Electroweak reactions with QMC methods

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Motivation

Fermi Lab / Sandbox Studio, Chicago

ENERGY DISTRIBUTION OF THE UNIVERSE

69%
DARK ENERGY

26%
NORMAL MATTER

5%
DARK MATTER

NASA / Chandra X-ray Center/ K. Divona

Symmetry Magazine / Sandbox Studio, Chicago
Nuclei for new physics

On-going effort to measure Standard Model (SM) allowed, SM forbidden, and known beyond SM phenomena to better understand new physics.
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Nuclear theory needed to aid in the interpretation of new physics
Nuclei for new physics

On-going effort to measure **Standard Model (SM) allowed**, **SM forbidden**, and **known beyond SM** phenomena to better understand new physics

Nuclear theory needed to aid in the interpretation of new physics

**Need an accurate and predictable model of nuclear physics at all relevant kinematics**
Outline

• Microscopic description of nuclei
  • The many-body problem
  • Quantum Monte Carlo methods
  • The NV2+3 nuclear model

• Model validation
  • $\beta$-decay matrix elements
  • Magnetic structure (moments, form factors)
  • Muon capture rates

• Prediction of the $^6$He $\beta$-decay spectrum

Recent review:
Microscopic description of nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions in terms of nucleon degrees of freedom

Requirements:
• A computational method to solve the nuclear many-body problem and compute observables
• An accurate understanding of the interactions/correlations between nucleons in pairs, triplets, … (two- and three-nucleon forces)
• An accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated pairs of nucleons, … (one- and two-body electroweak charges and currents)
Quantum many-body problem

Modeling physical phenomena in a system with many bodies interacting amongst themselves

\[ H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

Interaction generates *correlations* in solution of the Schrödinger equation

\[ H \left| \Psi \right\rangle = E \left| \Psi \right\rangle \]

Several approaches in nuclear physics:

**Quantum Monte Carlo**, No-core shell model (NCSM), couple cluster, ...
Quantum Monte Carlo (QMC)

Solving the many-body problem by stochastically solving the Schrödinger equation

Variational MC wave function $\left| \Psi_T \right\rangle = \mathcal{F} \left| \Phi \right\rangle$ contains model wave function and many-body correlations optimized by minimizing:

$$E_V = \min \left\{ \frac{\left\langle \Psi_T \middle| H \middle| \Psi_T \right\rangle}{\left\langle \Psi_T \middle| \Psi_T \right\rangle} \right\} \geq E_0$$

Green’s function MC improves by removing excited state contamination and gives the exact ground state

$$\lim_{\tau \to \infty} e^{-(H-E_0)\tau} \Psi_V = \lim_{\tau \to \infty} e^{-(H-E_0)\tau} \left( c_0 \psi_0 + \sum_{i=1}^{N} c_i \psi_i \right) \to c_0 \psi_0$$

Foulkes et al. Rev. Mod. Phys. 73, 33 (2001)
The Norfolk (NV2+3) interaction

\[ H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} \]

Model derived from \(\chi\)EFT with pion, nucleon, and delta degrees of freedom

NV2 contains 26 unknown LECs in contacts, two more from the NV3

Eight model classes arrived at from different procedures to constrain the unknown LECs

Piarulli et al. PRL 120, 052503 (2018)
Need nuclear electroweak charge and current operators as well

Schematically:

\[
\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots \\
j = \sum_{i=1}^{A} j_i + \sum_{i<j} j_{ij} + \ldots
\]

External field interacts with single nucleons and correlated pairs of nucleons

Pastore et al. PRC 80, 034004 (2009), Pastore et al. PRC 84, 024001 (2011), Piarulli et al. PRC 87, 014006 (2013), Schiavilla et al. PRC 99, 034005 (2019), Baroni et al. PRC 93, 049902 (2016), ...
Model Validation
Beta-decay: Gamow-Teller matrix elements

Calculations with NV2+3-1a* and NV2+3-1a compared to AV18+IL7 (◊) and exp (dashes)

Correlations provide bulk of quenching

Two-body almost always enhances

King et al. PRC 121, 025501 (2020)
Beta-decay: two-body densities

Different approaches to fitting the NV2+3 can result in different short-range behavior.

This alters the total two-body contribution depending on the model:

\[ M_{GT}^{2b} = \int dr_{ij} 4\pi r_{ij}^2 \rho_{GT}^{2b}(r_{ij}) \]
Magnetic structure: two-body currents

One-body picture:

\[ \mu^{LO} = \sum_i (L_{i,z} + g_p S_{i,z}) \frac{1 + \tau_{3,i}}{2} + g_n S_{i,z} \frac{1 - \tau_{3,i}}{2} \]

Two-body currents can play a large role (up to ~33%) in describing magnetic dipole moments.

