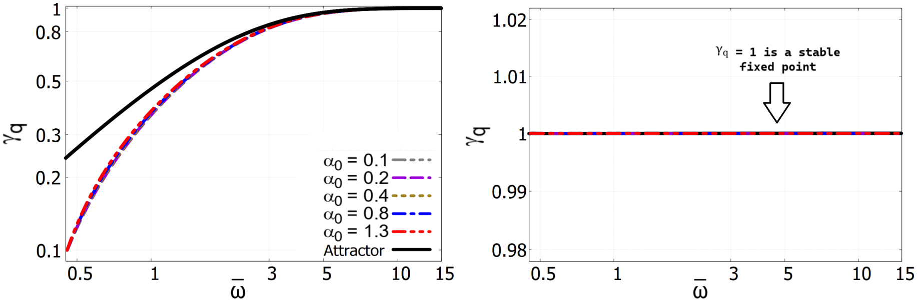
Evolution of the fugacity for an initially **underpopulated quark species** (left panel) and for quarks initially in chemical equilibrium (right panel)



- a gluon-dominated plasma is expected to be produced right after HIC ($\gamma_{q,0} < 1$)
- the **equilibrium solution** itself acts quickly as an **attractor** for the **fugacity**

The generalized Landau matching condition

The only physical constraint we need to require is the total (**quarks + gluons**) **energy-momentum conservation**, which reads:

$$\partial_\mu T^{\mu\nu} = 0 \longrightarrow \boldsymbol{\varepsilon_q} + \frac{\boldsymbol{\varepsilon_g}}{\boldsymbol{C_R}} = \boldsymbol{\varepsilon_{q,\mathrm{eq}}} + \frac{\boldsymbol{\varepsilon_{g,\mathrm{eq}}}}{\boldsymbol{C_R}} \,,$$

if quarks and gluons have **different relaxation times** ($\boldsymbol{C_R = 4/9}$).

- Landau frame: $T_a^{\mu\nu}\, u_\mu \equiv \varepsilon_a\, u^\nu$ (eigenvalue equation)
- In **equilibrium** $\varepsilon_{a,\mathrm{eq}} = \frac{3\, G_{a,\mathrm{eq}}}{\pi^2}\, \boldsymbol{T}^4$, this allows one to find an **effective temperature** for the whole mixture

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Kinetic definition of the **energy-momentum tensor** for a given particle species:

$$\boldsymbol{T_a^{\mu\nu} \equiv \int d\chi\; p^\mu\, p^\nu\, f_a(\tau; w, p_T)} \;,\;\; \text{where}\;\; d\chi = \frac{dw\, d^2 p_T}{(2\pi)^3\, v} \;\; \text{(invariant measure)}$$

and $v \equiv \sqrt{w^2 + p_T^2\, \tau^2}$ is another scalar quantity for longitudinal boosts.

The cooling of a two-component system

The **generalized Landau matching condition** can be employed to obtain a **dynamical equation for the effective temperature**:

$$T^4(\tau) = D(\tau,\tau_0)\left[\bar{r} + 2\,\gamma_{q,0}\left(D(\tau,\tau_0)\right)^{C_R-1}\right]\left(2\,\gamma_{q,0}+\bar{r}\right)^{-1} T_0^4\,\frac{\boldsymbol{H}\left(\boldsymbol{\alpha_0}\,\frac{\boldsymbol{\tau_0}}{\boldsymbol{\tau}}\right)}{\boldsymbol{H}(\boldsymbol{\alpha_0})} +$$

$$+\,\frac{C_R}{2+\bar{r}}\int_{\tau_0}^{\tau}\frac{d\tau'}{2\,\tau_{\rm eq}(\tau')}\,D(\tau,\tau')\left[\frac{\bar{r}}{C_R} + 2\left(D(\tau,\tau')\right)^{C_R-1}\right]T^4(\tau')\,\boldsymbol{H}\left(\frac{\boldsymbol{\tau'}}{\boldsymbol{\tau}}\right),$$

where we introduced: $\boldsymbol{H(y) = y\int_{-1}^{1} d\cos\theta\,\sqrt{y^2\,\cos^2\theta + \sin^2\theta}}$ and $\boldsymbol{r = g_g/g_q}$.

