Nuclear Matter properties from the ladder resummation method

Jose Manuel Alarcón

Universidad de Alcalá

Works done in collaboration with J. A. Oller

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- The interplay between astrophysical observations and nuclear calculations can help in improving our theoretical nuclear models.
- Neutron star equation of state explore a wide range of densities.

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  - In-medium effects
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  - Parameter-free
    - Based on NN observables.

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$$\mathcal{E}_V = \frac{i}{2}\mathrm{Tr}\left(\sum_{n=1}^{\infty}\frac{(t_m L_d)^n}{n}\right) = -\frac{i}{2}\mathrm{Tr}\,\log\left[\mathcal{I} - t_m L_d\right]$$

$$t_m = t_V\sum_{m=0}^{\infty}(L_m t_V)^m = (t_V^{-1} - L_m)^{-1}$$

$$L_m = -i\int\frac{d^3k_1}{(2\pi)^3}\theta(k_F - |\vec{k}_1|)\int\frac{d^4k_2}{(2\pi)^4}\frac{i}{k_2^0 - \frac{|\vec{k}_2|^2}{2m} + i\varepsilon}(2\pi)^4\delta(k_1 + k_2 - 2a)|\vec{k}_1,\vec{k}_2\rangle\langle\vec{k}_1,\vec{k}_2|$$

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- Cutoff and regulator-independent results.

Results

- Unitary limit [J. M. Alarcón, J. A. Oller, AOP 437 (2022)]

$$\mathcal{E}_V = \frac{8k_F^5}{m\pi^3}\int_0^1 s^2 ds \int_0^{\sqrt{1-s^2}} \kappa d\kappa \arctan\left(\frac{a_0 k_F I(s,\kappa)}{1 - a_0 k_F R(s,\kappa)/\pi}\right)$$

$$\xi = 1 - \frac{80}{\pi}\int_0^1 dss^2 \int_0^{\sqrt{1-s^2}} d\kappa\kappa \arctan\left(\frac{\pi I(s,\kappa)}{R(s,\kappa)}\right) = 0.5066$$

