Effect of hadron-quark phase transition on the Neutron star f-mode oscillations in general relativistic calculations

QNP2024, Barcelona

Sarmistha Banik

BITS Pilani, Hyderabad Campus, India

July 8, 2024

Introduction

Tools for studying compact objects & address the uncertainties in NS EoS: The structure parameters like

- Mass
- Radius
- Tidal deformability
- Quasinormal oscillations:r, p, g, f-modes

Eigenfrequencies of f-modes $\sim$ 2kHz ignoring the metric perturbations during fluid oscillations (relativistic Cowling approximation).

Highlights of our work

- Problem of stellar perturbation with a linearized fully General Relativistic formalism
- Using a Spectral Representation of the tabulated Equation of State (EoS)
- Calculation of $f-$mode frequencies for $\ell = 2$ modes and the corresponding gravitational wave damping time
- Explore correlations of the frequencies and damping times with nuclear matter parameters and stellar observables.

D. Guha Roy, T Malik, S. Bhattacharya, S. Banik, APJ, 2024

The Nuclear EoS set

- NS matter: RMF approach(Walecka Model with σ, ω, ρ mesons, non-linear self-interaction as well as cross-coupling terms)
- $\sim$9000 EoS, derived through Bayesian inference (Malik et al, 2023).
- Minimal constraints on nuclear saturation properties $\rho_0 = 0.153 \pm 0.005$ fm^{-3}, B.E.$= -16.1 \pm 0.2$ MeV, $K_0 = 230 \pm 40$ MeV, & $J_0 = 32.5 \pm 1.8$ MeV
- Constraints from chiral effective field theory (χEFT) relevant to low-density pure neutron matter
- Constraints derived from perturbative Quantum Chromodynamics (pQCD) at densities pertinent to the core of NS
- Maximum mass of $2M_\odot$

Spectral representation

- Adiabatic index:

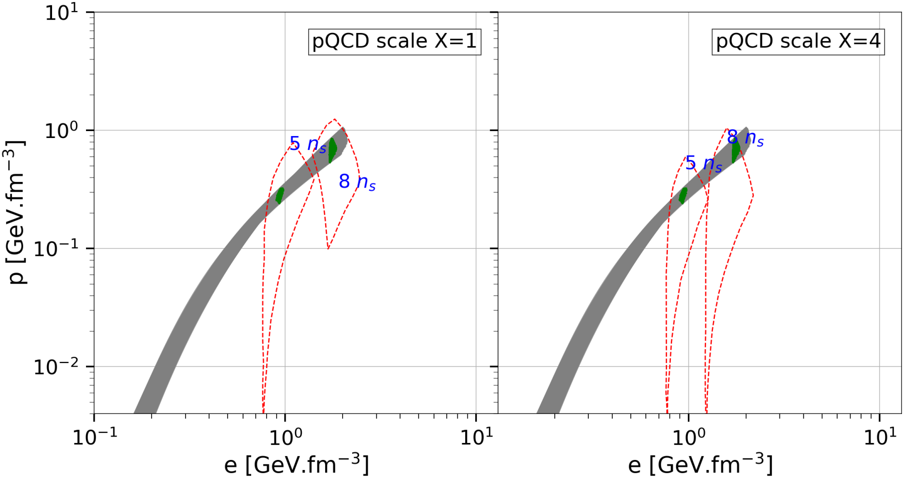
$$\Gamma(p) = \frac{\epsilon + p}{p} \frac{dp}{d\epsilon} \tag{1}$$

- To obtain a pressure-based form, we expand the adiabatic index $\Gamma(p)$ in terms of the linear combination basis functions $\Phi_k(p)$ as