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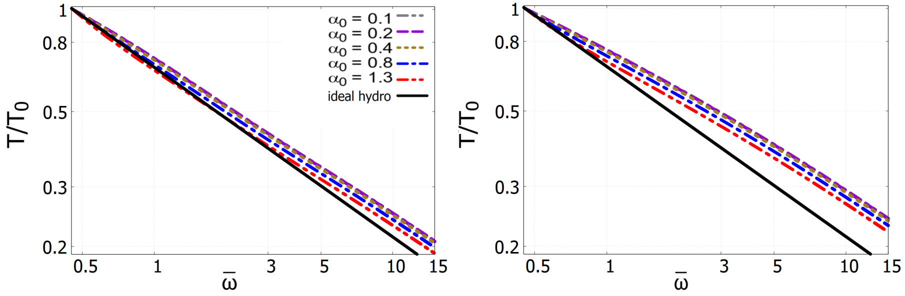

left panel: $\gamma_{q,0} = 0.1$, **right panel:** $\gamma_{q,0} = 1.0$

- The **temperature** of the mixture **does not possess a late-time attractor**

Evolution equation for the general moments

From the **exact solution of the BE**, one can derive an **evolution equation for the moments** associated with the distribution function:

$$M_a^{nm}(\tau) \equiv \int d\chi \; (p \cdot u)^n \; (p \cdot z)^{2m} \; f_a(\tau; w, p_T) \longrightarrow \begin{cases} n = 0\,,\, m = 1 \;:\; P_{L,a} \\ n = 1\,,\, m = 0 \;:\; n_a \\ n = 2\,,\, m = 0 \;:\; \varepsilon_a \end{cases}$$

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which reads, after considering **contributions from both quarks and gluons**:

$$M^{nm}(\tau) \equiv \sum_a M_a^{nm} = \boldsymbol{A}\Bigg\{ D(\tau,\tau_0)\left[r + 2\,\gamma_{q,0}\left(D(\tau,\tau_0)\right)^{C_R-1}\right]\left(\frac{2\ (2+\bar r)}{2\,\gamma_{q,0}+\bar r}\right)^{\frac{n+2m+2}{4}} \times$$

$$\times\, T_0^{\,n+2m+2}\ \frac{\boldsymbol{H^{nm}\left(\alpha_0\,\frac{\tau_0}{\tau}\right)}}{\left[\boldsymbol{H(\alpha_0)}\right]^{\frac{n+2m+2}{4}}} + C_R\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{\rm eq}(\tau')}\,D(\tau,\tau')\left[\bar r + 2\left(D(\tau,\tau')\right)^{C_R-1}\right]\times$$

$$\times\, T^{\,n+2m+2}(\tau')\ \boldsymbol{H^{nm}\left(\frac{\tau'}{\tau}\right)}\Bigg\}\ ,\ \text{with}\ \ \boldsymbol{A \equiv g_q\,\frac{\Gamma(n+2m+2)}{(2\pi)^2}}$$

In the above relation, hypergeometric special functions appear:

$$\boldsymbol{H^{nm}(y) = \frac{2}{2m+1}\,y^{2m+1}\ {}_2F_1\left(m+\frac{1}{2},\frac{1-n}{2},m+\frac{3}{2};1-y^2\right)}$$

