Gamow-Teller excitations and beta-decay within the Subtracted Second RPA

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Outline

Theoretical models:

- The Random Phase Approximation (RPA) and Second RPA (SRPA)
- SRPA Difficulties within EDF framework
- Improving on the SRPA: the Subtracted SRPA (SSRPA)

Applications

- Test case: RPA, SRPA and SSRPA in Quadrupole response of ^{16}O
- Gamow Teller Excitations
- Beta-decay half-life results

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Main References

- D. G., M. Grasso, and J. Engel Phys. Rev. Lett. 125, 212501, 2020
- D. G. and M. Grasso, Phys. Rev. C 105, 014321, 2022
- D. G. and M. Grasso, *in preparation*

The Random Phase Approximation (RPA)

- The RPA is a widely used tool for the description of collective excitations
- Very successful especially within the Energy Density Functional framework (interactions á la Skyrme or Gogny, covariant versions)
- It provides global properties: centroid energies and total strength

However, extensions of the RPA are required for:

- Strength Fragmentation
- Fine Structure
- Spreading Width
- ...

The Second RPA (SRPA): more general excitation operators are introduced

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_\nu^\dagger = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^\dagger a_h}_{\text{1p-1h}} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^\dagger a_p}_{\text{1h-1p}}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Second Random Phase Approximation (SRPA)

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \underbrace{\sum_{p_1<p_2, h_1<h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2}}_{\text{2p-2h}} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})}_{\text{2h-2p}}}_{\textbf{Spreading Width, Fragmentation}\text{, Double GRs and Anharmonicites, Low-Lying States}}$$

RPA Excitation Operators and Equations

RPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^{(\nu)} a_p^\dagger a_h - \sum_{ph} Y_{ph}^{(\nu)} a_h^\dagger a_p$$

RPA Equations of Motion (1$\mapsto$ 1p1h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix}$$

SRPA Excitation Operators and Equations

SRPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p)$$

$$+ \sum_{p_1<p_2, h_1<h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

SRPA Equations of Motion (1$\mapsto$ 1p1h, 2$\mapsto$ 2p2h)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

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Computationally very demanding

- The (few) applications performed in the past were done by using strong approximations and (very) small model spaces
- Only recently full large scale SRPA calculations have been performed

Large scale SRPA calculations have shown that:

- The SRPA strength distribution is sistematically shifted towards lower energies compared to the RPA one
- This shift is very strong ($\simeq$ 3-4 MeV), RPA description often spoiled

Origins and Causes:

1. Quasi Boson Approximation and stability problems in SRPA
2. Use of effective interactions in beyond-mean field methods

The Subtraction procedure (I. Tselyaev Phys. Rev. C 75, 024306 (2007))

- Designed for beyond RPA approaches
- It restores the Thouless theorem, e.g. instabilities are removed
- Static ($\omega = 0$) limit of the SRPA imposed to be equal to the RPA one

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

$$A_{1,1'} \mapsto \tilde{A}_{1,1'}(\omega) = A^{RPA}_{1,1'} + \sum_{2,2'} A_{1,2}(\omega + i\eta - A_{2,2'})^{-1} A_{2',1'} = A^{RPA}_{1,1'} + A^{Cor}_{1,1'}(\omega)$$

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The Subtraction procedure is SRPA (SSRPA)

- Subtraction of the zero–frequency limit of the SRPA correction

$$A^{Cor}_{1,1'} \mapsto \tilde{A}^{Cor}_{1,1'}(\omega) = A_{1,1'}(\omega) - A_{1,1'}(\omega = 0) \Rightarrow$$

$$\tilde{A}_{1,1'}(\omega = 0) = A^{RPA}_{1,1'}$$

$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

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- Subtraction of the zero–frequency limit of the SRPA correction

$$A^{Cor}_{1,1'} \mapsto \tilde{A}^{Cor}_{1,1'}(\omega) = A_{1,1'}(\omega) - A_{1,1'}(\omega = 0) \Rightarrow$$

