QCD sum rule approach to Okamoto-Nolen-Schiffer Anomaly

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1. Introduction
2. QCD-based Charge Symmetry Breaking (CSB) interaction and Okamoto-Nolen-Schiffer anomaly
3. Summary

RIKEN NiSHiNA CENTER

THE UNIVERSITY OF AIZU
to Advance Knowledge for Humanity
1993

What is ONS anomaly?

Mirror Nuclei Binding energy difference
(N+/-1, Z-/+1) with N=Z

Nuclei	^{17}F-^{17}O	^{15}O-^{15}N	^{41}Sc-^{41}Ca	^{39}Ca-^{39}K
Orbital	$1d_{5/2}$	$(1p_{1/2})^{-1}$	$1f_{7/2}$	$(1d_{3/2})^{-1}$
ΔE_{D} (Coulomb)	3.596	3.272	7.133	6.717
ΔE_{E} (Coulomb)	−0.203	0.026	−0.267	0.260
Sum	3.393	3.298	6.866	6.977
Expt. [29]	3.543	3.537	7.278	7.307

Binding energy difference
between mirror nuclei is always
$\Delta B(exp.) - \Delta B(theory) > 0$
from A=2~230

Nuclei	^{17}F-^{17}O	^{15}O-^{15}N	^{41}Sc-^{41}Ca	^{39}Ca-^{39}K
Particle (hole)	$1d_{5/2}$	$(1p_{1/2})^{-1}$	$1f_{7/2}$	$(1d_{3/2})^{-1}$
Finite size	−0.053	−0.070	−0.066	−0.082
Center-of-mass	0.023	0.030	0.014	0.018
δ^1_{NN}	0.014	0.006	0.034	0.021
δ^2_{NN}	0.050	−0.136	0.134	−0.176
Spin-orbit	−0.065	0.080	−0.126	0.142
pn mass difference	0.034	0.024	0.040	0.031
δ_{pol}	0.018	0.073	0.036	0.020
Vacuum polarization	0.019	0.021	0.036	0.037
Sum	0.040	0.028	0.102	0.011

How to cure ONS anomaly?

Binding energy difference between mirror nuclei is always $\Delta B(exp.)$ -$\Delta B(theory)$>0 from A=2~230

Suggest the existence of Charge symmetry breaking force (CSB force)

Okamoto-Nolen-Schiffer anomaly (Okamoto 1964, Nolen-Schiffer 1969) is very old problem, but not yet solved in any ab initio theory based on QCD dynamics

Nucleon mass ⟺ Spontaneous Chiral symmetry breaking (SSB)

Partial restoration of SSBin nuclear medium

↓

Charge symmetry breaking force derived from QCD sum rule ⟷ Universal Energy density functionals (EDF)

Goal: Cure ONS anomaly of light and heavy nuclei from QCD

Isospin Symmetry Breaking Interactions (ISB)

Concept of Isospin proposed by W. Heisenberg, 1932 and E. P. Wigner, 1937

Isospin conservation $[H, T] = 0$

$$[H, T] = [V_C + V_{CSB} + V_{CIB}, T] \neq 0$$

charge symmetry breaking

$V_{CSB} = V_{nn} - V_{pp}$

Charge independence breaking

$V_{CIS} = (V_{nn} + V_{pp})/2 - V_{np}$

Scattering Length

$$a^{pp}_{(S=0)} = -17.3 \pm 0.4\,\mathrm{fm},$$
$$a^{nn}_{(S=0)} = -18.7 \pm 0.6\,\mathrm{fm},$$
$$a^{pn}_{(S=0)} = -23.70 \pm 0.03\,\mathrm{fm}.$$

The difference between a_0^{pp} and a_0^{nn} is an evidence of CSB (charge symmetry breaking) nuclear force, while the difference between a_0^{pn} and the average $(a_0^{pp} + a_0^{nn})/2$ is due to CIB (charge invariance breaking) force. These negative

Chiral Symmetry

Hadrons: Nucleon and Pion mass difference

$m_n - m_p = 1.29\,\mathrm{MeV}$. $\pi^{\pm} - \pi^0 = 4.6\,\mathrm{MeV}$

Quarks: explicit chiral symmetry breaking

$$m_u \neq m_d$$

QCD-based Charge Symmetry Interaction and Okamoto-Nolen-Schiffer anomaly

HS, T. Naito, X. Roca-Maza and T. Hatsuda, PRC109, L011302 (2024)

Target. [Okamoto-Nolen-Schiffer anomaly]
Coulomb energy differences between mirror nuclei and Isobaric analogue states from A=3~220 are always 3-9 % larger than theoretical calculations of Independent particle model.

This anomaly suggests
1) 10-20% smaller proton radii of valence particles than those of cores
2) The effect of Charge symmetry breaking interaction (CSB)

Standard models on CSB interactions
1) Meson exchange model sigma-rho and pi-eta meson exchange potentials.
2) Phenomenological CSB EDF to reproduce ground state energy differences of mirror nuclei.