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$$\times\, T_0^{\,n+2m+2}\ \frac{H^{nm}\left(\alpha_0\,\frac{\tau_0}{\tau}\right)}{[H(\alpha_0)]^{\frac{n+2m+2}{4}}} + C_R\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{\rm eq}(\tau')}\,D(\tau,\tau')\left[\bar r + 2\Big(D(\tau,\tau')\Big)^{C_R-1}\right]\times$$
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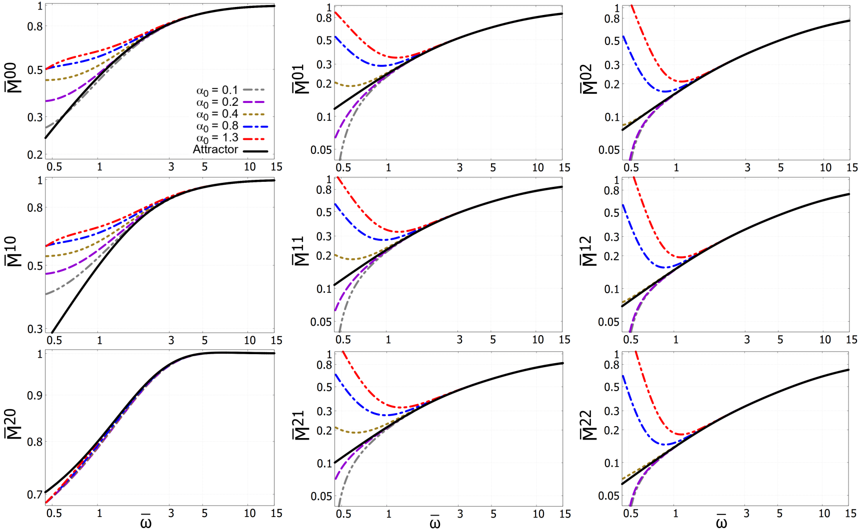
- These moments can be conveniently **re-scaled by their corresponding equilibrium value**:

$$M_{\rm eq}^{nm}(\tau) = g_q\,\frac{\Gamma(n+2m+2)}{(2\pi)^2}\,\frac{2\,(2+r)}{2m+1}\,T^{n+2m+2}(\tau)\ \longrightarrow\ \overline{M}^{\,nm} \equiv M^{nm}/M_{\rm eq}^{nm}$$

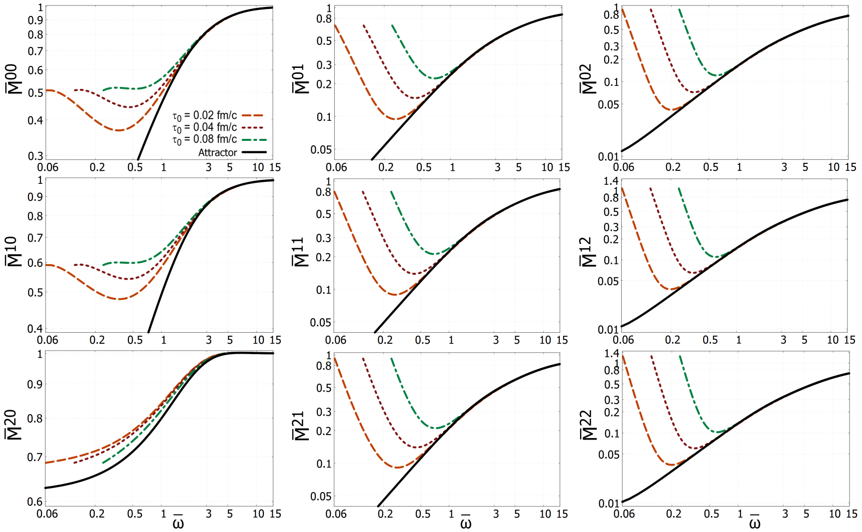
The late-time attractor for the general moments



$$\gamma_{q,0} = 0.1, \ \tau_0 = 0.15\,\mathrm{fm/c}, \ T_0 = 600\,\mathrm{MeV}, \ \eta/s = 0.2$$

- All scaled moments exhibit a **forward attractor**, except the scaled energy density

The early-time attractor for the general moments



$$\alpha_0 = 1,\ \gamma_{q,0} = 0.1,\ \mathrm{fm/c},\ T_0 = 600\,\mathrm{MeV},\ \eta/s = 0.2$$

