Low-energy K^+N scattering revisited and in-medium strange quark condensate

Daisuke JIDO

Department of Physics,
Tokyo Institute of Technology

Reference: Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

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Introduction

- revisit low energy KN scattering amplitude with S=+1
- kaon is one of Nambu-Goldstone bosons
- **chiral perturbation theory** describe KN amplitude in low energies

  low energy effective theory of QCD

  works for low energy hadron scattering unless resonances are present

  such as πN below Δ, $\pi\pi$ below $f_0(500)$ and ρ

  KN (S=+1) : no strong resonances nor no coupled channels

- **in-medium quark condensate** relates to KN amplitude in soft limit

  πN sigma term determines slope of reduction of quark condensate

  ChPT gives analytic function and allows us to take soft limit

  K^+N scattering amplitude reveals in-medium strange quark condensate

Correlation function approach
to evaluate in-medium quark condensate

in-medium strange quark condensate

- **correlation function approach**:

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

we consider a correlation function for K+ channel in nuclear medium.
K+ channel is relatively simpler than K- channel due to no resonances

$$\Pi_5(q;\rho) = \mathrm{F.T.}\,\partial^\mu \langle \Omega \,|\, \mathrm{T}[A_\mu(x) P^\dagger(0)\,|\,\Omega\rangle,$$ nuclear matrix element

K^+ $(\bar{s}u)$ channel: axial vector current $A_\mu = \frac{1}{\sqrt{2}} \bar{s}\gamma_\mu \gamma_5 u,$

pseudoscalar filed $P = \sqrt{2}\bar{s} i \gamma_5 u$

chiral Ward identity tells us

$$\Pi_5(0;\rho) = -i\langle \Omega \,|\, \bar{u}u + \bar{s}s \,|\, \Omega \rangle$$

in **soft limit** where four momentum $q \to 0$,

thanks to chiral algebra $[Q_5, P] = -i(\bar{u}u + \bar{s}s)$, $Q_5 = \int A_0\, d^3x$

in-medium strange quark condensate

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

- **chiral Ward identity**:

  $\Pi_5(0;\rho) = -i\langle\Omega|\bar{u}u+\bar{s}s|\Omega\rangle$ in **soft limit**

- correlation function can be also evaluated at low density expansion as

  $$\langle\Omega|\bar{J}(x)J(0)|\Omega\rangle = \langle 0|\bar{J}(x)J(0)|0\rangle + \rho\langle N|\bar{J}(x)J(0)|N\rangle + O(\rho^{n>1})$$

  $\to$ in-vacuum condensate $\quad\to T_{KN}$ using SVZ reduction formula

- linear density approximation, in-medium change of quark condensate

  $$\frac{\langle\bar{u}u+\bar{s}s\rangle^*}{\langle\bar{u}u+\bar{s}s\rangle_0} = \left(1+\frac{\rho}{M_K^2}\frac{T_{KN}(q=0)}{2M_N}\right),$$

  See also Drukarev, Levin, Prog. Part. Nucl. Phys. 27, 77 (1991)

  given by KN scattering amplitude **in (unphysical) soft limit**

- because analytic continuation to soft limit is necessary

  we use **ChPT amplitude**

K^+ nucleon elastic scattering revisited in chiral perturbation theory

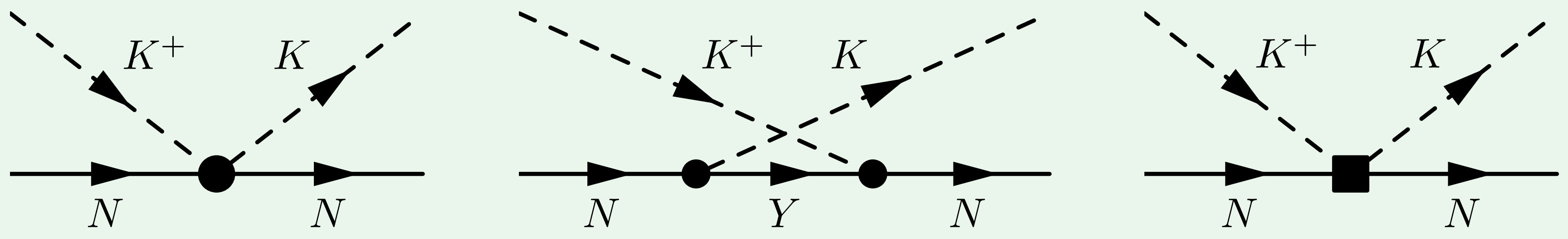
KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

- KN elastic scattering in **chiral perturbation theory**

  LO + NLO + (a part of) NNLO with m_s + Coulomb correction for K^+p

  12 low-energy constants (LECs), that are model parameters to be fitted



- **experimental data** up to P_{lab} = 800 MeV/c, where inelastic contribution are significant

  K^+p elastic differential cross section (P_{lab} = 145 to 726 MeV/c)

  $K^+n \to K^0p$ (charge exchange) diff. cross section (P_{lab} = 434 to 780 MeV/c)

  I=1 total cross section (P_{lab} = 145 to 788 MeV/c)

  I=0 total cross section (P_{lab} = 413 to 794 MeV/c, P_{lab} = 366 to 714 MeV/c)

  (we do not use K^+n elastic scattering data due to large ambiguities)

KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

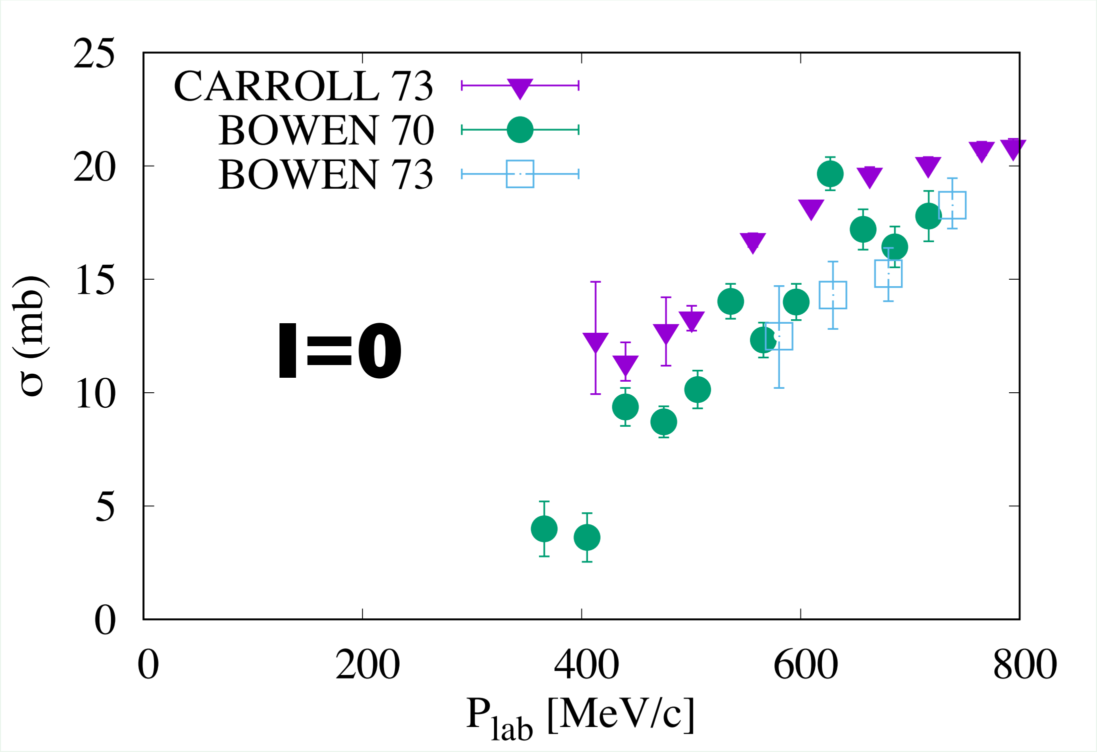
- **chi square fitting** for 12 parameters (LECs)

$$\chi^2_{\rm d.o.f} = \frac{1}{\mathcal{N}_{\rm d.o.f}} \sum_i^n \left(\frac{y_i - f(x_i)}{\sigma_i} \right)^2$$

- two remarks for fitting

  1) choice of data of I=0 total cross sections

  We use two data (CARROLL 73, BOWEN 70) separately → FIT 1, 2



KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

Aoki, DJ, PTEP2019,013D01(19)

2) resonance contribution

a previous work found a wide resonance in S=+1 and I=0 using chiral unitary approach with BOWEN 70 for I=0 total cross section two possible candidates: either P_{01} or P_{03} resonance

	channel	M [MeV]	Γ [MeV]	g [GeV^{-1}]
solution 1	P_{01} (1/2$^+$)	1617	305	5.26-2.62i
solution 2	P_{03} (3/2$^+$)	1678	463	4.46-2.62i

because chiral perturbation theory cannot generate resonances,

we account resonance contribution by adding $T_{\rm pole} = \dfrac{k^2 g^2}{W - M + i\Gamma/2}$