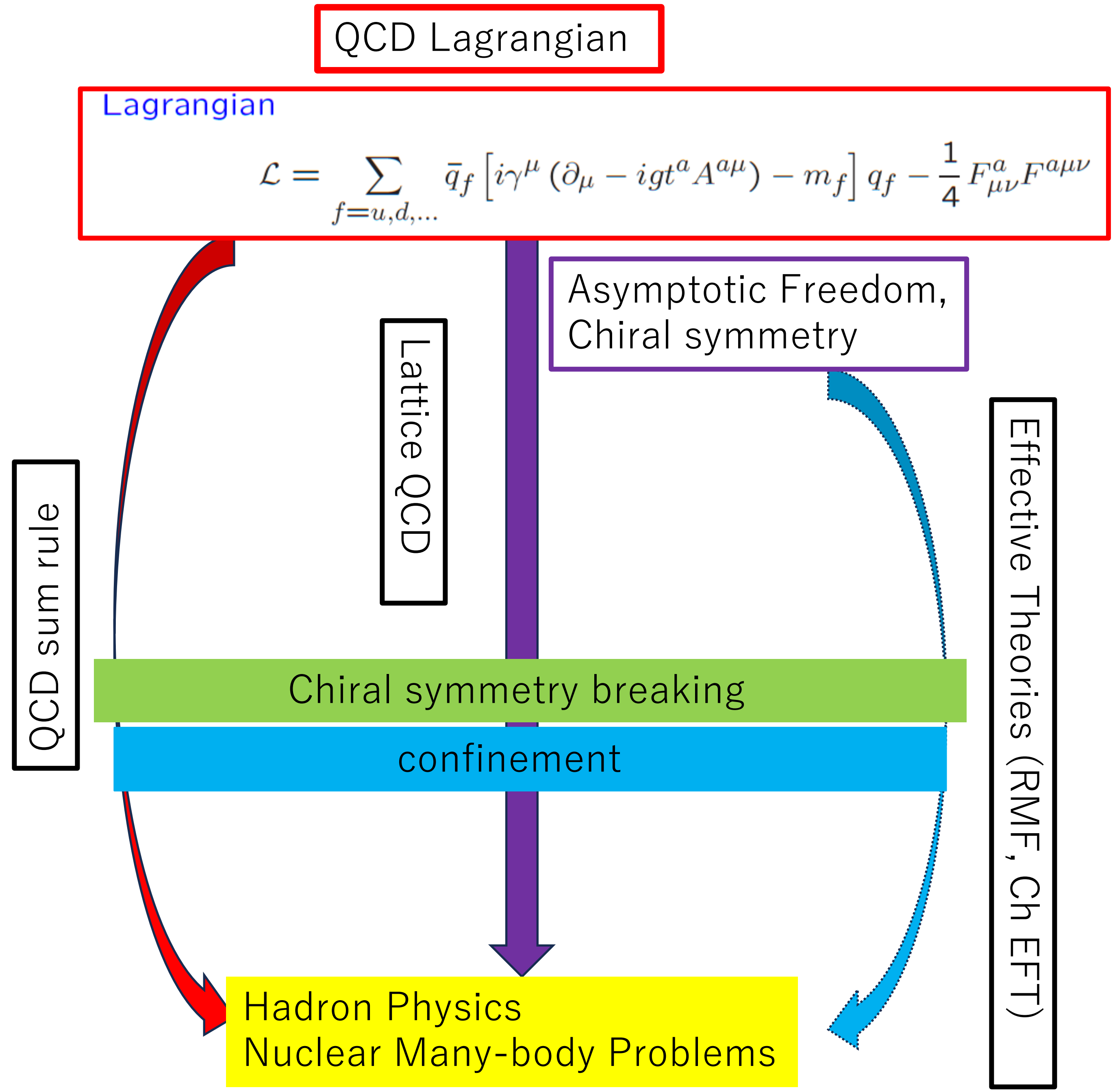
QCD –based approach

Proton=(uud) $m_u c^2 \sim 2.3$MeV
Neutron=(udd) $m_d c^2 \sim 4.8$MeV

Explicit Chiral symmetry breaking

QCD dynamics of strong interaction

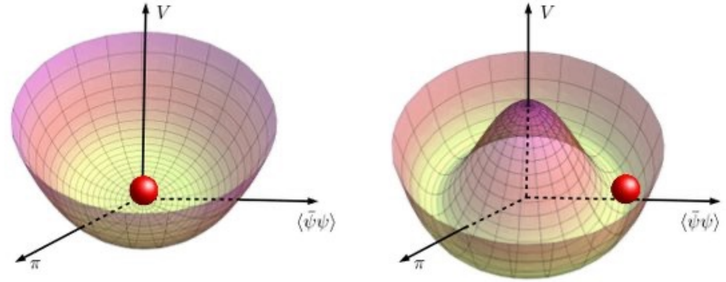
(Spontaneous chiral symmetry breaking)



