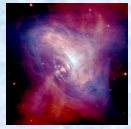


Probing properties of dense matter with neutron stars

Anthea F. Fantina ([anthea.fantina\[AT\]ganil.fr](mailto:anthea.fantina@ganil.fr))

QNP 2024,
08 – 12 July 2024, Barcelona (Spain)



Outline

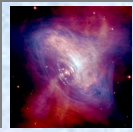
- ❖ Introduction:
 - Neutron-star (NS) properties and equation of state (EoS) modelling and constraints

- ❖ Selected results in :
 - Catalysed (“cold”) NSs ($T = 0$, full equilibrium)
 - EoS and NS observables

 - Proto-neutron-star (PNS) crust ($T \neq 0$, beta equilibrium)
 - multi-component plasma, impurity parameter

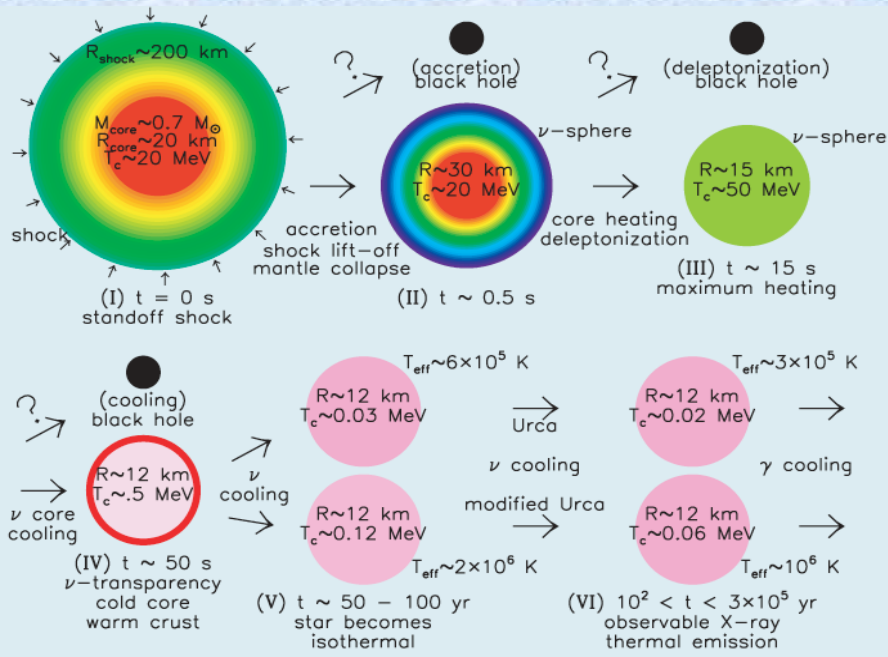
- ❖ Conclusions and open questions

N.B.: In this talk, beta-equilibrated matter
NS static properties



NS (isolated): formation

after shock formation



Lattimer & Prakash, Science 304, 536 (2004)

- NS born hot, $T \sim 10^{10}$ - 10^{11} K \sim 1 – tens MeV
- after few tens of sec – mins
 - \rightarrow beta equilibrium (e.g. [Camelio et al. 2017](#))
 - \rightarrow formation of crust (e.g. [Pons&Viganò 2019](#)) ($T < \sim 10^9 - 10^{10}$ K)
- cooling $\rightarrow T < \sim 10^8$ K
 - \rightarrow “cold catalysed” ($\rightarrow T = 0$)
 - full thermodynamic equilibrium, $P(n_B)$

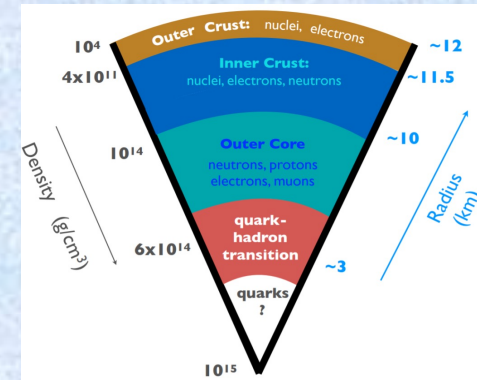
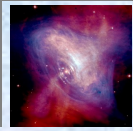
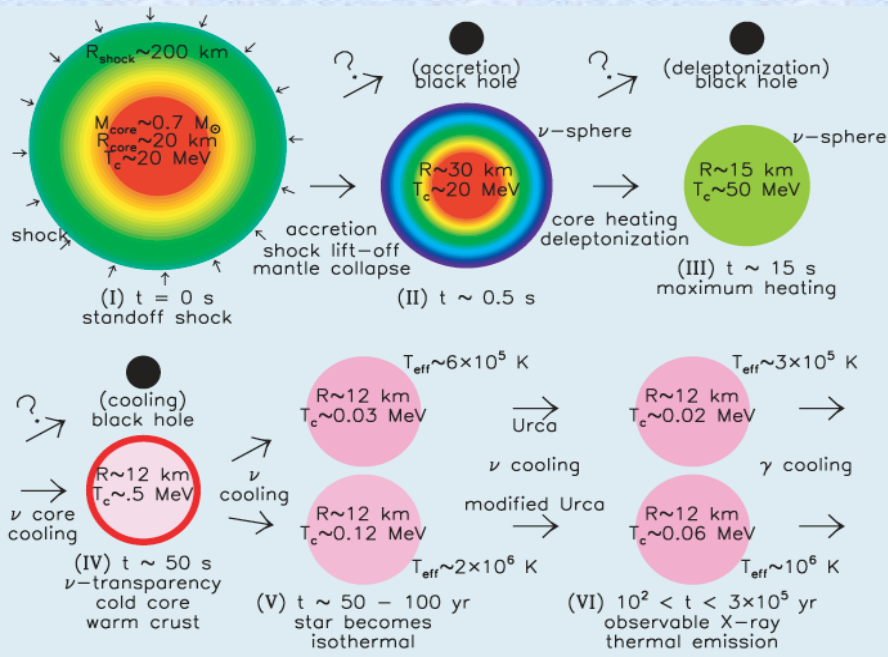


Image Credit: 3G Science White Paper



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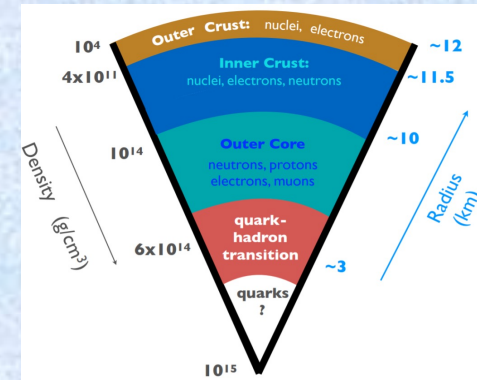


Image Credit: 3G Science White Paper

but: real picture can differ from cold catalysed one

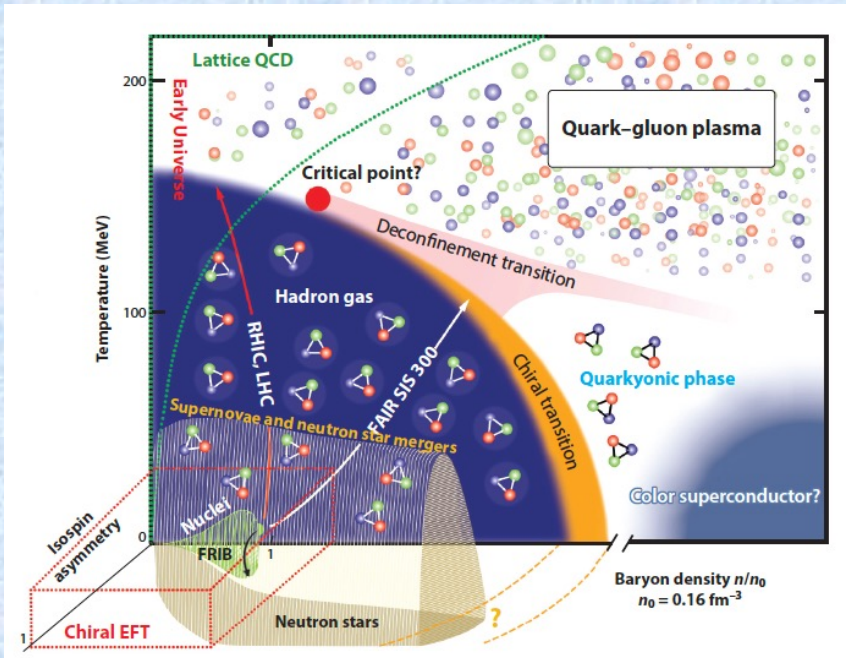
* **PNS** ($\rightarrow T > 0, P(n_B, T)$ if beta equilibrium)

N.B.: “General purpose” EoSs $P(n_B, T, Y_q)$, accretion & B effects not addressed here

(see e.g. Oertel et al., Rev. Mod. Phys. 2017; Burgio & Fantina, ASSL Springer 2018)



Probing extreme conditions in NSs



Drischler et al., Ann. Rev. Nucl. Part. Sci. 71, 403 (2021)

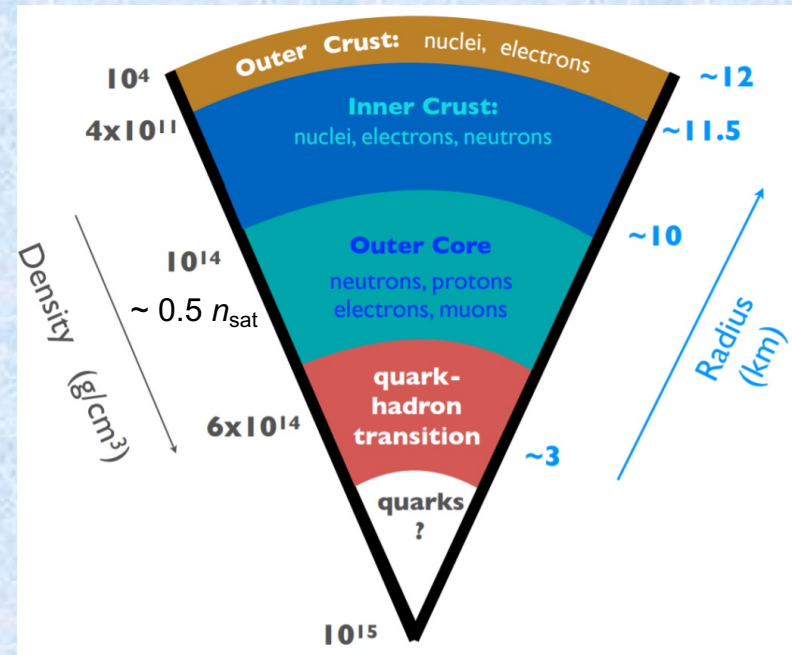


Image Credit: 3G Science White Paper

different states of matter spanned in NSs

→ inhomogeneous (crust), “pasta” phase, homogeneous (core), “exotic” particles (?) + superfluidity, (strong) magnetic field, etc.