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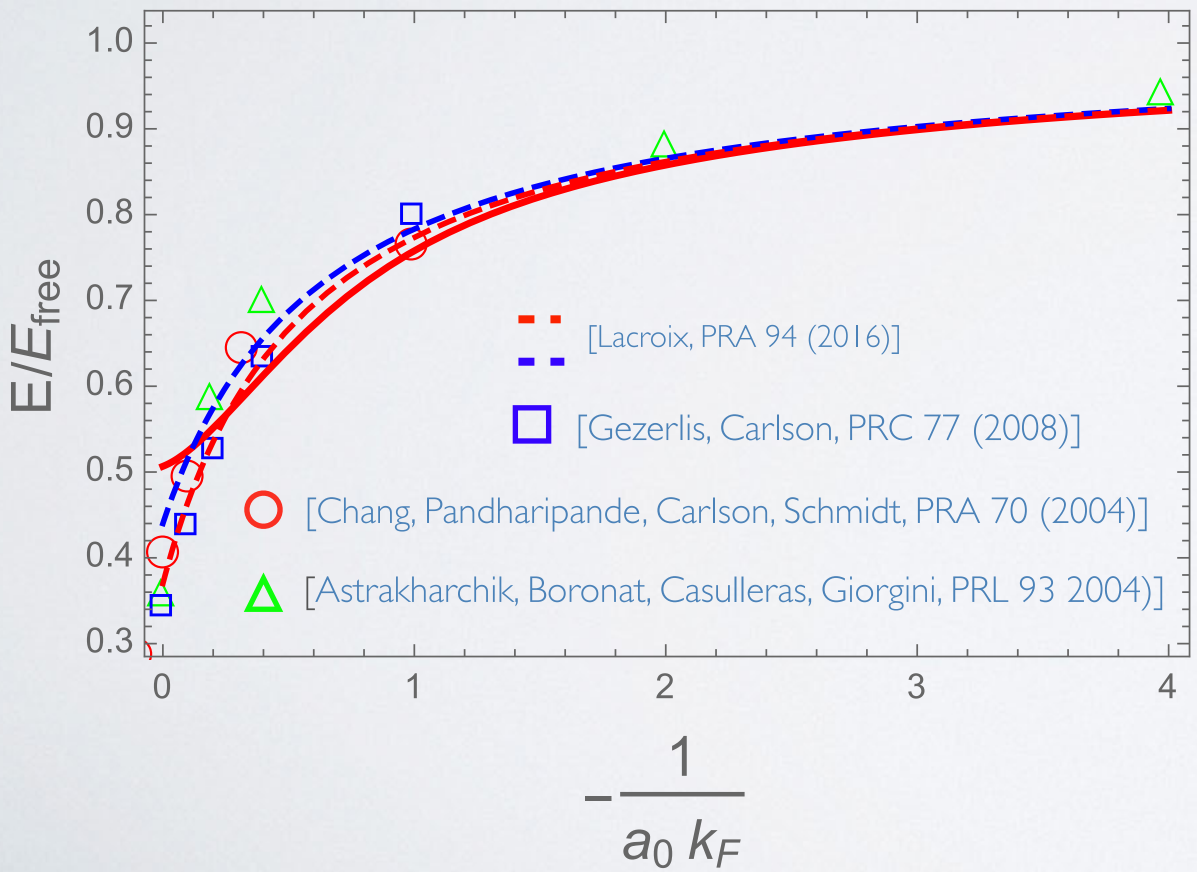
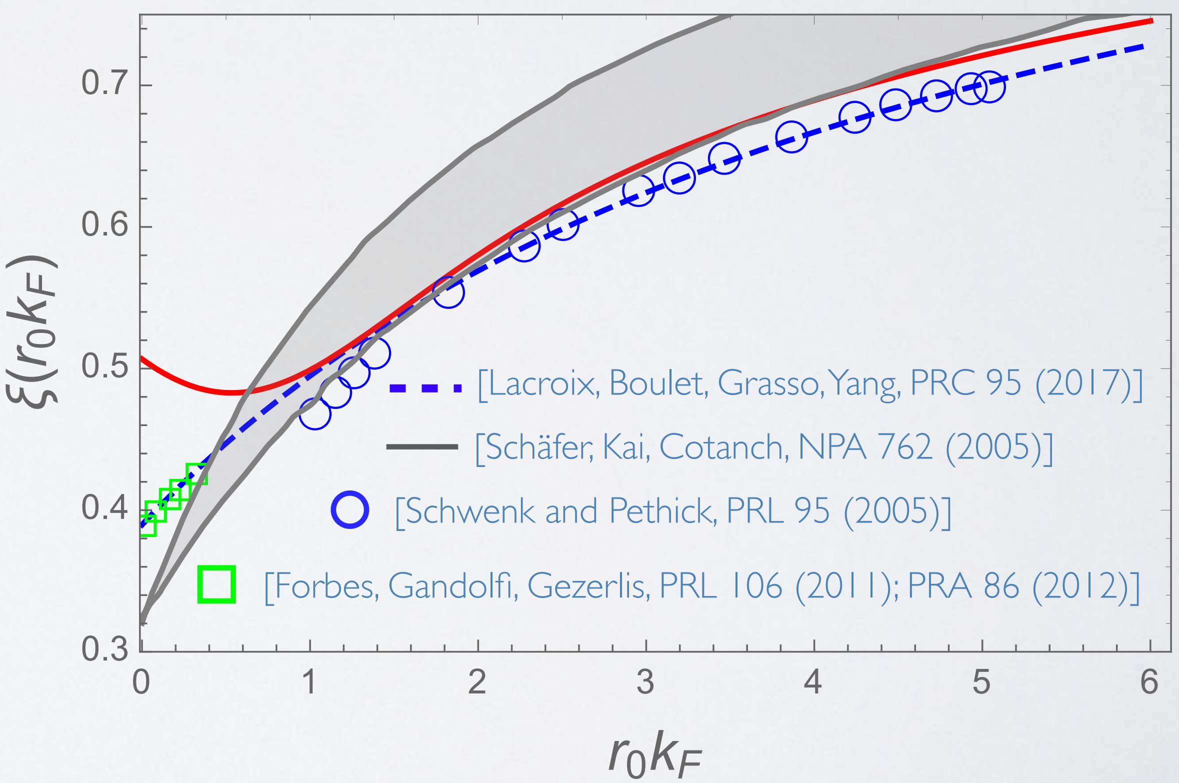
$$\xi(k_F) = 1 - \frac{80}{\pi}\int_0^1 dss^2 \int_0^{\sqrt{1-s^2}} d\kappa\kappa \arctan\left(\frac{\pi I(s,\kappa)}{\pi r_0 k_F \kappa^2/2 + R(s,\kappa)}\right)$$