LQCD
⇔
ab initio no core shell model

Quark Effective theories
(**Nambu–Jona-Lasinio model**)
⇔
BCS/HF in nuclear physics

QCD sum rule
⇔
Energy weighted sum rule for Giant resonances (RPA)
Spontaneous Symmetry Breaking



Ioffe's formula

$$m_N = \{ -2(2\pi)^2 \langle \bar{q} q \rangle \}^{1/3}$$

QCDの Lagrangian

$$L_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4} G_{\mu\nu}{}^{a} G^{a}{}_{\mu\nu}$$

HADRON PROPERTIES FROM QCD SUM RULES

L.J. REINDERS*, H. RUBINSTEIN** and S. YAZAKI***

PHYSICS REPORTS (Review Section of Physics Letters) 127, No. 1 (1985) 1–97.

Two-point function with nucleon current

$$\eta_N = \varepsilon_{abc}(u^a(x) C\gamma_\mu u^b(x))\gamma_5\gamma_\mu d^c(x),$$

$$\Pi_N(p) = i\int d^4x\, e^{ipx} < T(\eta_N(x)\, \eta_N(0)) > = \not{p} A(p) + B(p)$$

Operator product expansion (OPE) method

$$A(p) = \frac{1}{64\pi^2} p^4 \ln(-p^2) + \frac{1}{32\pi^2} < \frac{\alpha_s}{\pi} G^2 > \ln(-p^2) + \frac{2}{3} \frac{<\bar{q}q>^2}{p^2} + \cdots$$

$$B(p) = -\frac{1}{4\pi^2} < \bar{q}q > p^2 \ln(-p^2) + \cdots$$

Phenomenological hadron propagator

$$i\int d^4x\, e^{ipx} < T(\eta_N(x)\, \bar{\eta}_N(0)) > = \lambda_N^2 \frac{\not{p} + m_N}{p^2 - m_N^2} + \text{higher states}$$

$$\eta_N(x) \equiv \lambda_N N(x)$$

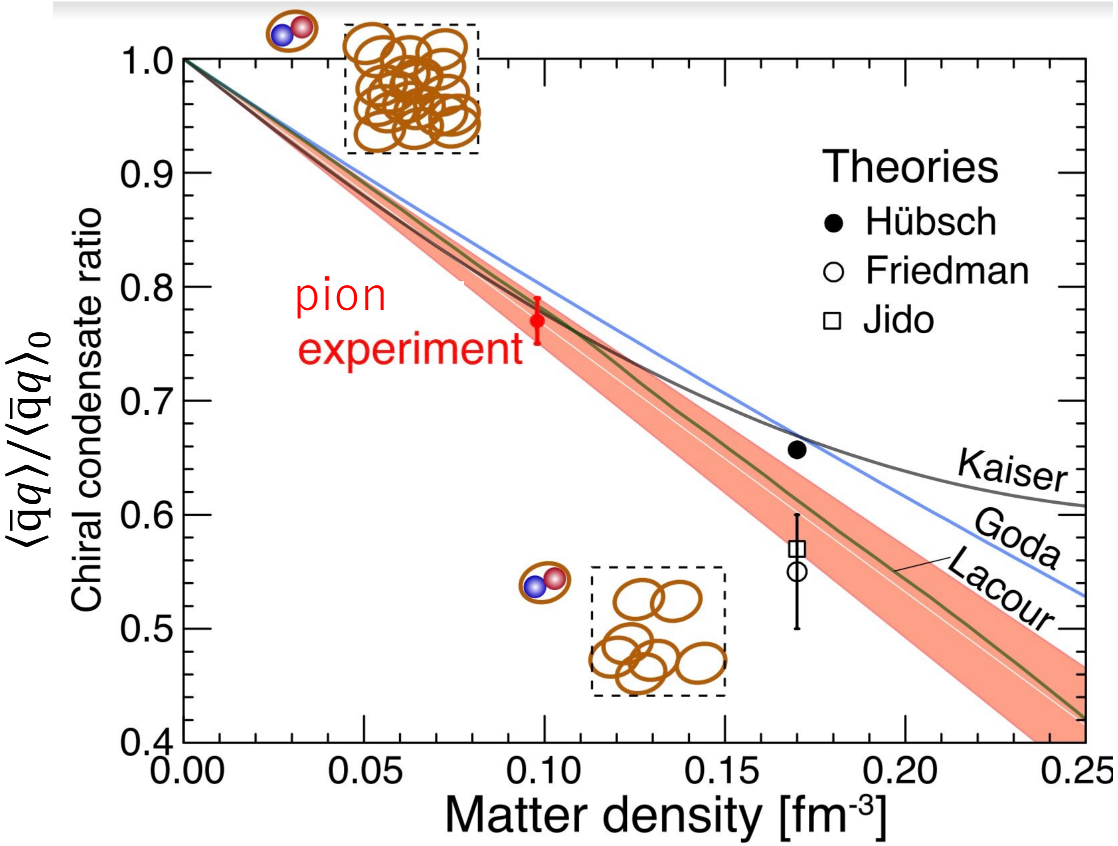
Derivatives after Borel transformation

$$m_N = F(<\bar{q}q>)$$

$$m_N = \{-2(2\pi)^2 < \bar{q}q >\}^{1/3}$$

QCD-based CSB interaction

HS, T. Naito, X. Roca-Maza and T. Hatsuda, PRC109, L011302 (2024)