→ Not all conditions can be probed in terrestrial labs → theoretical models !
→ Consistent description very challenging

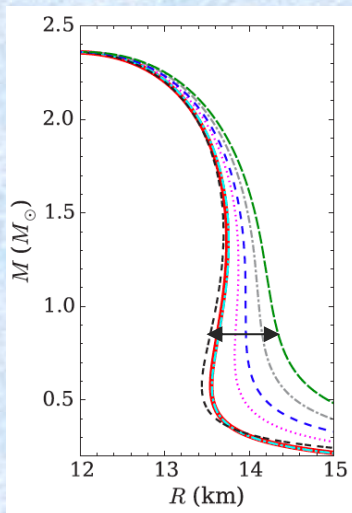
N.B.: T = 0 picture OK for cold isolated NSs and binary (pre-merger) NSs



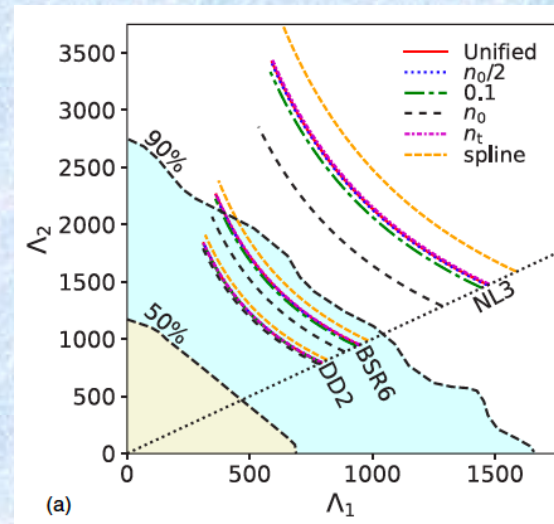
Why a unified treatment ?

Unified treatment of inhomogeneous & homogeneous matter
→ same nuclear model employed in different regions of star

- Challenging because of wide range of thermodynamic conditions
- Challenging because different states of matter
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016)



Suleiman et al., PRC 104, 015801 (2021)

see also Ferreira&Providencia 2020



Thermodynamically consistent and unified EoSs for astro modelling & inference

analyses (but not many available, e.g. Douchin&Haensel 2001; Fantina et al. 2013; Raduta&Gulminelli 2015; Viñas et al. 2021; Pearson et al. 2018; Grams et al. 2022; Xia et al. 2022; Scurto et al. 2024; see CompOSE database)



Micro to macro through modelling

Microphysics (inputs)

(e.g. EoS, nuclear processes)

Astrophysical (macrophysics)

hydrodynamic/static models

(simulations)

constraint

prediction

Nuclear theory (with model parameters)

constraint

prediction

Nuclear physics Experiments

e.g. nuclear masses, resonances, decay rates, ...

constraints

prediction

Astrophysical observations

(e.g. GW, NS masses, light curves,...)

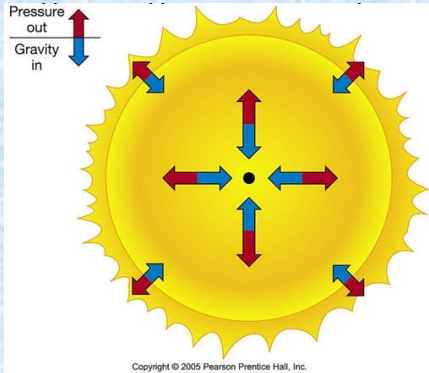


EoS \leftrightarrow NS (static) observables (1)

- **TOV $\rightarrow M(R)$** (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel, Potekhin, Yakovlev, Springer 2007)

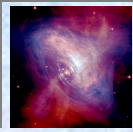
$$\frac{dP(r)}{dr} = -\frac{G\rho(r)\mathcal{M}(r)}{r^2} \left[1 + \frac{P(r)}{c^2\rho(r)} \right] \left[1 + \frac{4\pi P(r)r^3}{c^2\mathcal{M}(r)} \right] \left[1 - \frac{2G\mathcal{M}(r)}{c^2r} \right]^{-1}$$

$$\mathcal{M}(r) = 4\pi \int_0^r \rho(r')r'^2 dr' \quad \text{with b.c. } M(r=0) = 0; \rho(r=0) = \rho_c$$



➡ only EoS $P(\rho)$ is needed !

➡ for each ρ_c (or equivalently P_c) \rightarrow integration $\rightarrow R, M(r=R)$



EoS \leftrightarrow NS (static) observables (1)

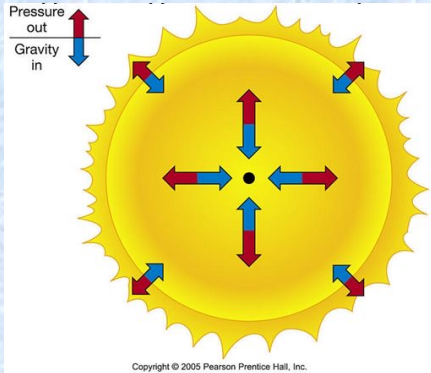
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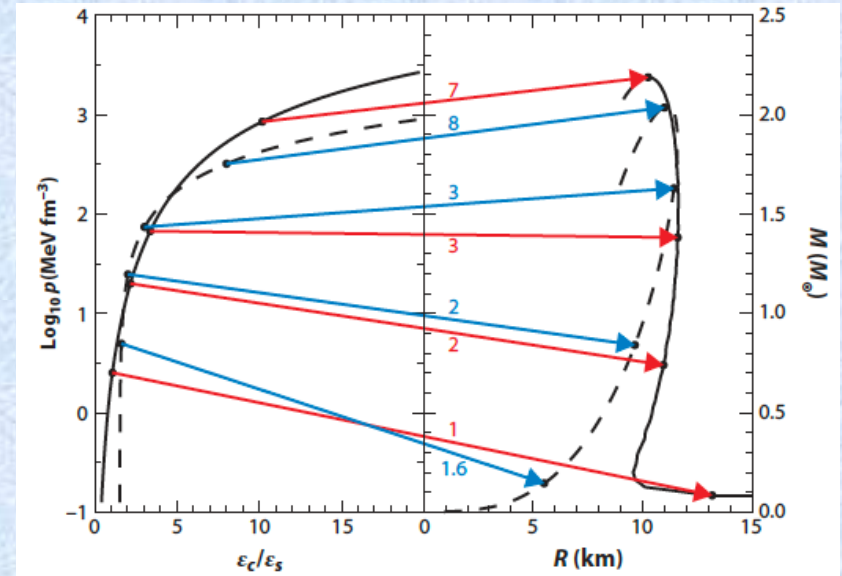
➡ only EoS $P(\rho)$ is needed !

➡ for each ρ_c (or equivalently P_c) \rightarrow integration $\rightarrow R, M(r=R)$



➡ GR \rightarrow direct correspondence
EoS \leftrightarrow NS static properties

➡ for each $\rho_c \rightarrow$ **rayon R , masse M**
 \rightarrow **tidal deformability Λ**



Lattimer, Annu. Rev. Part. Nucl. Sci. 62, 485 (2012)

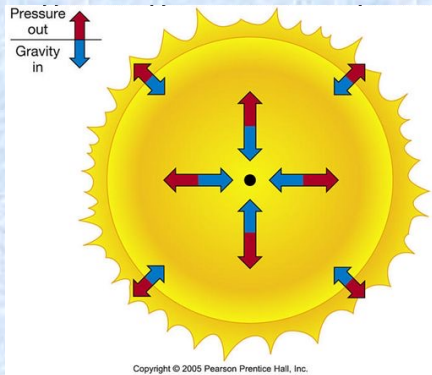


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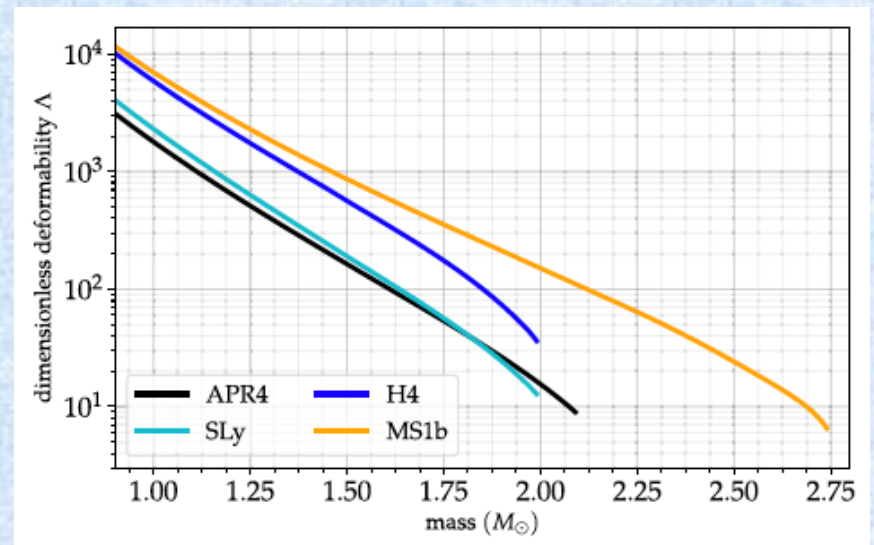
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?
 \rightarrow trace back to EoS and composition?



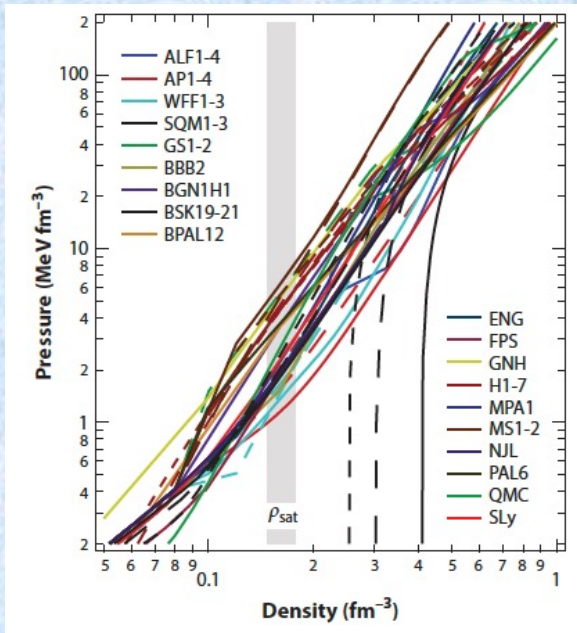
Dietrich et al., Gen. Rel. Gravit. 53, 27 (2021)



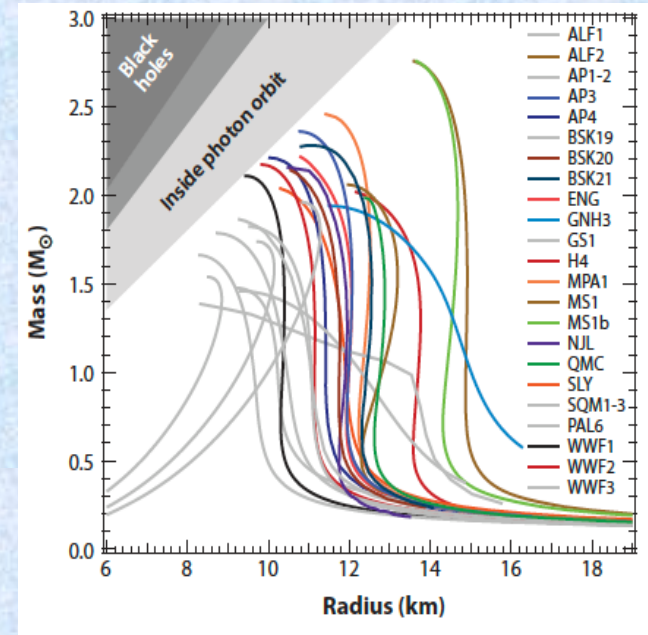
EoS \leftrightarrow NS (static) observables (2)

but:

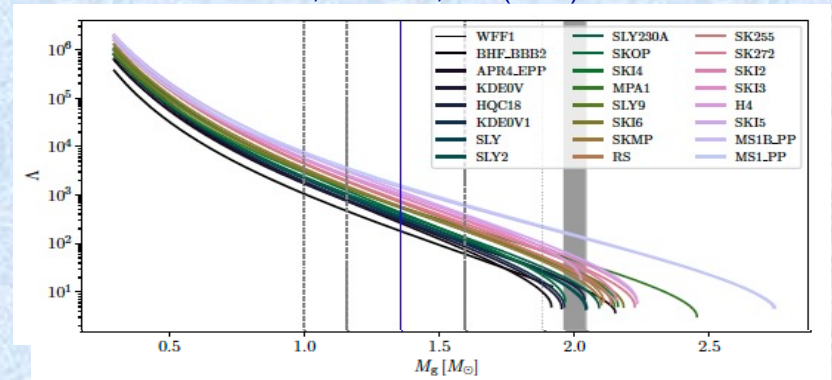
- X EoS model dependent !
- X no ab-initio dense-matter calculations in all regimes \rightarrow phenomenological models
- X composition \leftrightarrow EoS \rightarrow $M(R)$?



Ozel & Freire, ARAA 54, 401 (2016)



Ozel & Freire, ARAA 54, 401 (2016)

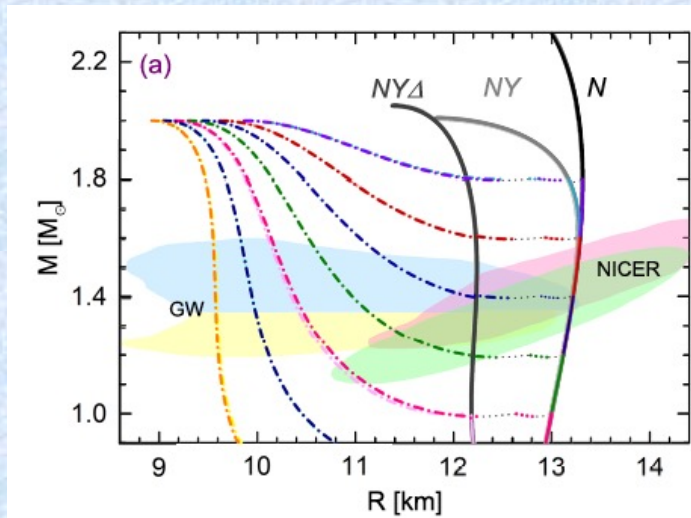


Abbott et al., Class. Quantum Grav, 37, 045006 (2020)

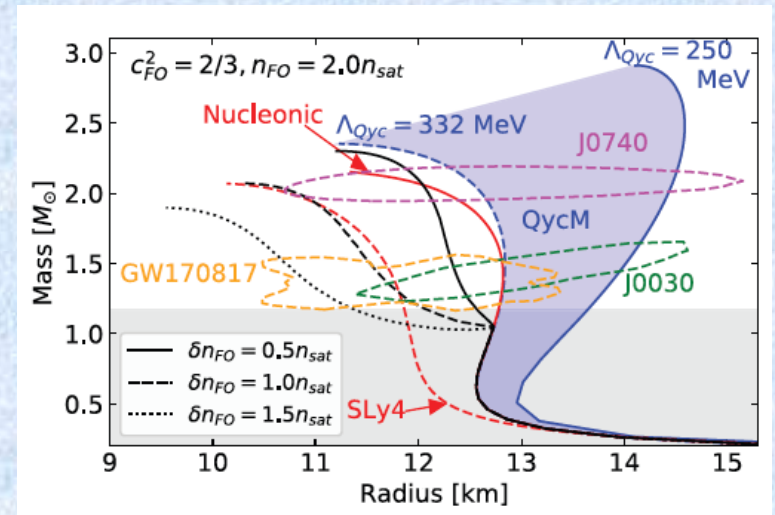


High-density EoS \rightarrow additional d.o.f.?

- Role of “exotic” degrees of freedom? (*not addressed in this talk, see talks Mon, session H*)
Hyperons \rightarrow softer EoS \rightarrow lower M_{\max} (+ reduction of R and Λ for intermediate-mass)
Quarks \rightarrow not clear



Li et al., PRD 101, 063022 (2020)

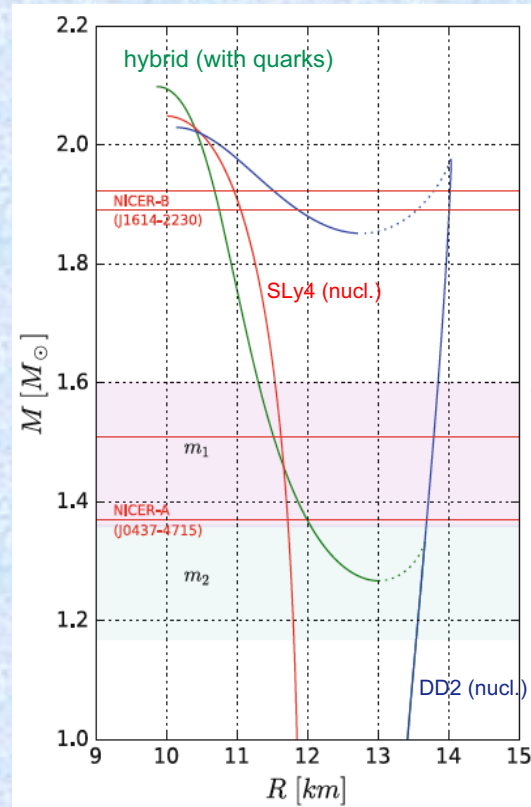


Somasundaram & Margueron, EPL 138, 14002 (2022)



High-density EoS \rightarrow additional d.o.f.?

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- “Masquerade” effect

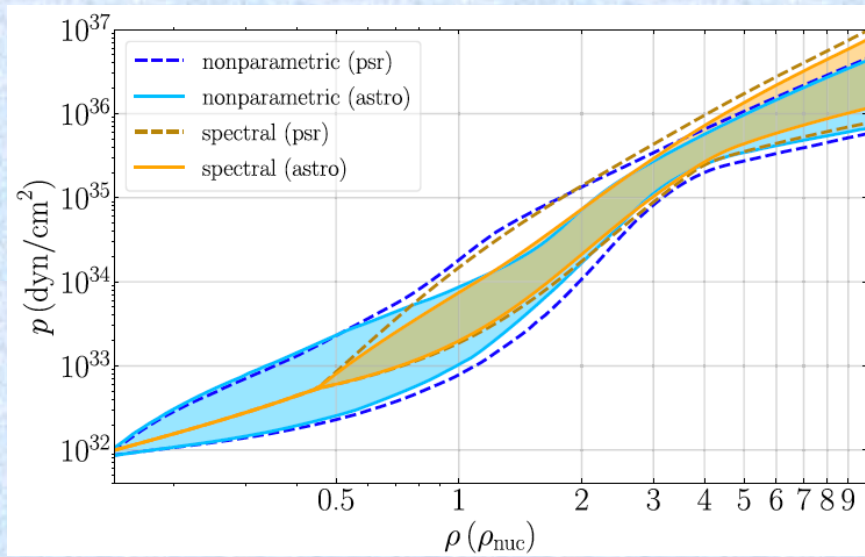


Blaschke & Chamel, *ASSL* 457, 337 (2018);



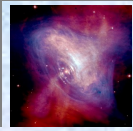
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Hyperons \rightarrow softer EoS \rightarrow lower M_{\max} (+ reduction of R and Λ for intermediate-mass)
Quarks \rightarrow not clear
- “Masquerade” effect
- *Agnostic* (“non-nuclear”) approaches for NS core (e.g. piecewise polytropes, c_s models,...)
(conditioned by astro)



Legred et al. PRD 105, 043016 (2022)

- ✓ powerful \rightarrow no underlying hypotheses
- ✗ what about nuclear physics \rightarrow composition ?
- ✗ often unique (non-consistent) low-density EoS \rightarrow uncertainties underestimated



EoS \leftrightarrow nuclear matter parameters

- Expansion in density and asymmetry around n_{sat} and $\delta = 0$

$$x = (n - n_{\text{sat}})/3n_{\text{sat}}$$

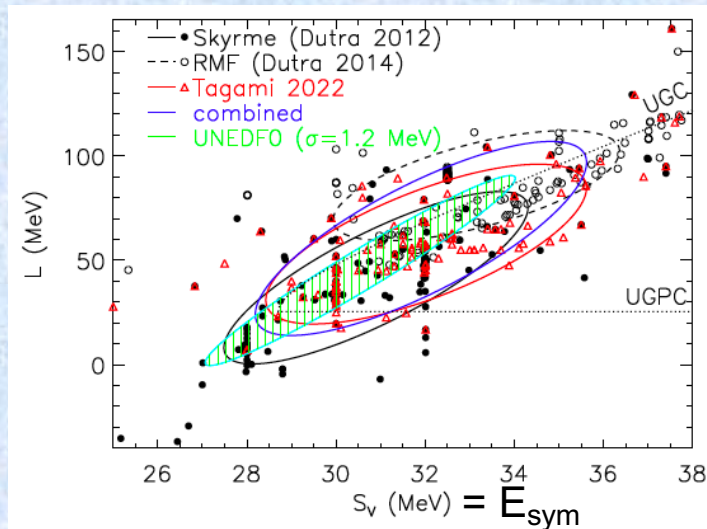
$$\delta = (n_n - n_p)/n$$

$$e_{\text{is}} = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{6}Q_{\text{sat}}x^3 + \dots \rightarrow e_{\text{sat}}(n, \delta=0)$$

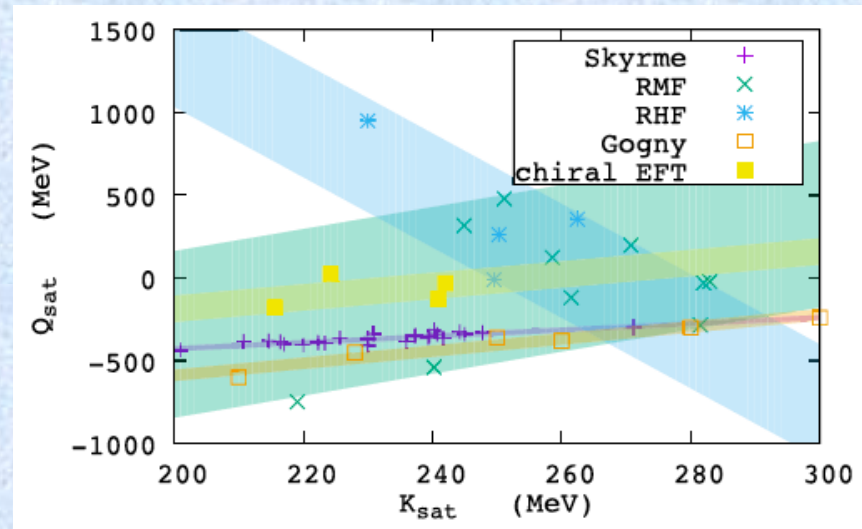
$$e_{\text{iv}} = E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 + \frac{1}{6}Q_{\text{sym}}x^3 + \dots \rightarrow e_{\text{sym}}(n) = e(n, \delta=1) - e(n, \delta=0)$$