to the I=0 amplitude → FIT 3, 4

$\bar{K}$N scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

- four fitting procedures to see systematic error of the method

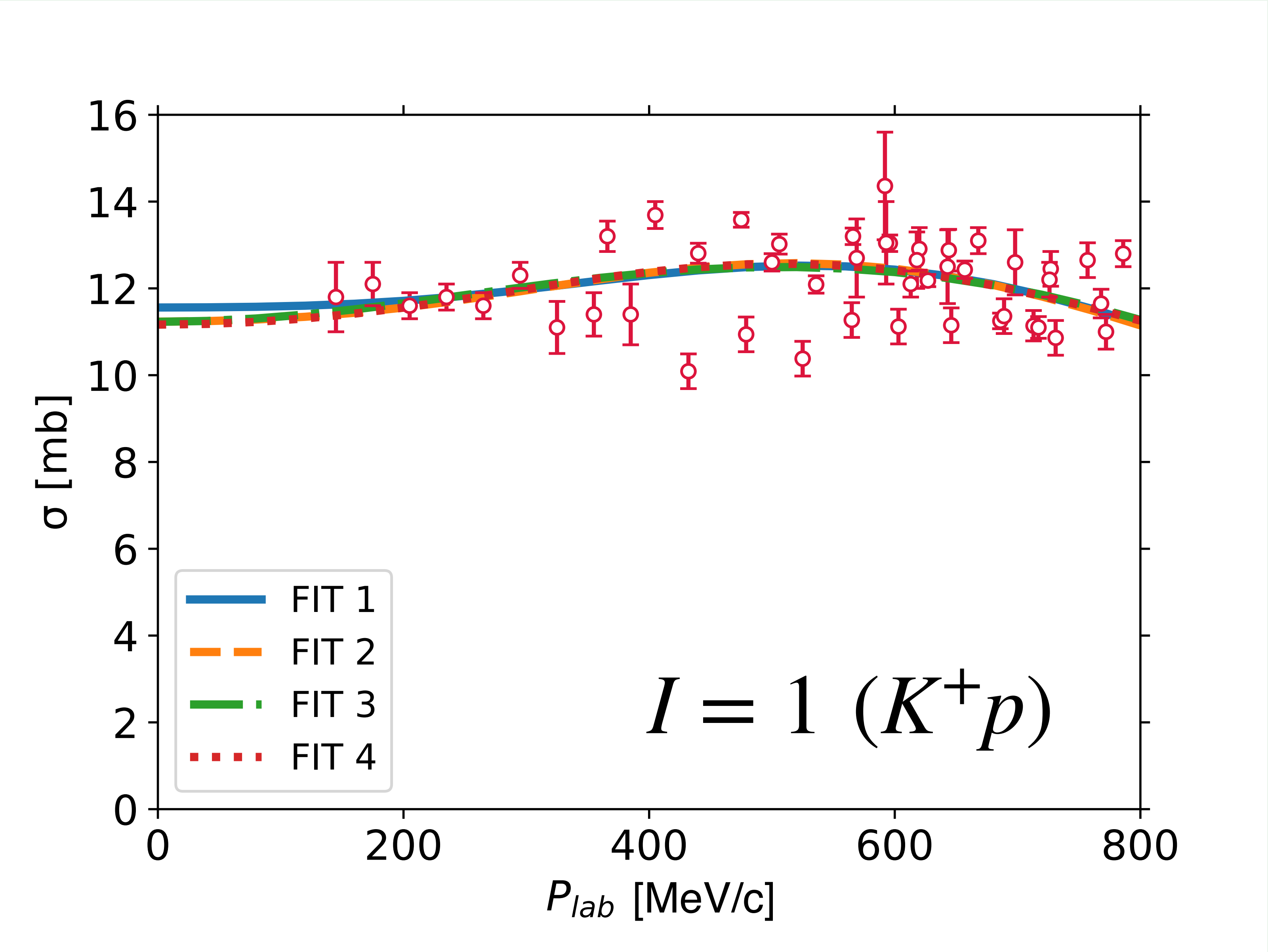
	I=0 total CS	S=+1 resonance
FIT 1	Carroll 73	no
FIT 2	Bowen 70	no
FIT 3	Bowen 70	P_{01} $(1/2^+)$
FIT 4	Bowen 70	P_{03} $(3/2^+)$

see theoretical uncertainties

KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

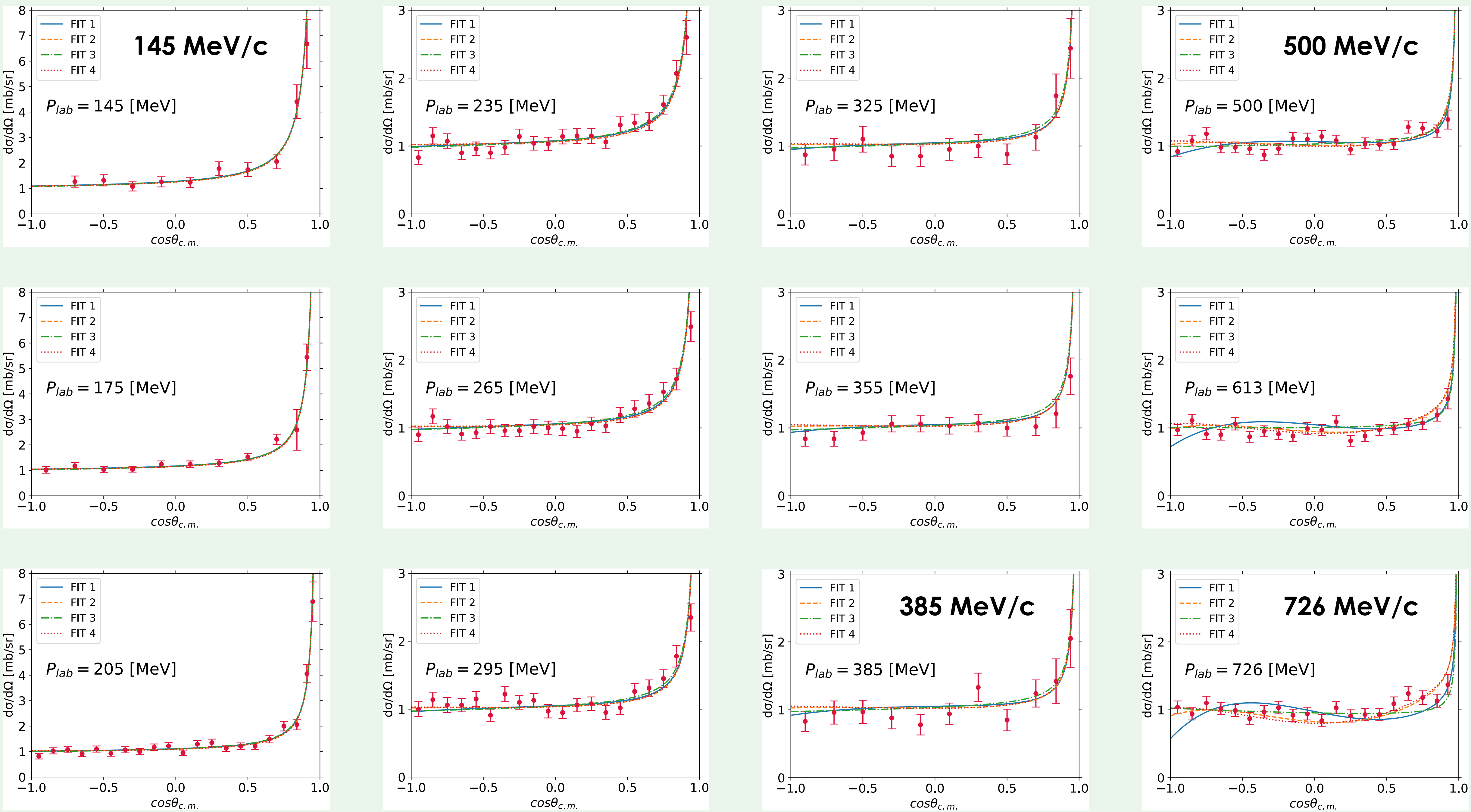
- I=1 (K^+p) cross section, fitted, is reproduced well



KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

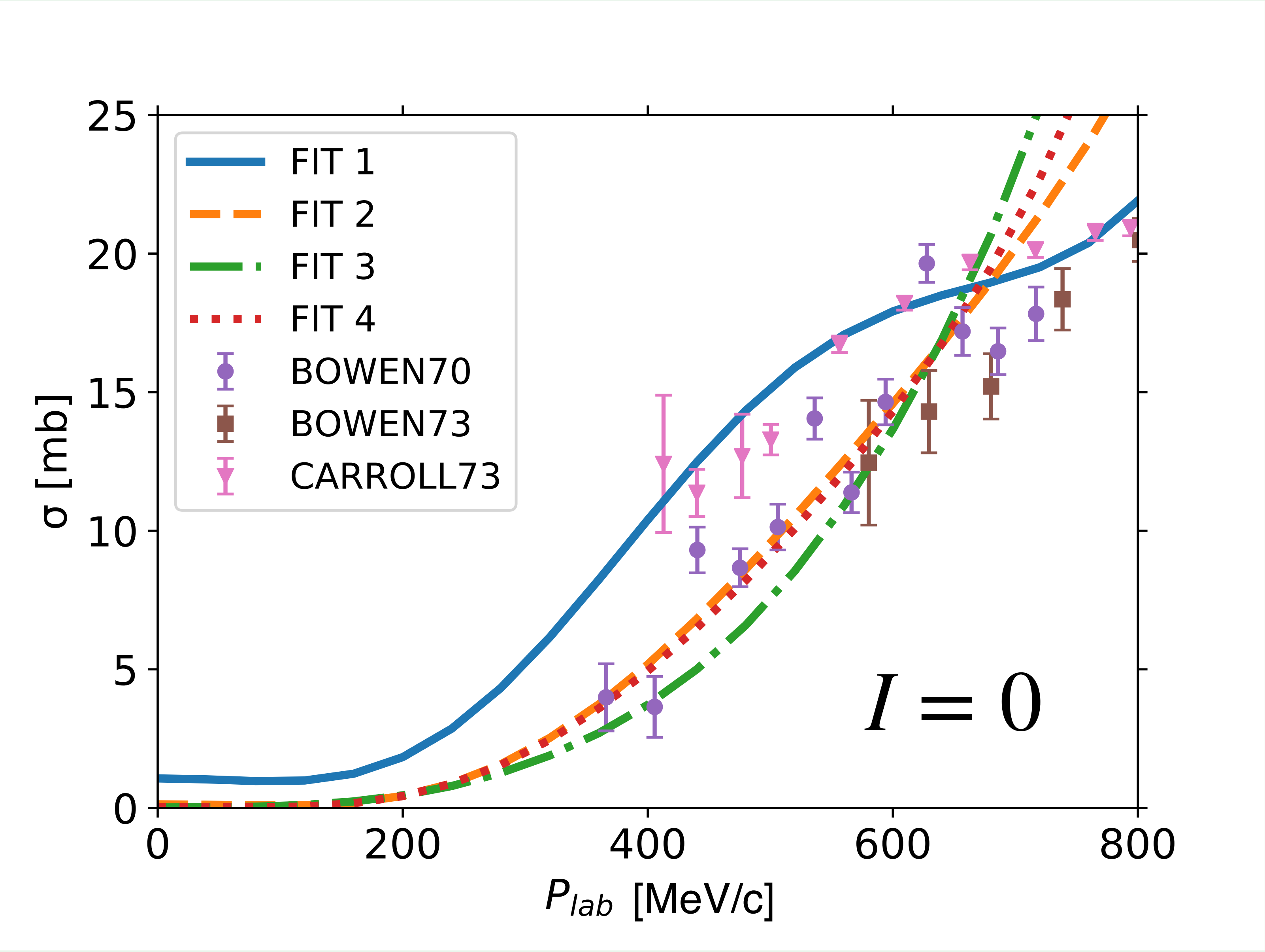
- I=1 (K^+p) differential cross sections, fitted, are also reproduced well



KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

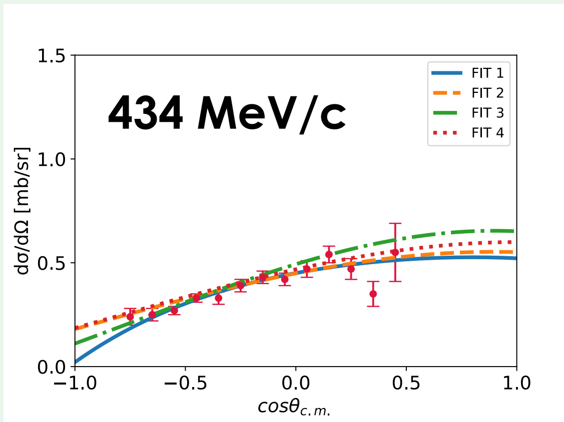
- I=0 total cross sections, fitted, are not consistent in different fits

