Study of forbidden β decays within the realistic shell model

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Outline

- The problem of the renormalization of στ operator
- The impact on 0νββ decay matrix element
- The β spectrum, the shape function
- The Realistic Shell Model
- Details of the Calculation
- Conclusions and perspectives

Outline

- The problem of the renormalization of $\sigma\tau$ operator

Renormalization of $\sigma\tau$ matrix elements

Gamow-Teller transitions (β-decay, EC, $2\nu\beta\beta$,charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

Renormalization of στ matrix elements

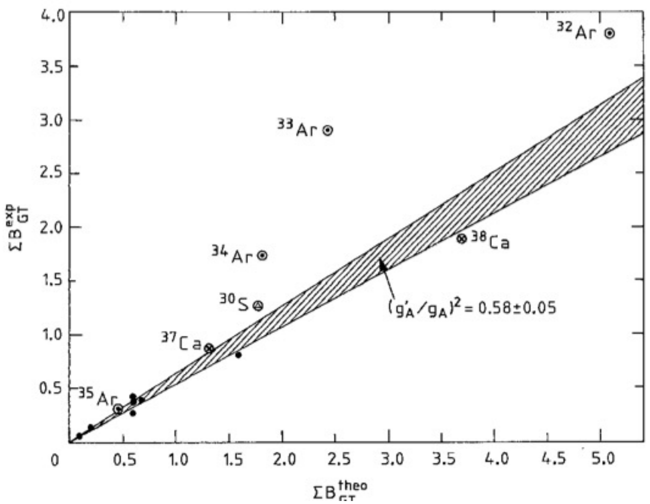
Gamow-Teller transitions (β-decay, EC, 2νββ,charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

Quenching of στ matrix elements is quite a general phenomenon in nuclear-structure physics.

$$g_A = g_A^{eff} = q\, g_A$$

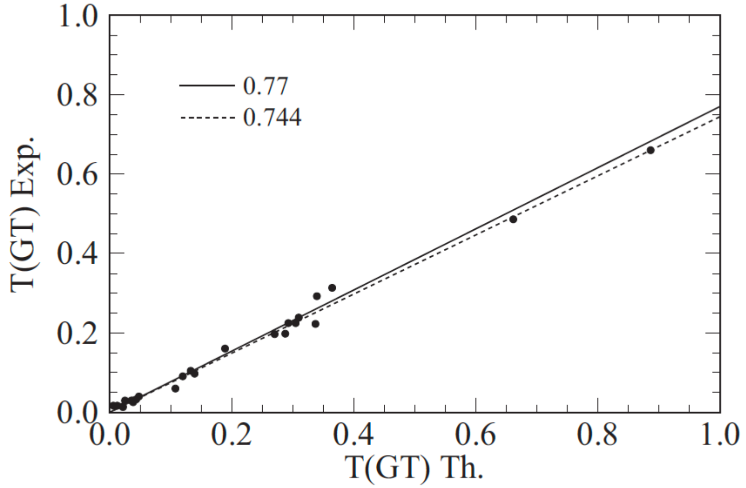
Renormalization of στ matrix elements

Z. Phys. A - Atomic Nuclei 332, 413417 (1989)



$$g_A = g_A^{eff} = q\, g_A$$

Martinez-Pinedo et al. PRC53 2602(1996)



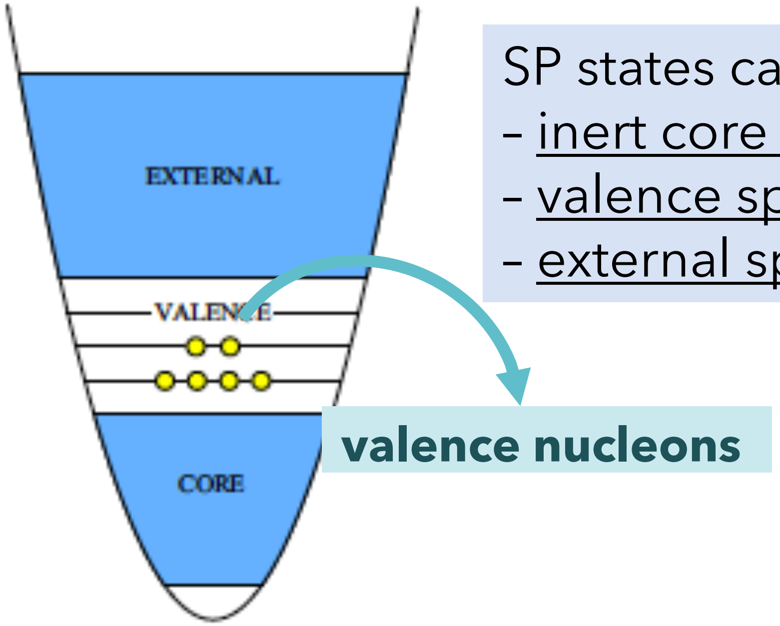
Suhonen, ACTA PHYSICA POLONICA B, 3 (2018)

Mass range	g_A^{eff}
Full $0p$ shell	$1.03^{+0.03}_{-0.02}$
$0p$–low $1s0d$ shell	1.18 ± 0.05
Full $1s0d$ shell	$0.96^{+0.03}_{-0.02}$
	1.0
$A = 41$–50 ($1p0f$ shell)	$0.937^{+0.019}_{-0.018}$
$1p0f$ shell	0.98
^{56}Ni	0.71
$A = 52$–67 ($1p0f$ shell)	$0.838^{+0.021}_{-0.020}$
$A = 67$–80 ($0f_{5/2}1p0g_{9/2}$ shell)	0.869 ± 0.019
$A = 63$–96 ($1p0f0g1d2s$ shell)	0.8
$A = 76$–82 ($1p0f0g_{9/2}$ shell)	0.76
$A = 90$–97 ($1p0f0g1d2s$ shell)	0.60
^{100}Sn	0.52
$A = 128$–130 ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.72
$A = 130$–136 ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.94
$A = 136$ ($0g_{7/2}1d2s0h_{11/2}$ shell)	0.57

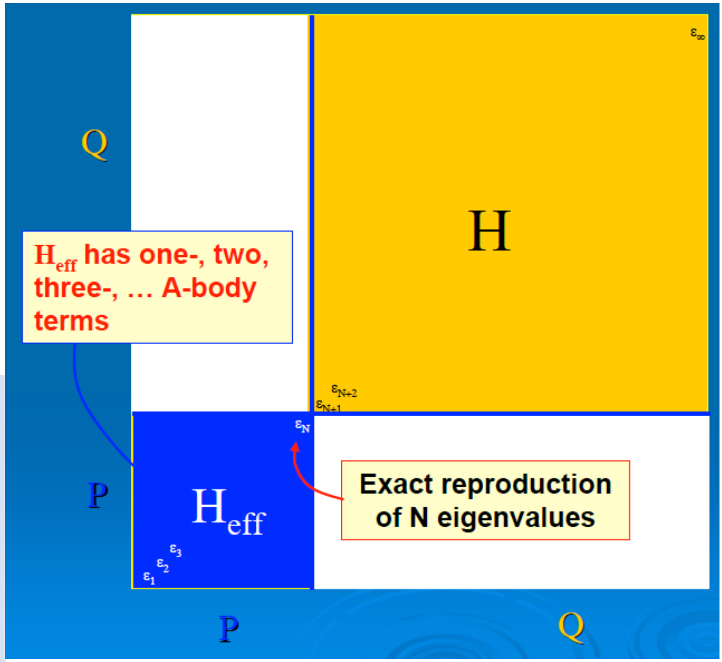
Quenching of στ matrix elements: theory

Two main sources:

1) LIMITED MODEL SPACE



SP states can be grouped into 3 sub-spaces, well separated in energy:
- inert core (*completely filled levels with neutrons and protons paired to J=0*)
- valence space (*partially filled levels*)
- external space (*empty levels*)



The only active degrees of freedom are given by nucleons inside the valence space (*valence nucleons)* while excitations of core nucleons and valence nucleons in the external space are "*frozen*" or, more properly, "not taken into account explicitly"

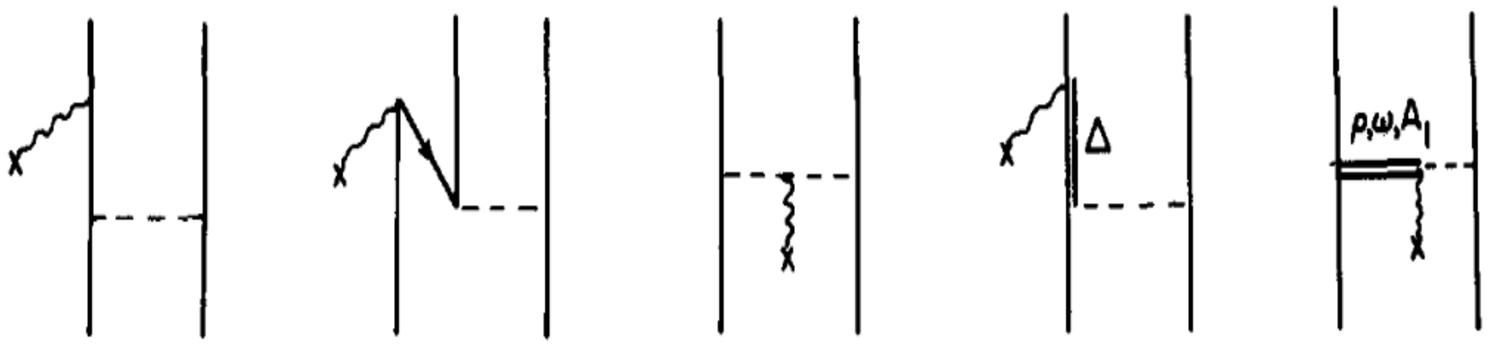
Quenching of στ matrix elements: theory

Two main sources:

2) NON-NUCLEONIC DEGREES OF FREEDOM

Processes in which the weak probe prompts a meson to be exchanged between two nucleons

→ meson-exchange two-body currents (2BC)

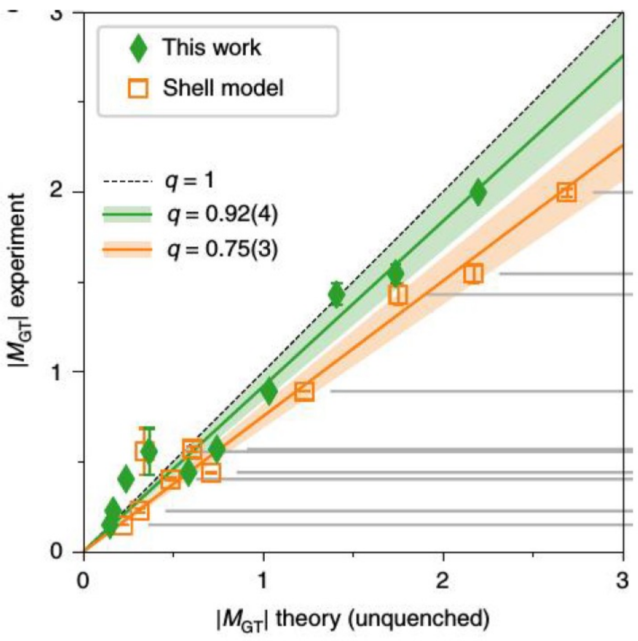


H. Hyuga and A. Arima, J. Phys. Soc. Jpn. Suppl. 34, 538 (1973)

Two-body e.w. currents effects

The contribution of 2BC improves systematically the agreement between theory and experiment

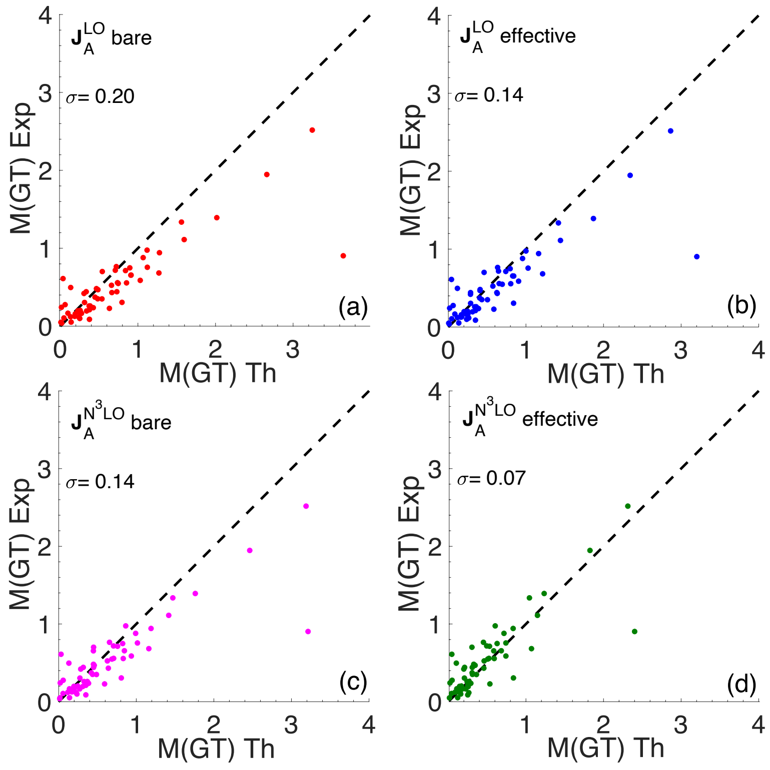
In-Medium SRG



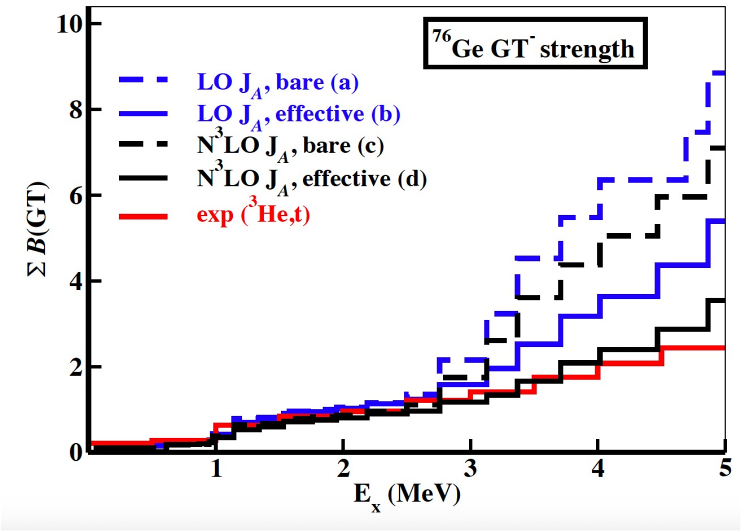
Gysbers et al. Nature Phys. 15 428 (2019)