- Only moments with $m > 0$ possess a **pullback attractor** (**no hydrodynamization**)

Attractor properties of the entropy density

We can calculate the **entropy density** from the following relation:

$$s(\tau) = -\sum_{a=g,q} G_a(\tau) \int d\chi\,(p\cdot u)\,\widehat{f}_a(\tau;w,p_T)\left\{\ln\left[\widehat{f}_a(\tau;w,p_T)\right]-1\right\},$$

where: $\boldsymbol{f_a(\tau;w,p_T)\equiv G_a(\tau)\,\widehat{f}_a(\tau;w,p_T)}$, with $\widehat{f}_a(\tau;0,0)=1$

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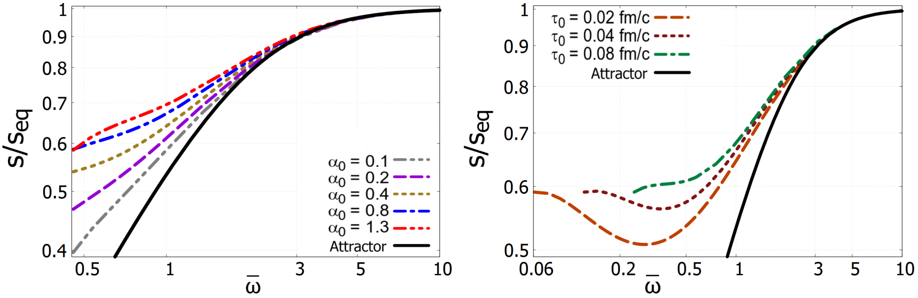
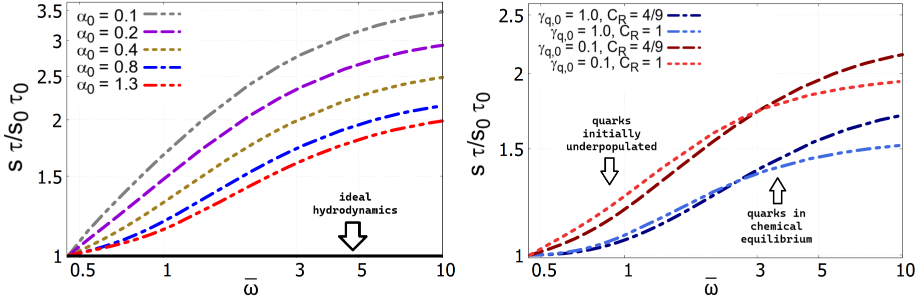


$T\,s_{\rm eq} = \varepsilon_{\rm eq} + P_{\rm eq} \longrightarrow s_{\rm eq}(\tau) = \frac{4}{\pi^2}\,g_q\,(2+r)\,T^3(\tau)$, here $\gamma_{q,0} = 0.1$

- **scaled entropy density** follows a **universal behavior at late times**, but it is <u>not</u> characterized by an early-time attractor

Entropy production in a Bjorken-expanding mixture

Even though we did not impose the **second principle of thermodynamics**, numerical simulations reveal a **rapid production of entropy**, especially during the **pre-hydrodynamic phase**



left panel: $\gamma_{q,0} = 0.1$

- after **hydrodynamization** ($\varpi \gtrsim 5$), the system's **entropy increases slowly**
- the more **quarks** initially deviate from **chemical equilibrium**, the **greater the entropy** generated
- no attractor properties for the entropy production

Conclusions and outlook

Summary and motivations for a two-component model

The **QGP**, soon after its formation ($\tau \gtrsim 1$ fm/c), behaves much like a **nearly perfect fluid**, effectively described by **hydrodynamic models**

- the **hydrodynamization process** (approach to the **late-time attractor**) can be appreciated also when considering a simple **(0+1)D expansion**