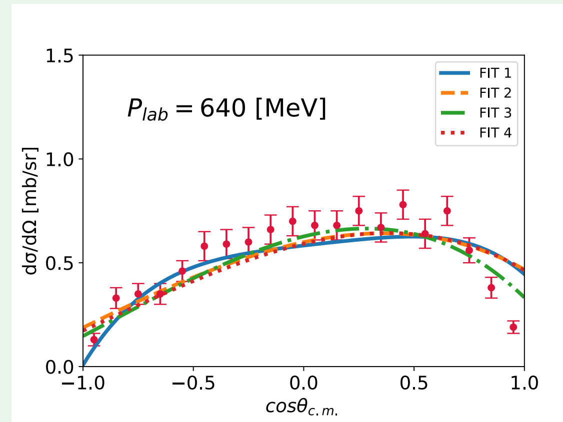


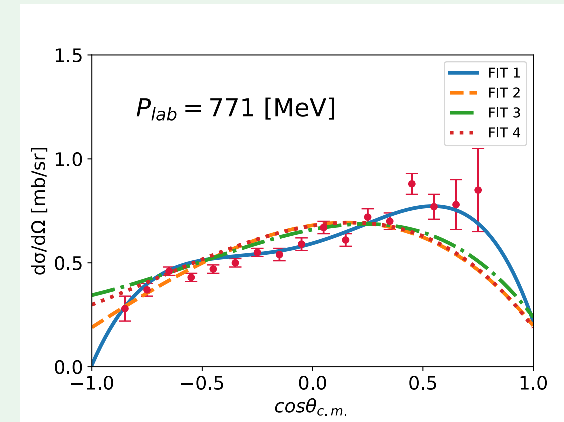
KN scattering in ChPT

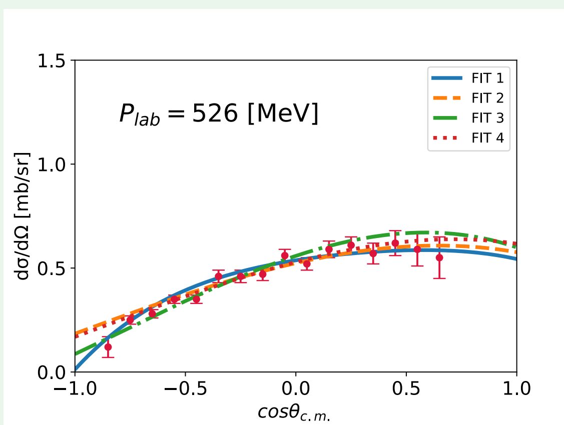
Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

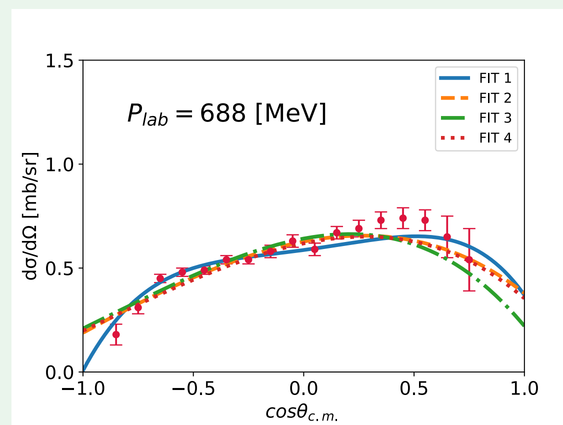
- $K^+n \to K^0p$ (charge exchange) diff. CS, fitted, are also reproduced well.

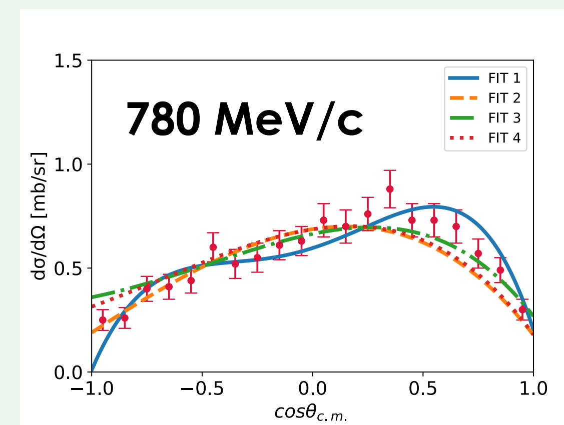


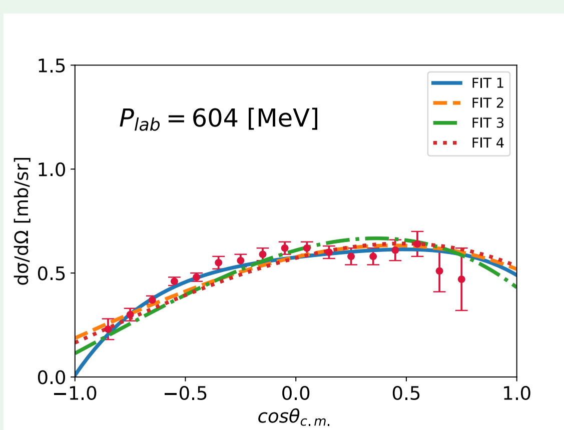


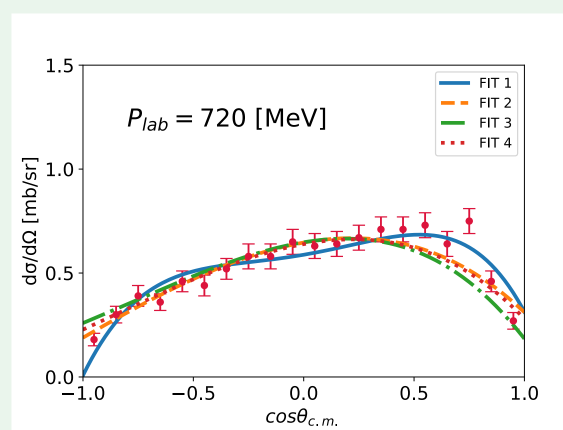












$$K^+n \to K^0p$$

KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

- I=1 parameters are consistently determined

LEC	unit	FIT 1	FIT 2	FIT 3	FIT 4	FIT 3′
$b^{I=1}$	[GeV^{-1}]	-1.07 ± 0.11	-1.10 ± 0.10	-0.11 ± 0.12	-1.08 ± 0.11	-0.39 ± 0.12
$d^{I=1}$	[GeV^{-1}]	-2.05 ± 0.20	-2.00 ± 0.17	-0.19 ± 0.19	-1.97 ± 0.17	-0.69 ± 0.18
$g^{I=1}$	[GeV^{-1}]	-0.82 ± 0.22	-0.93 ± 0.18	-0.80 ± 0.20	-1.01 ± 0.19	-1.07 ± 0.21
$h^{I=1}$	[GeV^{-1}]	3.67 ± 0.50	4.07 ± 0.60	0.91 ± 0.54	4.21 ± 0.60	2.07 ± 0.50
$w^{I=1}$	[GeV^{-2}]	-0.76 ± 0.11	-1.00 ± 0.10	-0.36 ± 0.10	-1.05 ± 0.10	-0.66 ± 0.10
$b^{I=0}$	[GeV^{-1}]	-3.66 ± 0.30	1.45 ± 0.40	2.36 ± 0.48	2.29 ± 0.40	-0.82 ± 0.50
$d^{I=0}$	[GeV^{-1}]	-9.21 ± 0.40	-0.20 ± 0.40	-1.42 ± 0.58	-0.63 ± 0.50	-1.95 ± 0.60
$g^{I=0}$	[GeV^{-1}]	1.46 ± 0.50	6.10 ± 0.70	8.27 ± 0.95	8.07 ± 0.80	1.03 ± 0.90
$h^{I=0}$	[GeV^{-1}]	16.29 ± 0.70	-3.99 ± 0.80	-1.64 ± 0.96	-4.91 ± 0.80	3.91 ± 0.90
$w^{I=0}$	[GeV^{-2}]	-0.57 ± 0.29	4.23 ± 0.35	4.92 ± 0.46	4.99 ± 0.40	-0.11 ± 0.40
v_-	[GeV^{-1}]	42.89 ± 1.70	12.32 ± 1.70	5.00 ± 0.19	10.12 ± 1.70	6.89 ± 0.19
v_+	[GeV^{-1}]	-7.55 ± 0.90	4.28 ± 0.90	-3.63 ± 0.93	4.74 ± 0.90	-1.98 ± 0.90
$\chi^2_{\rm dof}$		2.41	2.74	2.95	2.96	3.00