→ Nuclear empirical parameters (NEP, bulk)

$$\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, \dots, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, Q_{\text{sym}}, \dots$$



Lattimer, Particles 6, 30 (2023)



Margueron et al., PRC 97, 025805 (2018)



A semi-agnostic approach: meta-model

- **Meta-model (MM)** (Margueron et al., PRC 97, 025805 (2018); also e.g. Lim&Holt 2019, Tsang et al. 2020) → EDF-based but flexible. Based on a Taylor expansion in density and asymmetry.

$$\mathcal{E}_B(n_B, \delta) = \mathcal{E}_{\text{kin}}(n_B, \delta) + \mathcal{V}(n_B, \delta) \quad \mathcal{V}(n_B, \delta) = \sum_{k=0}^N \frac{n_B}{k!} (v_k^{\text{is}} + v_k^{\text{iv}} \delta^2) x^k u_k(x)$$

↑ functions of NEP ↑ zero-density limit ↑ zero-density limit

- For application of MM to NS crust → CLDM

e.g. Carreau et al., EPJA 2019; Dinh Thi et al., A&A 2021; Grams et al., EPJA 2022; Mondal et al., MNRAS 2023; Davis et al., A&A 2024 (for relativistic version, see Char et al., PRD 2023)

- ✓ Vary NEP → parameter exploration (without a priori correlations) → statistical (Bayesian) analysis (see Mon-Wed talks, session H)

$$p_{\text{post}}(\vec{X}) = \mathcal{N} p_{\text{prior}}(\vec{X}) e^{-\chi^2(\vec{X})/2} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X})$$

flat non-informative prior
→ large parameter space

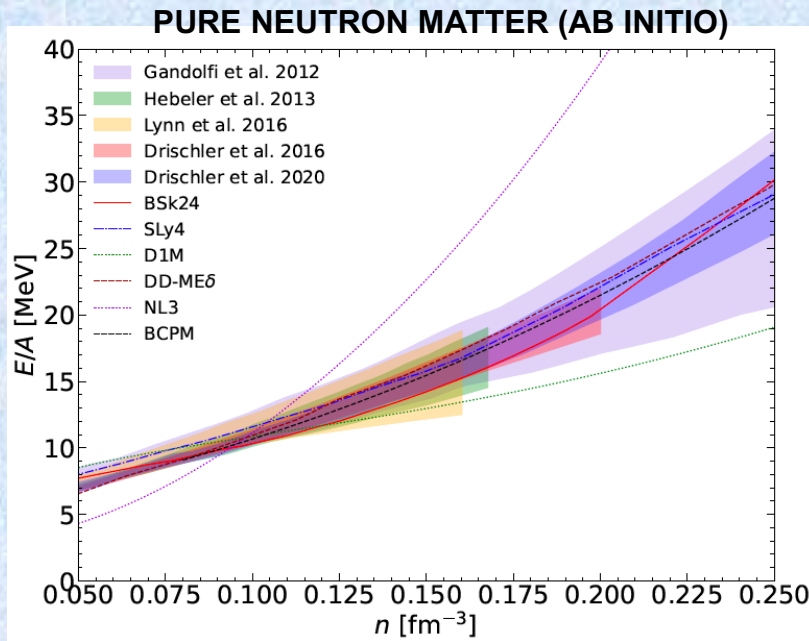
nuclear masses
(AME)

Low-Density filters
→ ab-initio (EFT)

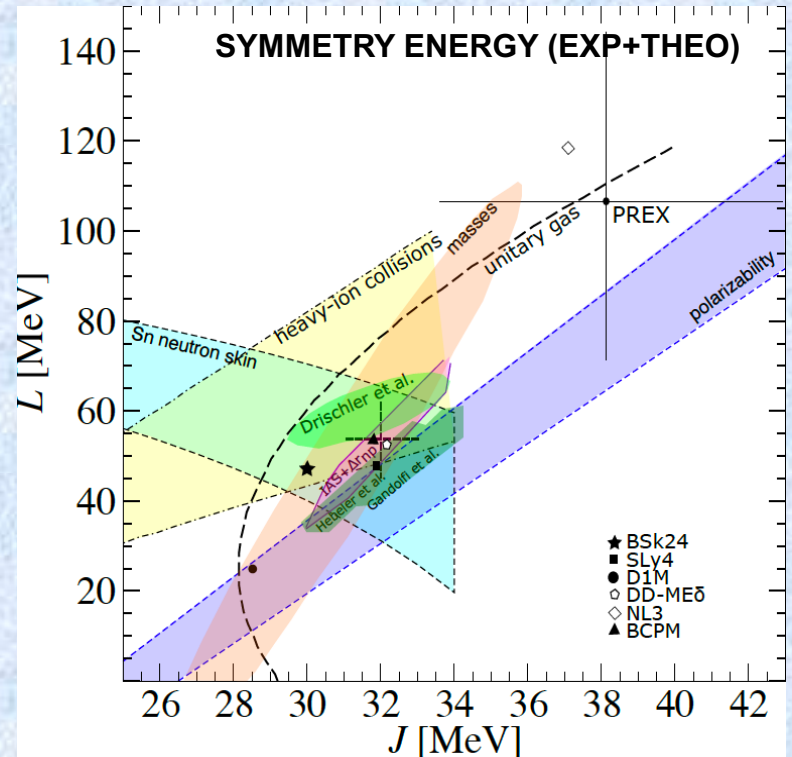
High-Density filters
→ causality, stability,
 $M_{\text{NS,max}}$ (+ NICER, GW)



Constraints from nuclear physics

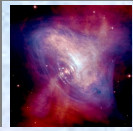


Fantina & Gulminelli, J.Phys. Conf. Ser. 2586, 012112 (2023);
see also Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)



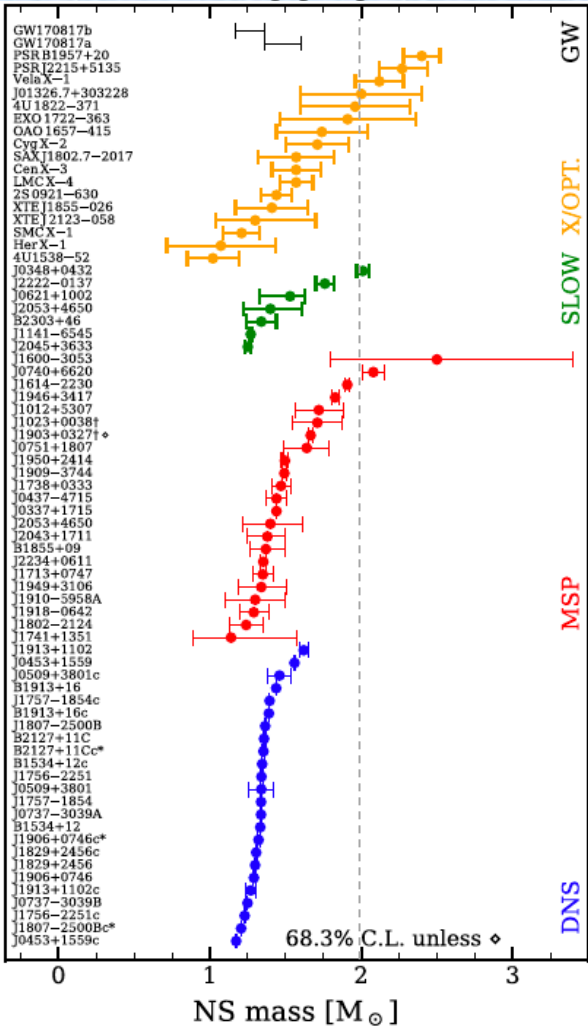
Gulminelli&Fantina, Nucl. Phys. News 31, 9 (2021);
Fantina&Gulminelli, J.Phys.Conf.Ser. 2586, 012112 (2023)

- PNM calculations benchmark / constraints
- not all popular models agree with ab-initio constraints!
- Exp. constraints at “lower” densities & more symmetric matter
- not always “clear” constraints → “tension” (data + modelling)



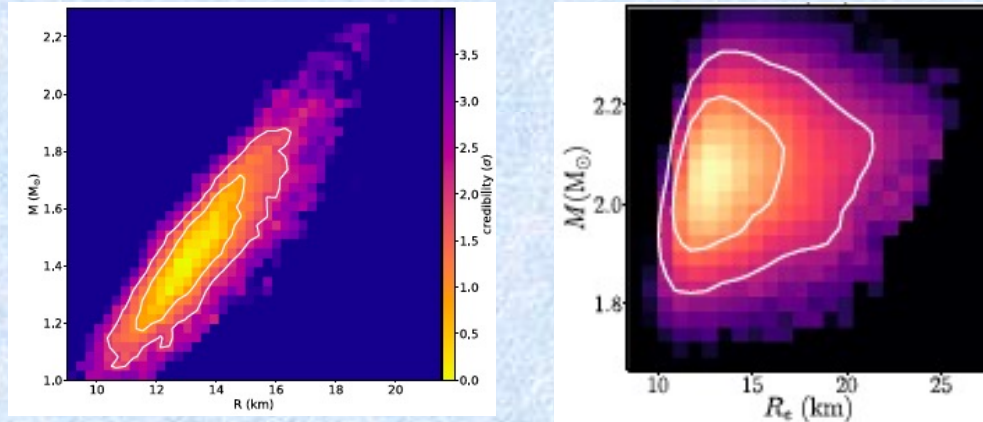
Constraints from astrophysics

MASSES



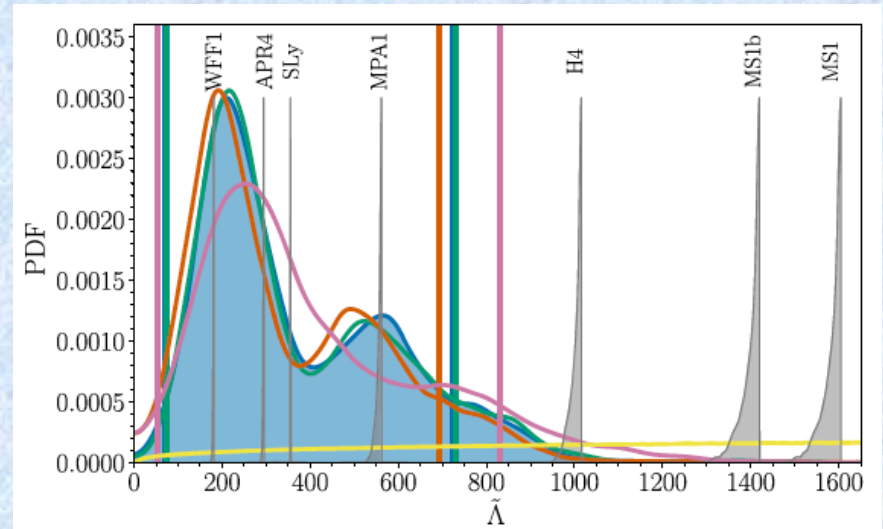
Suleiman et al., PRC 104, 015801 (2021)