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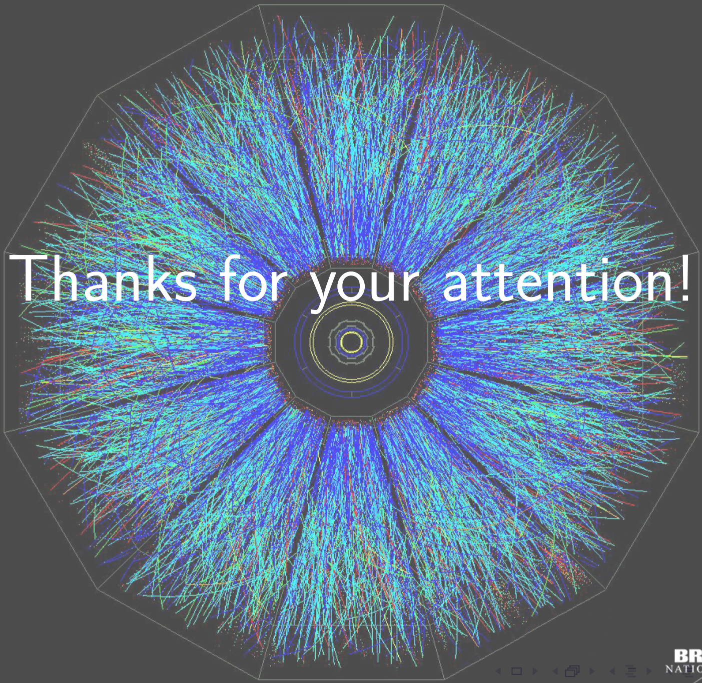
- the **hydrodynamization process** (approach to the **late-time attractor**) can be appreciated also when considering a simple **(0+1)D expansion**

Compared to the case of 1RTA ($C_R = 1$), we can observe:

1. a **slower isotropization** of the system
2. a **greater entropy production** during the expansion
3. a **less universal early-time dynamics**, partially "spoiled" by the process of **chemical equilibration** and by the **slower relaxation of quarks**

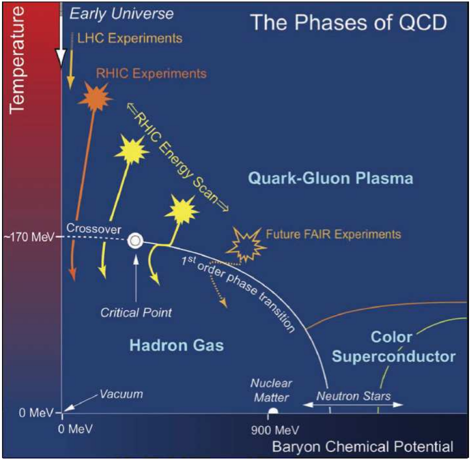
Thanks for your attention!



Backup slides

The QCD phase diagram

Quarks and **gluons** in **QGP** are not confined within hadrons, but actually experience **asymptotic freedom**

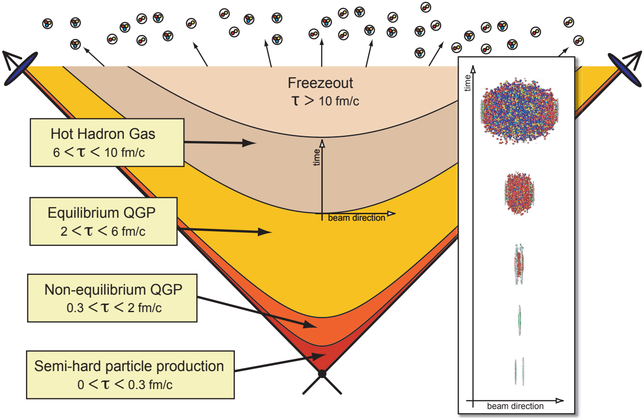


[1308.3328]

- At very high energies in the center of mass, the transition is a **crossover**

Timescales of heavy-ion collisions

Different stages after a collision between two heavy nuclei in a spacetime diagram



[1410.5786]

