Probing properties of dense matter with neutron stars

Anthea F. Fantina (anthea.fantina[AT]ganil.fr)
Outline

- Introduction:
  - Neutron-star (NS) properties and equation of state (EoS) modelling and constraints

- Selected results in:
  - Catalysed (“cold”) NSs ($T = 0$, full equilibrium)
    - EoS and NS observables
  - Proto-neutron-star (PNS) crust ($T \neq 0$, beta equilibrium)
    - multi-component plasma, impurity parameter

- Conclusions and open questions

**N.B.:** In this talk, beta-equilibrated matter
NS static properties
NS (isolated): formation

- NS born hot, $T \sim 10^{10}-10^{11}$ K $\sim$ 1 – tens MeV
- after few tens of sec – mins
  → beta equilibrium (e.g. Cameliò et al. 2017)
  → formation of crust (e.g. Pons & Viganò 2019)
    ($T < \sim 10^{9}-10^{10}$ K)
- cooling → $T < \sim 10^8$ K
  → “cold catalysed” ($\rightarrow T = 0$)
  full thermodynamic equilibrium, $P(n_B)$


Image Credit: 3G Science White Paper
NS (isolated): formation

- NS born hot, $T \sim 10^{10}-10^{11}$ K $\sim 1 –$ tens MeV
- after few tens of sec – mins
  $\rightarrow$ beta equilibrium (e.g. Camellio et al. 2017)
  $\rightarrow$ formation of crust (e.g. Pons&Viganò 2019)
  ($T < \sim 10^9 – 10^{10}$ K)
- cooling $\rightarrow T < \sim 10^8$ K
  $\rightarrow$ “cold catalysed” ($\rightarrow T = 0$)
  full thermodynamic equilibrium, $P(n_B)$


Image Credit: 3G Science White Paper

but: real picture can differ from cold catalysed one

$\star$ PNS ($\rightarrow T > 0$, $P(n_B, T)$ if beta equilibrium)

N.B.: “General purpose” EoSs $P(n_B, T, Y_q)$, accretion & B effects not addressed here

(see e.g. Oertel et al., Rev. Mod. Phys. 2017; Burgio & Fantina, ASSL Springer 2018)
Probing extreme conditions in NSs

Different states of matter spanned in NSs → inhomogeneous (crust), “pasta” phase, homogeneous (core), “exotic” particles (?) + superfluidity, (strong) magnetic field, etc.

→ Not all conditions can be probed in terrestrial labs → theoretical models!

→ Consistent description very challenging

N.B.: T = 0 picture OK for cold isolated NSs and binary (pre-merger) NSs


Image Credit: 3G Science White Paper

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Why a unified treatment?

*Unified* treatment of inhomogeneous & homogeneous matter
→ same nuclear model employed in different regions of star

- Challenging because of wide range of thermodynamic conditions
- Challenging because different states of matter
- But: essential to avoid spurious non-physical effects in numerical modelling

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**Thermodynamically consistent and unified EoSs for astro modelling & inference analyses** (but not many available, e.g. Douchin&Haensel 2001; Fantina et al. 2013; Raduta&Gulminelli 2015; Viñas et al. 2021; Pearson et al. 2018; Grams et al. 2022; Xia et al. 2022; Scurto et al. 2024; see CompOSE database)

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Fortin et al., PRC 94, 035804 (2016)
Suleiman et al., PRC 104, 015801 (2021)
see also Ferreira&Providencia 2020
Micro to macro through modelling

**Microphysics (inputs)**
(e.g. EoS, nuclear processes)

**Nuclear theory** (with model parameters)

**Nuclear physics Experiments**
e.g. nuclear masses, resonances, decay rates, ...

**Astrophysical (macrophysics)**
hydrodynamic/static models
(simulations)

**Astrophysical observations**
e.g. GW, NS masses, light curves, ...

Constraint
Prediction
Constraint
Prediction
EoS $\leftrightarrow$ NS (static) observables (1)

- **TOV $\rightarrow M(R)$** (Tolmann 1939; Oppenheimer&Volkoff 1939; see also Haensel, Potekhin, Yakovlev, Springer 2007)

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{r^2} \left[ 1 + \frac{P(r)}{c^2 \rho(r)} \right] \left[ 1 + \frac{4\pi P(r)r^3}{c^2 M(r)} \right] \left[ 1 - \frac{2GM(r)}{c^2 r} \right]^{-1}$$

$$M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$$

with b.c. $M(r=0) = 0$; $\rho(r=0) = \rho_c$

- only EoS $P(\rho)$ is needed!
- for each $\rho_c$ (or equivalently $P_c$) $\rightarrow$ integration $\rightarrow R$, $M(r = R)$

**N.B.:** GR in slow rotation limit w/o magnetic field!
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GR $\rightarrow$ direct correspondence
EoS $\leftrightarrow$ NS static properties
- for each $\rho_c$ $\rightarrow$ rayon $R$, masse $M$
  $\rightarrow$ tidal deformability $\Lambda$

N.B.: GR in slow rotation limit w/o magnetic field!