RADII

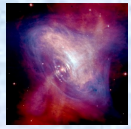


Miller et al., ApJL 887, L24 (2019); ApJL 918, L28 (2021); see also Riley et al., ApJL 2019, 2021

TIDAL DEFORMABILITY



Abbott et al., PRX 9, 011001 (2019)



Outline

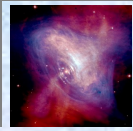
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 - Proto-neutron-star (PNS) crust ($T \neq 0$, beta equilibrium)
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- ❖ Conclusions and open questions

N.B.: In this talk, beta-equilibrated matter
NS static properties



Catalysed NSs: crustal properties

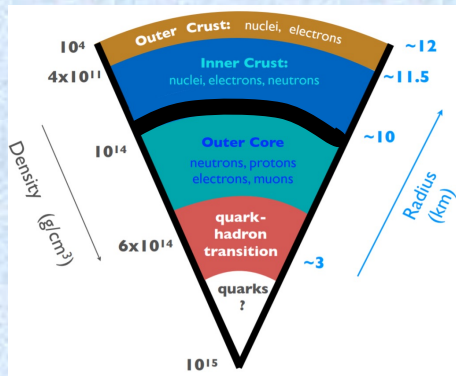
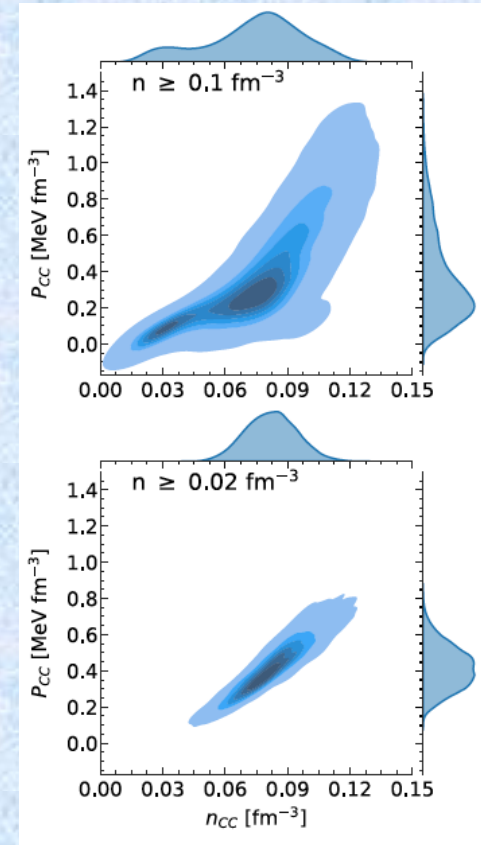


Image Credit: 3G Science White Paper

CRUST-CORE TRANSITION Meta-model + CLDM for crust

	n_{cc}														
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	-0.04	-0.07	0.11	-0.05	-0.02	-0.30	-0.57	-0.15	0.45	-0.15	0.05	0.52	-0.15	-0.04	0.51
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	-0.06	-0.06	0.33	-0.46	0.17	-0.15	-0.29	-0.10	0.39	-0.16	0.06	0.34	-0.11	-0.08	0.33
Prior	0.14	0.09	0.13	-0.18	0.02	0.08	-0.56	0.11	0.20	-0.05	-0.17	0.07	0.29	0.18	0.18
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p
	bulk										surface				

- importance of parameters (*bulk + surface*)
- importance of higher order parameters



- importance of low-density EoS

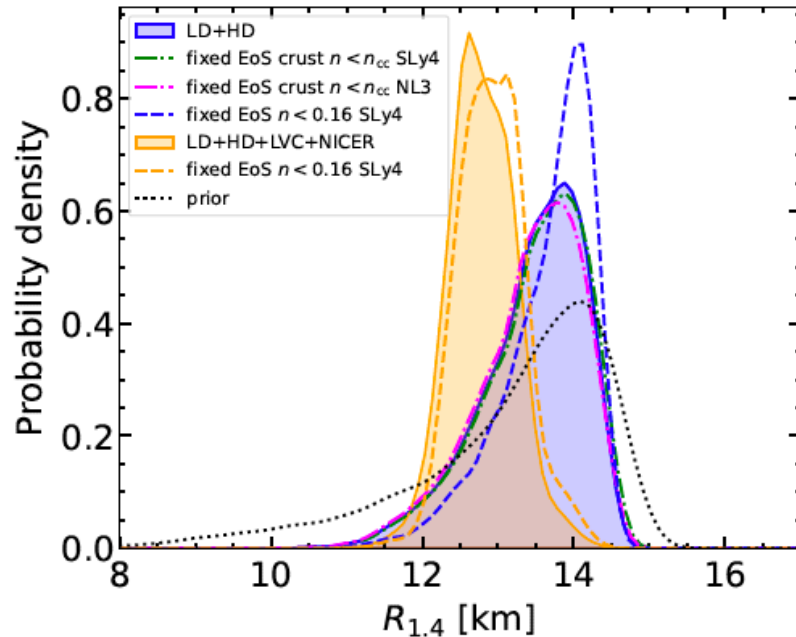
Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021)

see also Carreau et al., PRC 100, 055803 (2019), Balliet et al., ApJ 918, 79 (2021)



Effect of the (non-unified) crust

RADIUS



Davis, Dinh Thi, Fantina, et al., A&A 687, A44 (2024)

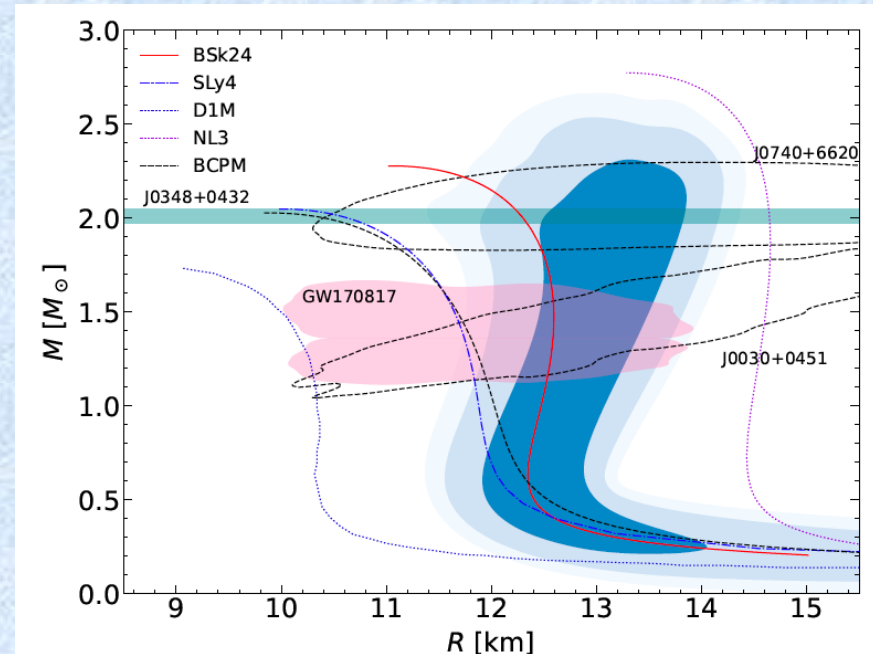
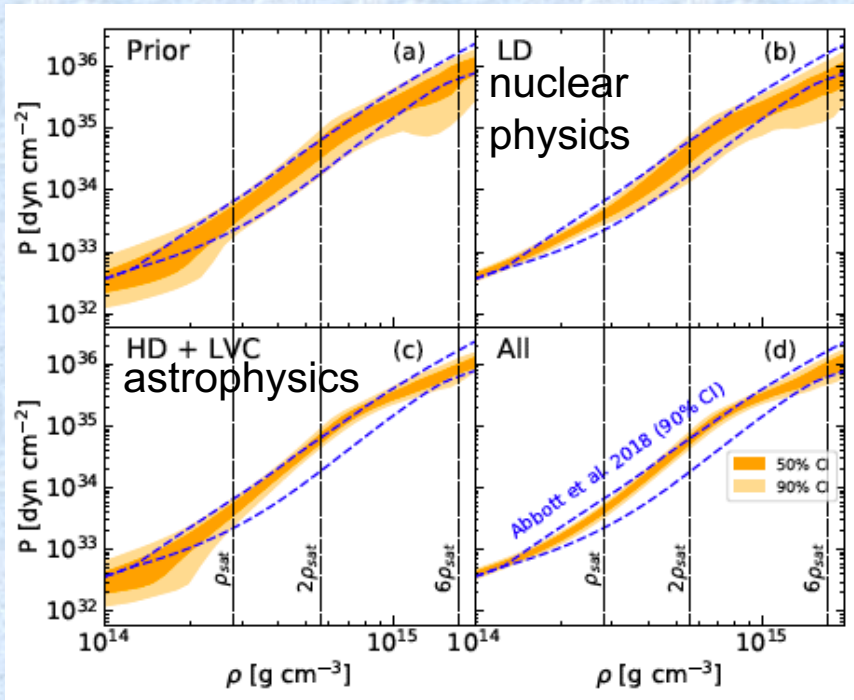
- use of unique crust does not change much averages (~ few %)
 - ok for current GW detectors, but next generation ?
(see also Gamba et al., Class. Quant. Grav. 37, 025008 (2020) → ~ 3%)
- underestimation of uncertainties in non-consistent approach
- quantitative error bars on NS properties can be addressed

→ **CUTER** code to reconstruct a *thermodynamically consistent and unified* low-density EoS from a (high-density) beta-equilibrium EoS

(available for LIGO-Virgo-KAGRA collab. and publicly available on Zenodo)



Catalysed NSs: observables



Gulminelli & Fantina, NPN 31, 9 (2021);

Fantina & Gulminelli, J.Phys. Conf.Ser. 2596, 012112 (2023)

Dinh Thi et al., Universe 7, 373 (2021);

Dinh Thi et al., A&A 654, A114 (2021)

→ posterior compatible with observations, but: some popular models are not !