The in-medium chiral condensate has a general form in the leading order of Fermi motion corrections;

Goda and Jido

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0} \right)^{5/3}, \tag{2a}$$

$$k_1 = -\frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} < 0, \qquad k_2 = -k_1 \frac{3k_{F0}^2}{10 m_N^2} > 0, \tag{2b}$$

where $\sigma_{\pi N}$ is the π-N sigma term, m_π (m_N) is the pion (nucleon) mass, and f_π is the pion decay constant. The

Partial restoration of Chiral condensation of quark pairs $\bar{q}q$

T. Nishi et al., Nature Physics, March 23, 2023
Pionic atom experiments

QCD-based CSB interaction

T. Hatsuda, H. Høgaasen, and M. Prakash, QCD sum rules in medium and the Okamoto-Nolen-Schiffer anomaly, Phys. Rev. Lett. **66**, 2851 (1991).

1. QCD sum rule approach to evaluate mass difference of proton and neutron in nuclear medium
2. Partial restoration of Spontaneous symmetry breaking (SSB) in nuclear medium

The mass difference between neutron and proton is formulated in nuclear matter by the QCD sum rule approach in leading order of the quark mass difference and QED effect

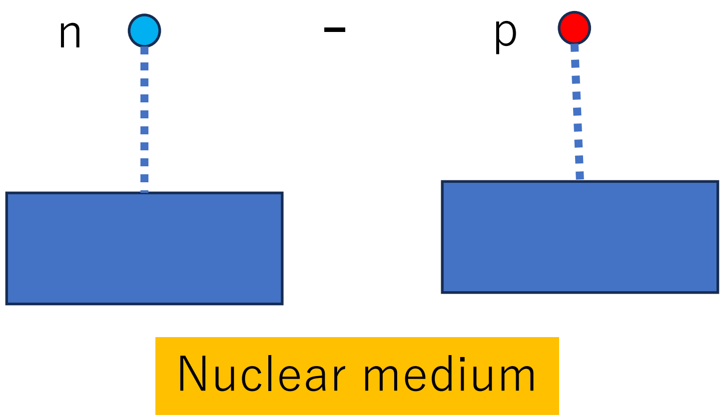
$$\Delta_{np}(\rho) \simeq C_1 G(\rho) - C_2,$$

$$G(\rho) = \left(\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \right)^{1/3}.$$

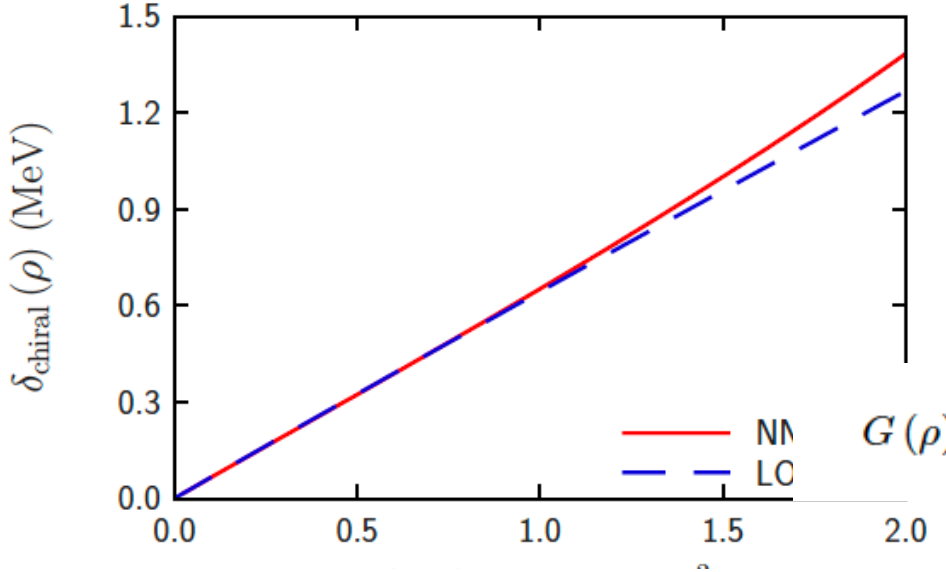
IN VACUUM,

$$\Delta_{np}(0) = m_n - m_p \simeq 1.29 \ \text{MeV}.$$

Here, $\langle \bar{q}q \rangle$ and $\langle \bar{q}q \rangle_0$ are, respectively, the isospin averaged in-medium and in-vacuum chiral condensate. The coefficient C_1 is proportional to the u-d quark mass difference δm [1], through the isospin-breaking constant $\gamma \equiv \langle \bar{d}d \rangle_0 / \langle \bar{u}u \rangle_0 - 1$ as $C_1 = -a\gamma$ with a positive numerical constant a determined by the Borel QSR method



