Consistent description of mean-field instabilities and clustering phenomena within a unified dynamical approach

10$^{\text{th}}$ International Conference on
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Outline of the presentation

1. **Many-body (MB) correlations and clustering phenomena in nuclear systems**
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)

2. **Extended EDF-based models: recent developments and results**
   - ✏ **Unified (thermodynamic) description of few-body correlations and clusters**
     - Embedding short-range correlations within relativistic mean-field approaches
     - Global mass-shift parameterization for a multi-purposes EOS
   - ✏ **Dynamical approach with light clusters as degrees of freedom (DOF)**
     - Quasi-analytical study of dilute NM with light clusters and in-medium effects
     - Characterization of spinodal instability and growth rate of unstable modes

3. **Further developments and outlooks**
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent descriptions of fragment formation mechanisms in heavy-ion collisions

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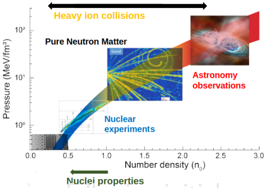
Heavy-ion collisions: clustering effects and EOS

- Heavy-ion collisions (**HIC**) at $E_{\mathrm{beam}} \approx (30-300)\,A\mathrm{MeV} \Rightarrow$ EOS



- **Expansion** following initial **compression**
  $\Rightarrow$ low density (ρ) & temperature (T)
  - **Spinodal** instabilities → **fragments**
  - **Few-body** correlations → light **clusters**

- **Phenomenological** EDF with **clusters** DOF
  - [illegible] NM → [illegible] (nucleons+nuclei)

Theoretical challenge

Consistent dynamical approach for **light clusters** and heavier **fragments**

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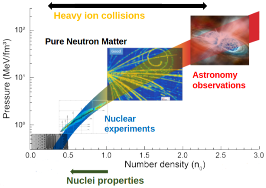

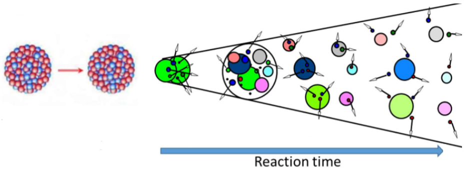

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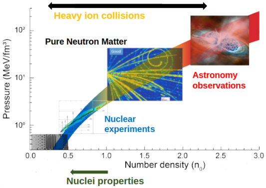

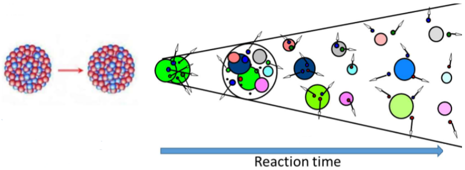

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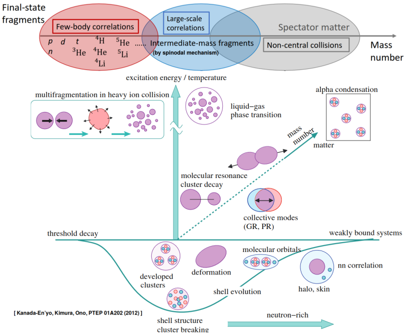

[Kanada-En'yo, Kimura, Ono, PTEP 01A202 (2012)]

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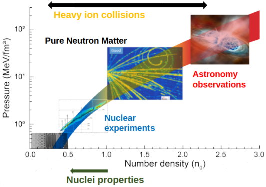

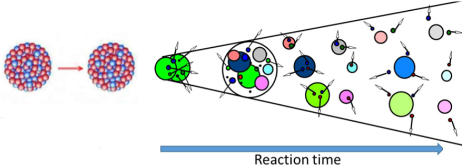

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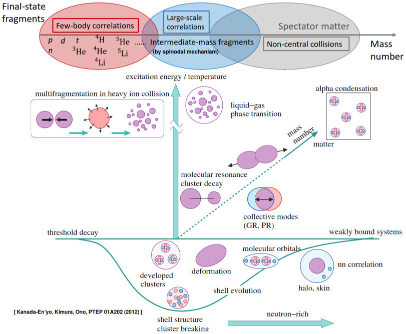

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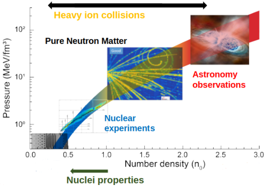