$$\tilde{A}_{1,1'}(\omega = 0) = A^{RPA}_{1,1'}$$

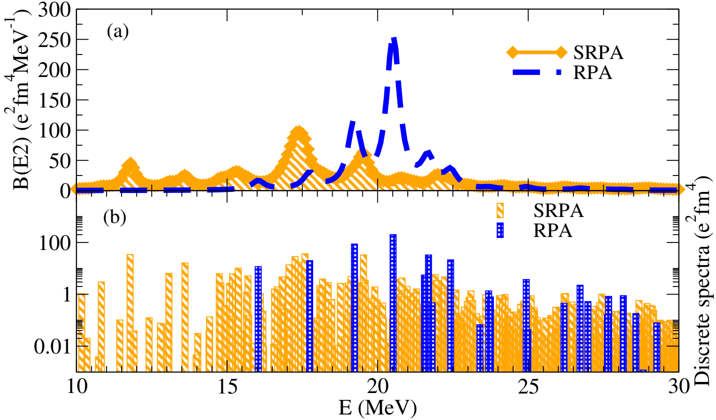
$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$

Numerical implementation

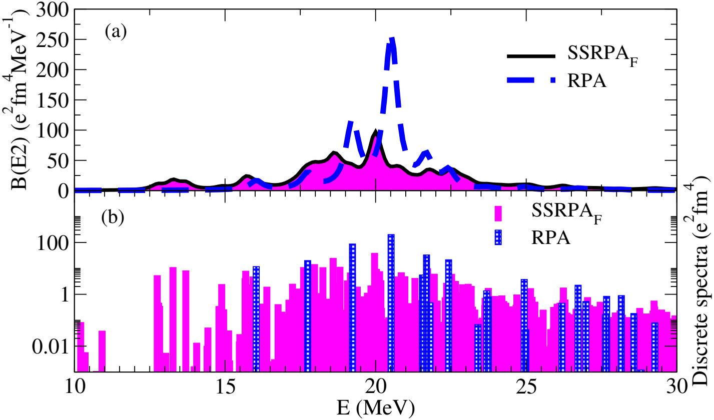
- Subtraction performed in diagonal approximation, e.g. $A_{2,2'} \approx \delta_{2,2'} A_{2,2}$
- Full subtraction recently performed (GT strength) [a]

[a] DG, M. Grasso, J. Engel, Physical Review Letters 125, 212501 (2020)

Quadrupole Strength Distribution in ^{16}O: RPA, SRPA and SSRPA



D. G., M. Grasso and J.Engel, Phys. Rev. C 92 , 034303 (2015)



D. G., M. Grasso and J.Engel, Phys. Rev. C 92 , 034303 (2015)

Recent extension to the charge exchange (CE) case

Second RPA for CE excitations

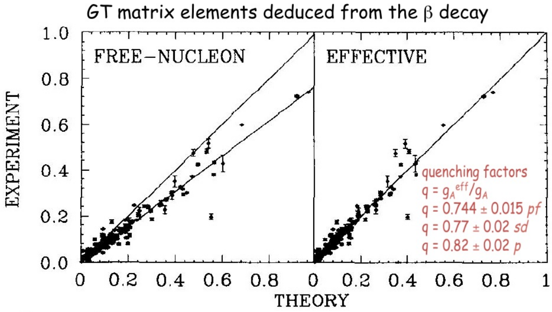
- Extension of the SSRPA for the description of **CE excitations**
- First applications to ^{48}Ca (lightest double-β emitter) and ^{78}Ni in Ref [1]
- More applications (^{14}C, ^{22}O, ^{90}Zr and ^{132}Sn) in Ref [2]

More details in:

[1] D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)
[2] D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.



(from Brown & Wildenthal, Ann.Rev.Nucl. Part.Sci.**38**,(1988)29)

Shell Model calculations

The quenching problem

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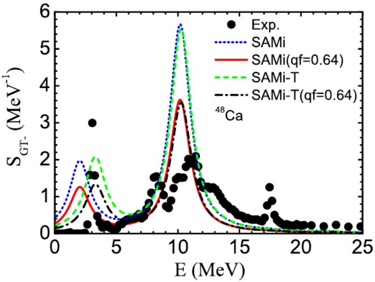
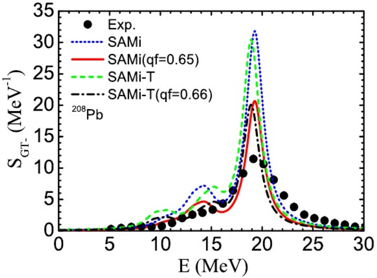




$$qf = \frac{\sum_{E_x=0}^{E_x(\max)} B(GT : E_x)_{\text{expt}}}{\sum_{E_x=0}^{E_x(\max)} B(GT)_{\text{calc}}}$$