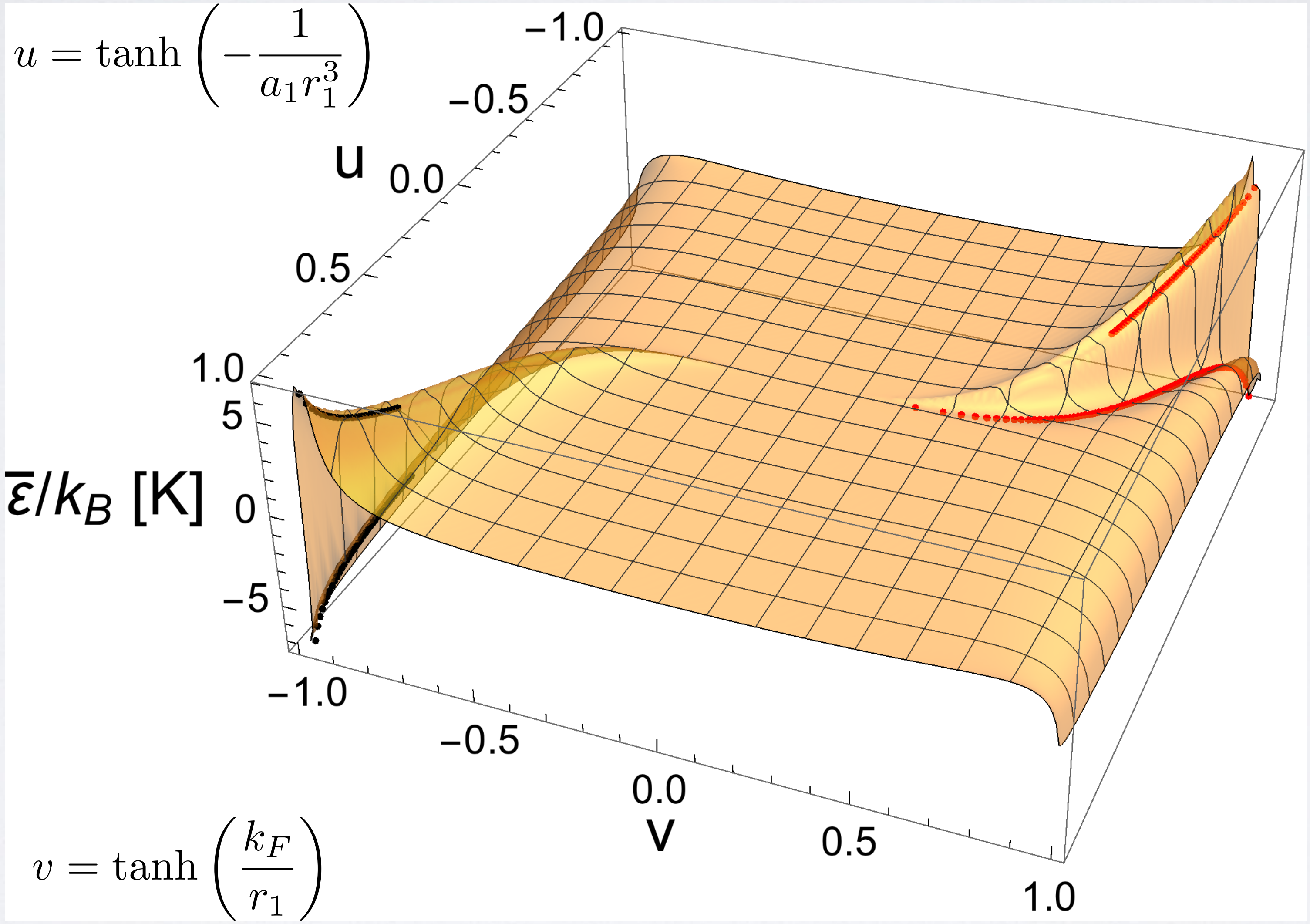




Results

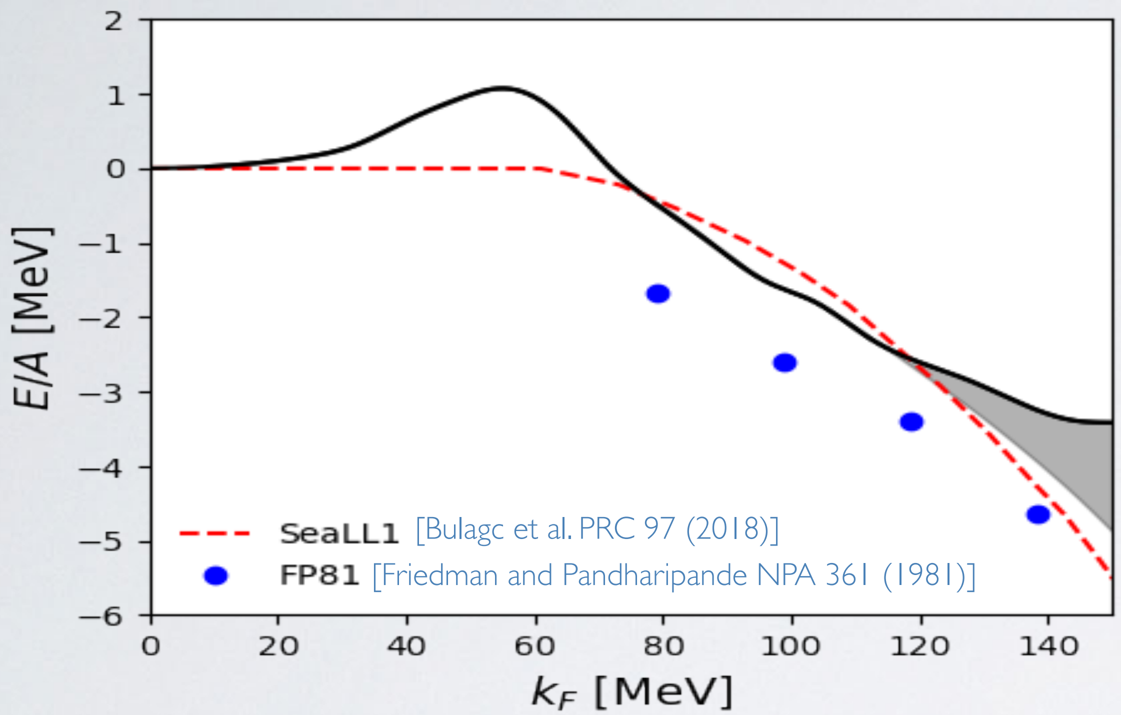
- Cold atoms [J. M. Alarcón, J. A. Oller, PRC 