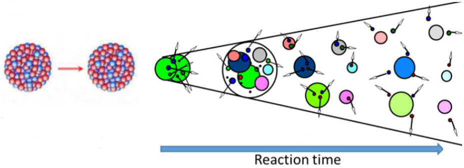

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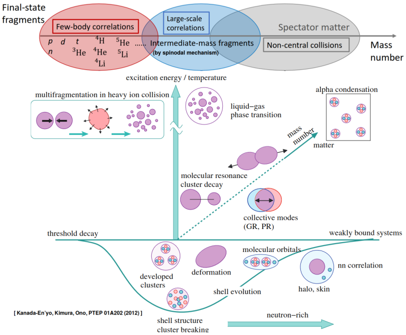

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Consistent dynamical approach for **light clusters** and heavier **fragments**

In-medium (Mott) effects and cluster dissolution

- Cluster **dissolution** approaching saturation from below
  $\Rightarrow$ **Mott effect** ruled by Pauli-blocking
- Generalized relativistic density functional (**GRDF**)
  [S. Typel et al., PRC 81, 015803 (2010)]
  - **Microscopic** in-medium effects
  - (Effective) binding energy $\rightarrow B^{\text{eff}} = B - \Delta m$
- $\Delta m^{(\text{low})}$ from in-medium MB **Schrödinger equation** [G. Röpke, NPA 867 (2011) 66–80]
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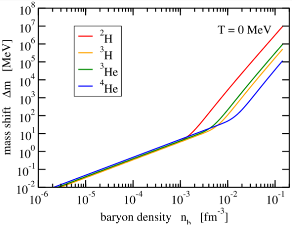

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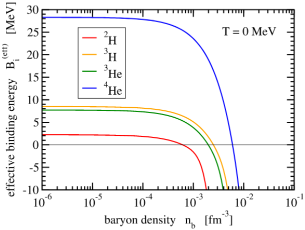

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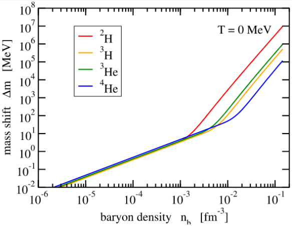

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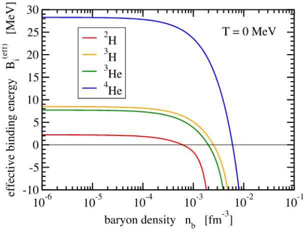

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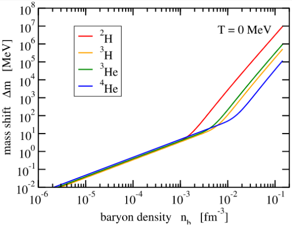

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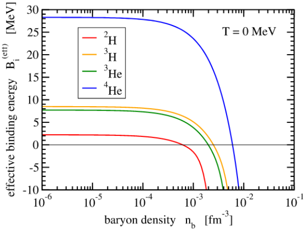

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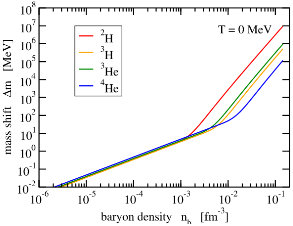

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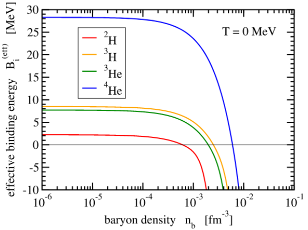

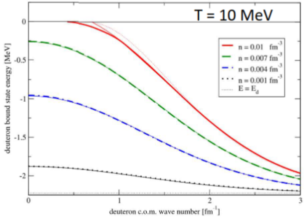

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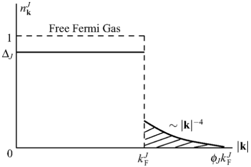
Short-range correlations within GRDF model

- Nucleon knock-out in **inelastic electron scattering**

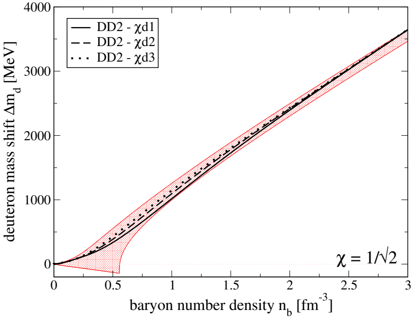
  [O. Hen et al. (CLAS Coll.), Science 346, 614 (2014)]