$$\Gamma(p) = \exp\left[\sum_k \gamma_k \Phi_k(p)\right]. \tag{2}$$

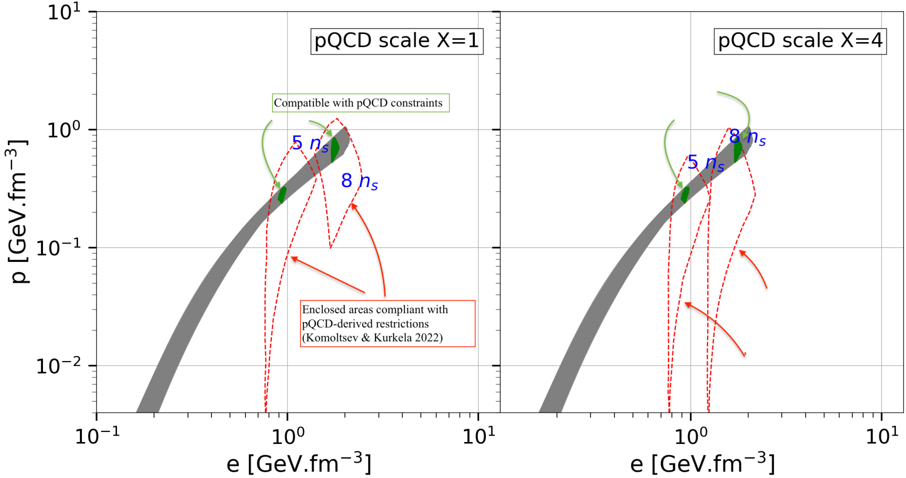
Lindblom, 2010

EoS Set



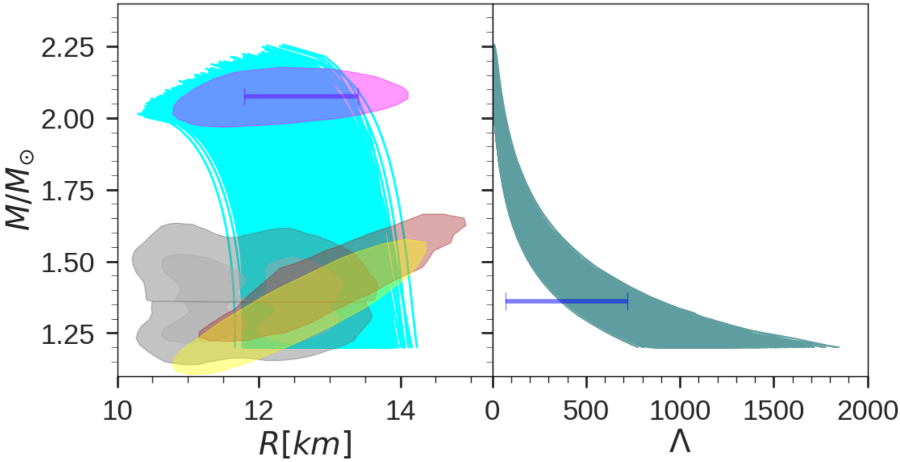
pQCD scale X=1
pQCD scale X=4
5 n_s
8 n_s
8 n_s
5 n_s
p [GeV.fm^{-3}]
e [GeV.fm^{-3}]
e [GeV.fm^{-3}]
10^{1}
10^{0}
10^{-1}
10^{-2}

EoS Set

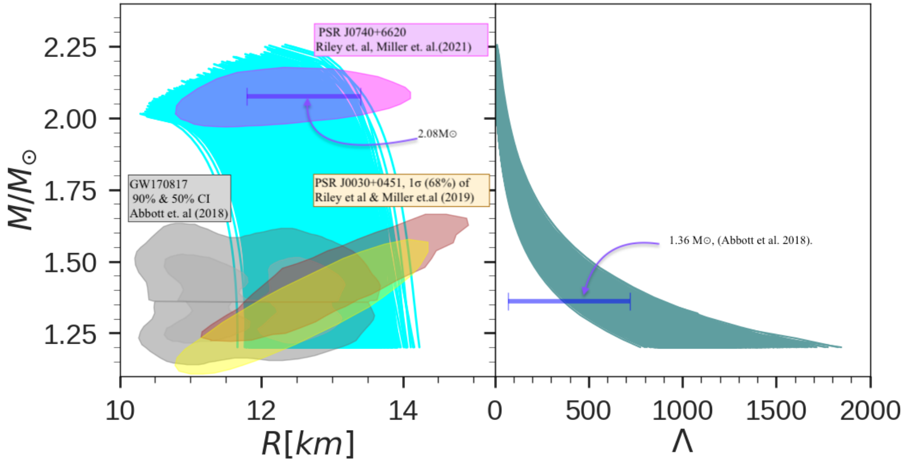


The central density for the maximum mass NS is modeled within $7n_s$ ($n_s = 0.16 fm^{-3}$) across our entire EoS set.

