QCD sum rule approach to Okamoto-Nolen-Schiffer Anomaly

Hiroyuki Sagawa    RIKEN/University of Aizu

QNP2024
July 8-12, 2024, Barcelona, Spain

1. Introduction
2. QCD-based Charge Symmetry Breaking (CSB) interaction and Okamoto-Nolen-Schiffer anomaly
3. Summary
Mirror Nuclei Binding energy difference (N+/-1, Z-/+1) with N=Z

What is ONS anomaly?

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$^{17}\text{F-}^{17}\text{O}$</th>
<th>$^{15}\text{O-}^{15}\text{N}$</th>
<th>$^{41}\text{Sc-}^{41}\text{Ca}$</th>
<th>$^{39}\text{Ca-}^{39}\text{K}$</th>
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<tr>
<td>Orbital</td>
<td>$1d_{5/2}$</td>
<td>$(1p_{1/2})^{-1}$</td>
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<td>$\Delta E_D$ (Coulomb)</td>
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<td>$\Delta E_E$ (Coulomb)</td>
<td>-0.203</td>
<td>0.026</td>
<td>-0.267</td>
<td>0.260</td>
</tr>
<tr>
<td>Sum</td>
<td>3.393</td>
<td>3.298</td>
<td>6.866</td>
<td>6.977</td>
</tr>
</tbody>
</table>

Expt. [29] 3.543 3.537 7.278 7.307

What is ONS anomaly?

Binding energy difference between mirror nuclei is always $\Delta B(\text{exp.}) - \Delta B(\text{theory}) > 0$
from $A=2$ to $230$
How to cure ONS anomaly?

Binding energy difference between mirror nuclei is always $\Delta B(\text{exp.}) - \Delta B(\text{theory}) > 0$ from $A=2\sim230$.

Suggest the existence of Charge symmetry breaking force (CSB force)

Okamoto-Nolen-Schiffer anomaly (Okamoto 1964, Nolen-Schiffer 1969) is a very old problem, but not yet solved in any ab initio theory based on QCD dynamics.

Nucleon mass $\leftrightarrow$ Spontaneous Chiral symmetry breaking (SSB)

Partial restoration of SSB in nuclear medium

Charge symmetry breaking force derived from QCD sum rule

Universal Energy density functionals (EDF)

Goal: Cure ONS anomaly of light and heavy nuclei from QCD
Isospin Symmetry Breaking Interactions (ISB)

Concept of Isospin proposed by W. Heisenberg, 1932 and E. P. Wigner, 1937

Isospin conservation

\[ [H, T] = 0 \]

\[ [H, T] = [V_C + V_{CSB} + V_{CIB}, T] \neq 0 \]

Scattering Length

\[ a_{(S=0)}^{pp} = -17.3 \pm 0.4 \text{fm}, \]
\[ a_{(S=0)}^{nn} = -18.7 \pm 0.6 \text{fm}, \]
\[ a_{(S=0)}^{pn} = -23.70 \pm 0.03 \text{fm}. \]

The difference between \( a_{0}^{pp} \) and \( a_{0}^{nn} \) is an evidence of CSB (charge symmetry breaking) nuclear force, while the difference between \( a_{0}^{pn} \) and the average \( (a_{0}^{pp} + a_{0}^{nn})/2 \) is due to CIB (charge invariance breaking) force. These negative

Charge independence breaking

\[ V_{CIS} = (V_{nn} + V_{pp})/2 - V_{np} \]

Chiral Symmetry

Hadrons: Nucleon and Pion mass difference

\( m_n - m_p = 1.29 \text{MeV} \)
\( \pi^\pm - \pi^0 = 4.6 \text{MeV} \)

Quarks: explicit chiral symmetry breaking

\( m_u \neq m_d \)
QCD-based Charge Symmetry Interaction and Okamoto-Nolen-Schiffer anomaly

Target. [Okamoto-Nolen-Schiffer anomaly]
Coulomb energy differences between mirror nuclei and Isobaric analogue states from A=3~220 are always 3-9 % larger than theoretical calculations of Independent particle model.