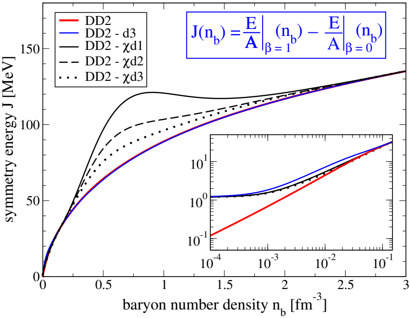
  - **Smearing** + high-**k** tail in **distribution** at T=0
- Nucleon-nucleon short-range correlations (**SRCs**)
  - **Tensor**/**repulsive** components of nuclear forces



- Embedding (effectively) SRCs in **GRDF** model using quasi-deuterons as **surrogate**

  [S. Burrello, S. Typel, EPJA 58, 120 (2022)]





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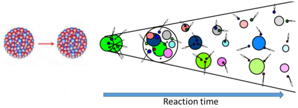
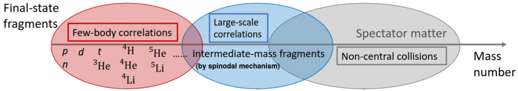
Kinetic approach for HIC with light-clusters DOF

- **Dynamical** processes modelizations ⇒ **Transport** theories
  - Lack of consistent description of light and heavier **fragments**





- **Kinetic** approach of light-nuclei production in HIC at intermediate energies
  - Boltzmann–Uehling–Uhlenbeck model + collision integral cut-off (Mott effect)

    [R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, PRC 108, L031601 (2023)]

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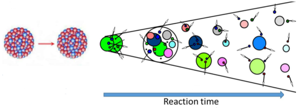

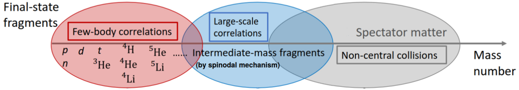

Final-state fragments

Few-body correlations

p d t ^{4}H ^{5}He ..... Intermediate-mass fragments (by spinodal mechanism)

n ^{3}He ^{4}He ^{5}Li ^{4}Li

Large-scale correlations

Spectator matter

Non-central collisions

Mass number

- **Kinetic** approach of light-nuclei **production** in HIC at intermediate energies
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$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \dots], \qquad \tau = n, p, d, t, h, \alpha$$

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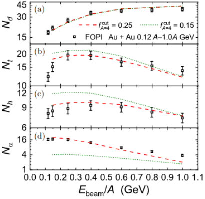

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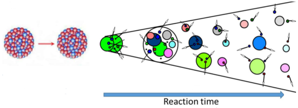

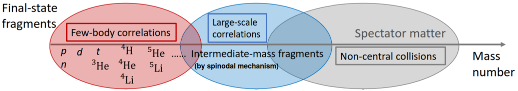

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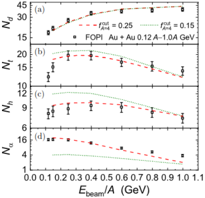

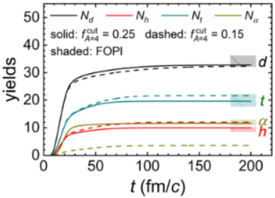

Our goal

Assess if light **clusters** (from **compression** phase) affect **spinodal** instability (**expansion** stage)

Density-dependent (Mott) momentum cut-off

- Non-relativistic framework ⇒ **dynamical** treatment more easily carried out
- Cut-off (Mott) momentum Λ_j for Pauli-blocking

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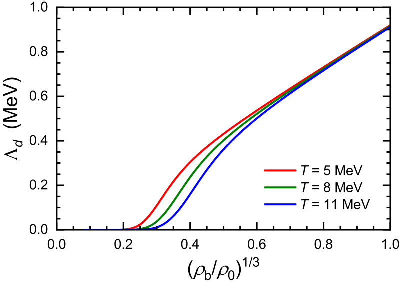

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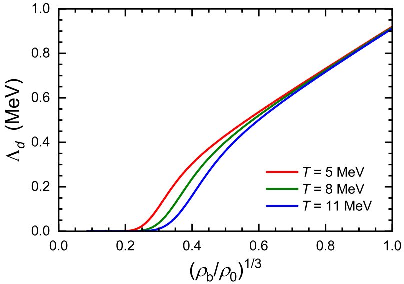

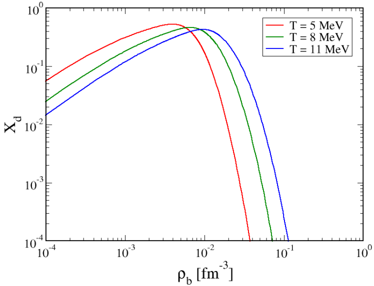

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Linearized Vlasov equations for NM+deuterons