2nd min. of FIT3

in-medium strange quark condensate

- quark condensate in symmetric nuclear matter

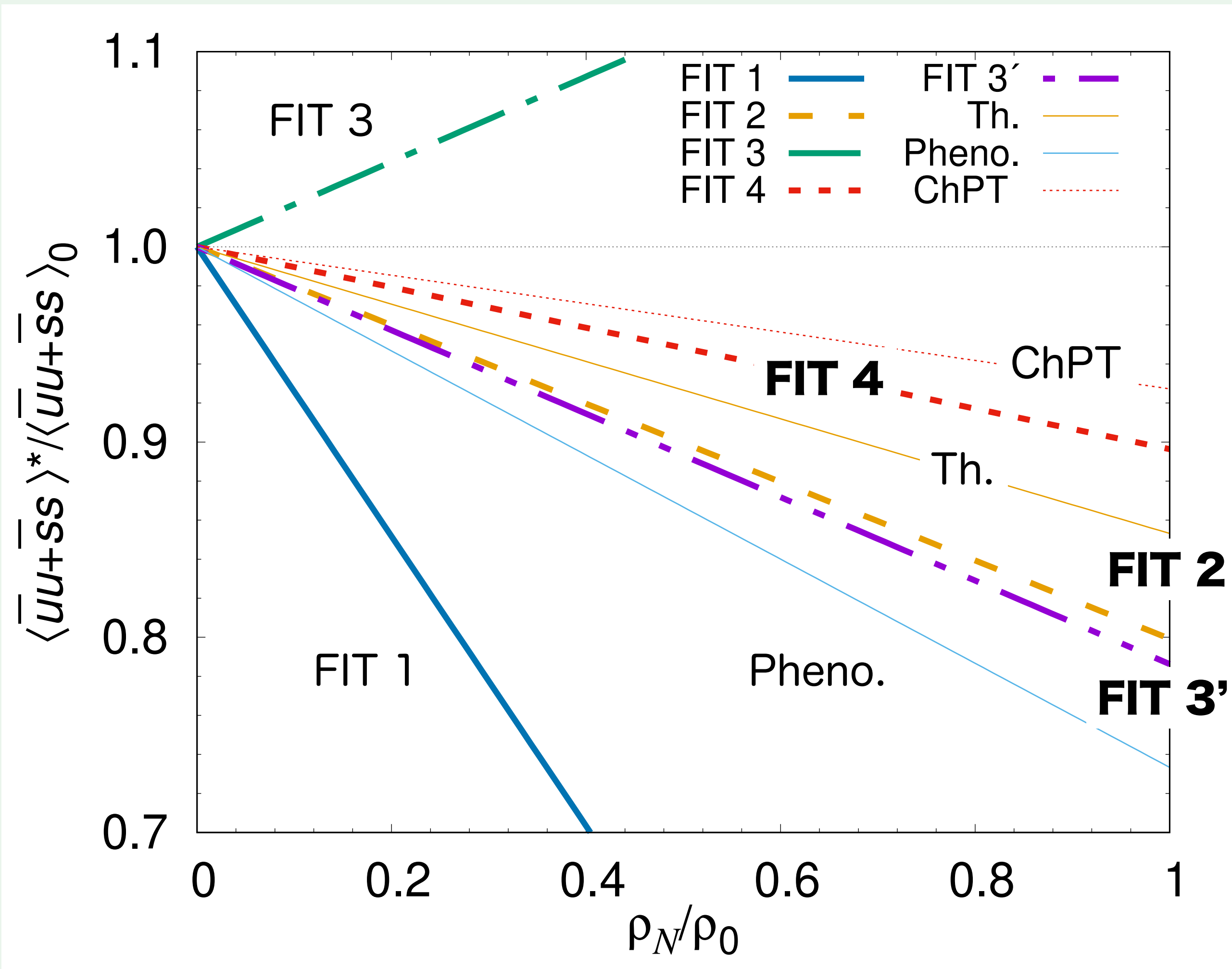
Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = 1 + \frac{\rho}{M_K^2} \frac{3T_{KN}^{I=1}(0) + T_{KN}^{I=0}(0)}{2M_N} = 1 + \frac{3b^{I=1} + b^{I=0}}{F_K^2} \rho$$

in-medium strange quark condensate

- quark condensate in symmetric nuclear matter

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)



$$\frac{\langle \bar{u}u + \bar{s}s\rangle^*}{\langle \bar{u}u + \bar{s}s\rangle_0} = 1 + \frac{3b^{I=1} + b^{I=0}}{F_K^2}\rho$$

10-20% reduction in FITs 2, 4 and 3'

FIT3' : second χ^2 minimum of FIT 3

Th.: LECs determined by baryon masses obtained in lattice calculation with various quark masses
L. Geng, Front. Phys. 8, 328 (13)

Pheno. : LECs fixed by observed baryon masses and σ term
B.Kubis, U.G.Meissner, EPJC18, 747 (01)
M.Holmberg, S.Leupold, EPJA54, 103 (18)

ChPT. : global fitting of LECs using πN and KN phase shift analyses
J.X.Lu et al. PRD99, 054024 (19)

- behavior of in-medium condensate is highly dependent on choice of FITs
- current status of K+N scattering data not be of sufficient quality for determination of LECs.

[GeV^{-1}]	FIT 1	FIT 2	FIT 3	FIT 4	FIT 3′	Th.	Pheno.	ChPT
$3b^{I=1} + b^{I=0}$	−6.87	−1.86	2.02	−0.96	−1.98	−1.36	−2.47	-0.674

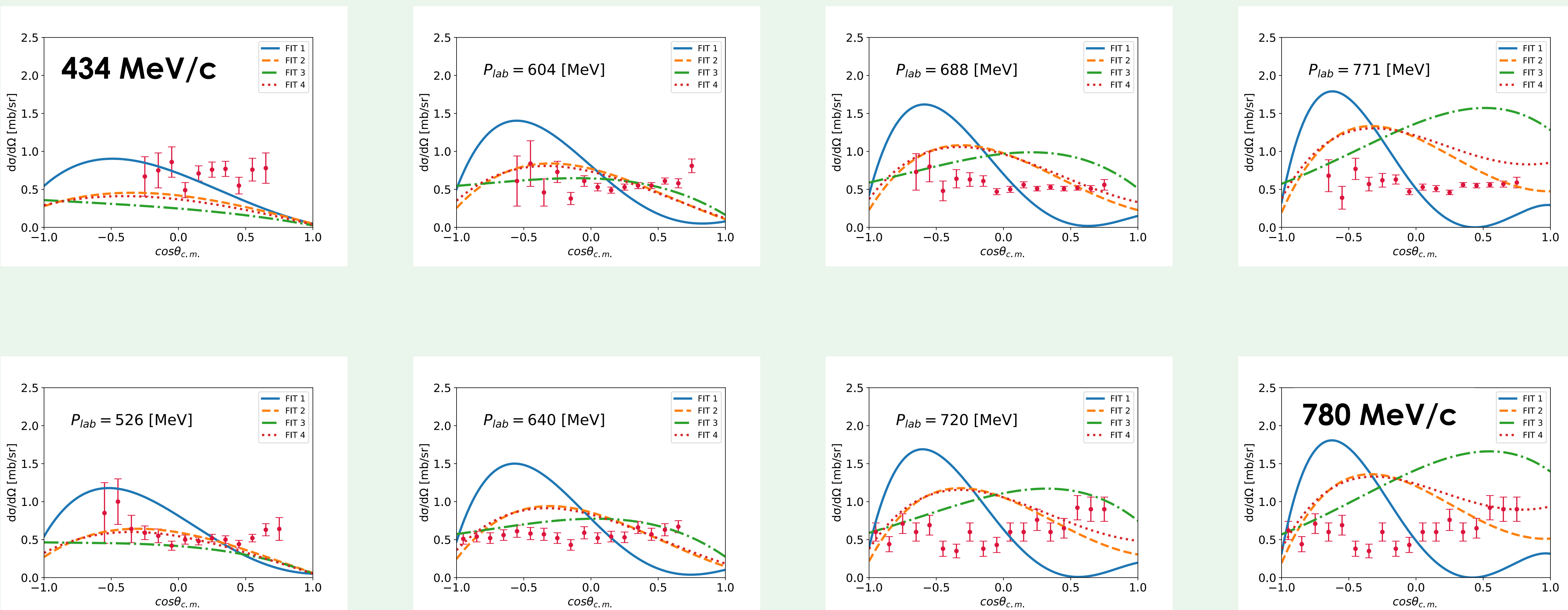
KN scattering in ChPT

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)

- K^+n elastic scattering, not fitted

  both I=1 and I=0 are fixed by experimental observation,

  but the reproduction of K^+n elastic scattering is poor



Summary

- in-medium quark condensate can be evaluated by correlation function in soft limit
  - it connects to quark condensate to the low energy scattering amplitude in low density
  - obtain reduction of $|\langle \bar{u}u + \bar{s}s \rangle|$ in nuclear medium as a qualitative conclusion
- NNLO chiral perturbation theory
  - perfect (nice) description of K^+p elastic scattering amplitude up to P_{lab} = 500 (800) MeV/c
  - unsatisfactorily reproduces I=0 scattering amplitudes
  - there are still ambiguities in low energy K^+n amplitudes to extrapolate to soft limit
- outlooks
  - for K^+n scattering, direct calculation of $K^+d \to KNN$ will be performed
  - $K_L p \to K^+n$ in K-long facility accesses I=0 KN amplitude
  - introduce SU(3) breaking to calculation of $\langle \bar{u}u + \bar{s}s \rangle^*$ to extract $\langle \bar{s}s \rangle^*$ alone
  - beyond linear density, calculate correlation function directly based on in-medium ChPT

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following the merger between Tokyo Tech and TMDU.

Backup slides

in-medium quark condensate

- **correlation function approach**

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

a correlation function (axial vector) in medium

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\,\partial^\mu \langle \Omega | \mathrm{T}[A_\mu^a(x) P^b(0) | \Omega \rangle, \quad |\Omega\rangle:$$ **nuclear matter ground state**

axial current $A_\mu^a = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5\tau^a q$, Noether current of chiral symmetry

pseudoscalar field $P^a = \bar{q} i \gamma_5 \tau^a q$, ChS trans. $[Q_5^a, P^b] = -i\delta^{ab} S$

according to chiral Ward identity

in the soft limit, $\Pi_5^{ab}(0) = \langle \Omega | [Q_5^a, P^b] | \Omega \rangle = -i\delta^{ab}\langle \bar{q}q\rangle^*$

Ward identity is an operator relation, applicable for any physical states

- if one evaluates the correlation function in medium, which is in-medium propagation of NG boson, we obtain the quark condensate in nuclear medium by **taking its soft limit.**

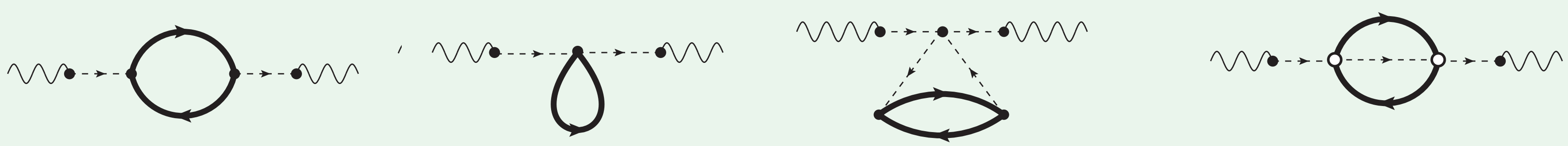
in-medium quark condensate

- correlation function approach

$$\lim_{q\to 0} \mathrm{F.T.}\, \partial^\mu \langle \Omega | \mathrm{T}[A_\mu^a(x) P^b(0) | \Omega \rangle = -i\delta^{ab} \langle \bar{q}q \rangle^*$$

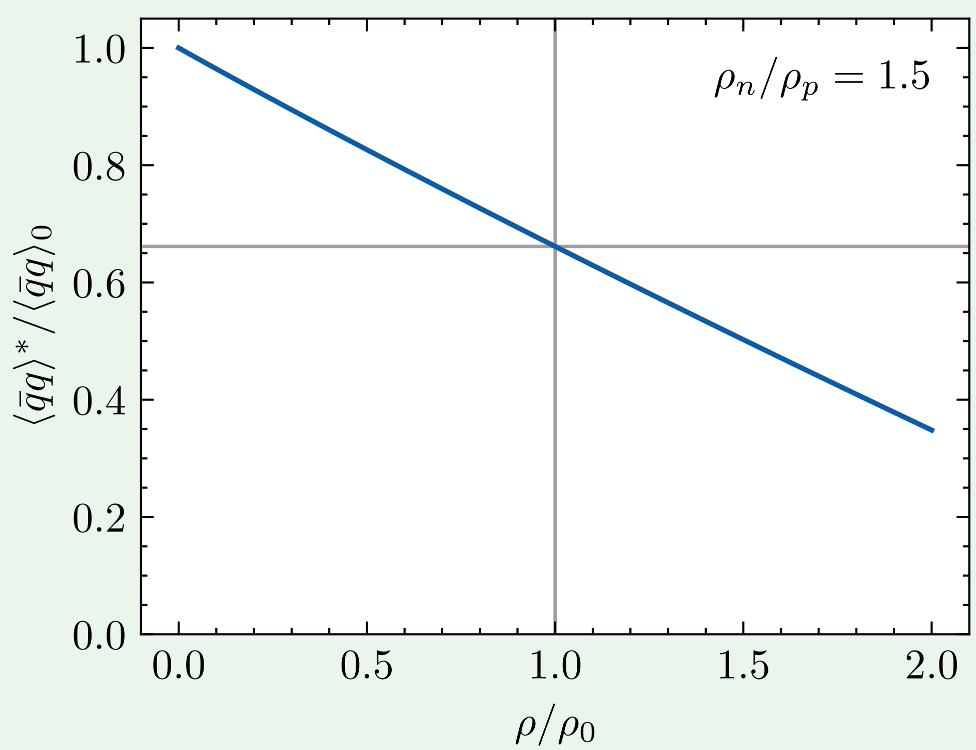
- in-medium chiral perturbation theory, diagrammatical calculation

Goda, DJ, PRC 88 (2013), 065204



etc.