Li-Gang Cao , Shi-Sheng Zhang, and H. Sagawa, PHYSICAL REVIEW C **100**, 054324 (2019)

Skyrme-RPA calculations

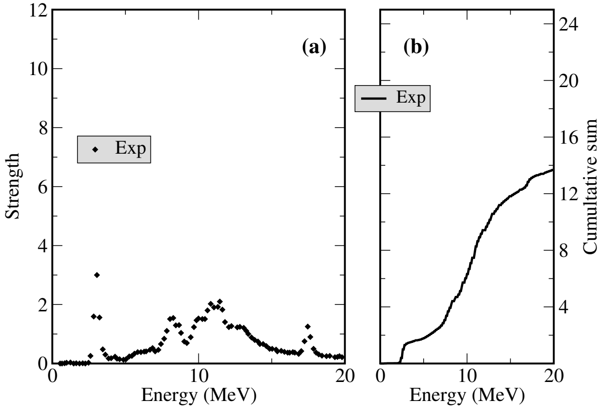
Theory vs Experiment: the quenching problem

The quenching problem

- Computed GT matrix elements **are larger** than the experimental ones.
- The problem is "cured" by **quenching** the strength by $q \sim 0.7$ or using effective axial constant g_A (~ 1) instead of the "bare" value ~ 1.27.

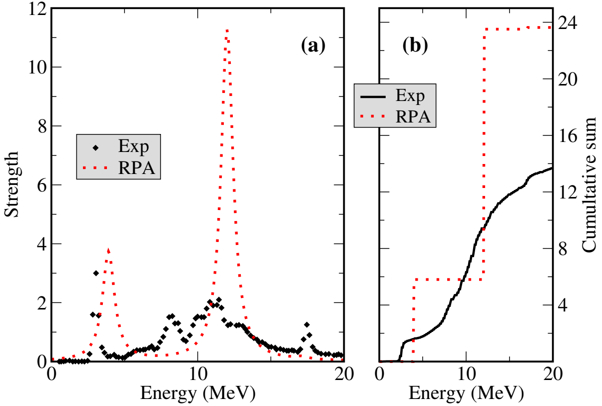
Possible causes fall in two main classes:

- **Nuclear many-body correlations not included in the calculations**: (truncation of the model space, short-range correlations, multi-phonon states, multi particle-hole excitations, ...)
- **Non-nucleonic degrees of freedom**: (Many-nucleon weak currents, Δ-isobar excitations, in-medium modification of pion physics, ...)



(a) GT^{-} strength in RPA and SSRPA compared with (GT^{-} plus IVSM) data.
(b) Cumulative strengths up to 20 MeV.
Data from: K. Yako *et al.*, Phys. Rev. Lett. 103, 012503 (2009)
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

GT$^-$ Strength Distribution ^{48}Ca, SGII interaction



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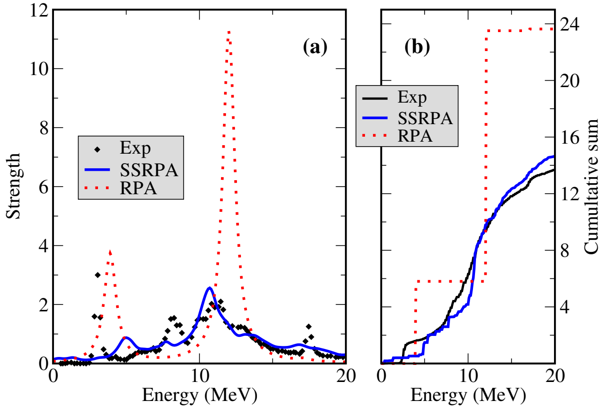