**EoS ↔ NS (static) observables (1)**

- **TOV → \( M(R) \)** (Tolman 1939; Oppenheimer&Volkoff 1939; see also Haensel, Potekhin, Yakovlev, Springer 2007)

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\frac{dP(r)}{dr} = -\frac{G\rho(r)M(r)}{r^2} \left[ 1 + \frac{P(r)}{c^2\rho(r)} \right] \left[ 1 + \frac{4\pi P(r)r^3}{c^2M(r)} \right] \left[ 1 - \frac{2GM(r)}{c^2r} \right]^{-1}
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- only EoS \( P(\rho) \) is needed!
- for each \( \rho_c \) (or equivalently \( P_c \)) → integration → \( R, M(r = R) \)

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**GR → direct correspondence**

**EoS ↔ NS static properties**

- for each \( \rho_c \) → rayon \( R \), masse \( M \)
- tidal deformability \( \Lambda \)

? → trace back to EoS and composition?

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N.B.: GR in slow rotation limit w/o magnetic field!
EoS ↔ NS (static) observables (2)

but:

- EoS model dependent!
- no ab-initio dense-matter calculations in all regimes → phenomenological models
- composition ↔ EoS → $M(R)$?

Ozel & Freire, ARAA 54, 401 (2016)

Abbott et al., Class. Quantum Grav, 37, 045006 (2020)
High-density EoS $\rightarrow$ additional d.o.f.?

- Role of “exotic” degrees of freedom? *(not addressed in this talk, see talks Mon, session H)*
  - Hyperons $\rightarrow$ softer EoS $\rightarrow$ lower $M_{\text{max}}$ (+ reduction of $R$ and $\Lambda$ for intermediate-mass)
  - Quarks $\rightarrow$ not clear

Li et al., PRD 101, 063022 (2020)

Somasundaram & Margueron, EPL 138, 14002 (2022)
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- “Masquerade” effect

Blaschke & Chamel, ASSL 457, 337 (2018);
High-density EoS $\Rightarrow$ additional d.o.f.?

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- **“Masquerade” effect**

  - **Agnostic (“non-nuclear”) approaches for NS core** (e.g. piecewise polytropes, $c_s$ models, etc.)
    (conditioned by astro)

  - ✓ powerful $\rightarrow$ no underlying hypotheses
  - ✗ what about nuclear physics $\rightarrow$ composition?
  - ✗ often unique (non-consistent) low-density EoS
    $\rightarrow$ uncertainties underestimated

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Legred et al. PRD 105, 043016 (2022)
EoS $\leftrightarrow$ nuclear matter parameters

- Expansion in density and asymmetry around $n_{\text{sat}}$ and $\delta = 0$
  
  $$e_{\text{is}} = E_{\text{sat}} + \frac{1}{2} K_{\text{sat}} x^2 + \frac{1}{6} Q_{\text{sat}} x^3 + \ldots \quad \Rightarrow e_{\text{sat}}(n, \delta = 0)$$

  $$e_{\text{iv}} = E_{\text{sym}} + L_{\text{sym}} x + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{6} Q_{\text{sym}} x^3 + \ldots \quad \Rightarrow e_{\text{sym}}(n) = e(n, \delta = 1) - e(n, \delta = 0)$$

$\Rightarrow$ Nuclear empirical parameters (NEP, bulk)

$$X_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, \ldots, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, Q_{\text{sym}}, \ldots$$

see e.g. Bulgac et al., PRC 97, 044313 (2018), Margueron et al., PRC 97, 025805 (2018), Carreau et al, EPJA 55, 188 (2019), Tews et al., EPJ A 55, 97 (2019), Dinh Thi et al., A&A 654, A114 (2021), Dinh Thi et al., EPJA 57, 296 (2021); Essick et al., PRC 104, 065804 (2021), …
A semi-agnostic approach: meta-model

- **Meta-model (MM)** (Margueron et al., PRC 97, 025805 (2018); also e.g. Lim&Holt 2019, Tsang et al. 2020) → EDF-based but flexible. Based on a Taylor expansion in density and asymmetry.

\[
\mathcal{E}_B(n_B, \delta) = \mathcal{E}_{\text{kin}}(n_B, \delta) + \mathcal{V}(n_B, \delta) \\
\mathcal{V}(n_B, \delta) = \sum_{k=0}^{N} \frac{n_B}{k!} (v_{ik}^{\text{is}} + v_{ik}^{\text{iv}} \delta^2) x^k u_k(x)
\]