Mass-Radius and Mass-Tidal deformability



Mass-Radius and Mass-Tidal deformability



Stellar perturbation

- Einstein Eqn: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- A small perturbation: $g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}$
- The static spherically symmetric background metric

$$
\begin{aligned}
ds^2 = &- e^{\nu(r)} \left[1 + r^l H_0(r) e^{i\omega t} Y_{lm}(\phi,\theta)\right] c^2 dt^2 \\
&+ e^{\lambda(r)} \left[1 - r^l H_0(r) e^{i\omega t} Y_{lm}(\phi,\theta)\right] dr^2 \\
&+ \left[1 - r^l K(r) e^{i\omega t} Y_{lm}(\phi,\theta)\right] r^2 d\omega^2 - 2i\omega r^{l+1} H_1(r) e^{i\omega t} Y_{lm}(\phi,\theta) dt dr
\end{aligned}
$$

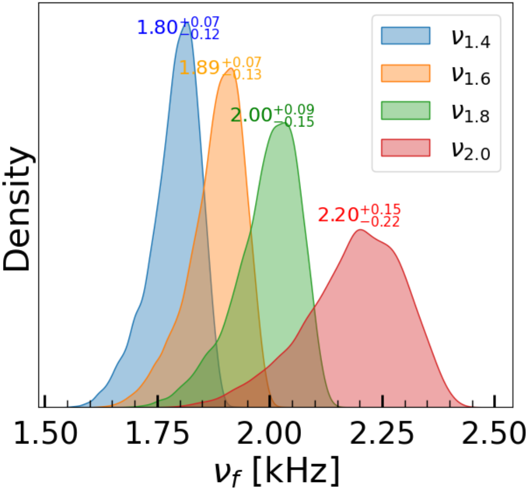
- $e^{i\omega t}$:time-dependence of the perturbed metric components
- H_0, H_1, and K are radial perturbations, Y^l_m contains the angular part
- This metric leads to four coupled linear differential equations for the four perturbation functions: H_1, K, W, and X over the variable r
- These equations describe non-radial oscillations inside the star

Lindblom & Detweiler, 1983 and Detweiler & Lindblom, 1985

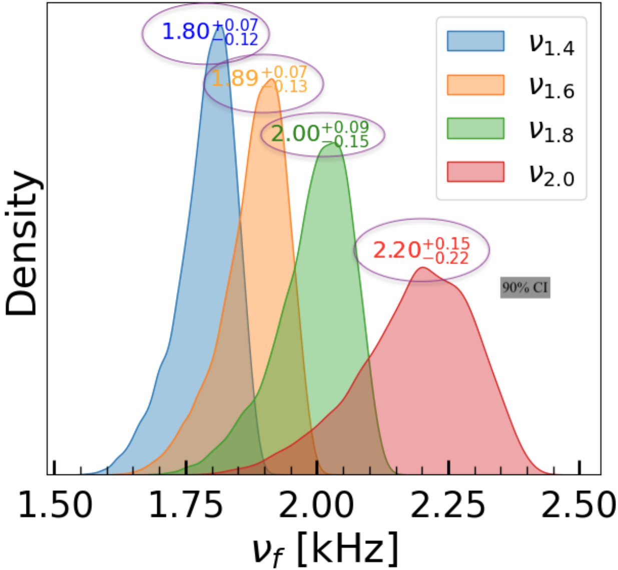
Solutions of perturbation equations in-out of NS

- Einstein equations govern the dynamics both inside and outside NS.
- Integrate the 4 perturbation equations inside the star with appropriate boundary conditions.
- Integrate the Zerilli equations outside NS to find the quasinormal mode frequency for a given star
- Outside the star, determination of frequency and damping time of the QNMs become an eigenfrequency problem, subject to appropriate boundary conditions both at NS surface and at spatial infinity.
- The real part of ω represents actual frequency of QNM oscillations, the imaginary part corresponds to an exponential damping.
- The internal structure and dynamics of the matter inside NS influence the frequencies and damping times of the QNMs via the boundary condition at the surface of the NS.

