Exploring Composition of Neutron Star Matter with a Relativistic Density Functional

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Outline

- A very brief introduction to neutron stars (NSs)
- Description of nuclear matter
- Models specific to this work and the constraints used
- Results
- Summary
Structure of a Neutron Star

- **Envelope**
- **Outer crust**
  - (Coulomb crystal of n-rich nuclei + relativistic degenerate e⁻)
- **Inner crust**
  - (Coulomb crystal of n-rich nuclei + dripped n + relativistic degenerate e⁻)
- **"Pasta" phases**
- **Core**
  - (uniform nuclear matter) \(n + p + e^- + [\mu^-]\)
  - \(\rho \sim \rho_0\)
  - Neutron drip: \(\rho \equiv 4 \times 10^{11}\) g cm\(^{-3}\)
  - Normal nuclear density: \(\rho \sim \rho_0 = 0.165 \text{ fm}^{-3} \equiv 3 \times 10^{14}\) g cm\(^{-3}\)
  - Hyperons?
  - Meson condensates?
  - Quark matter?

Figure: Schematic picture of a NS Interior

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NS Observations that an EOS must satisfy

- Precise mass-measurement of massive NSs:
  - $(2.01 \pm 0.04) M_\odot$ Antoniadis et al, Science 340, 448 (2013).
  - $(2.08 \pm 0.07) M_\odot$ E. Fonseca et al, ApJL 915 L12 (2021).

- BNS merger event GW170817 provides bounds on tidal deformability ($\Lambda$), and pressure at $2 \rho_0$; Abbott et al, PRL 121, 161101 (2018):
  $$\Lambda_{1.4} = 190^{+390}_{-120} \Rightarrow \Lambda_{1.4} \leq 580, P(2 \rho_0) = 3.5^{+2.7}_{-1.7} \times 10^{34} \text{ dyn/cm}^2$$

- NICER collaboration provided:
  1) Simultaneous mass-radius measurements of PSR J0030+0451
     $$M = 1.34^{+0.15}_{-0.16} M_\odot, \ R = 12.71^{+1.14}_{-1.19} \text{ km}$$ Riley et al, ApJL, 887, L21 (2019).
     $$M = 1.44^{+0.15}_{-0.14} M_\odot, \ R = 13.02^{+1.24}_{-1.06} \text{ km}$$ Miller et al, ApJL, 887, L24 (2019).
  2) Radius measurements of J0740+6620
     $$R = 12.39^{+1.30}_{-0.98} \text{ km}$$ Riley et al, ApJL, 918, L27 (2021).
     $$R = 13.7^{+2.6}_{-1.5} \text{ km}$$ Miller et al, ApJL, 918, L28 (2021).
Description of Nuclear Matter:

- Pressure as a function of energy:
  \[ P(\rho, \delta) = \rho^2 \frac{d}{d\rho} (E(\rho, \delta)) \]

- Energy per particle of nuclear matter:
  \[ E(\rho, \delta) \approx E_0(\rho) + E_{\text{sym}} \delta^2 \]

- Symmetric nuclear matter:
  \[ E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2} \chi^2 + \frac{Q_0}{6} \chi^3 \]

- Symmetry energy:
  \[ E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L_{\text{sym}} \chi + \frac{K_{\text{sym}}}{2} \chi^2 + \frac{Q_{\text{sym}}}{6} \chi^3 \]

- Isospin asymmetry:
  \[ \delta = \text{"Isospin asymmetry"} = (\rho_n - \rho_p)/\rho, \quad \chi = (\rho - \rho_0)/3\rho_0 \]
EOS of Dense Matter from Nuclear Physics

Difficulties

- Constituents are not known.
- Interaction between constituents are not fully known.
- Uncertainties in the many-body description.

⇒ EOS is model dependent.

Phenomenological approaches are most widely used.

- Based on effective Interaction.
  1. Non-relativistic Skyrme-Interaction ($\sim 240$)
  2. Relativistic Mean Field (RMF) models ($\sim 270$)

Dutra et al. PRC 85, 035201 (2012); Dutra et al. PRC 90, 055203 (2014);
Oertel et al. RMP 89, 015007 (2017)

Our main objective: Exploring the parameter space to quantify the uncertainties.
Nucleonic metamodelling

- Foundational aspects (Based on J. Margueron et. al., PRC 97, 025805 (2018))
- Flexible functional $e(\rho_n, \rho_p)$ able to reproduce existing effective nucleonic models and interpolate between them.
- Expansion in powers of the Fermi momentum or of the density.
- Expansion around saturation: Parameter space = emp. par. $\vec{X}$
- Beta-equilibrium!!!