NV2+3-IIb* is able to capture the shape of magnetic form factor

In some cases, good quantitative agreement at large momentum transfer

Two-body effects ~20% to 50% at large momentum transfer in various radioisotopes
Prediction: $^6$He $\beta$-decay spectrum
$^6\text{He beta decay spectrum: Overview}$

Differential rate:

$$d\Gamma_{\beta} = |M_{\beta}(q)|^2 \times \text{(kinematic factors)}$$

In the $q \to 0$ limit:

$$\frac{d\Gamma_{\beta}}{dE_e} = \frac{d\Gamma_0}{dE_e} \left[ 1 + b \frac{m_e}{E_e} \right]$$

SM ($q \to 0$):

$$b = 0$$
$^6$He beta decay spectrum: Overview

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SM ($q \to 0$):

$$b = 0$$

SM (with recoil):

$$b = 0 + \Delta b$$
$^6\text{He}$ beta decay spectrum: SM results

\[ \tau_{\text{VMC}} = 762 \pm 11 \text{ ms} \]
\[ \tau_{\text{GFMC}} = 808 \pm 24 \text{ ms} \]
\[ \tau_{\text{Expt.}} = 807.25 \pm 0.16 \pm 0.11 \text{ ms} \]

[Kanafani et al. PRC 106, 045502 (2022)]

\[ \varepsilon = \frac{E_e}{W_0} \]
$^6$He beta decay spectrum: BSM connections

Include new physics with strengths $\epsilon_i$ allowed from current analyses

With permille precision, it will be possible to further constrain new physics

$$\Lambda_{BSM} \sim \frac{\Lambda_{EW}}{\sqrt{\epsilon_i}} \sim 1\text{–}10 \text{ TeV}$$

King et al. PRC 107, 015503 (2023)
Many-body plus $\chi$EFT is a powerful tool to understand the impact of the nuclear dynamics on electroweak structure.

Impact of different approaches to fitting potential, currents on observables.

Ad-hoc uncertainty estimations have been performed for the model, but more robust UQ increasingly important (see Maria’s talk Thu).

**Future:** neutrino-nucleus scattering, radiative corrections to beta decay.
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Additional Slides
The Norfolk (NV2+3) Interaction

\[ H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} \]

Eight different Model classes:
- I [II]: NN scattering to fit two-body interaction from 0 to 125 [200] MeV
- a [b]: Long- and short-range regulators \((R_L, R_S) = (1.2 \text{ fm}, 0.8 \text{ fm}) [(1.0 \text{ fm}, 0.7 \text{ fm})]\)
- Unstarred: Three-body term constrained with strong data only
- Star: Three-body term constrained with strong and weak data

Piarulli et al. PRL 120, 052503 (2018)
Variational Monte Carlo (VMC)

$$|\Psi_T\rangle = \left[ S \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[ \sum_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTTz)\rangle$$

Slater determinant of nucleons in s- and p-shell coupled to the appropriate quantum numbers

Pair correlation operator encoding appropriate cluster structure

Two- and three-body correlation operator to reflect impact of nuclear interaction at short distances

Optimize when you minimize:

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Green’s Function Monte Carlo (GFMC)

Can expand in exact states $H$: $|\Psi_V\rangle = c_0\psi_0 + \sum_{i=1}^{n} c_n |\psi_n\rangle$

Imaginary time propagation:

$\Psi(\tau) = e^{-(H-E_0)\tau} \Psi_V = \left[e^{-(H-E_0)\Delta \tau}\right]^n \Psi_V$

Removes excited state contamination and gives the exact ground state

$\lim_{\tau \to \infty} e^{-(H-E_0)\tau} \Psi_V \rightarrow c_0 \psi_0$

Foulkes et al. Rev. Mod. Phys. 73, 33 (2001)
Mixed estimate

Assume small correction to VMC:

$$\Psi(\tau) = \Psi_V + \delta\Psi$$

To first order in the correction:

$$\langle \Psi(\tau)|\mathcal{O}|\Psi(\tau)\rangle = 2\frac{\langle \Psi(\tau)|\mathcal{O}|\Psi_V\rangle}{\langle \Psi(\tau)|\Psi_V\rangle} - \langle \mathcal{O} \rangle_{\text{VMC}}$$