Distribution: $f-$mode frequencies for different NS mass 1.4 - 2.0 $M_\odot$

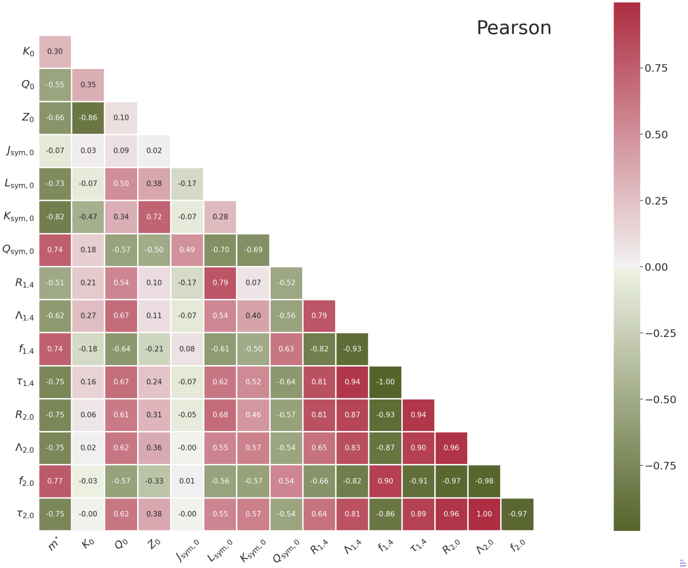


Distribution: $f-$mode frequencies for different NS mass

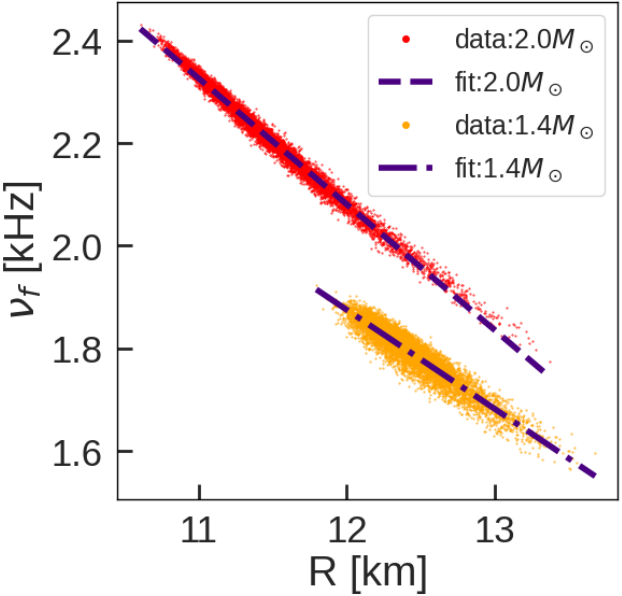


f-mode frequency measurement by the next generartion detectors could potentially lead to more constrained NS EoS domain

Heatmap: $f-$mode frequencies, nuclear matter parameters, NS observables



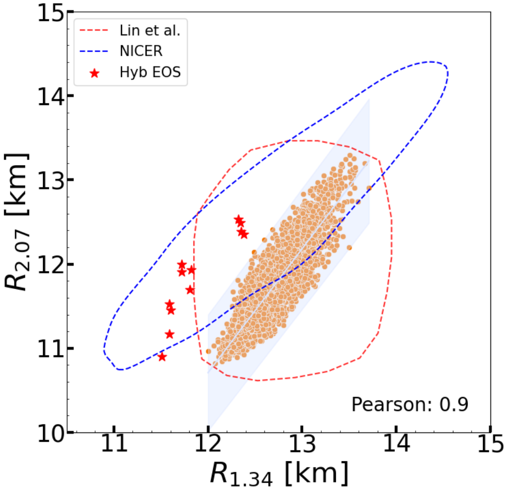
Correlation: $f-$mode frequency and NS radius