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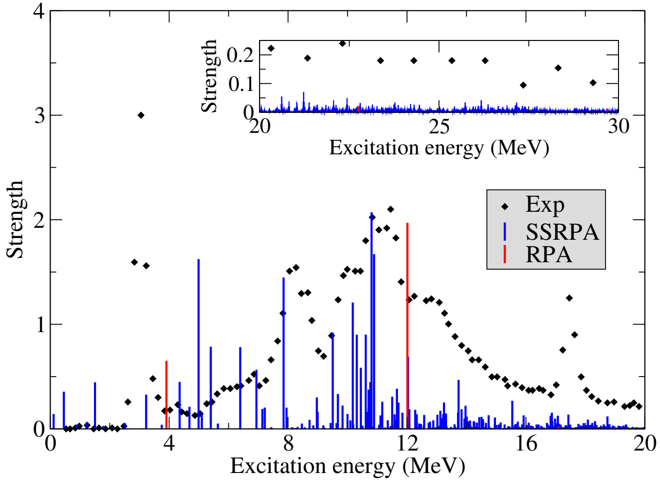

From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

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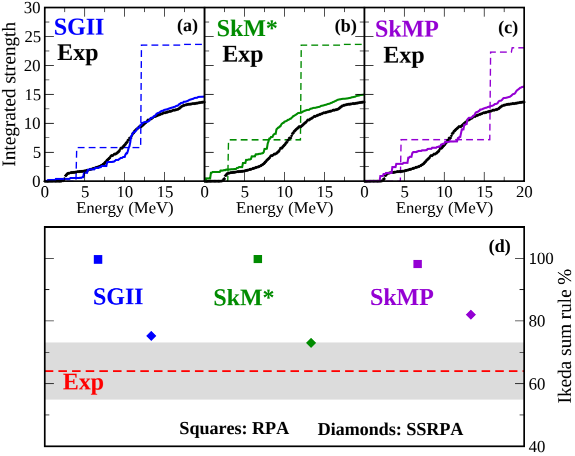


(a), (b), (c) Strengths integrated up to 20 MeV with different parameterizations.

(d) RPA and SSRPA percentages of the Ikeda sum rule below 30 MeV compared with the experimental one.

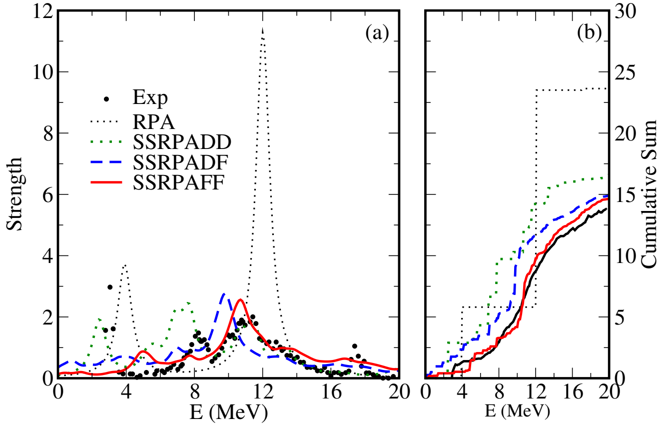
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

Numerical complexity

- The most demanding task is related to the treatment of the $A_{22'}$ matrix
- The number of 2p-2h configurations can be very large$\simeq 10^7, 10^8$
- We need to calculate the "full" spectrum
- Most demanding tasks:
  a) subtraction procedure, $A_{22'}$ inversion
  b) diagonalization of the SSRPA eigenvalue problem
- Strong simplification if $A_{22'}$ is assumed to be diagonal

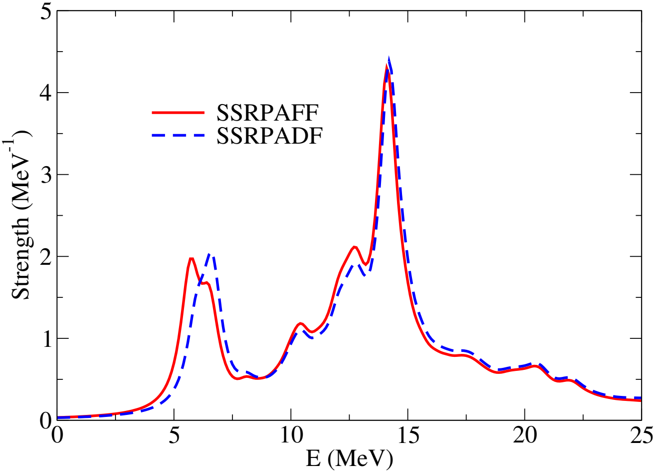
Different calculation scheme:

1. SSRPADD: $A_{22'}$ is Diagonal both in a) and b)
2. SSRPADF: $A_{22'}$ is Diagonal both in a) and Full in b)
3. SSRPAFF: $A_{22'}$ is Full both in a) and in b)



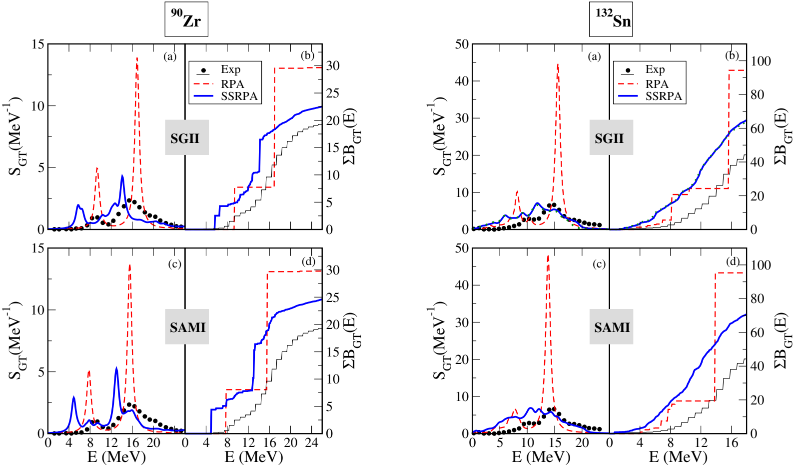
Comparison between SSRPADD, SSRPADF and SSRPAFF results .

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)

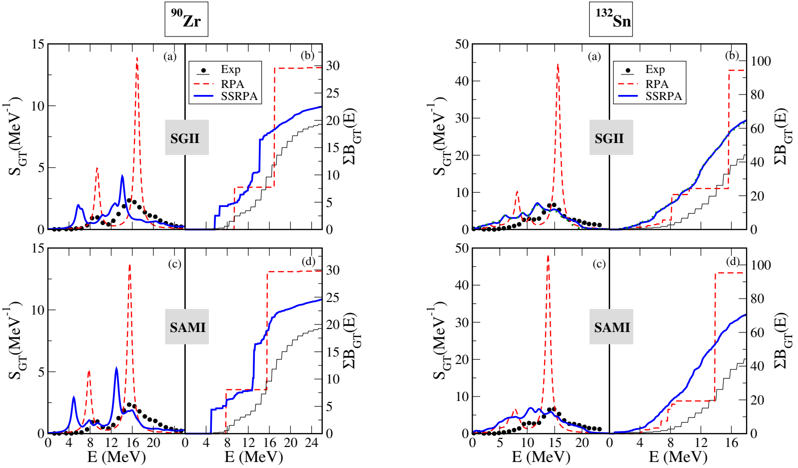


Comparison between SSRPADF and SSRPAFF results .

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)


90Zr
132Sn
(a)
(b)
(c)
(d)
Exp
RPA
SSRPA
SGII
SAMI
S_GT(MeV^-1)
ΣB_GT(E)
E (MeV)

GT^{-} Strength Distribution ^{90}Zr and ^{132}Sn, interaction dependence



Other sources of queenching may be needed ...

Beta-decay Life time

Beta-decay Half-life

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z,\omega) dE}.$$

$$f(Z,\omega) = \int_{m_e c^2}^{\omega} p_e E_e (\omega - E_e)^2 F_0(Z+1, E_e) dE_e,$$

Fermi function of the emitted electron

Gamow-Teller transition

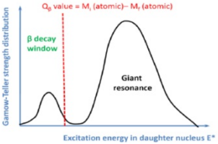
ΔS=1 ΔL=0 ΔT=1

operator

$$\hat{O}_{\text{GT-}} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_-(i)$$

Transition probability

$$B(\text{GT}^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$



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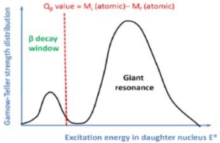
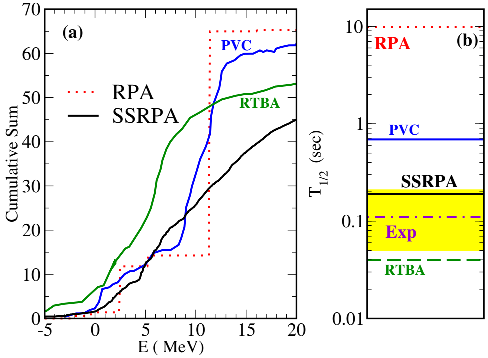