Pastore et al. PRC 87, 035503

Off-diagonal mixed estimate

$^{6}\text{He} \rightarrow ^{6}\text{Li}$ GT RME extrapolation

Mixed estimate for off-diagonal transitions:

$$\langle \mathcal{O}(\tau) \rangle = \frac{\langle \Psi^f(\tau)|\mathcal{O}|\Psi^i(\tau) \rangle}{\sqrt{\langle \Psi^f(\tau)|\Psi^f(\tau) \rangle} \sqrt{\langle \Psi^i(\tau)|\Psi^i(\tau) \rangle}}$$

$$\approx \langle \mathcal{O}(\tau) \rangle_{M_f} + \langle \mathcal{O}(\tau) \rangle_{M_i} - \langle \mathcal{O} \rangle_{VMC}$$

where

$$\langle \mathcal{O}(\tau) \rangle_{M_f} = \frac{\langle \Psi^f(\tau)|\mathcal{O}|\Psi^i_V \rangle}{\sqrt{\langle \Psi^f(\tau)|\Psi^f_V \rangle} \sqrt{\langle \Psi^i_V|\Psi^i_V \rangle}}$$

Pervin, Pieper, and Wiringa PRC 76, 064319 (2007)
Three-body LECs and N3LO-CT

The NV2+3-la model fits $c_D$ using *strong interaction data only*

The NV2+3-la* model fits $c_D$ with *strong and weak interaction data*

$$j_{5,a}^{\text{N3LO}}(q; \text{CT}) = z_0 \mathcal{O}_{ij}(q)$$

$$z_0 \propto (c_D + \text{known LECs})$$

Scaled two-body transition densities

\[ M_{2b}^{GT} = \int dr_{ij} 4\pi r_{ij}^2 \rho_{2b}^{GT}(r_{ij}) \]

King et al. PRC 121, 025501 (2020)

G.B. King, 7/8/2024
Explanation of universal scaling behaviors

ST=01 and 10 pairs dominate short distances due to suppression of P-waves

\[ N_{ST} = \int d\vec{r}_{ij} 4\pi r_{ij}^2 \rho_M^{2b}(r_{ij}) \]

King et al. PRC 121, 025501 (2020)
10C to 10B GT beta decay

Two states of the same quantum numbers nearby

The result depends strongly on the LS mixing of the p-shell

Particularly sensitive to the $^3S_1$ and $^3D_1$ mixing because S to S produces a larger m.e. and $^{10}$C is predominantly S wave

https://nucldata.tunl.duke.edu/
$^8$He to $^8$Li GT beta decay

Three ($1^+;1$) states within a few MeV

Different dominant spatial symmetries → sensitivity to the precise mixing of small components in the wave function

Improving the mixing of the small components in the ($1^+;1$) states is crucial to getting an improved m.e.

https://nucldata.tunl.duke.edu/
Isoscalar (IS) two-body magnetic densities

\[ \mu_{2b} = \int dr_{ij} 4\pi r_{ij}^2 \rho_M^{2b}(r_{ij}) \]

Chambers-Wall, King, et al. *in preparation*
Cutoff dependence: magnetic currents

Regulator choice between model Ia* and IIb* strongly influences the short-range dynamics

Chambers-Wall, King, et al. in preparation
Partial Muon Capture Rates with QMC

Assuming a muon at rest in a Hydrogen-like 1s orbital:

\[
\Gamma = \frac{G^2_V}{2\pi} \frac{|\psi_{1s}^{av}|^2}{(2J_i + 1)} \frac{E_{\nu}^*}{\text{recoil}} \sum_{M_f, M_i} |\langle J_f, M_f | \rho(E_{\nu}^* \hat{z}) | J_i, M_i \rangle|^2 + |\langle J_f, M_f | \mathbf{j}_z (E_{\nu}^* \hat{z}) | J_i, M_i \rangle|^2
\]

\[
+ 2 \text{Re} \left[ \langle J_f, M_f | \rho(E_{\nu}^* \hat{z}) | J_i, M_i \rangle \langle J_f, M_f | \mathbf{j}_z (E_{\nu}^* \hat{z}) | J_i, M_i \rangle^* \right] + |\langle J_f, M_f | \mathbf{j}_x (E_{\nu}^* \hat{z}) | J_i, M_i \rangle|^2
\]

\[
+ |\langle J_f, M_f | \mathbf{j}_y (E_{\nu}^* \hat{z}) | J_i, M_i \rangle|^2 - 2 \text{Im} \left[ \langle J_f, M_f | \mathbf{j}_x (E_{\nu}^* \hat{z}) | J_i, M_i \rangle \langle J_f, M_f | \mathbf{j}_y (E_{\nu}^* \hat{z}) | J_i, M_i \rangle^* \right]
\]
$^3\text{He}(1/2^+; 1/2) \rightarrow ^3\text{H}(1/2^+; 1/2)$ agrees with datum of Ackerbauer et al. Phys. Lett. B 417 (1998)