The mass difference between (Z+/-1,N) and (Z, N+/-1) with N=Z



$$G(\rho) = \left(\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0}\right)^{1/3}.$$

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0}\right)^{5/3}, \quad (2a)$$

$$k_1 = -\frac{\sigma_{\pi N}\rho_0}{f_\pi^2 m_\pi^2} < 0, \qquad k_2 = -k_1 \frac{3k_{F0}^2}{10 m_N^2} > 0, \quad (2b)$$

where $\sigma_{\pi N}$ is the π-N sigma term, m_π (m_N) is the pion (nucleon) mass, and f_π is the pion decay constant. The

$$\tilde{s}_0 = -\frac{4}{3}\frac{C_1\sigma_{\pi N}}{f_\pi^2 m_\pi^2}, \qquad \tilde{s}_1 + 3\tilde{s}_2 = \frac{1}{m_N^2}\frac{C_1\sigma_{\pi N}}{f_\pi^2 m_\pi^2}.$$

The Skyrme-type CSB and CIB interactions

$$V_{\mathrm{CSB}}(\boldsymbol{r}) = \Big[s_0 (1 + y_0 P_\sigma) \delta(\boldsymbol{r}) + \frac{s_1}{2}(1 + y_1 P_\sigma)\left(\boldsymbol{k}^{\dagger 2}\delta(\boldsymbol{r}) + \delta(\boldsymbol{r})\boldsymbol{k}^2\right) + s_2 (1 + y_2 P_\sigma)\boldsymbol{k}^\dagger \cdot \delta(\boldsymbol{r})\boldsymbol{k}\Big] \frac{\tau_{1z} + \tau_{2z}}{4},$$

$$\frac{E}{A} \simeq \varepsilon_0(\rho) + \varepsilon_1(\rho)\beta + \varepsilon_2(\rho)\beta^2.$$

$$\beta = (N - Z)/A$$

$$\mathcal{E}_{\mathrm{CSB}} = \frac{s_0(1-y_0)}{8}\left(\rho_n^2 - \rho_p^2\right) = \tilde{s}_0(\rho_n + \rho_p)(\rho_n - \rho_p)/8 + \text{---}$$

$$\Delta E|_{N=Z} = -2\varepsilon_1(\rho)$$

$$\delta_{\mathrm{Skyrme}} = -\frac{\tilde{s}_0}{4}\rho - \frac{1}{10}\left(\frac{3\pi^2}{2}\right)^{2/3}(\tilde{s}_1 + 3\tilde{s}_2)\rho^{5/3}, \quad (7)$$

where we have defined the effective coupling strengths,

$$\tilde{s}_0 \equiv s_0(1 - y_0), \quad \tilde{s}_1 \equiv s_1(1 - y_1), \quad \tilde{s}_2 \equiv s_2(1 + y_2). \quad (8)$$

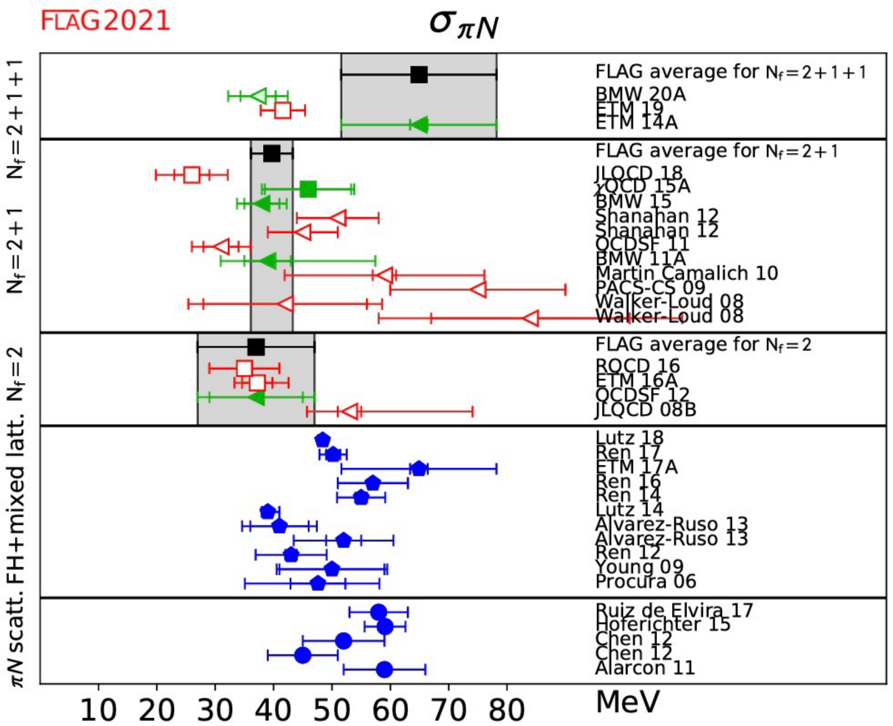


Figure 47: Lattice results and FLAG averages for the nucleon sigma term, $\sigma_{\pi N}$, for the $N_f = 2,\ 2+1$, and $2+1+1$ flavour calculations. Determinations via the direct approach are indicated by squares and the Feynman-Hellmann method by triangles. Results from calculations which analyze more than one lattice data set within the Feynman-Hellmann approach [204, 211–219] are shown for comparison (pentagons) along with those from recent analyses of π-N scattering [186–188, 220] (circles).