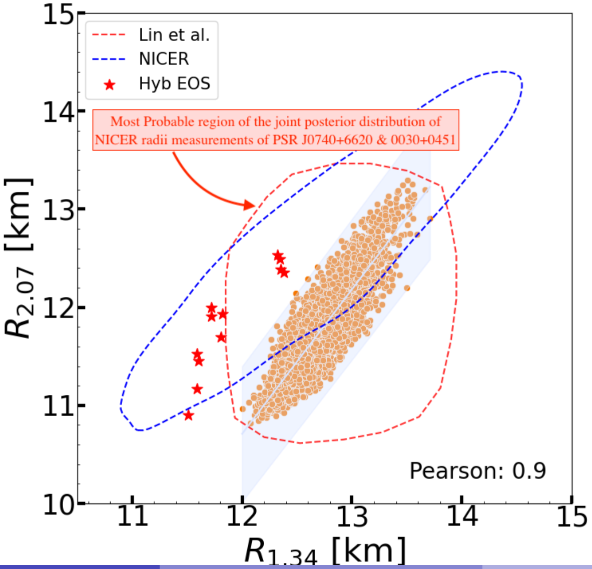
$\nu_{f,x} = aR_x + b$

- $1.4M_\odot$: $a = -0.1202 \pm 0.0007$, $b = 3.307 \pm 0.0096$
- $2.0M_\odot$: $a = -0.2367 \pm 0.0004$, $b = 4.948 \pm 0.0057$