Most sensitive to the 3N force

Two-body provide $\sim$9%-16% of the rate for different models
Partial muon capture rates with QMC

$^6$Li(g.s.$) \rightarrow ^6$He(g.s.$)$ disagrees with datum from Deutsch et al. Phys. Lett. B26, 315 (1968)

Subsequent NCSM evaluation agrees with QMC results

Could merit further attention

King et al. PRC 105, L042501 (2022)
The (standard model) matrix element may be decomposed into reduced matrix elements of four multipoles operators:

\[
\sum_{M_i} \sum_{M_f} |\langle f | H_W | i \rangle|^2 \propto \sum_{J=0}^{\infty} \left[(1 + \hat{\nu} \cdot \beta)|C_J(q)|^2 + (1 - \hat{\nu} \cdot \beta + 2(\hat{\nu} \cdot \hat{q})(\hat{q} \cdot \beta))|L_J(q)|^2 - \hat{q} \cdot (\hat{\nu} + \beta)2\text{Re}(L_J(q)M_J^*(q))\right] \\
+ \sum_{J=1}^{\infty} \left[(1 - (\hat{\nu} \cdot \hat{q})(\hat{q} \cdot \beta))(|M_J(q)|^2 + |E_J(q)|^2) + \hat{q} \cdot (\hat{\nu} - \beta)2\text{Re}(M_J(q)E_J^*(q))\right]
\]

With the standard operator definitions as [Walecka 1975, Oxford University Press]:

\[
C_{JM}(q) = \int d^3x [j_J(qx)Y_{JM}(\Omega_x)](\rho(x; V) + \rho(x; J))
\]

\[
L_{JM}(q) = \frac{i}{q} \int d^3x \{\nabla[j_J(qx)Y_{JM}(\Omega_x)]\} \cdot (j(x; V) + j(x; A))
\]

\[
E_{JM}(q) = \frac{1}{q} \int d^3x [\nabla \times j_J(qx)\mathbf{Y}_{JM}^*(\Omega_x)] \cdot (j(x; V) + j(x; A))
\]

\[
M_{JM}(q) = \int d^3x [j_J(qx)\mathbf{Y}_{JM}^*(\Omega_x)] \cdot (j(x; V) + j(x; A))
\]

Parity and angular momentum selection rules preserve only the four $J=1$, positive parity multipoles for $^6$He beta-decay.
6He beta decay spectrum: SM results

$$\tau_{\text{VMC}} = 762 \pm 11 \text{ ms}$$

$$\tau_{\text{GFMC}} = 808 \pm 24 \text{ ms}$$

$$\tau_{\text{Expt.}} = 807.25 \pm 0.16 \pm 0.11 \text{ ms}$$

[Kanafani et al. PRC 106, 045502 (2022)]

$$\varepsilon = \frac{E_e}{\omega}$$

King et al. PRC 107, 015503 (2023)
Standard Model Effective Field Theory (SMEFT) gives most general set of gauge-invariant operators complementing the SM

Tensor and pseudoscalar charged current interactions introduced at dimension-6

Matching the SMEFT to low-energy theory, one can investigate impact of BSM physics on the $^6$He beta-decay spectrum (effort led by Mereghetti+)
Muon capture: non-zero momentum transfer

Momentum transfer $\sim 100$ MeV/c

Two-body currents play a $\sim 9\%-16\%$ role for $A=3$, $\sim 3\%-7\%$ for $A=6$

Many-body calculations with $\chi$EFT based models not presently capturing the data

$$\Gamma \propto \sum_{\alpha\beta} |M_{\alpha\beta}|^2$$

King et al. PRC 105, L042501 (2022)
$^6$He beta decay spectrum: SM results

Fully retain two-body physics by leveraging low-$q$ behavior

$$C_1(q; A) = \frac{i}{\sqrt{4\pi}} \langle ^6\text{Li}, 10|\rho_+^\dagger(q\hat{z}; A)|^6\text{He}, 00 \rangle$$

$$L_1(q; A) = \frac{i}{\sqrt{4\pi}} \langle ^6\text{Li}, 10|\hat{z} \cdot j_+^\dagger(q\hat{z}; A)|^6\text{He}, 00 \rangle$$

$$E_1(q; A) = \frac{i}{\sqrt{2\pi}} \langle ^6\text{Li}, 10|\hat{z} \cdot j_+^\dagger(q\hat{x}; A)|^6\text{He}, 00 \rangle$$

$$M_1(q; V) = -\frac{1}{\sqrt{2\pi}} \langle ^6\text{Li}, 10|\hat{y} \cdot j_+^\dagger(q\hat{x}; V)|^6\text{He}, 00 \rangle$$

King et al. PRC 107, 015503 (2023)