- two-flavor ChPT at NNLO, density up to k_F^5, without N-N correlation



Hübsch, DJ, PRC 104 (2021), 015202.

reproduce 35% reduction at $\rho = \rho_0$

See also, Kaiser, Homont, Weise, PRC 77 (2008), 025204.

Possible wide resonance with S=+1 in unitarized amplitudes

Aoki, DJ, PTEP2019,013D01(19)

a wide resonance in I=0 and S=+1

Aoki, DJ, PTEP2019,013D01(19)

- to investigate possibility to have resonances in the amplitude

  unitarized amplitude T

$$T = V + VGT$$

  V: interaction kernel, given by ChPT

  G: KN loop function (I=0, I=1)

  one subtraction constant is fixed as **a natural value**

- **chiral Lagrangian**

  most general form up to next-leading-order

  L.-S. Geng, Frontiers of Physics 8, 328 (2013)

  8 LECs (4 LECs for I=1, 4 LECs for I=0)

- **data** up to 800 MeV, where inelastic contributions start to be significant

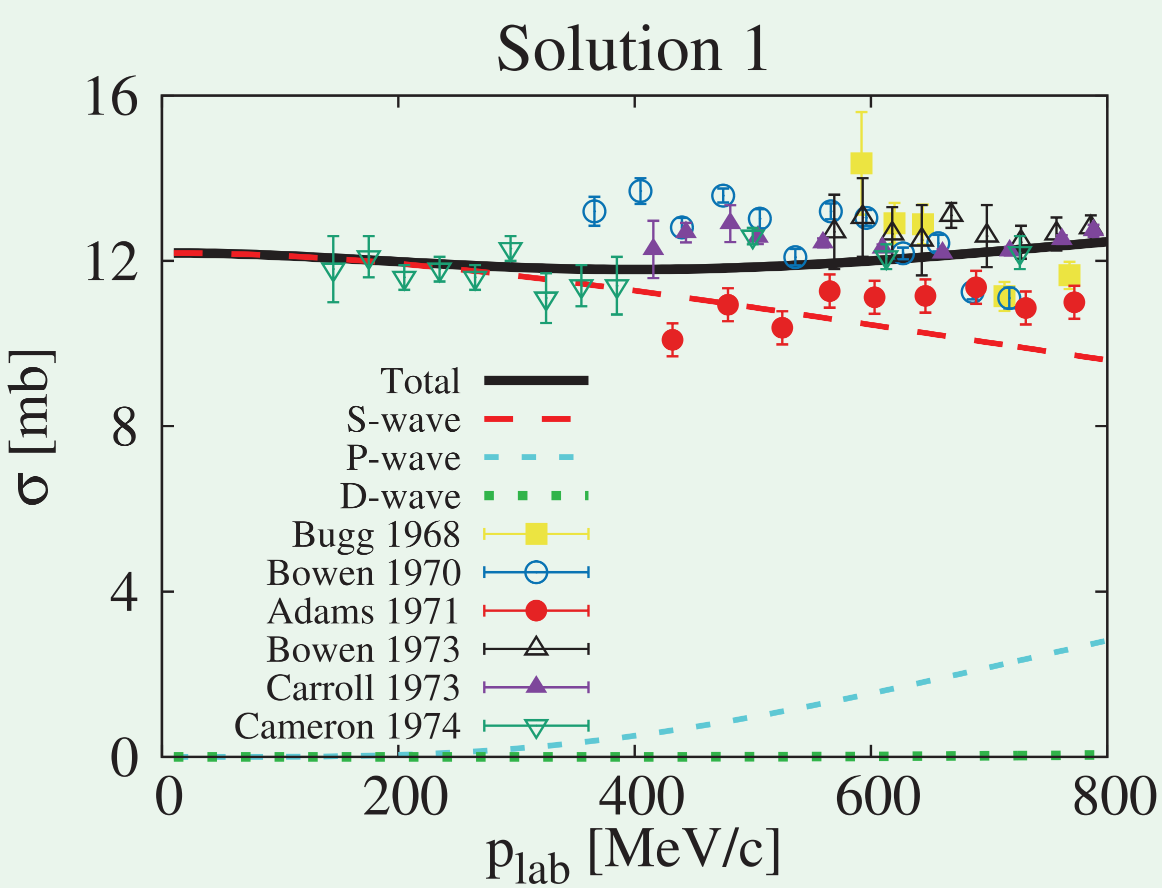
  $K^+p \to K^+p$, total and differential cross sections, p_{lab} =145 to 726 MeV, which determine I=1 amplitudes very well

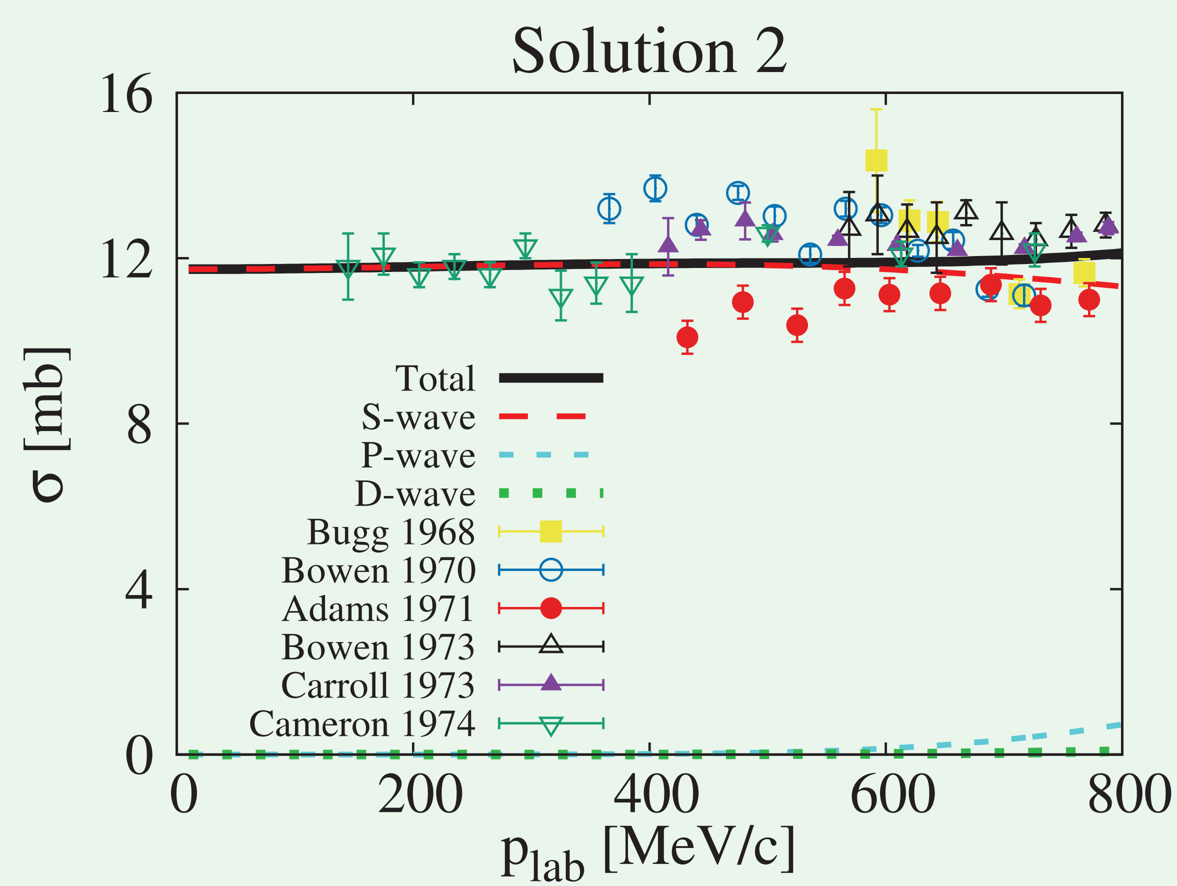
  $K^+n \to K^+n, K^0p$, differential cross sections, p_{lab} =526, 604, 640 MeV, total cross section I=0

I=1 total cross sections

Aoki, DJ, PTEP2019,013D01(19)