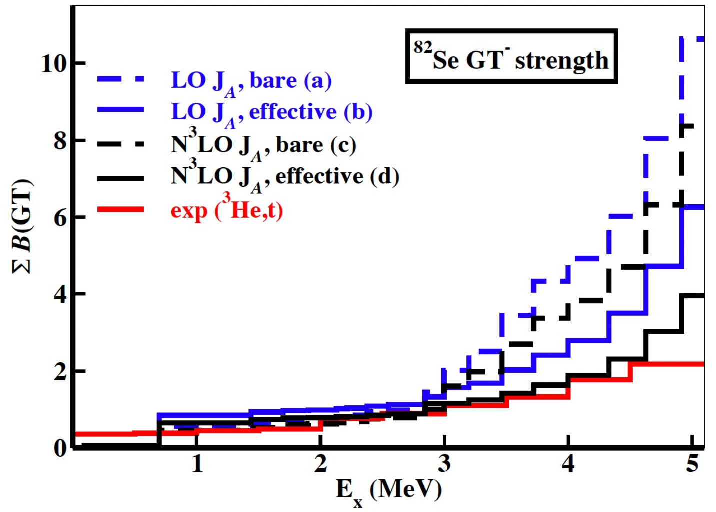
From ChEFT 2N +3N interaction

GT fp-shell nuclei

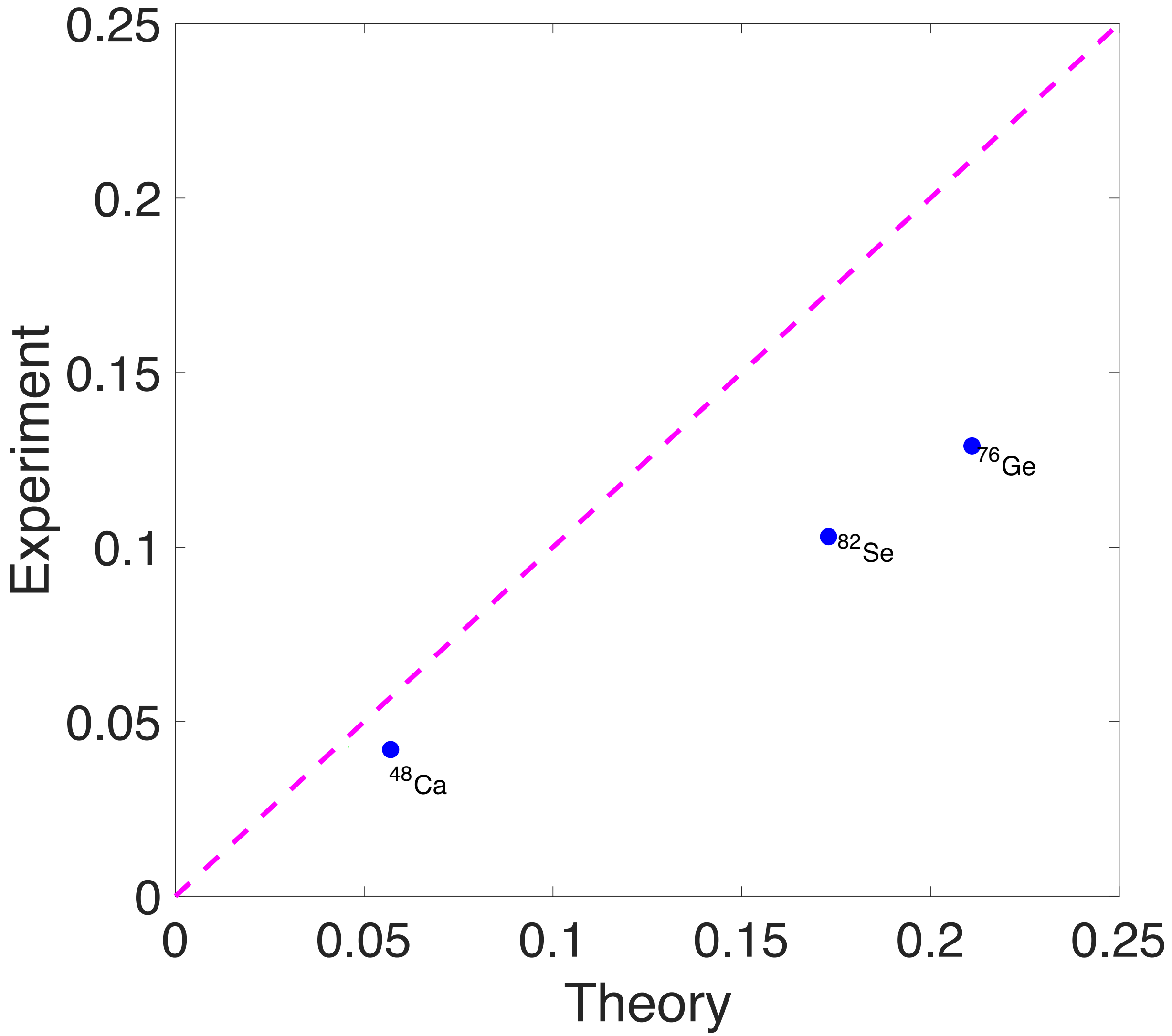


L. Coraggio et al. Physical Review C **109**, 014301 (2024)





2νββ decay

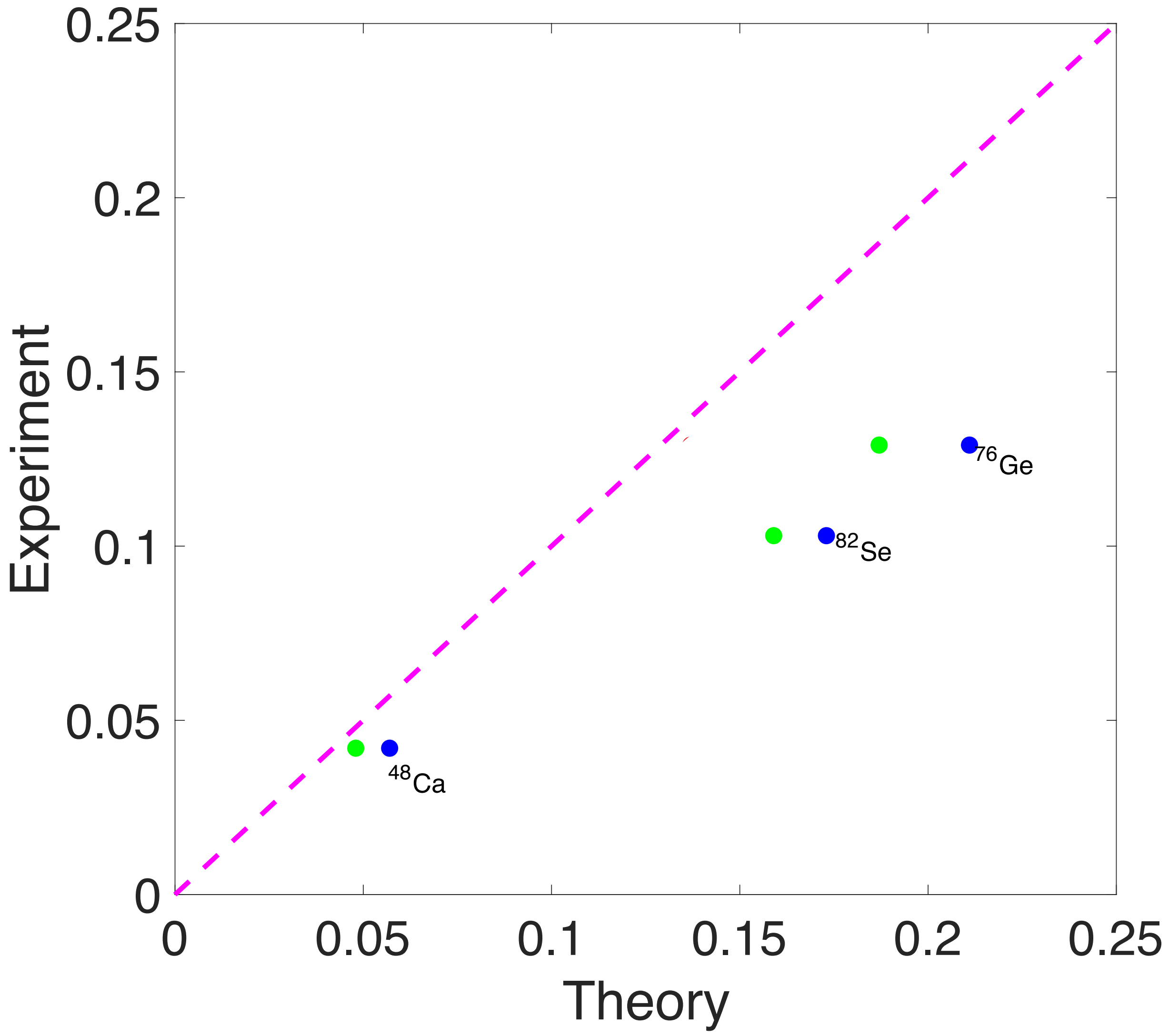


$$M^{2\nu}_{GT} = \sum_k \frac{\langle 0_f^+ || \vec{\sigma} \cdot \tau^- || k \rangle \langle k || \vec{\sigma} \cdot \tau^- || 0_i^+ \rangle}{E_k + E_0}$$

Blue: bare J_A at LO in ChPT (namely the GT operator g_A)

L. Coraggio et al. Physical Review C **109**, 014301 (2024)

2νββ decay



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Green: effective J_A at LO in ChPT

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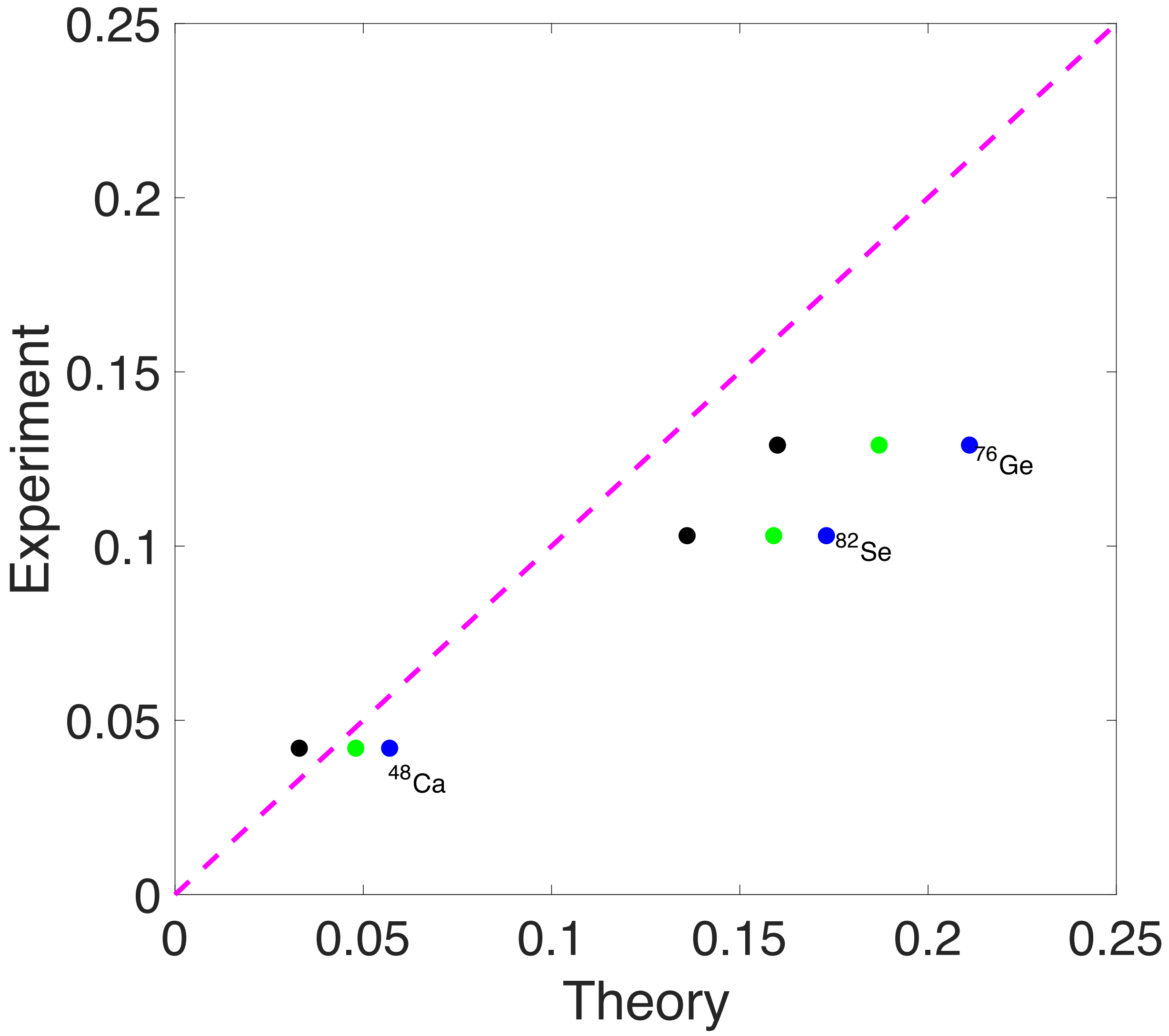