(a) Cumulative sum for the nucleus ^{78}Ni within the SSRPA, PVC and RTBA models;
(b) β-decay half-life for ^{78}Ni. **No quenching, bare $g_a = 1.27$;**
Data from: P. T. Hosmer *et al.* Phys. Rev. Lett. 94, 112501 (2005)
PVC: Y. F. Niu, G. Coló and E. Vigezzi, Phys. Rev. C 90, 054328 (2014)
RTBA:C. Robin and E. Litvinova, Phys. Rev. C 98, 051301(R), 2018

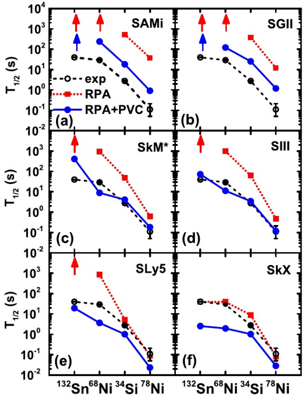
From D.Gambacurta, M. Grasso, J. Engel, Phys. Rev. Lett. 125, 212501 (2020)

β decay half-lives: PVC results

PRL **114**, 142501 (2015) PHYSICAL REVIEW LETTERS week ending 10 APRIL 2015

Particle-Vibration Coupling Effect on the β Decay of Magic Nuclei

Y. F. Niu (牛一斐),[1,2,*] Z. M. Niu (牛中明),[3,†] G. Colò,[1,4,‡] and E. Vigezzi[1,§]

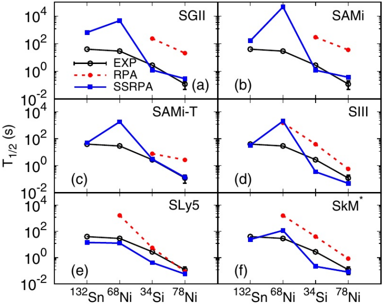


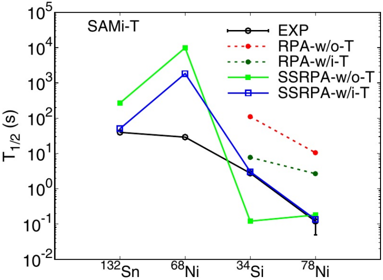
β decay half-lives: SSRPA results

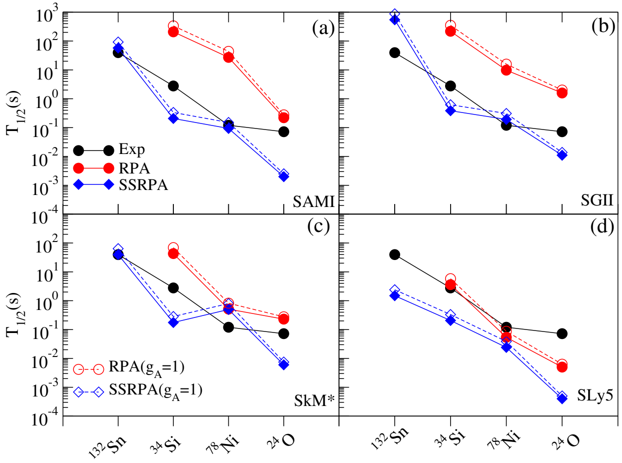
PHYSICAL REVIEW C **107**, 014325 (2023)

Effects of two-particle–two-hole configurations and tensor force on β decay of magic nuclei