$$e_{Elf}(\rho_n, \rho_p) = KE(\rho_n, \rho_p) + \sum_{\alpha \geq 0} \frac{1}{\alpha!} \left( v^{is}_\alpha + v^{iv}_\alpha \delta^2 \right) x^\alpha$$
RMF model

- Interaction between baryons is described via exchange of mesons.
- The most general form of the interaction Lagrangian density:

\[
L_{DD} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi + \Gamma_\sigma(\rho)\bar{\psi}\gamma^\mu \sigma \partial_\mu \psi + \Gamma_\omega(\rho)\bar{\psi}\gamma^\mu \omega_\mu \psi - \frac{\Gamma_\rho(\rho)}{2}\bar{\psi}\gamma^\mu \rho_\mu \cdot \tau_\psi \\
+ \frac{1}{2}(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}F^\mu_\nu F^\nu_\mu + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\tilde{B}^{\mu\nu}_{\nu\mu} + \frac{1}{2}m_\rho^2 \rho_\mu \cdot \rho^\mu,
\]

\(\sigma, \omega_\mu,\text{and}\ \rho_\mu\) are meson fields.
- For the density dependent (DD) models, the coupling parameters \(\Gamma_\sigma,\ \Gamma_\omega,\ \text{and}\ \Gamma_\rho\) are density dependent and do not have nonlinear terms.

\[\Gamma_i(\rho) = a_i + (b_i + d_i x^3)e^{-c_i x},\]

for \(i = \sigma, \omega, \rho,\) and \(x = n / n_0.\)

Saturation properties of nuclear matter:

Parameter sets are obtained by exploring the uncertainties of the saturation properties of nuclear matter:

- Saturation density: $\rho_{\text{sat}} = (0.135, 0.195) \text{ fm}^{-3}$
- Binding energy per nucleon: $E_{\text{sat}} = (-14, -17) \text{ MeV}$.
- Incompressibility: $K_{\text{sat}} = (150, 350) \text{ MeV}$.
- Symmetry energy: $E_{\text{sym}} = (20, 45) \text{ MeV}$.
- Symmetry energy slope: $L_{\text{sym}} = (20, 180) \text{ MeV}$.

Additionally, we use the constraints coming from chiral EFT calculations from Drischler et al., Phys. Rev. C 93, 054314 (2016)
Results: Unified EOS

- High density EOS is constructed for a set of model parameters corresponding to a unique set of nuclear matter parameters.
- Low density EOS is calculated within the compressible liquid drop model (CLDM) model for the aforementioned set of nuclear matter parameters.
- $\beta$-equilibrium is applied over the whole range.
- The crust and the core are matched with the continuity of pressure and chemical potential.
- For more details on unified crust with RMF approach, see Luigi’s talk!!!
Results:

Figure: Pressure (panel a) and proton fraction (panel b) as a function of baryon density $n_B$ for two example EOS models with negative and positive $K_{sym}$, respectively.
Results: SNM Parameters

PDF

Prior

χ-EFT

Posterior

$E_{\text{sat}}$(MeV)

$K_{\text{sat}}$(MeV)

$Q_{\text{sat}}$(MeV)

$Z_{\text{sat}}$(MeV)

$n_{\text{sat}}$(fm$^3$)

$E_{\text{sat}}$(MeV)

$K_{\text{sat}}$(MeV)

$Q_{\text{sat}}$(MeV)

$Z_{\text{sat}}$(MeV)
Results: Isospin Parameters

![Graph showing distribution of isospin parameters](image_url)
Results: EOS at $\beta$-equilibrium

![Graph showing EOS at $\beta$-equilibrium with various confidence intervals.](image-url)
### Results: Mass - Radius - Tidal deformability

<table>
<thead>
<tr>
<th>R [km]</th>
<th>M [M$_\odot$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>1.2</td>
<td>10$_2^{+3}$</td>
</tr>
<tr>
<td>1.4</td>
<td>10$_3^{+4}$</td>
</tr>
<tr>
<td>1.6</td>
<td>10$_4^{+5}$</td>
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<td>1.8</td>
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<tr>
<td>2.0</td>
<td>10$_6^{+7}$</td>
</tr>
<tr>
<td>2.2</td>
<td>10$_7^{+8}$</td>
</tr>
<tr>
<td>2.4</td>
<td>10$_8^{+9}$</td>
</tr>
</tbody>
</table>

**Figure**: Mass-radius (left) and mass-tidal deformability (right) relations along with the Model I and II obtained in the present study. 1$\sigma$ constraints from the NICER observations are also indicated in the mass-radius panel.
Results: Proton fraction