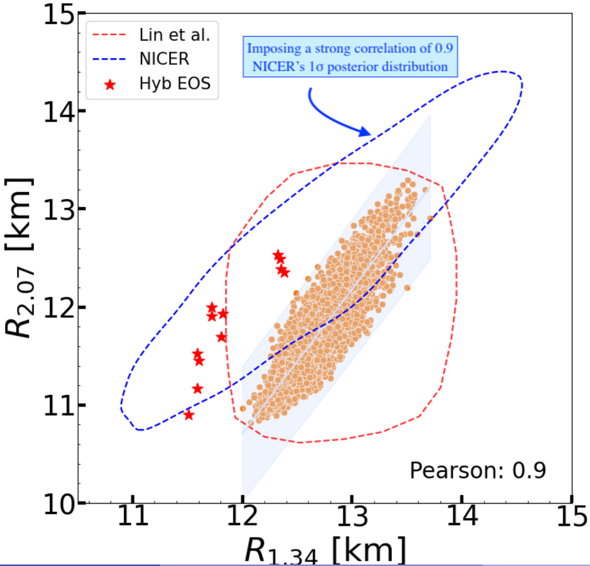
Correlation: Radius domain



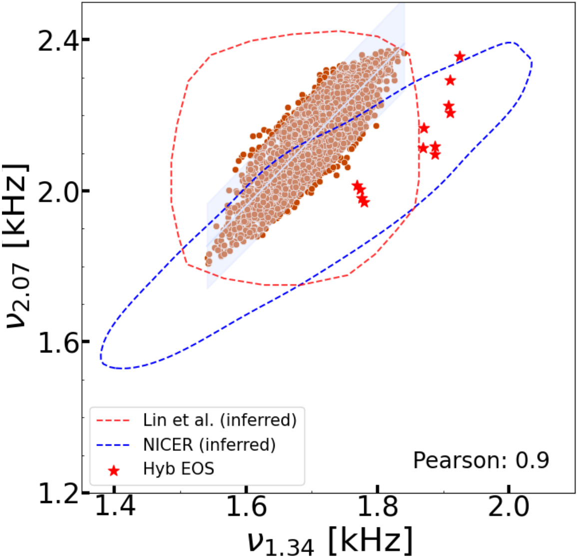
Correlation: Radius domain



Correlation: Radius domain



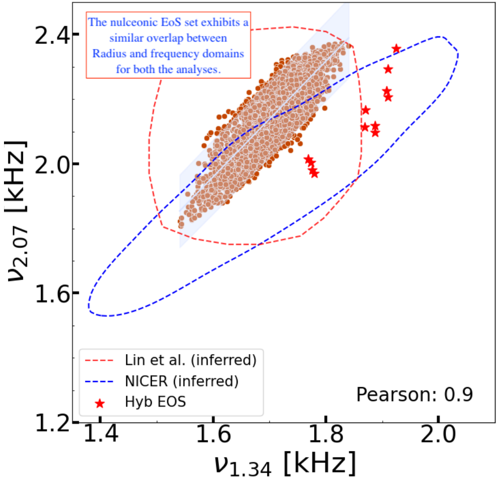
Correlation: Frequency domain



$$\nu_{f,x} = aR_x + b$$

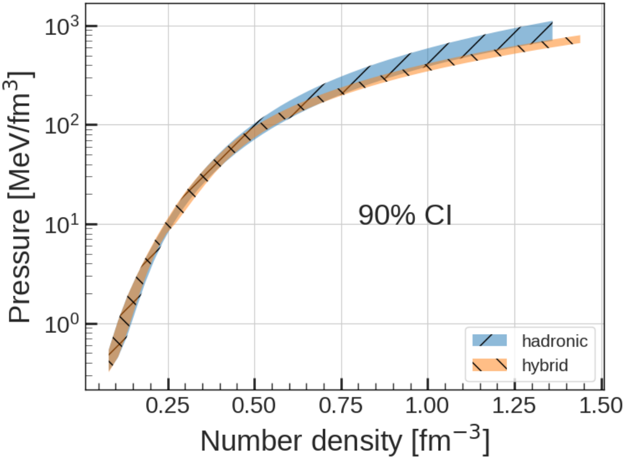
- $1.34 M_\odot$:
  $a = -0.1794$,
  $b = 3.9891$
- $2.07 M_\odot$:
  $a = -0.2360$,
  $b = 4.9283$

Correlation: Frequency domain



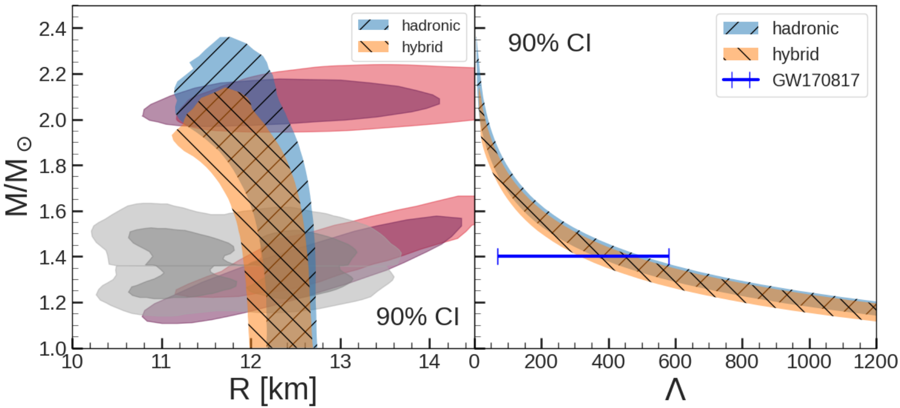
- However, the partial overlap between the nucleonic EoS and this NICER bound suggests nucleon-only bound for higher masses may not be sufficient.
- The stars are calculated values for Quark-Hadron hybrid EoS from CompOSE, they fit comfortably within the NICER bound.