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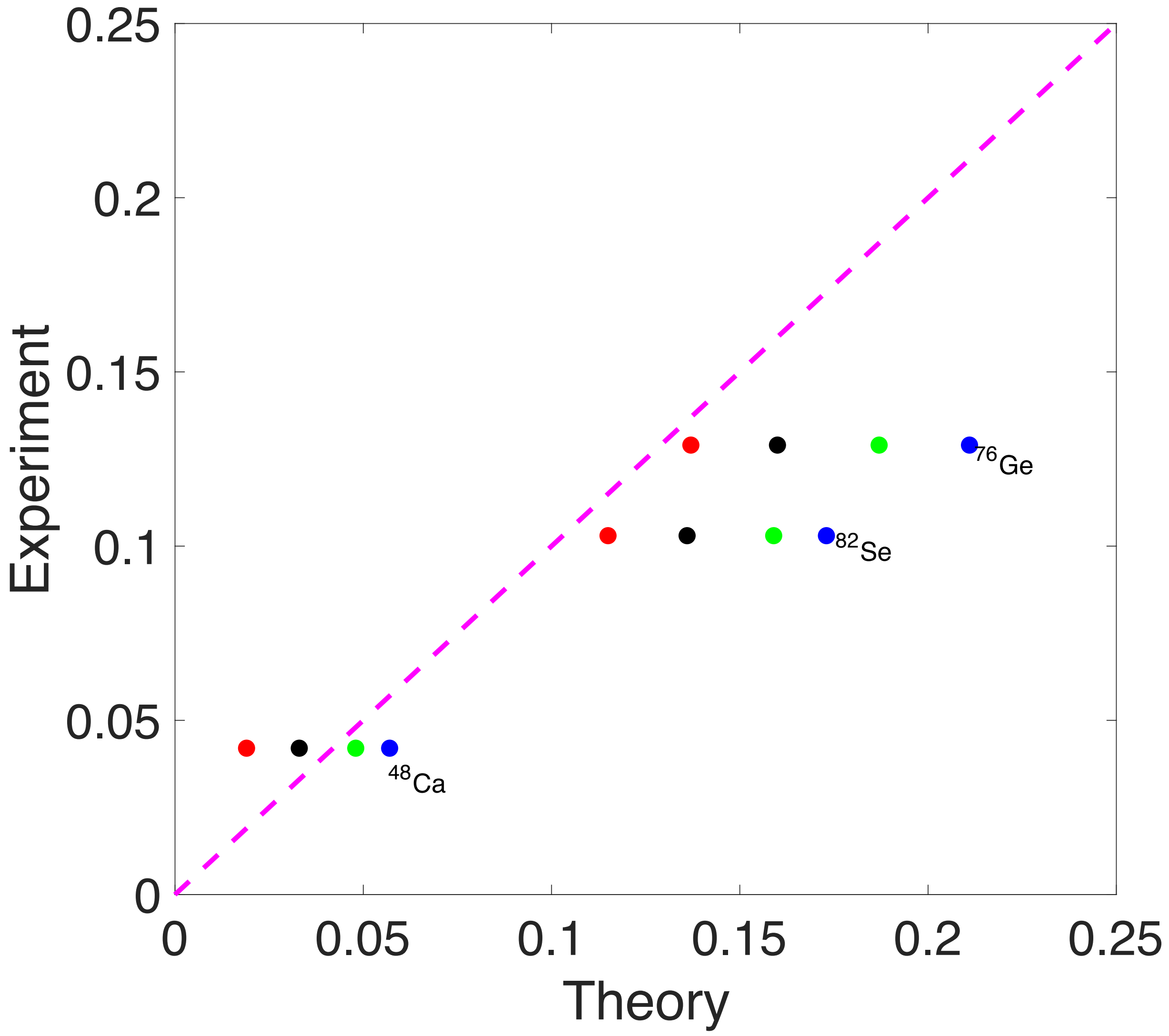
Green: effective J_A at LO in ChPT

Black: bare J_A at N3LO in ChPT

L. Coraggio et al. Physical Review C **109**, 014301 (2024)

2νββ decay



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Green: effective J_A at LO in ChPT

Black: bare J_A at N3LO in ChPT

Red effective J_A at N3LO in ChPT

L. Coraggio et al. Physical Review C **109**, 014301 (2024)

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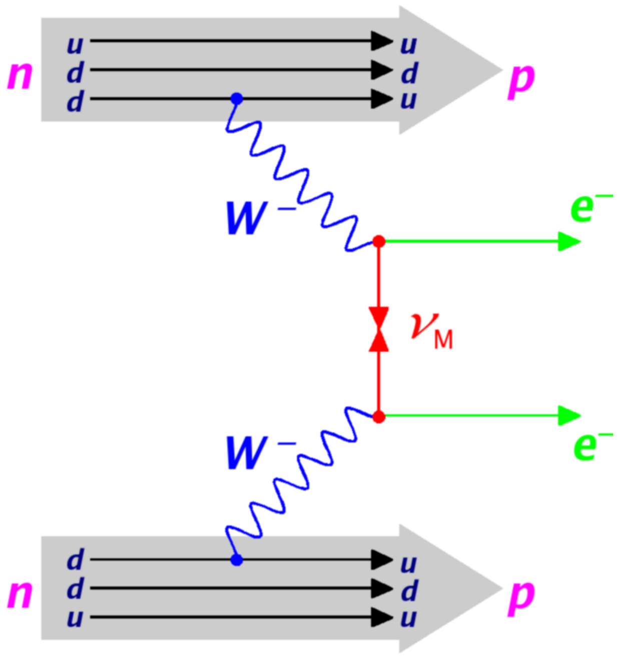
- The problem of the renormalization of $\sigma\tau$ operator
- The impact on $0\nu\beta\beta$ decay matrix element

Quenching of στ matrix elements & 0νββ decay

The inverse of the 0νββ-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME ($M^{0\nu}$)



$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\,|M^{0\nu}|^2\langle m_\nu\rangle^2 \propto g_A^4$$

- $G^{0\nu}$ → phase space factor
- $\langle m_\nu\rangle = |\sum_k m_k U_{ek}|$, effective mass of the Majorana neutrino
  U_{ek} being the lepton mixing matrix

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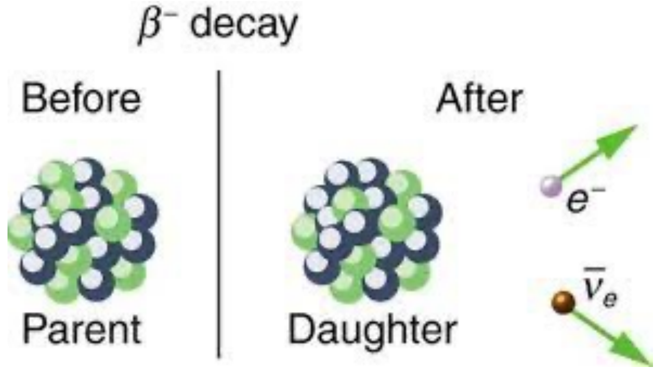
- The β spectrum, the shape function

The β decay spectrum

The total half-life of the β decay is expressed in terms of the k-th partial decay half-life as

$$\frac{1}{T_{1/2}} = \sum_k \frac{1}{t^k_{1/2}} \qquad t^k_{1/2} = \frac{\kappa}{\tilde{C}}$$



where κ = 6144 s

H. Behrens and W. Büring, Nuclear Physics A162 (1971) 11 1 – 144

G. De Gregorio et al. to be published on Phys. Rev. C

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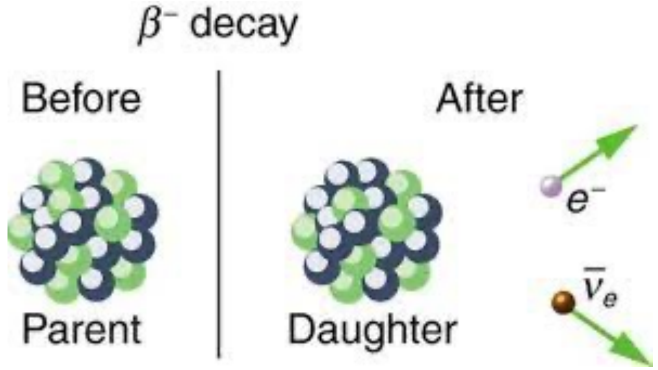

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$\tilde{C}$ is the integrated shape function, whose integrand defines the β-decay energy spectrum

$$\tilde{C} = \int_1^{w_0} p_e w_e (w_0 - w_e)^2 F(Z, w_e) C_n(w_e) dw_e$$

- $F(Z, w_e) = F_0(Z, w_e) L_0(Z, w_e)$, F is the Fermi function and takes into account the distortion of the electron wave function by the nuclear charge and L_0 accounts for the finite size effect.
- w_e is the electron energy
- $C_n(w_e)$ is the shape factor of the n-th forbidden transition which depends on the nuclear matrix elements (NMEs) of the decay operators.

H. Behrens and W. Büring, Nuclear Physics A162 (1971) 11 1 – 144

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The β decay spectrum

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In general C_n is function of W_e.

For allowed transition: $C_n(w_e)$=Const=B(GT)= ${g_A}^2 \frac{|<f|| \sum_k \sigma_k \tau_k^- ||i>|^2}{2J_i+1}$

Beta spectrum is insensitive to B(GT)

J. Kostensalo and J. Suhonen, Phys. Rev. C 96, 024317 (2017)

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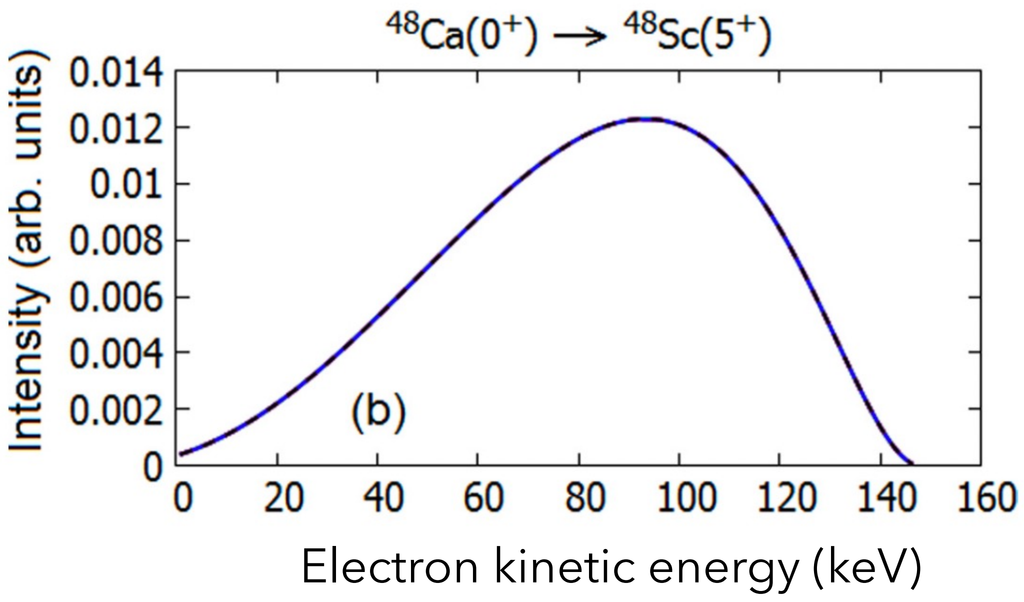
Beta spectrum is insensitive to L^th^-unique decays