→ nucleonic hp compatible with observations → observations not yet enough constraining!

similar conclusions in Lim&Holt, EPJA 2019, Malik et al., ApJ 2022

N.B.: Many works within Bayesian analysis trying to constrain NEP

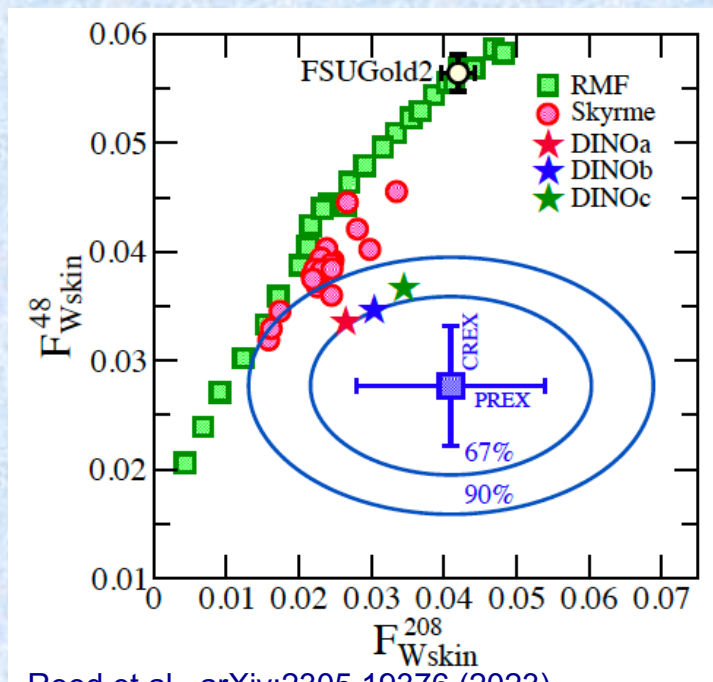
see also Beznogov & Raduta, PRC (2023); Ghosh et al., EPJA (2022); Char et al., PRD (2023); Imam et al., PRD (2024);²²

Zhu et al., ApJ (2023); Huang et al., arXiv:2303.17518, + see talks on Mon, Wed, session H

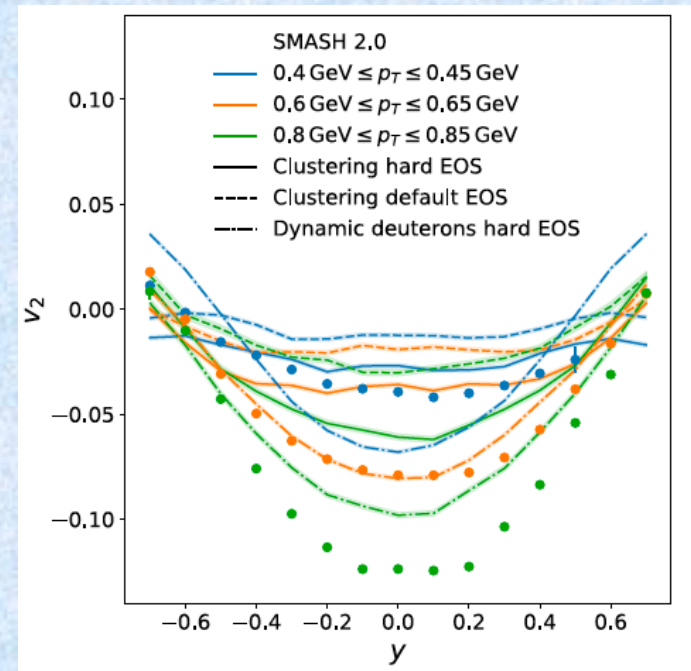


How to discriminate models? (exp)

- ❖ **Nuclear physics (theory + experiments)** → information up to $\sim 1.5 - 2 n_{\text{sat}}$
 - *reduced error bar in neutron skin measurements (e.g. PREX/CREX)*
→ constraints on low-order parameters in isospin sector
 - *constraints at high density e.g. HADES collaboration (transport model vs data)*
→ constraints on higher order parameters



Reed et al., arXiv:2305.19376 (2023)



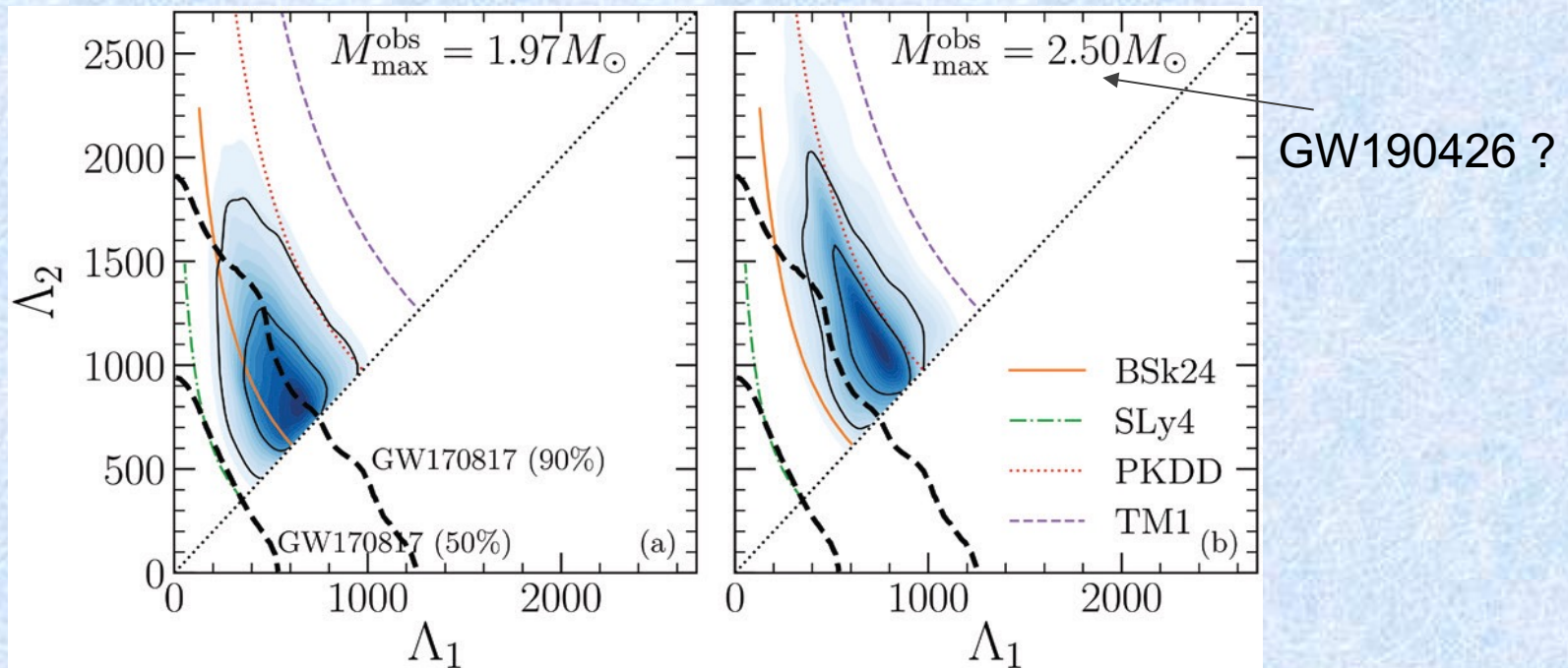
Mohs et al., PRC 105, 034906 (2022)



How to discriminate models ? (astro 1)

❖ Astrophysical observations (multi-messenger)

➤ “Smoking gun” observation



Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021); T. Carreau, PhD Thesis (2020)

→ posterior (nucleonic matter) compatible with observations

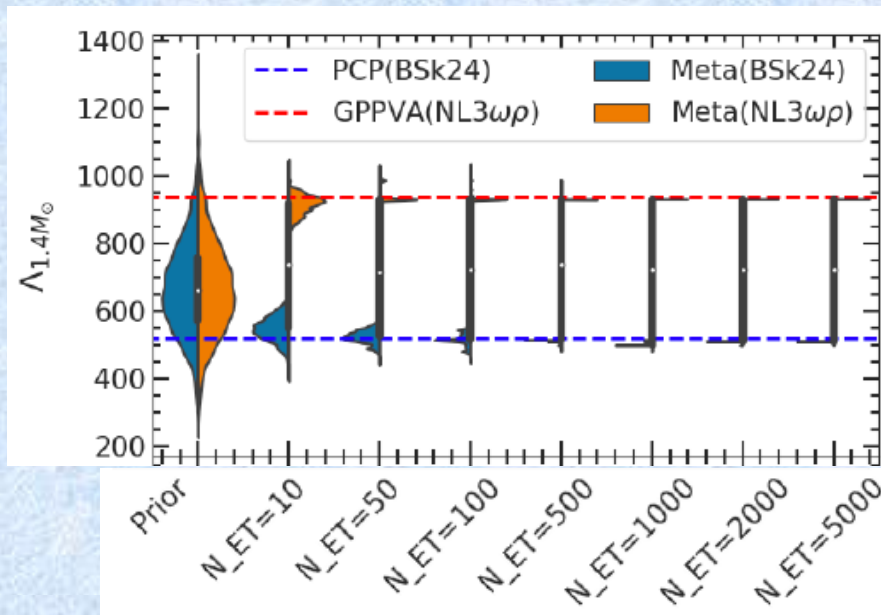
but: if $M_{\max} \sim 2.5 M_{\text{sun}}$ → challenge for nucleonic hypothesis ! → exotica !

→ Nucleonic hp can be used as null hp

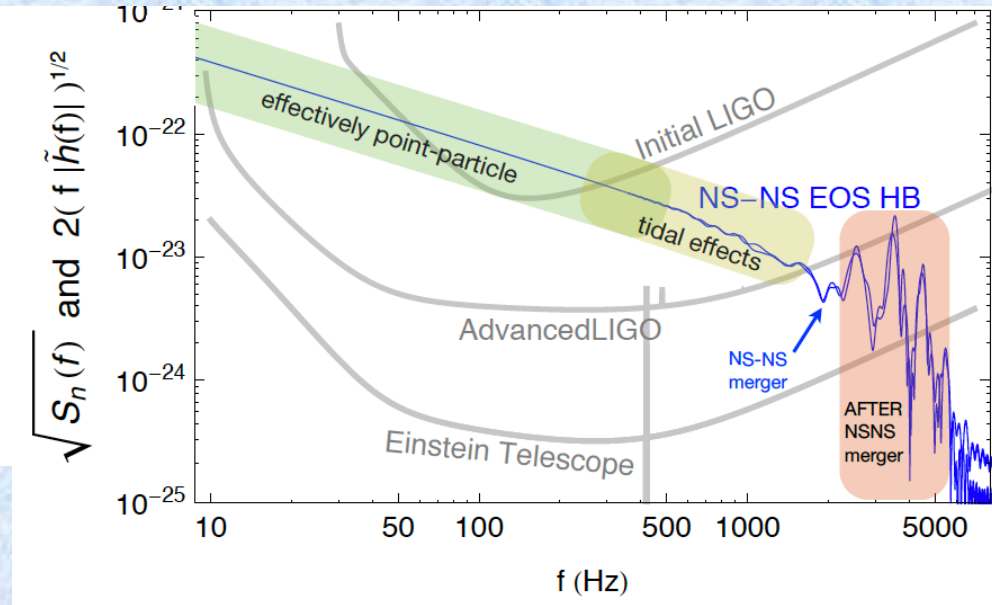


How to discriminate models? (astro 2)

- more and more precise data (e.g. M , R , Λ , ...)
- more sensitive detectors \rightarrow new generation (ET, CE) \rightarrow post-merger

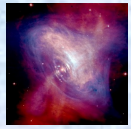


Iacovelli et al., PRD 2023



Read, CGWAS lecture (2015)

- More reliable prediction / interpretation of astrophysical observations
- Better knowledge of dense matter in compact stars :
Phase transition to deconfined matter (quarks, ...) ?
- Astrophysical sites of nucleosynthesis ?