TABLE II. Parameters of the Skyrme-type CSB interactions constrained from the low-energy constants in QCD. To evaluate the CSB effect in finite nuclei where $\tilde{s}_1$ and $\tilde{s}_2$ contribute independently, two characteristic parameter sets (Case I and Case II) are introduced.

$\tilde{s}_0$ (MeV fm^3)	$-15.5^{+8.8}_{-12.5}$	
$\tilde{s}_1+3\tilde{s}_2$ (MeV fm^5)	$0.52^{+0.42}_{-0.29}$	
	Case I	Case II
$\tilde{s}_0$ (MeV fm^3)	$-15.5^{+8.8}_{-12.5}$	$-15.5^{+8.8}_{-12.5}$
$\tilde{s}_1$ (MeV fm^5)	$0.52^{+0.42}_{-0.29}$	0.00
$\tilde{s}_2$ (MeV fm^5)	0.00	$0.18^{+0.14}_{-0.10}$

a conservative estimate

$$\sigma_{\pi N} = 45 \pm 15 \text{ MeV}$$

New LQCD value ($N_f=2+1$)
$\sigma_{\pi N}=43.7\pm3.6 MeV$

A. Agadjanov et al.,
Phys. Rev. Lett. **131**, 261902
– Published 27 December 2023

TABLE I. – *Strengths of the various Skyrme-like CSB interactions. LO and NLO refer to the leading-order ($\tilde{s}_0$) and the next-leading-order ($\tilde{s}_0$–$\tilde{s}_2$) CSB interactions, respectively. "Pheno" and "Theor", respectively, refer to results based on phenomenological fitting and on theoretical evaluation. The values shown here except ab initio determinations are taken from Refs.* [16-20].

Class	Method or Name	$\tilde{s}_0$ (MeV fm^3)	$\tilde{s}_1$ (MeV fm^5)	$\tilde{s}_2$ (MeV fm^5)
Pheno	SAMi-ISB	-52.6 ± 1.4	—	—
Pheno	SLy4-ISB (leading order)	-22.4 ± 4.4	—	—
Pheno	SkM*-ISB (leading order)	-22.4 ± 5.6	—	—
Pheno	SV$_{\mathrm{T}}$-ISB (leading order)	-29.6 ± 7.6	—	—
Pheno	SV$_{\mathrm{T}}$-ISB (next-leading order)	$+44 \pm 8$	-56 ± 16	-31.2 ± 3.2
Pheno	Estimation by isovector density	-17.6 ± 32.0	—	—
Theor	ΔE_{tot} (N^{2}LO$_{\mathrm{GO}}$ (394) & CC)	-4.2 ± 6.5	—	—
Theor	ΔE_{tot} (N^{2}LO$_{\mathrm{GO}}$ (450) & CC)	-5.1 ± 28.5	—	—
Theor	ΔE (AV18-UX & GFMC)	-6.413 ± 0.173	—	—
Theor	QCD sum rule (Case I)	$-15.5^{+8.8}_{-12.5}$	$+0.52^{+0.42}_{-0.29}$	—
Theor	QCD sum rule (Case II)	$-15.5^{+8.8}_{-12.5}$	—	$+0.18^{+0.14}_{-0.10}$

IAS of ^{208}Pb

Masses of mirror nuclei

$\rho_{IV} = \rho_n - \rho_p$

Mass of ^{48}Ca and 48Ni (CC)

Mass of ^{10}Be and ^{10}C (GFMC)

CSB interaction, s_0 can be determined by

$$s_0 = \frac{\Delta E_{\mathrm{tot}}^{\mathrm{w/\ CSB}} - \Delta E_{\mathrm{tot}}^{\mathrm{w/o\ CSB}}}{\overline{a}}. \qquad (6)$$

TABLE V. The breakdown of the mass difference of mirror nuclei ΔE into each contribution (Coulomb, Extra and CSB interaction (CSBI) for Case I with Skyrme EDF, SGII. Numbers are given in the unit of MeV.