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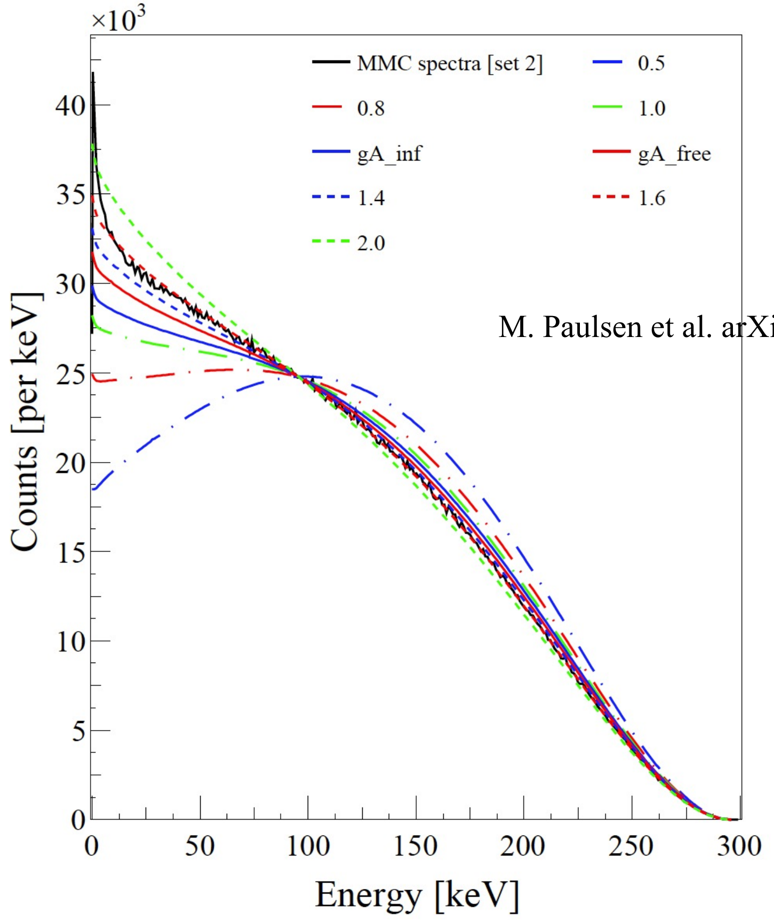
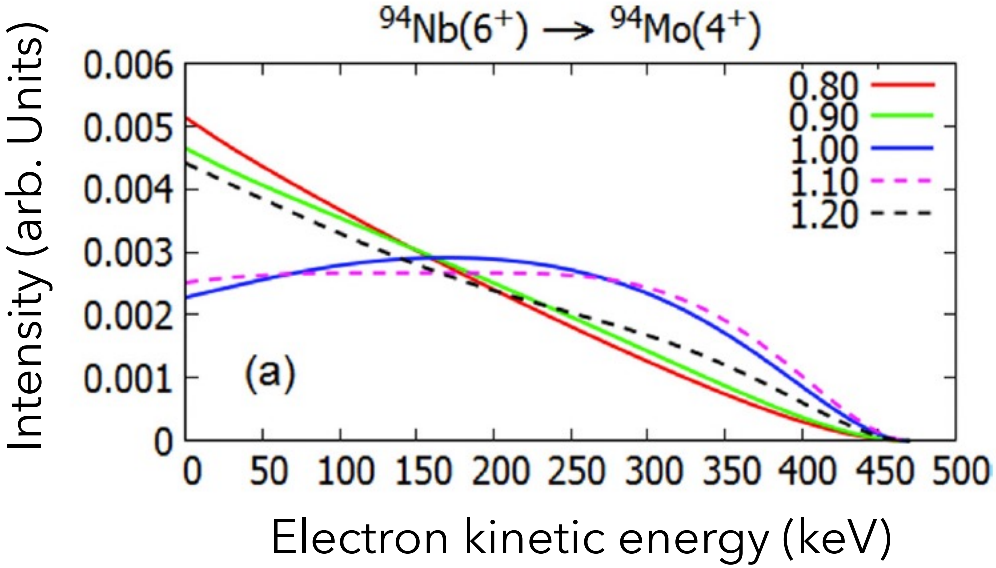
Beta spectrum is sensitive to Lth-nonunique decays



M. Paulsen et al. arXiv:2309.14014v1



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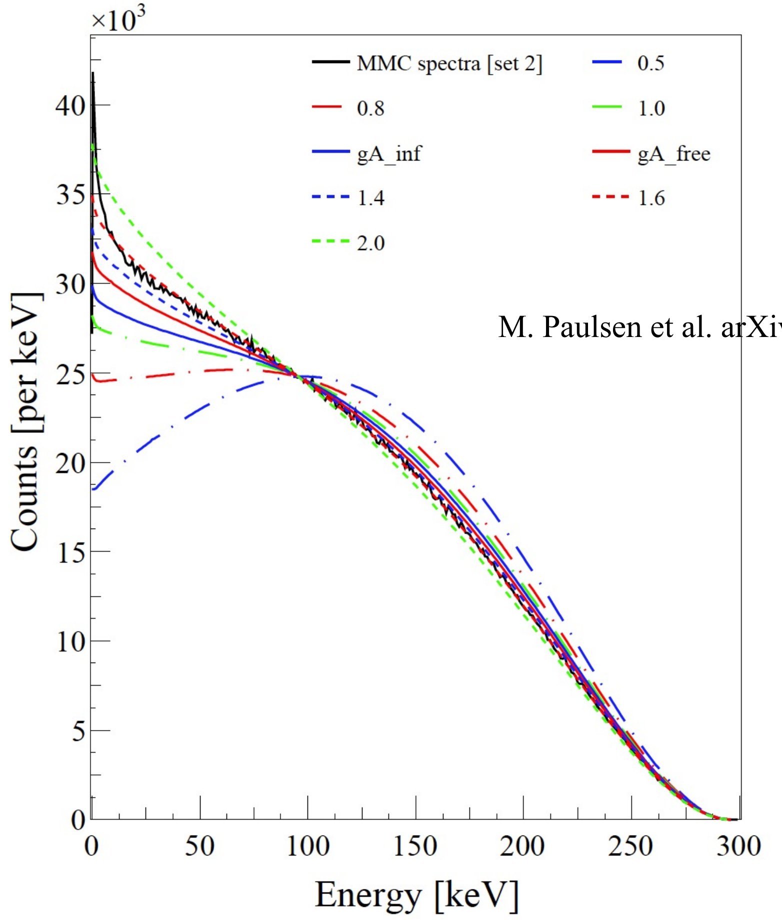

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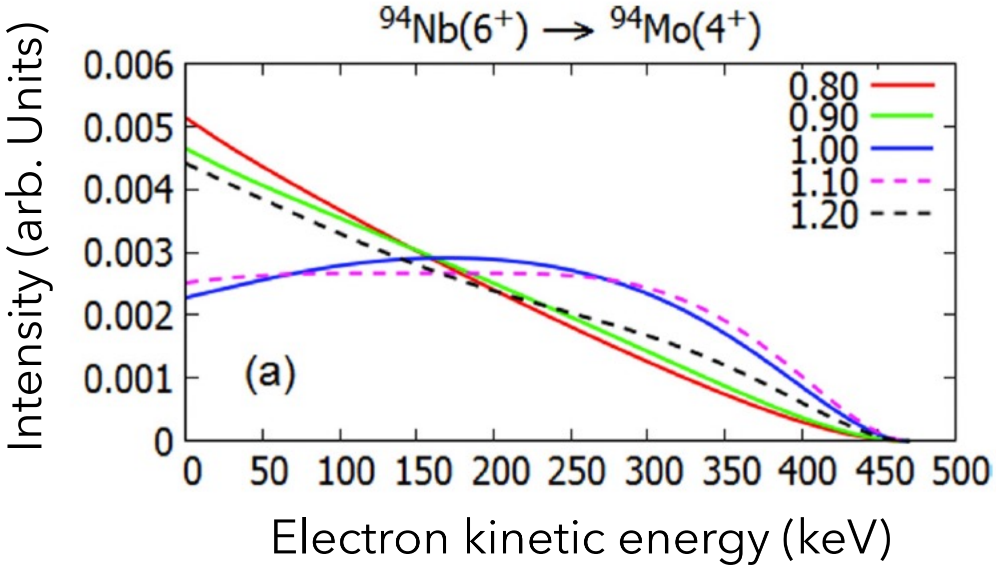

1. One can extract the empirical value of q from the β spectrum on forbidden non-unique decays.

2. Alternatively, having a microscopic theory without free parameters, β spectra offer a further benchmark of the theoretical framework.

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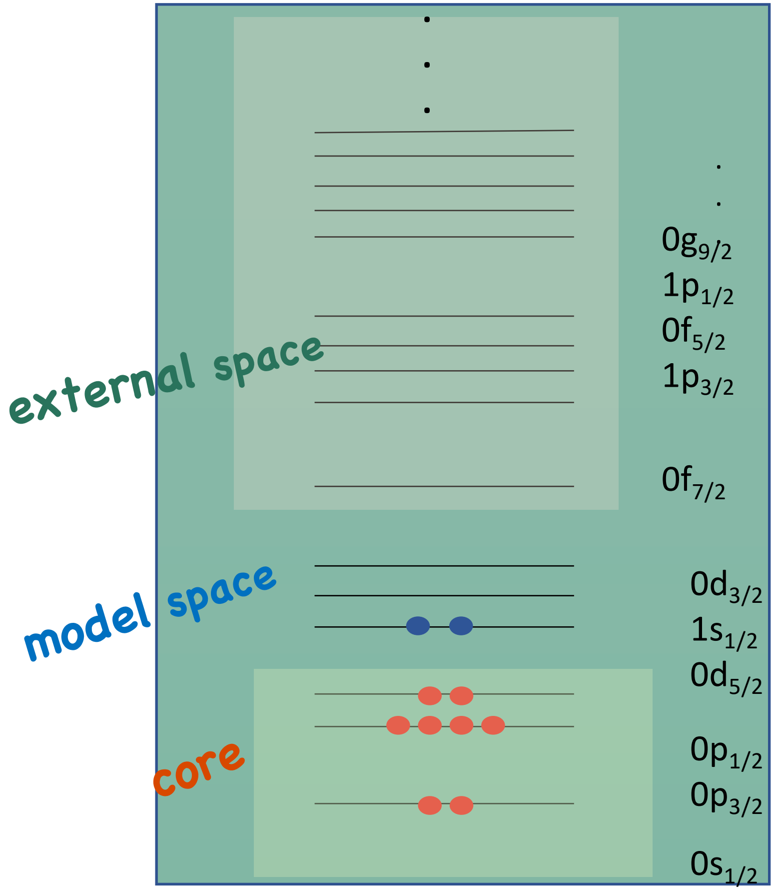
Realistic Shell-Model

Shell model: A well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei



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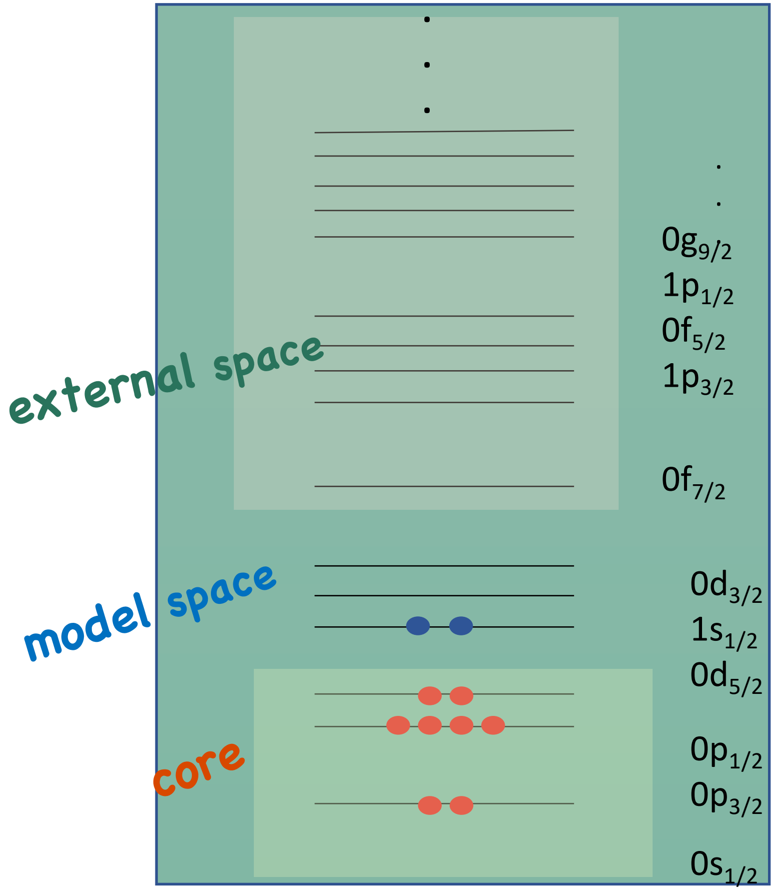