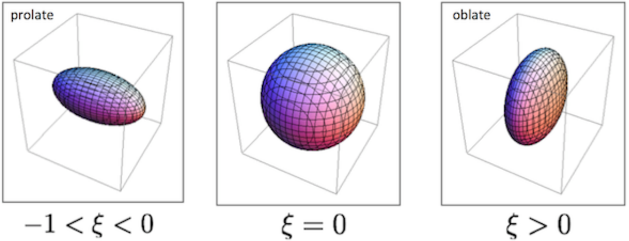
- for $T > T_{\rm FO}$, quarks and gluons expand like a **relativistic fluid**
- when the **inelastic**/**elastic** interactions become inefficient, the system experiences the process of **hadronization** (**chemical freeze-out**)/**momentum-distribution freezing** (**kinetic freeze-out**)

Why anisotropic distribution functions?

Bjorken flow $\longleftrightarrow$ **longitudinal boost-invariance** (we focus on the bulk of the fireball)

$\rightarrow$ particles close to the center $(z \rightarrow 0)$: **small p_z** $(v_z = z/t)$ **compared to p_T**

- **anisotropy in momentum space** implies a **deformed distribution function**, characterized by a **spheroidal shape**



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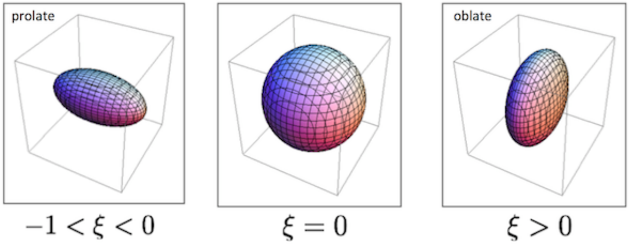

[1410.5786]

- **heavy-ion collisions** : $\boldsymbol{\xi \geq 0} \longleftrightarrow \langle p_z^2 \rangle \leq \langle p_T^2 \rangle / 2$ (**fast longitudinal expansion**, $\boldsymbol{\theta = 1/\tau}$)

$$\underbrace{f_{\text{eq}} \sim \exp\left(-\frac{p \cdot u}{T}\right)}_{\text{isotropic Maxwell-Boltzmann, } \xi = 0} \longrightarrow \underbrace{f(x;p) \sim \exp\left[-\frac{\sqrt{(p \cdot u)^2 + \boldsymbol{\xi}\ (\boldsymbol{p \cdot z})^2}}{\Lambda}\right]}_{\text{anisotropic distribution function}}$$

$\rightarrow \xi$ represents the **anisotropy parameter**, which is **not small** in the pre-hydrodynamic regime

Why quarks and gluons have different relaxation times?

Distribution functions for quarks and gluons satisfy the **Boltzmann equation** in **RTA**:

$$p^{\mu}\,\partial_{\mu}\,f_a(x;p) \simeq -\frac{(p\cdot u)}{\tau_{\mathrm{eq},a}}\Big[f_a(x;p) - f_a^{\mathrm{eq}}(x;p)\Big] \quad , \text{ with } \; a = q, g$$

In which we admit that quarks and gluons can possess **different relaxation times**.

- we work under the hypothesis of **Casimir scaling**, meaning that:

$$\boldsymbol{C_R \equiv \frac{\tau_{\mathrm{eq},g}}{\tau_{\mathrm{eq},q}} = \frac{C_F}{C_A} = \frac{4}{9}} \quad , \quad \text{QCD: } 2 \to 2 \text{ processes}$$

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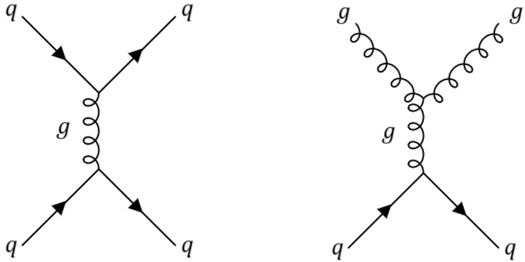
Quarks and **gluons**: different scattering amplitudes, due to **different color charges**