106 (2022)]

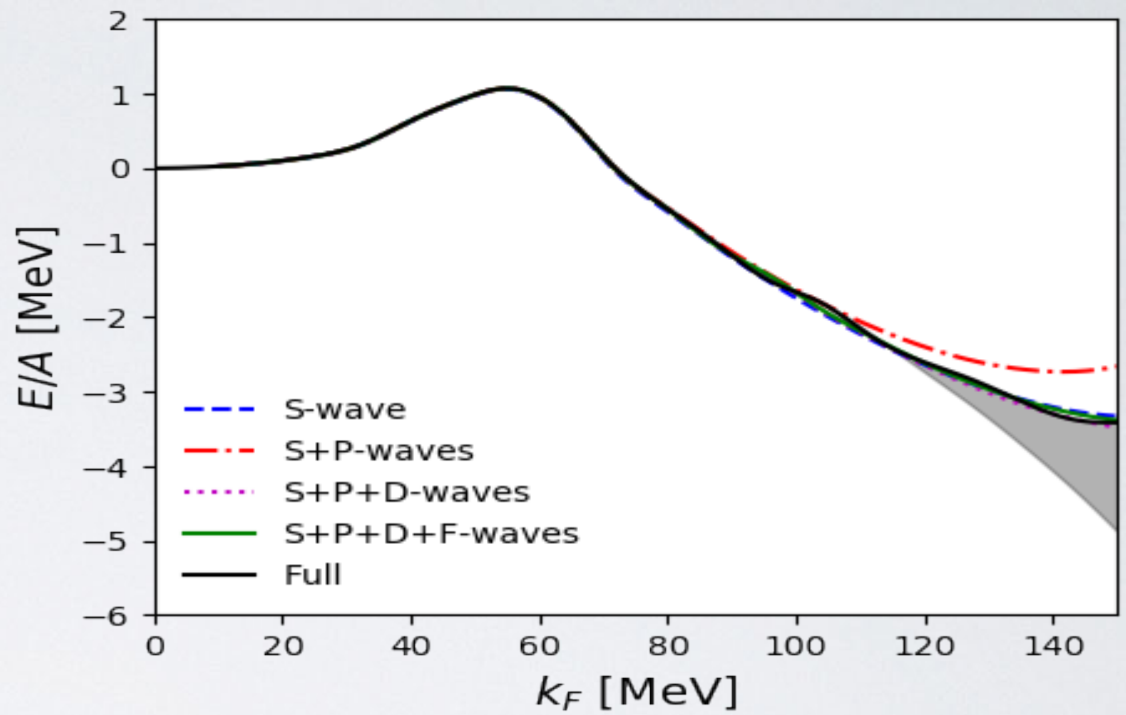
Spin-balanced fermonic quantum liquid with P-wave interactions