Nuclei	^{17}F-^{17}O	^{15}O-^{15}N	^{41}Sc-^{41}Ca	^{39}Ca-^{39}K
Orbital	$1d_{5/2}$	$(1p_{1/2})^{-1}$	$1f_{7/2}$	$(1d_{3/2})^{-1}$
$\Delta E_{\rm D}$ (Coulomb)	3.596	3.272	7.133	6.717
$\Delta E_{\rm E}$ (Coulomb)	−0.203	0.026	−0.267	0.260
Extra	0.040	0.028	0.102	0.011
CSBI (Case I)	0.224	0.264	0.287	0.315
Sum (w/o CSBI)	3.432	3.326	6.965	6.985
Sum (w/ CSBI)	3.656	3.590	7.252	7.300
Expt. [29]	3.543	3.537	7.278	7.307

TABLE VI. The same as Table V, but with Skyrme EDF, SAMi.

Nuclei	^{17}F-^{17}O	^{15}O-^{15}N	^{41}Sc-^{41}Ca	^{39}Ca-^{39}K
Orbital	$1d_{5/2}$	$(1p_{1/2})^{-1}$	$1f_{7/2}$	$(1d_{3/2})^{-1}$
$\Delta E_{\rm D}$ (Coulomb)	3.506	3.242	7.025	6.697
$\Delta E_{\rm E}$ (Coulomb)	−0.193	0.022	−0.259	0.281
Extra	0.043	0.075	0.104	0.092
CSBI (Case I)	0.206	0.269	0.271	0.321
Sum (w/o CSBI)	3.356	3.339	6.870	7.070
Sum (w/ CSBI)	3.562	3.608	7.141	7.391
Expt. [29]	3.543	3.537	7.278	7.307

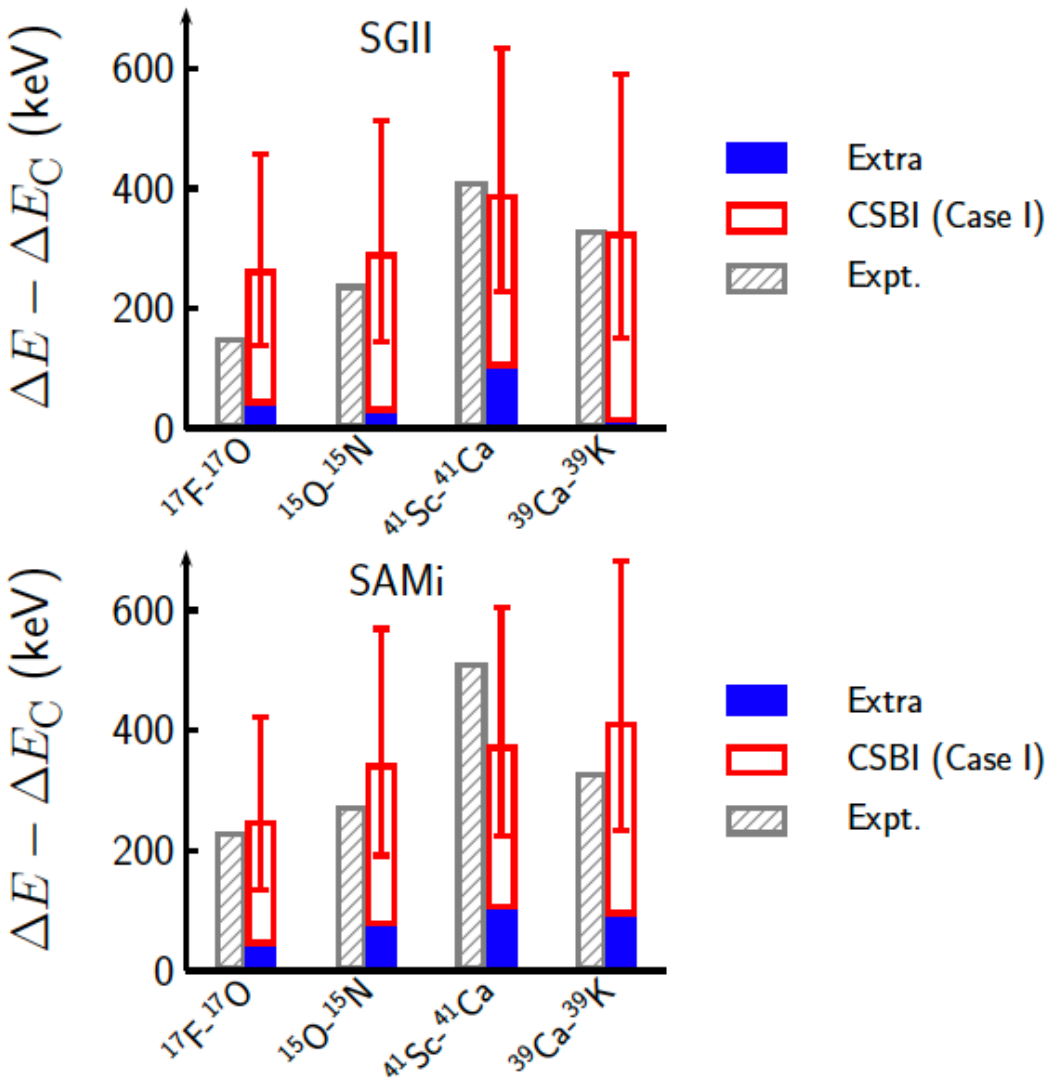


FIG. 2. Comparisons of the experimental ONS anomaly $\Delta E_{\rm Expt.} - \Delta E_{\rm C}$ (grey hatched bars) and the corresponding theoretical estimates in two EDFs (SGII and SAMi). The contribution from the QCD-based CSB interaction (CSBI) in Case I and the extra contributions are indicated by the red bars with error bars and the blue bars, respectively.