- For application of MM to NS crust → CLDM
e.g. Carreau et al., EPJA 2019; Dinh Thi et al., A&A 2021; Grams et al., EPJA 2022; Mondal et al., MNRAS 2023; Davis et al., A&A 2024 (for relativistic version, see Char et al., PRD 2023)

- Vary NEP → parameter exploration (without a priori correlations) → statistical (Bayesian) analysis (see Mon-Wed talks, session H)

\[
p_{\text{post}}(\vec{X}) = \mathcal{N} p_{\text{prior}}(\vec{X}) e^{-\chi^2(\vec{X})/2} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X})
\]

- Flat non-informative prior → large parameter space
- Nuclear masses (AME)
- Low-Density filters → ab-initio (EFT)
- High-Density filters → causality, stability, \(M_{\text{NS,max}} \) (+ NICER, GW)
Constraints from nuclear physics

PURE NEUTRON MATTER (AB INITIO)

SYMMETRY ENERGY (EXP+THEO)

→ PNM calculations benchmark / constraints
→ not all popular models agree with ab-initio constraints!
→ Exp. constraints at “lower” densities & more symmetric matter
→ not always “clear” constraints → “tension” (data + modelling)

Fantina & Gulminelli, J.Phys. Conf. Ser. 2586, 012112 (2023); see also Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

Constraints from astrophysics

MASSES

RADIIS

TIDAL DEFORMABILITY

NB: most are inferred, not “direct” observations, so model dependent!
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Catalysed NSs: crustal properties

**CRUST-CORE TRANSITION**
Meta-model + CLDM for crust

→ importance of parameters (*bulk + surface*)
→ importance of higher order parameters

→ importance of low-density EoS

**Dinh Thi et al., A&A 654, A114 (2021); EPJA 57, 296 (2021)**

see also Carreau et al., PRC 100, 055803 (2019), Balliet et al., ApJ 918, 79 (2021)
Effect of the (non-unified) crust

RADIUS

- **CUTER** code to reconstruct a *thermodynamically consistent and unified* low-density EoS from a (high-density) beta-equilibrium EoS
  (available for LIGO-Virgo-KAGRA collab. and publicly available on Zenodo)

- use of unique crust does not change much averages (~ few %)
- ok for current GW detectors, but next generation?
  (see also Gamba et al., Class. Quant. Grav. 37, 025008 (2020) → ~ 3%)
- underestimation of uncertainties in non-consistent approach
- quantitative error bars on NS properties can be addressed

**CUTER** = Crust Unified Tool for Equation-of-state Reconstruction

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Catalysed NSs: observables

Dinh Thi et al., Universe 7, 373 (2021); Dinh Thi et al., A&A 654, A114 (2021)

→ posterior compatible with observations, but: some popular models are not!
→ nucleonic hp compatible with observations → observations not yet enough constraining!

similar conclusions in Lim&Holt, EPJA 2019, Malik et al., ApJ 2022

N.B.: Many works within Bayesian analysis trying to constrain NEP
see also Beznogov & Raduta, PRC (2023); Ghosh et al., EPJA (2022); Char et al., PRD (2023); Imam et al., PRD (2024); Zhu et al., ApJ (2023); Huang et al., arXiv:2303.17518, …. + see talks on Mon, Wed, session H
How to discriminate models? (exp)

- **Nuclear physics (theory + experiments)** → information up to ~ 1.5 – 2 \( n_{\text{sat}} \)

- **reduced error bar in neutron skin measurements (e.g. PREX/CREX)** → constraints on low-order parameters in isospin sector

- **constraints at high density e.g. HADES collaboration (transport model vs data)** → constraints on higher order parameters

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Reed et al., arXiv:2305.19376 (2023)

Mohs et al., PRC 105, 034906 (2022)

→ **Better extrapolation of models**
How to discriminate models? (astro 1)

- Astrophysical observations (multi-messenger)
  - “Smoking gun” observation

GW190426?

→ posterior (nucleonic matter) compatible with observations
  but: if $M_{\text{max}} \sim 2.5 \, M_{\odot}$ → challenge for nucleonic hypothesis! → exotica!