Results

- Symmetric Nuclear Matter [J. M. Alarcón, J. A. Oller, PRC 107 (2023)]



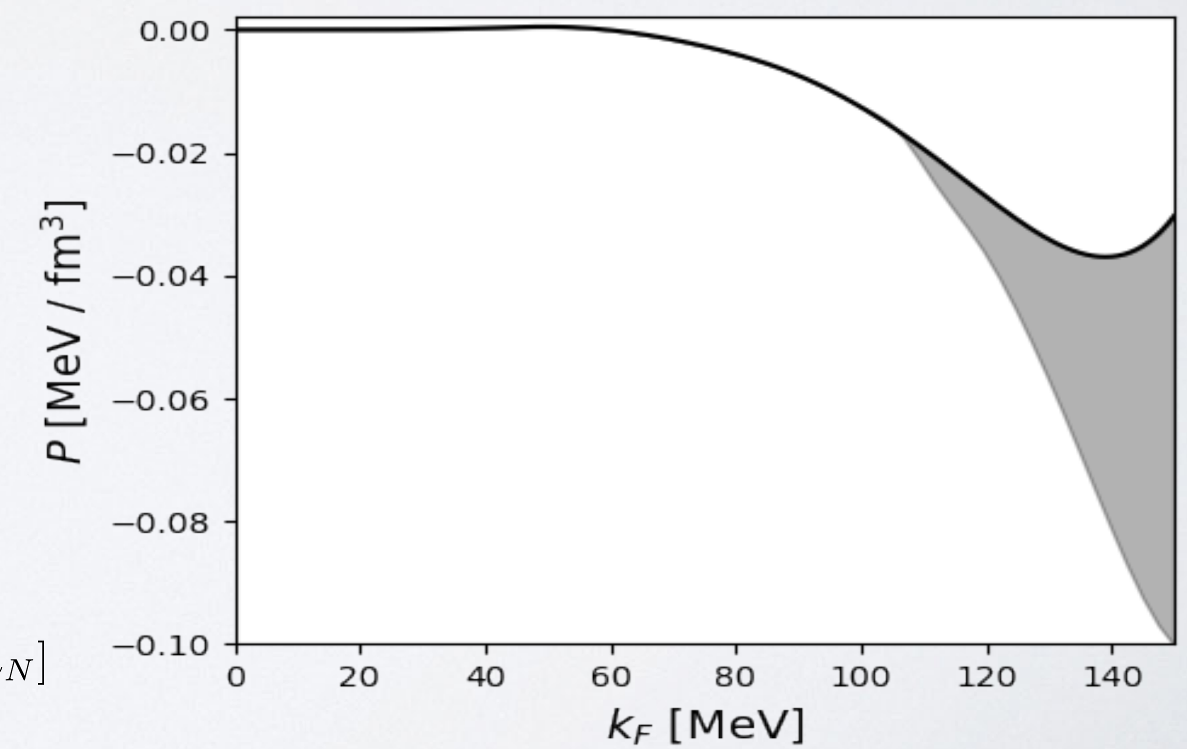


- To estimate the uncertainty:
  - Multiply the off-shell dependence of Lm times the gaussian regulator

$$\exp[-\frac{(k-M_\pi/2)^2}{\lambda^2}] \qquad \lambda > M_\pi$$

  - Consider a density-dependence nucleon mass

$$m_N(n) = \frac{m_N}{1+\frac{n}{n_s}\left(\frac{m_N}{m_N^*}-1\right)} \qquad m_N^* \in [0.7 m_N, 0.9 m_N]$$