- **Linear** response to **collision-less** Boltzmann ⇒ linearized **Vlasov** equations for NMd

$$\partial_t\left(\delta f_j\right)+\nabla_{\mathbf{r}}(\delta f_j)\cdot\nabla_{\mathbf{p}}\varepsilon_j-\nabla_{\mathbf{p}}f_j\cdot\nabla_{\mathbf{r}}(\delta\varepsilon_j)=0 \quad\Rightarrow\quad \boxed{\delta\rho_j=-\chi_j\textstyle\sum_l\left(F_0^{jl}+\tilde{F}_\lambda^{jl}\right)\delta\rho_l-\delta_{jd}\sum_l\Phi_\lambda^{dl}\delta\rho_l}$$

- **Single-particle** energy $\varepsilon_j\equiv\dfrac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (from **EDF** $\mathcal{E}=\mathcal{K}+\mathcal{U}$)

$$\varepsilon_j=\frac{p^2}{2m_j}+U_j+\tilde{\varepsilon}_j^\lambda \qquad (\tilde{\varepsilon}_j^\lambda\propto\Phi_\lambda^{dl})$$

- Momentum-independent **Skyrme**-like interaction (= for bound and free nucleons)

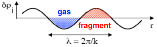
$$\mathcal{U}=\frac{A}{2}\frac{\rho_b^2}{\rho_0}+\frac{B}{\alpha+2}\frac{\rho_b^{\alpha+2}}{\rho_0^{\alpha+1}}+\frac{C(\rho)}{2}\frac{\rho_3^2}{\rho_0}+\frac{D}{2}(\nabla_{\mathbf{r}}\rho_b)^2-\frac{D_3}{2}(\nabla_{\mathbf{r}}\rho_3)^2$$

- **Density-dependent** (Mott) momentum **cut-off** ⇒ extra-terms in both $\delta\rho_j$ and ε_j

$$\rho_j=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_j \quad j=n,p,d \quad\rightarrow\quad \delta\rho_j(\mathbf{r},t)=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j-\delta_{jd}\sum_{l=n,p,d}\Phi_\lambda^{dl}\delta\rho_l$$

- $\Phi_\lambda^{dl}\neq 0$ ⇒ adding **in-medium** effects for cluster appearance/dissolution in **dynamics**

- **Landau** procedure $\left(F_0^{jl}\sim\dfrac{\partial U_j}{\partial\rho_l},\tilde{F}_\lambda^{jl}\sim\dfrac{\partial\tilde{\varepsilon}_j^\lambda}{\partial\rho_l}\right)$ for $\delta f_j\sim\sum_{\mathbf{k}}\delta f_j^{\mathbf{k}}\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$



Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations ⇒ **dispersion relation** $\omega = \omega(k)$

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- $\omega = \mathrm{Im}(\omega) \Leftrightarrow$ **unstable** mode (**spinodal region**)

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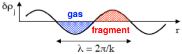

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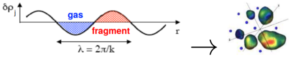

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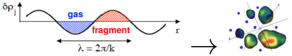
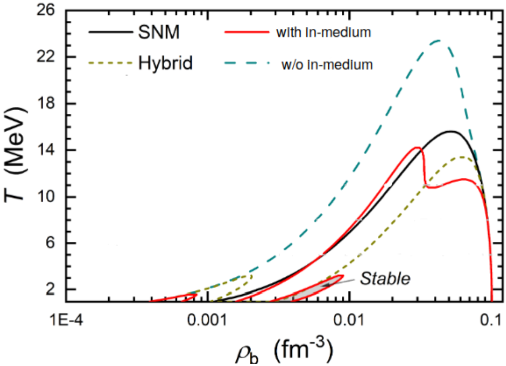


- $\omega = 0$ (**Lindhard** functions $\chi_j = 1$) ⇒ border



Legend

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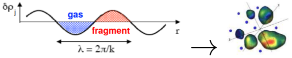

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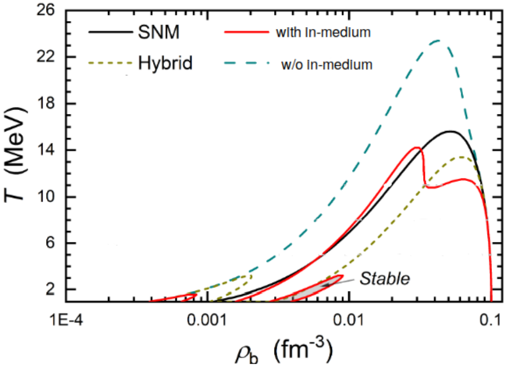