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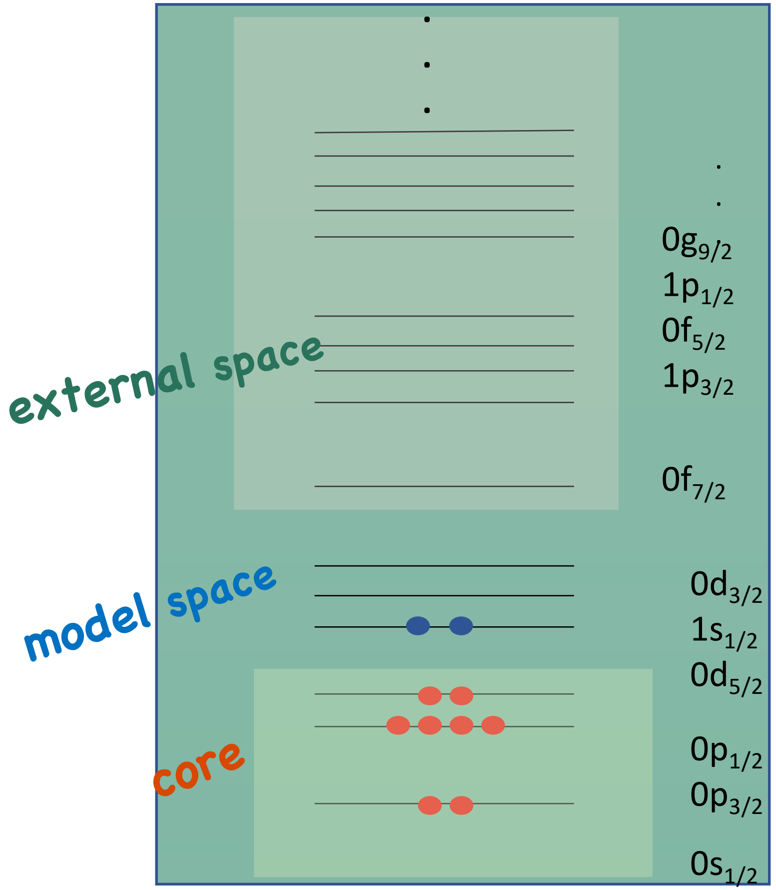

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$$V_{NN}\ (+V_{NNN})$$
$$\downarrow$$
$$MANY\ BODY\ PERTURBATION\ THEORY$$
$$\downarrow$$
$$H_{eff}$$

Realistic shell-model calculations starting from a nuclear Hamiltonian and decay operators derived consistently

Realistic Shell-Model

$$H \rightarrow H_{eff}$$

$$H|\psi_\nu\rangle = E_\nu|\psi_\nu\rangle \rightarrow H_{eff}|\varphi_\alpha\rangle = E_\nu|\varphi_\alpha\rangle$$

$|\varphi_\alpha\rangle$ = eigenvectors obtained diagonalizing H_{eff} in the reducedmodel space $|\varphi_\alpha\rangle$ = P $|\psi_\nu\rangle$

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$$\langle\varphi_\nu|\Theta|\varphi_\lambda\rangle \neq \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

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$$\langle\varphi_\nu|\Theta|\varphi_\lambda\rangle \neq \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

We then require an effective operator Θ_{eff} defined as follows

$$\Theta_{eff} = \sum_{\nu\lambda} |\varphi_\nu\rangle\langle\Psi_\nu|\Theta|\Psi_\lambda\rangle\langle\varphi_\lambda|$$

$$\langle\varphi_\nu|\Theta_{eff}|\varphi_\lambda\rangle = \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

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Details of the Calculations

Calculations for nuclei in the neighborhood of 0νββ candidates above 78Ni core:

- Second forbidden non-unique gs to gs β-decay of ^{94}Nb and ^{99}Tc
- Fourth forbidden non-unique gs to gs β-decay of ^{113}Cd and ^{115}In

One major proton and neutron shell above ^{78}Ni

Interaction

$V_{\text{low-k}}$ of CD-Bonn with Λ=2.6 fm^{-1} + Coulomb force for protons

↓

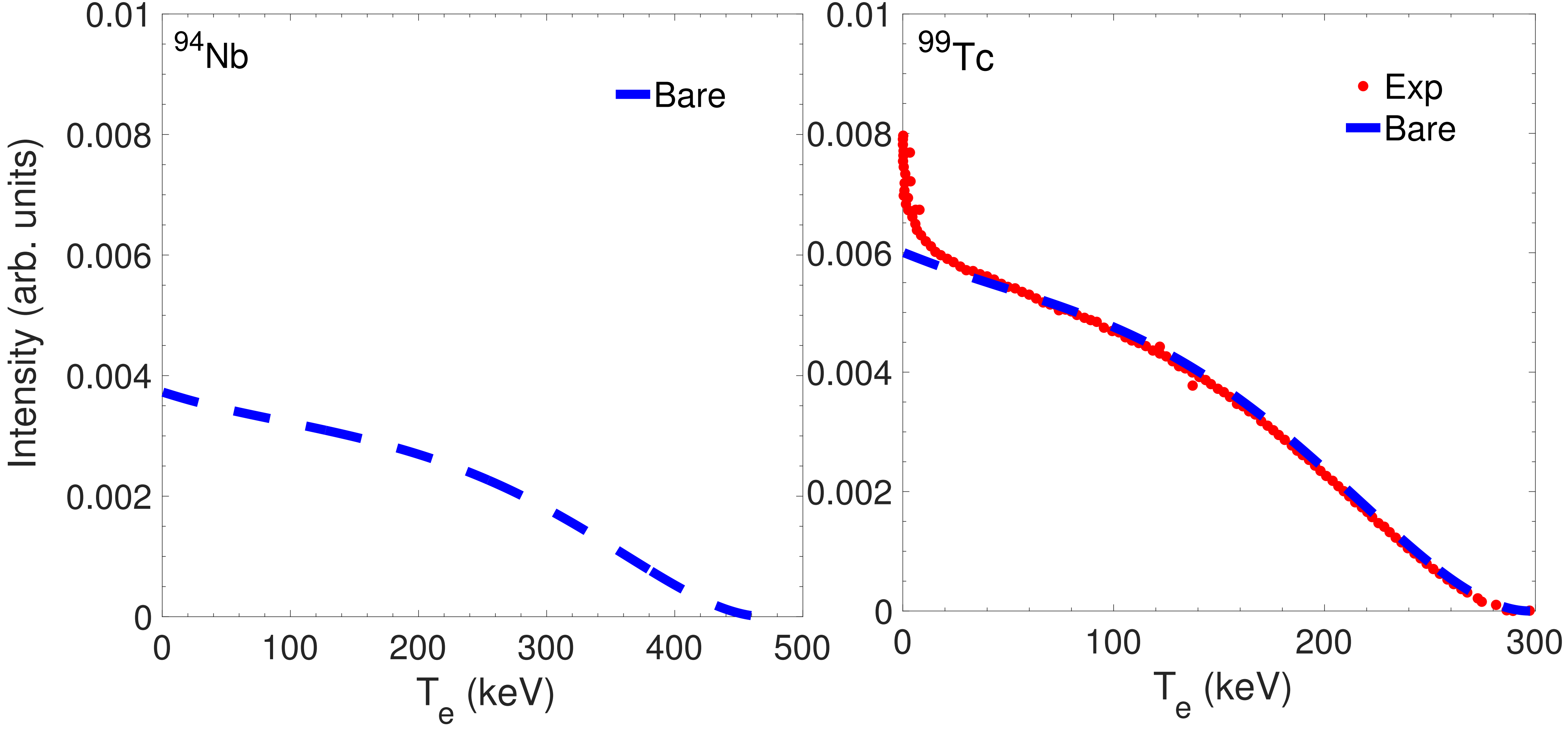
TBME +SP energies of H_{eff} & Θ_{eff} @ third order +3-body correlations

Log*ft*

$$ft = \frac{\kappa}{\tilde{C}} \int_1^{w_0} p_e w_e (w_0 - w_e)^2 F(Z, w_e)\, dw_e$$

	Bare	Effective	Exp
^{94}Nb	11.30	11.58	11.95 (7)
^{99}Tc	11.580	11.876	12.325 (12)
^{113}Cd	21.902	22.493	23.127 (14)
^{115}In	21.22	21.64	22.53 (3)

RSM β decay spectra



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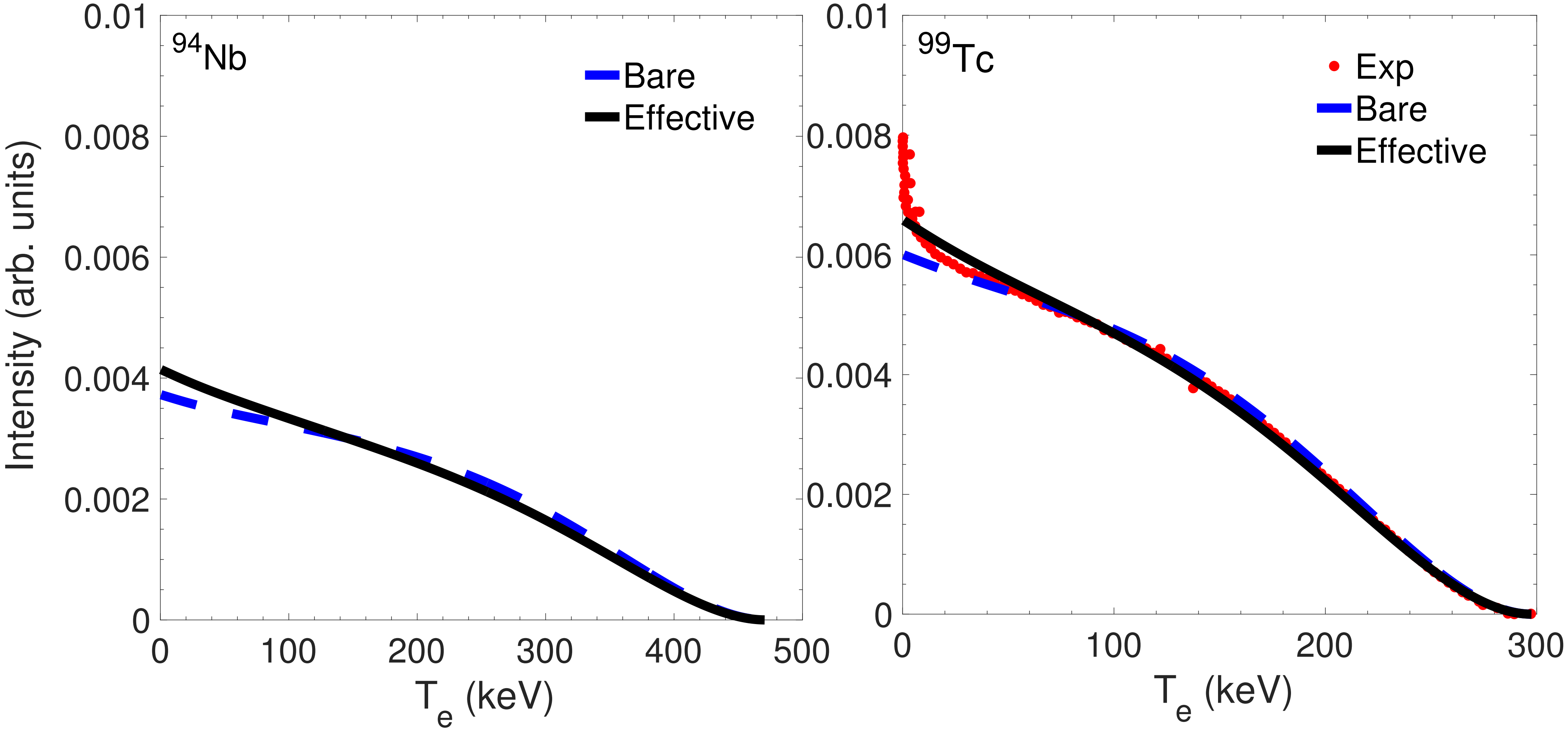

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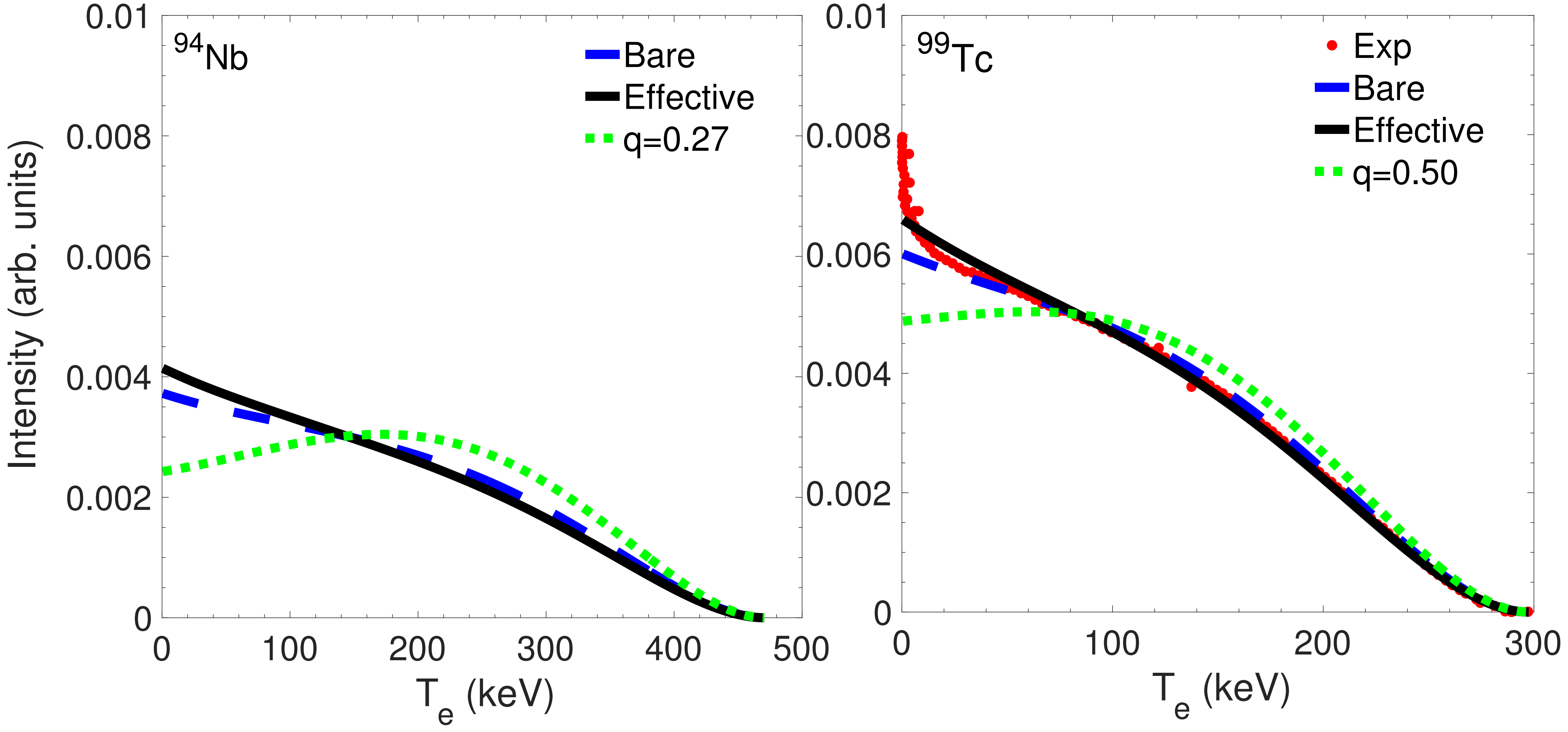