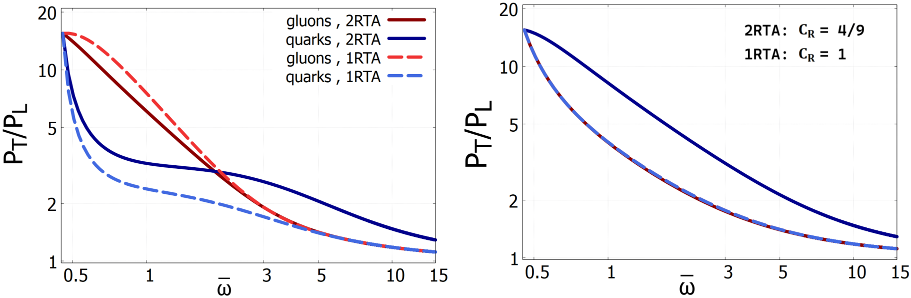
This fact implies **two relaxation times** for partons within the expanding mixture:

$$\tau_{\mathrm{eq},q} \sim 1/C_F \quad , \quad \tau_{\mathrm{eq},g} \sim 1/C_A$$

Quarks slow down the dynamics of the mixture

- The **momentum-anisotropy parameter** and the **(inverse) pressure anisotropy** are actually related quantities

$$\underbrace{1+\xi}_{\text{momentum anisotropy}} \propto \underbrace{\frac{P_T}{P_L}}_{\text{spatial anisotropy}} \longrightarrow \underline{\text{equilibrium}} : \xi = 0\,,\ P_L/P_T = 1$$



left panel: $\gamma_{q,0} = 0.1$, **right panel:** $\gamma_{q,0} = 1.0$

$\alpha_0 \equiv (1+\xi_0)^{-1/2} = 0.2\,,\ \tau_0 = 0.15$ fm/c , $T_0 = 600$ MeV , $\eta/s = 0.2$

- The **quark component experiences a slower isotropization** than gluons (2RTA), so its **momentum anisotropy is systematically higher** at any time

How far from equilibrium is the system?

One possible parameter to **measure the deviation from equilibrium** corresponds to the so-called **inverse Reynolds number**:

$$\mathrm{Re}^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{P_{\mathrm{eq}}} = \sqrt{\frac{3}{2}}\,|\overline{\phi}| \;\;, \text{ where } \;\overline{\phi} \equiv \frac{P - P_L}{P_{\mathrm{eq}}} = \underbrace{\overline{M}^{20} - \overline{M}^{01}}_{\text{scaled moments}}$$

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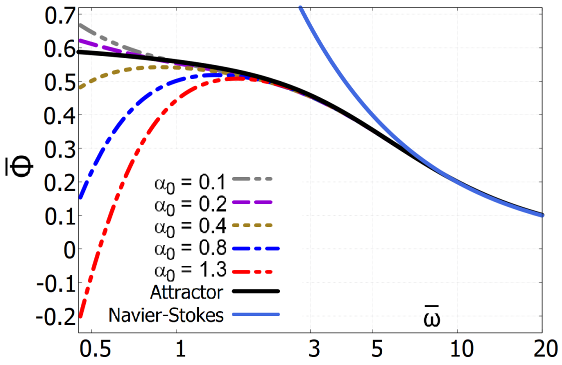

$\gamma_{q,0} = 0.1,\ \tau_0 = 0.15\,\mathrm{fm/c},\ T_0 = 600\,\mathrm{MeV},\ \eta/s = 0.2$

Navier-Stokes

↓

indicator of the validity of **hydrodynamic schemes**

$$\overline{\phi}_{\mathrm{NS}} = \frac{16}{15}\,\frac{r + \frac{2}{C_R}}{r+2}\,\frac{1}{\overline{\omega}}$$

HYDRO: $\overline{\phi} \lesssim 0.25$