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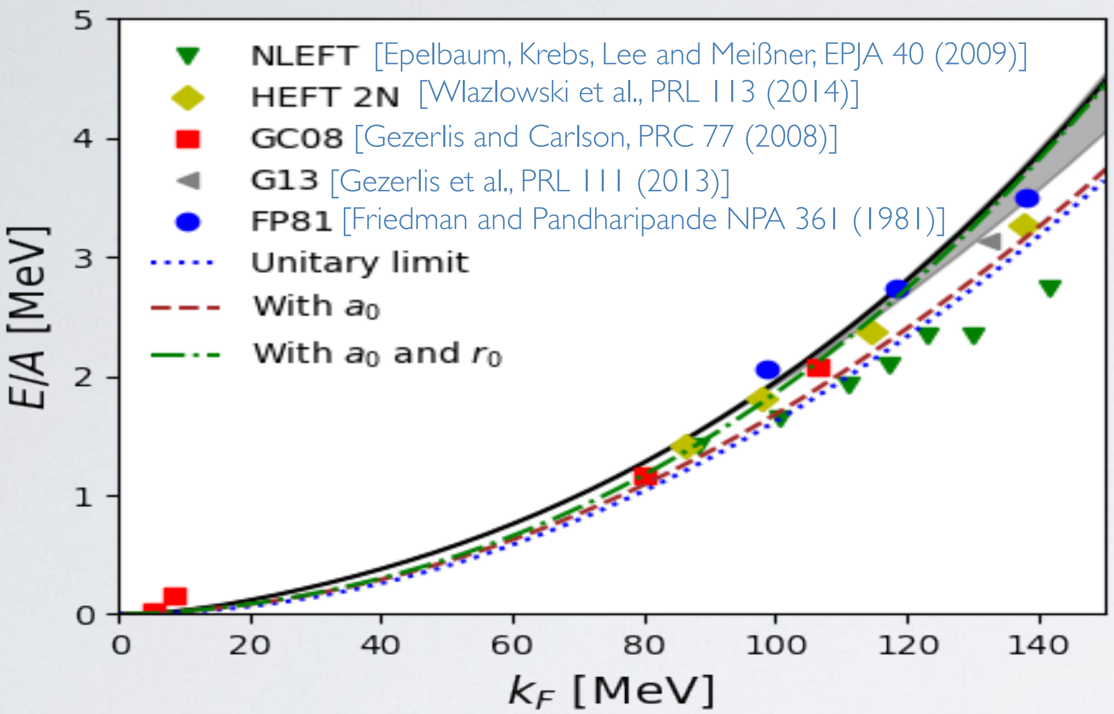
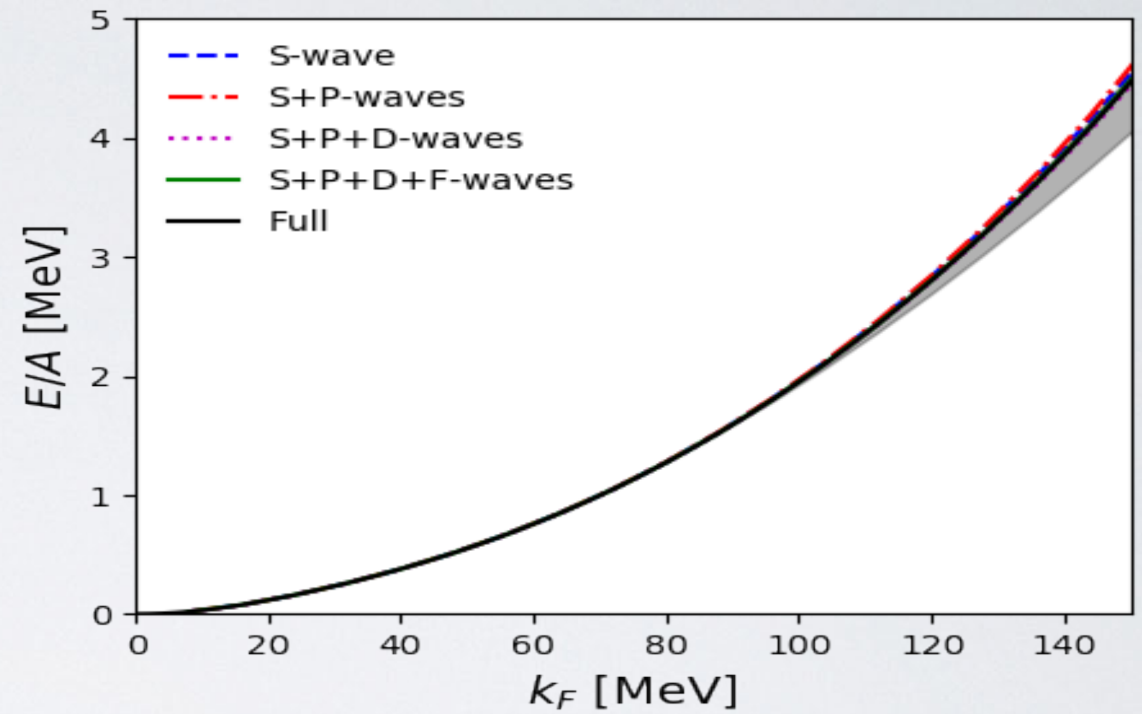
- Pure Neutron Matter [J. M. Alarcón, J. A. Oller, PRC 107 (2023)]