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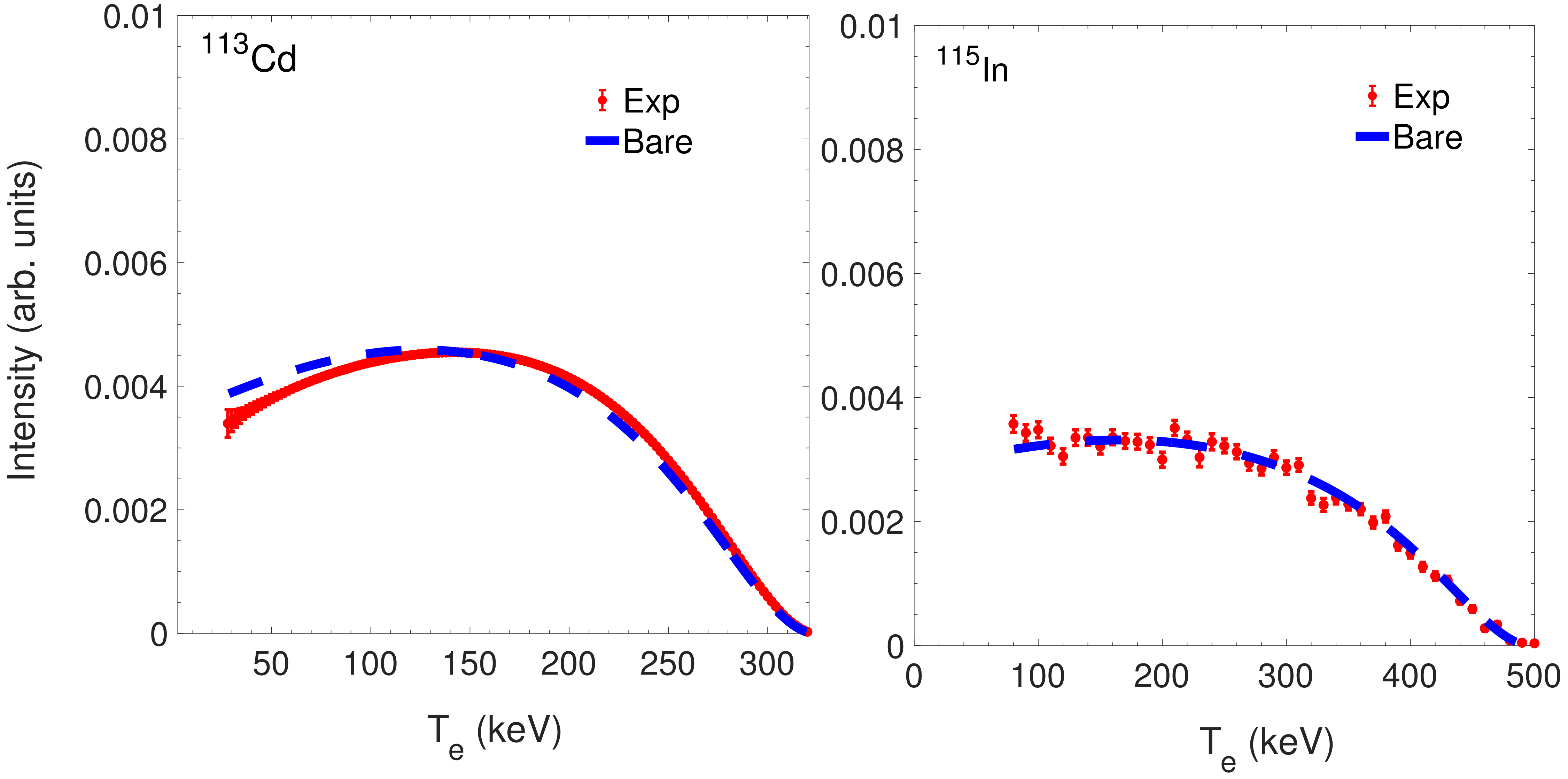

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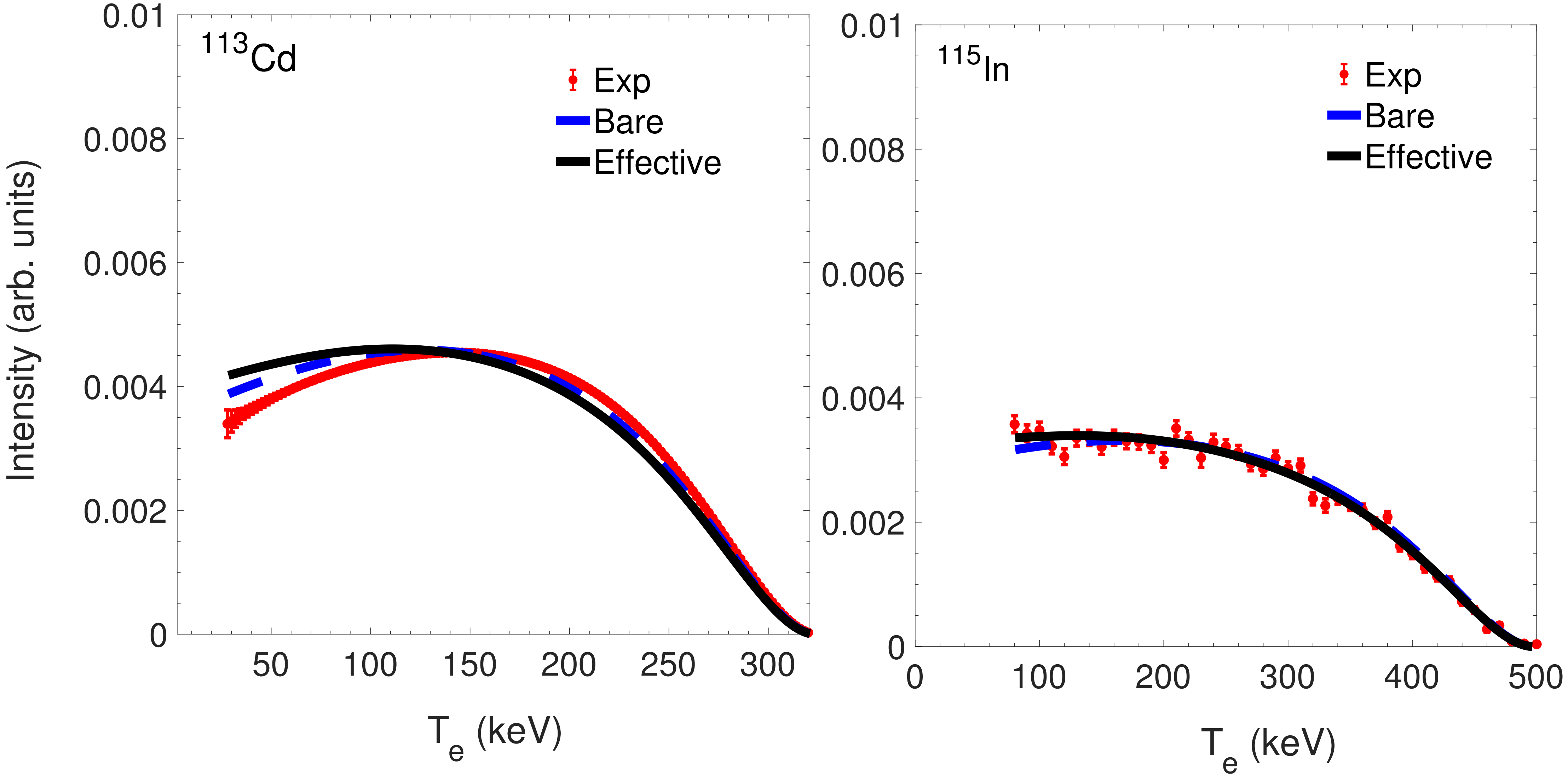

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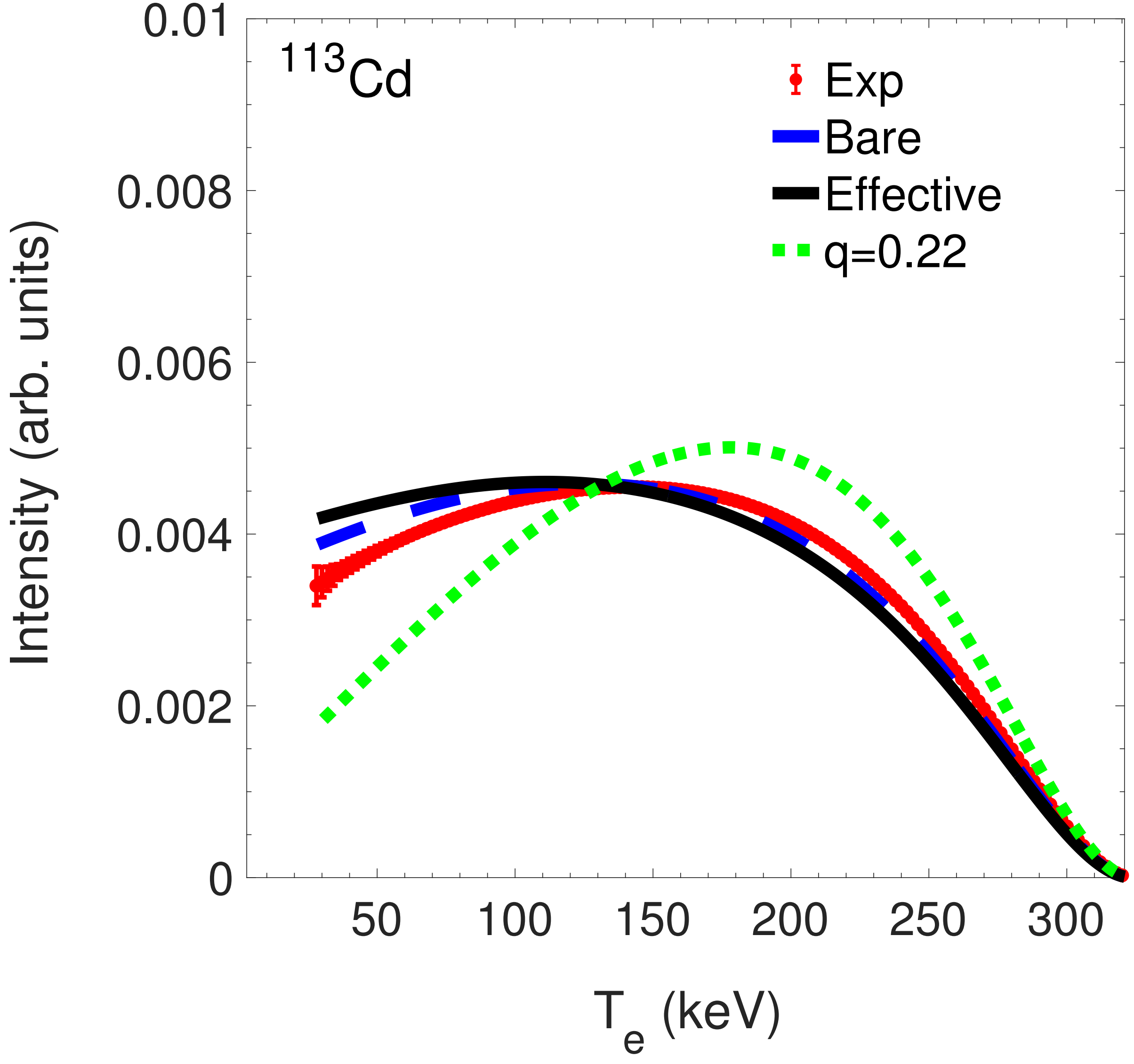