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In-medium effects in dynamics

- Dawn of meta-stable region

  [G. Röpke et al, NPA 970, 224 (2018)]

- Slowdown of instability rate

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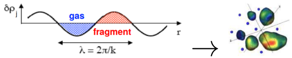
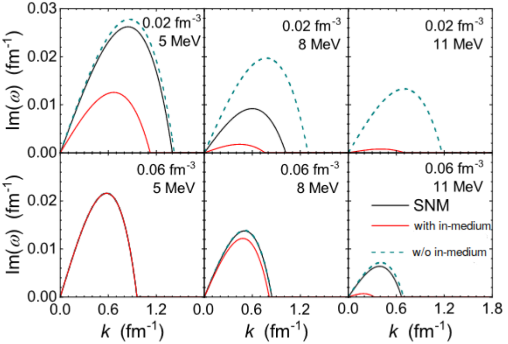


- $\mathrm{Im}(\omega) \Rightarrow$ growth rate of density fluctuations



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Instability direction: “distillation” mechanism

- **Direction of instability** in space of density fluctuations: $\frac{\delta\rho_S}{\delta\rho_d}$ $(\rho_S = \rho_n + \rho_p)$
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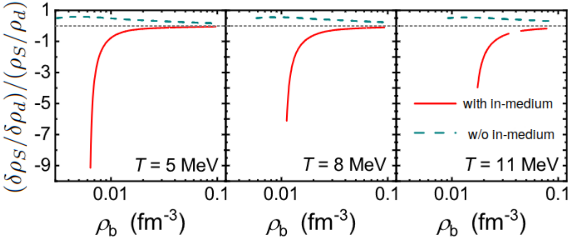

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  - Favored growth of instabilities
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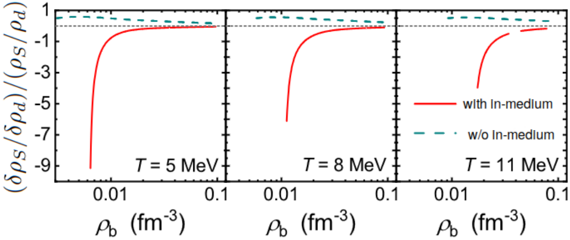

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    - Understanding Equation of State (EOS) for nuclear matter (NM)
    - Phenomenological models based on energy density functionals (EDF)
2. Extended EDF-based models: recent developments and results
    - ✏ Unified (thermodynamic) description of few-body correlations and clusters
    - ✏ Dynamical approach with light clusters as degrees of freedom (DOF)
3. **Further developments and outlooks**
    - Connection between hydrodynamical and linearized Vlasov approach
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  - $\eta_d < 1 \Rightarrow$ in-medium effects and description of chemical **equilibrium constant**

[L. Qin et al., PRL 108, 172701 (2012); R. Bougault et al., J. Phys. G 47, 025103 (2020)]

- **Alternative** framework for spinodal instability $\Rightarrow$ **Hydrodynamical** approach
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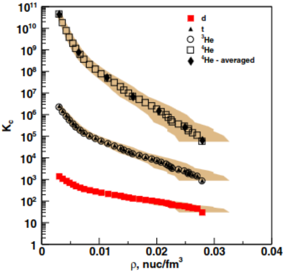
[S. Burrello, M. Colonna, F. Matera, R. Wang, in preparation]



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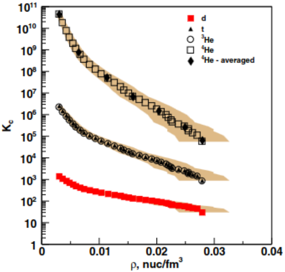

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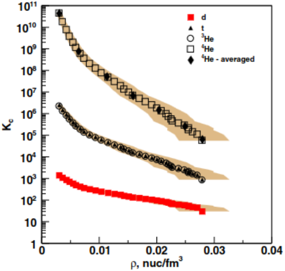

Work in progress

- **Extensive** calculations (other light clusters, ANM)
  - Different parameterizations for interaction & **cut-off**
- **Consistent** description of HIC **fragmentation** mechanisms
  - Beyond quasi-analytical $\Rightarrow$ numerical calculations

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THANK YOU FOR YOUR ATTENTION!