Pressure vs Number Density: Hybrid-Hadronic Comparison

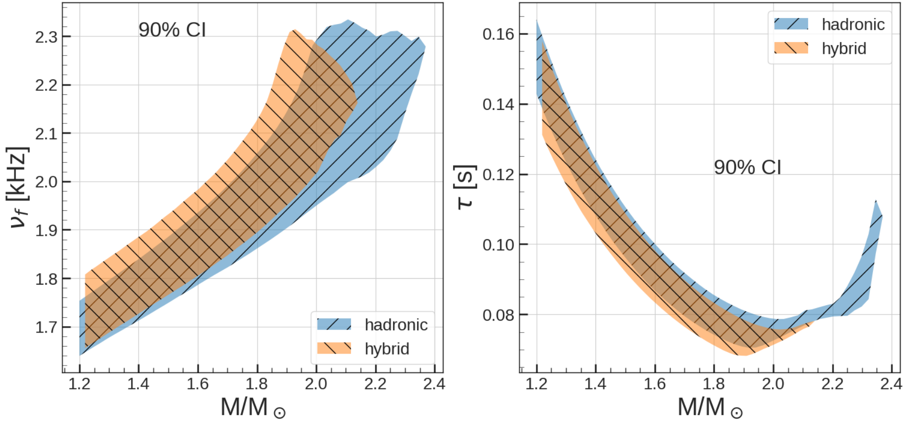


QUARK EoS: MFTQCD formalism Albino et al, 2021

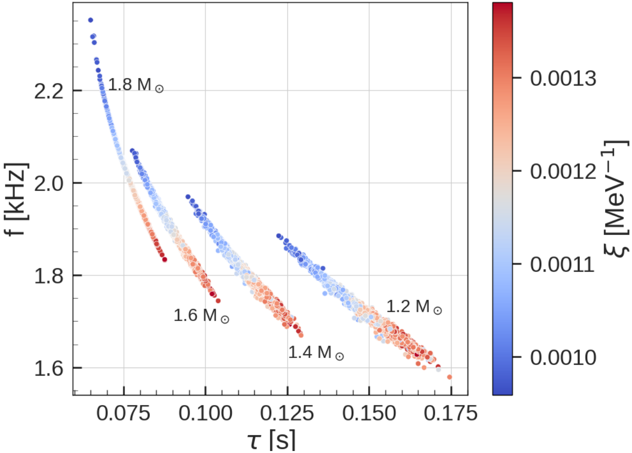
Mass-Radius and Mass-Tidal deformability: Hybrid-Hadronic Comparison



$f-$mode frequencies and GW damping times against stellar mass: Hybrid-Hadronic Comparison



Correlation between $f-$mode frequencies and damping times with variation of the MFTQCD model parameter ξ



$\xi \equiv g/m_G$, g strong coupling constant, m_G dynamical gluon mass

Summary

- Significant correlation between $f-$mode frequency and NS radius with our set of EoS.
- Notable correlation between the radii of NS $1M_{solar}$ & $2.0M_{solar}$ using a robust Nucleonic EoS set.
- Assuming this correlation on joint NICER measurements not fully compatible; slopes of radius differ from nucleonic set predictions. Similar discrepencies were noted in the f-mode domain.
- Hybrid EoS shows promising results, potentially resolving inconsistencies with NICER data.

Acknowledgement

- This work is supported by DST-SERB, Govt. of India through the Core Research Grant (CRG/2020/003899).
- T.M. is supported by Fundaço para a Ciência e a Tecnologia, IEP, Portugal & "EUROpean Laboratories for Accelerator Based Science".

THANK YOU

Lagrangian

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{NL} + \mathcal{L}_{leptons} \tag{3}$$

where

$$\mathcal{L}_N = \bar{\Psi}\Big[\gamma^\mu \left(i\partial_\mu - g_\omega \omega_\mu - \frac{1}{2} g_\varrho \boldsymbol{t} \cdot \boldsymbol{\varrho}_\mu\right) - (m_N - g_\sigma \sigma)\Big]\Psi$$

$$\begin{aligned}\mathcal{L}_M &= \frac{1}{2}\left[\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right] - \frac{1}{4} F^{(\omega)}_{\mu\nu} F^{(\omega)\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ &- \frac{1}{4} \boldsymbol{F}^{(\varrho)}_{\mu\nu} \cdot \boldsymbol{F}^{(\varrho)\mu\nu} + \frac{1}{2} m_\varrho^2 \boldsymbol{\varrho}_\mu \cdot \boldsymbol{\varrho}^\mu\end{aligned}$$

where $F^{(\omega,\varrho)\mu\nu} = \partial^\mu A^{(\omega,\varrho)\nu} - \partial^\nu A^{(\omega,\varrho)\mu}$ are the vector meson tensors.

$$\begin{aligned}\mathcal{L}_{NL} &= -\frac{1}{3} b\ m_N\ g_\sigma^3 (\sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 + \frac{\xi}{4!} g_\omega^4 (\omega_\mu \omega^\mu)^2 \\ &+ \Lambda_\omega g_\varrho^2 \boldsymbol{\varrho}_\mu \cdot \boldsymbol{\varrho}^\mu g_\omega^2 \omega_\mu \omega^\mu\end{aligned}$$

$$\mathcal{L}_{leptons} = \bar{\Psi}_l \Big[\gamma^\mu \left(i\partial_\mu - m_l\right) \Psi_l\Big]$$