This anomaly suggests
1) 10-20% smaller proton radii of valence particles than those of cores
2) The effect of Charge symmetry breaking interaction (CSB)

Standard models on CSB interactions
1) Meson exchange model sigma-rho and pi-eta meson exchange potentials.
2) Phenomenological CSB EDF to reproduce ground state energy differences of mirror nuclei.

Proton=(uud) \( m_u c^2 \sim 2.3 \text{MeV} \)
Neutron=(udd) \( m_d c^2 \sim 4.8 \text{MeV} \)

Explicit Chiral symmetry breaking
QCD dynamics of strong interaction
(Spontaneous chiral symmetry breaking)
QCD Lagrangian

\[ \mathcal{L} = \sum_{f=u,d,...} \bar{q}_f \left[ i\gamma^\mu \left( \partial_\mu - ig \tau^a A^{a\mu} \right) - m_f \right] q_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

LQCD ⇔ ab initio no core shell model

*Effective Theories (RMF, Ch EFT) *

QCD sum rule ⇔

Energy weighted sum rule for Giant resonances (RPA)

**Spontaneous Symmetry Breaking**

Chiral symmetry breaking

Hadron Physics
Nuclear Many-body Problems

**Lattice QCD**

Asymptotic Freedom, Chiral symmetry

Spontaneous Symmetry Breaking Based on a slide of T. Hatsuda

Ioffe’s formula

\[ m_N = \left\{ -2 \left( 2\pi \right)^2 \langle \bar{q} q \rangle \right\}^{1/3} \]
Two-point function with nucleon current

\[ \Pi_N(p) = i \int d^4x e^{ipx} \langle T(\eta_N(x)\eta_N(0)) \rangle = \not{p}A(p) + B(p) \]

Operator product expansion (OPE) method

\[ A(p) = \frac{1}{64\pi^2} p^4 \ln(-p^2) + \frac{1}{32\pi^2} \frac{\alpha_s}{\pi} G^2 \ln(-p^2) + \frac{2}{3} \frac{<\bar{q}q>^2}{p^2} + \ldots \]

\[ B(p) = -\frac{1}{4\pi^2} <\bar{q}q> p^2 \ln(-p^2) + \ldots \]

Phenomenological hadron propagator

\[ i \int d^4x e^{ipx} \langle T(\eta_N(x)\eta_N(0)) \rangle = \lambda_N^2 \frac{\not{p} + m_N}{p^2 - m_N^2} + \text{higher states} \]

\[ \eta_N(x) \equiv \lambda_N N(x) \]

Derivatives after Borel transformation

\[ m_N = F\left(<\bar{q}q>\right) \]

\[ m_N = \left\{ -2(2\pi)^2<\bar{q}q> \right\}^{1/3} \]
The in-medium chiral condensate has a general form in the leading order of Fermi motion corrections;

\[
\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \approx 1 + \frac{k_1}{\rho \rho_0} + k_2 \left( \frac{\rho}{\rho_0} \right)^{5/3},
\]

(2a)

\[
k_1 = -\frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} < 0, \quad k_2 = -k_1 \frac{3k_{F_0}^2}{10m_N^2} > 0,
\]

(2b)

where \(\sigma_{\pi N}\) is the \(\pi-N\) sigma term, \(m_\pi\) (\(m_N\)) is the pion (nucleon) mass, and \(f_\pi\) is the pion decay constant. The

Partial restoration of Chiral condensation of quark pairs \(\bar{q}q\)

*T. Nishi et al., Nature Physics, March 23, 2023*  
*Pionic atom experiments*
QCD-based CSB interaction

1. QCD sum rule approach to evaluate mass difference of proton and neutron in nuclear medium
2. Partial restoration of Spontaneous symmetry breaking (SSB) in nuclear medium

The mass difference between neutron and proton is formulated in nuclear matter by the QCD sum rule approach in leading order of the quark mass difference and QED effect

\[
\Delta_{np}(\rho) \simeq C_1 G(\rho) - C_2,
\]

\[
G(\rho) = \left( \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \right)^{1/3}.
\]

Here, \( \langle \bar{q}q \rangle \) and \( \langle \bar{q}q \rangle_0 \) are, respectively, the isospin averaged in-medium and in-vacuum chiral condensate. The coefficient \( C_1 \) is proportional to the \( u-d \) quark mass difference \( \delta m \), through the isospin-breaking constant \( \gamma \equiv \langle \bar{d}d \rangle_0/\langle \bar{u}u \rangle_0 - 1 \) as \( C_1 = -a\gamma \) with a positive numerical constant \( a \) determined by the Borel QSR method.