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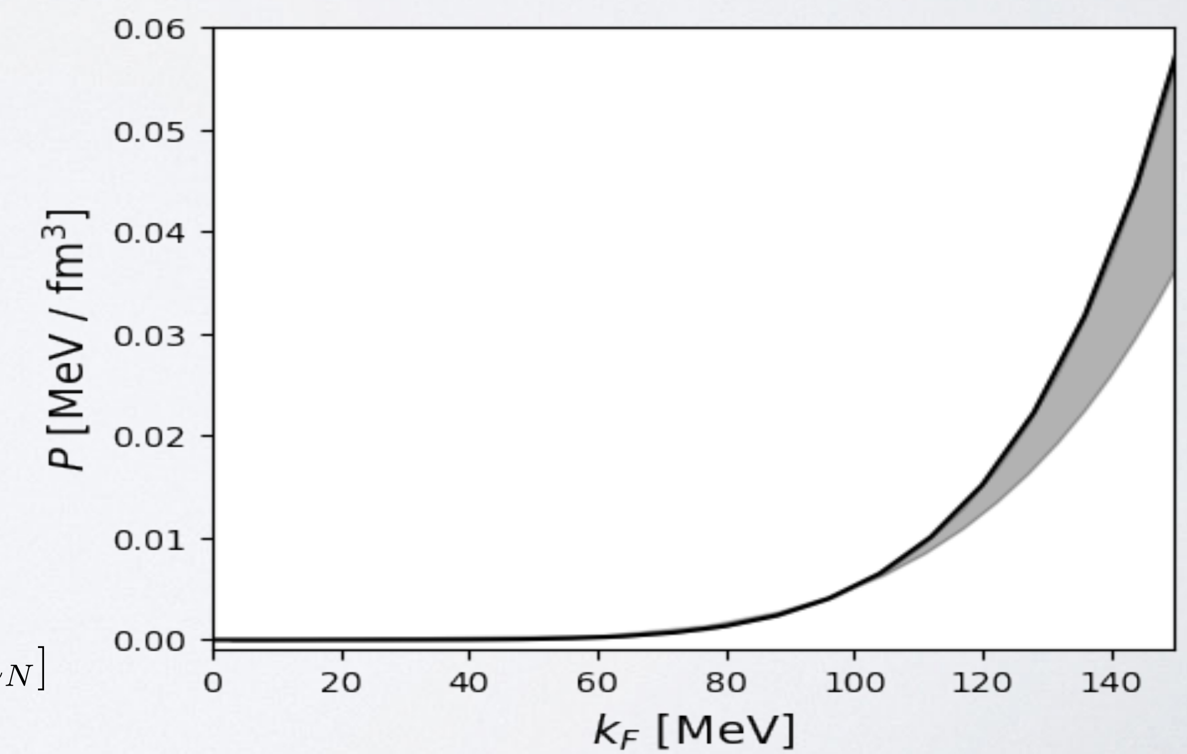