Lindblom-Detweiler equations 1

The gravitational wave equations can be written as a set of four coupled linear differential equations for the four perturbation functions: H_1, K, W, and X, which do not diverge inside the star for any given value of ω (Lindblom, 1983).

$$r\frac{dH_1}{dr} = -\left[l + 1 + 2be^{\lambda}4\pi r^2 e^{\lambda}(p - \epsilon)\right] H_1 + e^{\lambda}\left[H_0 + K - 16\pi(\epsilon + p)V\right],$$

$$r\frac{dK}{dr} = H_0 + (n+1)H_1 + \left[e^{\lambda}Q - l - 1\right] K - 8\pi(\epsilon + p)e^{(\lambda/2)}W$$

Lindblom-Detweiler equations 2

$$
\begin{aligned}
r\frac{dW}{dr} &= -(l+1)\left[W + le^{\lambda/2}V\right] \\
&\quad + r^2 e^{\lambda/2}\left[\frac{e^{-\nu/2}X}{(\epsilon+p)c_{eq}^2} + \frac{H_0}{2} + K\right], \\
r\frac{dX}{dr} &= -lX + \frac{(\epsilon+p)e^{(\nu/2)}}{2}\Bigg\{(3e^{\lambda}Q - 1)K \\
&\quad - \frac{4(n+1)e^{\lambda}Q}{r^2}V + (1 - e^{\lambda}Q)H_0 \\
&\quad + (r^2\omega^2 e^{-\nu} + n + 1)H_1 - \Bigg[8\pi(\epsilon+p)e^{\lambda/2} + 2\omega^2 e^{\lambda/2-\nu} r^2 \\
&\quad \times \frac{d}{dr}\left(\frac{e^{-\lambda/2}}{r^2}\frac{d\nu}{dr}\right)\Bigg] W\Bigg\}.
\end{aligned}
$$

where $c_{eq}^2 = dp/d\epsilon$ is the equilibrium speed of sound of NS matter undergoing oscillations.

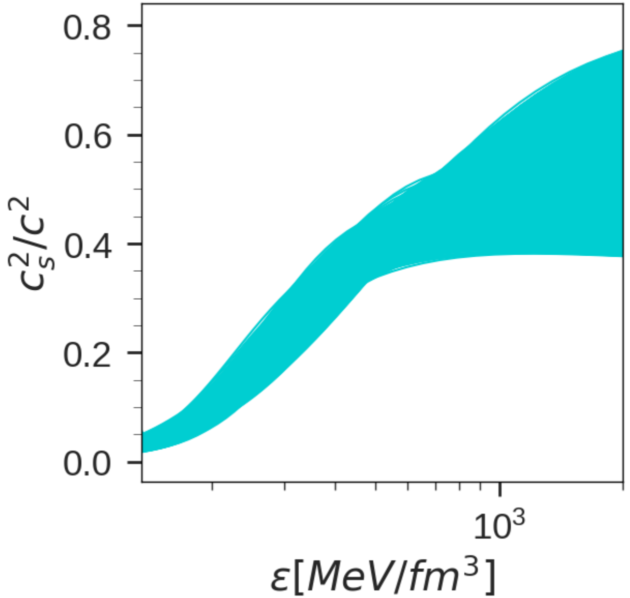
Fluid perturbations

- Perturbations of the energy-momentum tensor of the fluid also need to be taken into account in the Einstein equation.
- Components of Lagrangian displacement vector $\xi^a(r, \theta, \phi)$ describes the perturbations of the fluid inside the star:

$$\xi^r = r^{l-1} e^{-\lambda/2} W Y^l_m e^{i\omega t}$$
$$\xi^\theta = -r^{l-2} V \partial_\theta Y^l_m e^{i\omega t}$$
$$\xi^\phi = -\frac{r^{l-2}}{sin^2\theta} V \partial_\theta Y^l_m e^{i\omega t},$$

where W and V are functions with respect to r that describe fluid perturbations. Fluid perturbations exist only inside the star.

Speed of Sound



Damping time