In vacuum,

\[
\Delta_{np}(0) = m_n - m_p \simeq 1.29 \text{ MeV}.
\]
The mass difference between \((Z+/\text{-}1,N)\) and \((Z, N+/\text{-}1)\) with \(N=Z\)

\[
\left( \frac{\bar{q}q}{\langle \bar{q}q \rangle_0} \right) \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left( \frac{\rho}{\rho_0} \right)^{5/3}, \tag{2a}
\]

\[
k_1 = - \frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} < 0, \quad k_2 = -k_1 \frac{3k_0^2}{10m_N^2} > 0, \tag{2b}
\]

where \(\sigma_{\pi N}\) is the \(\pi-N\) sigma term, \(m_\pi\) (\(m_N\)) is the pion (nucleon) mass, and \(f_\pi\) is the pion decay constant. The

\[
\tilde{s}_0 = -\frac{4}{3} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2}, \quad \tilde{s}_1 + 3\tilde{s}_2 = \frac{1}{m_N^2} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2}.
\]

The Skyrme-type CSB and CIB interactions

\[
V_{\text{CSB}}(r) = \left[ s_0 \left( 1 + y_0 P_\sigma \right) \delta(r) + \frac{s_1}{2} \left( 1 + y_1 P_\sigma \right) \left( k^2 \delta(r) + \delta(r) k^2 \right) \right. \\
\left. + s_2 \left( 1 + y_2 P_\sigma \right) k^\dagger \cdot \delta(r) k \right] \frac{\tau_1 z + \tau_2 z}{4},
\]

\[
\frac{E}{A} \simeq \varepsilon_0(\rho) + \varepsilon_1(\rho) \beta + \varepsilon_2(\rho) \beta^2.
\]

\[
\beta = (N - Z) / A
\]

\[
\mathcal{E}_{\text{CSB}} = \frac{s_0(1 - y_0)}{8} \left( \rho_n^2 - \rho_p^2 \right) = \tilde{s}_0(\rho_n + \rho_p)(\rho_n - \rho_p) / 8 + \ldots
\]

\[
\Delta E|_{N=Z} = -2\varepsilon_1(\rho)
\]

\[
\delta_{\text{Skyrme}} = -\frac{\tilde{s}_0}{4} \rho - \frac{1}{10} \left( \frac{3\pi^2}{2} \right)^{2/3} (\tilde{s}_1 + 3\tilde{s}_2) \rho^{5/3}, \tag{7}
\]

where we have defined the effective coupling strengths,

\[
\tilde{s}_0 \equiv s_0 (1 - y_0), \quad \tilde{s}_1 \equiv s_1 (1 - y_1), \quad \tilde{s}_2 \equiv s_2 (1 + y_2).
\]
Figure 47: Lattice results and FLAG averages for the nucleon sigma term, $\sigma_{\pi N}$, for the $N_f = 2, 2+1$, and $2+1+1$ flavour calculations. Determinations via the direct approach are indicated by squares and the Feynman-Hellmann method by triangles. Results from calculations which analyze more than one lattice data set within the Feynman-Hellmann approach [204, 211–219] are shown for comparison (pentagons) along with those from recent analyses of $\pi$-$N$ scattering [186–188, 220] (circles).

**a conservative estimate**

$$\sigma_{\pi N} = 45 \pm 15 \text{ MeV}$$

New LQCD value ($N_f=2+1$)

$$\sigma_{\pi N}=43.7\pm3.6\text{MeV}$$

A. Agadjanov et al., Phys. Rev. Lett. 131, 261902 – Published 27 December 2023
Table I. — Strengths of the various Skyrme-like CSB interactions. LO and NLO refer to the leading-order ($\bar{s}_0$) and the next-leading-order ($\bar{s}_0 - \bar{s}_2$) CSB interactions, respectively. “Pheno” and “Theor”, respectively, refer to results based on phenomenological fitting and on theoretical evaluation. The values shown here except ab initio determinations are taken from Refs. [16-20].