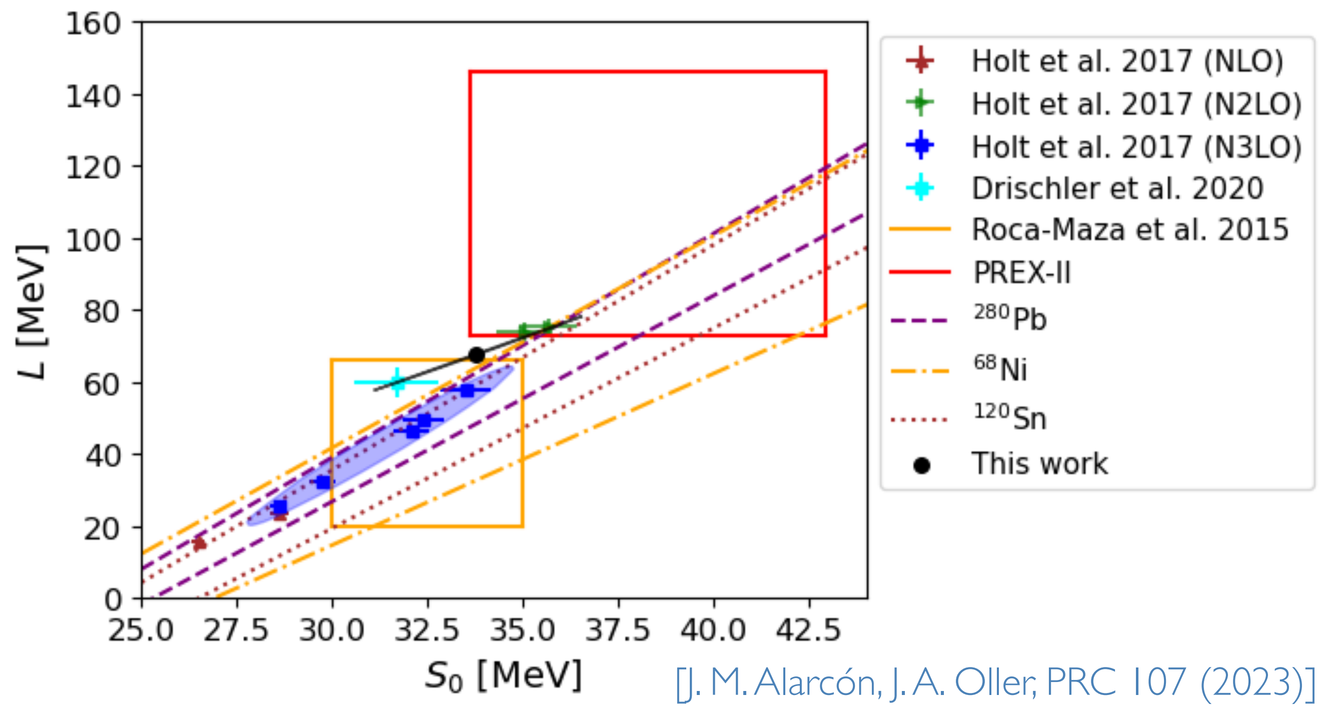
Results

- To study the properties of nuclear matter at saturation density we use the parametization [Gandolfi et al., Mon. Not. Roy. Astron. Soc. 404 (2010)]

$$S(n) = (E/A)_{PNM}(n) - (E/A)_{SNM}(n) \qquad S(n) = C_s \left(\frac{n}{n_s}\right)^{\gamma_s} \qquad S_0 = S(n_s) \qquad L = 3n_s \left.\frac{dS(n)}{dn}\right|_{n_s}$$

- We fit this parametrization to our results at low densities.

$$C_s = 34.77(15) \text{ MeV} \qquad 31.10 \leq S_0 \leq 36.57 \text{ MeV}$$
$$\gamma_s = 0.667(3) \qquad 57.82 \leq L \leq 78.29 \text{ MeV}$$



[J. M. Alarcón, J. A. Oller, PRC 107 (2023)]

Summary and Conclusions

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- We develop a method to compute the energy per particle in nuclear matter at low densities.
- E/A is directly related to in-vacuum NN scattering amplitude.
- Regularizator-independent results
- Can be used to study:
  - Unitary limit
  - Cold atoms
  - Equation of state of symmetric nuclear matter and pure neutron matter.
- Constraints on equation of state of neutron stars at low densities

→ Important for interpolation of the EoS of neutron stars (see Eva Lope Oter's talk)

FIN