- we have two solutions



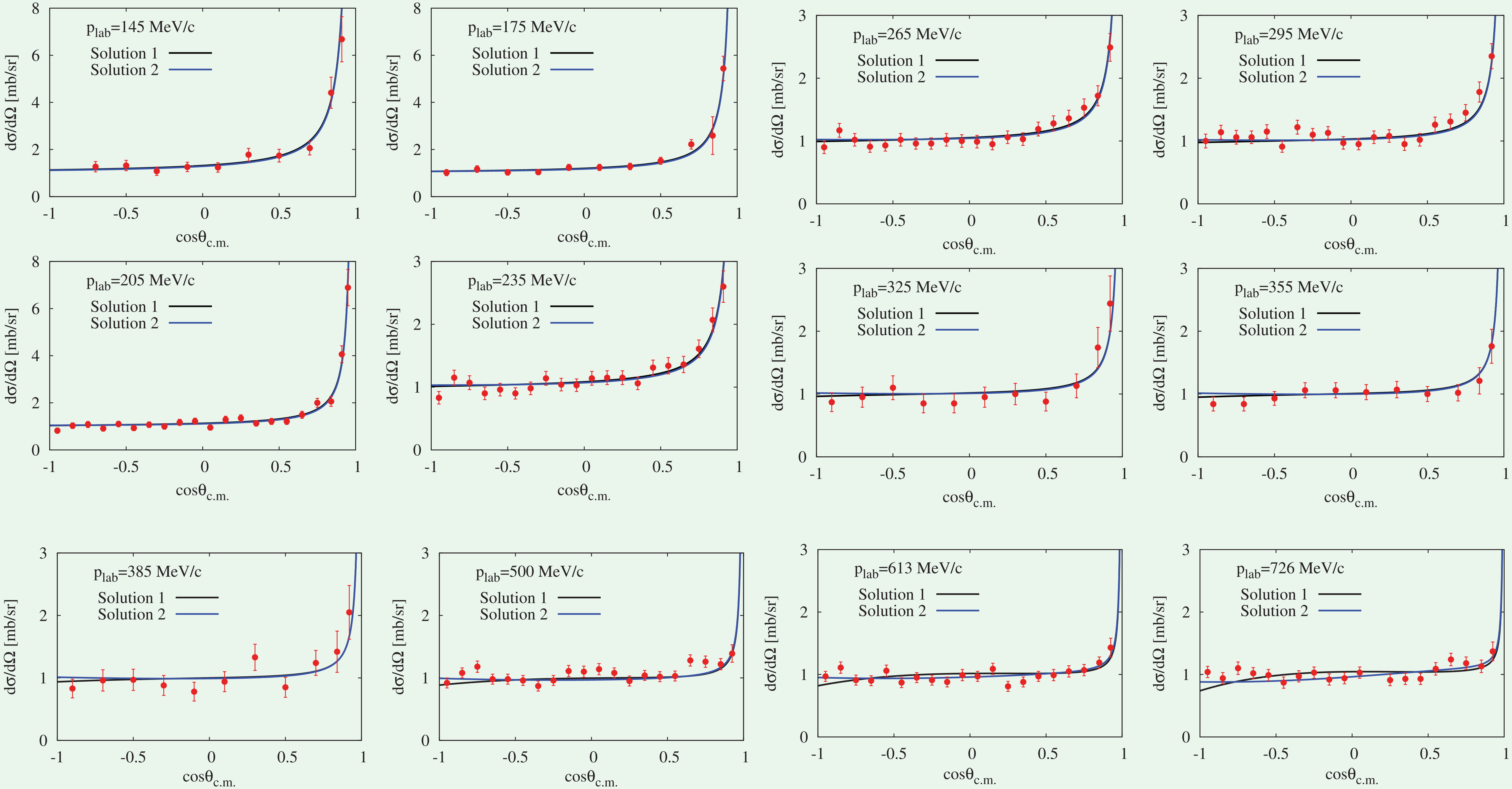


good agreements

solution 1 is consistent with Martin' amplitude and SAID

K+p differential cross sections

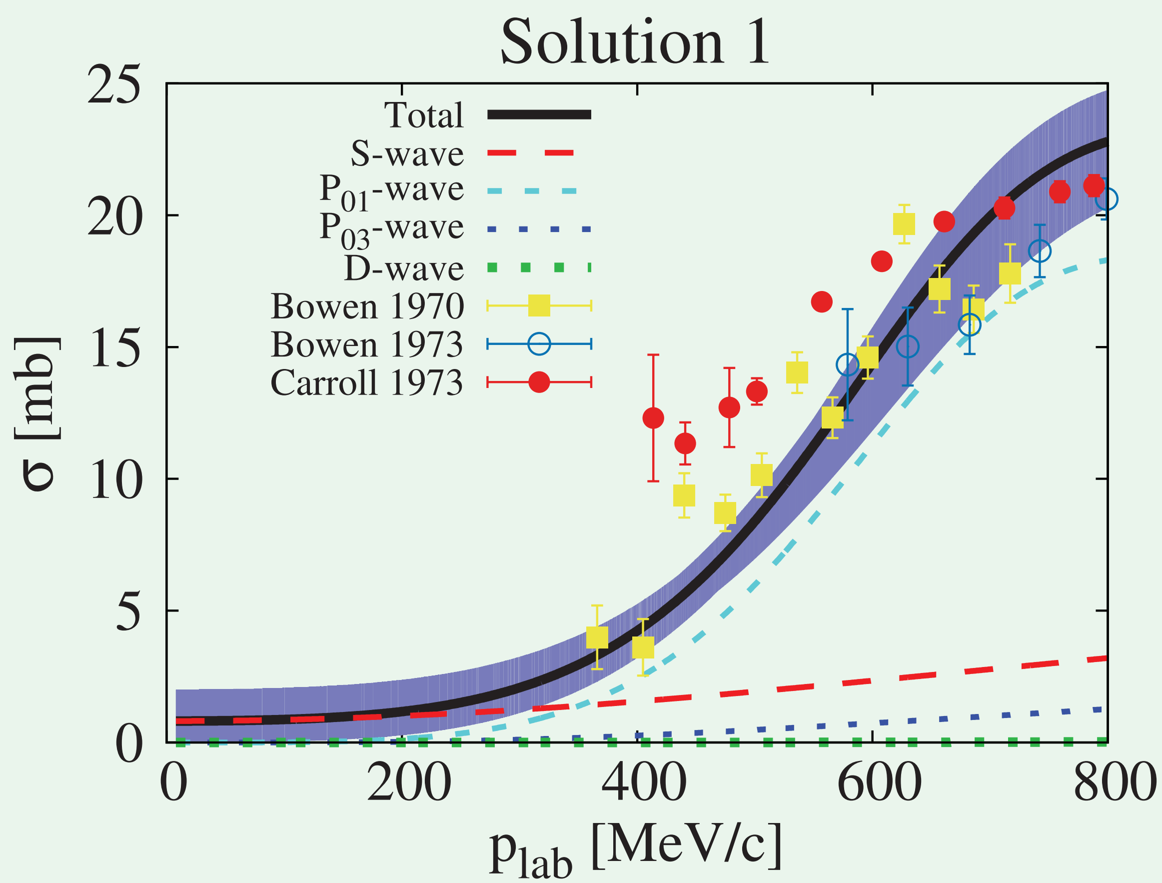
Aoki, DJ, PTEP2019,013D01(19)

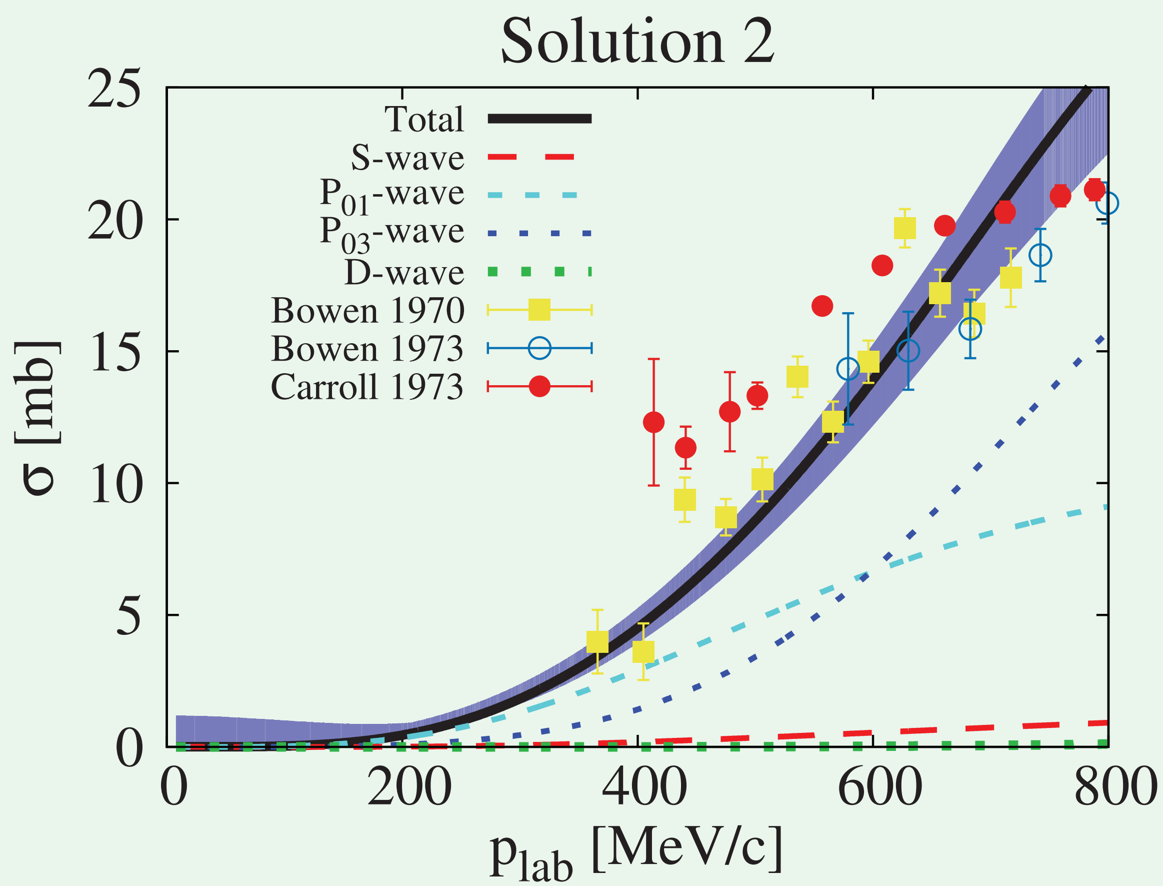


I=0 total cross sections

Aoki, DJ, PTEP2019,013D01(19)