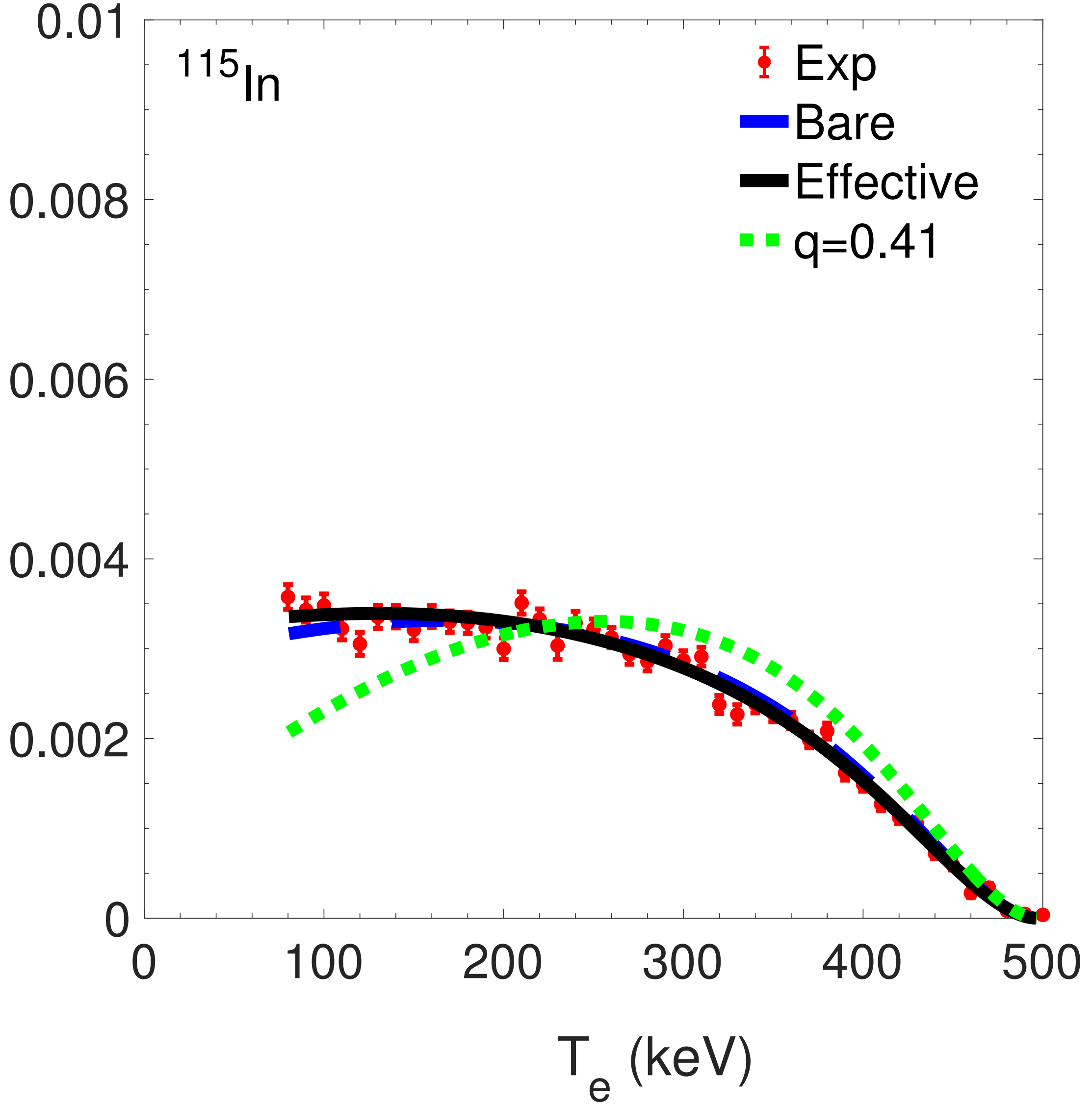

Conclusions and Perspectives

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- Study of forbidden β decays in other mass regions, close to nuclei that are candidates for detecting the 0νββ decay

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- Tackle the forbidden β-decay problem starting from the derivation of electroweak currents by way of the chiral Perturbation theory

Details of the Calculations

- Nuclear Hamiltonian: Entem-Machleidt N3LO two-body potential plus N2LO three-body potential (Λ = 500 MeV)

- Axial current J_A calculated at N3LO in ChPT

- Heff obtained calculating the Q-box up to the 3rd order in V_{NN} (up to 2p-2h core excitations) and up to the 1st order in V_{NNN}

- Effective operators are consistently derived by way of the MBPT

- fp-shell nuclei: four proton and neutron orbitals outside ^{40}Ca: 0f7/2, 0f5/2, 1p3/2, 1p1/2

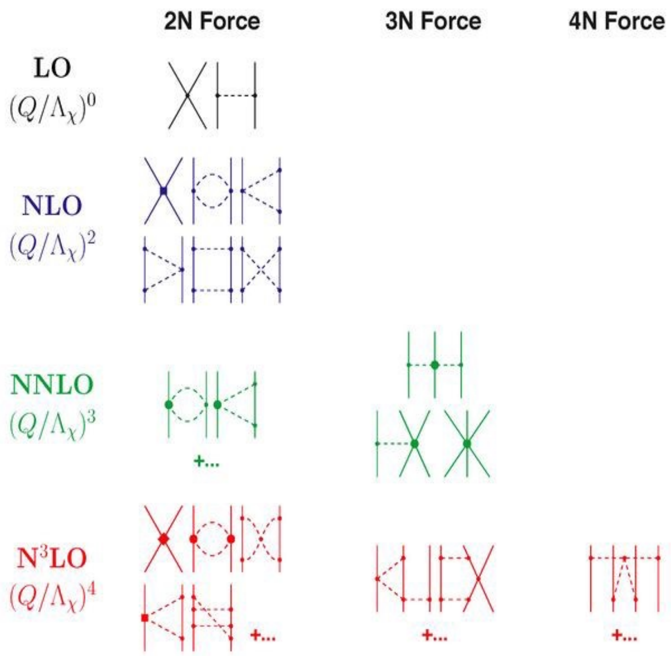
- fpg-shell nuclei: four proton and neutron orbitals outside ^{56}Ni: 0f5/2, 1p3/2, 1p1/2, 0g9/2

RSM calculations, starting from ChPT two- and three-body potentials and two-body meson-exchange currents for spectroscopic and spin-isospin dependent observables of ^{48}Ca, ^{76}Ge, ^{82}Se

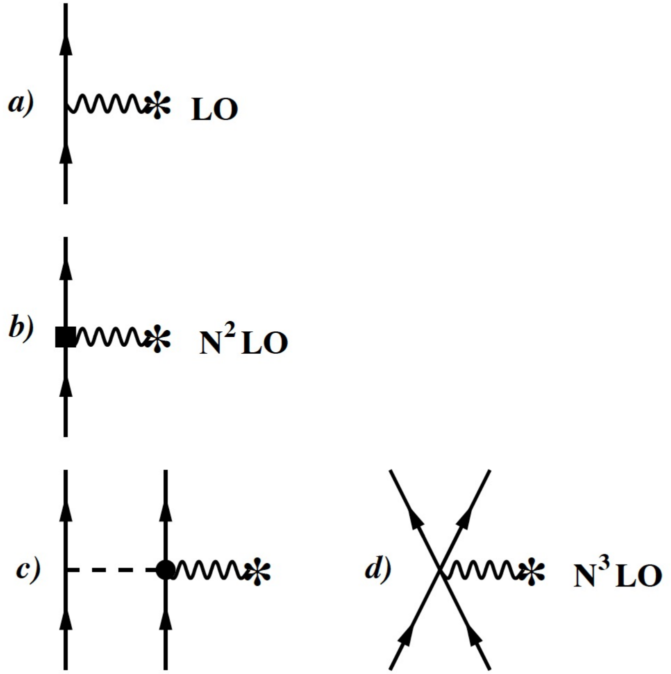
Quenching of $\sigma\tau$ matrix elements: meson exchange currents

Nowadays, EFT provides a powerful approach where both nuclear potentials and two electroweak currents may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator $\boldsymbol{\sigma\tau}^{\pm}$

Nuclear potential



Electroweak axial currents



The axial current J_A

The matrix elements of the axial current J_A are derived through a chiral expansion up to N3LO, and employing the same LECs as in 2NF and 3NF

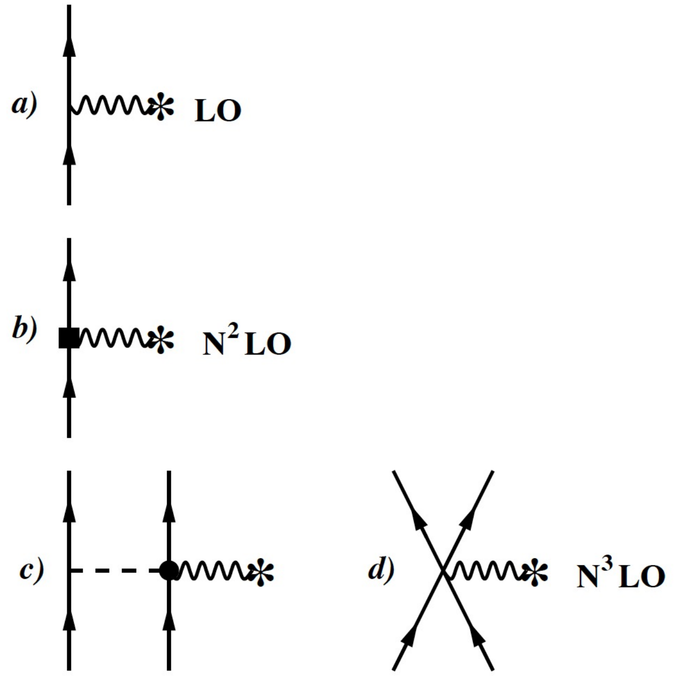
$$J_A^{LO} = -g_A \sum_i \sigma_i \tau_{i,\pm}$$

$$J_A^{N^2LO} = \frac{g_A}{2m_N^2} \sum_i K_i \times (\sigma_i \, x \, K_i) \, \tau_{i,\pm}$$

$$J_A^{N^3LO}(1PE;\boldsymbol{k}) = \sum_{i<j} \frac{g_A}{2f_\pi^2} \Big\{ 4c_3 \, \tau_j \boldsymbol{k} + \left(\tau_i \times \tau_j \right)_\pm \times \left[\left(c_4 + \frac{1}{4m} \sigma_i \times \boldsymbol{k} - \frac{i}{2m} K_i \right) \right] \Big\} \, \sigma_j \cdot \boldsymbol{k} \, \frac{1}{\omega_k^2}$$

$$J_A^{N^3LO}(CT;\boldsymbol{k}) = \sum_{i<j} z_0 \left(\tau_i \times \tau_j \right)_\pm (\sigma_i \times \sigma_j)$$

$$z_0 = \frac{g_A}{2f_\pi^2 m_N^2} \left[\frac{m_N}{4 g_A \Lambda_\chi} c_D + \frac{m_N}{3} (c_3 + 2c_4) + \frac{1}{6} \right]$$



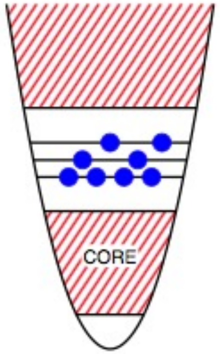
A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C 93, 015501 (2016)

Effective Shell-Model Hamiltonian

$$H|\Psi_\nu\rangle = E_\nu \, |\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A} T_i + U_i + \sum_{i<j} \left(V_{ij} - U_i\right)$$

Effective Shell-Model Hamiltonian



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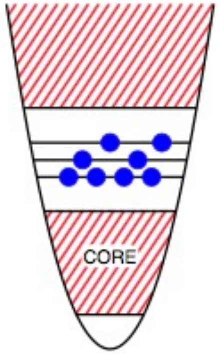

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$$|\phi_i\rangle = \left[a_1^\dagger a_2^\dagger a_3^\dagger \ldots a_n^\dagger\right]_i |c\rangle$$

i=1,2,…d

GOAL: $H|\Psi_\nu\rangle = E_\nu\,|\Psi_\nu\rangle$ → $H_{eff}P|\Psi_\nu\rangle = E_\nu P|\Psi_\nu\rangle$

Effective Shell-Model Hamiltonian

We are looking for a new hamiltonian $\mathcal{H}$ that has the same eigenvalues of the A-nucleon system hamiltonian H, and satisfies the decoupling equation between the model space P and its complement Q:

$QHP = 0$ **so that** $H_{eff} = P\mathcal{H}P$

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$$\mathcal{H} = \Omega^{-1} H \Omega$$

$$\begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \rightarrow \begin{pmatrix} P\mathcal{H}P & P\mathcal{H}Q \\ 0 & Q\mathcal{H}Q \end{pmatrix}$$

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Suzuki-Lee $\quad \Omega = e^{\omega} \quad \omega = \begin{pmatrix} 0 & 0 \\ Q\omega P & 0 \end{pmatrix}$

$$H_{eff}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} Q H_1P - PH_1Q \frac{1}{\epsilon - QHQ} QH_1^{eff}(\omega)$$