<table>
<thead>
<tr>
<th>Class</th>
<th>Method or Name</th>
<th>$\bar{s}_0$ (MeV fm$^3$)</th>
<th>$\bar{s}_1$ (MeV fm$^5$)</th>
<th>$\bar{s}_2$ (MeV fm$^5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pheno</td>
<td>SAMi-ISB</td>
<td>$-52.6 \pm 1.4$</td>
<td>$-22.4 \pm 4.4$</td>
<td></td>
</tr>
<tr>
<td>Pheno</td>
<td>SLY4-ISB (leading order)</td>
<td>$-22.4 \pm 5.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pheno</td>
<td>SkM*-ISB (leading order)</td>
<td>$-29.6 \pm 7.6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pheno</td>
<td>SVT-ISB (leading order)</td>
<td>$+44 \pm 8$</td>
<td>$-56 \pm 16$</td>
<td>$-31.2 \pm 3.2$</td>
</tr>
<tr>
<td>Pheno</td>
<td>Estimation by isovector density</td>
<td>$-17.6 \pm 32.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>$\Delta E_{tot}$ (N$^2$LO$_{CC}$, CC)</td>
<td>$-4.2 \pm 6.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>$\Delta E_{tot}$ (N$^2$LO$_{CC}$, CC)</td>
<td>$-5.1 \pm 28.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>$\Delta E$ (AV18-UX &amp; GFMC)</td>
<td>$-6.413 \pm 0.173$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>QCD sum rule (Case I)</td>
<td>$-15.5^{+8.5}_{-12.5}$</td>
<td>$+0.52^{+0.42}_{-0.29}$</td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>QCD sum rule (Case II)</td>
<td>$-15.5^{+8.5}_{-12.5}$</td>
<td></td>
<td>$+0.18^{+0.14}_{-0.10}$</td>
</tr>
</tbody>
</table>

CSB interaction, $s_0$ can be determined by

$$s_0 = \frac{\Delta E_{tot}^{w/CSB} - \Delta E_{tot}^{w/o\,CSB}}{\bar{a}}.$$
TABLE V. The breakdown of the mass difference of mirror nuclei $\Delta E$ into each contribution (Coulomb, Extra and CSB interaction (CSBI)) for Case I with Skyrme EDF, SGII. Numbers are given in the unit of MeV.

<table>
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<tr>
<th>Nuclei</th>
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<td>$\Delta E_E$ (Coulomb)</td>
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<td>0.200</td>
</tr>
<tr>
<td>Extra</td>
<td>0.040</td>
<td>0.028</td>
<td>0.102</td>
<td>0.011</td>
</tr>
<tr>
<td>CSBI (Case I)</td>
<td>0.224</td>
<td>0.264</td>
<td>0.287</td>
<td>0.315</td>
</tr>
<tr>
<td>Sum (w/o CSBI)</td>
<td>3.432</td>
<td>3.326</td>
<td>6.965</td>
<td>6.985</td>
</tr>
<tr>
<td>Sum (w/ CSBI)</td>
<td>3.656</td>
<td>3.590</td>
<td>7.252</td>
<td>7.300</td>
</tr>
<tr>
<td>Expt. [29]</td>
<td>3.543</td>
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<td>7.278</td>
<td>7.307</td>
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TABLE VI. The same as Table V, but with Skyrme EDF, SAMi.

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<th>Nuclei</th>
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<tr>
<td>Extra</td>
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<td>0.075</td>
<td>0.104</td>
<td>0.092</td>
</tr>
<tr>
<td>CSBI (Case I)</td>
<td>0.206</td>
<td>0.269</td>
<td>0.271</td>
<td>0.321</td>
</tr>
<tr>
<td>Sum (w/o CSBI)</td>
<td>3.356</td>
<td>3.339</td>
<td>6.870</td>
<td>7.070</td>
</tr>
<tr>
<td>Sum (w/ CSBI)</td>
<td>3.562</td>
<td>3.608</td>
<td>7.141</td>
<td>7.391</td>
</tr>
<tr>
<td>Expt. [29]</td>
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<td>7.307</td>
</tr>
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FIG. 2. Comparisons of the experimental ONS anomaly $\Delta E_{\text{Expt.}} - \Delta E_C$ (grey hatched bars) and the corresponding theoretical estimates in two EDFs (SGII and SAMi). The contribution from the QCD-based CSB interaction (CSBI) in Case I and the extra contributions are indicated by the red bars with error bars and the blue bars, respectively.
Summary