M. J. Yang,[1] H. Sagawa,[2,3] C. L. Bai,[1] and H. Q. Zhang[4]

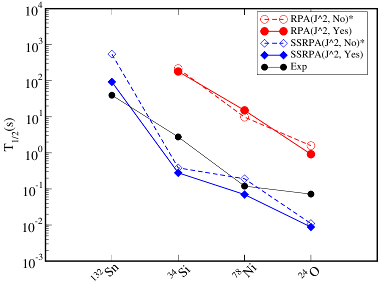




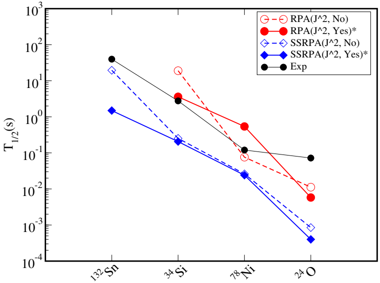


From D.Gambacurta and M. Grasso, *in preparation*

Effect of J^2 terms



SGII interaction



SLy5 interaction

Conclusions and Outlook

Conclusions

- The SRPA: richer and more general description of excited states
- The Subtracted SRPA:more reliable results within the EDF framework
- GT strength and β-decay half-life, considerable improvement with respect to the RPA

Outlook

- Extension to superfluid case, Second quasi-particle RPA
- Neutrino-less Double Beta Decay studies

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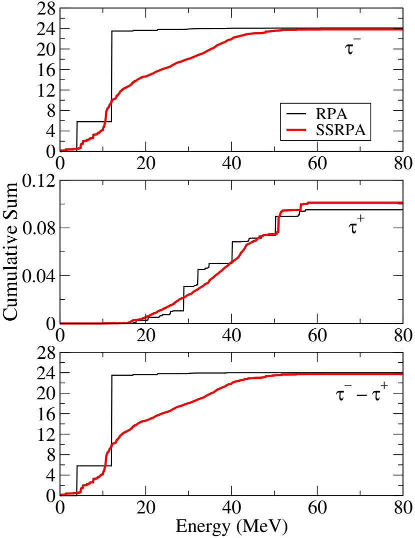
- Extension to superfluid case, Second quasi-particle RPA
- Neutrino-less Double Beta Decay studies
- Hibrid PVC plus SRPA calculations

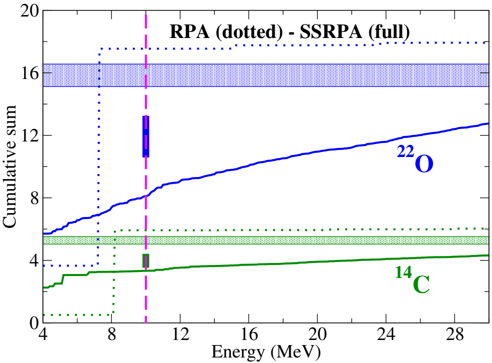
Thanks For Your Attention !!!

Backup Slides

GT$^-$ Strength Distribution ^{48}Ca, sum rules in the two channels

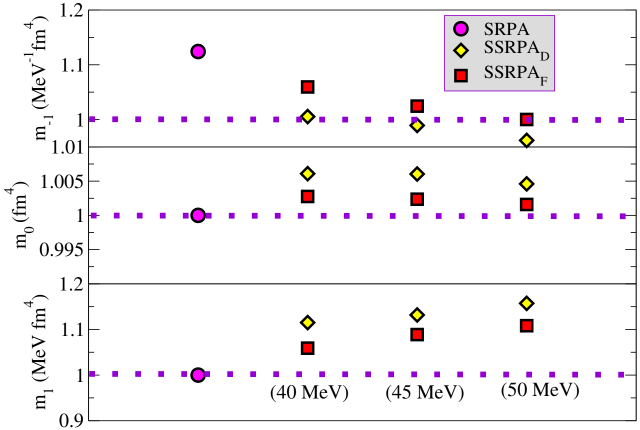


GT^- Strength Distribution for the nuclei ^{22}O (blue) and ^{14}C (green): SSRPA versus *ab initio* Coupled Cluster including two-body currents [1].



The blue and green horizontal areas represent the reduction of the total Ikeda sum rule $S_{GT^-} - S_{GT^+}$ from *ab initio* results [1].
The blue and green vertical intervals correspond to a reduction of (70-80 %) of the sum rule exhausted at 10 MeV.
[1] A. Ekstrom *et al.* Phys. Rev. Lett. 113, 262504 (2014)
See also Gysbers et al. Nature Phys. 15 428 (2019)

From D.Gambacurta and M. Grasso, Phys. Rev. C 105, 014321, (2022)



Ratios of the moments of the isoscalar quadrupole strength distribution in ^{16}O