Effective Shell-Model Hamiltonian

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}Q\,H_1P$$

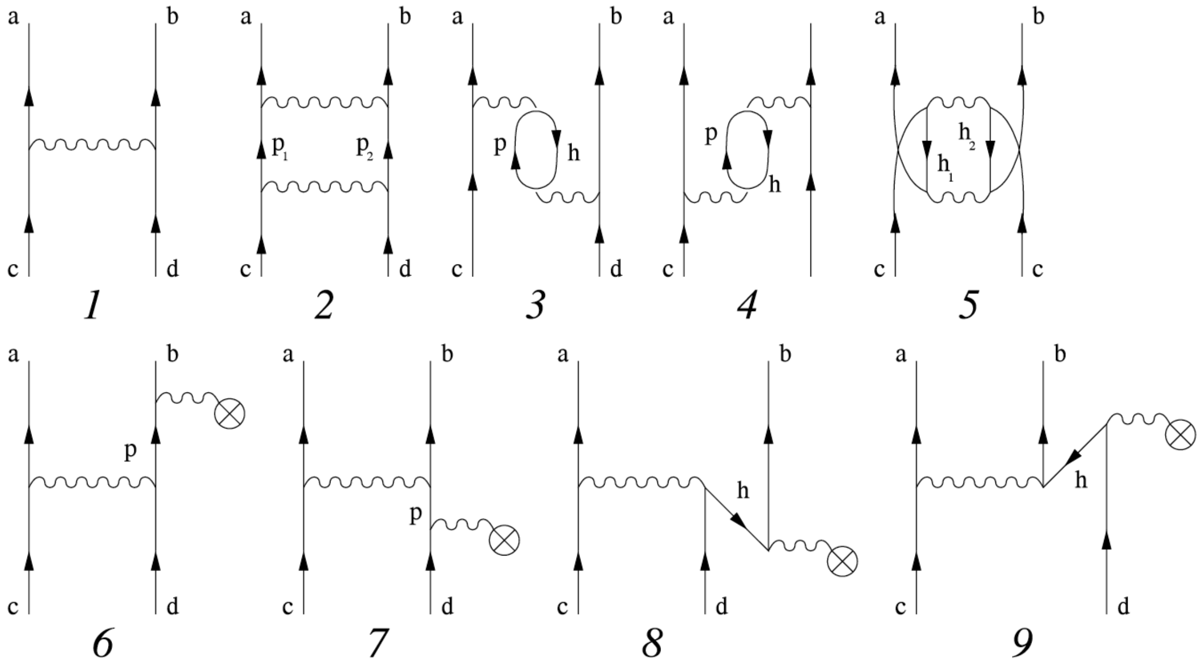
Effective Shell-Model Hamiltonian

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}Q\,H_1P$$

Exact calculation of the $\hat{Q} - box$ is computationally prohibitive (for manybody system) we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty}\frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

Second order diagrams



Effective Shell-Model Operators

Consistently with the derivation of the effective Hamiltonian we have to derive effective transition operators, by way of the many-body perturbation theory

$$|\varphi_\nu\rangle = P|\Psi_\nu\rangle$$

Obviously, for any decay-operator Θ $\quad \langle\varphi_\nu|\Theta|\varphi_\lambda\rangle \neq \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$

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$$|\varphi_\nu\rangle = P|\Psi_\nu\rangle$$

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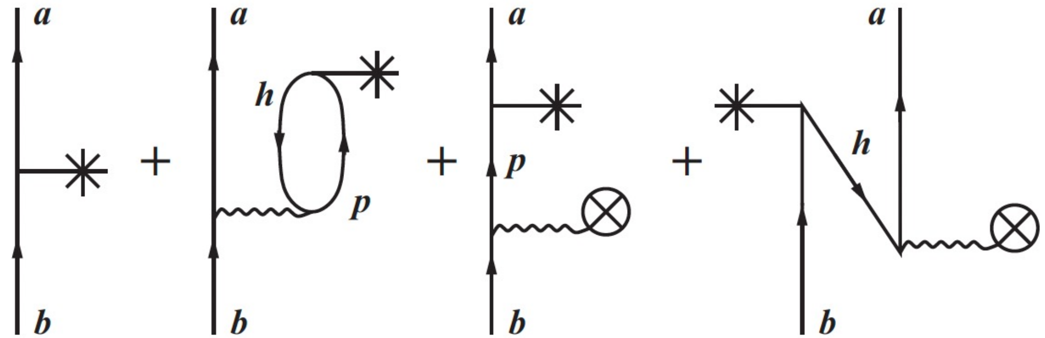
We then require an effective operator Θ_{eff} defined as follows

$$\Theta_{eff} = \sum_{\nu\lambda} |\varphi_\nu\rangle\langle\Psi_\nu|\Theta|\Psi_\lambda\rangle\langle\varphi_\lambda| \qquad \langle\varphi_\nu|\Theta_{eff}|\varphi_\lambda\rangle = \langle\Psi_\nu|\Theta|\Psi_\lambda\rangle$$

Second order one body diagrams for a one-body operator



K. Suzuki and R. Okamoto, Prog. Theor. Phys. 93 , 905 (1995)

Forbidden β-decay ME

$$\phi_{nljm} = \begin{pmatrix} G_{nljm}(r) \\ F_{nljm}(r) \end{pmatrix}$$

$$\frac{dg_\kappa(r)}{dr} + \frac{\kappa+1}{r} g_\kappa(r) - (E+m-V(r)) f_\kappa(r) = 0$$
$$\frac{df_\kappa(r)}{dr} - \frac{\kappa-1}{r} f_\kappa(r) + (E-m-V(r)) g_\kappa(r) = 0$$

$${}^{V/A}M^{(N)}_{KLS}(k_e, m, n, \rho) = \frac{1}{\hat{J}_i} \sum_{\pi,\nu} {}^{V/A}m^{(N)}_{KLS}(\pi, \nu)(k_e, m, n, \rho) \times \mathrm{OBTD}(\Psi_f, \Psi_i, \pi, \nu, K)$$

$${}^{V}m^{(N)}_{KLS}(k_e, m, n, \rho) = \langle \phi_{\kappa_\pi \mu} \, || \left(\frac{r}{R}\right)^{L+2N} \mathcal{I}(k_e, m, n, \rho, r) T_{KLS} \, || \, \phi_{\kappa_\nu \mu} \rangle$$

$${}^{A}m^{(N)}_{KLS}(k_e, m, n, \rho) = \langle \phi_{\kappa_\pi \mu} \, || \left(\frac{r}{R}\right)^{L+2N} \mathcal{I}(k_e, m, n, \rho, r) \gamma_5 T_{KLS} \, || \, \phi_{\kappa_\nu \mu} \rangle$$

$$T^M_{KLS} = \begin{cases} Y_{LM}\delta_{KL} & S = 0 \\ (-1)^{L-K+1}\gamma_5 [Y_L \otimes \sigma]_{KM} & S = 1 \end{cases}$$

Non-relativistic ME

$$\phi_{nljm} = \begin{pmatrix} G_{nljm}(r) \\ F_{nljm}(r) \end{pmatrix}$$

$$\frac{dg_\kappa(r)}{dr} + \frac{\kappa+1}{r} g_\kappa(r) - (E+m-V(r)) f_\kappa(r) = 0$$
$$\frac{df_\kappa(r)}{dr} - \frac{\kappa-1}{r} f_\kappa(r) + (E-m-V(r)) g_\kappa(r) = 0$$

$$\begin{aligned} {}^V m^{(N)}_{KK0}(\pi,\nu)(k_e,m,n,\rho) = \sqrt{2} g_V \Big[& G_{KK0}(\kappa_\pi,\kappa_\nu) \int_0^\infty g_\pi(r,\kappa_p) \left(\tfrac{r}{R}\right)^{K+2N} \mathcal{I}(k_e,m,n,\rho,r) g_\nu(r,\kappa_\nu) r^2 dr \\ & + G_{KK0}(-\kappa_\pi,-\kappa_\nu) \int_0^\infty f_\pi(r,\kappa_\pi) \left(\tfrac{r}{R}\right)^{K+2N} \mathcal{I}(k_e,m,n,\rho,r) f_\nu(r,\kappa_\nu) r^2 dr \Big] \end{aligned} \quad (27)$$

$$\begin{aligned} {}^A m^{(N)}_{KL1}(\pi,\nu)(k_e,m,n,\rho) = \mathrm{sign}(K-L+\tfrac{1}{2})\sqrt{2} g_A \Big[& G_{KK0}(\kappa_\pi,\kappa_\nu) \int_0^\infty g_\pi(r,\kappa_\pi) \left(\tfrac{r}{R}\right)^{L+2N} \mathcal{I}(k_e,m,n,\rho,r) g_\nu(r,\kappa_\nu) r^2 dr \\ & + G_{KK0}(-\kappa_\pi,-\kappa_\nu) \int_0^\infty f_{i_p}(r,\kappa_p) \left(\tfrac{r}{R}\right)^{L+2N} \mathcal{I}(k_e,m,n,\rho,r) f_\nu(r,\kappa_\nu) r^2 dr \Big]. \end{aligned} \quad (28)$$

Relativistic ME

$$\phi_{nljm} = \begin{pmatrix} G_{nljm}(r) \\ F_{nljm}(r) \end{pmatrix}$$

$$\frac{dg_\kappa(r)}{dr} + \frac{\kappa+1}{r} g_\kappa(r) - (E+m-V(r)) f_\kappa(r) = 0$$
$$\frac{df_\kappa(r)}{dr} - \frac{\kappa-1}{r} f_\kappa(r) + (E-m-V(r)) g_\kappa(r) = 0$$

$$\begin{aligned} {}^{V}m^{(N)}_{KL1}(\pi,\nu)(k_e,m,n,\rho) = \operatorname{sign}(K-L+\tfrac{1}{2})\sqrt{2}g_V \Big[& G_{KL1}(\kappa_\pi,-\kappa_\nu)\int_0^\infty g_\pi(r,\kappa_\pi)\left(\tfrac{r}{R}\right)^{L+2N}\mathcal{I}(k_e,m,n,\rho,r) f_\nu(r,\kappa_\nu) r^2 dr \\ & - G_{KK0}(-\kappa_\pi,\kappa_\nu)\int_0^\infty f_\pi(r,\kappa_\pi)\left(\tfrac{r}{R}\right)^{L+2N}\mathcal{I}(k_e,m,n,\rho,r) g_\nu(r,\kappa_\nu) r^2 dr \Big] \end{aligned} \tag{29}$$

$$\begin{aligned} {}^{A}m^{(N)}_{KK0}(\pi,\nu)(k_e,m,n,\rho) = \sqrt{2}g_A \Big[& G_{KK0}(\kappa_\pi,-\kappa_\nu)\int_0^\infty g_\pi(r,\kappa_\pi)\left(\tfrac{r}{R}\right)^{K+2N}\mathcal{I}(k_e,m,n,\rho,r) f_\nu(r,\kappa_\nu) r^2 dr \\ & - G_{KK0}(-\kappa_\pi,\kappa_\nu)\int_0^\infty f_\pi(r,\kappa_\pi)\left(\tfrac{r}{R}\right)^{K+2N}\mathcal{I}(k_e,m,n,\rho,r) g_\nu(r,\kappa_\nu) r^2 dr \Big]. \end{aligned} \tag{30}$$

Treatment of relativistic ME within a non-relativistic framework

1) In then non-relativistic limit the small and large component are connected

$$\frac{dg_\kappa(r)}{dr} + \frac{\kappa+1}{r} g_\kappa(r) - (E + M - V(r)) f_\kappa(r) = 0,$$
$$\frac{df_\kappa(r)}{dr} - \frac{\kappa-1}{r} f_\kappa(r) + (E - M - V(r)) g_\kappa(r) = 0$$

T (E-M) << 2M and V(r) << 2M

$$f_\kappa(r) = \frac{1}{2M_N}\left(\frac{d}{dr} + \frac{\kappa+1}{r}\right) g_\kappa(r)$$

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\kappa(\kappa+1)}{r^2} + 2M_N[T - V(r)]\right) g_\kappa(r) = 0.$$

H. Behrens and W. Bu¨hring, Nucl. Phys. A 162, 111 (1971).

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T (E-M) << 2M and V(r) << 2M

$$f_\kappa(r) = \frac{1}{2M_N}\left(\frac{d}{dr} + \frac{\kappa+1}{r}\right) g_\kappa(r) \qquad \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\kappa(\kappa+1)}{r^2} + 2M_N[T - V(r)]\right) g_\kappa(r) = 0.$$

2) Resort to CVC relations

$$-\sqrt{L(2L+1)}\,{}^V F_{LL-11} + \frac{(qR)^2}{2L+3}\sqrt{\frac{L+1}{2L+1}}\,{}^V F_{LL+11} - W_0 R\,{}^V F_{LL0} = RC_L.$$

$${}^V F_{211} = -\frac{1}{\sqrt{10}} R E_\gamma \,{}^V F_{220}, \quad {}^V F_{431} = -\frac{1}{\sqrt{36}} R E_\gamma \,{}^V F_{440}$$

H. Behrens and W. Bu¨hring, Nucl. Phys. A 162, 111 (1971).

RME within CVC

TABLE III. ^{99}Tc, ^{113}Cd, and ^{115}In β decay relativistic form factors determined with and without recurring to the CVC relations, and the non-relativistic form factors connected with the relativistic ones by CVC. The values are in adimensional units.

^{99}Tc	Bare	Effective
$^{V}F_{211}$	0.000	0.008
$^{V}F_{211}^{CVC}$	-0.030	-0.017
$^{V}F_{220}$	0.286	0.161
^{113}Cd	Bare	Effective
$^{V}F_{431}$	0.0003	-0.008
$^{V}F_{431}^{CVC}$	0.032	0.015
$^{V}F_{440}$	-0.521	-0.237
^{115}In	Bare	Effective
$^{V}F_{431}$	-0.0004	-0.009
$^{V}F_{431}^{CVC}$	0.031	0.017
$^{V}F_{440}$	-0.473	-0.267