QCD sum rule approach is adopted to obtain EDF CSB parameters without introducing any free parameters; all QCD parameters are determined by experimental observables.

QCD-based CSB interactions are applied to solve ONS anomaly of $A=16+/−1$ and $40+/−1$ mirror nuclei and cured all experimental observed values within the theoretical uncertainties.

Neutron skins of $N>Z$ and also $N<Z$ nuclei are very sensitive to CSB interactions.

Future perspectives

CIB interaction in nuclear medium
QCD based $Λ − N$ CSB interaction for hypernuclei
CSB and CIB effects of Isobaric Analogue states
CSB in Exotic nuclei near proton drip lines
Collaborators

QCD-CSB,QCD-CIB

T. Naito, iTHEMS, RIKEN
T. Hatsuda, iTHEMS, RIKEN
Xavi Roca-Maza, INFN, University of Milano, Italy
University of Barcelona, Spain
Gianluca Colo, INFN, University of Milano, Italy

Skyrme CSB and neutron skin

T. Naito, iTHEMS, RIKEN
Xavi Roca-Maza, INFN, University of Milano, Italy
University of Barcelona, Spain
Gianluca Colo, INFN, University of Milano, Italy
H. Z. Liang, University of Tokyo
Isospin Breaking interactions

Charge symmetry breaking (CSB) interaction
\[ V_{nn} \neq V_{pp} \]

Charge Independence breaking (CIB) interaction
\[ V_{np} \neq \frac{(V_{nn} + V_{pp})}{2} \]

Hadrons: Nucleon and Pion mass difference
\[ m_n - m_p = 1.29 \text{MeV} \]
\[ \pi^\pm - \pi^0 = 4.6 \text{MeV} \]

Quarks: explicit chiral symmetry breaking
\[ m_u \neq m_d \]

Skyrme type ISB interactions

Energy density functionals

\[ E_{\text{CSB}} = \frac{s_0(1 - y_0)}{8} (\rho_n^2 - \rho_p^2) \]
\[ E_{\text{CIB}} = \frac{u_0}{8} \left[ \left( 1 - \frac{z_0}{2} \right) (\rho_n + \rho_p)^2 - 2(2 + z_0)\rho_n\rho_p \right] \]

Observables

Okamoto-Nolen-Schiffer (ONS) anomaly
Energy of IAS
mass differences in isobar and isotriplet nuclei
Two-point function with nucleon current

\[\Pi_N(p) = i \int d^4x \ e^{ipx} \langle T(\eta_N(x) \eta_N(0)) \rangle = \not p A(p) + B(p)\]

Operator product expansion (OPE) method

\[A(p) = \frac{1}{64 \pi^2} \ p^4 \ln(-p^2) + \frac{1}{32 \pi^2} \ \frac{a_0}{z} G^2 \ ln(-p^2) + \frac{2}{3} \ \frac{\langle \bar{q}q \rangle^2}{p^2} + \ldots\]

\[B(p) = -\frac{1}{4 \pi^2} \langle \bar{q}q \rangle \ p^2 \ ln(-p^2) + \ldots\]

Phenomenological hadron propagator

\[i \int d^4x \ e^{ipx} \langle T(\gamma_N(x) \bar{\gamma}_N(0)) \rangle = \lambda_N^2 \ \frac{\not p + m_N}{p^2 - m_N^2} + \text{higher states}\]

\[\eta_N(x) \equiv \lambda_N N(x)\]

Derivatives after Borel transformation

\[m_N = F(\langle \bar{q}q \rangle)\]

\[m_N = \{ -2 (2 \pi)^2 \langle \bar{q}q \rangle \}^{1/3}\]