- increase at p_{lab} ~ 500 MeV/c is reproduced



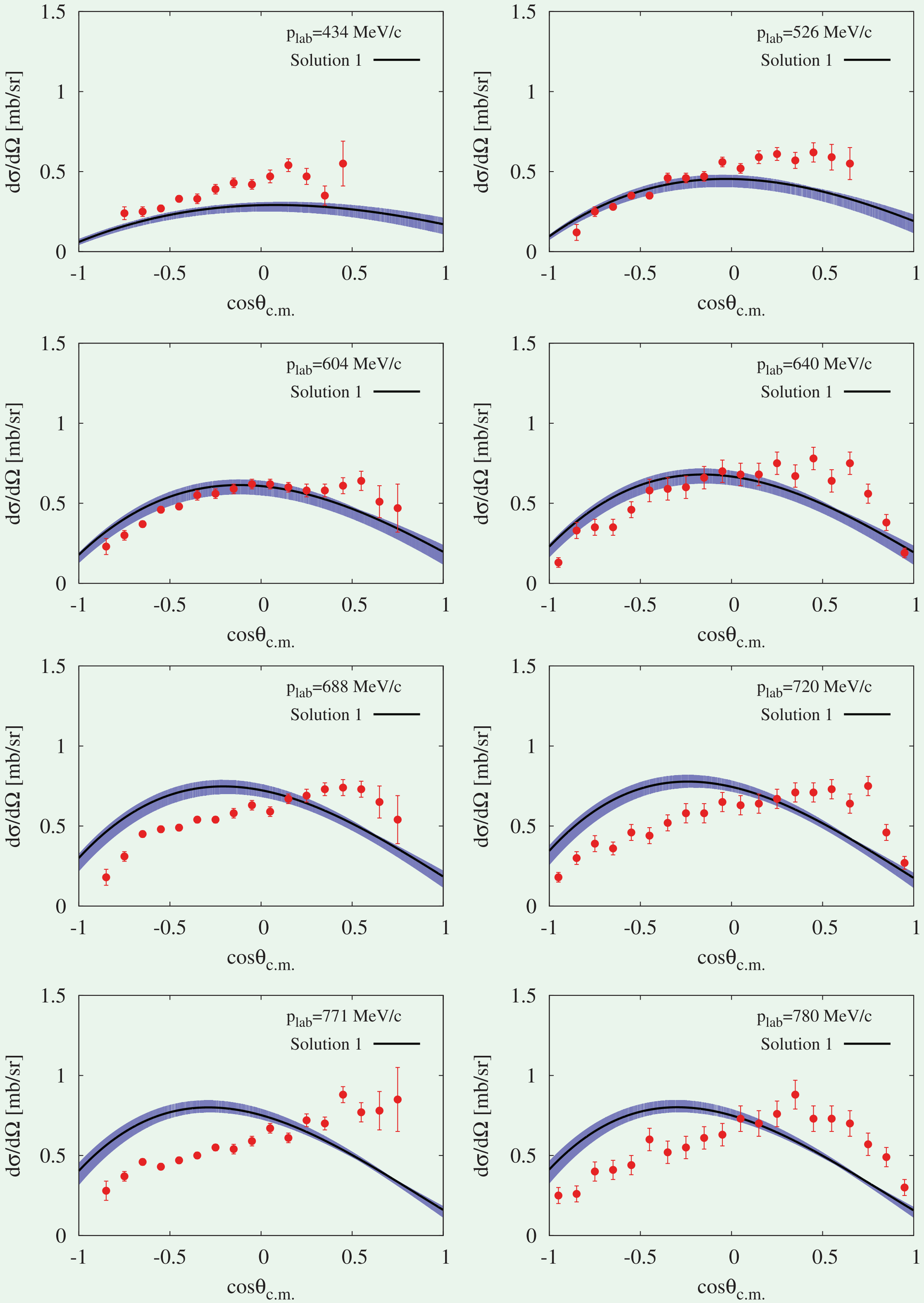


- solution 1: P_{01} amplitude dominate
- solution 2: P_{03} amplitude largely contributed

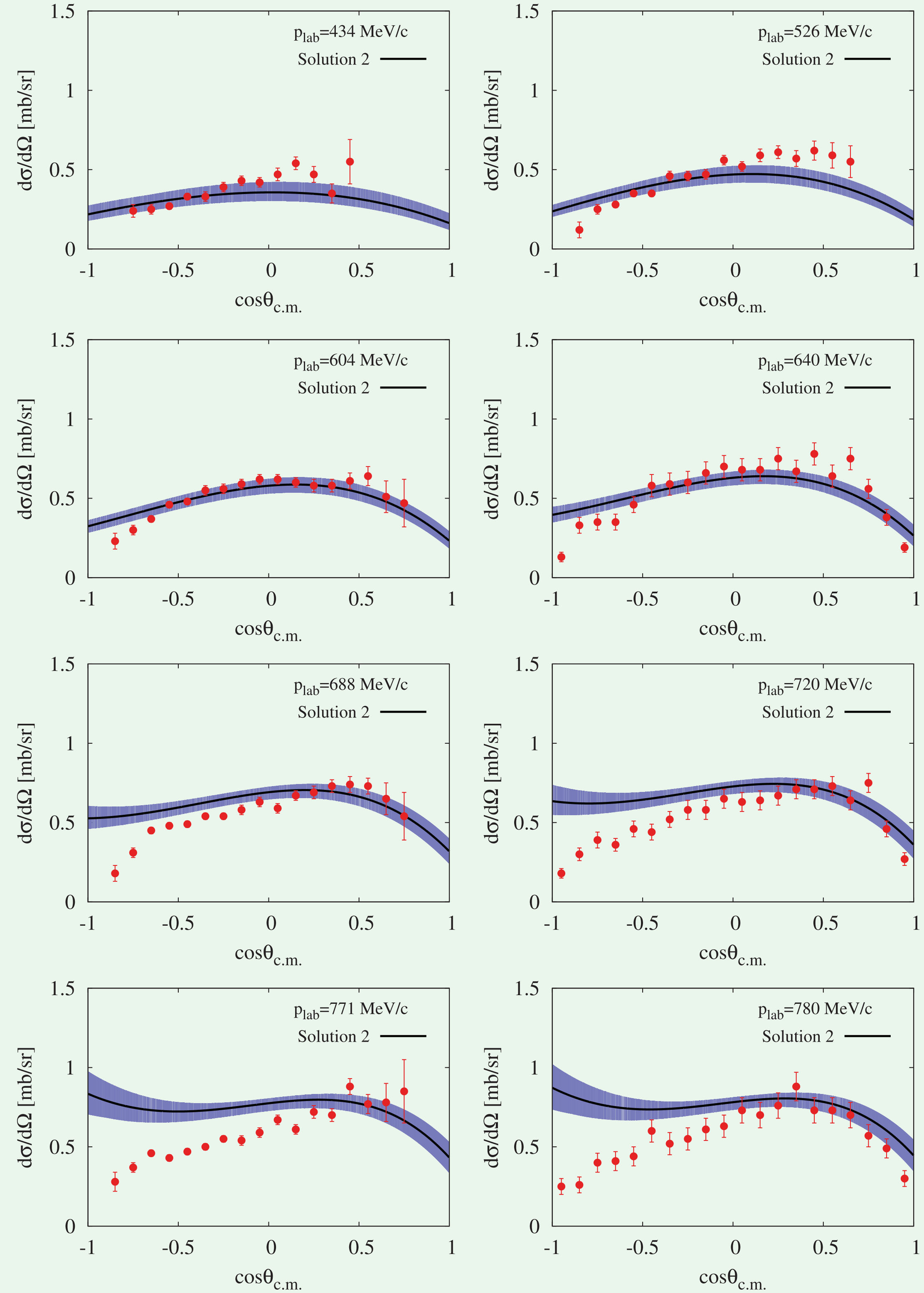
$K^+ n \to K^0 p$ charge exchange scatt.

Aoki, DJ, PTEP2019,013D01(19)