→ Nucleonic hp can be used as null hp

How to discriminate models? (astro 2)

- more and more precise data (e.g. $M$, $R$, $\Lambda$, ... )
- more sensitive detectors $\rightarrow$ new generation (ET, CE) $\rightarrow$ post-merger

More reliable prediction / interpretation of astrophysical observations
Better knowledge of dense matter in compact stars: *Phase transition to deconfined matter (quarks, ...)?*  
Astrophysical sites of nucleosynthesis?
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NS static properties
Proto-NS (finite temperature)

NS formation from CCSN

- $R_{\text{shock}} \approx 200$ km
- $M_{\text{core}} \approx 0.7 M_\odot$
- $R_{\nu} \approx 20$ km
- $T_{\nu} \approx 20$ MeV

(II) $t \approx 0.5$ s
- accretion shock lift-off
- mantle collapse
- $R \approx 30$ km
- $T_{\nu} \approx 20$ MeV

(III) $t \approx 15$ s
- maximum heating
- $R \approx 15$ km
- $T_{\nu} \approx 50$ MeV

(IV) $t \approx 50$ s
- $\nu$ core cooling
- $R \approx 12$ km
- $T_{\nu} \approx 0.03$ MeV

(V) $t \approx 50 - 100$ yr
- star becomes isothermal
- $R \approx 12$ km
- $T_{\nu} \approx 0.12$ MeV

(VI) $10^8 < t < 3 \times 10^9$ yr
- observable X-ray thermal emission
- $R \approx 12$ km
- $T_{\nu} \approx 0.06$ MeV

$T_{\text{eff}} \approx 2 \times 10^8$ K

$\rho_B = 10^{-6}$ fm$^{-3}$


At finite $T$ $\rightarrow$ need to go beyond OCP

Gulminelli & Raduta, PRC 92, 055803 (2015)

$\Rightarrow$ NS are born hot ($T > 1$ MeV) $\Rightarrow$ ensemble of nuclei (MCP) expected

$\Rightarrow$ NS crust crystallises at $T_m \sim 0.1 - 1$ MeV $\Rightarrow$ composition of the crust “frozen”

but: depending on cooling timescales, composition can be frozen at $T > T_m$

(e.g. Goriely et al., A&A 531, A78 (2011)) or other reactions possible below $T_m$?

(e.g. Potekhin & Chabrier, A&A 645, A102 (2021))

for a review, see Haensel, Potekhin, Yakovlev (Springer 2007)
PNS: composition and impurities

1. Composition can be different from $T = 0$ & OCP one!

2. Co-existence of nuclear species → "impurity factor" (usually free parameter adjusted on cooling data)

$$Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2$$

→ impact dynamic, magneto-rotational and transport properties

Dinh Thi et al., A&A 677, A174 (2023)
see also Fantina et al., A&A 633, A149 (2020);
Carreau et al., A&A 640, A77 (2021)

see e.g. Schmidt&Shternin, ASSL 457, 455 (2018) for a review; Jones, PRL 83, 3589 (1999), MNRAS 321, 167 (2001), PRL 93, 221101 (2001); Pons et al., Nat. Phys. 9, 431 (2013)
PNS crust (MCP): composition

- OCP less reliable at higher density and temperature
  → (self-consistent) MCP
- appearance of bi-modal distribution → light clusters!
  → importance of light cluster already highlighted, e.g. Typel et al., PRC 2010; Hempel et al., PRC 2011

Dinh Thi et al., A&A 677, A174 (2023) – CLDM with BSk24
(P)NS crust: impurities

✓ **Self-consistent** calculations of \( Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2 \)

Outer crust

- HFB-24 masses

Inner crust

- MM + CLDM

- **consistent calculations of** \( Q_{\text{imp}} \) **throughout the crust** (data available)

Fantina et al., A&A 633, A149 (2020) + data on CDS

Dinh Thi et al., A&A 677, A174 (2023); see also Carreau et al., A&A 640, A77 (2020) + data on CDS
(P)NS crust: impurities

- Self-consistent calculations of $Q_{\text{imp}} = \langle Z^2 \rangle - \langle Z \rangle^2$

**Consistent calculations of $Q_{\text{imp}}$ throughout the crust → impact on transport coefficient/properties**

Dinh Thi et al., A&A 677, A174 (2023); see also Carreau et al., A&A 640, A77 (2020) + data on CDS

Conclusions & open questions

- Nuclear inputs needed for neutron-star modelling → extrapolation of data / theory
- Nuclear physics + astrophysics → constraints on EoS but still hard to discriminate
  - ✓ need of (microscopic) reliable theoretical model when no data
  - ✓ need of experimental data to calibrate the models
  - ✓ need of (more precise / numerous) astrophysical observations
- Importance of MCP treatment at finite temperature
Conclusions & open questions

- Nuclear inputs needed for neutron-star modelling → extrapolation of data / theory
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- Extrapolation from raw data → model dependence of the constraints
- Lab. exper. mostly “low” density (~ saturation density), low $T$ probed; matter in astro sites different from lab → extrapolation to astro conditions (high $T$ and density, asymmetry, charge neutral)?
- Uncertainties in high-density EoS → blurring of different effects?
- Astro simulations vs microphysics inputs → uncertainties, consistency of inputs and relative effects of microphysics inputs in astro modelling? → systematic studies / bayesian analysis needed
Thank you