Outline

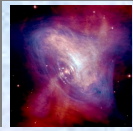
- ❖ Introduction:
 - Neutron-star (NS) properties and equation of state (EoS) modelling and constraints

- ❖ Selected results in :
 - Catalysed (“cold”) NSs ($T = 0$, full equilibrium)
 - EoS and NS observables

 - Proto-neutron-star (PNS) crust ($T \neq 0$, beta equilibrium)
 - multi-component plasma, impurity parameter

- ❖ Conclusions and open questions

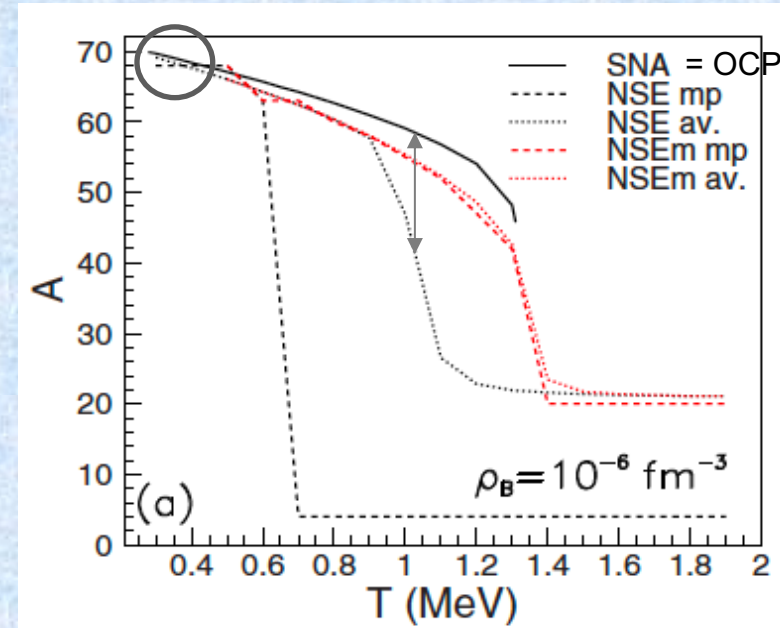
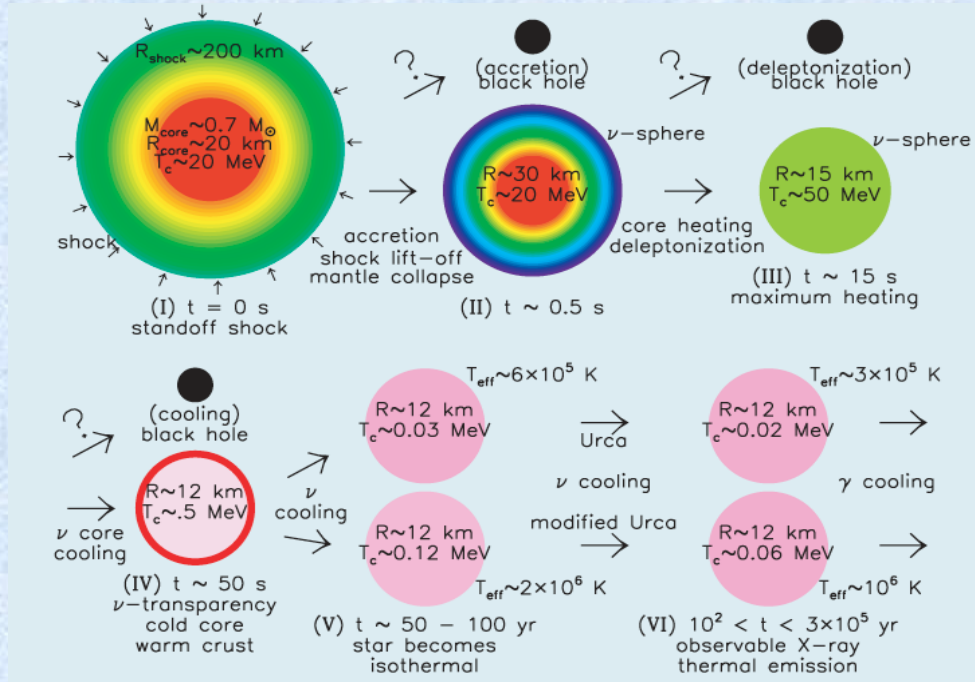
N.B.: In this talk, beta-equilibrated matter
NS static properties



Proto-NS (finite temperature)

NS formation from CCSN

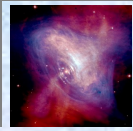
At finite T \rightarrow need to go beyond OCP



Gulminelli & Raduta, PRC 92, 055803 (2015)

Lattimer & Prakash, Science 304, 536 (2004)

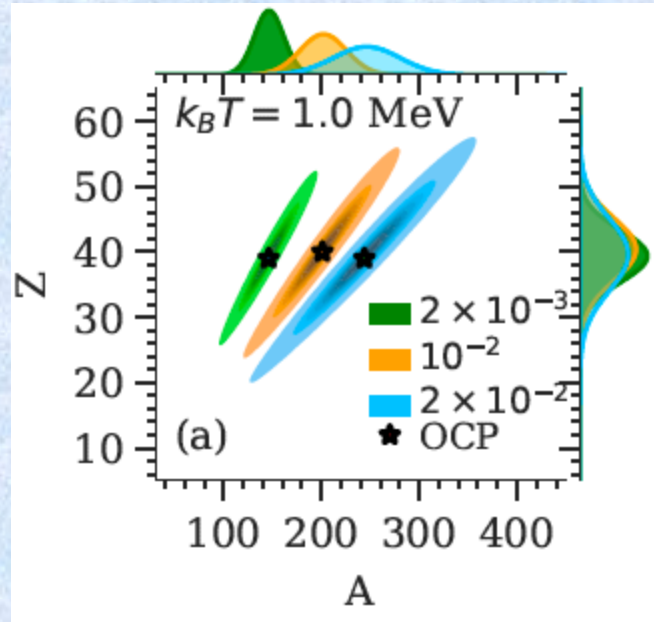
- \rightarrow NS are born hot ($T > 1$ MeV) \rightarrow ensemble of nuclei (MCP) expected
- \rightarrow NS crust crystallises at $T_m \sim 0.1 - 1$ MeV \rightarrow composition of the crust “frozen”
- but: depending on cooling timescales, composition can be frozen at $T > T_m$
- (e.g. Goriely et al., A&A 531, A78 (2011)) or other reactions possible below T_m ?
- (e.g. Potekhin & Chabrier, A&A 645, A102 (2021))



PNS: composition and impurities



1. Composition can be different from $T = 0$ & OCP one !

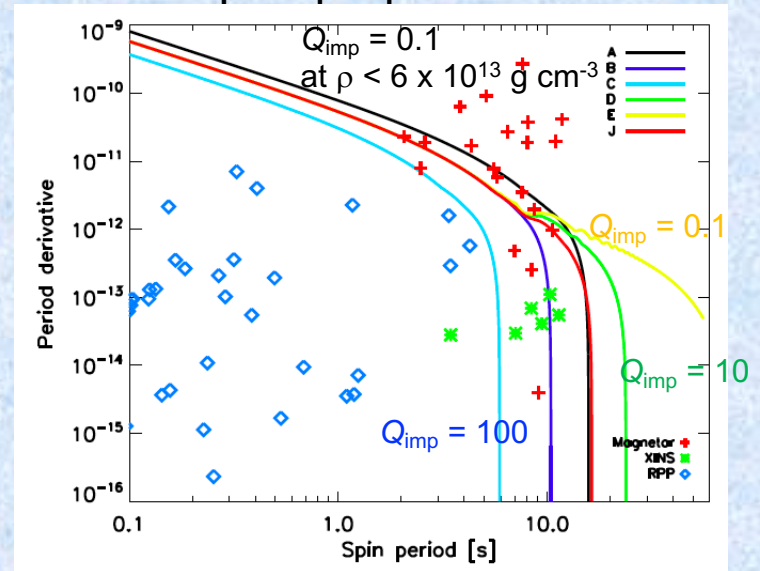


Dinh Thi et al., A&A 677, A174 (2023)
 see also Fantina et al., A&A 633, A149 (2020);
 Carreau et al., A&A 640, A77 (2021)

2. Co-existence of nuclear species
 → “impurity factor” (usually free parameter adjusted on cooling data)

$$Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2$$

→ impact dynamic, magneto-rotational and transport properties

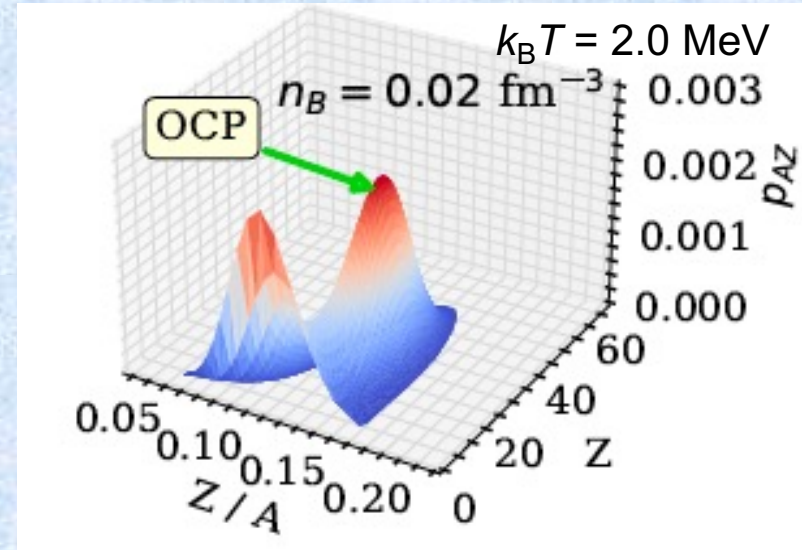
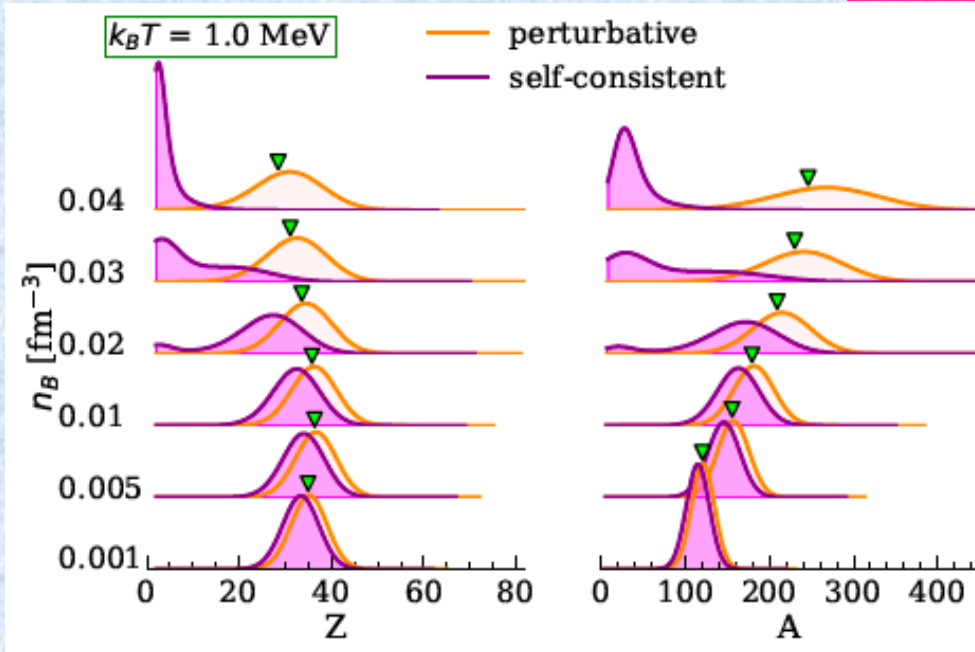