solution 1



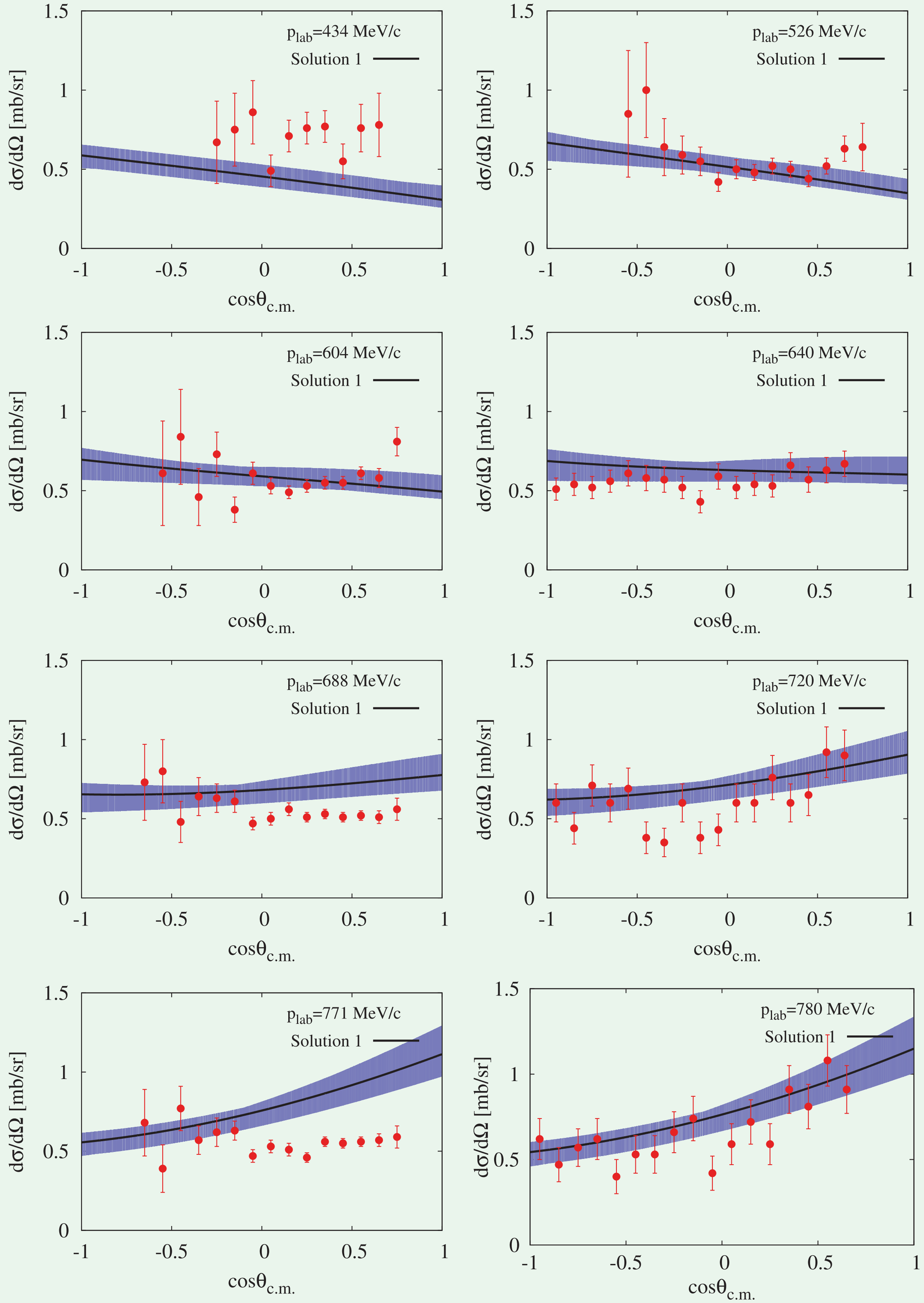
solution 2



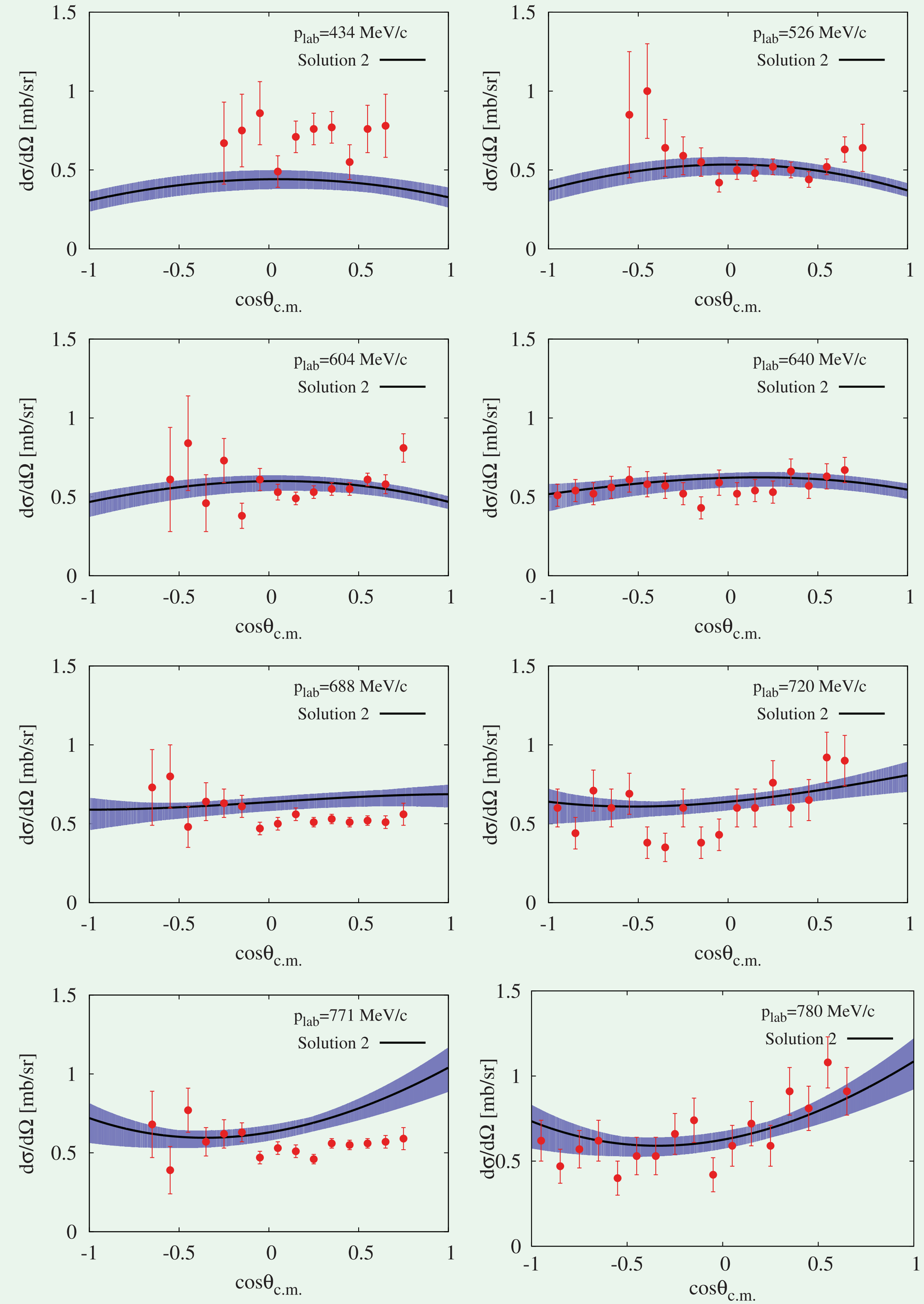
K+n elastic scattering

Aoki, DJ,
PTEP2019,013D01(19)

solution 1



solution 2



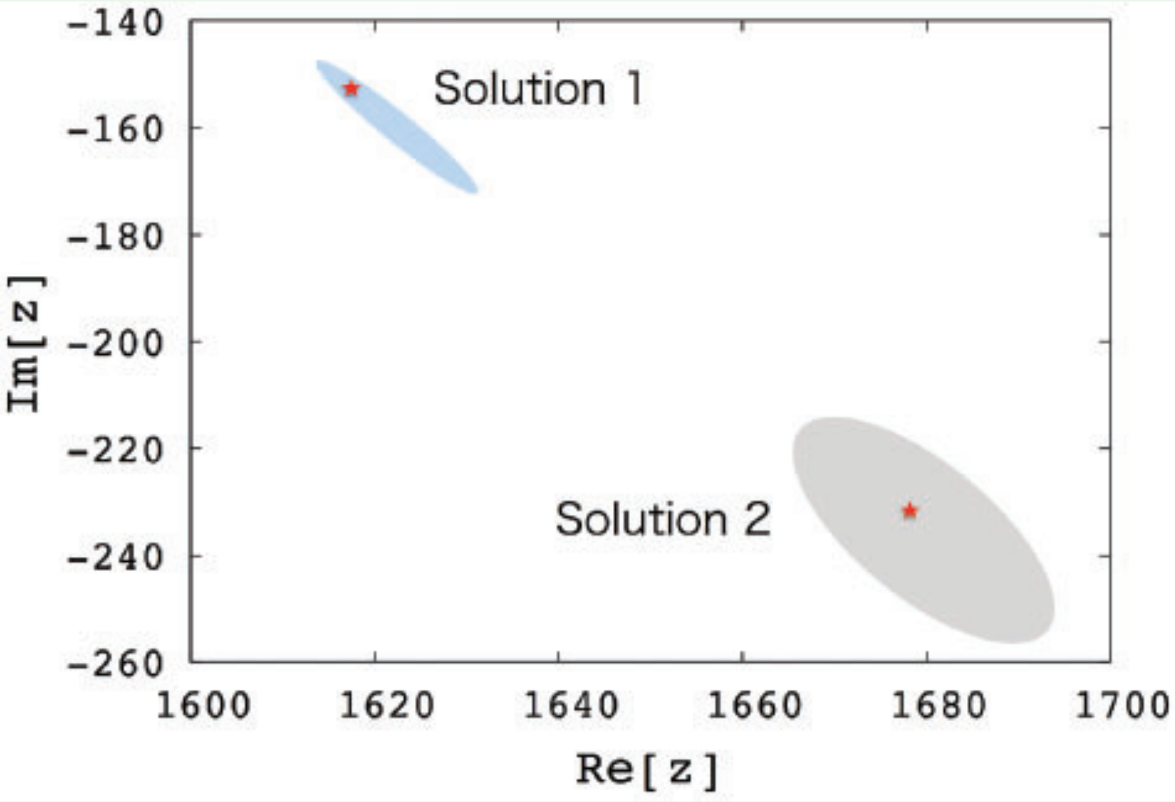
Possible broad resonance with S=+1

- a resonance pole around 1650 MeV (p_{lab} = 400 MeV) with a large width

Aoki, DJ, PTEP2019,013D01(19)

Table 3. The resonance states of Solutions 1 and 2.

amplitude (J^P)		mass [MeV]	width [MeV]
Solution 1	P_{01} $\left(\frac{1}{2}^+\right)$	1617	305
Solution 2	P_{03} $\left(\frac{3}{2}^+\right)$	1678	463



wavefunction renormalization

Aoki, DJ, PTEP2017,103D01(17)

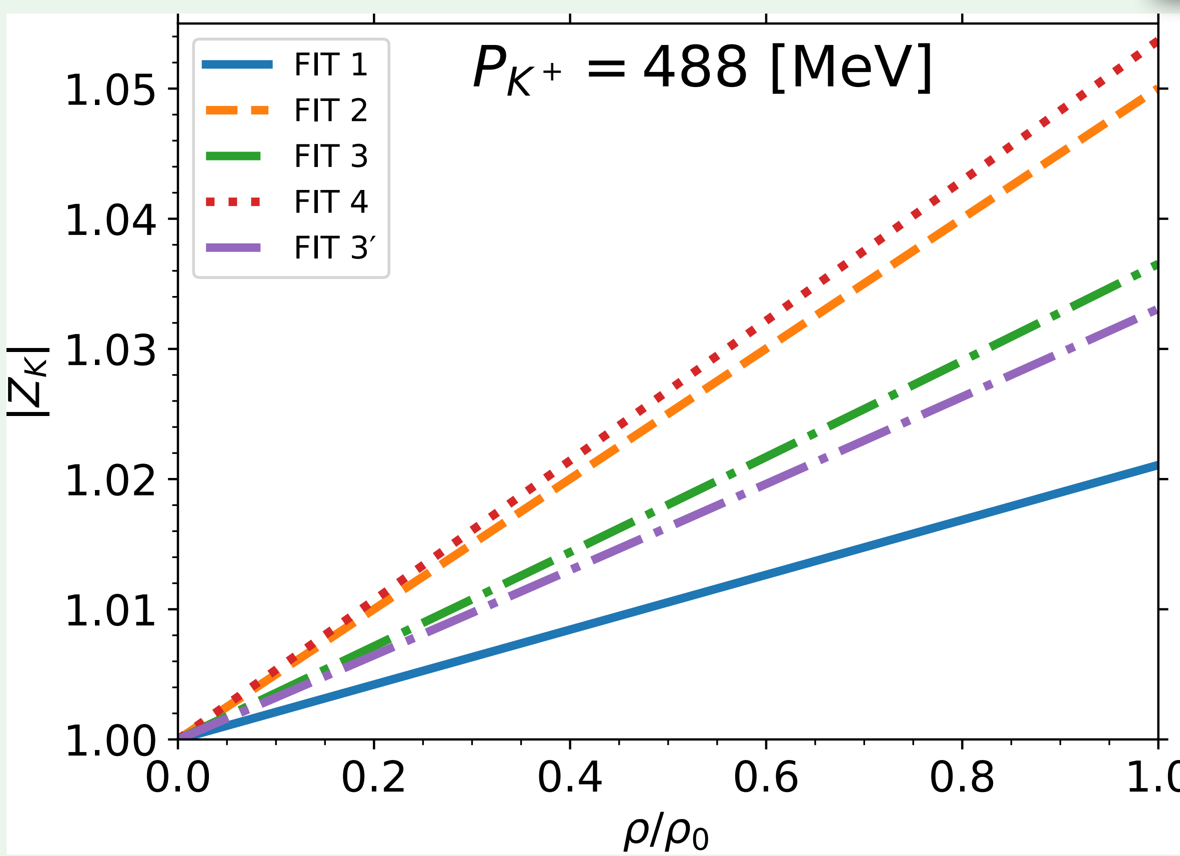
- leading order (Weinberg-Tomozawa term)

$$Z = 1 + \frac{3\rho_0}{8M_K f_K^2}\frac{\rho}{\rho_0} = 1 + 0.082\frac{\rho}{\rho_0},$$

8% enhancement at $\rho = \rho_0$

- + next-to-leading order (without medium modification on kaon)

Iizawa, DJ, Hübsch, PTEP 2024, 053D01 (2024)



a few % enhancement with $p_{K^+} \sim 500$ MeV