![Graph showing the proton fraction as a function of neutron density. The graph includes 99% Prior, 99% Posterior, 95% Posterior, and 68% Posterior confidence intervals. The x-axis represents neutron density ($n_B$) in fm$^{-3}$, and the y-axis represents the proton fraction ($x_p$).]
Results: Including Λ hyperons

\[ n_B \text{ [fm}^3\text{]} \]

99% Posterior
95% Posterior
68% Posterior

\[ x_p \]

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Results: Including $\Lambda$ hyperons

\[ n_B [\text{fm}^3] \]

\[ x^\Lambda \]

99% Posterior
95% Posterior
68% Posterior
Summary:

- Any study of dense matter EOS is heavily model dependent. Therefore, a metamodelling approach to dense matter is very helpful to refine our knowledge.
- Within the GDFM type density-dependent RMF model, a wide range of EOSs can be generated with diverse nuclear matter properties that will be able to satisfy present observational constraints.
- One key finding is the large variation of proton fraction within this model.
- Ongoing project: Inclusion of hyperons
- Our future objective is to apply this model to study finite nuclei properties

Thank You
Backup: Speed of Sound

\[(c_s/c)^2\]

- 99% Prior
- 99% Posterior
- 95% Posterior
- 68% Posterior

\[n_B \text{ [fm}^{-3}\text{]}\]

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Backup: Correlations among NMPs

<table>
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<tr>
<th></th>
<th>$E_{\text{sat}}$</th>
<th>$n_{\text{sat}}$</th>
<th>$K_{\text{sat}}$</th>
<th>$Q_{\text{sat}}$</th>
<th>$Z_{\text{sat}}$</th>
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<td>$n_{\text{sat}}$</td>
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<tr>
<td>$Q_{\text{sat}}$</td>
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<td>0.03</td>
<td>0.07</td>
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<td>-0.07</td>
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<td>-0.31</td>
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<tr>
<td>$Z_{\text{sym}}$</td>
<td>0.17</td>
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<td>-0.41</td>
<td>-0.61</td>
<td>-0.77</td>
<td>0.27</td>
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</table>

Correlations are shown as follows: Posterior values are indicated in red, and Prior values are indicated in blue.
## Backup: Correlations among NMPs and selected Observables

![Correlation Matrix Image]

<table>
<thead>
<tr>
<th></th>
<th>( R_{1.4} )</th>
<th>( \Lambda_{1.4} )</th>
<th>( \chi^1_{p} )</th>
<th>( R_{2.0} )</th>
<th>( \Lambda_{2.0} )</th>
<th>( \chi^2_{p} )</th>
<th>( M_{\text{max}} )</th>
<th>( n_{B, c} )</th>
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<td>( R_{1.4} )</td>
<td>0.14</td>
<td>-0.53</td>
<td>0.05</td>
<td>0.32</td>
<td>0.45</td>
<td>0.04</td>
<td>0.41</td>
<td>0.35</td>
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<tr>
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<td>0.52</td>
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<td>( \chi^1_{p} )</td>
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<td>0.10</td>
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<td>0.62</td>
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<td>0.07</td>
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<tr>
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<td>-0.48</td>
<td>0.66</td>
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<td>0.68</td>
<td>-0.49</td>
<td>-0.42</td>
<td>-0.47</td>
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<tr>
<td>( n_{B, c} )</td>
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<td>-0.03</td>
<td>-0.40</td>
<td>-0.64</td>
<td>0.42</td>
<td>0.31</td>
<td>0.34</td>
</tr>
</tbody>
</table>

- \( E_{\text{sat}} \)
- \( n_{\text{sat}} \)
- \( K_{\text{sat}} \)
- \( Q_{\text{sat}} \)
- \( Z_{\text{sat}} \)
- \( E_{\text{sym}} \)
- \( L_{\text{sym}} \)
- \( K_{\text{sym}} \)
- \( Q_{\text{sym}} \)
- \( Z_{\text{sym}} \)
Backup: With HESS J1731-347

\[
\begin{array}{ccc}
\text{R [km]} & \text{0.50} & \text{0.75} & \text{1.00} & \text{1.25} & \text{1.50} & \text{1.75} & \text{2.00} & \text{2.25} & \text{2.50} \\
\text{M [M}_\odot\text{]} & \text{J0030+0451} & (1) & \text{J0740+6620} & (1) & \text{full prior} & \text{xray only} & 99\% \text{ Prior} & 95\% \text{ Posterior} & 68\% \text{ Posterior} \\
\end{array}
\]