Summary

QCD sum rule approach is adopted to obtain EDF CSB parameters without introducing any free parameters;
all QCD parameters are determined by experimental observables.

QCD-based CSB interactions are applied to solve ONS anomaly of A=16+/-1 and 40+/-1 mirror nuclei and cured all experimental observed values within the theoretical uncertainties.

Neutron skins of N>Z and also N<Z nuclei are very sensitive to CSB interactions.

Future perspectives

CIB interaction in nuclear medium
QCD based $\Lambda - N$ CSB interaction for hypernuclei

CSB and CIB effects of Isobaric Analogue states
CSB in Exotic nuclei near proton drip lines

Collaborators

<u>QCD-CSB,QCD-CIB</u>

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University of Barcelona, Spain
Gianluca Colo, INFN, University of Milano, Italy

<u>Skyrme CSB and neutron skin</u>

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Gianluca Colo, INFN, University of Milano, Italy
H. Z. Liang, University of Tokyo

Isospin Breaking interactions

Coulomb interaction

Charge symmetry breaking (CSB) interaction

$V_{nn} \neq V_{pp}$

Charge Independence breaking (CIB) interaction

$V_{np} \neq (V_{nn} + V_{pp})/2$

Hadrons: Nucleon and Pion mass difference

m_n-m_p=1.29MeV. $\pi^{\pm}$-π^0=4.6MeV

Quarks: explicit chiral symmetry breaking

$m_u \neq m_d$

Skyrme type ISB interactions

$$v_{\text{Sky}}^{\text{CSB}}(\vec{r}) = s_0(1 + y_0 P_\sigma)\delta(\vec{r})\frac{\tau_{z1} + \tau_{z2}}{4},$$

$$v_{\text{Sky}}^{\text{CIB}}(\vec{r}) = u_0(1 + z_0 P_\sigma)\delta(\vec{r})\frac{\tau_{z1}\tau_{z2}}{2}.$$

Energy density functionals

$$\mathcal{E}_{\text{CSB}} = \frac{s_0(1 - y_0)}{8}\left(\rho_n^2 - \rho_p^2\right),$$

$$\mathcal{E}_{\text{CIB}} = \frac{u_0}{8}\left[\left(1 - \frac{z_0}{2}\right)\left(\rho_n + \rho_p\right)^2 - 2(2 + z_0)\rho_n\rho_p\right].$$

Observables

Okamoto-Nolen-Schiffer (ONS) anomaly

Energy of IAS

mass differences in isobar and isotriplet nuclei

Lagrangian

$$\mathcal{L} = \sum_{f=u,d,...} \bar{q}_f \left[i\gamma^\mu \left(\partial_\mu - i g t^a A^{a\mu} \right) - m_f \right] q_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

HADRON PROPERTIES FROM QCD SUM RULES

L.J. REINDERS*, H. RUBINSTEIN** and S. YAZAKI***

PHYSICS REPORTS (Review Section of Physics Letters) 127, No. 1 (1985) 1–97.

Two-point function with nucleon current

$$\eta_N = \varepsilon_{abc} (u^a(x) C \gamma_\mu u^b(x)) \gamma_5 \gamma_\mu d^c(x) ,$$

$$\Pi_N(p) = i \int d^4x \, e^{ipx} < T(\eta_N(x) \, \eta_N(0)) > = \not{p} A(p) + B(p)$$

Operator product expansion (OPE) methed

$$A(p) = \frac{1}{64\pi^2} p^4 \ln(-p^2) + \frac{1}{32\pi^2} < \frac{\alpha_s}{\pi} G^2 > \ln(-p^2) + \frac{2}{3} \frac{<\bar{q}q>^2}{p^2} + \cdots$$

$$B(p) = -\frac{1}{4\pi^2} <\bar{q}q> p^2 \ln(-p^2) + \cdots$$

Phenomenological hadron propagator

$$i \int d^4x \, e^{ipx} < T(\eta_N(x) \, \bar{\eta}_N(0)) > = \lambda_N^2 \frac{\not{p} + m_N}{p^2 - m_N^2} + \text{higher states}$$

$$\eta_N(x) \equiv \lambda_N N(x)$$

Derivatives after Borel transformation

$$m_N = F(<\bar{q}q>)$$

$$m_N = \{ -2(2\pi)^2 <\bar{q}q> \}^{1/3}$$