Pons et al., Nature Phys., 9, 431 (2013)
 (see also Viganò et al., MNRAS 2013)



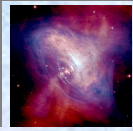
PNS crust (MCP): composition

MM + CLDM



Dinh Thi et al., A&A 677, A174 (2023) – CLDM with BSk24

- OCP less reliable at higher density and temperature
→ (self-consistent) MCP
- appearance of bi-modal distribution → light clusters !
→ importance of light cluster already highlighted, e.g. Typel et al., PRC 2010; Hempel et al., PRC 2011

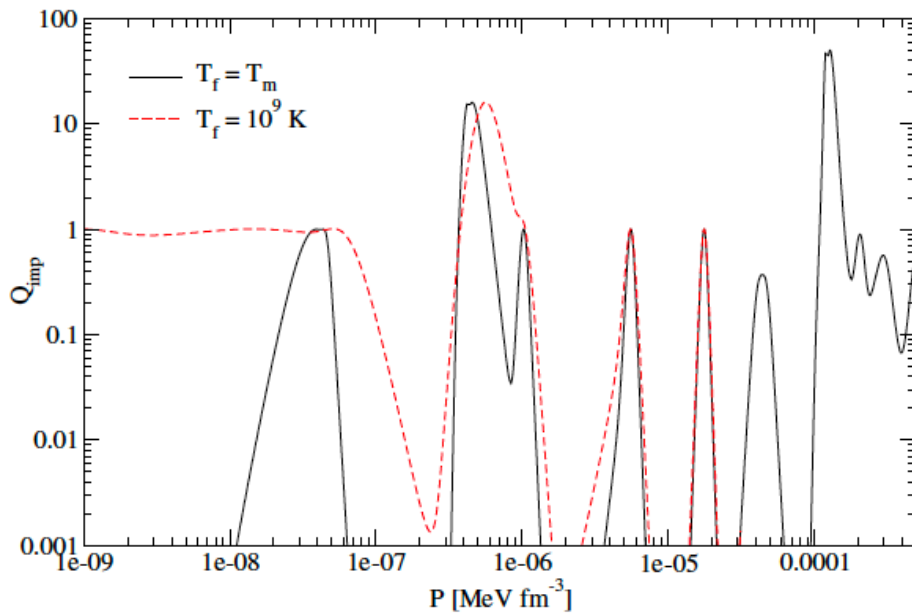


(P)NS crust: impurities

✓ Self-consistent calculations of $Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2$

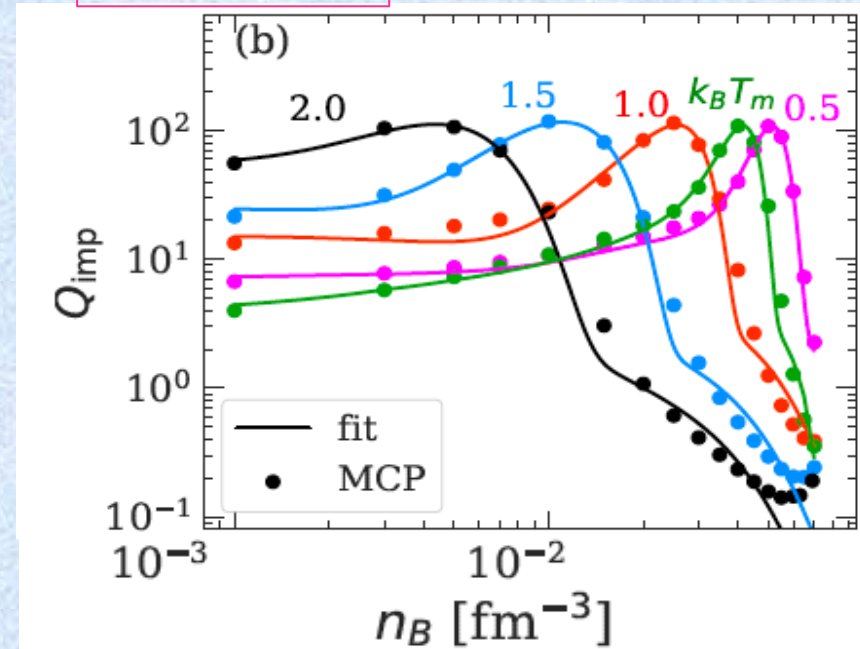
Outer crust

HFB-24 masses



MM + CLDM

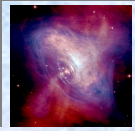
Inner crust



Fantina et al., A&A 633, A149 (2020) + data on CDS

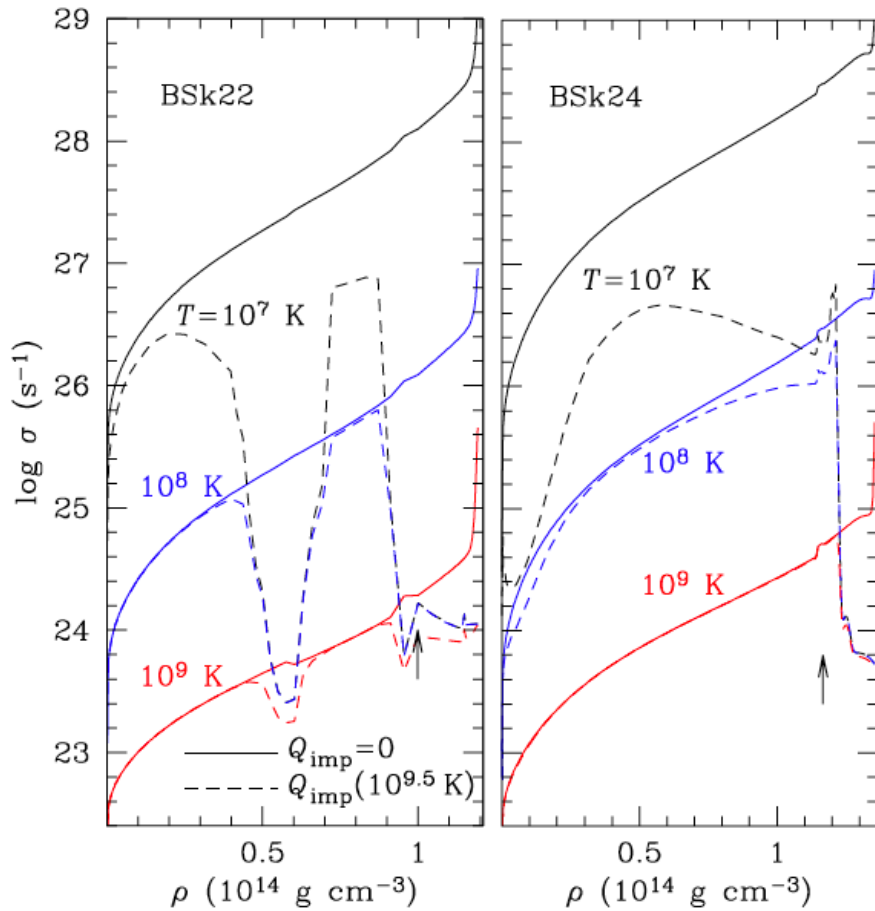
Dinh Thi et al., A&A 677, A174 (2023); see also Carreau et al., A&A 640, A77 (2020) + data on CDS

➤ consistent calculations of Q_{imp} throughout the crust (data available)

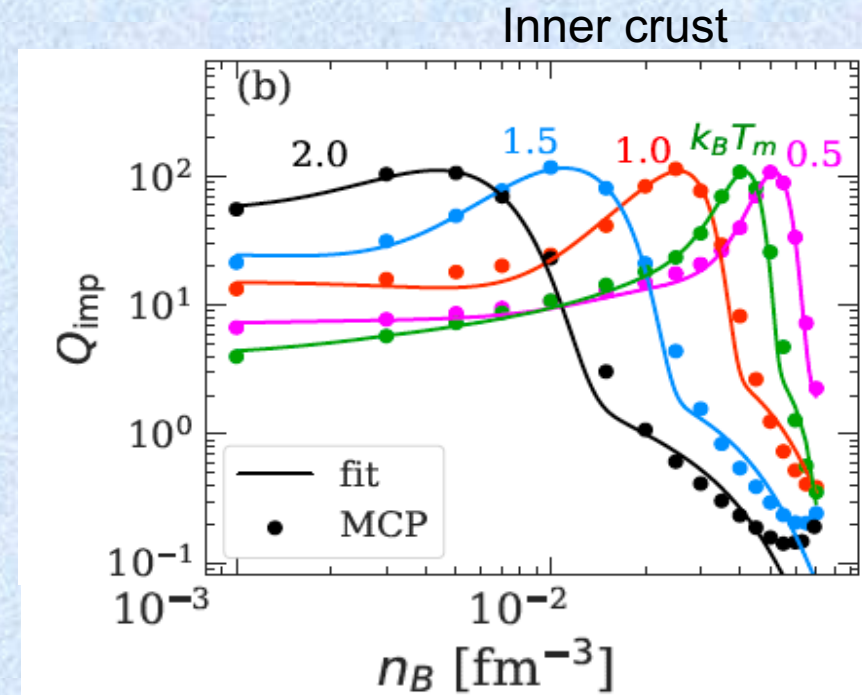


(P)NS crust: impurities

✓ Self-consistent calculations of $Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2$



Potekhin & Chabrier, A&A 645, A102 (2021)



Dinh Thi et al., A&A 677, A174 (2023); see also Carreau et al., A&A 640, A77 (2020) + data on CDS

- consistent calculations of Q_{imp} throughout the crust
- ➔ impact on transport coefficient/properties



Conclusions & open questions

- ❖ Nuclear inputs needed for neutron-star modelling → extrapolation of data / theory
 - ❖ Nuclear physics + astrophysics → constraints on EoS but still hard to discriminate
 - ➔ ✓ need of (microscopic) reliable theoretical model when no data
 - ✓ need of experimental data to calibrate the models
 - ✓ need of (more precise / numerous) astrophysical observations
 - ❖ Importance of MCP treatment at finite temperature
-



Conclusions & open questions

- ❖ Nuclear inputs needed for neutron-star modelling → extrapolation of data / theory
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-
- Extrapolation from raw data → **model dependence of the constraints**
 - Lab. exper. mostly “low” density (\sim saturation density), low T probed; matter in astro sites different from lab → **extrapolation to astro conditions (high T and density, asymmetry, charge neutral) ?**
 - Uncertainties in high-density EoS → **blurring of different effects ?**
 - Astro simulations vs microphysics inputs → **uncertainties, consistency of inputs and relative effects of microphysics inputs in astro modelling ?**
→ systematic studies / bayesian analysis needed



Thank you